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Joint elicitation of elasticity of intertemporal substitution, risk and time preferences

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Abstract

The elicitation of the elasticity of intertemporal substitution (EIS), discount factor and risk attitude parameters in dynamic models is of central importance to economics, finance and public policy. This paper suggests an alternative method to jointly elicit and estimate these three parameters using experimental data. We employ a new model based on dynamic quantile preferences, where individuals maximize the stream of future τ-quantile utilities, for $\tau \in (0,1)$. These preferences are simple, dynamically consistent and monotonic. In the quantile model, the risk attitude is captured by the quantile τ of the payoff distribution, while the EIS and the discount factor are related to the utility function describing individual's intertemporal behaviour, hence allowing for complete separability between risk, EIS and discount factor. The estimation of the parameters of interest uses a structural maximum likelihood method. Individual's risk aversion is estimated below the median. The discount factor is marginally smaller than estimates reported in the literature, and the EIS is slightly larger than one, which suggests that utility over time is concave. The estimates for the elasticity contrast with those reported by the existing studies using observational disaggregated data, which in general find an elasticity smaller than one.

KEYWORDS

discount factor, elasticity of intertemporal substitution, experiment, quantile preferences, risk attitude

1 | INTRODUCTION

Elasticity of intertemporal substitution (EIS), discount factor and risk attitude are central to many branches of economics, finance and public policy. See, among many others, Bansal and Yaron (2004), Guvenen (2006), Epstein et al. (2014), Brown and Kim (2014), Havranek (2015), Thimme (2017), Crump et al. (2019), Meissner and Pfeiffer (2022) and references therein, for some recent contributions. These three parameters are relevant to characterize individuals' preferences over risk and time, and affect how consumers transfer wealth across periods and respond to monetary and fiscal policies.

Given their importance, eliciting their values has attracted the effort of many researchers in macroeconomics, econometrics and experimental economics. For instance, the metanalysis conducted by Havranek (2015) included 169 articles with 2735 estimates of the EIS using consumption Euler equations. The experimental literature on the topic is already sizable and continues to grow at a fast pace. This includes, among others, Von Gaudecker et al. (2011), Brown and Kim (2014), Masatlioglu et al. (2023), Andersen et al. (2018), Nielsen (2020) and Meissner and Pfeiffer (2022).

Many (but not all) of those articles use the Epstein-Zin-Weil model of dynamic choices (see Epstein and Zin (1989) and Weil (1989)), instead of the more familiar dynamic expected utility (EU) model with exponential discount. The reason for this choice is the fact that in the EU model, risk aversion and the EIS cannot be disentangled, a drawback that has been at the centre of economists' attention at least since Hall (1978, 1988). Epstein-Zin-Weil preferences are able to separate those parameters, but may seem complex and fail to satisfy desirable properties, such as monotonicity.[1] Epstein et al. (2014) investigate the magnitude of timing premia in the Epstein-Zin (EZ) model adopted by Bansal and Yaron (2004) and conclude that the implied levels of timing premia 'seem implausible' (p. 2693). Meissner and Pfeiffer (2022) confirm with an experiment that the time premia for EZ is not realistic.

This paper contributes to this growing experimental literature by adopting an alternative framework based on dynamic quantile preferences (QP) to elicit and estimate the time discount factor, risk aversion and EIS using experimental data. Under QP, individuals maximize the stream of future τ-quantile utilities, for $\tau \in (0,1)$.[2] To understand how QP work, consider a random stream of consumption $\widetilde{c} = (\widetilde{c}_1, \widetilde{c}_2, ..., \widetilde{c}_t, ...)$ that, for simplicity, the c_t's are independent (but not necessarily identically distributed).[3] Consumption in period t is evaluated through a utility function, say, $U(\widetilde{c}_t) = \frac{(\widetilde{c}_t)^{1-\gamma}}{1-\gamma}$. While an expected utility maximizer evaluates this stream of consumption according to $\sum_{t=0}^{\infty} \beta^t E[U(\widetilde{c}_t)]$, a τ-quantile maximizer evaluates this as $\sum_{t=0}^{\infty} \beta^t Q_\tau[U(\widetilde{c}_t)]$, where $Q_\tau[X]$ the τ-quantile of X, that is, $Q_\tau[X] = F_X^{-1}(\tau)$ for F_X denoting the cumulative distribution function (c.d.f.) of X.

QP are simple, dynamically consistent and monotonic. In the QP setting, one can disentangle individuals' risk attitude from their intertemporal preferences, which are jointly determined by a diminishing marginal utility function and a positive time preference parameter.[4] Thus the risk attitude is entirely captured by the single-dimensional parameter τ, whereas in most models it depends on the whole utility function. The EIS is determined by the concavity of the instantaneous utility function, and individuals' time preference is determined by the discount factor.

Rather than devising a new experiment to elicit the parameters of interest, we take advantage of the field experiment conducted by Andersen et al. (2008) and use their data.[5] The experiment has two parts. In the first, participants engage in decisions between two risky lotteries using a multiple price list (MPL) design. This experimental design was originally introduced in Holt and Laury (2002). The second part of the experiment corresponds to the basic experimental design for eliciting individual discount introduced in Coller and Williams (1999) and Harrison et al. (2002), where participants engage in multiple horizon treatment binary decisions. It is important to notice that the same subjects perform both parts of the experiment. Hence, we employ the terminology 'joint elicitation' for all three parameters.

