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Description	

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On the Convergence Property of an MMSE Based Multiuser MIMO Turbo Detector with Uplink Precoding

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***Abstract*—In this the paper, impact of minimum mean squared error (MMSE) based linear precoding on the convergence of iterative multiuser detector in multiuser multiple input multiple output (MIMO) uplink single carrier communications is studied. The influence of linear precoding on iterative MMSE based multiuser detector is investigated via the multi-dimensional EXIT analysis. It is shown that the use of the linear precoding enhances the separability of the EXIT planes of the simultaneous streams over without precoding; This invokes the idea that different code rate be allocated to the each transmitted streams at the transmitter. Especially, in the case of multiuser communication precoding has significant roles in the convergence property of iterative detector.**

I. Introduction

Since the advent of the multiple input multiple output (MIMO) system concept [1], it has been quite common to assume that transmitter has the knowledge about the channels, referred to as channel state information (CSI), between the transmit and receive antennas e.g., [2]; the use of CSI brings significant flexibility in designing the transmission chain, resulting in improved performance and efficiency.

Despite the volume of publications dealing with the joint optimization of linear transmitter and receiver for downlink MIMO systems [3][4] and for uplink MIMO systems [5][6] not many has been focusing on joint optimization of linear transmitter and iterative receiver for uplink. Recently, an iterative waterfilling algorithm has been proposed in [7] to maximize the sum capacity of multiple access channel (MAC). The chain rule of mutual information in information theory indicates that successive interference cancellation (SIC) is the optimum receiver for the sum capacity optimized uplink transmission. However, in practice SIC based receivers are suffering from the error propagation. Therefore, e.g. iterative receivers can be used to mitigate error propagation problems. Thus, the goal of this paper is to apply minimum mean squared error (MMSE) based linear precoding technique for up-link in multi-user (MU) MIMO systems with an iterative multiuser detector, of which purpose is to investigate impact of precoding on convergence property of a frequency domain soft cancellation minimum mean squared error (FD SC MMSE) iterative (turbo) equalizer.

Currently, single carrier frequency division multiple access (FDMA) has been recognized as one of the most attractive candidates for uplink transmission scheme in the 3GPP long term evolution scenario making framework [8]. One of the main arguments towards single carrier transmission is that in uplink the battery life longevity of user terminals is a crucial requirement, and the power consumption for single carrier signal transmission is obviously much less than that with orthogonal frequency domain multiplexing (OFDM).

In this paper we focus on single carrier transmission based on centralized precoder design where receiver at the base station has perfect knowledge about all the users' CSIs at the both transmitter and the receiver sides; Base station determines the optimal precoding matrices for the multiple users, and forward the matrix to each user.

In this paper, the optimality of precoding is defined as to minimize the sum of mean squared error totaling over the all users, referred to as MinSum-MSE criterion without assuming decoder feedback. The solution to the optimization problem is derived by following [6], where the Semidefinte Definitive Programming (SDP) based convex optimization technique is used.

As noted above, the precoding matrices are determined according to the MinSum-MSE criterion without assuming decoder feedback. At the receiver side, however, the MMSE filter weight matrices change, iteration by iteration. Therefore, the use of precoding makes impacts on the convergence property of the iterative detector. Mismatch between the channel code and channels inherent signal transmission capability results in two detrimental scenarios, described below, which can be predicted through extrinsic information transfer (EXIT) analysis [9]; If the EXIT curves with the channel decoder and detector intersects before attaining high enough mutual information, high bit error rate (BER) results; on the contrary if the gap between the two EXIT curves is unacceptably

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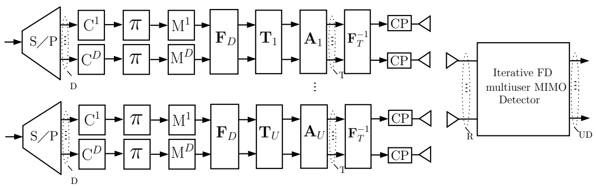


Fig. 1: Linearly precoded multiuser MIMO system for uplink communications.

large, the signalling chain does not well exploit the channels signal transmission capability. In MU MIMO uplink, the mutual information exchange via the Turbo loop for a user is affected by the other users, and thus, the EXIT analysis has to be multi-dimensional [10]. Therefore, impact of the use of precoding on the convergence properties of each user's turbo loop has to be evaluated via multi-dimensional EXIT analysis. A series of simulations was conducted to evaluate the convergence properties of MU MIMO up-link systems emplying this paper's proposed precoding techniques, where quaternary phase shift keying (QPSK) is used as a modulation scheme. The most crucial observation of the outcomes is that for the each user precoding clearly separates the EXIT surfaces of the streams.

This paper is organized as follows: Section 2 describes the system model used in this paper, where frequency domain notation of the MU MIMO signalling chain used in this paper is provided. Section 3 provides mathematical expressions of the optimality definitions for the precoding and weighting matrices used at the transmitter and the receiver sides, respectively. Section 3 then derives the solutions to those optimization problems. Section 4 presents results of the simulations, where non-systematic repeat accumulate (RA) code [11] is used as the channel coding scheme of each user, because of its simplicity in encoding and decoding as well as flexibility in the rate adjustment. Section 5 concludes this paper.

II. System Model

In this paper the uplink of a single cell system with U synchronous users is considered. The both users and the base station are equipped with multiple antennas, T transmit and R receive antennas, respectively. Each of the simultaneous uplink users multiplexes its fixed number D of data streams through its T transmit antennas. A model of considered linearly precoded multiuser MIMO uplink system is depicted in Fig. 1. The system uses cyclic-prefix single carrier burst transmission. Since the cyclic-prefix burst transmission technique is very well known [12], its details are not described in this paper. After guard period removal,[1] a space-time presentation of the signal vector $\tilde{\mathbf{r}} \in \mathbb{C}^{RK_B\times 1}$ received by the R received antennas is given by

$$\mathbf{r} = \hat{\mathbf{H}}\mathbf{F}_U^{-1}\mathbf{ATFb} + \mathbf{v}, \tag{1}$$

[1]We restrict ourselves to the case where the length of guard period is larger than or as large as the channel memory length.

