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A group nonadditive multiattribute consumer-oriented Kansei evaluation model with an application to traditional crafts

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Abstract The aesthetic aspects of products have become crucial factors in achieving higher consumer satisfaction. This study deals with evaluation of commercial products according to the Kansei, which is an individual subjective impression reflecting the aesthetic appeal of products. To do so, we have proposed a three-phase group nonadditive multiattribute Kansei evaluation model. Particularly, a novel approach is first proposed to generate Kansei profiles involving fuzzy uncertainty as well as semantic overlapping of Kansei data. Second, a target-oriented Kansei evaluation function is proposed to induce Kansei satisfaction utility according to a consumer's personal Kansei preference, which provides a good description of the consumer's preference. Third, after formulating a general multiattribute target-oriented (MATO) Kansei evaluation function, a nonadditive MATO Kansei evaluation function is proposed based on an analogy between the general MATO Kansei evaluation function and the Choquet integral, in which an entropy-based method is utilized to estimate the fuzzy measure on a subset of Kansei attributes. The main advantages of our model are its abilities to deal with semantic overlapping of Kansei data, different types of personalized Kansei preferences, as well as mutual dependence among multiple Kansei preferences. An application to Kansei evaluation for hand-painted Kutani cups, one of the traditional craft items in Japan, is conducted to illustrate how our model works as well as its effectiveness and advantages.

Keywords Kansei data · semantic overlapping · target-oriented Kansei evaluation · nonadditive multiattribute Kansei evaluation · hand-painted Kutani cups.

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1 Introduction

With the development of global markets and modern technologies, it is very likely that many similar products are functionally equivalent, thus it is difficult for consumers to distinguish and choose their desired products. In this sense, the aesthetic quality of products has become a key consideration in today's consumer marketplace. Therefore, it is important for manufacturers to have a consumer-oriented approach in order to improve the attractiveness of products, which should not only satisfy the functional requirements of products, defined objectively, but also the aesthetic needs, by essence subjective (Petiot and Yannou, 2004). For example, the Apple's iMac was heralded as an "aesthetic revolution in computing", which indicates that the aesthetics of computers has become a key factor in purchase decisions (Postrel, 2001).

The aesthetic aspects of products have actually received much attention since 1970s from the research community of consumer-focused design and Kansei Engineering (KE). Particularly, KE is developed as a methodology to "translate the technology of the consumer's feeling and image for a product into the design elements of the product" (Nagamachi, 2002). Kansei is a Japanese word reflecting a multifaceted expression that is closely related to Japanese culture and has no direct corresponding word in English. A specific Kansei arises when a person is exposed to an artifact in a certain environmental context. According to M. Nagamachi, the founder of KE, Kansei is "an individual subjective impression from a certain artifact, environment or situation using all senses of sight, hearing, feeling, smell, taste [and sense of balance] as well as their recognition," quoted from Grimsæth (2005). Since its foundation, KE has been widely applied to the process of new product design in industries such as automotive, home electronics, office machines, cosmetics, building products, and other sectors (Grimsæth, 2005); especially in Japan and Korea. The main aim of many KE studies is to develop new product prototypes that generate specific aesthetics of products. These studies utilize multivariate statistical analysis such as principle component analysis and regression analysis, in order to discover the relationship between design elements (physical attributes) and Kansei attributes. Different Kansei elements have been studied in the literature, e.g., form/shape Kansei (Petiot and Yannou, 2004), color Kansei (Lai et al., 2006), and image Kansei (Chen and Chang, 2009). Kansei design studies concluded that the aesthetic quality of a design can greatly enhance the desirability of a product and influence consumer satisfaction in terms of perceived product quality. However, the relationship between Kansei aspects and consumer satisfaction is seldom discussed (Chen and Chuang, 2008).

This study regards Kansei as one aspect of quality of products and focuses on Kansei evaluation of existing commercial products in order to capture the relationship between Kansei attributes and consumer satisfaction. As consumers' preferences on Kansei attributes vary from person to person according to character, feeling, aesthetic and so on, we assume that a potential consumer has provided his personal preferences on a subset of Kansei attributes. We refer to our Kansei evaluation as *consumer-oriented Kansei evaluation*. Such an evaluation is helpful for marketing and recommendation purposes, since consumers can make purchase decisions according to the Kansei aspect of products. Furthermore, by integrating with the relationship between design elements and Kansei attributes, consumer-oriented Kansei evaluation may provide a support for consumer satisfactory-oriented design (Yadav and Goel, 2008), i.e., personalized design, since designers are able to design new products best satisfying consumers' Kansei preferences. Fig. 1 shows the integrated framework of KE, in which Kansei

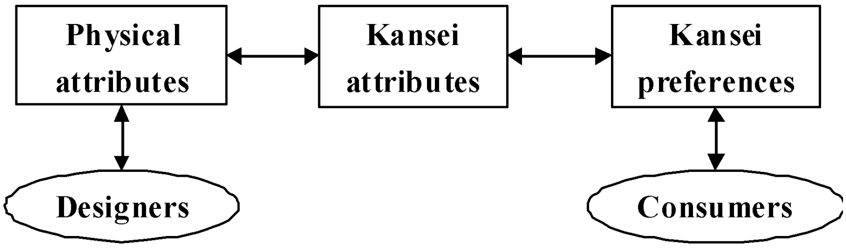

Fig. 1 An integrated framework of Kansei Engineering

acts as a bridge between designers and consumers. Note that many Kansei design studies have involved an evaluation process, in which a design could be selected for production (Chen and Chuang, 2008; Petiot and Yannou, 2004, e.g.). However, Kansei evaluation of existing commercial products for the purpose of purchase decision has generally received less attention (Mondragón et al., 2005). Also, a similar problem with Kansei evaluation is *sensory evaluation* (Ruan and Zeng, 2004), in which knowledge is acquired from a panel of experts by means of the five senses of *sight*, *taste*, *touch*, *smell* and *hearing*. In fact, KE is also referred to as *sensory engineering* or *emotional usability* (Grimsæth, 2005). Since our research context is closely related to Japanese culture, we shall use "Kansei" in this study.

In this study, we aim at proposing and developing decision analysis approaches to consumer-oriented Kansei evaluation by addressing the following three questions:

1. **Kansei profile generation**: How to generate a Kansei profile involving the fuzzy uncertainty as well as semantic overlapping of Kansei data?
2. **Single attribute Kansei evaluation function**: How to quantify a consumer satisfaction based on our Kansei profile generated?
3. **Multiattribute Kansei evaluation function**: How to quantify mutual dependence among multiple Kansei preferences in our considered context?

Toward this end, Sec. 2 begins with the preparatory experiment in KE and follows with detailed formulations of our research problems. To answer question 1, Sec. 3 proposes a novel approach to generating Kansei profiles involving fuzzy uncertainty and semantic overlapping of Kansei data, the generated Kansei profile results with a probability distribution on Kansei data. To answer question 2, Sec. 4 proposes a target-oriented Kansei evaluation function to quantify a consumer satisfaction. Our evaluation function is based on the appealing idea of target-oriented decision analysis (Bordley and LiCalzi, 2000; Huynh et al., 2010; Yan et al., 2009, 2008). To answer question 3, in Sec. 5, target-oriented Kansei evaluation function is extended to multiple Kansei attributes. Particularly, after formulating the general multiattribute target-oriented (MATO) Kansei evaluation function based on Bordley and Kirkwood (2004); Tsetlin and Winkler (2007), an analogy between general MATO Kansei evaluation and Choquet integral is then proposed. An entropy-based method is used to estimate the fuzzy measure on a subset of Kansei attributes. Sec. 6 uses hand-painted Kutani cups, one of the traditional crafts in Japan, to show the effectiveness of our model. Sec. 7 compares our model with related work. Finally, some concluding remarks are presented in Sec. 8.

2 The Preparatory Experiment and Problems Formulation

2.1 The Preparatory Experiment

The first step in KE is to select a product domain and collect product samples. It is easy to collect product images in the marketplace such as websites, producers, catalogs, and magazines. Researchers then need to eliminate duplicate or similar ones. Let

$$\mathcal{O} = \{O_1, \ldots, O_m, \ldots, O_M\}$$

be a set of representative products.

Second, we have to identify and measure Kansei attributes used by people to express their psychological feelings. Usually, Kansei attributes are identified by a panel of experts (people familiar with the product types and KE) via a brainstorming process (Grimsæth, 2005). There are different ways of measuring the Kansei: *words*, *physiological response*, *people's behaviors and actions*, and *facial and body expressions*. Most KE studies which have been published in English, use words when measuring the Kansei. The words reflect elements of the Kansei and are also used to measure the Kansei in this study. Each Kansei attribute is defined by a bipolar pair of Kansei words, which describe the product domain and can be collected from many sources (Grimsæth, 2005): *magazines*, *manuals*, *product reviews*, and *users*. Researchers then need to eliminate duplicate or similar Kansei words. The refined bipolar pairs of Kansei words can be formally expressed as follows:

- let $\mathcal{X} = \{X_1, \ldots, X_n, \ldots, X_N\}$ be a set of Kansei attributes;
- let $\mathrm{KW}_n = \langle \mathrm{kw}_n^-, \mathrm{kw}_n^+ \rangle$ be the bipolar pair of Kansei words with respect to Kansei attribute X_n;
- let **KW** be the set of bipolar pairs of Kansei words such that

$$\mathbf{KW} = \left\{ \mathrm{KW}_n = \left\langle \mathrm{kw}_n^-, \mathrm{kw}_n^+ \right\rangle | n = 1, \ldots, N \right\}. \quad (1)$$

Third, a questionnaire is designed by means of the semantic differential (SD) method (Osgood et al., 1957). The questionnaire consists of listing N Kansei attributes, each of which corresponds to a bipolar pair of Kansei words with a G-point odd qualitative scale, denoted by

$$\mathcal{V} = \left\{ \mathrm{V}_1, \ldots, \mathrm{V}_{(G+1)/2}, \ldots, \mathrm{V}_G \right\}, \quad (2)$$

with the middle point $\mathrm{V}_{(G+1)/2}$ being neutral Kansei $\mathrm{kw}^\sim$, and the rest of the points being placed symmetrically around it. The left-most hand point V_1 stands for left Kansei word kw^-, and the right-most hand point V_G expresses right Kansei word kw^+. Also, we have $\mathrm{V}_{g+1} - \mathrm{V}_g = 1$, where $g = 1, \ldots, G-1$. In practice, people can reasonably manage to keep about seven points in mind (Miller, 1956). For example, a 7-point qualitative scale can be denoted as $\mathcal{V} = \{1, 2, 3, 4, 5, 6, 7\}$ (Petiot and Yannou, 2004) or $\mathcal{V} = \{-3, -2, -1, 0, 1, 2, 3\}$ (Llinares and Page, 2007).

Finally, the questionnaire is distributed to a number K of subjects $\mathcal{E}$, who are asked to express their subjective assessments for the products in $\mathcal{O}$ on Kansei attributes in $\mathcal{X}$ via the G-point odd qualitative scale simultaneously. Formally, the Kansei assessment provided by subject $E_k \in \mathcal{E}$ for product $O_m \in \mathcal{O}$ on Kansei attribute $X_n \in \mathcal{X}$ is denoted as $x_n^m(E_k)$, where for all $x_n^m(E_k) \in \mathcal{V}, m = 1, \ldots, M$; $n = 1, \ldots, N$; and $k = 1, \ldots, K$. Table 1 shows the Kansei database of product O_m on N Kansei attributes.

Table 1 Kansei database of product O_m on N Kansei attributes

Subjects	Kansei attributes				
	X_1	...	X_n	...	X_N
E_1	$x_1^m(E_1)$	...	$x_n^m(E_1)$	...	$x_N^m(E_1)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
E_k	$x_1^m(E_k)$	...	$x_n^m(E_k)$	...	$x_N^m(E_k)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
E_K	$x_1^m(E_K)$	...	$x_n^m(E_K)$	...	$x_N^m(E_K)$

2.2 Formulations of Research Problems

Based on the Kansei database in Table 1, our main objective is to get the product(s) best satisfying a consumer's individual Kansei preference(s). Assume a potential consumer is interested in looking for a product that meets his Kansei preferences given by a proper subset $\mathbb{KW}$ of the set $\mathbf{KW}$ of N bipolar pairs of Kansei words. Particularly, we are concerned with consumer-specified Kansei requests that can be stated generally in form of the following statement:

> "I like products which best meet my Kansei preferences specified in $\mathbb{KW} \subseteq \mathbf{KW}$."