We first discuss the identification of the risk aversion, the discount factor and the EIS using QP. The identification of these three parameters use variations on both parts of the experiment. The first part of the experiment described above—decisions between two risky lotteries—allows us to elicit the risk attitude parameter, which is given by the representative quantile. We are able to identify the quantile because of the variability in the MPL probabilities and payoffs. We use the second part of the experiment to identify the discount factor and EIS. It is possible to identify these two parameters because there are variations on both time-horizon and interest rates, but no uncertainty. In particular, these variations allow us to elicit the time preferences together with the intertemporal substitution by considering different interest rates and time horizons across tasks.[6]

To estimate the parameters of interest, we adapt the structural estimation methods of Holt and Laury (2002) (see also Moffatt, 2016) to the quantile setting. We specify a parametric functional form for the underlying latent choice models from an index defined by the ratio of the lifetime utilities of each option offered to the individual in the different experiments. In addition, we specify an isoelastic utility function for modelling time preferences. Estimation is then implemented using a maximum likelihood estimator.

The empirical results show a risk attitude estimate, which is captured by the quantile τ, of 0.45. Hence, the result reveals evidence of mild risk aversion of individuals participating in the experiment. The estimate of the discount rate δ is 0.075, which implies a discount factor $1/(1+\delta)$ of 0.93. These estimates suggest that individuals are patient and discount similarly income received in different instances into the future.[7] The discount rate (factor) estimate obtained in this paper is slightly smaller (larger) than the estimate obtained in the related literature. Andersen et al. (2008) report a discount rate of 0.101 (0.908), which is computed using a standard EU model and likelihood function that estimate jointly the relative risk aversion coefficient and the discount factor, while Andersen et al. (2018) find a discount rate of 0.114 (0.898) using a weakly separable nonadditive intertemporal utility function.

In addition, the QP model also identifies the EIS coefficient that is not necessarily the reciprocal of the coefficient of relative risk aversion, but related to the curvature of the utility function. The QP model reports an estimate of the EIS coefficient of 1.083. This EIS estimate suggests that the instantaneous utility is somewhat close to linear, but significantly concave. Such finding is consistent with similar results regarding concavity of the utility, using different experimental data, such as Andreoni and Sprenger (2012a) and Cheung (2020). Moreover, this elasticity estimate has important implications to the debate. On the one hand, it is in line with recent empirical literature on long-risks asset pricing models, but on the other hand, it contrasts with estimates reported by studies using observational disaggregated data, which in general, find an elasticity smaller than one—see, for example, Thimme (2017) for a survey of the literature on the EIS coefficient.

The empirical estimates are robust to variations in several parameters of the intertemporal consumption experiment. We find that small changes in background income and the number of periods defining the intertemporal consumption problem do not affect the estimation of risk aversion. Similarly, the EIS and discount factor estimates show only small changes with respect to the benchmark model. Following the literature, we also consider potential heterogeneity of preferences due to individuals' observable characteristics; in particular, we focus on gender and age. Overall, the three parameter estimates show little variation with respect to the benchmark model once individuals' observable characteristics are accounted for. The empirical results in this paper highlight the importance of considering QP as an alternative paradigm to the EU model in dynamic settings.

The remainder of the paper is structured as follows. Section 2 reviews the QP model. Section 3 summarizes the experiments. Section 4 discusses the use of the QP model to separately identify risk aversion, time preferences and the EIS using data from these experiments. Section 5 introduces the methodology to estimate the model using structural maximum likelihood procedures. Section 6 presents the empirical results. Finally, Section 7 concludes. We conclude this introduction with a literature review, below.

1.1 | Literature review

Economists have developed different empirical methods to elicit dynamic preferences' parameters, and in different ways to estimate risk aversion, discount factor and EIS, although in most cases not all of them jointly. We focus here only on dynamic models, and we do not review the very extensive literature on estimating risk aversion in static models.

Among the many techniques employed recent studies have favoured multiple price lists (MPL) with monetary payments to elicit discount rates; see Harrison et al. (2002), Harrison et al. (2005), Andersen et al. (2008, 2014, 2018), Cheung (2015, 2020) among many others. Sources and types of data used in the different papers vary from field experiments (such as Andersen et al. (2008, 2014, 2018)) to lab experiments (such as Andreoni and Sprenger (2012a, 2012b)).

Despite the relative success of the MPL methodology, Andreoni and Sprenger (2012a) note that experimentally elicited discount rates from MPLs are frequently higher (implying lower discount factors) than what seems reasonable for economic decision making and propose to use convex time budgets.[8] Cheung (2015) investigates the robustness of this result to the experimental design and finds that the effect disappears when a MPL instrument is used instead of a convex time budget design. In a related study, Andreoni et al. (2015) compare the predictive ability of convex time budgets (CTB) with double multiple price lists (DMPL) for eliciting time preferences. These authors find that each method performs equally well within sample, however, the CTB significantly outperforms the DMPL on out-of-sample measures.

A related issue in dynamic models is the separation between the discount factor and the EIS. Both parameters together determine the consumer's willingness to substitute consumption over time. However, each parameter has a different role and interpretation as discussed above. This separation is of major importance for disentangling the effect of individual's impatience, which is given by the present value of future consumption and characterized by the discount factor, from the marginal rate of substitution as a result of individual's preferences over time. Similarly, risk aversion in dynamic models is usually identified as the inverse of the EIS. However, imposing the same utility function under risk and over time, that is, fixing the EIS as the reciprocal of risk aversion, may not only bias the estimates of risk aversion but also the estimation of the discount factor. To correct this, Von Gaudecker et al. (2011), Brown and Kim (2014) and Meissner and Pfeiffer (2022) estimate the three parameters using a EZ model, although the last paper also presents results that are 'model free'. In contrast, Andersen et al. (2018) try to perform a similar task using a generalized expected discounted utility (GEDU) model, which is discussed in more detail in Appendix A. However, the GEDU preference is not dynamically consistent and the preference at any given point in time depends on past consumption. Thus, given these significant shortcomings, empirical results using this preference are difficult to interpret and compare. In a related setting, Kapteyn and Teppa (2003) disentangle the discount factor from the EIS, but do not consider risk in their experimental setup. Other experimental papers have investigated the preference for the resolution of uncertainty and the timing of information, such as Masatlioglu et al. (2023) and Nielsen (2020).