where $\mathbf{v} \in \mathbb{C}^{RK_B\times 1}$ is a white additive independent identically distributed (i.i.d) Gaussian noise vector with variance σ^2 per dimension, with K_B being the length of discrete Fourier transform (DFT) over entire bandwidth shared by all the users, and $\mathbf{b} \in \mathbb{C}^{UDK_S\times 1}$ is the transmitted multiuser signal vector

$$\mathbf{b} \;=\; [\mathbf{b}^1, ..., \mathbf{b}^u, ..., \mathbf{b}^U]^\dagger \tag{2}$$

with $\mathbf{b}^u \in \mathbb{C}^{DK_S\times 1}$, K_S being the length of discrete Fourier transform (DFT) over each user's bandwidth and $u = 1, ..., U$. The sub-vectors $\mathbf{b}^u$ of $\mathbf{b}$ is given by

$$\mathbf{b}^u \;=\; [\mathbf{b}^{u,1}, ..., \mathbf{b}^{u,d}, ..., \mathbf{b}^{u,D}]^\dagger \tag{3}$$

denoting the u^{th} user's transmitted streams over the T transmit antennas. $\mathbf{b}^{u,d} \in \mathbb{C}^{K_S\times 1}$ is given by

$$\mathbf{b}^{u,d} \;=\; [b_1^{u,d}, ..., b_k^{u,d}, ..., b_{K_s}{}^{u,d}]^\dagger, \tag{4}$$

where $t = 1, ..., T$ and $k = 0, ..., K_s - 1$ contain transmitted symbols of the u^{th} user's t^{th} layer. The precoder matrix $\mathbf{T} \in \mathbb{C}^{UTK_S\times UDK_S}$ is given by

$$\mathbf{T} \;=\; bdiag\{[\mathbf{T}_1 ... \mathbf{T}_u ... \mathbf{T}_U]^\dagger\}, \tag{5}$$

where $\mathbf{T}_u \in \mathbb{C}^{TK_S\times DK_S}$ is each user's precoder matrix and the operator $bdiag\{\}$ generates block diagonal matrix from its argument components. The frequency bin allocation matrix $\mathbf{A} \in \mathbb{R}^{UTK_B\times UTK_S}$ for all the users is defined as

$$\mathbf{A} \;=\; bdiag\{[\mathbf{A}_1 ... \mathbf{A}_u ... \mathbf{A}_U]^\dagger\}, \tag{6}$$

where $\mathbf{A}_u = bdiag\{[\mathbf{A}_u^1 ... \mathbf{A}_u^t ... \mathbf{A}_u^T]^\dagger\} \in \mathbb{R}^{TK_B\times TK_S}$ denotes each user's frequency bin allocation with $\mathbf{A}_u^t \in \mathbb{R}^{K_B\times K_S}$ being the bin allocation matrix for the t^{th} transmit antenna of the u^{th} user. [2] It should be noticed that depending on the positions of zeros and ones in bin allocation matrix $\mathbf{A}_u$, both SDMA/FDMA based multiple-access methods can be expressed with a unified notation using the matrix $\mathbf{A}$. However, it should be noted that the optimization of frequency bin allocation is out of the scope of this study.

The circulant block channel matrix $\hat{\mathbf{H}} \in \mathbb{C}^{RK_B\times UTK_B}$ is then given as

$$\hat{\mathbf{H}} \;=\; [\hat{\mathbf{H}}_1, ..., \hat{\mathbf{H}}_u, ..., \hat{\mathbf{H}}_U], \tag{7}$$

where $\hat{\mathbf{H}}_u \in \mathbb{C}^{RK_B\times TK_B}$ with $u = 1, \cdots, U$ is a circulant block matrix corresponding to the u^{th} user. The circulant block matrix for the u^{th} user is denoted as

$$\hat{\mathbf{H}}_u = \begin{bmatrix} \hat{\mathbf{H}}_u^{1,1} & \dots & \hat{\mathbf{H}}_u^{1,T} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{H}}_u^{R,1} & \dots & \hat{\mathbf{H}}_u^{R,T} \end{bmatrix}, \tag{8}$$

where the channel submatrices $\hat{\mathbf{H}}_u^{r,t} \in \mathbb{C}^{K_B\times K_B}$ between the t^{th} transmit and the r^{th} receive antennas, $r = 1, ..., R$, are also circulant, as

$$\hat{\mathbf{H}}_u^{r,t} = circ\left\{\left[h_{u,1}^{r,t}, h_{u,2}^{r,t} \dots h_{u,L}^{r,t}\right]^\dagger\right\}. \tag{9}$$

[2]All the frequency bin allocation matrices of u^{th} user, $\mathbf{A}_u^t$, are assumed to be equivalent for each transmit antenna.

The operator $circ\{\ \}$ generates matrix that has a circulant structure of its argument. L denotes the length of the channel, and $h^{r,t}_{u,l}$, $l = 1, ..., L$, the fading gains of multipath channel between the u^{th} user's t^{th} transmit antenna and the r^{th} receive antenna. For the each user's transmit-receive antenna pair the sum of the average power of fading gains is normalized to one. It is well known that the circulant matrices can be diagonalized by the unitary DFT matrix $\mathbf{F_B} \in \mathbb{C}^{K_B \times K_B}$ with the elements $f_{m,k} = \exp^{\frac{j2\pi m_B k_B}{K}}$, where $m_B, k_B = 0, ..., K_B - 1$. Similarly, the circulant block matrices can be block-diagonalized by using block diagonal DFT matrices. The block-diagonalization of $\hat{\mathbf{H}}$ is performed as

