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RESEARCH ARTICLE

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Optimal placement of switched capacitors equipped with stand-alone voltage control systems in radial distribution networks

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Summary
This study presents a method for optimally selecting the location and size of switched shunt capacitors equipped with stand-alone voltage control systems and fixed shunt capacitors in radial distribution networks. The main contribution of this paper is the introduction of a novel algorithm specifically designed for placement of capacitors equipped with stand-alone voltage control systems. The considered objective function is the net saving calculated for every potential solution by using a backward-forward load flow technique. A genetic algorithm is used to maximize the objective function while taking into account the technical constraints of the distribution network and the maximum capacitors' sizes depending on the set points of the stand-alone voltage control system and on the locations of capacitors. The effectiveness of the proposed optimization method is verified and compared with other prevalent methods by simulation results carried on 69-bus and 28-bus distribution networks with three load levels.

KEYWORDS
capacitor placement, distribution system, genetic algorithm, optimization, stand-alone voltage control system

Nomenclature: $I_{i,\,j}$, The current magnitude for ith branch and for jth load level; I_{ij}, The current of the line connected between buses i and j; $I_{\max,\,i}$, The maximum allowable current for ith branch; K_c^{fixed}, The cost of fixed capacitor per kVA; K_c^{switched}, The cost of switched capacitor per kVAr; K_e, The cost of 1 KWh of energy losses; K_f, The fixed installation cost; N_C, The number of capacitors set to be installed; N_{LL}, The number of load levels; N_s, The net savings due to the capacitor placement in the distribution network; P_{ij}, The active power flow of the line connected between buses i and j; P_j, The power losses at jth load level; P_j^c, The power losses at the jth load level after capacitors' placement,; Q_i^{fixed}, The size of installed fixed capacitor at node i; Q_{ij}, The reactive power flow of the line connected between buses i and j; Q_i^{switched}, The size of installed switched capacitor at node i; R_{ij}, The resistance of the line connected between buses i and j; T_j, The time duration of jth load level; $U_{\text{Ref. max}}$, Upper set point of stand-alone voltage controller; $U_{\text{Ref. min}}$, Lower set point of stand-alone voltage controller; $U_{i,\,j}$, The voltage magnitude for ith bus and for jth load level; U_i, The voltage of the bus i; $U_{\max}$, The maximum voltage constraint; $U_{\min}$, The minimum voltage constraint; X_{ij}, The reactance of the line connected between buses i and j; X_t, Total network reactance in the location of the switched capacitor bank; Z_{ij}, The impedance of the line connected between buses i and j; ΔU_{ij}, The difference between voltages of buses i and j

Operators: *Imag* (Z), Give the imaginary part of Z; *Real* (Z), Give the real part of Z; $|Z|$, Give the magnitude of Z; Z^*, Give the conjugation of Z

List of abbreviations: GA, Genetic algorithm; kV, Kilo volt; kWh, Kilo watt hour; p.u., Per unit; VAr, Volt-ampere reactive

Highlights

- An algorithm for placement of capacitors equipped with stand-alone voltage controllers.
- A model for the stand-alone voltage control system.
- Suitable values of fixed and switched capacitors are determined.
- A comparison of loss reduction, savings, and maximum voltage loss for the two cases.

1 | INTRODUCTION

Placement of capacitors in distribution networks is a key factor in reducing the power losses in a power system. By installing shunt capacitors, the portion of distribution system power losses due to reactive power flows may be reduced.[1] In addition to power loss reduction, shunt capacitors can also contribute to system stability, power system control, power factor correction, and voltage profile management. The effectiveness of shunt capacitors depends on the location, size, type, number, type, and design of the control system; therefore, a designing a proper optimization method for their selection is crucial.

Early methods used for capacitor placement problems carried out the maximization of an objective function without taking into account operational constraints. The distribution network was often simplified to a great extent and used in combination with analytical methods[2] or iterative methods[3] to make the calculations more practical and less time-consuming. Duran[4] pioneered the utilization of discrete variables for determining the optimal capacitor size by using a numerical method. Salama et al[5] in an effort to generalize the proposed method modeled nonuniform feeders as uniform feeders and took into account nonuniformly distributed loads, load variations, and switched capacitors. Baran and Wu introduced mixed integer[6] and nonlinear[7] programming to solve the capacitor placement problem. The objective function used in more recent works were more sophisticated, and with the utilization of powerful computational devices, the modeling of power networks and the considered operative constraints became more accurate.[8]

Metaheuristic methods are used to solve a wide spectrum of problems in engineering, and many optimization methods based on metaheuristic methods have been used to solve capacitor placement problems. Among such as genetic algorithms (GAs), simulating annealing (SA), particle swarm optimization (PSO), and fuzzy set theory (FST) have particularly been favored. PSO is used in Singh and Rao[9] to find the location, size, and type of shunt capacitors in a distribution network. Chang et al[10] proposed a fuzzy-immune algorithm to minimize a cost function consisting of energy losses and capacitor costs. Gallego et al[11] combined metaheuristic methods, Tabu search, simulated annealing, and GA with heuristic methods in an effort to reduce the computational cost of the optimization problem.

In many studies to reduce the computation time, a two-stage method was implemented whereby busses with maximum overall impact are identified first and in the second step metaheuristic methods such as improved harmonic search (IHS)[12] and flower pollination algorithm (FPA)[13-15] are employed to reduce the search space. This method is not limited to capacitors and has been utilized for the allocation of other units such as in Ali et al[16] where ant lion optimization algorithm (ALOA) is used for the optimal allocation of distributed generation systems.

In Bhattacharya and Goswami,[17] a two-stage solution is achieved for the capacitor placement problem. In the first step of the solution algorithm, a number of probable capacitor locations are selected based upon their fuzzy membership values. In the second phase, SA technique is used to determine the capacitor sizes at the locations selected in the first step. In Ali et al,[12] the most appropriate busses are identified by injecting a token reactive load in each bus in turn and ranking the total active power loss. After identifying busses with largest impact, authors proceed to utilize harmonic search method to optimally allocate capacitor banks. FPA was used.

GAs like many other metaheuristic methods are based on natural phenomena, the natural process of selection being the inspiration for GAs. In contrast to many traditional optimization methods, GAs start with a large diverse group of potential solutions (initial population), exploring large parts of the search space in parallel and thus reducing the optimization program's chance of being stuck in a local optima.[18] Sundhararajan and Pahwa[19] were the first to apply GA to find the optimum size of capacitors for a network modeled with an arbitrary number of load levels. GAs are used in Das[20] for minimization of total costs, including those related to energy losses and those associated with the cost of installation of capacitors, and the voltage constraint was added to the objective function as a penalty factor.