2 | DYNAMIC QUANTILE MODEL

This section describes the quantile model. Section 2.1 reviews the economic model of intertemporal allocation of consumption considering a QP framework. Section 2.2 describes the measure of risk attitude under dynamic QP. In Section 2.3, we briefly discuss the separation of risk attitude and elasticity of intertemporal substitution (EIS) under QP.

2.1 | Dynamic model for quantile preferences

Before describing the economic model, recall the definition of the quantile function. Let X be a random variable and F_X (or simply F) denote its cumulative distribution function (CDF) such that $F_X(\alpha) \equiv \Pr[X \le \alpha]$. The quantile is the generalized inverse of F_X; more formally:

$$Q_\tau[X] \equiv \begin{cases} \inf\{\alpha \in \mathbb{R} : F_X(\alpha) \geqslant \tau\}, & \text{if } \tau \in (0,1] \\ \sup\{\alpha \in \mathbb{R} : F_X(\alpha) = 0\}, & \text{if } \tau = 0. \end{cases}$$

It is clear that if F is invertible, $Q_\tau[X] = F_X^{-1}(\tau)$. For convenience, throughout the paper we will focus on $\tau \in (0,1)$, unless explicitly stated otherwise.

A well-known and useful property of quantiles is 'invariance' with respect to monotonic transformations, that is, if $g : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function, then

$$Q_\tau[g(X)] = g(Q_\tau[X]). \tag{1}$$

In the dynamic QP setting, the economic agent decides on the intertemporal consumption and savings (assets to hold) over an infinity horizon economy, subject to a budget constraint. Let c_t denote the amount of consumption in period t. At the beginning of period t, consumer has x_t units of the risky asset, which produced real rate of return r_t. With wealth $(1+r_t)x_t$ at the beginning of period t, the consumer decides how many units of the risky asset x_{t+1} to save for the next period and her consumption c_t.[9]

The dynamic problem of interest for the consumer is to choose a sequence $(x_t)_{t=1}^{\infty}$ to maximize the following recursive equation:

$$V(x_t, r_t) = \max_{(x_s)_{s=t+1}^{\infty}} \{U(c_t) + \beta Q_\tau[V(x_{t+1}, r_{t+1}) | \Omega_t]\}, \tag{2}$$

subject to

$$\begin{aligned} &c_t + x_{t+1} \leqslant (1+r_t) \cdot x_t, \\ &c_t, x_{t+1} \geqslant 0. \end{aligned} \tag{3}$$

In this problem, $V(\cdot,\cdot)$ is the value-function, the quantile-τ is given, $\beta \in (0,1)$ is the discount factor, $U : \mathbb{R}_+ \to \mathbb{R}$ is the utility function, and Ω_t is the information set at time t.

The interpretation of the recursive problem in (2) is very similar to the standard expected utility model. The value function at time t is equal to the utility of consumption at time t plus the discounted value of the *τ-quantile*—instead of the expectation—of the value function at time $t+1$. An alternative representation for the quantile model uses recursive substitution as following. The value function at $t=0$ is given by

$$V(x_0, r_0) = U(c_0) + \beta Q_\tau[V(x_1, r_1) | \Omega_0], \tag{4}$$

and at $t=1$

$$V(x_1, r_1) = U(c_1) + \beta Q_\tau[V(x_2, r_2) | \Omega_1]. \tag{5}$$

By substituting (5) into (4)

$$\begin{aligned} V(x_0, r_0) &= U(c_0) + \beta Q_\tau[U(c_1) + \beta Q_\tau[V(x_2, r_2) | \Omega_1] | \Omega_0] \\ &= Q_\tau\left[Q_\tau\left[U(c_0) + \beta U(c_1) + \beta^2 V(x_2, r_2) | \Omega_1\right] | \Omega_{i0}\right], \end{aligned}$$

where we can move the terms $U(c_0)$ and $U(c_1)$ into $Q_\tau[\cdot|\Omega_1]$ because they are constant given the conditioning information. By recursively repeating this procedure, we obtain

$$\begin{aligned} V(x_0, r_0) &= U(c_0) + \beta Q_\tau[U(c_1) + \beta Q_\tau[U(c_2) + \cdots \\ &\quad + \cdots + \beta Q_\tau[U(c_{T-1}) + \beta Q_\tau[V(x_T, r_T) | \Omega_{T-1}] | \Omega_{T-2}] \,|\, \cdots \,|\, \Omega_1] \,|\, \Omega_0] \\ &= Q_\tau\left[Q_\tau\left[\cdots\left[Q_\tau\left[U(c_0) + \beta U(c_1) + \beta^2 U(c_2) + \cdots \right.\right.\right.\right. \\ &\quad \left.\left.\left.\left. + \cdots + \beta^{T-1} U(c_{T-1}) + \beta^T V(x_T, r_T) \,|\, \Omega_{T-1}\right] \,|\, \Omega_{T-2}\right] \,|\, \cdots \,|\, \Omega_1\right] \,|\, \Omega_0\right]. \end{aligned} \tag{6}$$

At this point, it is convenient to use the notation introduced by de Castro and Galvao (2019). We refer the reader to their paper for details. Let $Q_\tau^T[\cdot]$ denote T applications of the quantile operator, conditioned on the information sets Ω_{T-1}, Ω_{T-2}, ..., Ω_1, Ω_0, consecutively, as above. With this notation, (6) can be written as

$$V(x_0,r_0) = Q_\tau^T\left[\sum_{t=0}^{T-1}\beta^t U(c_t) + \beta^T V(x_T,r_T)\right]. \tag{7}$$

It should be noted that in the case that shocks r_t are independent, the successive conditional quantiles in (6) and (7) become simple quantiles and can be applied directly to each term $U(c_t)$. Therefore, in the case of independent shocks the right hand side of Equation (7) simplifies to $\sum_{t=0}^{T-1}\beta^t Q_\tau[U(c_t)] + \beta^T Q_\tau[V(x_T,r_T)]$.