$$\hat{\mathbf{H}} = \mathbf{F}_R^{-1}\mathbf{\Gamma}\mathbf{F}_U, \tag{10}$$

where $\mathbf{\Gamma} \in \mathbb{C}^{RK_B \times UTK_B}$ is the corresponding diagonal block matrix, and $\mathbf{F}_R^{-1} = \frac{1}{K_B}\mathbf{F}_R^{\ddagger} \in \mathbb{C}^{RK_B \times RK_B}$ is the unitary block inverse discrete fourier transform (IDFT) matrix. $\ddagger$ indicates the Hermitian transpose, and $\mathbf{F}_R \in \mathbb{C}^{RK_B \times RK_B}$ is block-diagonal DFT matrix given by $\mathbf{F}_R = \mathbf{I}_R \otimes \mathbf{F_B}$ for the R received antennas, where $\mathbf{F}_B \in \mathbb{C}^{K_B \times K_B}$ is the unitary DFT matrix with $\mathbf{I}_R \in \mathbb{R}^{R \times R}$ being the identity matrix and the symbol $\otimes$ indicates the Kronecker product. $\mathbf{F}_U \in \mathbb{C}^{UTK_B \times UTK_B}$ is given by $\mathbf{F}_U = \mathbf{I}_U \otimes \mathbf{F}_T$ for the transmit antennas of all users, where $\mathbf{I}_U \in \mathbb{R}^{U \times U}$ is the identity matrix and $\mathbf{F}_T = \mathbf{I}_T \otimes \mathbf{F}_B$ with $\mathbf{I}_T \in \mathbb{R}^{T \times T}$ being the identity matrix. Correspondingly, the block-diagonal DFT matrices $\mathbf{F}_D \in \mathbb{C}^{DK_S \times DK_S}$ and $\mathbf{F} \in \mathbb{C}^{UDK_S \times UDK_S}$ are defined as $\mathbf{F}_D = \mathbf{I}_D \otimes \mathbf{F}_S$ and $\mathbf{F} = \mathbf{I}_U \otimes \mathbf{F}_D$, respectively. The matrix $\mathbf{F}_S \in \mathbb{C}^{K_S \times K_S}$ is the unitary DFT matrix with $\mathbf{I}_D \in \mathbb{R}^{D \times D}$ being the identity matrix. Average signal-to-noise ratio per receiver antenna is defined as ratio of information bit power and noise power, as $SNR = \frac{\tilde{P}_u}{2\sigma^2}$, where, $\tilde{P}_u$ is the average transmitted symbol energy per user.

III. The Joint MMSE Based Transceiver Design

Our goal is to design transmitter-receiver pairs for all the users that minimize the total Mean Square Error (MSE) $\mathbb{E}_{tot}$ of the system subject to transmit power constraint for the each user. Let the MMSE optimization problem for the design be expressed as follows [6]

$$\begin{aligned}
[\mathbf{T}_1, \mathbf{\Omega}_1, ..., \mathbf{T}_u, \mathbf{\Omega}_u, ..., \mathbf{T}_U, \mathbf{\Omega}_U] &= \min_{[\mathbf{T}_1, \mathbf{\Omega}_1, ..., \mathbf{T}_u, \mathbf{\Omega}_u, ..., \mathbf{T}_U, \mathbf{\Omega}_U]} \mathbb{E}_{tot} \\
\mathbb{E}_{tot} &= \sum_{u=1}^{U} \mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) \quad (11)\\
s.t: &\quad Tr(\mathbf{T}_u\mathbf{T}_u^{\ddagger}) \le p_u
\end{aligned}$$

where $\mathbf{\Omega}_u \in \mathbb{C}^{RK_S \times DK_S}$ corresponds to the u^{th} user receive MMSE filter and p_u is the transmission power for u^{th} user. $\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u)$ is the MSE of the u^{th} user given by

$$\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) = Tr\{E\{\mathbf{e}_u\mathbf{e}_u^{\ddagger}\}\} \tag{12}$$

and $\mathbf{e}_u = \mathbf{b}_u - \hat{\mathbf{b}}_u \in \mathbb{C}^{DK_S \times 1}$ is error vector with $\hat{\mathbf{b}}_\mathbf{u} \in \mathbb{C}^{DK_S \times 1}$ being the estimate of transmitted streams at the output of MMSE filter, given by

$$\hat{\mathbf{b}}_u = \mathbf{F}_D^{-1}\mathbf{\Omega}_u^{\ddagger}\mathbf{F}_R\hat{\mathbf{r}}_u \tag{13}$$

where the vector $\hat{\mathbf{r}}_u \in \mathbb{C}^{RK_B \times 1}$ combines the soft-cancellation outputs for the linearly precoded transmitted streams, as

$$\hat{\mathbf{r}}_u = \hat{\mathbf{r}} + \tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)\tilde{\mathbf{b}}^u. \tag{14}$$

Here, the matrix $\tilde{\mathbf{\Gamma}}_u = \widehat{\mathbf{A}}_u^{\ddagger}\mathbf{\Gamma}_u\mathbf{A}_u \in \mathbb{C}^{RK_S \times TK_S}$ is the effective channel matrix corresponding the frequency bins of the channel allocated u^{th} user with $\mathbf{\Gamma}_u \in \mathbb{C}^{RK_B \times TK_B}$ being the u^{th} desired user's frequency domain channel matrix. The receiver side frequency bin allocation matrix $\widehat{\mathbf{A}}_u \in \mathbb{R}^{RK_B \times RK_B}$ is given as

$$\widehat{\mathbf{A}}_u \quad = \quad bdiag\{[\widehat{\mathbf{A}}_u^1 ... \widehat{\mathbf{A}}_u^r ... \widehat{\mathbf{A}}_u^R]^{\dagger}\}, \tag{15}$$

where $\widehat{\mathbf{A}}_u^r = \mathbf{A}_u^t \in \mathbb{R}^{K_B \times K_S}$. The output of soft-cancellation $\hat{\mathbf{r}} \in \mathbb{C}^{RK_B \times 1}$ and u^{th} user soft estimate of the transmitted streams $\tilde{\mathbf{b}}^u \in \mathbb{C}^{DK_S \times 1}$ are described more in detailed in the Appendix.

A. Iterative Equalization

In the following derivation it is assumed that all the precoders $\mathbf{T}_u$ to be fixed. It should be also noted that the receivers are independent of other user's receive filters. Therefore, the receive MMSE filters can be optimized independently, user-by-user.

