

Possible ion-acoustic soliton formation in the ionospheric perturbations observed on DEMETER before the 2007 Pu'er earthquake*

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Abstract The data of ionospheric perturbations observed on DEMETER before the 2007 Pu'er earthquake are analyzed. The three-component plasma (ions, electrons and heavy ions) is studied in the fluid concept. The linear dispersion relation for ion-acoustic wave is found in the presence of heavy ions. The nonlinear dynamics is studied for arbitrary amplitude of the wave. The Sagdeev potential is calculated, which shows that solitary structure exists for Mach number within a range defined by the presence of heavy ions. The developed ion-acoustic solitons may be used as precursor for earthquake prediction.

Key words: earthquake; ionosphere; plasma; dispersion; ion-acoustic soliton

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1 Introduction

Detection of electromagnetic emission transmitted from earthquake regions (DEMETER) is a micro-satellite built by the Centre National d'Etudes Spatiales (CNES) of French launched on 29 June 2004 on a nearly circular ~710 km altitude quasi-heliosynchron at 98° inclination. Measurements of the electric and magnetic field components of the electromagnetic waves are performed by the instrument champ électrique (ICE) (Berthelier et al, 2006) and by the instrument magnetic search coil (IMSC) (Parrot et al, 2006), respectively.

A destructive shock with magnitude of 6.3 occurred on 2 June 2007 at 21:34:56 (UTC) in Pu'er region (23.0°N, 101.1°E), Yunnan province, China. The data from DEMETER satellite during the period from 23 May to 2 June, i.e., ten days before the earthquake and one day just on the day of earthquake occurrence, were analyzed (Zhu et al, 2008). Among the 284 orbits of DEMETER during the period, 29 orbits with the trace passing through the region within 1 888 km from the epicenter were selected for the study. Seven anomalous

events were found on the data set of the seven orbits among the 29 ones. There existed synchronous perturbations on the variations of the spectrogram of the electric field and the variations of the density and temperature of the ions and electron, in contrast with the variations of its surrounding area. Electrostatic turbulences were also recorded with the synchronous perturbations of the electron density and ion density in plasma in the region near the epicenter for the five events, which seems to suggest that there be some physical relation between these events and the preparation processes of the Pu'er earthquake.

In this paper, we will make further research on the density perturbations and electrostatic field turbulence by analyzing the three component plasma, i.e., ions, electrons and heavy ions, in the low temperature hydrodynamical fluid plasma concept. The linear dispersion relation of ion acoustic wave in the presence of heavy ion is derived. It is found that the phase velocity of the ion-acoustic wave decreases with heavy ion density increasing. For arbitrary amplitude of the perturbations, the nonlinear dynamics of the perturbed plasma is studied. The Sagdeev potential is calculated, which shows that ion-acoustic solitons are formed within a range of Mach number defined by the presence of heavy ions. We predict that the detection of the ion-acoustic solitons

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may be a precursor for earthquake prediction.

2 Linear dynamics of ion-acoustic waves

Consider a collisionless and unmagnetized three component plasma in the perturbed ionosphere due to seismic emission. The components are electrons (e), singly charged ions (i) and heavy ions (hi). The dynamics of the multi-component plasma is governed by the following fluid equations:

$$m_j n_j \left[\frac{\partial \mathbf{V}_j}{\partial t} + (\mathbf{V}_j \cdot \nabla) \mathbf{V}_j \right] = -\nabla P_j + q_j n_j \mathbf{E}, \quad (1)$$

where, $j=i, e, hi$; P_j is the pressure, q_j is the charge, m_j is the mass and n_j is the density of each species. \mathbf{E} is the electrostatic field developed to balance the quasi-neutrality of the plasma.