Notice that as $T\to\infty$, $\beta^T V(x_T,r_T)\to 0$ if V is bounded. In fact, the right side of (7) also converges to a function that is denoted using the notation $Q_\tau^\infty[\cdot]$, that is,

$$V(x_0,r_0) = Q_\tau^\infty\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right] \equiv \lim_{T\to\infty} Q_\tau^T\left[\sum_{t=0}^{T-1}\beta^t U(c_t) + \beta^T V(x_T,r_T)\right]. \tag{8}$$

This recursive substitution problem is the exact analogue to that of the expected utility model with two main differences. First, differently from the expectation, the quantile is not a linear operator, and second, the law of iterated expectations is not valid for quantiles. If shocks are independent, however, the expression in the right hand side of (8) simplifies to $\sum_{t=0}^{\infty}\beta^t Q_\tau[U(c_t)]$.

The general theoretical properties of the quantile maximization model are established in de Castro and Galvao (2019). They show that the QP are *dynamically consistent*, monotone and that the optimization problem leads to a contraction, which therefore has a unique fixed point. This fixed point is the value function of the problem and satisfies the Bellman equation. They also prove that the value function is concave and differentiable, thus establishing the quantile analogue of the envelope theorem. Additionally, they derive the corresponding Euler equation.

2.2 | Risk attitude in the dynamic quantile model

Before we discuss the separation between risk attitude and EIS in the dynamic quantile model, we present the notion of risk attitude. Risk attitudes in the static quantile model were first studied by Mendelson (1987), Manski (1988) and Rostek (2010). de Castro and Galvao (2022) provide a discussion of the risk attitude in dynamic quantile models.

In order to discuss the risk attitude under QP, let us introduce the concept of quantile-preserving spreads, as introduced by Mendelson (1987). This is related to (and inspired by) the familiar Rothschild and Stiglitz (1970)'s mean-preserving spreads, which captures the notion of 'added noise'. That is, the intuition that Y is equal to X plus noise can be formalized either as Y is a mean-preserving spread of X or that Y is a quantile-preserving spread. The choice of the formalization is a subjective matter. We follow Mendelson (1987) and define:

Definition 2.1. (Quantile-preserving spread). *We say that Y is a τ-quantile-preserving spread of X if $Q_\tau[Y]=Q_\tau[X]=q$ and the following holds:*

i. $t<q \Rightarrow F_Y(t) \geqslant F_X(t)$;
ii. $t>q \Rightarrow F_Y(t) \leqslant F_X(t)$.

Y is a quantile-preserving spread of X if it is a τ-quantile-preserving spread of X for some $\tau\in(0,1)$.

Figure 1 below illustrates the c.d.f.'s of random variables Y and X when Y is a $\overline{\tau}$-quantile-preserving spread of X. This figure suggests that the choice of a τ-quantile maximizer or τ-decision maker (τ-DM) depends on whether τ is below or above the quantile $\overline{\tau}$ where the two c.d.f.'s cross. That is, when $\tau<\overline{\tau}$ as in Figure 1, a τ-DM prefers the safer asset X, $Q_\tau[X]\geqslant Q_\tau[Y]$. On the other hand, if $\tau>\overline{\tau}$, a τ-DM prefers the riskier asset Y, $Q_\tau[X]\leqslant Q_\tau[Y]$. Note that if $Q_\tau[Y]=q$ and X is equal to q with probability 1, then Y is a τ-quantile-preserving spread of X. In other words, any risky asset Y with τ-quantile q is a quantile-preserving spread of any riskless asset X that takes the value q (with certainty).

Mendelson (1987) formalizes other four conditions and shows that they are all equivalent to the above definition; see his paper for further discussion and intuition. Notice that this definition captures the notion that Y is riskier than X, since it puts weight in more extreme values than X. Manski (1988) uses a different terminology for the same concept referring to the property of 'single crossing from below': F_X crosses F_Y from below when Y is a quantile-preserving spread of X. The following result formalizes this intuition for the simple static case.

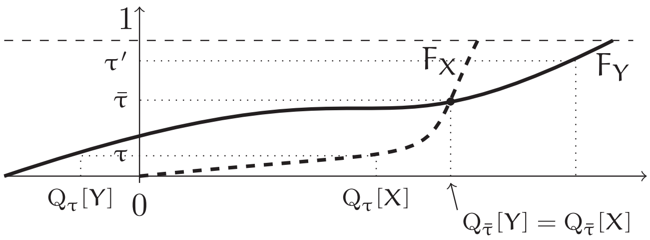

FIGURE 1 Y is a $\overline{\tau}$-quantile-preserving spread of X.

Proposition 2.2. (Manski). *Let Y be a $\overline{\tau}$-quantile-preserving spread of X for $\overline{\tau} \in (0,1)$. Then*:

i. $\tau \leqslant \overline{\tau} \Rightarrow Q_\tau[X] \geqslant Q_\tau[Y]$, *that is, a τ-DM prefers the less risky asset X if $\tau \leqslant \overline{\tau}$;*
ii. $\tau \geqslant \overline{\tau} \Rightarrow Q_\tau[X] \leqslant Q_\tau[Y]$, *that is, a τ-DM prefers the riskier asset Y if $\tau \geqslant \overline{\tau}$.*