Formally, the problem can be formulated as follows. Given

$$\mathbb{KW} = \left\{ \mathrm{kw}^*_{n_1}, \ldots, \mathrm{kw}^*_{n_n}, \ldots, \mathrm{kw}^*_{n_N} \right\}$$

corresponding to the Kansei request linguistically specified by a consumer, where $*$ stands for $-$, $\sim$, or $+$. Here, $-, \sim, +$ represent *left Kansei word preference*, *neutral Kansei preference (neither left Kansei word nor right Kansei word)*, and *right Kansei word preference*, respectively. The problem now is how to evaluate the products in $\mathcal{O}$ using the Kansei database based on consumer-specified request $\mathbb{KW}$?

In order to achieve this goal, we consider three subproblems. The first subproblem is the Kansei profile generation. Traditional KE studies treat the qualitative scale in Eq. (2) as numerical data and then use the mean scale of questionnaire-collected Kansei data as Kansei profile. However, the subjective assessments provided by the subjects are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms (Zadeh, 1975). Therefore, it is more appropriate to treat the qualitative scale $\mathcal{V}$ as a linguistic variable with a set of G linguistic labels. Formally, the set of linguistic labels for Kansei attribute X_n can be denoted by

$$\mathcal{L}^n = \left\{ L_1^n, \ldots, L_g^n, \ldots, L_G^n \right\} \tag{3}$$

with $L_1^n < \cdots < L_g^n < \cdots < L_G^n$. We shall call $\mathcal{L}^n$ the "Kansei linguistic variable". In this sense, if a subject assesses the product O_m on Kansei attribute X_n using V_g, it implies that the subject chooses Kansei label L_g^n as his assessment.

With the linguistic interpretation, if one subject assesses a product O_m on an attribute X_n using L_g^n, it implies that the subject makes an assertion "O_m on X_n is L_g^n". From the philosophical viewpoint of the epistemic stance (Lawry, 2008), humans posses some kind of mechanism for deciding whether or not to make certain assertions. Furthermore, although the underlying concepts are often vague, the decisions about assertions are, at a certain level, bivalent. That is to say for a product O_m on an attribute X_n and a description L^n_g, the subject is willing to assert that "O_m on X_n is L^n_g" or not. However, the dividing line between those Kansei labels are and those are not appropriate to use may be uncertain. Therefore, if one subject assesses a product using L^n_g, other Kansei labels $L^n_l (l \neq g)$ in $\mathcal{L}^n$ may also be appropriate for describing O_m on X_n. Such a phenomenon is referred to as the *semantic overlapping* of Kansei data. The sensory evaluation model (Martínez, 2007) based on 2-tuple linguistic model (Herrera and Martínez, 2000) cannot capture the *semantic overlapping* of Kansei data. Our first subproblem is how to obtain Kansei profile based on the linguistic interpretation and semantic overlapping of Kansei?

The second one is single attribute Kansei evaluation function. With a consumer's Kansei request $\mathrm{kw}^*_{n_n}$ on a Kansei attribute X_{n_n}, previous KE studies have usually applied the linear satisfaction utility functions to model consumers' preferences (Petiot and Yannou, 2004, e.g.) using the mean scale based Kansei profile. However, the relationship between Kansei attribute and consumer may be nonlinear (Chen and Chuang, 2008). Also, it is difficult to build rigorous utility functions based on attributes and the conventional attribute utility function often does not provide a good description of individual preferences (Bordley and Kirkwood, 2004). In addition, the sensory evaluation model (Martínez, 2007) based on 2-tuple linguistic decision analysis assumes a consistent preference order relation on a linguistic variable, thus it will be inappropriate for our problem. Our second subproblem is how to quantify a product on a Kansei attribute X_{n_n} meets $\mathrm{kw}^*_{n_n}$ based on our Kansei profile?

The final subproblem is the nonadditive multiattribute Kansei evaluation function. Extending single Kansei preference to multiple Kansei preferences

$$\mathbb{KW} = \{\mathrm{kw}^*_{n_1}, \ldots, \mathrm{kw}^*_{n_n}, \ldots, \mathrm{kw}^*_{n_N}\},$$

either the traditional KE studies (Chen and Chuang, 2008; Llinares and Page, 2007; Petiot and Yannou, 2004) or the sensory evaluation model (Martínez, 2007), have usually applied classical weighted arithmetic mean (WAM) method to obtain global evaluation without further considering mutual dependence among Kansei attributes. However, it is well-known that the mutual independence among multiple Kansei aspects is rarely verified. Our third subproblem is how to quantify mutual dependence among $\mathbb{KW}$ in our considered context?

Due to the above three subproblems, a group nonadditive multiattribute Kansei evaluation model is given in Fig. 2. Once the preparatory experiment has been conducted, our model consists of three phases: Kansei profile generation involving semantic overlapping, target-oriented Kansei evaluation, and nonadditive multiattribute target-oriented Kansei evaluation.

3 Kansei Profile Generation Involving Semantic Overlapping

Having obtained the Kansei database, as shown in Table 1, we have to aggregate the opinions of all subjects to a final value (Kansei profile) for each product on each attribute. Due to the linguistic interpretation of Kansei data discussed in Sec. 2.2, the ordered structure approach (Martínez, 2007) can be used to choose linguistic descriptors, e.g., "fairly" and "very", for Kansei attributes. For example, the Kansei attribute

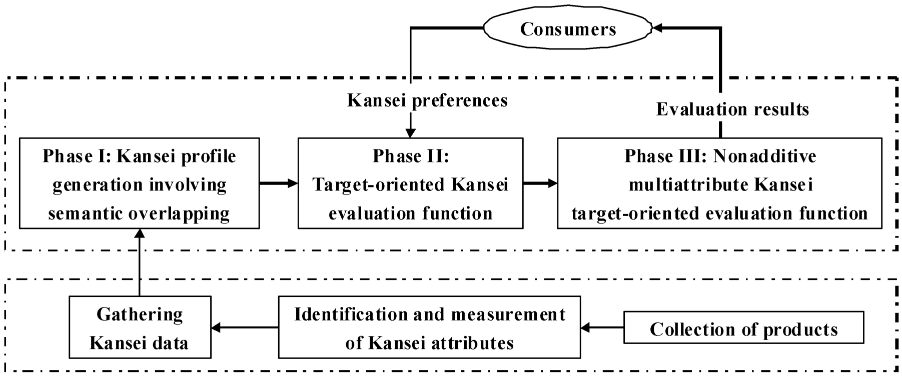

Fig. 2 A group nonadditive multiattribute Kansei evaluation model

fun having a bipolar pair of Kansei words <*solemn*, *funny*> with a 7-point qualitative scale, can be implicitly defined by

$$\mathcal{L} = \{\text{Very solemn}, \text{Solemn}, \text{Fairly solemn}, \text{Neutral}, \text{Fairly funny}, \text{Funny}, \text{Very funny}\}.$$

In the sequel, we shall propose a probabilistic approach to generating Kansei profile involving semantic overlapping of Kansei data.

3.1 A Probabilistic Approach to Generating Kansei profiles

The Kansei assessment provided by a subject implies that he makes an assertion. Motivated by the *epistemic stance* (Lawry, 2008), we assume that any neighboring Kansei labels have partial semantic overlapping. Thus, when one subject $E_k \in \mathcal{E}$ assesses a product $O_m \in \mathcal{O}$ on an attribute X_n using Kansei label $x_n^m(E_k) \in \mathcal{L}^n$, other Kansei labels in $\mathcal{L}^n$ may also be appropriate for describing O_m on X_n, but which of these Kansei labels is uncertain. The Kansei label $x_n^m(E_k)$ will be referred to as a *prototype label*. Lawry (2008) has introduced a new framework for label semantics where the semantics of linguistic labels are described by appropriateness degree, which means the belief that a linguistic label is appropriate for describing a product. The appropriateness distribution can be represented by a possibility distribution, which is convenient for representing consonant imprecise knowledge. A possibility distribution, π, describes the more or less plausible values of some uncertain variable. If subjects can directly assign the appropriateness degrees of all Kansei labels, then we can obtain a possibility distribution. However, the need of subjects' involvement creates the burden of assessment process. Therefore, we assume that the appropriate labels are distributed around the prototype label $x_n^m(E_k)$ with a possibility distribution.

Returning back to the Kansei database in Table 1, given the Kansei data $[x_n^m(E_1), \ldots, x_n^m(E_k), \ldots, x_n^m(E_K)]$ of product O_m on attribute X_n provided by K subjects, it is very rare that all subjects share the same opinion, since a diversity of opinions commonly exists. For product O_m on Kansei attribute X_n, we first define

$$\begin{aligned} L_{\min}(x_n^m) &= \min_{k=1,\ldots,K}\left\{x_n^m(E_k)\right\}, \\ L_{\max}(x_n^m) &= \max_{k=1,\ldots,K}\left\{x_n^m(E_k)\right\}, \end{aligned} \tag{4}$$

where $x_n^m(E_k) \in \mathcal{L}^n$ and $L_{\min}(x_n^m)$, $L_{\max}(x_n^m)$ are the smallest and largest Kansei labels of product O_m on Kansei attribute X_n, respectively. The label indices of the smallest and largest labels are expressed as $\mathrm{Ind}_{\min}(x_n^m)$ and $\mathrm{Ind}_{\max}(x_n^m)$, respectively. Also, the label index of the prototype label $x_n^m(E_k)$ is denoted as $\mathrm{Ind}_k(x_n^m)$. In the sequel, without possibility of confusion, we shall drop the subscripts m and n to simplify the notations.

The aggregation result of $[x(E_1), \ldots, x(E_k), \ldots, x(E_K)]$ should lie between $L_{\min}$ and $L_{\max}$. Accordingly, we define a possibility distribution of "around the prototype label $x(E_k) \in \mathcal{L}$" as follows:

$$\pi\left(L_g|x(E_k)\right) = \begin{cases} \left(\frac{g-\mathrm{Ind}_{\min}}{\mathrm{Ind}_k-\mathrm{Ind}_{\min}}\right)^\gamma, & \text{if } g \in [\mathrm{Ind}_{\min}, \mathrm{Ind}_k]; \\ \left(\frac{\mathrm{Ind}_{\max}-g}{\mathrm{Ind}_{\max}-\mathrm{Ind}_k}\right)^\gamma, & \text{if } g \in [\mathrm{Ind}_k, \mathrm{Ind}_{\max}]; \\ 0, & \text{if } g \notin [\mathrm{Ind}_{\min}, \mathrm{Ind}_{\max}]. \end{cases} \tag{5}$$

Here, γ is a linguistic modifier and $\gamma > 0$. When $\gamma > 1$, it means that the subject has an optimistic attitude (he is more sure that the prototype label is appropriate enough to describe a product); when $\gamma = 1$, it means that the subject has a neutral attitude; when $\gamma < 1$, it means that the subject has a pessimistic attitude (he is less sure that the prototype label is appropriate enough to describe a product). Without additional information, this study assumes that each subject has a neutral attitude, i.e. $\gamma = 1$.

Since $\pi(L_g|x(E_k))$ is the possibility distribution of "around prototype label $x(E_k)$" on a set of G Kansei labels, then a consonant mass assignment $\mathbf{m}_{x(E_k)}$ can be derived as follows.

Definition 1 Given the possibility distribution $\pi(L_g\,|x(E_k))$, the possibility degrees are reordered as

$$\{\pi_1(x(E_k)), \ldots, \pi_j(x(E_k)), \ldots, \pi_J(x(E_k))\}$$

such that

$$1 = \pi_1(x(E_k)) > \pi_2(x(E_k)) > \cdots > \pi_J(x(E_k)) \geq 0,$$

then the consonant mass selection function identifies the mass assignment

$$\begin{aligned} &\mathbf{m}_{x(E_k)}(\phi) = 1 - \pi_1(x(E_k)), \\ &\mathbf{m}_{x(E_k)}(F_j) = \pi_j(x(E_k)) - \pi_{j+1}(x(E_k)), j = 1, \ldots, J-1, \\ &\mathbf{m}_{x(E_k)}(F_J) = \pi_J(x(E_k)), \end{aligned} \tag{6}$$

where $F_j = \{\pi(L_g|x(E_k)) \geq \pi_j(x(E_k))\}, j = 1, \ldots, J$ and $\{F_j\}_{j=1}^J$ are the focal elements of $\mathbf{m}_{x(E_k)}$. The mass $\mathbf{m}_{x(E_k)}(F)$ means one's belief that F is the extension of the prototype Kansei label $x(E_k)$.

The notion of mass assignment suggests a means of defining probability distribution (Lawry, 2004). For any prototype Kansei label, we can derive a probability distribution as follows.

Definition 2 We can then obtain the least prejudiced distribution of "around the prototype label $x(E_k)$" on the set of Kansei labels as follows:

$$p(L_g|x(E_k)) = \sum_{F_j : L_g \in F_j} \frac{\mathbf{m}_{x(E_k)}(F_j)}{|F_j|}, L_g \in \mathcal{L}, \tag{7}$$

where $\mathbf{m}_{x(E_k)}$ is the mass assignment of $\pi(x(E_k))$ and $\{F_j\}_j$ is the corresponding set of focal elements.