For a quasi-neutral plasma, we have $n_i + n_{hi} = n_e \equiv n$, which is maintained when $\mathbf{V}_i = \mathbf{V}_{hi} = \mathbf{V}_e \equiv \mathbf{V}$ and $q_i = q_{hi} = -q_e$. We consider the departure from charge neutrality is very small, then from momentum balance equation (1), we get

$$-\frac{1}{m_i} \nabla P_i + \frac{e}{m_i} n_i \mathbf{E} - \frac{1}{m_{hi}} \nabla P_{hi} + \frac{e}{m_{hi}} n_{hi} \mathbf{E} = -\frac{1}{m_e} \nabla P_e - \frac{e}{m_e} n_e \mathbf{E}, \quad (2)$$

which can be solved to get the electric field

$$\mathbf{E} = \frac{\nabla \left(P_i + \frac{m_i}{m_{hi}} P_{hi} \right) - \frac{m_i}{m_e} \nabla P_e}{e \left[\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi} \right]}. \quad (3)$$

The motion equation of ions is

$$m_i n_i \left[\frac{\partial \mathbf{V}_i}{\partial t} + (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i \right] = -\nabla P_i + e n_i \mathbf{E}, \quad (4)$$

which, with the substitutions $n_i = n - n_{hi}$ and $\mathbf{V}_i \sim \mathbf{V}$, gives

$$m_i (n - n_{hi}) \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla P_i + e (n - n_{hi}) \mathbf{E}. \quad (5)$$

Then substituting \mathbf{E} from equation (3) into equation (5), we get

$$m_i (n - n_{hi}) \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] =$$

$$-\left[1 - \frac{n - n_{hi}}{\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi}} \right] \nabla P_i - \frac{m_i}{m_e} \frac{n - n_{hi}}{\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi}} \nabla P_e + \frac{m_i}{m_{hi}} \frac{n - n_{hi}}{\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi}} \nabla P_{hi}. \quad (6)$$

We consider the ion compression is adiabatic, electron compression is isothermal and the heavy ions are uncompressed. Then

$$\nabla P_i = \gamma T_i \nabla n_i = \gamma T_i \nabla (n - n_{hi}) = \gamma T_i \nabla n \quad (n_{hi} = n_{hi0}), \quad (7)$$

$$\nabla P_e = T_e \nabla n, \quad (8)$$

and

$$\nabla P_{hi} = 0, \quad (9)$$

where γ is the adiabatic constant and for one degree of freedom we consider $\gamma=3$.

With these assumptions, equation (6) reduces to

$$m_i (n - n_{hi}) \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -(\lambda_1 \gamma T_i + \lambda_2 T_e) \nabla n, \quad (10)$$

where

$$\lambda_1 = 1 - \frac{n - n_{hi0}}{\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi0}} \quad (11)$$

and

$$\lambda_2 = \frac{m_i}{m_e} \frac{n - n_{hi0}}{\left(1 + \frac{m_i}{m_e} \right) n - \left(1 - \frac{m_i}{m_{hi}} \right) n_{hi0}}. \quad (12)$$

The continuity equation of the fluid is

$$\frac{\partial n}{\partial t} + \nabla \cdot (n - n_{hi0}) \mathbf{V} = 0. \quad (13)$$

Equations (10) and (13) now can be used to determine the dispersion relation for the ion acoustic wave. We assume no average velocity of the medium, so that \mathbf{V} is the velocity associated with the wave motion. Both the velocity and the density perturbation are considered to be small. If the undisturbed plasma density is n_0 , then

$n=n_0+\delta n$. We consider a plane wave propagation in the medium so that both δn and V are proportional to $\exp[i(\omega t - \mathbf{k} \cdot \mathbf{r})]$. Then we find the following dispersion relation:

$$\omega^2 = k^2 C_s^2 \left(\lambda_2 + \lambda_1 \gamma \frac{T_i}{T_e} \right), \tag{14}$$

where $C_s=(T_e/m_i)^{1/2}$ is the sound speed. The phase velocity of the ion-acoustic wave is

$$\frac{\omega}{k} = C_s \sqrt{\lambda_2 + \lambda_1 \gamma \frac{T_i}{T_e}}, \tag{15}$$