When solving the problem of optimal switched capacitors' placement, it is generally assumed that the capacitors are in some way connected to a central command and control unit and that this control system has the capability to control the switched capacitors at any time. In this approach, capacitors' size in each load level (injected reactive power to the distribution system by each switched capacitor bank and in each load level) is defined as decision variables and is determined optimally by the optimization method. This control system is generally referred to as a universal control system. In many cases, it is assumed that the universal control system has complete knowledge of the status of the distribution network at all times[21] and able to calculate the most effective status of the switched capacitors. In other cases, it is assumed that the universal control system has access to limited data regarding the distribution network.[22]

In contrast to universal control systems, stand-alone control units are not connected to a central control unit or any other control units, whether stand-alone or not, and work autonomously. Stand-alone control systems only have access to local data such as the voltage magnitude of the bus where the capacitor is placed in and only utilize these data for controlling the switched capacitor banks.

Despite the fact that the capacitor placement problem has been studied continuously for several decades, little attention has been given to capacitors equipped with local autonomous control systems. The main and important drawback of all of the previously proposed approaches for placement of switched capacitor banks is that in those approaches without considering stand-alone voltage control systems, capacitor size in each load level is defined as decision variable and is determined optimally by the optimization method. The prevalent approach of optimal placement of switched capacitor banks is flawed, in the sense that it does not differentiate between stand-alone systems and universal systems. In the other words, prevalent capacitor placement technique does not consider the model of stand-alone voltage controller. In this paper, a novel method for placement of capacitors equipped with stand-alone voltage control systems is presented for the first time.

Even though these kinds of capacitors have been commercially available for years, only recently an algorithm for the optimal placement of capacitors equipped with stand-alone VAr control systems was presented in Moradian et al[23] where the equivalent of this problem has been resolved for capacitors equipped with VAr control systems. Although the nature of these problems is similar for the two cases, the solutions are very different.

In this paper, a GA-based optimization algorithm is used to find the optimal location and sizes of switched capacitors equipped with stand-alone voltage control systems and fixed capacitors. GA was chosen mainly because (*a*) GA has been tested by many researchers on the capacitor placement problem with good results[20,24]; (*b*) GA is very well known among the power engineering community and as the key novelty of this paper is the introduction of capacitors with stand-alone voltage control systems and not the type of metaheuristic algorithm, using GA facilitates the comprehension of the key novelty of this paper for readers; and (*c*) allocation of capacitors with stand-alone voltage control systems does not significantly alter the complexity of the optimization problem as compared with a typical capacitor placement problem; therefore, GA which is extensively studied for the capacitor placement problem was found to be satisfactory for this paper.

Voltage deviations and the total cost of energy losses and capacitors as well as of the voltage control system are taken into account for determining the objective function and optimization procedure. Furthermore, a model is provided for the stand-alone voltage control system. To the best of our knowledge, capacitors with stand-alone voltage control systems have not been previously modeled and optimally allocated before. Therefore, the effectiveness of the proposed optimization method is verified and compared with other prevalent methods by simulation results carried on 69-bus and 28-bus distribution networks with three load levels. The importance of using an optimization algorithm specifically designed for capacitors equipped with stand-alone voltage control systems is shown in the section containing the simulation results.

The main contributions of this work are highlighted as follows:

1. Introducing a novel method for the placement of switched capacitors equipped with stand-alone voltage control systems.
2. Identifying and solving the problem associated with the placement of switched capacitors equipped with stand-alone voltage control systems.
3. Establishing the importance of modeling the voltage control systems in the capacitor placement problem.
4. Confirming the efficiency of capacitors equipped with stand-alone voltage control systems in terms of cost savings and voltage profile improvement.

A description of the capacitor placement problem regarding stand-alone voltage control system is provided in Section 2. Sections 3 and 4 explain the objective function used in this study and the prevalent approach for placement of switched capacitors, respectively. The proposed optimization algorithm is described in Section 5. Simulation results are provided in Section 6, and finally, some conclusions are presented in Section 7.

2 | PROBLEM DESCRIPTION

For switched capacitors to switch at the appropriate time and in the correct size, data regarding the distribution network are needed. By studying the loads of the distribution network, it is possible to predetermine the switch time and amount; however, because of the volatile nature of the loads at the distribution level, the switch settings must be altered frequently, and at times, the capacitors will not operate in the their optimal state. Another method to solve this problem is that of implementing a universal control system. In these approaches, the optimal banks of the capacitors are calculated for the following day[24] or in real time.[22] The latter is the most complete method for solving the capacitor placement problems as the capacitors are always at their optimal states.

Despite this method's efficiency, in many cases, implementing such a system is either too expensive or impractical due to the remote location of some or all busses and the lack of communication between control systems or between control systems and the central control unit. Accordingly, a proper approach is proposed here that considers the optimal placement of capacitors equipped with local (stand-alone) control systems. By only using local data, such as voltage, current, active, and reactive power flows, each control system can operate autonomously from all the other systems.

2.1 | Stand-alone voltage control system

A stand-alone voltage control system operates by constantly reading the voltage of the bus where it is placed in and comparing it with the nominal voltage of the system.[25] When a voltage deviation of more than a predefined reference value is read, a bank is added or removed depending on whether the voltage is lower or higher than the nominal voltage. This continues until the deviation is no longer higher than the predefined value or all the installed capacitor banks are added or removed. An unstable distribution network can induce the control system to add or remove capacitor banks prematurely. Therefore, a simple delay mechanism is considered such that when a capacitor is added or removed, the control system is momentarily disabled so that, regardless of the voltage of the bus, no capacitor is added or removed. After the delay period, the control system is enabled and continues its operations normally.

The relation between the voltage and current of a two-bus system (Figure 1) is shown in Equation 1. The combination of Equations 2 and 3 results in the active (Equation 4) and reactive (Equation 5) feeder power losses. Readers are referred to Grainger et al[26] for further explanation of these equations. As shown in Equation 6, the active power losses of a given line are proportional to the square of $|\Delta U_{ij}|$. It is shown by Equation 6 that a higher amount of line power losses is due to a higher amount of voltage deviation. This is not a clear-cut relationship; however, it can be roughly assumed that a higher amount of voltage deviation indicates higher total power losses.