Table 2 Collective probability distributions for O_m.

Attribute	G-scale Kansei data				
	V_1	$\dots$	V_g	$\dots$	V_G
X_1	$p_1^m(L_1^1)$	$\dots$	$p_1^m(L_g^1)$	$\dots$	$p_1^m(L_G^1)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
X_n	$p_n^m(L_1^n)$	$\dots$	$p_n^m(L_g^n)$	$\dots$	$p_n^m(L_G^n)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
X_N	$p_N^m(L_1^N)$	$\dots$	$p_N^m(L_g^N)$	$\dots$	$p_N^m(L_G^N)$

The idea underlying this probability distribution is that, for each focal set F containing Kansei label L, a uniform proportion $\frac{1}{|F|}$ is allocated to L. In other words, the value $p(L_g|x(E_k))$ reflects the probability that $L_g \in \mathcal{L}$ belongs to the extensions of the prototype label $x(E_k)$.

Therefore, we obtain a probability distribution

$$[p(L_1|x(E_k)), \dots, p(L_g|x(E_k)), \dots, p(L_G|x(E_k))]$$

of "around the prototype label $x(E_k)$" on the set of Kansei labels. In KE, a number K of subjects $\mathcal{E}$ are asked to provide their judgements for the products. We assume that each subject is assigned a degree of importance or weight w_k, the weighting vector is denoted as

$$\mathrm{W} = [\mathrm{w}_1, \dots, \mathrm{w}_k, \dots, \mathrm{w}_K]$$

such that $\sum_{k=1}^{K} \mathrm{w}_k = 1$. The assignment of weight information for subjects is useful when different groups of subjects are selected in Kansei evaluation. This is motivated by the fact there still remains a gap between designers' and consumers' perception, due to the fact that subjective functions and criteria are often neither named nor objectively assessed (Hsu et al., 2000). With the weighting vector, a collective probability distribution under different prototype Kansei labels $x(E_k)(k = 1, \dots, K)$ is then defined as follows:

$$p(L_g) = \sum_{k=1}^{K} p(L_g|x(E_k)) \cdot \mathrm{w}_k, g = 1, \dots, G. \tag{8}$$

Extending this definition to M products and N Kansei attributes, we can obtain a G-tuple probability distribution on the set of Kansei labels $\mathcal{L}^n$ for product O_m regarding Kansei attribute X_n such that

$$X_n(O_m) = [p_n^m(L_1^n), \dots, p_n^m(L_g^n), \dots, p_n^m(L_G^n)],$$

which will be called *Kansei profile*. Table 2 shows the Kansei profiles of product O_m on the set of N Kansei attributes.

With the Kansei data provided by the subjects for product O_m on Kansei attribute X_n, and the importance weight w_k for subject E_k, we can obtain a weighting vector of all the possible prototype Kansei linguistic labels $L_{\mathbf{p}}^n$ for O_m on X_n. The weight of each prototype Kansei label is denoted as $\varpi_n^m(L_{\mathbf{p}}^n)$ and can be obtained by

$$\varpi_n^m(L_{\mathbf{p}}^n) = \sum_{\substack{E_k \in \mathcal{E}, \\ x_n^m(E_k) = L_{\mathbf{p}}^n}} \mathrm{w}_k, \mathbf{p} = 1, \dots, G \tag{9}$$

where $\mathbf{p}$ denotes the index of a prototype label. Especially, when all the subjects are equally important, i.e., $\mathrm{w}_k = 1/K$, $\varpi_n^m(L_{\mathbf{p}}^n)$ reduces to

$$\varpi_n^m(L_{\mathbf{p}}^n) = \frac{|E_k \in \mathcal{E} : x_n^m(E_k) = L_{\mathbf{p}}^n|}{K}, \mathbf{p} = 1, \ldots, G. \quad (10)$$

Similar to Eq. (8), we obtain the collective probability distribution for O_m on X_n as follows:

$$p_n^m(L_g^n) = \sum_{\mathbf{p}=1}^{G} p_n^m(L_g^n|L_{\mathbf{p}}^n) \cdot \varpi_n^m(L_{\mathbf{p}}^n), \quad (11)$$

where $p_n^m(L_g^n|L_{\mathbf{p}}^n), g = 1 \ldots, G$, is the probability distribution of "around the prototype Kansei label $L_{\mathbf{p}}^n$" on the set $\mathcal{L}^n$ of Kansei labels, and can be derived by Eqs. (4)-(7).

3.2 Properties of Kansei profiles Generated

Our Kansei profile has several interesting properties.

Property 1 Instead of viewing Kansei data as numerical data, our approach treats Kansei data as linguistic labels, in which fuzzy uncertainty of Kansei data can be taken into account.

Property 2 It can treat different groups of subjects by incorporating a weighing vector for the subjects such that

$$\mathrm{W} = [\mathrm{w}_1, \ldots, \mathrm{w}_k, \ldots, \mathrm{w}_K], \sum_{k=1}^{K} \mathrm{w}_k = 1.$$

Property 3 The maximum probability in the Kansei profile generated for product O_m on Kansei attribute X_n could only happen between $L_{\min}(x_n^m))$ and $L_{\max}(x_n^m)$.

Proof From Eqs. (4)-(5), it is obvious that our Kansei profile generation approach is bounded, thus the maximum probability in the Kansei profile generated could only happen between $L_{\min}(x_n^m))$ and $L_{\max}(x_n^m)$. □

Property 4 Our approach to generating Kansei profile involves the semantic overlapping of Kansei data, which results with a probability distribution on the set of Kansei labels. The sum of probability distributions of all the Kansei labels in the Kansei profile generated is equivalent to one.

Proof

$$\begin{aligned}
\sum_{g=1}^{G} p_n^m(L_g^n) &= \sum_{g=1}^{G} \left\{ \sum_{\mathbf{p}=1}^{G} p_n^m(L_g^n|L_{\mathbf{p}}^n) \cdot \varpi_n^m(L_{\mathbf{p}}^n) \right\} \\
&= \sum_{\mathbf{p}=1}^{G} \left\{ \sum_{g=1}^{G} p_n^m(L_g^n|L_{\mathbf{p}}^n) \cdot \varpi_n^m(L_{\mathbf{p}}^n) \right\} \\
&= \sum_{\mathbf{p}=1}^{G} \left\{ \varpi_n^m(L_{\mathbf{p}}^n) \cdot \sum_{g=1}^{G} p_n^m(L_g^n|L_{\mathbf{p}}^n) \right\} \\
&= \sum_{\mathbf{p}=1}^{G} \varpi_n^m(L_{\mathbf{p}}^n) = 1
\end{aligned}$$

Thus, $\sum_{g=1}^{G} p_n^m(L_g^n) = 1$. Such a probability distribution reflects the fuzzy uncertainty as well as the semantic overlapping of Kansei data. □

4 Single Attribute Kansei Evaluation Function

The Kansei profile $X_n(O_m) = \left[p_n^m(L_1^n), \dots, p_n^m(L_g^n), \dots, p_n^m(L_G^n)\right]$ of product O_m on Kansei attribute X_n can viewed as a decision making under uncertainty problem, described as follows. $O_m (m = 1, \dots, M)$ represent the alternatives available to a consumer, one of which must be selected. Kansei attribute X_n has G possible values ($\mathcal{L}^n$=$\{L_1^n, \dots, L_g^n, \dots, L_G^n\}$) corresponding to the so-called state space $\mathcal{S} = \{S_1, \dots, S_g, \dots, S_G\}$, which is characterized by a probability distribution $p_{\mathcal{S}}$ on the state space $\mathcal{S}$. $p_n^m(L_g^n)(g = 1, \dots, G)$ on $\mathcal{L}^n$ acts as the the probability distribution on the state space $\mathcal{S}$.

Due to the inconsistent preference order relations on $\mathcal{L}^n$, the sensory evaluation model (Martínez, 2007) based on the 2-tuple linguistic representation model (Herrera and Martínez, 2000) cannot be used in our problems. Also, we can use the expected utility value function (discrete case) for our problems such that

$$U_n^m = \sum_{g=1}^{G} U(L_g^n) \cdot p_n^m(L_g^n), \tag{12}$$

where U is a utility function. For example, Petiot and Yannou (2004) have defined three types of satisfaction utility functions in Kansei evaluation problems. However, substantial empirical evidence has shown that it is difficult to build mathematically rigorous utility functions based on attributes (Bordley and Kirkwood, 2004).

Assume a consumer has specified his preference toward Kansei attribute X_n in terms of Kansei words kw_n^*, where $*$ stands for $-, \sim$ or $+$. Intuitively, if a consumer expresses his preference for Kansei attribute X_n with kw_n^-, he might implicitly assume a preference order on the Kansei data toward L_1^n where the left Kansei word is placed. Conversely, if the consumer's preference for Kansei attribute X_n is kw_n^+, the preference order on the Kansei data corresponding to the Kansei attribute X_n should be determined adaptively according to particular consumer's preference toward the end L_G^n where right Kansei word is placed. Moreover, sometimes a consumer may express his preference for the Kansei attribute with $\mathrm{kw}_n^\sim$. Given a consumer's Kansei preference kw_n^* for Kansei attribute X_n, the preference relation $\succeq^n$ can be formally expressed as follows:

$$\succeq^n \Leftrightarrow \begin{cases} L_1^n \succeq \cdots \succeq L_{(G+1)/2}^n \succeq \cdots \succeq L_G^n, \text{ if } \mathrm{kw}_n^* = \mathrm{kw}_n^-; \\ L_1^n \preceq \cdots \preceq L_{(G+1)/2}^n \succeq \cdots \succeq L_G^n, \text{ if } \mathrm{kw}_n^* = \mathrm{kw}_n^\sim; \\ L_1^n \preceq \cdots \preceq L_{(G+1)/2}^n \preceq \cdots \preceq L_G^n, \text{ if } \mathrm{kw}_n^* = \mathrm{kw}_n^+. \end{cases} \tag{13}$$

In addition, due to the vagueness inherent in consumer's expression of preference in terms of Kansei words, each is considered as a Kansei target of X_n, denoted by T_n, which can be represented as a possibility variable defined as

$$\pi_{T_n}(L_g^n) = \begin{cases} \left[\frac{(G-g)}{(G-1)}\right]^\lambda, & \text{if } \mathrm{kw}_n^* = \mathrm{kw}_n^-; \\ \left[1 - \frac{|(G+1)/2-g|}{(G+1)/2-1}\right]^\lambda, & \text{if } \mathrm{kw}_n^* = \mathrm{kw}_n^\sim; \\ \left[\frac{(g-1)}{(G-1)}\right]^\lambda, & \text{if } \mathrm{kw}_n^* = \mathrm{kw}_n^+, \end{cases} \tag{14}$$

where $g = 1, \ldots, G$, and $\lambda > 0$ represents a degree of intensity toward the Kansei target of a consumer. Intuitively, when a consumer expresses his Kansei targets using Kansei words combined with linguistic modifiers (e.g. *very*, *slightly*) to emphasize his intensity about target, the degree of intensity λ can then be determined similarly as in Zadeh's method of modeling linguistic modifiers via power functions in approximate reasoning (Zadeh, 1975). Without additional information, λ is set to one for default.

With the Kansei targets and their preference order relations, we are able to evaluate how well the Kansei profile $X_n(O_m)$ meets the Kansei target T_n. We define the following value function for a single Kansei attribute

$$\Pr(X_n(O_m) \succeq T_n) = \sum_{g=1}^{G} \Pr(L_g^n \succeq T_n) \cdot p_n^m(L_g^n), \tag{15}$$

where T_n is a consumer's Kansei target toward Kansei attribute X_n, and T_n is stochastically independent of $L_g^n(g = 1, \ldots, G)$ and $O_m(m = 1, \ldots, M)$. Eq. (15) is referred to as target-oriented Kansei evaluation model, and Pr is called target-oriented utility. We denote $\Pr(X_n(O_m) \succeq T_n)$ as $\Pr_n^m$.