or, we can write

$$\frac{\omega}{k} = C'_s \quad \text{with} \quad C'_s = C_s \lambda, \tag{16}$$

where

$$\lambda = \lambda(n_0, n_{hi0}, T_i, T_e) = \frac{m_i}{m_e} \frac{1 - \frac{n_{hi0}}{n_0}}{\left(1 + \frac{m_i}{m_e}\right) - \left(1 - \frac{m_i}{m_{hi}}\right) \frac{n_{hi0}}{n_0}} + \left[1 - \frac{1 - \frac{n_{hi0}}{n_0}}{\left(1 + \frac{m_i}{m_e}\right) - \left(1 - \frac{m_i}{m_{hi}}\right) \frac{n_{hi0}}{n_0}} \right]^{1/2} \gamma \frac{T_i}{T_e}. \tag{17}$$

We consider the values of plasma parameters in the ionospheres as: $m_i/m_e=2\,000$, $m_i/m_{hi}=1/16$, $\gamma=3$, $T_e=2eV$, $m_i=m_p=1.672 \times 10^{-27}$ kg. Then we find $C_s=1.38 \times 10^4$ m·s⁻¹ and due to m_i/m_e is large enough, the formula (17) can be simplified as following:

$$\lambda = \lambda(n_0, n_{hi0}, T_i, T_e) = \frac{m_i}{m_e} \frac{1 - \frac{n_{hi0}}{n_0}}{\left(1 + \frac{m_i}{m_e}\right) - \left(1 - \frac{m_i}{m_{hi}}\right) \frac{n_{hi0}}{n_0}} + \left[1 - \frac{1 - \frac{n_{hi0}}{n_0}}{\left(1 + \frac{m_i}{m_e}\right) - \left(1 - \frac{m_i}{m_{hi}}\right) \frac{n_{hi0}}{n_0}} \right]^{1/2} \gamma \frac{T_i}{T_e} \approx \frac{1 - \frac{n_{hi0}}{n_0}}{1 + \frac{n_{hi0}}{n_0}} + \left[1 - \frac{1 - \frac{n_{hi0}}{n_0}}{\frac{m_i}{m_0} \frac{1 - n_{hi0}/n_0}{1 + n_{hi0}/n_0}} \right]^{1/2} \gamma \frac{T_i}{T_e} \approx$$

$$\left[\frac{1 - \frac{n_{hi0}}{n_0}}{1 + \frac{n_{hi0}}{n_0}} + \gamma \frac{T_i}{T_e} \right]^{1/2}. \tag{18}$$

In the observations of DEMETER $T_i \approx T_e$. Figure 1 shows the decrease of the ion acoustic speed with increase of heavy ion density.

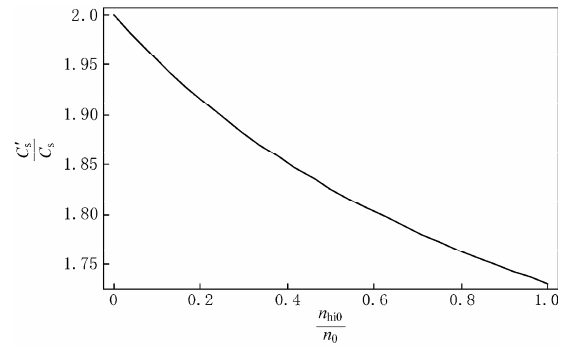


Figure 1 Ion acoustic speed C'_s/C_s as a function of heavy ions n_{hi0}/n_0 .

3 Nonlinear dynamics of ion-acoustic solitons

For the arbitrary amplitude of density perturbations we study the nonlinear dynamics. It is considered that the earthquake emission produces the density perturbations that creates an electrostatic field $\mathbf{E}=-\nabla\phi$, where ϕ is the electrostatic potential. The electrons and the heavy ions obey the Boltzmann distribution:

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right) \tag{19}$$

and

$$n_{hi} = n_{hi0} \exp\left(\frac{-e\phi}{T_{hi}}\right), \tag{20}$$

where, T_e and T_{hi} are the electron's and heavy ion's temperatures, respectively.