The importance of this relationship is revealed when optimally placing capacitors equipped with stand-alone voltage control systems. This is a more difficult task than usual capacitor placement problems as the control system has to be modeled and taken into account. The voltage control system only reads the voltage and does not take into account active power flow and current magnitude. Adding or removing capacitor banks is carried out with the aim of modifying the reactive power flow and reducing the voltage deviations. Therefore, the voltage profile is improved, and total power losses are reduced.

$$U_i - U_j = Z_{ij} \times I_{ij} \tag{1}$$

$$Z_{ij} = R_{ij} + jX_{ij} \tag{2}$$

$$P_{ij} + jQ_{ij} = \left(U_i - U_j\right) \times I_{ij}^{*} \tag{3}$$

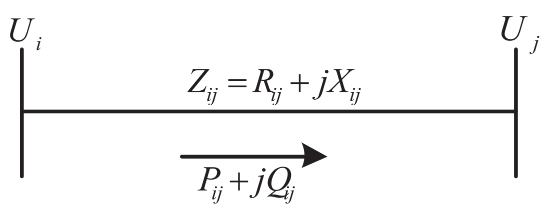

FIGURE 1 Typical two-bus system

$$P_{ij}^{\text{loss}} = \text{Real}\left(\Delta V_{ij} \times \frac{\Delta U_{ij}^{*}}{Z_{ij}^{*}}\right) \tag{4}$$

$$Q_{ij}^{\text{loss}} = \text{Imag}\left(\Delta V_{ij} \times \frac{\Delta U_{ij}^{*}}{Z_{ij}^{*}}\right) \tag{5}$$

Therefore, P_{ij}^{loss} is given as follows:

$$P_{ij}^{\text{loss}} = \frac{R_{ij} \times \left|\Delta U_{ij}\right|^{2}}{\left|Z_{ij}\right|^{2}} \tag{6}$$

$$Q_{ij}^{\text{loss}} = \frac{X_{ij} \times \left|\Delta U_{ij}\right|^{2}}{\left|Z_{ij}\right|^{2}} \tag{7}$$

The reactive power supplied by the shunt capacitors varies with bus voltage, as the shunt capacitors are modeled as capacitances and not as constant loads (ie, a voltage-independent capacitor bank).[27] This point is taken into account when performing the load flow.

2.2 | Maximum allowable capacitor size

According to the model of the control system given in Section 2.1, if the voltage of the bus where the capacitor is placed is lower than the minimum voltage reference value, a capacitor bank is added to the network. Let us assume, for example, a case where the addition of a capacitor increases the voltage of the bus where the capacitor is placed in such a way that the voltage of this bus exceeds the maximum voltage reference value. According to the model of the control system, the control system will remove the capacitor bank and the capacitor will return to its initial state. The voltage of the bus where the capacitor is placed will be again lower than the minimum voltage reference value thus completing a cycle which theoretically can continue indefinitely.

In order to prevent this cycle, its occurrence should be accurately predicted. It is therefore required to estimate the effect the addition or removal of a single capacitor bank has on the voltage of the bus where the capacitor banks are located. For this purpose, Equation 8, recently used by Elkhatib et al[28] and Homaee et al,[22] can be utilized. Equation 8 shows the relation between the voltage of the bus where the capacitor is placed and the size of the capacitor bank. As this equation shows, the change in voltage depends on the characteristics of the distribution network, namely the impedance, the voltage of the bus, and the size of the capacitor bank. According to Equation 8, in order for this cycle to be effectively predicted, the increase (or decrease) of the voltage at the bus where the capacitor is placed has to be larger than the difference between the maximum and minimum voltage reference values (Equation 9). In the worst-case scenario when the voltage of the bus where the capacitor is placed is equal to the minimum voltage reference value, the addition of a capacitor bank must not result in an increase in the bus voltage such that the increased voltage exceeds the maximum voltage reference value.

$$\Delta Q = \frac{\Delta U \times U}{X} \tag{8}$$

$$\Delta Q_{\max} = \frac{\left(U_{\text{Ref. max}} - U_{\text{Ref. min}}\right) \times U_{\text{Ref. min}}}{X} \tag{9}$$

where $U_{\text{Ref. max}}$ and $U_{\text{Ref. min}}$ are, respectively, upper and lower set points of stand-alone voltage controller which controls the switching of the switched capacitor bank. X is the total network reactance in the location of the switched capacitor bank. $\Delta Q_{\max}$ is the maximum size of capacitor banks which guarantee prevention of occurrence of the previous mentioned cycle.

In this paper, in order to prevent the occurrence of the previous mentioned cycle, the busses with associated capacitor banks which give rise to the cycle have been identified before the placement of the capacitors. Initially, all network busses are analyzed using Equation 9, and the $\Delta Q_{\max}$ for each bus is recorded. Then, these data are compared with the available capacitor bank sizes, and, for each bus, the capacitor banks lower than $\Delta Q_{\max}$ are selected, and in the final step, after the addition of the capacitors, the new $\Delta Q_{\max}$ is calculated using the new voltage. This technique benefits from a straightforward approach that by predicting adverse effect of switching on or switching off capacitors minimizes the risk of its implementation in the distribution network.

3 | OBJECTIVE FUNCTION

The typical cost function[20] is shown by Equation 10.

$$S = K_e \times \sum_{j=1}^{N_{LL}} T_j \times P_j + \sum_{i=1}^{N_C} K_c^{\text{fixed}} \times Q_i^{\text{fixed}} + \sum_{i=1}^{N_C} K_c^{\text{switched}} \times Q_i^{\text{switched}} \tag{10}$$

where $K_e \times \sum_{j=1}^{N_{LL}} T_j \times P_j$ represents the cost of energy loss after capacitor placement while purchase cost of the shunt capacitors is represented by $\sum_{i=1}^{N_C} K_c^{\text{fixed}} \times Q_i^{\text{fixed}} + \sum_{i=1}^{\text{ncap}} K_c^{\text{switched}} \times Q_i^{\text{switched}}$. T_j is the time duration of jth load level; K_e is the cost of 1 kWh of energy losses; P_j is the power loss at jth load level; Q_i^{fixed} is the size of installed fixed capacitor at node i; Q_i^{switched} is the size of installed switched capacitor at node i; K_c^{fixed} is the cost of fixed capacitor per kVAr; K_c^{switched} is the cost of switched capacitor per kVAr; N_C is the number of capacitors set to be installed; N_{LL} is the number of load levels.

The objective function of the optimization method proposed in Das[20] that should be minimized consists of the total costs, including those related to energy losses and those associated with the cost of installation of capacitors. The variable of the optimization problem are the size, location, type, and number of shunt capacitors.