In other words, risk-attitude can be related to the quantile rather than to the concavity of the utility function. However, the static model does not have the concept of intertemporal substitution. Risk attitudes are shown similarly in the dynamic model by de Castro and Galvao (2022). To fix ideas, consider QP $\succeq^i$, $i \in \{1,2\}$ and $a = (a_0, a_1) \in \mathcal{A}$ a consumption path, so that they are represented by V^i satisfying the following recursive equation:

$$V^i(a) = u(a_0) + \beta Q_{\tau^i}\left[V^i(a_1)\right].$$

As Epstein and Zin (1989), de Castro and Galvao (2022) adapt definition in Rostek (2010) from the static to the dynamic case as follows: we say that $\succeq^1$ is more risk averse than $\succeq^2$ if, for all $c^\infty \in C^\infty$ and $a \in \mathcal{A}$,

$$c^\infty \succeq^2 a \Rightarrow c^\infty \succeq^1 a. \tag{9}$$

Observe that if Y is a deterministic prospect, it crosses from below any other distribution. Since c^∞ is a deterministic prospect, this justifies (9). We have the following:

Lemma 2.3. (de Castro & Galvao, 2022). *$\succeq^1$ is more risk averse than $\succeq^2$ if and only if $Q_{\tau^1}[\cdot] \leq Q_{\tau^2}[\cdot]$, which is equivalent to $\tau^1 \leq \tau^2$.*

Hence, as in Manski (1988) and Rostek (2010), the dynamic quantile model admits a notion of comparative risk attitude, which is captured by τ. This implies that an agent with a quantile given by τ_1 is more risk averse than another agent with quantile given by τ_2 if $\tau_1 < \tau_2$, independently of the functional form of the utility. Thus, a decision maker that maximizes a lower quantile is more risk averse than one who maximizes a higher quantile. In other words, the risk attitude is defined by the quantile rather than by the concavity of the utility function. Moreover, this definition of risk allows for the risk attitudes to be disentangled from the degree of intertemporal substitutability, as we discuss next.

2.3 | Separation of EIS and risk attitude

It is illustrative to compare the quantile maximization model in Equation (2) with its counterpart from the expected utility maximization. It is well known that the value function for the expected utility has the following recursive representation:

$$V(x_t, r_t) = U(c_t) + \beta E[V(x_{t+1}, r_{t+1}) | \Omega_t]. \tag{10}$$

When comparing Equations (2) and (10), one can notice that these equations share similarities and differences. Regarding the similarities, both equations describe recursive models, and both expressions are similar. First, naturally, a conditional quantile function captures uncertainty in the quantile model, while a conditional expectation captures uncertainty in the expected utility case. Second, the expressions inside the conditional quantile and conditional expectation are essentially the same. Regarding the differences, whereas the standard expected utility model (10) is not able to separate the EIS from the risk aversion (see, e.g. Hall (1988)), the quantile utility model is able to do this. Since the seminal works of Kreps and Porteus (1978) and Epstein and Zin (1989), it is well understood that a separation between risk and intertemporal attitudes is possible only if the timing of the resolution of uncertainty matters. More recently, Bommier et al. (2017, Proposition 3) show that scale-invariant certainty equivalents generate what they call restricted indifference towards the timing of resolution of uncertainty. This is, in a sense, the weakest form of indifference towards the timing of the resolution of uncertainty that still accommodates the separation between risk and intertemporal substitution attitudes. Thus, it is important to note that since the quantile certainty equivalent operator is scale-invariant, it belongs to this selected class and thus allows for this separation.

Here we briefly illustrate how this separation can be achieved with the quantile model. Consider an isoelastic utility function $U(c) = c^{1-\gamma}$. When $\gamma \in (0,1)$ we have the case of risk aversion in the standard expected utility model. In particular, when $\gamma^1 > \gamma^2$, individual 1 is more risk averse than individual 2, in the sense that individual 1 has a higher coefficient of relative risk aversion. However, under static QP, any $\gamma > 0$ leads to exactly the same choices, as discussed above, since $Q_\tau\left[c_1^{1-\gamma}\right] = (Q_\tau[c_1])^{1-\gamma}$ by (1). In other words, the parameter γ does not capture any aspect of the decision maker attitude towards risk for the static QP.[10] On the other hand, under a multiperiod horizon the parameter γ plays an important role. In fact, consider the quantile recursive Equation (2) with the same utility index above, that is,

$$V(c_0, c_1) = c_0^{1-\gamma} + \beta Q_\tau\left[c_1^{1-\gamma}\right]. \tag{11}$$

Applied to a deterministic prospect, this yields $V(c_0, c_1) = c_0^{1-\gamma} + \beta c_1^{1-\gamma}$ being an intertemporal isoelastic utility function. Recall that EIS measures the elasticity of the ratio (c_1/c_0) to a change in the marginal rate of substitution between c_0 and c_1, that is $MRS_{c_1,c_0} = U'(c_1)/U'(c_0)$, with $U'(\cdot)$ denoting marginal utility. Hence, using the standard definition, $\text{EIS} = -d\ln(c_1/c_0)/d\ln MRS_{c_1,c_0}$, it is easy to see that the EIS in this case is simply $\frac{1}{\gamma}$. The EIS measures how willing individuals are to substitute intertemporally between consumption this period and consumption next period.[11]

It is useful to compare this quantile method with the most widely used method to separate risk aversion and the EIS, which is the following specification of Epstein and Zin (1989) and Weil (1990), with $\gamma \neq 0$, $\alpha \neq 0$:

$$V^{EZ}(c_0, c_1) = \left(c_0^\alpha + \beta\left(E\left[c_1^{1-\gamma}\right]\right)^{\frac{\alpha}{1-\gamma}}\right)^{\frac{1}{\alpha}},$$

where the parameter α determines the EIS, given by $1/1-\alpha$, and the parameter γ captures risk aversion, with larger values of γ, other things equal, implying a stronger aversion to risk. As observed by Bommier et al. (2017), this model satisfies monotonicity if and only if $\gamma = \alpha$, in which case the model collapses to the standard expected utility model and fails to satisfy the separation of risk aversion and EIS. For achieving its goal the popular Esptein-Zin-Weil preferences are necessarily non-monotonic as previously discussed.