The approach has been used to get Boltzmann distributions for the case of electron:

$$n_e m_e \left[\frac{\partial \mathbf{u}_e}{\partial t} + \mathbf{u}_e \cdot \nabla \mathbf{u}_e \right] \approx n_e m_e \frac{\partial \mathbf{u}_e}{\partial t} \approx -en_e \mathbf{E} - \nabla p.$$

For $m_e \rightarrow 0$,

$$e\nabla\phi = \frac{\nabla p}{n_e} = \frac{\gamma T_e}{n_e} \nabla n_e = \gamma T_e \cdot \nabla \ln n_e,$$

$$\ln n_e = \frac{e\phi}{\gamma T_e} + c$$

and

$$n_e = n_{e0} \exp\left(\frac{e\phi}{\gamma T_e}\right) = n_{e0} \exp\left(\frac{e\phi}{T_e}\right),$$

where $n_{e0} = \exp c$ and $\gamma = 1$.

However, for m_{hi} that is much larger than m_e , this approach would not be suitable to get the same expression as mentioned above. But we consider the slow motion of heavy ions ($u_{hi} \rightarrow 0$) for what it may be acceptable.

Ion dynamics is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i V_i) = 0, \tag{21}$$

$$\frac{\partial V_i}{\partial t} + (V_i \cdot \nabla) V_i = -\frac{e}{m_i} \nabla \phi \tag{22}$$

and

$$\nabla^2 \phi = 4\pi e(n_e - n_i - n_{hi}). \tag{23}$$

We introduce the following dimensionless variables:

$$\begin{aligned} \bar{n}_i &= \frac{n_i}{n_0}, \quad \bar{n}_{hi0} = \frac{n_{hi0}}{n_0}, \quad \mathbf{u}_i = \frac{\mathbf{V}}{C_s}, \quad \bar{\phi} = \frac{e\phi}{T_e}, \\ \tau &= \omega_p t, \quad \bar{x} = \frac{x}{\lambda_D}, \quad \alpha = \frac{T_e}{T_{hi}}, \end{aligned} \tag{24}$$

where $\omega_p = (4\pi e^2 n_0 / m_i)^{1/2}$ and $\lambda_D = (T_e / 4\pi e^2 n_0)^{1/2}$. Considering the one dimensional perturbations ($\nabla \rightarrow \partial / \partial \bar{x}$), the normalized governing equations of ion dynamics are given by

$$\frac{\partial \bar{n}_i}{\partial \tau} + \frac{\partial}{\partial \bar{x}} (\bar{n}_i u_i) = 0, \tag{25}$$

$$\frac{\partial u_i}{\partial \tau} + u_i \frac{\partial u_i}{\partial \bar{x}} = -\frac{\partial \bar{\phi}}{\partial \bar{x}} \tag{26}$$

and

$$\frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} = \exp \bar{\phi} - \bar{n}_i - \bar{n}_{hi0} \exp(-\alpha \bar{\phi}). \tag{27}$$

To study the properties of stationary arbitrary amplitude of electrostatic waves, we transform the above set of equations to a stationary frame moving with velocity V , the phase velocity of the wave, i.e., $\xi = \bar{x} - M\tau$, where $M = V/C_i$ is the Mach number with respect to the

ion thermal velocity $C_i = (T_i/V_i)^{1/2}$. Then with the imposed appropriate boundary conditions for the localized disturbances, viz, $u \rightarrow 0$, $\bar{n}_i \rightarrow 1 - \bar{n}_{hi0}$ at $\xi \rightarrow \pm\infty$ (i.e., away from the centre of localization), from equations (25) and (26), we obtain