In the new method proposed in this paper, instead, as also shown in Equation 11, the objective function to be maximized consists of the difference between the net savings, due to the reduction of energy losses, and the installation costs of added capacitors.

$$\text{Maximize } N_s = \sum_{j=1}^{N_{LL}} P_j^{0} \times T_j \times K_e - \sum_{j=1}^{N_{LL}} P_j^{c} \times T_j \times K_e - \sum_{i=1}^{N_C} K_c^{\text{fixed}} \times Q_i^{\text{fixed}} - \sum_{i=1}^{N_C} K_c^{\text{switched}} \times Q_i^{\text{switched}} - N_C \times K_f \tag{11}$$

where N_s is the net savings due to capacitor placement in the distribution network, P_j^{0} is the power loss at the jth load level before capacitor placement, P_j^{c} is the power loss at the jth load level after capacitor placement, N_C is the number of locations where capacitors are installed, and K_f is the fixed installation cost.

3.1 | Constraints

For any potential solution to be considered, the resulting voltage magnitude for all busses and for all load levels should be within the given limits. The maximum and minimum nodal voltage magnitudes assumed in this study are 0.9 and 1.05 p.u., respectively.

$$U_{\min} \le U_{i,j} \le U_{\max} \quad \forall i \text{ and} \forall j \tag{12}$$

where $U_{i,\,j}$ is the voltage magnitude for ith bus and for jth load level and $U_{\min}$ and $U_{\max}$ are the minimum and maximum voltage constraints, respectively.

The resulting current flow for all branches and for all load levels should be lower than given limits.

$$I_{i,j} \le I_{\max,i} \qquad \forall i \text{ and} \forall j \tag{13}$$

where $I_{i,\,j}$ is the current magnitude for ith branch and for jth load level and $I_{\max,\,i}$ is the maximum allowable current for ith branch.

To make the optimal capacitor placement results more realistic, a limit has been imposed on the number of installed capacitor banks or capacitor size for a given location. For this study, the maximum number of banks is 10.

A limit has also been placed on the number of times a capacitor bank can be continuously switched on and off per unit of time. This constraint is aimed at preventing both the destabilization of the distribution network, and damage to the capacitor from overswitching. This constraint is closely related to the discussion in Section 2.2. In this paper, this limitation has been set to six.

4 | PREVALENT APPROACH FOR PLACEMENT OF SWITCHED CAPACITORS EQUIPPED WITH STAND-ALONE VOLTAGE CONTROL SYSTEMS

In a distribution network with a universal control system, the control system has real-time access to all the data of the distribution network and adjusts the capacitors accordingly. Therefore, capacitors optimally placed for this control system, are guaranteed to have maximum impact as long as they are controlled by this system. Optimal placement of capacitors with this control system is explained in subsection 4.1.

To the best of the authors' knowledge no method has been designed for optimizing capacitors with stand-alone voltage control systems. Without knowledge of the method proposed in this paper, the only available optimization method for capacitors with stand-alone voltage control systems is the optimization method designed for universal control systems. The implementation of this optimization method for stand-alone voltage control systems is explained in Section 4.2.

The proposed method for optimization of stand-alone voltage systems is explained in Section 5. The results of the proposed method are compared with that of the above two methods in Section 6.

4.1 | Optimization algorithm for placement of switched capacitors with a universal control system

In order to optimally size and place capacitors with a universal control system in a distribution network, an algorithm similar to Das[29] is used. Both this algorithm and the algorithm proposed in the following section have been developed to be solved by using a GA; nonetheless, both can be easily adapted for other search methods. For this algorithm, Equation 11 is used as the objective function, and nodal voltage magnitudes, power flow limits, and capacitor sizes are taken into account as constraints. Every potential solution is represented by a chromosome consisting of $(N_{LL}+1)\times N_C$ genes where N_{LL} and N_C are the number of load levels and the number of capacitor banks, respectively. Each chromosome contains the capacitor size for each load level in addition to the location of the capacitors. In each location, a capacitor placed in the minimum size of capacitor is designated as fixed, and the difference between the maximum and minimum amount of capacitors is designated as switched.

The method can be summarized as follows:

1. Randomly create an initial population containing N chromosomes.
2. Calculate the voltage of nodes and the power losses of the lines for all individuals of the current generation by using the load flow program.
3. Designate the minimum size of capacitor at each location as fixed and the remaining as switched.
4. Evaluate the members of the current generation by using the data from steps 2 and 3 to calculate the objective function and check the constraints for every member.
5. If the maximum number of iterations or any stopping criteria is met, then go to step 8.
6. Produce the next generation by using genetic operators selection, mutation, and migration.
7. Go to step 2
8. Report the best individual of the current generation as the optimal solution.

4.2 | Placement of switched capacitors equipped with stand-alone voltage control systems using the method defined in Section 4.1

Research done by the authors revealed that even though these systems are used today specially in developing countries, no article exists in the relevant literature on the topic of the optimal placement of capacitors equipped with stand-alone voltage control systems in distribution networks. Without an algorithm designed for these control systems, they are either placed using optimization algorithms designed for universal control systems same as the method defined in Section 4.1 or not optimized at all.

In this method, assuming that all stand-alone capacitors are instead capacitors with a universal control system, the optimum size and location of the switched and fixed banks are calculated using the algorithm given in Section 4.1. By this step, even though the stand-alone capacitors have been optimized in the system, the resulting power flow and node voltages are still unknown as the optimization method does not take into account the model of stand-alone control system. Following this step, using the model of the stand-alone voltage control system and this data, the total savings and the voltage profiles of the network are calculated. This last step is not included in the optimization process, and it merely calculates the result of the optimization.

Switched capacitors with a universal control system when added or removed from the system will have maximum impact on power loss reduction while preventing the voltage profile from violating the voltage constraints. However, stand-alone voltage control system uses local voltage data to add or remove banks; therefore, there is no guarantee that for a given load level, the number of banks added to or removed from the system is the optimal number. Furthermore, since it is not possible to calculate the voltage profile without modeling the stand-alone voltage control system, potential solution that results in constraint violations cannot be excluded without taking the models into account.[23]

5 | PROPOSED APPROACH FOR PLACEMENT OF CAPACITORS EQUIPPED WITH STAND-ALONE VOLTAGE CONTROL SYSTEMS

In this study, the following method is proposed for optimal placement of capacitors equipped with stand-alone voltage control systems. As in the algorithm explained in the previous section, Equation 11 is used as the objective function. The constraints are also the same as those in the previous section. For this optimization algorithm, each potential solution is represented by a chromosome consisting of three types of genes: those representing the location of the potential solution, those representing the fixed capacitors at each location, and those representing the maximum switched capacitor size at each location, summing the total number of genes in a chromosome to $N_C \times 3$, N_C being the number of capacitors. The modeled control system uses the given data to calculate the status of switched banks for different load levels. As in the previous section, the minimum amount of switched capacitor in each location is distinguished as fixed and the difference between the minimum amount and the maximum amount is distinguished as switched. The fixed capacitor calculated here is added to the fixed capacitor given by the chromosome to make up the total size of fixed capacitor for a given location. The placement algorithm here is illustrated by Figure 2 and given as below:

1. Randomly create an initial population containing N chromosomes.
2. For every member of the generation, select the suitable bank for the given locations by modeling the control system as below:

2.A. Save the current capacitor banks.

