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## Specific Type of Knowledge Map: Mathematical Model

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### ABSTRACT

The article deals with relationships between mathematical models and knowledge maps. The goal of the article is to suggest how to use the mathematical model as a knowledge map and/or as a part (esp. the inference mechanism) of the knowledge system. The results are demonstrated on the case study, when the knowledge from a story is expressed by mathematical model. The model is used for both knowledge warehousing and inferencing new artificially derived knowledge.

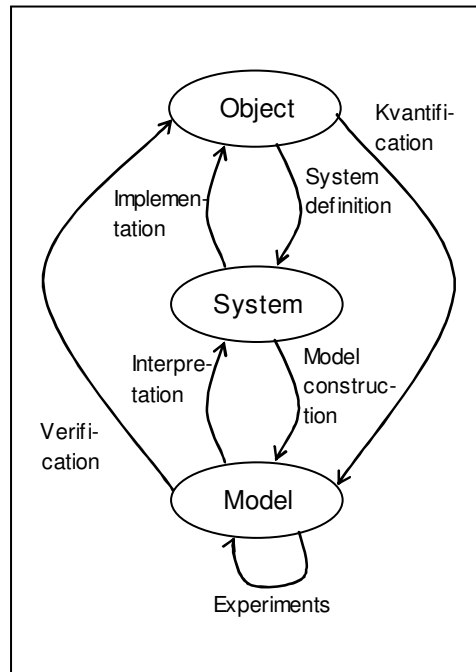
**Keywords:** knowledge management, knowledge map, system analysis, mathematical model

### 1. INTRODUCTION

Problem situations can be solved by the systems analysis. The standard procedure contains following steps:

1. Problem definition
2. Abstract system definition for the problem situation
3. Model construction
4. Model experiments
5. Interpretation of model outputs
6. Implementation of suggested solution

Systems analysis process is in the picture 1.



Picture 1 Process of systems modeling

Mathematical models are being used during system analysis of real object problems. They use mathematical apparatus for description of abstract system elements and relationships.

From the other point of view the mathematical model can be understood as a special type of knowledge map. Davenport and Prusak [2] note that developing a knowledge map involves locating important knowledge in the organization and then publishing some sort of list or picture that shows where to find it. Knowledge maps typically point to people as well as to document and databases.

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## 2. DEFINITIONS

### 2.1. Knowledge and Knowledge Map

There are many definitions of these terms. Stuhlman [6] defines knowledge and knowledge map as follows:

*Knowledge* is the result of learning. Knowledge is the internalization of information, data, and experience. *Tacit Knowledge* is the personal knowledge resident within the mind, behavior and perceptions of individual members of the organization. *Explicit Knowledge* is the formal, recorded, or systematic knowledge in the form of scientific formulae, procedures, rules, organizational archives, principles, etc., and can easily be accessed, transmitted, or stored in computer files or hard copy.

*Knowledge map* is a tangible representation or catalog of the concepts and relationships of knowledge. The catalog is a navigational aid that enables a user to find the desired concept, and then retrieve relevant knowledge sources. Knowledge (including data and information) is often stored in a text and/or numerical format and must be accessible for everyone, who needs it. That's why the knowledge maps of the organization are constructed.

According to Grey [1], a *knowledge map* is a navigation aid to explicit and tacit knowledge, illustrating how knowledge flows throughout an organization. The knowledge map portrays the sources, flows, constraints and terminations of knowledge within an organization. Knowledge mapping helps to understand the relationships between knowledge stores and dynamics.

### 2.2. Mathematical model

Mathematical model is the use of mathematical language to describe the behavior of a system. Mathematical models are used in particularly in the sciences such biology, electrical engineering, physics but also in other fields such as economics, sociology and political science.

Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations.

A mathematical model usually describes a system by a set of variables and a set of equations that establish

relationships between the variables. The values of the variables can be practically anything; real or integer numbers, boolean values or strings, for example. The variables represent some properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, event occurrence (yes/no). The actual model is the set of functions that describe the relations between the different variables.

According to Berka [7], the most important properties of mathematical models are as follows:

1. Generality – It is possible to use one mathematical model for many object properties and intra-relationships description.
2. Shortness and accuracy – There are a lot of implicit knowledge about object in mathematical models, which can be converted to explicit using exact mathematical tools.
3. Simple verification of hypothesis – Hypothesis can be formulated exactly and proved by mathematical methods.

These properties of mathematical modes will be used for knowledge description and mapping.

The game theory model will be used in the case study.

### 2.3. The game theory model

The game theory is a branch of mathematics that uses models to study interactions with formalized incentive structures ("games"). It has applications in a variety of fields, including economics, evolutionary biology, political science, and military strategy. Game theorists study the predicted and actual behavior of individuals in games, as well as optimal strategies. Seemingly different types of interactions can exhibit similar incentive structures, thus all exemplifying one particular game.

The game theory model consists of several elements:

*Players* – Subjects of decision-making process. They are interested in the results of the conflict situation.

*Actions* – Decision alternatives for both players.

*Payoffs* – Results of the conflict situation. They are written into payoff matrix, which gives the result of game for each combination of strategies.

*Zero-sum game* describes a situation in which a participant's gain (or loss) is exactly balanced by the losses (or gains) of the other participant(s). It is so named because when it is added up the total gains of the participants and subtract the total losses then they will

sum to zero. Cutting a cake is zero-sum because taking a larger piece for reducing the amount of cake available for others. Situations where participants can all gain or suffer together, such as a country with an excess of bananas trading with an other country for their excess of apples where both benefit from the transaction, are referred to as non-zero-sum.