By using (10) for each user's channel matrix, the MSE of the uth user using (12) is given by[3]

$$\begin{aligned}
\mathbb{E}_u(\mathbf{T}_u, \mathbf{\Omega}_u) = &\ Tr\{\mathbf{\Sigma}_{b_u}\} - Tr\{\mathbf{\Sigma}_{b_u}\mathbf{S}(n)^{\ddagger}\mathbf{F}_D^{\ddagger}\mathbf{T}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u^{\ddagger}\mathbf{\Omega}_u\mathbf{F}_D\} \\
&- Tr\{\mathbf{F}_D^{\ddagger}\mathbf{\Omega}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)^{\ddagger}\mathbf{\Sigma}_{b_u}^{\ddagger}\} \\
&+ Tr\{\mathbf{F}_D^{\ddagger}\mathbf{\Omega}_u^{\ddagger}\mathbf{\Sigma}_{\hat{r}_u}\mathbf{\Omega}_u\mathbf{F}_D\}
\end{aligned} \tag{16}$$

where $\mathbf{\Sigma}_{b_u} = E\{\mathbf{b}_u\mathbf{b}_u^{\ddagger}\}$ and $\mathbf{\Sigma}_v = E\{\mathbf{v}\mathbf{v}^{\ddagger}\}$ with $\mathbf{\Sigma}_{\hat{r}_u} \in \mathbb{C}^{RK_S \times RK_S}$ being the covariance matrix of the residual and desired signal components, given by

$$\mathbf{\Sigma}_{\hat{r}_u} = \mathbf{\Sigma}_{\hat{r}} + \tilde{\mathbf{\Gamma}}_u\mathbf{T}_u\mathbf{F}_D\mathbf{S}(n)_u\breve{\mathbf{\Lambda}}_u\mathbf{S}(n)_u^{\ddagger}\mathbf{F}_D^{\ddagger}\mathbf{T}_u^{\ddagger}\tilde{\mathbf{\Gamma}}_u^{\ddagger}. \tag{17}$$

The covariance matrix of the residual $\mathbf{\Sigma}_{\hat{r}} \in \mathbb{C}^{RK_S \times RK_S}$ is given by

$$\mathbf{\Sigma}_{\hat{r}} = \tilde{\mathbf{\Gamma}}\mathbf{T}\mathbf{A}\mathbf{F}\mathbf{\Lambda}\mathbf{F}^{\ddagger}\mathbf{A}^{\ddagger}\mathbf{T}^{\ddagger}\tilde{\mathbf{\Gamma}}^{\ddagger} + \sigma^2\mathbf{I}, \tag{18}$$

where $\tilde{\mathbf{\Gamma}} \in \mathbb{C}^{RK_S \times UTK_S}$ is determined in Appendix. In Eq. (18) it has been assumed that transmitted streams are statistically independent. By using u^{th} user's soft estimate of transmitted layers given in Appendix, the soft-feedback term $\breve{\mathbf{\Lambda}}_u \in \mathbb{C}^{DK_S \times DK_S}$ in (17) is obtained as

$$\breve{\mathbf{\Lambda}}_u = diag\left\{\left[\ddot{\mathbf{b}}_u^1 \dots \ddot{\mathbf{b}}_u^d \dots \ddot{\mathbf{b}}_u^D\right]^{\dagger}\right\}, \tag{19}$$

with $\ddot{\mathbf{b}}^{u,d} \in \mathbb{C}^{K_S \times 1}$ being

$$\ddot{\mathbf{b}}^{u,d} = \left[|\tilde{b}_1^{u,d}|^2 \dots |\tilde{b}_{k_S}^{u,d}|^2 \dots |\tilde{b}_{K_S}^{u,d}|^2\right]^{\dagger}, \tag{20}$$

where soft-symbol estimates $\tilde{b}_{k_S}^{u,d}$ are obtained calculating the first moment of soft-symbols similarly as in [13].

[3] $Tr\{\mathbf{AB}\} = Tr\{\mathbf{BA}\}$

The diagonal matrix $\mathbf{\Lambda} \in \mathbb{C}^{UDK_S \times UDK_S}$ expresses the mean residual interference energy after soft cancellation, as

$$\mathbf{\Lambda} = diag\left\{\left[\hat{\mathbf{d}}_1 \ldots \hat{\mathbf{d}}_u \ldots \hat{\mathbf{d}}_U\right]^\dagger\right\}, \quad (21)$$

where $\hat{\mathbf{d}}_u \in \mathbb{C}^{DK_S \times DK_S}$ is given by

$$\hat{\mathbf{d}}_u = \dot{\mathbf{b}}_u - \ddot{\mathbf{b}}_u \quad (22)$$

with $\dot{\mathbf{b}}_u \in \mathbb{C}^{DK_S \times 1}$ being

$$\dot{\mathbf{b}}_u = \left[\dot{\mathbf{b}}^{u,1} \ldots \dot{\mathbf{b}}^{u,d} \ldots \dot{\mathbf{b}}^{u,D}\right]^\dagger \quad (23)$$

and $\dot{\mathbf{b}}^{u,d} \in \mathbb{C}^{K_S \times 1}$ is given by

$$\dot{\mathbf{b}}^{u,d} = \left[E\left\{|b_1^{u,d}|^2\right\} \ldots E\left\{|b_{k_S}^{u,d}|^2\right\} \ldots E\left\{|b_{K_S}^{u,d}|^2\right\}\right]^\dagger. \quad (24)$$

The second moment of the soft-symbol estimates $E\left\{|b_{k_S}^{u,d}|^2\right\}$ is then computed similarly as in [13].

Since Hessian matrix of the objective function in (16) is not positive definite [6], the objective function is nonconvex respect to $\mathbf{T}_u, \mathbf{\Omega}_u$. Thus, it is difficult to minimize $\mathbb{E}_{tot}$ due to problems with local solutions [6]. However, for the given set of precoders $\mathbf{T}_u$ the MSE for uth user is convex respect to $\mathbf{\Omega}_u$. Therefore, the standard optimal Wiener solution for the receive filters is determined as

$$\mathbf{\Omega}_u \mathbf{F}_D \;=\; \mathbf{\Sigma}_{\hat{r}_u}^{-1} \tilde{\mathbf{\Gamma}}_u \mathbf{T}_u \mathbf{F}_D \mathbf{S}(n)^\ddagger \mathbf{\Sigma}_{b_u}^\ddagger. \quad (25)$$

Now, let us write the block circulant Hermitian covariance matrix $\mathbf{\Delta} \in \mathbb{C}^{UDK_S \times UDK_S}$ of the feedback soft estimates by

$$\mathbf{\Delta} \;=\; \mathbf{F}\mathbf{\Lambda}\mathbf{F}^\dagger. \quad (26)$$

It should be noted at this stage that since $\mathbf{\Delta}$ is a block circulant Hermitian matrix, the covariance matrix $\mathbf{\Sigma}_{\hat{r}}$ of the residual interference does not have diagonal structure. Therefore, it requires unacceptable computational efforts to strictly invert (17). Moreover, the sampling matrix, $\mathbf{S}(n)$, still remains in the filter output expression of (25), which necessitates the whole chain of equations for the algorithm to be calculated at every symbol timing.