We then proceed as follows. First, we derive a consonant mass assignment function $\mathbf{m}_{\pi_{T_n}}$ for the possibility distribution $\pi_{T_n}(L_g^n)(g = 1, \ldots, G)$. The possibility degrees are reordered as $\{\pi_1(T_n), \ldots, \pi_j(T_n), \ldots, \pi_J(T_n)\}$ such that $1 = \pi_1(T_n) > \pi_2(T_n) > \cdots > \pi_J(T_n) \geq 0$. Second, we obtain the least prejudiced distribution of T_n such that

$$p_{T_n}(L_g^n) = \sum_{F_j : L_g^n \in F_j} \frac{\mathbf{m}_{\pi_{T_n}}(F_j)}{|F_j|}, \tag{16}$$

where $L_g^n \in \mathcal{L}^n, g = 1, \ldots, G$, $\mathbf{m}_{\pi_{T_n}}$ is the mass assignment of π_{T_n}, $F_j = \{\pi_{T_n}(L_g) \geq \pi_j(T_n)\}, j = 1, \ldots, J$ and $\{F_j\}_{j=1}^J$ are referred to as the focal elements of $\mathbf{m}_{\pi_{T_n}}$. Third, we obtain the target-oriented utility for each Kansei label L_g^n of X_n based on the continuous target-oriented decision model with different types of target preferences (Yan et al., 2009) as follows. In case of left and right Kansei preferences, the consumer has monotonic preferences, the probability of L_g^n meeting T_n is

$$\Pr(L_g^n \succeq T_n) = \begin{cases} \sum_{l=g}^{G} p_{T_n}(L_l^n), \text{ if } \mathrm{kw}_n^* = \mathrm{kw}_n^-; \\ \sum_{l=1}^{g} p_{T_n}(L_l^n), \text{ if } \mathrm{kw}_n^* = \mathrm{kw}_n^+. \end{cases} \tag{17}$$

In case of neutral Kansei preference ($\mathrm{kw}_n^* = \mathrm{kw}_n^\sim$), the probability of L_g^n meeting T_n is

$$\Pr(L_g^n \succeq T_n) = \begin{cases} \frac{\sum_{l=1}^{g} p_{T_n}(L_l^n)}{\sum_{l=1}^{\frac{(G+1)}{2}} p_{T_n}(L_l^n)}, & g < \frac{G+1}{2}; \\ 1, & g = \frac{G+1}{2}; \\ \frac{\sum_{l=g}^{G} p_{T_n}(L_l^n)}{\sum_{l=\frac{(G+1)}{2}}^{G} p_{T_n}(L_l^n)}, & g > \frac{G+1}{2}. \end{cases} \tag{18}$$

Substituting Eqs. (17)-(18) into Eq. (15), we are able to obtain the probability of product O_m on Kansei attribute X_n meeting three types of Kansei targets kw_n^-, $\mathrm{kw}_n^\sim$, kw_n^+, respectively.

Generally, our Kansei evaluation function is based on the appealing idea of target-oriented decision model (Bordley and LiCalzi, 2000; Yan et al., 2009). Interestingly, despite the differences in approach and interpretation, both the utility-based procedure Eq. 12 and target-oriented procedure Eq. 15 essentially lead to only one basic model for decision making (Bordley and LiCalzi, 2000). Note that Eq. 15 is strictly more general than Eq. 12, in the sense that equivalence holds under stochastic independence of the target.

5 Nonadditive Multiattribute Kansei Evaluation Function

For the sake of simplicity of notation, we assume the consumer prefers N Kansei attributes. A natural question that arises is how to extend our target-oriented Kansei evaluation function to multiple Kansei attributes?

5.1 General Multiattribute Target-Oriented Kansei Evaluation Evaluation

Assume that a consumer has linguistically specified his Kansei preferences toward N Kansei attributes in terms of Kansei words such that $\{\mathrm{kw}_1^*, \ldots, \mathrm{kw}_n^*, \ldots, \mathrm{kw}_N^*\}$, we can then build a set of Kansei targets $\{T_1, \ldots, T_n, \ldots, T_N\}$. Following Bordley and Kirkwood (2004), we have

Definition 3 With N Kansei attributes $\mathcal{X} = \{X_1, \ldots, X_n, \ldots, X_N\}$ and N Kansei targets $T = (T_1, \ldots, T_n, \ldots, T_N)$, a consumer is defined to be *target oriented* if his utility for a product $O_m = (X_1(O_m), \ldots, X_n(O_m), \ldots, X_N(O_m))$ depends only on which Kansei targets are met by that O_m. The utility function for a Kansei target-oriented consumer is completely specified by 2^N constants where these constants are the utilities of achieving specific combinations of the various Kansei targets.

Let $\mathcal{I} = \{I_1, \ldots, I_n, \ldots, I_N\}$ be a set of indicator variables, where $I_n = 1$, if $X_n(O_m) \succeq T_n$; 0, otherwise. Then a target-oriented consumer has a function $U_I(\mathcal{I})$ assigning utilities to the 2^N possible values of $\mathcal{I}$. Note that, here, we first assume there is no uncertainty about the Kansei profile $X_n(O_m)$ of product O_m on Kansei attribute X_n and the Kansei target T_n, i.e., $X_n(O_m)$ and T_n are specific Kansei labels in $\mathcal{L}^n$. Similar to Tsetlin and Winkler (2007), let $U_I(\mathcal{I}) = \nu_A$, where A is the set of indices $\{n|I_n = 1\}$ corresponding to the attributes in $\mathcal{I}$ for which the targets are met. For example, $U_I(1, 0, \ldots, 0) = \nu_1$, $U_I(0, 1, 1, \ldots, 0) = \nu_{2,3}$ and so on. If $A_1 \subseteq A_2$, then $\nu_{A_1} \leq \nu_{A_2}$; utility can never be reduced by meeting additional targets. We also know that $0 \leq \nu_A \leq 1$ for all A, with $\nu_\varnothing = U_I(0, \ldots, 0, \ldots, 0)$ and $\nu_{1,\ldots,n,\ldots,N} = U_I(1, \ldots, 1, \ldots, 1) = 1$, leaving $2^N - 2$ utilities ν_A to be assessed. Consider a simple example with $N = 2$, we know

$$U_I(\mathcal{I}) = \nu_\varnothing I_\varnothing + \nu_1 I_1 + \nu_2 I_2 + (1 - \nu_1 - \nu_2) I_1 I_2.$$

Recall that I_n depends on whether $X_n(O_m) \succeq T_n$, $\nu_\varnothing = 0$, and $\nu_{12} = 1$, and that T_n is independent of $X_n(O_m)$. Integrating out the uncertainty about T_n and $X_n(O_m)$, we can get

$$\mathrm{Val}(O_m) = \nu_1 \mathrm{Pr}_1^m + \nu_2 \mathrm{Pr}_2^m + (1 - \nu_1 - \nu_2) \mathrm{Pr}_{1,2}^m, \tag{19}$$

where $\mathrm{Pr}_{1,2}^m$ is the joint probability of meeting targets T_1 and T_2, Pr_1^m and Pr_2^m are the probabilities of meeting targets T_1 and T_2, respectively. Extending this to N targets, the target-oriented Kansei evaluation function for the product O_m is as follows

$$\mathrm{Val}(O_m) = \sum_A \omega_A \cdot \mathrm{Pr}_{\{n|n \in A\}}^m, \text{ where } \sum_A \omega_A = 1. \tag{20}$$

With the mutual independent and additive preferences of Kansei targets, Eq. (20) reduces to

$$\mathrm{Val}(O_m) = \sum_{n=1}^{N} \mathrm{Pr}_n^m \cdot \omega_n, \tag{21}$$

where for all $n = 1, \ldots, N$, $\omega_n \geq 0$ is the weight of attribute X_n and $\sum_{n=1}^{N} \omega_n = 1$. The assumption of mutual independence among Kansei targets is however rarely verified.

5.2 An Analogy between MATO Kansei Evaluation function and Choquet Integral

The fuzzy measure and Choquet integral have been widely applied in multiattribute decision making problems. One natural question is that whether we can apply them directly in our MATO Kansei evaluation framework. In this section, we shall provide an axiomatic approach to interdependent multiattribute target-oriented (MATO) Kansei evaluation model based on fuzzy measure and Choquet integral. The fuzzy measure and Choquet integral are briefly introduced in the appendix part.

Proposition 1 *The utility function ν in MATO Kansei evaluation function is a fuzzy measure.*

Proof The utility function ν in MATO evaluation function Eq. (20) satisfies the axioms of the fuzzy measure: boundary, $\nu_\varnothing = 0$ and $\nu_{1,2,\ldots,N} = 1$; and monotonic, if $A_1 \subseteq A_2$, then $\nu_{A_1} \leq \nu_{A_2}$. Thus, consumer's utility function ν_A over A is a fuzzy measure. □

Proposition 2 *The weight ω_A in Eq. (20) acts as the interaction among Kansei targets.*

Proof Following Eq. (19), with two Kansei attributes we know that $\nu_n = \omega_n (n = 1, 2)$ and $\omega_{1,2} = 1 - \nu_1 - \nu_2$. With 3 attributes, Eq. (20) becomes

$$\begin{aligned}\mathrm{Val}(O_m) =& \nu_1 \mathrm{Pr}_1^m + \nu_2 \mathrm{Pr}_2^m + \nu_3 \mathrm{Pr}_3^m + \\ & (\nu_{1,2} - \nu_1 - \nu_2)\mathrm{Pr}_{1,2}^m + (\nu_{1,3} - \nu_1 - \nu_3)\mathrm{Pr}_{1,3}^m + (\nu_{2,3} - \nu_2 - \nu_3)\mathrm{Pr}_{2,3}^m + \\ & (1 - \nu_{1,2} - \nu_{1,3} - \nu_{2,3} + \nu_1 + \nu_2 + \nu_3)\mathrm{Pr}_{1,2,3}^m,\end{aligned}$$

which implies that $\omega_n = \nu_n (n = 1, 2, 3)$; $\omega_{n,l} = \nu_{n,l} - \nu_n - \nu_l (n \neq l, n, l = 1, 2, 3)$, and $\omega_{1,2,3} = 1 - \nu_{1,2} - \nu_{1,3} - \nu_{2,3} + \nu_1 + \nu_2 + \nu_3$. Recursively extending this to N attributes, we can have

$$\omega_A = \sum_{B \subseteq A} (-1)^{|A|-|B|} \cdot \nu_B, A \subseteq \mathcal{X}. \tag{22}$$

Since ν is a fuzzy measure and Eq. (22) is equivalent to Möbius transform Eq. (36) of ν, ω_A is the interaction index among Kansei targets. □

Proposition 3 *MATO Kansei evaluation function Eq. (20) is linear with respect to the consumer's utility function ν_A.*

Proof Following Propositions 1-2, a consumer's utility function ν can be expressed in a unique way as

$$\nu_A = \sum_{B \subseteq A} \omega_B, A \subseteq \mathcal{X},$$

which is equivalent to (Eq. 36). $\Pr(X(O_m) \succeq T) = \sum_A \omega_A \cdot \Pr_A^m$ is linear with respect to the weight information ω_A. Since conversion formulas between ν and ω are linear, Eq. (20) can also be expressed as

$$\mathrm{Val}(O_m) = \sum_{A \subseteq \mathcal{X}} \nu_A \cdot f_A^m,$$

where there exist 2^N functions f_A^m. Thus $\Pr(X(O_m) \succeq T)$ is linear with respect to consumer's utility function ν_A. □

Proposition 4 *Nonlinear MATO Kansei evaluation can be modeled by the Choquet integral.*

Proof MATO Kansei evaluation function can be expressed as

$$\begin{aligned} \Pr(X(O_m) \succeq T) &= \sum_{A \subseteq \mathcal{X}} \omega_A \cdot \Pr_A^m \\ &= \sum_{A \subseteq \mathcal{X}} \nu_A \cdot f_A^m, \end{aligned}$$

where there exist 2^N functions $\Pr_A^m$ and f_A^m. Such two equivalent functions can be modeled by the Choquet integral, see Marichal (2000). □

5.3 Entropy-Based Fuzzy Measure in Kansei Evaluation

Before being able to use the Choquet integral in our Kansei evaluation, it is clearly necessary to identify the weights of all subsets of Kansei attributes. However, unlike WAM, it is rather unrealistic to identify $2^N - 2$ coefficients of fuzzy measures, especially if the number of Kansei attributes is large. The need of the consumer's involvent in specifying the fuzzy measures on the subsets of Kansei attributes creates the burden of evaluation process. The products in Kansei evaluation are usually representative with a large number showing a statistical properties of the product domain. This is why many KE studies have applied statistical analysis. In this sense, it is possible to consider non-additive MATO Kansei evaluation from a probabilistic view. Fortunately, Kojadinovic (2008) has proposed an identification method by means of information-theoretic functionals to cope with exponentially increasing complexity of fuzzy measures, in which each attribute is viewed as a random variable taking a finite number of values. Such a fuzzy measure is referred to as *entropy-based fuzzy measure* and will be used in our framework. One basic assumption of the entropy-based fuzzy measure is that each random variable can only take a finite number of values. Should the attributes happen to have a continuous nature, they can be straightforwardly transformed into discrete random variables by discretizing the value domain. The discretization procedure is equivalent to considering that the associated discrete random variable can take only a finite set of values.