*Minimax* is a method in decision theory for minimizing the expected maximum loss. It started from two player zero-sum game theory, covering both the cases where players take alternate moves and those where they make simultaneous moves. It has also been extended to more complex games and to general decision making in the presence of uncertainty.

*The minimax algorithm* is a recursive algorithm for choosing the next move in a two-player game. A value is associated with each position or state of the game. This value is computed by means of a position evaluation function and it indicates how good it would be for a player to reach that position. The player then makes the move that maximizes the minimum value of the position resulting from the opponent's possible following moves. If it is A's turn to move, A gives a value to each of his legal moves.

The result of the minimax algorithm is determination of *saddle point* of the game. Saddle point gives the payoff for combination of optimal (pure) strategies of both players. If some of them choose some other strategy than his optimal, he lost more than he have to or win less than he can.

### 3. CASE STUDY

#### 3.1. The Story

Knowledge can be described by a story. Let us quote the old King's Solomon story from the Holy Bible [8].

"Solomon returned to Jerusalem, stood before the ark of the Lord's covenant and sacrificed burnt offerings and fellowship offerings. Then he gave a feast for all his court.

Now two prostitutes came to the king and stood before him. One of them said, "My lord, this woman and I live in the same house. I had a baby while she was there with me. The third day after my child was born; this woman also had a baby. We were alone; there was no one in the house but the two of us.

"During the night this woman's son died because she lay on him. So she got up in the middle of the night and took my son from my side while I your servant was asleep. She put him by her breast and put her dead son

by my breast. The next morning, I got up to nurse my son — and he was dead! But when I looked at him closely in the morning light, I saw that it wasn't the son I had borne."

The other woman said, "No! The living one is my son; the dead one is yours." But the first one insisted, "No! The dead one is yours; the living one is mine." And so they argued before the king.

The king said, "This one says, 'My son is alive and your son is dead,' while that one says, 'No! Your son is dead and mine is alive.' ". Then the king said, "Bring me a sword." So they brought a sword for the king. He then gave an order: "Cut the living child in two and give half to one and half to the other."

The woman whose son was alive was filled with compassion for her son and said to the king, "Please, my lord, give her the living baby! Don't kill him!" But the other said, "Neither I nor you shall have him. Cut him in two!"

Then the king gave his ruling: "Give the living baby to the first woman. Do not kill him; she is his mother." When all Israel heard the verdict the king had given, they held the king in awe, because they saw that he had wisdom from God to administer justice."

#### 3.2. Mathematical model

The knowledge included in the previous story can be described by mathematical model based on the game theory.

The model is defined as follows:

Player 1: right mother of living child (P1)

Player 2: mother of dead child (P2)

Strategies: Each woman has three strategies:

1. "The living child is mine!" (Give me)
  2. "Cut the child in two!" (Cut him)
  3. "Give her baby, do not kill him!" (Give her)
- These strategies are the same for both women.

Payoffs: Result of Solomon's decision. There are three possible results:

- a) The child returns to his right mother (right)
- b) The false mother gets the child (false)
- c) The child will be killed (death)

Let's suppose following payoff matrix based on Solomon's decision in the story:

	P2	Give me	Cut him	Give her
P1				
Give me		death	right	right
Cut him		false	death	false
Give her		false	right	death

Now it is possible to try to find the saddle point of this game. The saddle point will be searched respecting the fact that the utility of the payoffs are not symmetric. The Solomon's decisions are sorted from the point of view both women in the table:

Utility	Right mother	False mother
The best	right	false
Worse	false	death
The worst	death	right

In such case, the saddle point is determined as

	P2	Give me	Cut him	Give her	Result
P1					
Give me		death	right	right	death
Cut him		<b>false</b>	death	false	<i>false</i>
Give her		<b>false</b>	right	death	<i>false</i>
Result		<i>false</i>	right	right	

The game is solvable in pure strategies. It has two saddle points. If both women choose their optimal strategy, the false mother will obtain child.

It is obvious that the false mother chose bad (even dominated!!!) strategy. Thus, the right mother won more and the baby returned to her.

### 3.3. Knowledge inference

There was described how it is possible to transform one specific story, which includes knowledge, into the form of the mathematical model in the chapter 4.2. Respect to Stuhlman's definition of knowledge map, that mathematical model can be declared as a specific type of it. The knowledge from the story is described, stored and represented in terms of game theory model. So the knowledge is accessible to everyone, who needs it.

The story, as it is described by text, is very instructive. But the mathematical form of the story allows to the user to inference and to derive new knowledge from it.

The most important general consequences, which can be derived from the story in the form of knowledge, are:

1) Compare your possibilities with your opponent's ones.

2) Do not decide under emotion stress.

3) Check the dominance relationships between your strategies. Do not choose dominated strategy.

4) When the saddle point of the game exists, do not use any other strategy than your optimal is. You can not win more; you can only loose and obtain some worse result.

## 4. CONCLUSIONS

Mathematical models are often used during systems analysis procedure for solving many complex problems. Their potential as some specific type of knowledge map is demonstrated in the article. The case study showed how to use them for description and storing knowledge as well as a tool for inferencing new ones.

The critical point of utilization of this access seems to be the interface factor. The user has to be able to recognize the relevant type of the mathematical model, create it, configure and transform to the knowledge map. Farther research will be dealt with this problem.

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