It is shown in [13] that the calculation of exact optimal solution (25) results prohibitive computational complexity. The major computationally complexity is due to inversion of the covariance matrix $\mathbf{\Sigma}_{\hat{r}_u}$. Therefore, an approximation is required. We follow closely the technique presented in [13] to reduce the computational complexity of the algorithm. In reference [13] the matrix inversion lemma is used to invert the covariance matrix of (17). Moreover, it approximates the matrix $\mathbf{\Delta}$ matrix with a diagonal matrix by replacing the symbol-wise residual interference energy terms in (22) with the each streams's corresponding time-average. With this approximation, the necessity for the symbol-by-symbol computation of the algorithm can be avoided, because the residual interference energy is assumed to be the constant over one received frame within each stream.

With the time-average approximation, $\mathbf{\Delta}$ is replaced by a diagonal matrix

$$\mathbf{\Delta} \;\approx\; diag\left\{\left[\bar{\hat{\mathbf{d}}}_1 \ldots \bar{\hat{\mathbf{d}}}_u \ldots \bar{\hat{\mathbf{d}}}_U\right]^\dagger \otimes \mathbf{1}\right\} \quad (27)$$

where $\mathbf{1} \in \mathbb{R}^{K_S \times 1}$ is a vector having all the elements being one, and $\bar{\mathbf{d}}_u \in \mathbb{C}^{D \times 1}$ is defined as

$$\bar{\hat{\mathbf{d}}}_u = \left[\bar{\hat{d}}_u^1 \ldots \bar{\hat{d}}_u^d \ldots \bar{\hat{d}}_u^D\right]^\dagger. \quad (28)$$

The scalar $\bar{d}_u^d$ in (28) is given by using (20) and (24), as

$$\bar{\hat{d}}_u^d = avg\left\{\dot{\mathbf{b}}^{u,d} - \ddot{\mathbf{b}}^{u,d}\right\} \quad (29)$$

where the operator $avg\{\}$ calculates the vectorwise average from its argument vector as $avg\{\} = \frac{1}{K_S}\sum$. With this approximation, significant computational complexity reduction can be expected.

Due to the time-invariance of the residual interference energy over the frame and the necessity of using the sampling matrix $\mathbf{S}(n)$ can be now eliminated. Now, the MMSE filter output can be written as

$$\hat{\mathbf{b}}_u = \bar{\mathbf{\Xi}}_u \mathbf{\Pi}_u (\mathbf{F}_D^{-1} \mathbf{T}_u^\ddagger \tilde{\mathbf{\Gamma}}_u^\ddagger \mathbf{\Sigma}_{\hat{r}}^{-1} \hat{\mathbf{r}} + \mathbf{\Upsilon}_u \tilde{\mathbf{b}}^u). \quad (30)$$

With the diagonal structure of $\mathbf{\Delta}$, the matrices $\mathbf{\Upsilon}_u$ and $\mathbf{\Pi}_u$, respectively, can be written as

$$\mathbf{\Upsilon}_u = \begin{bmatrix} \varphi_u^{1,1}\mathbf{I}_{K_S} & \ldots & \varphi_u^{1,D}\mathbf{I}_{K_S} \\ \vdots & \ddots & \vdots \\ \varphi_u^{D,1}\mathbf{I}_{K_S} & \ldots & \varphi_u^{D,D}\mathbf{I}_{K_S} \end{bmatrix} \quad (31)$$

and [4][5]

$$\mathbf{\Pi}_u = \mathbf{I}_{DK_S} - \mathbf{\Upsilon}_u \bar{\bar{\mathbf{\Lambda}}}_u (\mathbf{\Upsilon}_u \bar{\bar{\mathbf{\Lambda}}}_u + \mathbf{I}_{DK_S})^{-1}, \quad (32)$$

where $\mathbf{I}_{DK_S} \in \mathbb{R}^{DK_S \times DK_S}$ is an identity matrix. The scalar $\varphi_u^{d,f}$, $d, f = 1, \ldots, D$, in (31) is given by [6]

$$\varphi_u^{d,f} = \frac{1}{K_S} Tr\left\{\mathbf{T}_u^{d^\ddagger} \tilde{\mathbf{\Gamma}}_u^\ddagger \mathbf{\Sigma}_{\hat{r}}^{-1} \tilde{\mathbf{\Gamma}}_u^f \mathbf{T}_u^f\right\}, \quad (33)$$

where the matrix $\mathbf{T}_u^d \in \mathbb{C}^{TK_S \times DK_S}$ contains all rows from the $(d-1)K_S + 1$ to dK_S-th columns in $\mathbf{T}_u$. The matrix $\bar{\mathbf{\Xi}}_u \in \mathbb{C}^{DK_S \times DK_S}$ can be computed as

$$\begin{aligned} \bar{\mathbf{\Xi}}_u \;=\;& diag\left\{\left[avg\left\{\dot{\mathbf{b}}_u^1\right\} \ldots \right.\right. \\ & \left.\left. avg\left\{\dot{\mathbf{b}}_u^d\right\} \ldots avg\left\{\dot{\mathbf{b}}_u^D\right\}\right]^\dagger \otimes \mathbf{1}\right\}. \end{aligned} \quad (34)$$

The matrix $\bar{\bar{\mathbf{\Lambda}}}_u \in \mathbb{C}^{DK_S \times DK_S}$ in (32) can be computed by using (19) by averaging over each $\ddot{\mathbf{b}}_u^d$ vector. After this, the diagonal matrix $\bar{\bar{\mathbf{\Lambda}}}_u$ can be generated from the averaged values similarly as in (34).