In order to apply the entropy-based fuzzy measure in our framework, each Kansei attribute will be viewed as a random variable. It then follows that the partial target-oriented utility $\Pr_n^m$ of product O_m on Kansei attribute X_n can be interpreted as a realization of the random variable X_n. Despite the linguistic interpretation of Kansei data, the partial utilities $\Pr_n^m$ of products $O_m (m = 1, \ldots, M)$ on attributes $X_n (n =$

$1,\ldots,N)$ have a continuous nature regarding our Kansei profiles. According to the G-point qualitative scale, we divide the domain of each attribute, into $G-1$ classes $\mathcal{D}$. Accordingly, we can transform the partial utilities $\mathrm{Pr}_n^m(m=1,\ldots,M; n=1,\ldots,N)$ into associated range levels r_n^m according to the division classes such that $r_n^m \in \mathcal{D}$. The division classes are expressed as

$$\mathcal{D}=\left\{d_1=\left[0,\frac{1}{G-1}\right),\ldots,d_{G-2}=\left[\frac{G-3}{G-1},\frac{G-2}{G-1}\right),d_{G-1}=\left[\frac{G-2}{G-1},1\right]\right\}.$$

Using the range level matrix r_n^m, we can then obtain a probability distribution on the finite number of values for a subset of Kansei attributes. For a single Kansei attribute, the probability distribution $p_{\{X_n\}}$ can be obtained as

$$p_{\{X_n\}}(d_{g_n})=\frac{|\{d_{g_n}\in\mathcal{D}: r_n^m=d_{g_n}\}|}{M}, \tag{23}$$

where $m=1,\ldots,M$, and $d_{g_n}(g_n=1,\ldots,G-1)$ denote the range levels in $\mathcal{D}$. In case of two attributes X_n and X_l, their probability distribution can be obtained as

$$p_{\{X_n,X_l\}}(d_{g_n},d_{g_l})=\frac{|\{d_{g_n},d_{g_l}\in\mathcal{D}: r_n^m=d_{g_n}, r_l^m=d_{g_l}\}|}{M}, \tag{24}$$

where $m=1,\ldots,M$, and $g_n,g_l\in\{1,2,\ldots,G-1\}$. Recursively extending this to a more general situation, we can obtain the probability distribution p_A of any subset $A=\{X_1,X_2,\ldots,X_l\}$ of Kansei attributes $\mathcal{X}$.

The fundamental concept of entropy of a probability distribution is initially proposed by Shannon (1948). It can be interpreted as measure of the uncertainty or the information or, equivalently, the structure contained in a probability distribution, and is defined as follows.

Definition 4 Let p be a discrete probability distribution on a set Θ. The Shannon entropy of p is defined by

$$H(p)=-\sum_{\theta\in\Theta}p(\theta)\ln\theta.$$

The quantity $H(p)$ can also be seen as measure of the uniformity of the discrete probability distribution p. The entropy of Kansei attribute X_n with the probability distribution $p_{\{X_n\}}$ is given by $H(p_{\{X_n\}})$. More generally, the entropy of a subset of Kansei attributes $A=(X_1,\ldots,X_l)$ with the probability distribution p_A is given by

$$H(\{X_1,X_2,\ldots,X_l\})=H\left(p_A\right),l=1,\ldots,N. \tag{25}$$

From the properties of the Shannon entropy (Shannon, 1948), it is easy to check that H as a set of function on $\mathcal{X}$ is always non-negative and monotonic. Also, Kojadinovic (2008) has proved an analogy between mutual information and the Möbius representation of fuzzy measure. Thus H is a fuzzy measure on $\mathcal{X}$ that simply does not satisfy the boundary condition $H(\mathcal{X})=1$. It is natural to define the weights of the subsets of attributes of $\mathcal{X}$ using

$$\widehat{\nu}_A=\frac{H(A)}{H(\mathcal{X})}. \tag{26}$$

The Choquet integral of $\mathrm{Pr}^m = [\mathrm{Pr}_1^m, \ldots, \mathrm{Pr}_n^m, \ldots, \mathrm{Pr}_N^m]$ with respect to the entropy based fuzzy measure $\widehat{\nu}$ is defined by

$$\mathrm{Val}(O_m) = \frac{\sum_{n=1}^{N} \mathrm{Pr}_{(n)}^m \left[H(A_{(n)}) - H(A_{(n+1)})\right]}{H(\mathcal{X})}, \tag{27}$$

where $(\cdot)$ indicates a permutation of the partial utilities regarding product O_m such that $\mathrm{Pr}_{(1)}^m \leq \cdots \leq \mathrm{Pr}_{(n)}^m \leq \cdots \leq \mathrm{Pr}_{(N)}^m$. Also $A_{(n)} = \{X_{(n)}, \ldots, X_{(N)}\}$, for all $n \in \{1, \ldots, N\}$, $A_{(N+1)} = \varnothing$, and $H(A_{(n)}) = H(p_{A_{(n)}})$.

6 Kansei Evaluation for Hand-Painted Kutani Cups

In this section, we use the hand-painted Kutani cups in Japan to illustrate how our model works in practice as well as its effectiveness.

6.1 An Experimental Study

In Japan, there are a large number of traditional craft items which are closely related to Japanese traditional culture. These items can be traced back through the ages, each is unique fostered through regional differences and loving dedication and provides a continual wealth of pleasure. In this study, a particular emphasis is laid on the hand painted Kutani cups in Ishikawa Prefecture, Japan. The Kutani cup is a traditional craft item with a long history of over 400 years. Hand-painted Kutani cups are functionally equivalent and have high prices ranging from \$10 to \$100, as searched from the website [1]. Due to the equivalent physical quality as well as high price, the aesthetic aspects play a crucial role in human purchase choice of these items, especially in the era of e-commerce. Kansei evaluation of the hand-painted Kutani cups would be of great help for purchase and recommendation purposes. Within the framework of our research project, a total of 35 representative products of hand painted Kutani cups were first selected for Kansei evaluation, as shown in Fig. 3.

Second, 26 refined bipolar pairs of Kansei words were selected through a brainstorming process by consulting with local manufacturers and selling shops. The 26 refined bipolar pairs of Kansei words were first used in Japanese, and then approximately translated into English, as shown in Table 3.

A 7-point scale was used to put a value for each Kutani cup with respect to 26 Kansei attributes such that $\mathcal{V} = \{\mathrm{V}_1 = 1, \mathrm{V}_2 = 2, \mathrm{V}_3 = 3, \mathrm{V}_4 = 4, \mathrm{V}_5 = 5, \mathrm{V}_6 = 6, \mathrm{V}_7 = 7\}$. Finally, a total of 60 people, including relevant researchers of KE, senior residents in Ishikawa, and certified masters of traditional crafts, were chosen as subjects. The 60 subjects were asked to provide their Kansei assessments for the 35 Kutani cups on the 26 Kansei attributes simultaneously.

6.2 Kansei Evaluation

Assume a consumer prefers four Kansei attributes $\{X_1, X_9, X_{11}, X_{17}\}$ and specifies his Kansei requests as $\{\mathrm{kw}_1^-, \mathrm{kw}_9^+, \mathrm{kw}_{11}^\sim, \mathrm{kw}_{17}^-\}$. Verbally, the consumer would like

[1] http://search.borderless.rakuten.com

Fig. 3 The 35 hand-painted Kutani cups to be evaluated

Table 3 26 Kansei attributes with bipolar pairs of Kansei words

X_n	Bipolar Kansei words $< \mathrm{kw}_n^-, \mathrm{kw}_n^+ >$	X_n	Bipolar Kansei words $< \mathrm{kw}_n^-, \mathrm{kw}_n^+ >$
X_1	<conventional,unconventional>	X_{14}	<delicate,large-hearted>
X_2	<simple,compound>	X_{15}	<luxurious,frugal>
X_3	<solemn,funny>	X_{16}	<gentle,pithy>
X_4	<formal,casual>	X_{17}	<bright,dark>
X_5	<serene,forceful>	X_{18}	<reserved,imperious>
X_6	<still,moving>	X_{19}	<free,regular>
X_7	<pretty,austere>	X_{20}	<level,indented>
X_8	<friendly,unfriendly>	X_{21}	<lustrous,matte>
X_9	<soft,hard>	X_{22}	<transpicuous,dim>
X_{10}	<blase,attractive>	X_{23}	<warm,cool>
X_{11}	<flowery,quiet>	X_{24}	<moist,arid>
X_{12}	<happy,normal>	X_{25}	<colorful,sober>
X_{13}	<elegant,loose>	X_{26}	<plain,gaudy-loud>

Table 4 Preferred 4 Kansei attributes with 4 Kansei requests

Attribute	Bipolar Kansei words	Kansei preference
X_1	<conventional,unconventional>	*conventional*
X_9	<soft,hard>	*hard*
X_{11}	<flowery,quiet>	*neutral*
X_{17}	<bright,dark>	*bright*

to ask for a Kutani cup meeting his Kansei preferences of *conventional*, *hard*, *neither flowery nor quiet*, and *bright*, as shown in Table 4.

As discussed in Sec. 3, a linguistic variable with a set of 7 Kansei linguistic labels can be defined implicity for each Kansei attribute, denoted as $\mathcal{L}^n$={L_1^n, L_2^n, L_3^n, L_4^n, L_5^n, L_6^n, L_7^n}. Assuming all the 60 subjects are equivalently important, i.e., $w_k = 1/60(k = 1, \ldots, 60)$, we can derive a weighting vector $\varpi_n^m(L_{\mathbf{p}}^n)(\mathbf{p} = 1, \ldots, 7)$ of 7 Kansei labels for product O_m on Kansei attribute X_n based on the Kansei assessments provided by the 60 subjects such that

$$\varpi_n^m(L_{\mathbf{p}}^n) = \frac{|\{E_k \in \mathcal{E} : x_n^m(E_k) = L_{\mathbf{p}}^n\}|}{60}, \mathbf{p} = 1, \ldots, 7,$$

where $n = 1, 9, 11, 17$. The weights $\varpi_n^m(L_{\mathbf{p}}^n)$ are then used to generate a Kansei profile for product O_m on Kansei attribute X_n. To do so, we first obtain the bounded domain of product O_m on attribute X_n via Eq. (4). We then build the possibility distributions of "around prototype Kansei labels" on the set of 7 Kansei labels via Eq. (5). Finally, we can obtain the Kansei profile via Eqs. (5)-(8), which results with a probability distribution $p_n^m(L_g^n), g = 1, \ldots, 7$. Such a probability distribution reflects the uncertainty of Kansei data and acts as a decision making under uncertainty problem.

According to Table 4, the consumer has a *left Kansei preference* on Kansei attributes X_1 and X_{17}, a *neutral Kansei preference* on Kansei attribute X_{11}, and a *right Kansei preference* on Kansei attribute X_9, respectively. We then determine preference orders on $\mathcal{L}^n$ for X_1, X_9, X_{11}, and X_{17} via Eq. (13). Particularly, the preference order relations are expressed as follows:

$$\succeq^n \Leftrightarrow \begin{cases} L_1^n \succeq L_2^n \succeq L_3^n \succeq L_4^n \succeq L_5^n \succeq L_6^n \succeq L_7^n, \text{ if } n = 1, 17; \\ L_1^n \preceq L_2^n \preceq L_3^n \preceq L_4^n \preceq L_5^n \preceq L_6^n \preceq L_7^n, \text{ if } n = 9; \\ L_1^n \preceq L_2^n \preceq L_3^n \preceq L_4^n \succeq L_5^n \succeq L_6^n \succeq L_7^n, \text{ if } n = 11. \end{cases}$$

With the Kansei preference order relations on $\mathcal{L}^n$, we are now able to define Kansei targets T_1, T_9, T_{11}, and T_{17} for attributes X_1, X_9, X_{11}, and X_{17} via Eq. (14), respectively. Each Kansei target derives a possibility distribution on Kansei labels, as shown in Table 5 (indexed by π). Furthermore, we can derive a probability distribution on 7 Kansei labels for each Kansei target via Eq. (16), as shown in Table 5 (indexed by p). Based on the preference orders on $\mathcal{L}^n$ for X_1, X_9, X_{11}, and X_{17} and the probability distribution on 7 Kansei labels for each Kansei target, we can induce the target-oriented utility of each Kansei label with respect to three types of Kansei targets via Eqs. (15)-(18), as shown in Table 5 (indexed by **Pr**). Using the Kansei profiles $X_n(O_m)$ of 35 Kutani cups on the 4 attributes and the derived utilities of the 7 Kansei labels, we obtain the target-oriented utility $\Pr_n^m$ for each Kutani cup on each of the four attributes via Eq. (15). The rows (indexed by "Target-oriented utility") in Table 6 show the target-oriented utility of product O_m on attribute X_n.