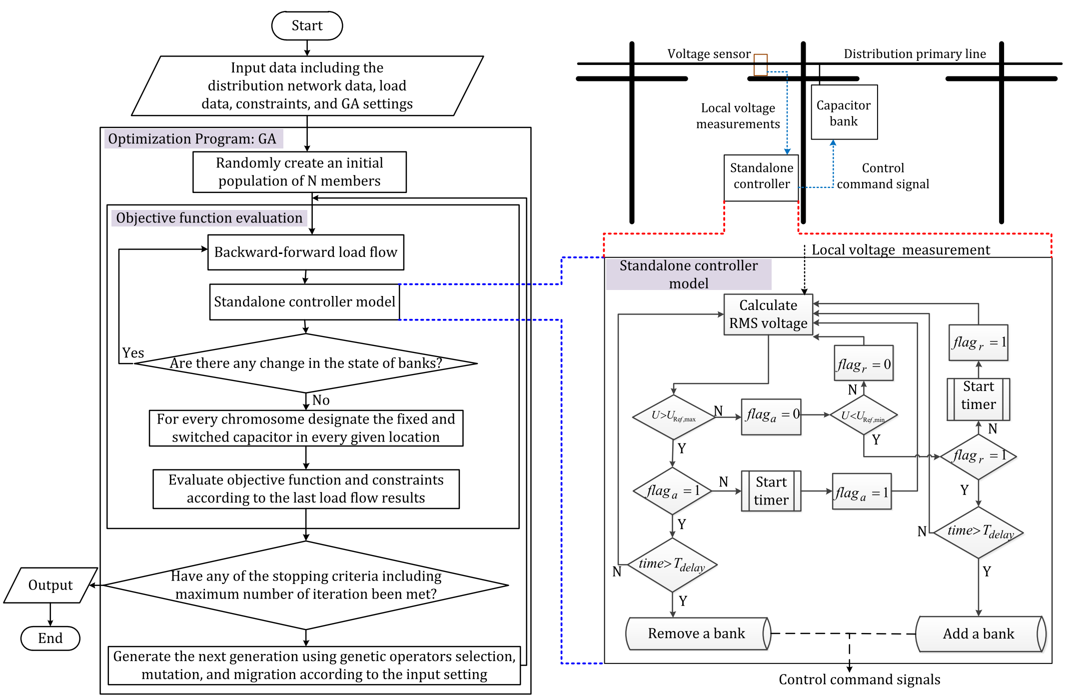

FIGURE 2 Optimization flow chart

2.B. Perform load flow.
2.C. For every given location, if the voltage deviation is more than $Z\%$ and the maximum number of permissible banks for this location has not been reached, switch on a capacitor bank.
2.D. For every selected capacitor location, if the voltage deviation is less than $-Z\%$ switch off an existing capacitor bank.
2.E. If the current capacitor banks differ from the saved banks go to step 2.A.
2.F. Report the current banks as the selected capacitor size for this member.
3. For every chromosome, designate the fixed capacitors at each location from the switched capacitors.
4. Evaluate the members by calculating the objective function for every member using the data given by the load flow program in step 2.B.
5. If the maximum number of iterations or any of the stopping criteria has been met, then go to step 8.
6. Produce the next generation by using genetic operators selection, mutation, and migration.
7. Go to step 2.
8. Report the best individual of the current generation as the optimal solution.

6 | CASE STUDY

The optimal placement of capacitors equipped with stand-alone voltage control systems using the proposed GA-based method is applied to two distribution networks. The optimization method seeks to maximize the objective function given in Equation 11. The GA parameters used are the same as those utilized by Moradian et al.[23] Key parameters are summarized below.

The population size is 150, and is divided into two subpopulations. Roulette wheel method is utilized for selection with the selection probability being 90%. Single crossover operator with a crossover probability of 75% is used. The migration rate is 10%, with the migration interval being 25 generations. Single-point mutation operator with a mutation probability of 10% is utilized. The maximum number of generations (iterations) is 100. In both case studies, after 100 generations, the algorithm converged to the same solution after five consecutive runs. The optimization algorithm was repeated 50 times to obtain the statistical distribution of the optimal solution.

The variables which are all discrete include the number of banks in each capacitor, and the capacitor location. A maximum number of three capacitors with a maximum of 10 banks (each bank being 200 kVAr) are set to be installed in the 69- and 28-bus distribution networks. For both cases, the binary coding technique for the GA, as utilized in Singh and Rao,[9] is used for encoding. The cost for 1 kWh is $0.06, and the purchase cost for fixed and switched capacitors are $3/kVAr and $3.2/kVAr, respectively.[30] A fixed installation cost of $1000[30] is added to the savings function for every location a capacitor is placed in. The nodal voltage magnitude is limited between 0.9 and 1.05 p.u., and the maximum permissible capacitor size is 2 MVAr. The stand-alone control system adds or removes a bank of 200 kVAr whenever the voltage deviation is lower or higher than −10% and 5% of the nominal line voltage, respectively, thus satisfying the voltage limits.

The resulting voltage profile and amount of savings are compared with those resulting from the optimal placement of capacitors with a universal control system using the method explained in Section 4.1. In order to establish the importance of taking into account the local control system, the results are also compared with those obtained with the method used for placement of capacitors with a universal control system.

6.1 | 69-Bus distribution system

The proposed method has been tested on a 69-bus, 12.66 kV distribution network. Three load levels, each with different durations, are considered. The details for the load levels are shown in Table 1 while the line data and nominal load data for this system are given by Das.[29] Convergence curve for 69-bus network is shown in Figure 3. The results are shown in Tables 2–5.