[4] $(\mathbf{AB})^{-1}) = (\mathbf{B})^{-1}(\mathbf{A})^{-1}$
[5] $(\mathbf{I} + \mathbf{A}^{-1})^{-1} = \mathbf{A}(\mathbf{A} + \mathbf{I})^{-1}$
[6] Note that only diagonal entries of each sub-matrice have to be considered, because of the sampling matrix $\mathbf{S}$(n).

The gains of the equivalent Gaussian channel $\mathbf{\Phi}_u^j \in \mathbb{C}^{DK_S \times DK_S}$, at the output of MMSE filter (30), are expressed as [14]

$$\mathbf{\Phi}_u = \overline{\mathbf{\Xi}}_u^{-1} \mathbf{\Pi}_u \mathbf{\Upsilon}_u. \tag{35}$$

The variance of equivalent channel $\mathbf{\Psi}_u \in \mathbb{R}^{DK_S \times DK_S}$ is expressed as

$$\mathbf{\Psi}_u = \overline{\mathbf{\Xi}}_u^{-1} \mathbf{\Pi}_u \dot{\mathbf{\Upsilon}}_u \mathbf{\Pi}_u \overline{\mathbf{\Xi}}_u^{-1}. \tag{36}$$

The matrix $\dot{\mathbf{\Upsilon}}_u \in \mathbb{C}^{DK_S \times DK_S}$ is defined as

$$\dot{\mathbf{\Upsilon}}_u = \begin{bmatrix} \mathbf{I}_{K_S} & \cdots & \dot{\varphi}_u^{1,D} \mathbf{I}_{K_S} \\ \vdots & \ddots & \vdots \\ \dot{\varphi}_u^{D,1} \mathbf{I}_{K_S} & \cdots & \dot{\varphi}_u^{D,D} \mathbf{I}_{K_S} \end{bmatrix} \tag{37}$$

with the scalar $\dot{\varphi}_u^{d,f}$ is given by

$$\begin{aligned} \dot{\varphi}_u^{d,f} &= \frac{1}{K_S} Tr \Big\{ \mathbf{T}_u^{d\ddagger} \tilde{\mathbf{\Gamma}}_u^{\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1} (\tilde{\mathbf{\Gamma}} \mathbf{T} \mathbf{\Delta} \mathbf{T}^{\ddagger} \tilde{\mathbf{\Gamma}}^{\ddagger} \\ &+ \sigma^2 \widehat{\mathbf{A}}_u^{\ddagger} \widehat{\mathbf{A}}_u) \mathbf{\Sigma}_{\hat{r}}^{-1} \tilde{\mathbf{\Gamma}}_u^f \mathbf{T}_u^f \Big\}. \end{aligned} \tag{38}$$

Demapper is a part of the iterative equalizer and it performs symbol-to-soft bit conversion. Since the demapper algorithm itself is very well known based on MAP algorithm its details are not given in this paper. The reference [13] provides more details of the demapper algorithm used in this work .

B. Linear Precoding

In this subsection the design of a set of precoders for the U users is considered. In the precoder design it is assumed that a-priori information provided by each user's channel decoder is not utilized. Now, let us rewrite the total MSE of the system using (25), (16) and (11) as:

$$\begin{aligned} \mathbb{E}_{tot} &= \sum_{u=1}^{U} Tr\{\mathbf{\Sigma}_{b_u}\} \\ & - Tr\{\mathbf{\Sigma}_{b_u} \mathbf{S}(n) \mathbf{F}_D^{-1} \mathbf{T}_u^{\ddagger} \tilde{\mathbf{\Gamma}}_u^{\ddagger} \mathbf{\Sigma}_{\hat{\mathbf{r}}_u}^{-1} \tilde{\mathbf{\Gamma}}_u \mathbf{T}_u \mathbf{F}_D \mathbf{S}(n) \mathbf{\Sigma}_{b_u}^{\ddagger}\} \end{aligned} \tag{39}$$

Recall that *a*-priori information is not utilized, which results in $\mathbf{\Sigma}_{b_u} = \mathbf{I}$, $\mathbf{\Sigma}_{\hat{\mathbf{r}}_u} = \mathbf{\Sigma}_{\hat{\mathbf{r}}}$ and $\mathbf{\Delta} = \mathbf{I}$. Moreover, the sampling matrices $\mathbf{S}(n)$ can also be eliminated, due to the time-invariance assumption of the residual interference over the frame. Now, the total MSE in (39) can be re-written as follows

$$\mathbb{E}_{tot} = UDK_s - RK_S + \sigma^2 Tr\{\mathbf{\Sigma}_{\hat{r}}^{-1}\}. \tag{40}$$

It can be also observed that the total MSE in (40) is not jointly convex with respect to $\mathbf{T}_u$. Therefore, the problem has to be reformulate into convex one to find the global optimum. In this paper, we follow closely the technique presented in [6] to reformulate the problem. First of all, an auxiliary matrix $\mathbf{U}_u \in \mathbb{C}^{TK_S \times TK_S}$ is introduced as

$$\mathbf{U}_u = \mathbf{T}_u \mathbf{T}_u^{\ddagger}. \tag{41}$$

Now, by using (41) the total MSE in (40) can be re-written as

$$\mathbb{E}_{tot} = UDK_s - RK_B + \sigma^2 Tr\{\digamma\} \tag{42}$$

where $\digamma \in \mathbb{C}^{RK_S \times RK_S}$ is given as

$$\digamma = (\sigma^2 \mathbf{I} + \tilde{\mathbf{\Gamma}} \mathbf{U} \tilde{\mathbf{\Gamma}}^{\ddagger})^{-1}. \tag{43}$$