In sum, the simple standard expected utility model in (10) is not able to separate the EIS from risk aversion, Epstein-Zin preferences are able to do so at the expense of violating the condition of monotonicity of preferences. In contrast, the simple QP is able to disentangle the EIS from risk attitude as well as preserving the monotonicity of preferences.

3 | EXPERIMENTAL PROCEDURES

The experimental procedures used in this paper are documented in detail in both Harrison et al. (2005) and Andersen et al. (2008).[12] In this section we just review the basics. In summary, the experiment is divided into two parts. In the first part each subject was asked to answer to static tasks related to risk attitude. In the second part subjects respond to intertemporal tasks, which are related to discount factor and the elasticity of intertemporal substitution (EIS).[13] Each such task involved a series of binary choices that we will use to infer risk, EIS and time preferences.

The first part of the experiment involves multiple price list (MPL) risky lotteries with immediate reward and contributes to identify the risk aversion coefficient for an isoelastic utility function. Each subject is presented with a choice between two lotteries, which we can call A or B. Table 1 illustrates a basic payoff matrix presented to subjects in the experiments. The first row shows that lottery A offered a 10% chance of receiving 2000 Danish kroner (DKK) and a 90% chance of receiving 1600 DKK. Similarly, lottery B in the first row has chances of payoffs of 3850 and 100 DKK. The columns $Q_{0.5}[A]$ and $Q_{0.5}[B]$ show the quantile $\tau = 0.5$ for lotteries A and B, respectively, although these columns were not presented to subjects. The two lotteries have a relatively large difference in median values.

The subject chooses A or B in each row, and one row is later selected at random for payout for that subject. The logic behind this test for risk attitude is that, for a fixed quantile, $\tau = 0.5$ for example, subjects would take lottery B for the last six rows and only would take lottery A in the first four rows. Now, by varying the probabilities in the MPL we are able to identify the underlying quantile. We take each of the binary choices of the subject as the data, and estimate the latent quantile parameter that explains those choices using an appropriate error structure to account for the panel nature of the data. For a candidate value of the τ parameter, we can construct the quantile of the two gambles and then use a linking function to infer the likelihood of the observed choice. We discuss statistical specifications in more detail below.

The second experiment uses lotteries and tasks that involve no risk and pay off at different periods that are rewarded after a few months. This part of the experiment was introduced in Coller and Williams (1999) and Harrison et al. (2002) for eliciting individual discount rates. Table II in Andersen et al. (2008) – which we reproduce below as Table 2 for completeness—presents an example of the payoffs for the discount rate experiment when the

TABLE 1 Typical payoff matrix in the risk aversion experiments.

Lottery A				Lottery B						Difference
p	DKK	p	DKK	p	DKK	p	DKK	$Q_{0.5}[A]$	$Q_{0.5}[B]$	$Q_{0.5}[B]-Q_{0.5}[A]$
0.1	2000	0.9	1600	0.1	3850	0.9	100	1600	100	−1500
0.2	2000	0.8	1600	0.2	3850	0.8	100	1600	100	−1500
0.3	2000	0.7	1600	0.3	3850	0.7	100	1600	100	−1500
0.4	2000	0.6	1600	0.4	3850	0.6	100	1600	100	−1500
0.5	2000	0.5	1600	0.5	3850	0.5	100	2000	3850	1850
0.6	2000	0.4	1600	0.6	3850	0.4	100	2000	3850	1850
0.7	2000	0.3	1600	0.7	3850	0.3	100	2000	3850	1850
0.8	2000	0.2	1600	0.8	3850	0.2	100	2000	3850	1850
0.9	2000	0.1	1600	0.9	3850	0.1	100	2000	3850	1850
1	2000	0	1600	1	3850	0	100	2000	3850	1850

TABLE 2 Payoff table for 6 month time horizon in the discount rate experiments.

Payoff alternative	Payment option A (Pays amount below in 1 month)	Payment option B (Pays amount below in 7 months)	Annual interest rate (AR in person)	Annual effective interest rate (AR in person)	Preferred payment options (Circle A and B)	
1	3000 DKK	3075 DKK	5	5.09	A	B
2	3000 DKK	3152 DKK	10	10.38	A	B
3	3000 DKK	3229 DKK	15	15.87	A	B
4	3000 DKK	3308 DKK	20	21.55	A	B
5	3000 DKK	3387 DKK	25	27.44	A	B
6	3000 DKK	3467 DKK	30	33.55	A	B
7	3000 DKK	3548 DKK	35	39.87	A	B
8	3000 DKK	3630 DKK	40	46.41	A	B
9	3000 DKK	3713 DKK	45	53.18	A	B
10	3000 DKK	3797 DKK	50	60.18	A	B

time horizon is 6 months. Subjects are presented with a payoff table with 10 symmetric intervals that provides the annual and annual effective interest rates of each different payment option. This is important because it introduces changes in the interest rates. Moreover, the experiment uses multiple-horizon treatment, where the subjects are presented with six discount rate tasks, corresponding to six different time horizons: 1 month, 4 months, 6 months, 12 months, 18 months and 24 months. In each task, subjects are provided two future income options rather than one 'instant income' option and one future income option. The experiment uses a delay of 1 month to the early income option in all tasks. Thus, there is variation on the time horizon that subjects face. Subjects were asked to choose between two options for each of the 10 payoff alternatives. We highlight that these two types of variation—on both interest rates and time horizon—are very important for our purposes of identifying the discount rate and EIS because they allow us to identify both discount factor and EIS, see also Barsky et al. (1997) for a similar identification strategy.