TABLE 1 Load level and load duration time

Load level	1 (Nominal)	1.6 (Peak)	0.5 (Light)
Time duration (h)	5260	1500	2000

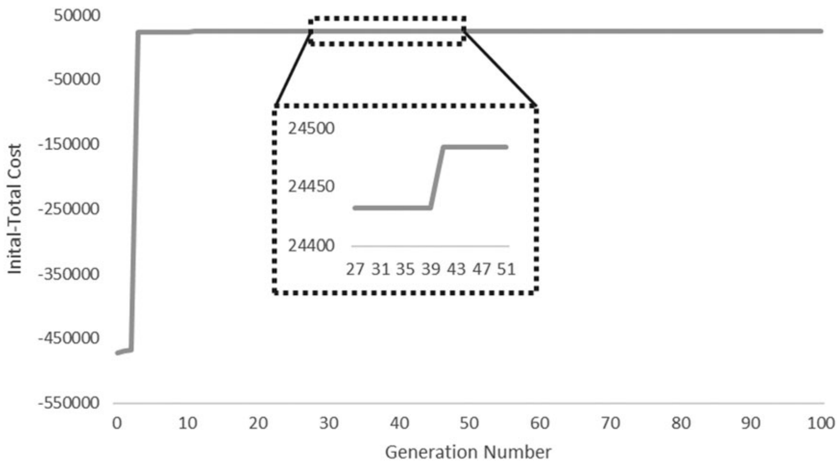

FIGURE 3 Convergence curve for 69-bus network

TABLE 2 Simulation results for the 69-bus distribution system: costs

		The Proposed Method in Section 5	Prevalent Approaches Explained in Section 4: Universal Control System, Optimized for Universal Control System (Section 4.1)	Prevalent Approaches Explained in Section 4: Stand-Alone Voltage Control System, Optimized for Universal Control System (Section 4.2)	Without Compensation
Total cost (best solution) ($)		111 371	106 970	114 710	135 855
Worst solution ($)		114 024	107 953	118 235	
Mean solution ($)		111 724	107 176	115 021	
Variance		0.0942	0.0381	0.0894	
Standard deviation		0.3069	0.1952	0.2990	
Net savings ($)		24 484	28 887	21 142	-
Cost of energy losses in each load level ($)	Nominal load level	49 244	46 150	53 839	70 969
	Peak load level	41 746	39 782	39 837	58 697
	Light load level	5460	4157	4157	6189
Total cost of energy losses ($)		96 451	90 088	97 833	135 855
Capacitor costs ($)		14 920	16 880	16 880	-

TABLE 3 Simulation results for the 69-bus distribution system: number of capacitor banks at each location

		The Proposed Method in Section 5			Prevalent Approaches Explained in Section 4: Universal Control System, Optimized for Universal Control System (Section 4.1)			Prevalent Approaches Explained in Section 4: Stand-Alone Voltage Control System, Optimized for Universal Control System (Section 4.2)		
Location		65	62	61	64	61	12	64	61	12
Number of installed capacitor banks at each location (200 kVAr)	Fixed	3	2	3	0	3	2	0	3	2
	Switched	1	6	6	9	5	3	9	5	3
Number swiched capacitor banks at each location and at each load level (200 kVAr)	Nominal load level	0	0	0	1	3	1	0	0	0
	Peak load level	1	6	6	9	5	3	9	5	0
	Light load level	0	0	0	0	0	0	0	0	0
Injected reactive power at each location and at each load level (KVAr)	Nominal load level	600	400	600	200	1200	600	0	600	400
	Peak load level	800	1600	1800	1800	1600	1000	1800	1600	400
	Light load level	600	400	600	0	600	400	0	600	400

TABLE 4 Simulation results for the 69-bus distribution system: power losses

		The Proposed Method in Section 5	Prevalent Approaches Explained in Section 4: Universal Control System, Optimized for Universal Control System (Section 4.1)	Prevalent Approaches Explained in Section 4: Stand-Alone Voltage Control System, Optimized for Universal Control System (Section 4.2)	Without Compensation
Power losses at each load level (kW)	Nominal load level	156.03	146.23	170.59	224.89
	Peak load level	463.85	442.02	442.64	652.21
	Light load level	45.50	34.64	34.37	51.58

TABLE 5 Simulation results for the 69-bus distribution system: lowest voltage

		Proposed Method (Section 5)		Prevalent Approaches Explained in Section 4: Universal Control System, Optimized for Universal Control System (Section 4.1)		Prevalent Approaches Explained in Section 4: Stand-Alone Voltage Control System, Optimized for Universal Control System (Section 4.2)		Without Compensation	
		Bus	Voltage	Bus	Voltage	Bus	Voltage	Bus	Voltage
Lowest voltage at each load level (p.u.)	Nominal load level	64	0.9280	65	0.9316	65	0.9192	61	0.9092
	Peak load level	64	0.9004	65	0.9006	65	**0.8984**	65	0.8445
	Light load level	61	0.947	65	0.9664	65	0.9664	65	0.9567

As is evident from Table 2, by optimally placing three capacitors equipped with stand-alone voltage control systems using the proposed algorithm (in Section 5), it is possible to annually save \$24 484. The effectiveness of the proposed optimization method is shown once it is compared with the method explained in Section 4, which does not take into account the stand-alone control system. In fact, net savings are limited to \$21 142, or 15% less than those resulting from the proposed method.

This is due to the fact that the locations of the capacitors obtained with the optimization algorithm explained in Section 4.1 are selected so that an injection of reactive power has the maximum effect on power loss reduction. The approach explained in Section 4.1 is correct when placing capacitors with control systems not relying on local data for the decision-making process. However, this is not the case when considering stand-alone voltage control systems as these systems only add (or remove) a bank when the voltage magnitude of the bus where the capacitor is placed is lower (or higher) than certain amounts. Therefore, even though the capacitors will have maximum effect in power loss reduction when they are switched on, there is no guarantee they will be switched on when required. This is shown for the 69-bus network at the nominal load level. As shown in Table 3, none of the switched capacitors have been switched on in the case where capacitors equipped with stand-alone voltage control systems have been placed using the algorithm designed for capacitors with a universal control system, as explained in Section 4.2. Without taking into account the costs related to the capacitors, this resulted in the equivalent of \$2192.742 savings due to in energy loss reduction. As is evident from Table 3, in addition to this, the switched capacitors placed in bus number 12 are never added to the system, and therefore do not contribute to power loss reduction and only act as an unnecessary economic burden, adding \$1800 to the total costs in capacitor costs.

In addition to the problems related to power losses and the reduction in the amount of savings, using algorithms not specifically designed for capacitors equipped with stand-alone control systems can result in constraint violation. Table 5 shows that the lowest voltage in the capacitors equipped with stand-alone control systems placed using the method explained in Section 4.2 is 0.8984 p.u. (in boldface) whereas the minimum acceptable voltage is 0.9 p.u. As can be seen, when using the prevalent optimization method for placement of capacitors equipped with stand-alone voltage control systems, constraint violation occurs. This is because the prevalent optimization technique is flawed, in the sense that it does not differentiate between stand-alone systems and universal systems. In other words, prevalent capacitor placement technique does not consider the model of stand-alone voltage controller. It optimizes for a universal system even though it is placing stand-alone systems. The constraint violation occurs because the stand-alone system does not understand the voltage of other busses besides itself; therefore, unless preparation is made (the size of capacitors or their locations is limited), capacitors will be added and removed irrespective of voltage constraints.