We now consider the nonadditive aggregation. According to the entropy-based fuzzy measure introduced in Sec. 5.3, the target-oriented utilities of the 35 hand-painted Kutani cups on each of the selected four Kansei attributes can be divided into 6 range levels such that $\{[0, 1/6), [1/6, 2/6), [2/6, 3/6), [3/6, 4/6), [4/6, 5/6), [5/6, 1]\}$. A range level matrix can then be derived for all cups on the four Kansei attributes, as shown in Table 6 (indexed by "Range levels"). Using the range level matrix, we can obtain the probability distributions of the subsets of Kansei attributes via Eqs. (23)-(24) and their extensions. The associated probability distributions of the subsets of Kansei attributes are then used to derive the fuzzy measures via Eq. (26). The derived fuzzy measures on the subsets of Kansei attributes are shown in Table 7. Based on the partial utilities and derived fuzzy measures, we can obtain the results of nonlinear aggregation for cups

Table 5 Possibility, probability, and target-oriented utility of each Kansei label regarding different Kansei targets

Request	Value	L_1^n	L_2^n	L_3^n	L_4^n	L_5^n	L_6^n	L_7^n
kw_n^-	π	1	5/6	4/6	3/6	2/6	1/6	0
	p	49/120	29/120	19/120	37/360	11/180	1/36	0
	Pr	1	71/120	7/20	23/120	4/45	1/36	0
kw_n^+	π	0	1/6	2/6	3/6	4/6	5/6	1
	p	0	10/360	22/360	37/360	19/120	29/120	49/120
	Pr	0	1/36	4/45	23/120	7/20	71/120	1
$\mathrm{kw}_n^\sim$	π	0	1/3	2/3	1	2/3	1/3	0
	p	0	3/45	8/45	23/45	8/45	3/45	0
	Pr	0	3/34	11/34	1	11/34	3/34	0

Table 6 Induced utilities and range levels of all the 35 Kutani cups

No.	Target-oriented utility				Range level				No.	Target-oriented utility				Range level			
	X_1	X_9	X_{11}	X_{17}	X_1	X_9	X_{11}	X_{17}		X_1	X_9	X_{11}	X_{17}	X_1	X_9	X_{11}	X_{17}
O_1	0.249	0.405	0.317	0.329	2	3	2	2	O_{19}	0.115	0.281	0.424	0.180	1	2	3	2
O_2	0.121	0.281	0.378	0.245	1	2	3	2	O_{20}	0.135	0.393	0.358	0.143	1	3	3	1
O_3	0.273	0.238	0.381	0.362	2	2	3	3	O_{21}	0.236	0.223	0.397	0.288	2	2	3	2
O_4	0.133	0.122	0.343	0.436	1	1	3	3	O_{22}	0.208	0.228	0.377	0.319	2	2	3	2
O_5	0.318	0.243	0.290	0.133	2	2	2	1	O_{23}	0.335	0.325	0.298	0.122	3	2	2	1
O_6	0.226	0.149	0.343	0.468	2	1	3	3	O_{24}	0.199	0.132	0.337	0.510	2	1	3	4
O_7	0.306	0.101	0.409	0.461	2	1	3	3	O_{25}	0.203	0.213	0.389	0.331	2	2	3	2
O_8	0.269	0.075	0.388	0.406	2	1	3	3	O_{26}	0.165	0.226	0.409	0.369	1	2	3	3
O_9	0.262	0.113	0.389	0.401	2	1	3	3	O_{27}	0.228	0.200	0.339	0.249	2	2	3	2
O_{10}	0.244	0.070	0.394	0.470	2	1	3	3	O_{28}	0.201	0.239	0.383	0.225	2	2	3	2
O_{11}	0.244	0.106	0.403	0.466	2	1	3	3	O_{29}	0.189	0.113	0.438	0.466	2	1	3	3
O_{12}	0.272	0.381	0.398	0.210	2	3	3	2	O_{30}	0.176	0.288	0.387	0.266	2	2	3	2
O_{13}	0.202	0.063	0.439	0.520	2	1	3	4	O_{31}	0.144	0.142	0.371	0.448	1	1	3	3
O_{14}	0.091	0.089	0.435	0.393	1	1	3	3	O_{32}	0.162	0.268	0.389	0.278	1	2	3	2
O_{15}	0.291	0.154	0.402	0.374	2	1	3	3	O_{33}	0.175	0.196	0.316	0.475	2	2	2	3
O_{16}	0.194	0.150	0.378	0.300	2	1	3	2	O_{34}	0.205	0.294	0.361	0.139	2	2	3	1
O_{17}	0.314	0.278	0.373	0.296	2	2	3	2	O_{35}	0.158	0.150	0.36	0.35	1	1	3	3
O_{18}	0.134	0.377	0.258	0.341	1	3	2	3									

Table 7 Entropy based capacities

No.	X_1	X_9	X_{11}	X_{17}	Capacity	No.	X_1	X_9	X_{11}	X_{17}	Capacity
1	0	0	0	0	0.0	9	0	0	0	1	0.474
2	1	0	0	0	0.299	10	1	0	0	1	0.733
3	0	1	0	0	0.404	11	0	1	0	1	0.741
4	1	1	0	0	0.689	12	1	1	0	1	0.950
5	0	0	1	0	0.171	13	0	0	1	1	0.621
6	1	0	1	0	0.446	14	1	0	1	1	0.860
7	0	1	1	0	0.529	15	0	1	1	1	0.803
8	1	1	1	0	0.784	16	1	1	1	1	1.0

$O_m (m = 1, \ldots, 35)$ via Eq. (27). The top 3 hand-painted Kutani cups are shown in Fig. 4 such that $O_{13} \succ O_7 \succ O_{24}$.

O_{13} $\succ$ O_7 $\succ$ O_{24}

Fig. 4 Top 3 recommended Kutani cups

Table 8 Population distributions of the top 3 Kutani cups on the four Kansei attributes

$\mathcal{V}$	O_{13}				O_7				O_{24}			
	X_1	X_9	X_{11}	X_{17}	X_1	X_9	X_{11}	X_{17}	X_1	X_9	X_{11}	X_{17}
V_1	3	25	1	16	9	9	2	9	3	14	14	15
V_2	5	17	10	30	15	32	6	24	7	26	16	28
V_3	10	16	26	13	10	16	20	25	9	19	16	16
V_4	1	1	5	0	3	2	4	0	1	0	2	0
V_5	14	1	15	1	11	0	16	2	6	0	4	1
V_6	17	0	3	0	9	1	11	0	18	0	6	0
V_7	10	0	0	0	3	0	1	0	16	1	2	0

6.3 Analysis of the Results Obtained

For the sake of facilitating the analysis of the results obtained, the original Kansei assessments of the top 3 Kutani cups on the selected 4 Kansei attributes X_1, X_9, X_{11}, and X_{17}, are shown in Table 8, in which the number stands for the population number of subjects who assess a Kutani cup on an attribute using the same scale value.

6.3.1 Kansei profile generation

Due to subjective assessment, our approach treats $\mathcal{V}$ as a linguistic variable with a set of 7 linguistic labels for each Kansei attribute. Taking Kutani cup O_{13} on Kansei attribute X_{17} as an example, the set of linguistic labels are $\{L_1^{17}, L_2^{17}, L_3^{17}, L_4^{17}, L_5^{17}, L_6^{17}, L_7^{17}\}$. From Table 8, it is obvious that the population distribution of subjects' assessments on $\mathcal{V}$ is $[16, 30, 13, 0, 1, 0, 0]$, which indicates that no subject assesses O_{13} on X_{17} using L_4^{17}. However, due to the interpretation of semantic overlapping introduced in Sec. 3, it is undeniable that if one subject assesses O_{13} on X_{17} using a prototype label L_g^{17}, other labels is also appropriate to describe O_{13} on X_{17}. Also, we know that $L_{\min}(x_{17}^{13}) = L_1^{17}$ and $L_{\max}(x_{17}^{13}) = L_5^{17}$ via Eq. (4). Then we have 5 possible prototype Kansei linguistic labels and can build the possibility distributions of "around prototype Kansei labels" via Eq. (5), as shown in Table 9 (indexed by "Possibility distribution"). We can then derive the least prejudiced distribution of "around prototype Kansei labels" via Eqs. (6)-(7), as shown in Table 9 (indexed by "Probability distribution").

In addition, the population distribution of subjects' assessments defines a weighting vector for prototype Kansei labels such that

$$\varpi_{17}^{13}(L_{\mathbf{p}}^{17}) = \left[0.267/L_1^{17}, 0.5/L_2^{17}, 0.217/L_3^{17}, 0.0/L_4^{17}, 0.017/L_5^{17}, 0/L_6^{17}, 0/L_7^{17}\right].$$

Table 9 Possibility and probability distributions of "around prototype label $L_{\mathbf{p}}$" for O_{13} on X_{17}

$L_{\mathbf{p}}^{17}$	Possibility distribution							Probability distribution						
	L_1^{17}	L_2^{17}	L_3^{17}	L_4^{17}	L_5^{17}	L_6^{17}	L_7^{17}	L_1^{17}	L_2^{17}	L_3^{17}	L_4^{17}	L_5^{17}	L_6^{17}	L_7^{17}
L_1^{17}	1	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	0	0	0	$\frac{25}{48}$	$\frac{13}{48}$	$\frac{7}{48}$	$\frac{3}{48}$	0	0	0
L_2^{17}	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	$\frac{11}{18}$	$\frac{5}{18}$	$\frac{2}{18}$	0	0	0
L_3^{17}	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	0	0	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0	0	0
L_4^{17}	0	$\frac{1}{3}$	$\frac{2}{3}$	1	0	0	0	0	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{11}{18}$	0	0	0
L_5^{17}	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	0	0	0	$\frac{3}{48}$	$\frac{7}{48}$	$\frac{13}{48}$	$\frac{25}{48}$	0	0

With the probability distributions in Table 9 and the weighting vector $\varpi_{17}^{13}(L_{\mathbf{p}}^{17})$, we are able to induce a collective probability distribution for the products O_{13} regarding X_{17} on the set of 7 Kansei labels. Particularly, the collective probability of O_{13} on Kansei label L_2^{17} is calculated as

$$\begin{aligned} p_{17}^{13}(L_2^{17}) &= 0.267 \cdot \frac{13}{48} + 0.5 \cdot \frac{11}{18} + 0.217 \cdot \frac{1}{6} + 0.017 \cdot \frac{1}{16} \\ &= 0.415. \end{aligned}$$

The collective probabilities on other Kansei labels can be obtained as

$$p_{17}^{13}(L_{\mathbf{p}}^{17}) = \left[0.139/L_1^{17}, 0.415/L_2^{17}, 0.325/L_3^{17}, 0.113/L_4^{17}, 0.009/L_5^{17}, 0/L_6^{17}, 0/L_7^{17}\right],$$

which means our Kansei profile generation approach results with a probability distribution around L_2^{17} having the maximal probability 0.415. Also, Kansei label L_4^{17} has a probability 0.113 reflecting the semantic overlapping of Kansei data. This probability distribution represents the uncertainty of subjects' assessments. Fig. 5 shows the Kansei profiles of the top 3 Kutani cups on the 4 Kansei attributes, where each Kansei profile is a probability distribution around a Kansei linguistic label.

6.3.2 Single Attribute Kansei Evaluation Function

With a consumer's Kansei preference kw_n^* on X_n, our target-oriented Kansei evaluation is expressed as

$$\text{Pr}_n^m = \sum_{g=1}^{7} \Pr(L_g^n \succeq T_n) \cdot p_n^m(L_g^n),$$

where T_n is the Kansei target corresponding to kw_n^*. It is clearly that Pr_n^m depends on two factors: an uncertain Kansei profile $X_n(O_m)=[p_n^m(L_1^n), \ldots, p_n^m(L_g^n), \ldots, p_n^m(L_7^n)]$ and target-oriented utility $\Pr(L_g^n \succeq T_n)$ of Kansei label L_g^n meeting a consumer's Kansei target T_n. Such an evaluation function acts as a decision making under uncertainty problem.