4 | IDENTIFYING RISK, EIS AND THE DISCOUNT RATE

In this section we present the methods used to disentangle risk attitude from the elasticity of intertemporal substitution (EIS) and discount factor under QP. We compare our methodology with that in Andersen et al. (2008) for the standard EU model. Under the EU framework, the parameters of interest—risk aversion and discount rate—characterize the following time separable intertemporal utility function to describe individuals' preferences over their lifetime:

$$V_t^{EU} = \sum_{i=0}^{\infty} \frac{1}{(1+\delta)^i} E[U(M_{t+i})], \quad (12)$$

where δ is the discount rate, M_{t+i} denotes income received by the individual at period $t+i$, and $U(\cdot)$ is a period utility function. The risk aversion parameter is captured by the curvature of the utility function in (12), for instance, for the isoelastic utility function specification,

$$U(M) = \frac{M^{1-\gamma}}{1-\gamma}, \quad (13)$$

the risk aversion is γ.

In this paper, we use the QP model, as discussed in Section 2.1, to elicit and identify individual's risk attitude, EIS and associated discount rate. Individuals with QP have the following intertemporal objective function

$$V_t^{QP} = Q_\tau^\infty \left[\sum_{i=0}^{\infty} \frac{1}{(1+\delta)^i} U(M_{t+i}) \right],$$

where Q_τ^∞ is defined in (8). For ease of notation, we use the same parameter δ to denote the discount rate under QP. Thus, under independence of the sequence of shocks the intertemporal objective function can be expressed as

$$V_t^{QP} = \sum_{i=0}^{\infty} \frac{1}{(1+\delta)^i} Q_\tau[U(M_{t+i})]. \quad (14)$$

In the QP case, we have three parameters of interest. First, the quantile τ captures the risk attitude. Second, the discount rate is δ. Finally, as discussed in Equation (11), Section 2.3, the EIS is given by the curvature of the utility function, which under the isoelastic utility is $1/\gamma$.

The first multiple price list contains tasks that involve some risk and are immediately rewarded, and the second list is defined by tasks that involve no risk and are rewarded with at least a one-month delay. With an appeal to the dual-selves model of choice (Benhabib and Bisin (2005), Fudenberg and Levine (2006)), the responses to the first set of tasks are probably temptation driven, while the responses to the latter set are probably self-controlled. In this setting, the two different behaviours given by risk aversion and discounting are revealed in the responses to each list.

4.1 | Decision between pairwise risky lotteries

We consider the first experiment (risk aversion) and describe the decision of an agent when making a pairwise choice between two risky lotteries A and B, and how it relates to the risk attitude parameter τ.

The payoffs of both lotteries are paid immediately so individuals' decision is driven by temptation. First, to fix ideas, we present the condition determining indifference between A and B under a EU setting. The objective function (12) reduces to a single-period utility function and the EU preference can be defined as

$$A \succeq B \Leftrightarrow EU^A \equiv E\left[U\left(M^A\right)\right] \geqslant E\left[U\left(M^B\right)\right] \equiv EU^B, \quad (15)$$

with M^A and M^B the payments from the risky lotteries A and B. The agent maximizing EU prefers B to A when the expected value of lottery B is larger than A, that is, when $\Delta EU = EU^B - EU^A > 0$.

The decision of an agent with QP differs. In this setting, the agent maximizes a given quantile τ of the distribution of the risky lotteries A and B. To show this, we note a very important feature of the static model that is the invariance property with respect to the utility function. Let $U(X)$, where $U:\mathbb{R}\to\mathbb{R}$, be an increasing utility function describing an individual's preferences. Then, for a given quantile $\tau \in (0,1)$, the optimization problem is

$$\max_{X \in \mathscr{R}^*} Q_\tau[U(X)], \quad (16)$$

where $\mathscr{R}^* \subset \mathscr{R}$ is the subset of random variables (lotteries) available. Given the invariance property described in (1) it can be directly seen that the maximization argument x^* solves (16) if and only if it solves

$$\max_{X \in \mathscr{R}^*} Q_\tau[X]. \quad (17)$$

Equations (16) and (17) show that the quantile optimization problem, for a given utility function, is equivalent to maximizing the quantile of the distribution of the random variable X. Hence, the optimal choice under QP does *not* depend on any particular specification of the utility function. Within the class of QP models, an agent's choices are determined by the characteristic quantile, that is,

$$A \succeq B \Leftrightarrow QP_\tau^A \equiv Q_\tau\left[U\left(M^A\right)\right] \geqslant Q_\tau\left[U\left(M^B\right)\right] \equiv QP_\tau^B$$
$$\Leftrightarrow Q_\tau\left[M^A\right] \geqslant Q_\tau\left[M^B\right].$$

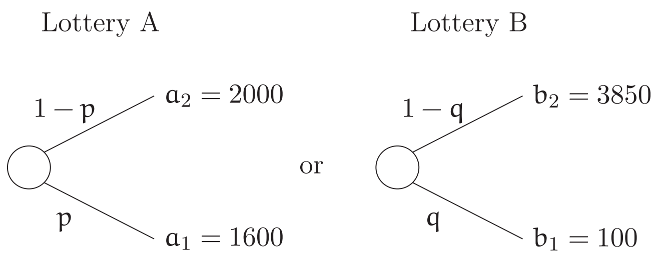

FIGURE 2 Example of lottery choice task as in Table 1.