Although the proposed method is highly effective in placing capacitors equipped with stand-alone control systems, the results are still not comparable to those of optimally placed capacitors with the universal control systems. The reason for this is simply the lack of a complete perspective of the distribution network in the former case.

The control settings have a substantial effect on the efficiency of the optimal solution. By setting a high value for the deviation needed for a bank to be added or removed, we risk inflexibility while a small value may result in an excessive number of installed banks and therefore in an unacceptable economic burden.

Figure 4 illustrates the voltage profile of the 69-bus network for the peak load level for the case with capacitors equipped with stand-alone voltage control systems placed using the proposed algorithm. The three capacitors placed at busses 61, 62, and 65 are all located in the sublateral with the highest amount of voltage deviation. The capacitors contribute to the improvement of the voltage profile of this branch and of the system as a whole, and more importantly to maintain the voltage magnitude of nodes with high-voltage deviations, such as busses 65 and 61 within the constraint limits. The voltage profile of the three cases has been compared for the 69-bus network for the peak load level in Figure 5. This figure illustrates the importance of taking into account the model of the control system for placing

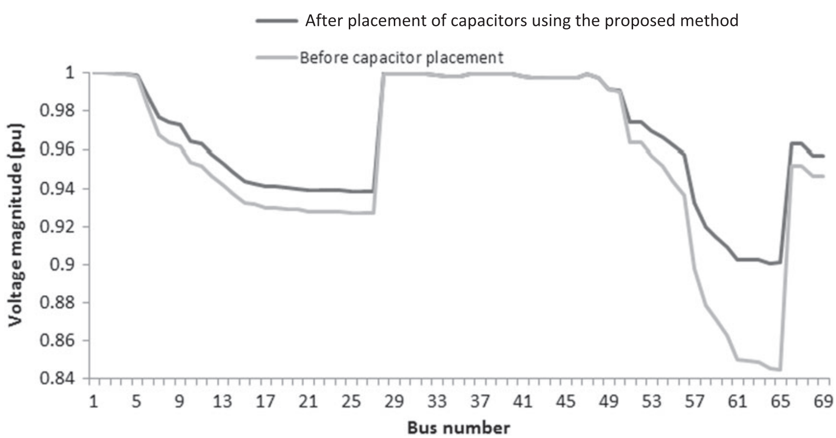

FIGURE 4 Voltage profile of 69-bus distribution network

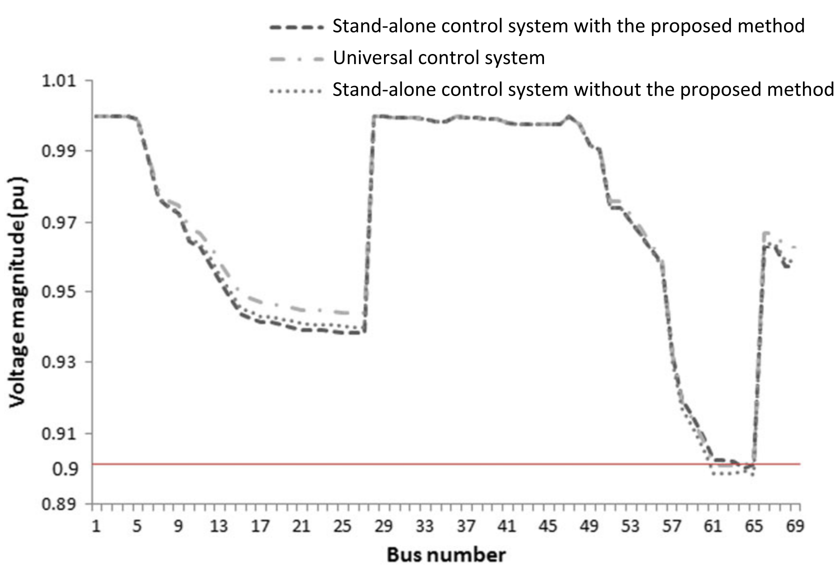

FIGURE 5 Voltage profile of the three cases for the 69-bus network for the peak load level

capacitors equipped with stand-alone voltage control systems in terms of voltage constraint violation. It is clearly shown that busses from 61 to 65 have violated the voltage constraint for this load level.

6.2 | 28-Bus distribution system

The second system is a 28-bus, 11 kV distribution network with one main feeder and four laterals. Line data and the nominal loads of the network are given in Das et al.[31] The load levels for this system are given by Table 1. Convergence curve for 28-bus network is shown in Figure 6. The results are shown in Tables 6 to 9.

Table 6 shows that the proposed placement algorithm results in higher savings compared with the case in which capacitors equipped with stand-alone voltage control systems are placed using the method designed for capacitors with

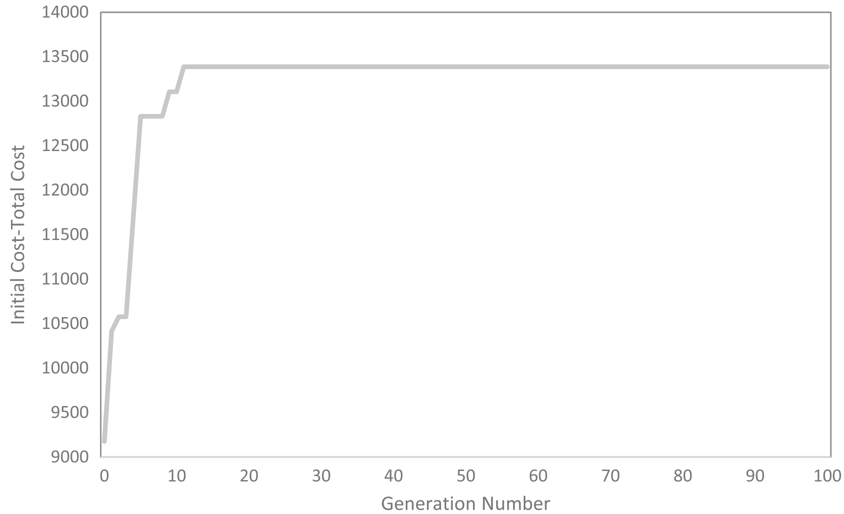

FIGURE 6 Convergence curve for 28-bus network

TABLE 6 Simulation results for the 28-bus distribution system: costs

			Prevalent approaches explained in section 4		
		Proposed method (section 5)	**Universal control system, optimized for universal control system (section 4.1)**	**Stand-alone voltage control system, optimized for universal control system (section 4.2)**	**Without compensation**
Total cost (best solution) ($)		28,036	27,551	29,362	41,424
Worst solution ($)		29,680	29,324	31,459	
Mean solution ($)		28,196	28,031	29,774	
Variance		0.0871	0.015	0.0982	
Standard deviation		0.295	0.1226	0.3134	
Net savings ($)		13,388	13,873	12,062	
Cost of energy losses in each load level ($)	Nominal load level	11,410	10,561	12,473	21,717
	Peak load level	9,351	8,606	9,145	17,804
	Light load level	1,596	1,025	1,025	1,902
Total cost of energy losses ($)		22,356	20,191	22,642	41,424
Capacitor costs ($)		5680	7360	7360	