Fig. 6 shows the target-oriented utility $\Pr(L_g^n \succeq T_n)$ of each Kansei label with respect to three types of Kansei preferences. The derived target-oriented utility function $\Pr(L_g^n \succeq T_n)$ is convex-shaped, which could be justified by the psychological finding of target-oriented decision model (Bordley and LiCalzi, 2000) and fuzzy target-oriented decision model (Yan et al., 2009). To explain this point in more detail, consider the case of left Kansei preference. With a left Kansei preference kw_n^-, we assume that the

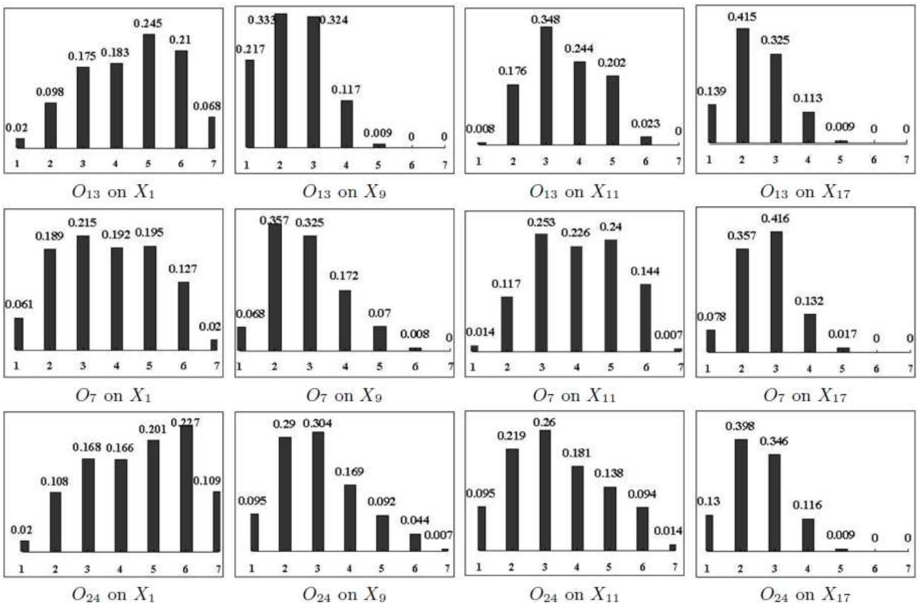

Fig. 5 Kansei profiles of top 3 recommended Kutani cups

consumer assesses higher possibility about his target toward the L_1^n, which corresponds to the fact that the consumer believes that 'best thing may happen', L_1^n is viewed as the reference label and other labels $L_g^n (g > 1)$ will be viewed as a loss. The convex-shaped function is consistent with the psychological finding that people tend to be risk seeking over looses (convex over losses) (Kahneman and Tversky, 1979). In this regard, target-oriented utility acts as a psychological distance. The same reasoning can be applied to the right and neutral Kansei preferences.

We know that the consumer has a *left Kansei preference* on Kansei attributes X_1 and X_{17}. Seen from the Kansei profiles of cup O_{13} on X_1 and X_{17} in Fig. 5, it is clear that $X_1(O_{13})$ is distributed around the 5th Kansei label L_5^1 on $[L_1^1, L_7^1]$, whereas $X_{17}(O_{13})$ is mainly distributed around the second Kansei label L_2^{17} on $[L_1^{17}, L_4^{17}]$. Also, $\Pr(L_g^{17} \succeq T_{17})$ and $\Pr(L_g^1 \succeq T_1)$ are monotonically decreasing with g, thus we can conclude that $\Pr_1^{13} < \Pr_{17}^{13}$. From Table 6, we know $\Pr_1^{13} = 0.202 < \Pr_{17}^{13} = 0.52$. As discussed previously, the Kansei profile generated by our approach can capture the fuzzy uncertainty as well as semantic overlapping of Kansei data.

6.3.3 Nonadditive multiattribute Kansei Evaluation Function

The estimated fuzzy measures of the subsets of Kansei attributes by means of the entropy-based method are given in Table 7. Note that, because of the way $\nu_{\{A\}}$ was defined, the weight of a nonempty subset of Kansei attributes directly depends on the uniformity of the distribution of the target-oriented utilities for these attributes. To explain this point in more detail, consider the case of a subset reduced to a single Kansei attribute. If most of the 35 Kutani cups have a similar utility for the considered attribute, the weight of the attribute will be low, which could be justified by the fact it does not clearly discriminate between 'good' and 'bad' Kutani cups. On the contrary,

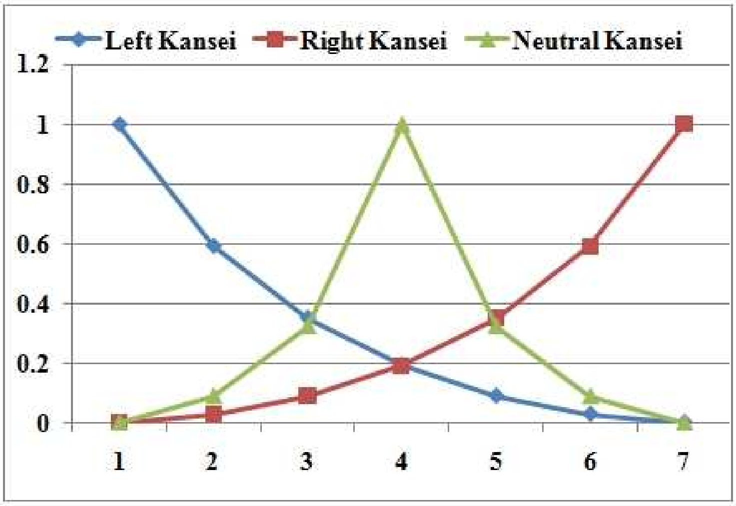

Fig. 6 Target-oriented utility of each label w.r.t. three types of Kansei preferences

the more the utilities are uniformly distributed, the higher the weight of the attribute. For example, from Table 7, we know $\nu_{\{X_{17}\}} = 0.474 > \nu_{\{X_9\}} = 0.404 > \nu_{\{X_1\}} = 0.299 > \nu_{\{X_{11}\}} = 0.171$. Seen from Table 6, almost all of the cups have the same range level 3 regarding attribute X_{11}, so the weight $\nu_{\{X_{11}\}}$ is low. The same reasoning can be applied to subsets containing more one Kansei attributes.

In addition, the sum of fuzzy measures of each attribute is (much) higher than one such that

$$\nu_{\{X_1\}} + \nu_{\{X_9\}} + \nu_{\{X_{11}\}} + \nu_{\{X_{17}\}} = 1.348 > 1,$$

which indicates the substitutivity (also called redundancy) among attributes. The same observations are suitable to the subsets of Kansei attributes. Such phenomena could be explained by the fact that in our Kansei evaluation framework, an unsupervised approach is used to estimate fuzzy measures of the subsets of Kansei attributes. As observed in Kojadinovic (2008), indeed, in an unsupervised setting, two attributes should be able to interact only in a redundant way since, to detect multiplicative effects between two Kansei attributes, initial preferences on weights would be necessary.

Now that the weights of the nonempty subsets of Kansei attributes are estimated, the global evaluations of the cups can be computed by means of the Choquet integral w.r.t. $\nu_{\{A\}}$. These global evaluations are given in Fig. 7 (indexed by Choquet integral). The global evaluations by means of the WAM will also be used to in order to compare with the Choquet integral. Similar to Mon et al. (1994), the entropy-based weight for each of the four Kansei attributes can be defined as $w_n = \frac{H(X_n)}{\sum_n H(X_n)}$, and then the global evaluations by means of the WAM are

$$\mathrm{Val}(O_m) = \mathrm{Pr}_n^m \cdot w_n = \mathrm{Pr}_n^m \cdot \frac{H(X_n)}{\sum_n H(X_n)}, \tag{28}$$

and are shown in Fig. 7.

By considering Fig. 7, one can notice that the global evaluations computed by the Choquet integral are always superior to that by the WAM. This disjunctive behavior of

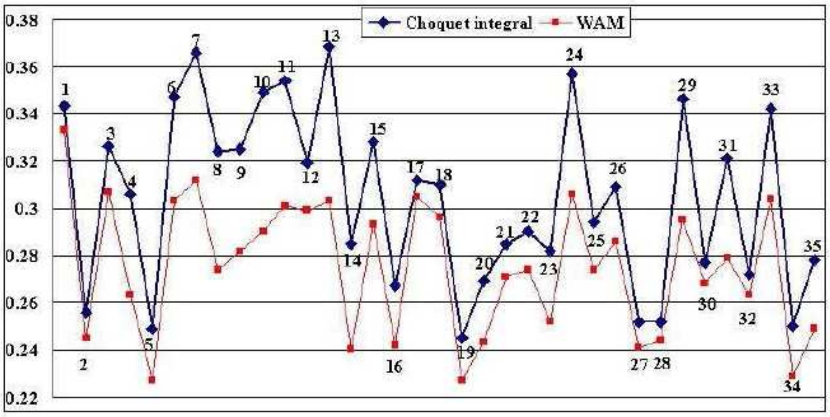

Fig. 7 Global utility computed by Choquet integral and WAM

Table 10 Partial utility scores of the cup designated best by WAM and Choquet integral

Aggregation type	Partial utility			
	X_1	X_9	X_{11}	X_{17}
WAM (O_1)	0.249	0.405	0.317	0.329
Choquet integral (O_{13})	0.202	0.063	0.439	0.52

the Choquet integral is due to the strong redundancy among Kansei attributes modeled by $\nu_{\{A\}}$. To study the effects of the redundant interaction phenomena among Kansei attributes modeled by $\nu_{\{A\}}$, we compare the partial utilities of the cup designated best by the WAM to that designated best by the Choquet integral w.r.t. $\nu_{\{A\}}$. By observing Fig. 7, it appears that, the best cup designated by WAM is O_1 and the best cup designated by Choquet integral is O_{13}. Their partial utilities are given in Table 10. By observing Table 10, one can see that cup O_1 has good results on average, but that the utilities of cup O_{13} are globally superior, except in X_9 where its utility 0.063 is extremely low. The fact that O_{13} is designated better than O_1 by the Choquet integral can be explained by O_{13}'s high score in X_{17} and the disjunctive behavior of the Choquet integral due, among other attributes, to the negative interaction between X_9 and X_{17}. In fact, by means of the Pearson correlation computation method, the correlation between X_9 and X_{17} is -0.748. In other terms, a high score in X_9 or in X_{17} is sufficient to significantly influence the global evaluation. In summary, we could say that, globally, the WAM tends to underestimate the Kutani cups since it does not take into account the redundancy effects among Kansei attributes.

7 Comparative Analysis with Related Work

In this section, we shall compare our model with related work in the literature. Two types of models, namely numerical model and linguistic model, are briefly introduced.

7.1 Two Models in KE

7.1.1 Numerical model

Almost all KE studies treat the G-point odd qualitative scale $\mathcal{V}$ as numerical data, i.e. the Kansei assessment provided by subject $E_k \in \mathcal{E}$ for product $O_m \in \mathcal{O}$ on Kansei attribute $X_n \in \mathcal{X}$, denoted as $x_n^m(E_k)$, is a numerical data, where for all $x_n^m(E_k) \in \mathcal{V}, m = 1, \ldots, M$; $n = 1, \ldots, N$; and $k = 1, \ldots, K$. We refer to this type of approaches as numerical model. The numerical model consists of three phases as follows.

- **Phase I: Mean scale based Kansei profile**
  Prior KE studies use the mean scale of the questionnaire-collected data as Kansei profile (Chen and Chuang, 2008; Grimsæth, 2005; Petiot and Yannou, 2004, e.g.) such that
$$\overline{x_n^m} = \sum_{g=1}^{G} \left(\frac{|\{E_k \in \mathcal{E} : x_n^m(E_k) = V_g\}|}{K} \times V_g \right), \tag{29}$$
  where $x_n^m(E_k) \in \mathcal{V}$, $E_k \in \mathcal{E}$, $K = |\mathcal{E}|$, and $g = 1, \ldots, G$.
- **Phase II: Consumer satisfaction utility**
  Prior KE studies define three linear utility functions to model the three types of Kansei preferences such that (Petiot and Yannou, 2004, e.g.)
$$u_n^m = \begin{cases} \frac{\mathrm{V}_G - \overline{x_n^m}}{\mathrm{V}_G - 1}, & \text{left Kansei;} \\ 1 - \frac{|\overline{x_n^m} - x_n^{\mathrm{ob}}|}{\max\{\mathrm{V}_G - x_n^{\mathrm{ob}}, x_n^{\mathrm{ob}} - \mathrm{V}_1\}}, & \text{neutral Kansei;} \\ \frac{\overline{x_n^m} - \mathrm{V}_1}{\mathrm{V}_G - \mathrm{V}_1}, & \text{right Kansei.} \end{cases} \tag{30}$$
  where x_n^{ob} is the target value toward Kansei attribute X_n and $\mathrm{V}_1 \leq x_n^{\mathrm{ob}} \leq \mathrm{V}_G$. In our problem, x_n^{ob} is set to $\mathrm{V}_{(G+1)/2}$.
- **Phase III: Global satisfaction utility**
  Using the WAM method (Petiot and Yannou, 2004; Chen and Chuang, 2008) to obtain global satisfaction utility for each product O_m such that
$$U(O_m) = \sum_{n=1}^{N} w_n \cdot u_n^m, \tag{31}$$
  where w_n is the weight of Kansei attribute X_n. The weighting vector can be obtained by various ways such as AHP, OWA, and the entropy-based method.