$$\bar{n}_i = \frac{(1 - \bar{n}_{hi0})M}{\sqrt{M^2 - 2\bar{\phi}}}. \tag{28}$$

Substituting above equation into equation (27) and multiplying both sides of the resulting equation by $\partial \bar{\phi} / \partial \xi$, integrating once, and taking into account the appropriate boundary conditions, i.e., $\bar{\phi} \rightarrow 0$ and $\partial \bar{\phi} / \partial \xi \rightarrow 0$ at $\xi \rightarrow \pm\infty$, we obtain the energy integral

$$\frac{1}{2} \left(\frac{\partial \bar{\phi}}{\partial \xi} \right)^2 + \Psi(\bar{\phi}, M) = 0, \tag{29}$$

where $\Psi(\bar{\phi}, M)$ is the Sagdeev potential, that is,

$$\begin{aligned} \Psi(\bar{\phi}, M) &= 1 + (1 - \bar{n}_{hi0})M^2 - \exp \bar{\phi} - \\ &(1 - \bar{n}_{hi0})M \sqrt{M^2 - 2\bar{\phi}} + \frac{\bar{n}_{hi0}}{\alpha} [1 - \exp(-\alpha \bar{\phi})]. \end{aligned} \tag{30}$$

The solitonic solution of equation (29) exists when the usual conditions, namely, $\Psi(\bar{\phi}, M) = 0$ and $\partial \Psi(\bar{\phi}, M) / \partial \bar{\phi} = 0$ at $\bar{\phi} = 0$, and $\Psi(\bar{\phi}, M) < 0$ for $0 < |\bar{\phi}| < |\bar{\phi}_0|$, where $|\bar{\phi}_0|$ is the maximum amplitude of the soliton, are satisfied. Equation (28) defines that $|\bar{\phi}_0| = M^2 / 2$. The condition $\partial^2 \Psi(\bar{\phi}, M) / \partial \bar{\phi}^2 < 0$ at $\bar{\phi} = 0$ is satisfied provided $M > M_0$, where $M_0 = (1 - \bar{n}_{hi0})^{1/2}$ is the lower critical Mach number. For $\bar{n}_{hi0} = 0.01$, we have $M_0 = 0.99$.

The upper critical Mach number can be determined from the conditions that at $\bar{\phi} = \bar{\phi}_0 = M^2 / 2$, $\partial \bar{\phi} / \partial \xi = 0$ (i.e., at the maximum amplitude of $\bar{\phi}$ the derivative is zero). Then from equation (29) $\Psi(\bar{\phi}_0, M) = 0$, which yields the following equation for M :

$$\begin{aligned} \exp\left(\frac{M^2}{2}\right) - 1 - \left(1 - \frac{\bar{n}_{hi0}}{2}\right)M^2 - \\ \frac{\bar{n}_{hi0}}{\alpha} \left[1 - \exp\left(-\alpha \frac{M^2}{2}\right)\right] = 0. \end{aligned} \tag{31}$$

Numerically we solve the equation (31) for particular values of $\bar{n}_{hi0} = 0.01$ and $\alpha = 0.1$. We find the

upper Mach number $M=M_{\max}=1.6$. Thus it seems that ion acoustic soliton may be developed in the perturbed ionospheric plasma with the Mach number between $0.99 < M < 1.6$. Figure 2 shows the Sagdeev potential $\Psi(\bar{\phi}, M)$ for $\alpha=0.1$, $\bar{n}_{\text{hi}0} = 0.01$ and $M=1.5$.

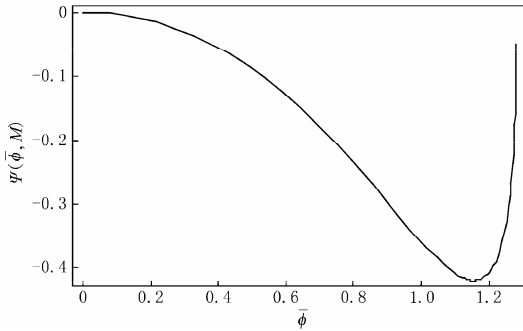


Figure 2 Sagdeev potential $\Psi(\bar{\phi}, M)$ for $\bar{n}_{\text{hi}0} = 0.01$, $\alpha=0.1$ and $M=1.5$.