TABLE 7 Simulation results for the 28-bus distribution system: Number of capacitor banks at each location

		Proposed method (section 5)		Prevalent approaches explained in section 4: Universal control system, optimized for universal control system (section 4.1)			Prevalent approaches explained in section 4: Stand-alone voltage control system, optimized for universal control system (section 4.2)		
Location		24	6	11	8	24	11	8	24
Number of installed capacitor banks at each location (200 kVAr)	Fixed	1	3	1	1	1	1	1	1
	Switched	3	0	1	2	1	1	2	1
Number switched capacitor banks at each location and at each load level (200 kVAr)	Nominal load level	0	0	1	1	0	0	0	0
	Peak load level	3	0	1	2	1	0	2	1
	Light load level	0	0	0	0	0	0	0	0
Injected reactive power at each location and at each load level (KVAr)	Nominal load level	200	600	400	400	200	200	200	200
	Peak load level	800	600	400	600	400	200	600	400
	Light load level	200	600	200	200	200	200	200	200

TABLE 8 Simulation results for the 28-bus distribution system: Power losses

		Proposed method (section 5)	Prevalent approaches explained in section 4: Universal control system, optimized for universal control system (section 4.1)	Prevalent approaches explained in section 4: Stand-alone voltage control system, optimized for universal control system (section 4.2)	Without compensation
Power losses at each load level (kW)	Nominal load level	36.15	33.46	39.52	68.81
	Peak load level	103.90	95.62	101.61	197.82
	Light load level	13.30	8.54	8.54	15.85

TABLE 9 Simulation results for the 28-bus distribution system: Lowest voltage

		Proposed Method (Section 5)		Prevalent Approaches Explained in Section 4: Universal Control System, Optimized for Universal Control System (Section 4.1)		Prevalent Approaches Explained in Section 4: Stand-Alone Voltage Control System, Optimized for Universal Control System (Section 4.2)		Without Compensation	
		Bus	Voltage	Bus	Voltage	Bus	Voltage	Bus	Voltage
Lowest voltage at each load level (p.u.)	Nominal load level	26	0.9429	26	0.9471	26	0.9348	26	0.9125
	Peak load level	28	0.9025	26	0.9056	26	0.9014	26	0.8511
	Light load level	15	0.9873	21	0.9795	21	0.9795	26	0.9581

a universal control system. As is evident from Table 9, for this system, there has been no constraint violation for the latter case. Therefore, it can be concluded that using the later method is not always associated with constrain violation. For this method and other methods not specifically designed for stand-alone systems, the occurrence of constraint violation depends on a number of factors such as line data, load levels, constraint limits, and controller settings, making the prediction of constraint violations very difficult without testing.

Figure 7 shows the voltage profile of the 28-bus system before and after the compensation with capacitors equipped with stand-alone voltage control systems placed with the proposed algorithm.

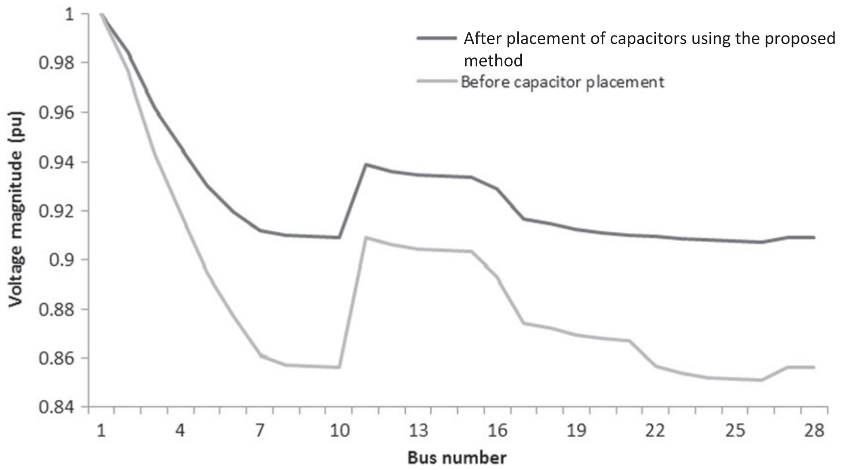

FIGURE 7 Voltage profile of 28-bus distribution network

7 | CONCLUSION

This paper proposes an optimization method for the placement of capacitors equipped with stand-alone voltage control systems. The proposed method uses GA to maximize the net savings while taking into account voltage limits, feeders' thermal limits, and the maximum permissible capacitor size. The method is implemented on a 69- and a 28-bus radial distribution network. The results are compared with those of the optimal placement of the same number and sizes of capacitors with a universal control system and to networks for which placement of capacitors with stand-alone control systems was performed without modeling the control system.

- It is shown that, despite the very limited information local control systems use, the savings are comparable to that of the universal control system, as the savings for the former are about 84.8% and 96.1% that of the latter for the 69- and 28-bus networks, respectively.
- Despite the commercial availability of stand-alone voltage, control systems have been commercially available for some time; until now, no optimization method has been established for the optimal placement of capacitors equipped with such control systems.
- We for the first time developed a placement method for capacitors with stand-alone voltage control systems and demonstrate the importance of modeling and taking into account the local control system in the optimization algorithm; the proposed method is compared with a similar method that does not model the control system.
- It is shown that for the same number of capacitors and same constraints, the amount of savings for the proposed method is $3342 and $1326 higher than that of the method that does not model the control system for the 69- and 28-bus distribution networks, respectively.

The utility of the proposed method or the capacitors themselves is not inherently limited by the size of the network. The proposed method is dependent on feasibility of power flow calculation and cannot be implemented otherwise. Similar to other optimization methods, computation time increases with size of the network. This is chiefly due to the increased time to perform power flow.

Results of this paper suggest that decentralized and autonomous control of components of the distribution network is a viable option and in many cases can perform on par with centralized control systems. In the future, we will seek to enhance its performance by including categories of units common in distribution networks and experimenting with different control systems.

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