As a results of this, our objective function in (42) becomes convex with respect to $\mathbf{U}$ and constraints are convex as well after the Schur's complement computation. Therefore, convex optimization methods, e.g. semidefinite programming (SDP) can be used to find the global optimum. SDP can be solved efficiently using e.g standard convex optimization package [15]. Finally, by using (42) and via Schur's complement the joint transmitter-receiver MMSE design problem can be stated as an SDP problem, as

$$\begin{aligned} \min_{\digamma, \mathbf{U}_1, \ldots, \mathbf{U}_u, \ldots, \mathbf{U}_U} \quad & Tr\{\digamma\} \\ s.t. \quad & Tr\{\mathbf{U}_u\} \leq p_u \\ & \digamma \ satisfies (45) \\ & \mathbf{U}_u \succeq \mathbf{0}, \qquad u = 1, ..., U \end{aligned} \tag{44}$$

with

$$\begin{bmatrix} \digamma & \mathbf{I} \\ \mathbf{I} & \sigma^2 \mathbf{I} + \sum_{u=1}^{U} \tilde{\mathbf{\Gamma}}_u \mathbf{U}_u \tilde{\mathbf{\Gamma}}_u^{\ddagger} \end{bmatrix} \succeq \mathbf{0}. \tag{45}$$

Now, the set of optimal precoders $\mathbf{U}_u$ for all users can be found by using (44). Therefore, in the MMSE sense the optimal linear precoders $\mathbf{T}_u$ of all the users are obtained by applying the singular value decomposition separately to each $\mathbf{U}_u$, resulting in

$$\mathbf{T}_u = \mathbf{V}_u \mathbf{P}_u^{\frac{1}{2}}. \tag{46}$$

The diagonal matrix $\mathbf{P}_u^{\frac{1}{2}} \in \mathbb{R}^{DK_S \times DK_S}$ is the power allocation matrix of the u^{th} user with diagonal elements corresponding to the square root of power allocated on each frequency bin. Correspondingly, $\mathbf{V}_u \in \mathbb{C}^{TK_S \times DK_S}$ is the beamformer matrix of the u^{th} user.

IV. Numerical Results

In the following, simulation parameters are summarized as; The number of users $U = 1, 3$, receiver antennas at base station $R = 2, 4, 6$, transmit antennas per user $T = 2, 4$, streams per user $D = 2, 4$, the size of FFT' are $F_B = F_S = 256$, QPSK $M = 4$ with Gray mapping and a rate 1/2 non systematic RA channel code [11] for all streams in the system. The decoding is performed with the well-known sum-product algorithm. The number of decoding iterations is set at 6. A Quasistatic Rayleigh fading channel with $L = 3$ is assumed where each path has equal average gains. In the case of multiuser transmission, SDMA is assumed as a multiple access method.

In this paper, we use the EXIT chart [9] as well as its projection [10] techniques to analyze convergence properties of the proposed iterative multiuser detector with uplink precoding. The results of analysis were then averaged over channel realizations.

Let us now define the following multidimensional EXIT functions that describe the convergence properties of the iterative multiuser detector. The equalizer mutual information is measured at the output of demapper. The extrinsic information at the output of equalizer output for the d^{th} stream of u^{th} user is given by

$$I(u)^{E}_{E_d} = f(\mathbf{r}, \mathbf{I}(1)^{E}_{A}, ..., \mathbf{I}(u)^{E}_{A}, ..., \mathbf{I}(U)^{E}_{A}), \quad (47)$$

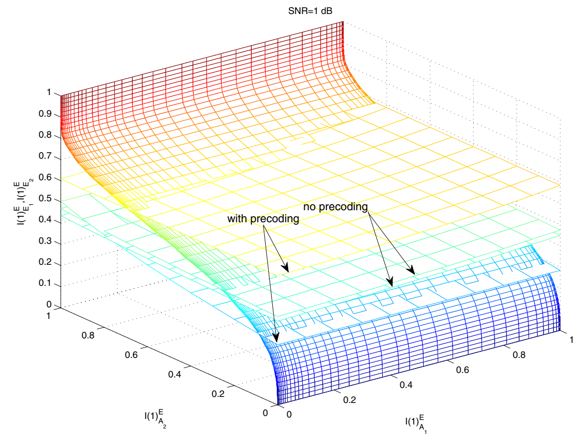
where the equalizer *a* priori mutual information vector of the u^{th} user is defined as $\mathbf{I}(u)^{E}_{A} = [I(u)^{E}_{A_1} ... I(u)^{E}_{A_d} ... I(u)^{E}_{A_D}]$ with $I(u)^{E}_{A_d}$ being *a* priori information of the equalizer for the d^{th} stream of the u^{th} user. By contrast, the decoder output information for the d^{th} stream of the u^{th} user is given by

$$I(u)^{De}_{E_d} = f(I(u)^{De}_{A_d}), \quad (48)$$

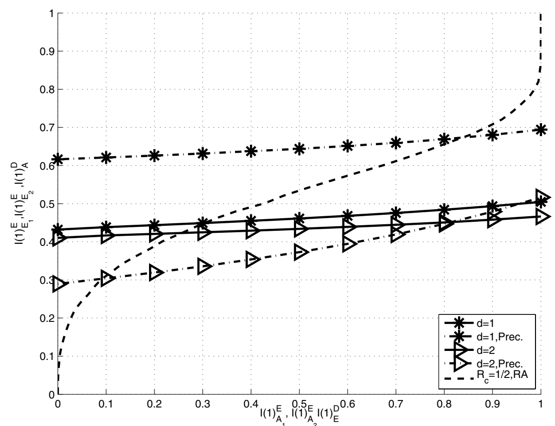
where $I(u)^{De}_{A_d}$ is *a* priori information for the d^{th} stream of the u^{th} user's decoder.

We first of all consider the single user case in which $T = D = R = 2$. Figure 2a shows the three dimensional EXIT surfaces of equalizer for the transmission with and without precoding. Since the both streams are using a half rate channel code and are independent of each other, it is sufficient to present the decoder extrinsic information with one surface. It can be seen that with precoding the surfaces of $I(1)^{E}_{E_1}$ and $I(1)^{E}_{E_2}$ have been more separated from each others compared to the case without precoding. In fact, in the case of without precoding the extrinsic information surfaces of the equalizer are almost identical. Moreover, it can be observed that the slope of equalizer surfaces is steeper with precoding. Similarly, as in [10] EXIT projection was used for the single user case to transform the multidimensional EXIT chart into the two dimensional EXIT chart. Figure 2b presents the two dimensional EXIT chart obtained by projection. Now, it can be observed that with precoding equalized streams have different intersection points with each other. However, without precoding equalized streams have almost equivalent convergence points. Figure 3 shows the comparison of EXIT projection results with and without precoding in the single user's case when $T = D = R = 4$ and SNR= 4dB. As expected, the precoding enhances significantly the separability of the streams. This is due to the fact that all the streams can be perfectly decoupled from each other with precoding in single user's case . However, inter symbol interference remains still in the system which has to be eliminated with iterative equalizer. Moreover, it should be noted that due to the increased amount of degrees of freedom compared to $T = D = R = 2$ the power allocation can be performed more efficiently.