To confirm the above prediction, we numerically solved the equation

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} = \exp \bar{\phi} - \frac{(1 - \bar{n}_{\text{hi}0})M}{\sqrt{M^2 - 2\bar{\phi}}} - \bar{n}_{\text{hi}0} \exp(-\alpha \bar{\phi}) \quad (32)$$

with the boundary conditions:

$$\left. \frac{\partial \bar{\phi}}{\partial \xi} \right|_{\xi=0} = 0 \quad (33)$$

and

$$\bar{\phi} \Big|_{\xi=0} = \frac{M^2}{2}. \quad (34)$$

Figure 3 shows the numerical solution which is an ion acoustic soliton. It confirms that ion-acoustic solitons may develop in the ionospheric plasma due to earthquake emission.

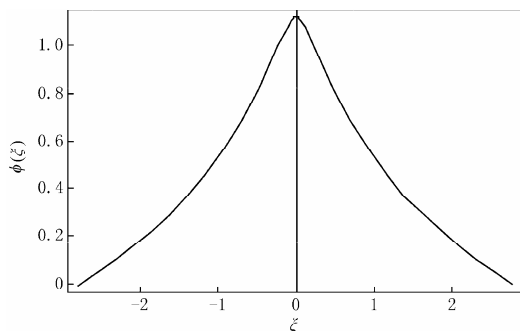


Figure 3 Ion-acoustic soliton of $\bar{\phi}(\xi)$ for $M=1.5$ and $n_{\text{hi}0}=0.01$.

4 Discussion and conclusions

DEMETER satellite launched is already more than four years. The national oceanic and atmospheric administration (NOAA) and the national aeronautics and space administration (NASA) jointly developed a series of polar operational environmental satellites (POES). A decade of NOAA-15 particle flux data is now available. Presently, it offers an opportunity to test the correlations between seismic activity and effects on the ionosphere. In this study, we try to find the correlation in detecting the soliton like structure rather than the particle flux. For this we took the available data collected on DEMETER satellite during the Pu'er earthquake occurred in Pu'er region, Yunnan province, China in 2007. The data shows that the ionospheric perturbations occurred in a three-component plasma consists of ions, electrons and heavy ions. We consider a low temperature, un-magnetized and collisionless plasma perturbations due to earthquake emission. Since electrons are highly oscillating, so ion dynamics is more predominant in the slow plasma response. Therefore, in this study we concentrate on ion acoustic waves, which are related with the compression of plasma fluids. The dispersion relation for the ion acoustic wave is derived, which shows that the presence of heavy ions decreases the phase velocity of the wave. For arbitrary amplitude of the perturbations, we investigate the nonlinear dynamics. Sagdeev potential is calculated and it is found that soliton like structure may develop within a range of Mach number determined by the presence of heavy ions. The corresponding equation is solved numerically for a particular value of the Mach number and the heavy ions. The solution is an ion-acoustic soliton, which may be detected as a precursor of earthquake prediction. Thus, our earlier study (Mofiz, 2006) on soliton formation in the ionosphere by earthquake emission is further confirmed through this investigation.

There has been published many papers about similar anomalies (Parrot, 1995; Sarkar et al, 2007; Ouyang et al, 2008). But the data of Pu'er earthquake (Zhu et al, 2008) is found easier to develop the multi-component plasma fluid concept in the ionosphere. The electric field wave, electron, ion (H^+) and heavy ion (O^+) density and temperature have been detected on DEMETER. These data will be utilized for a real analysis on soliton formation in our further investigations. It is to be noted that there may be other factors that can result in the variation of plasma but the formation of solitary pulses

may be one of them.

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