7.1.2 Linguistic Model

In a different, but similar context, Martínez (2007) has proposed a sensory evaluation based on the linguistic 2-tuple representation model (Herrera and Martínez, 2000). The 2-tuple Linguistic representation model are briefly introduced in the appendix 2. The sensory evaluation model considers the evaluation problem as a multiexpert/multiattribute decision making problem, and assumes a consistent order relation over the qualitative evaluation scale treated as a linguistic variable with a set of linguistic labels. In addition, the sensory evaluation model assumes all attributes are represented by only one linguistic variable $\mathcal{L} = \{L_1, \ldots, L_G\}$. With the sensory assessment provided by subject $E_k \in \mathcal{E}$ for product $O_m \in \mathcal{O}$ on attribute $X_n \in \mathcal{X}$, denoted as $x_n^m(E_k)$, where for all $x_n^m(E_k) \in \mathcal{L}, m = 1, \ldots, M$; $n = 1, \ldots, N$; and $k = 1, \ldots, K$. The sensory evaluation model consists of two steps as follows.

– **Phase I: Computing collective evaluations for each attribute**

$$V_n^m = \Delta \left(\sum_{g=1}^{G} \left(\frac{|\{E_k \in \mathcal{E} : x_n^m(E_k) = L_g\}| \times \Delta^{-}\left(x_n^m(E_k), 0\right)}{K} \right) \right) \tag{32}$$

where $x_n^m(E_k) \in \mathcal{L}$, $E_k \in \mathcal{E}$, and $K = |\mathcal{E}|$.

– **Phase II: Computing collective evaluations for each product**
The collective performance of O_m is calculated by aggregating all of them such that

$$V(O_m) = \mathcal{F}(V_1^m, \ldots, V_n^m, \ldots, V_N^m), \tag{33}$$

where $\mathcal{F}$ is an aggregation function with respect to the 2-tuple linguistic model. For more detail about the aggregation of 2-tuples linguistic mode, see appendix 2.

7.2 Comparative Analysis

7.2.1 Kansei profile generation

First, let us consider the Kansei profile generation. The numerical model uses Eq. (29) to derive a Kansei profile for each product on each Kansei attribute. The 2-tuple linguistic model uses Eq. (32) to derive a Kansei profile for each product on each Kansei attribute. It is obvious that the Kansei profile derived by either the numerical model or linguistic model is a numerical number or 2-tuple Kansei linguistic representation. Taking Kutani cup O_{13} on Kansei attribute X_{17} as an example. The Kansei profiles generated by the two model are 2.0 and $(L_2^{17}, 0)$, respectively. Whereas, our approach results with a probability distribution around a specific Kansei label, as shown in Fig. 5. Such a probability distribution reflects the semantic overlapping of Kansei data. In summary, the numerical model cannot model the fuzzy uncertainty as well as semantic overlapping of Kansei data. The 2-tuple model cannot capture the semantic overlapping of Kansei data.

7.2.2 Consumer satisfaction utility

Second, let us consider the consumer satisfaction utility. Numerical model uses Eq. (30) to get partial utility. The main question in Eq. (30) is that the relationship between Kansei attribute and consumer is usually nonlinear (Chen and Chuang, 2008). Thus, it is too arbitrary to specify a linear utility function. Also, it is difficult to build a rigorous utility function based on attribute (Bordley and Kirkwood, 2004) and traditional utility often does not provide a good description of the consumer. Since the 2-tuple linguistic model assumes a consistent preference order relation such that $\succeq^n = L_1^n \preceq \ldots L_{(G+1)/2}^n \preceq L_G^n$, it directly uses the Kansei profile V_n^m in Eq. (32) as the utility value. However, in our research context, a consumer may have three types of preference order relations, namely left Kansei preference, neutral Kansei preference, and right Kansei preference. Thus, the 2-tuple linguistic model is not suitable to our research.

Our Kansei profile generated perform well than those generated by the numerical model and 2-tuple linguistic model, thus we now analyze the satisfaction utility function based on our Kansei profile generated. Our satisfaction utility is expressed as

$$\mathrm{Pr}_n^m = \sum_{g=1}^{G} p_n^m(L_g^n) \cdot \mathrm{Pr}(L_g^n \succeq T_n),$$

where T_n a Kansei target corresponding to the consumer-specified Kansei request kw_n^* toward Kansei attribute X_n. The traditional utility theory used in the numerical model can also be applied to our Kansei profile generated such that

$$U_n^m = \sum_{g=1}^{G} p_n^m(L_g^n) \cdot U(L_g^n), \tag{34}$$

where $U(L_g^n)$ is a consumer's utility toward Kansei label L_g^n. Similar to the linear satisfaction utility function in Eq. (30), it is easy to specify three discrete linear utility functions $U(L_g^n)$. Moreover, if we assume the possibility distribution target T_n is expressed as $\pi_{T_n}(L_g^n) = 1$, according to the single attribute Kansei evaluation function discussed in Sec. 4, we will also have three linear satisfaction utility functions. In this sense, the linear utility function reflects a neutral attitude of the consumer. As discussed in Sec. 6, our target-oriented Kansei evaluation function provide a support for psychological preference.

7.2.3 Multiattribute Kansei evaluation function

Third, let us consider the multiattribute evaluation function. Either the numerical model or the 2-tuple linguistic model uses a WAM method to obtain a global evaluation result. However, they cannot model the interaction among multiple Kansei attributes. As we have mentioned in the analysis of the results obtained in Sec. 6, the WAM tends to underestimate the Kutani cups since it does not take into account the redundancy effects among Kansei attributes.

8 Concluding Remarks

This paper focuses on evaluation of commercial products according to the Kansei, which is an individual subjective impression reflecting the aesthetic appeal of products. To do so, a preparatory experimental study was carried out to obtain Kansei database of the products to be evaluated. Based on the Kansei database obtained, a probabilistic approach to generating Kansei profiles was first proposed to involve the fuzzy uncertainty as well as partial semantic overlapping of Kansei data. It resulted with a probability distribution on the Kansei labels. Second, the single attribute Kansei evaluation function was formulated to induce consumer's satisfaction utility function based on the appealing idea of target-oriented decision model. Third, after formulating multiattribute target-oriented Kansei evaluation function, an analogy between the nonadditive multiattribute Kansei evaluation and Choquet integral was given. Based on the analogy, an entropy based method fuzzy measure was chosen to induce the fuzzy measure for each subset of attributes. Finally, as the aesthetic aspects play a crucial role in human choice of traditional crafts, an application to evaluating hand-painted

Kutani cups, one of the traditional crafts in Japan with a long history, was conducted to illustrate the effectiveness of our model. A comparative analysis with prior research was also given.

Although our model only focuses on Kansei evaluation of commercial products, it has some relationships with Kansei design problems in KE. In Kansei design, discovering relationships between Kansei data and design elements is essential. For this task, traditional KE methodologies utilize multivariate statistical analysis by treating Kansei data as numerical data. This paper treats the Kansei data as linguistic variables and proposes a Kansei profile generation method, which results with a set of Kansei labels having a probability distribution. The main advantage of our proposed Kansei profile generation method is its ability to deal with fuzzy uncertainty as well as semantic overlapping of Kansei labels. A possible solution to discover the relationships between Kansei data and design elements using our Kansei profiles is to conduct regression analysis by means of belief functions (Petit-Renaud and Denœux, 2004). This study is left for the future work.

Appendix 1: Fuzzy Measures and Choquet Integral

Given a finite attributes set $\mathcal{X}$, the power set $P(\mathcal{X})$ is a class of all of the subsets of $\mathcal{X}$. Although the fuzzy measure can be continuous and discrete, only the discrete case is considered in this paper.

Definition 5 A discrete fuzzy measure on $\mathcal{X}$ is a set function $\mu : P(\mathcal{X}) \rightarrow [0,1]$ satisfying the following conditions: *Axiom 1: boundary*, $\mu_{\varnothing} = 0$ ($\varnothing$ is the empty set) and $\mu_{\mathcal{X}} = 1$; *Axiom 2: monotonic*, if $A_1 \subseteq A_2$, then $\mu_{A_1} \leq \mu_{A_2}$, $\forall A_1, A_2 \in P(\mathcal{X})$.

For each subset of the attributes $A \subseteq \mathcal{X}$, μ_A can then be interpreted as the weight or the importance of the coalition A. The monotonicity of μ means that the weight of a subset of the attributes can only increase when one adds new attributes to it. For all $A_1, A_2 \subseteq \mathcal{X}, A_1 \cap A_2 = \varnothing$, the discrete fuzzy measure is further said to be (Marichal, 2000): *additive* whenever $\mu_{A_1 \cup A_2} = \mu_{A_1} + \mu_{A_2}$; *multiplicative* whenever $\mu_{A_1 \cup A_2} > \mu_{A_1} + \mu_{A_2}$; *substitutive* whenever $\mu_{A_1 \cup A_2} < \mu_{A_1} + \mu_{A_2}$.

When using a fuzzy measure to model the importance of each subset of attributes, a suitable aggregation operator is the discrete Choquet integral, which is defined as follows (Marichal, 2000).

Definition 6 Let μ be a discrete fuzzy measure on $\mathcal{X}$ and $h : \mathcal{X} \rightarrow [0,1]$, where $\mathcal{X}$ is a finite attributes set. The Choquet integral C_μ of h with respect to μ is defined by

$$C_\mu(h) = \sum_{n=1}^{N} h(X_{(n)}) \left[\mu_{A_{(n)}} - \mu_{A_{(n+1)}}\right], \tag{35}$$

where $(\cdot)$ indicates a permutation of $\mathcal{X}$ such that $h(X_{(1)}) \leq \cdots \leq h(X_{(n)}) \leq \cdots \leq h(X_{(N)})$. Also $A_{(n+1)} = \varnothing$ and $A_{(n)} = \{X_{(n)}, \ldots, X_{(N)}\}$.

A fuzzy measure μ on $\mathcal{X}$ and its Möbius representation $a^\mu : P(\mathcal{X}) \rightarrow \mathbb{R}$ can be mutually expressed by (Marichal, 2000)

$$\mu(A) = \sum_{B \subseteq A} a^\mu(B), \; a^\mu(A) = \sum_{B \subseteq A} (-1)^{|A|-|B|} \mu(B), \tag{36}$$

where $\forall A \subseteq \mathcal{X}$ and the set function a^μ is called the *Möbius transform or Möbius representation* of μ. $a^\mu(A)$ can be interpreted as the *interaction index* of the attributes in the subset A.

Appendix 2: Computational Model Based on 2-Tuple Linguistic Model

The 2-tuple linguistic representation model has been proposed by Herrera and Martínez (2000) in order to provide an appropriate tool for computing with words, which aims at overcoming the limitation of the loss of information caused by the process of linguistic approximation in the conventional fuzzy set-based and symbolic approaches.

Let $\mathcal{L} = \{L_1, \ldots, L_G\}$ be a linguistic term set on which a total order is defined as $L_i \leq L_j \Leftrightarrow i \leq j$. In general, applying a symbolic method for aggregating linguistic information often yields a value $\beta \in [1, G]$ and $\beta \notin \{1, \ldots, G\}$, then a symbolic approximation must be used to get the results expressed in $\mathcal{L}$. Alternatively, the 2-tuple linguistic representation model takes $\mathcal{L} \times [0.5, 0.5)$ as the underlying space for representing information. Under such a representation, if a value $\beta \in [1, G]$ representing the result of a linguistic aggregation operation, then the two-tuple that expresses the equivalent information of β is obtained by means of the following transformation:

$$\begin{aligned} \triangle\colon [1, G] &\longrightarrow \mathcal{L} \times [-0.5, 0.5) \\ \beta &\longrightarrow (L_g, \alpha) \end{aligned} \tag{37}$$

with $n = \texttt{round}(\beta)$ and $\alpha = \beta - g$, where $\texttt{round}(\cdot)$ is the usual round operator and L_g means the linguistic label having the closest index to β. Inversely, a two-tuple $(L_g, \alpha) \in \mathcal{L} \times [-0.5, 0.5)$ can be equivalently represented by a numerical value in $[1, G]$ by the following transformations:

$$\triangle^{-1}\colon \mathcal{L} \times [-0.5, 0.5) \longrightarrow [1, G] \tag{38}$$

such that $\triangle^{-1}(L_g, \alpha) = g + \alpha$. Under such transformations, it should be noted here that any original linguistic term L_g in $\mathcal{L}$ is then represented by its equivalent 2-tuple $(L_g, 0)$ in the 2-tuple linguistic model.

When K linguistic information expressed by 2-tuple is available, the aggregation result can be derived by using weighted average operator as follows. Let $x = \{(r_1, \alpha_1), \ldots, (r_k, \alpha_k), \ldots, (r_K, \alpha_K)\}$ be a set of linguistic 2-tuples, the 2-tuple weighted average is computed as

$$\overline{x}^e = \triangle\left(\sum_{k=1}^{K} (r_k + \alpha_k) \cdot w_k\right), \tag{39}$$

where $W = [w_1, \ldots, w_K]$ is the weighting vector associated with x.

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