Finally, we consider a multiuser MIMO case. Figure 4 shows the comparison between with and without precoding for $U = 3$, $T = D = 2$, $R = 6$, and SNR= 3dB. As can be seen, the both starting and ending points have significantly larger difference in multiuser case than in the single user. The reason for this is due to increased multiaccess interference compared to the single user case. Moreover, it should be also noticed that in the multiuser transmission all the streams can not be perfectly decoupled from each other in the spatial domain with beamforming as in the case of single user transmission with precoding.



(a) Three dimensional EXIT surface with and without precoding.



(b) EXIT projection with and without precoding.

Fig. 2: $U = 1, T = R = 2, D = 2$, SNR= 1.

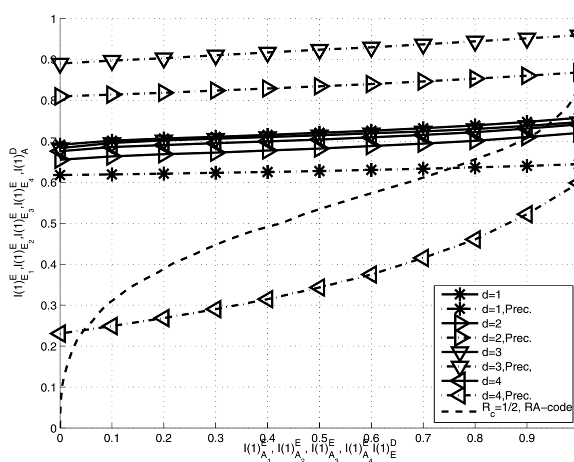


Fig. 3: EXIT projection with and without precoding for single user MIMO, $U = 1, T = 4, R = 4, D = 4$, SNR= 4 .

V. Conclusion

In this paper impact of minimum sum MSE optimized linear precoding on the convergence of iterative MMSE based

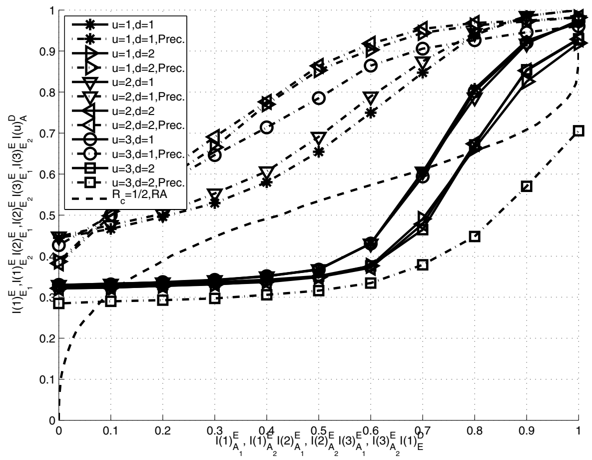


Fig. 4: EXIT projection with and without precoding for multiuser MIMO, $U = 3, T = 2, R = 6, D = 2$, SNR= 3.

multiuser MIMO detector has been discussed. It is shown that the use of the linear precoding enhances the separability of the EXIT planes of the simultaneous streams over without precoding; this invokes the idea that different code rate be allocated to the each transmitted streams at the transmitter. Especially, in the case of multiuser communication precoding has more significant role in the convergence property of iterative detector. In fact, the EXIT separability supported by the precoding, which is a major finding of this paper, opens new vistas in uplink transmission control with adaptive coding, which is, however, left as future study.

APPENDIX

The frequency domain residual interference, $\hat{\mathbf{r}} \in \mathbb{C}^{RK_B \times 1}$, after the cancellation of signal components to be detected from the received signal is given by

$$\hat{\mathbf{r}} = \widehat{\mathbf{A}}_u^{\dagger}\mathbf{F}_R\mathbf{r} - \tilde{\mathbf{\Gamma}}\mathbf{T}\mathbf{F}\tilde{\mathbf{b}}, \tag{49}$$

where $\tilde{\mathbf{\Gamma}} = \widehat{\mathbf{A}}_u^{\dagger}\mathbf{\Gamma}\mathbf{A} \in \mathbb{C}^{RK_S \times UTK_S}$ is the effective channel matrix with corresponding frequency bins. $\tilde{\mathbf{b}} \in \mathbb{C}^{UDK_S \times 1}$ represents the soft-estimate of the multiple user's transmitted signal vector $\tilde{\mathbf{b}} = [\tilde{\mathbf{b}}^1, ..., \tilde{\mathbf{b}}^u, ..., \tilde{\mathbf{b}}^U]^{\dagger}$ with $\tilde{\mathbf{b}}^u \in \mathbb{C}^{DK_S \times 1}$ being the u^{th} user soft estimate of the transmitted layers $\tilde{\mathbf{b}}^u = [\tilde{\mathbf{b}}^{u,1}, ..., \tilde{\mathbf{b}}^{u,d}, ..., \tilde{\mathbf{b}}^{u,D}]^{\dagger}$. $\tilde{\mathbf{b}}^{u,d} \in \mathbb{C}^{K_S \times 1}$ is given by $\tilde{\mathbf{b}}^{u,d} = \left[\tilde{b}_1^{u,d} \ldots \tilde{b}_{k_S}^{u,d} \ldots \tilde{b}_{K_S}^{u,d}\right]^{\dagger}$ with $\tilde{b}_{k_S}^{u,d}$ being soft estimate of k_S^{th} transmitted symbol of the u^{th} user's d^{th} stream. Reference [13] describes in detail the first two moments of soft-symbol estimates, $\tilde{b}_{k_S}^{u,d} = E\left\{b_{k_S}^{u,d}\right\}$ and $E\left\{|b_{k_S}^{u,d}|^2\right\}$.

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