Thus, the agent maximizing a given quantile τ prefers B to A when, for that τ, the quantile of lottery B is larger than A. That is,

$$\Delta Q(\tau) \equiv Q_\tau\left[M^B\right] - Q_\tau\left[M^A\right] > 0. \tag{18}$$

We now explore the suitability of the QP methodology for the experimental data in Harrison et al. (2005). There are two lotteries, A and B, each of them have two possible payoffs, $M^A = \{a_1, a_2\}$ and $M^B = \{b_1, b_2\}$, and corresponding outcome probabilities p and q. As an illustrative example, one of the pairs of risky payoffs that these authors consider are $a_1 = 1600$, for $0 < \tau \le p$, and $a_2 = 2000$, for $p < \tau \le 1$, and $b_1 = 100$, for $0 < \tau \le q$, and $b_2 = 3850$, for $q < \tau \le 1$. The choice between lotteries A and B for a $\tau-$individual is as follows;

- Lottery A yields $a_1 = 1600$ with probability p, and $a_2 = 2000$ with probability $1-p$.
- Lottery B yields $b_1 = 100$ with probability q, and $b_2 = 3850$ with probability $1-q$.

This lottery can be represented graphically, as illustrated in Figure 2.

The calculation of $\Delta Q(\tau) = Q_\tau\left[M^B\right] - Q_\tau\left[M^A\right]$ depends on the quantile τ, the payoffs $\{a_1, a_2, b_1, b_2\}$, and the probabilities (p, q). Specifically,

$$\Delta Q(\tau) = \begin{cases} b_1 - a_1, & \text{if } \tau \leqslant \min\{p, q\} \\ b_1 - a_2, & \text{if } p < \tau \leqslant q \\ b_2 - a_1, & \text{if } q < \tau \leqslant p \\ b_2 - a_2, & \text{if } \tau > \max\{p, q\}. \end{cases} \tag{19}$$

To make the example more specific, fix the quantile at the median, $\tau = 0.5$, and the outcome probabilities at $p = 0.3$ and $q = 0.1$. The quantile functions of lotteries A and B are plotted in the left panel of Figure 3. Lottery A (solid line) pays $a_1 = 1600$ with probability 0.3 and $a_2 = 2000$ with probability 0.7, and lottery B (dashed line) pays $b_1 = 100$ with probability 0.1, and $b_2 = 3850$ with probability 0.9. The solid vertical line at 0.5 represents the quantile of interest. To complete the example of an agent maximizing the median, $\tau = 0.5$, and choosing between lotteries A and B, we compute $\Delta Q(\tau)$. The calculation is simple and only requires one to subtract the quantile of A from that of B. From the left panel in Figure 3, we can see that $\Delta Q(0.5) = Q_{0.5}[B] - Q_{0.5}[A] = 3850 - 2000 = 1850$. Therefore, the agent chooses lottery B.

Suppose now that we modify the lotteries by changing the probability q in lottery B, so that we have:

- Lottery A: $a_1 = 1600$ with probability $p = 0.3$, and $a_2 = 2000$ with probability $1 - p = 0.7$;
- Lottery B: $b_1 = 100$ with probability $q = 0.8$, and $b_2 = 3850$ with probability $1 - q = 0.2$.

The quantile functions of the new A and B lotteries are displayed in the right panel of Figure 3. Lottery A (solid line) pays $a_1 = 1600$ with probability 0.3 and $a_2 = 2000$ with probability 0.7, and lottery B (dashed line) pays $b_1 = 100$ with probability 0.8, and $b_2 = 3850$ with probability 0.2. In this case, we can see that the calculation of $\Delta Q(0.5)$ for the median (solid vertical line) is $\Delta Q(0.5) = Q_{0.5,t}[B] - Q_{0.5,t}[A] = 100 - 2000 = -1900$, and hence, the agent chooses lottery A.

This subsection shows a very important difference between the QP and the EU models. The risk attitude in the QP is captured by the quantile τ and does not depend on the parameters of the utility function $U(\cdot)$. But in the EU model, risk is described by the parameters of the utility function, in particular, for an isoelastic utility function, the risk parameter is the curvature of the utility function, which captures simultaneously both risk and EIS by construction. The next section clarifies this point.

4.2 | Decision between pairwise certain lotteries in different time periods

The second experiment discussed in Andersen et al. (2008) is to elicit the discount factor. In this case, we consider the decision problem of an agent with QP when making a pairwise choice between options A and B, and how it relates to the parameters of interest, the discount factor and the EIS. There is no uncertainty in this part of the experimental design so individuals know exactly the income received in each period. The absence of uncertainty implies that the objective function of EU and QP are equivalent, that is, Equations (12) and (14) are equal under no uncertainty. Nevertheless, it is important to highlight that both equations are characterized by the discount rate δ and the parameter of the utility function $U(\cdot)$.

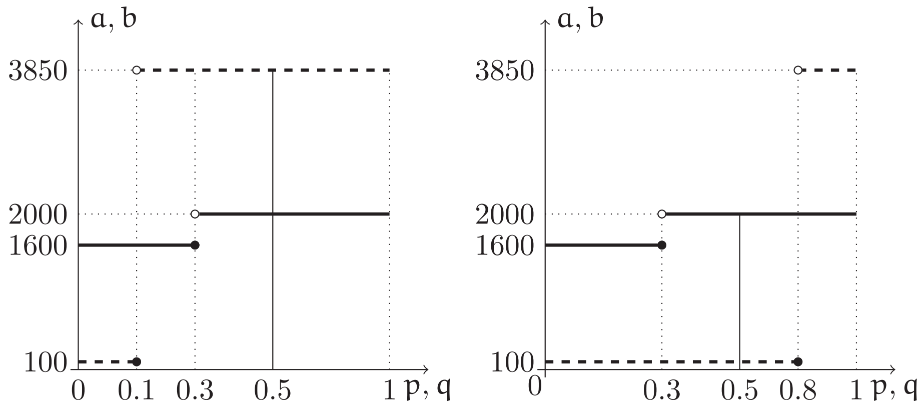

FIGURE 3 Quantile function of lotteries A (solid line) and B (dashed line). The left plot considers $p=0.3$ and $q=0.1$. The right plot considers $p=0.3$ and $q=0.8$.

In this experiment individuals have to choose between Options A and B. In Option A the individual receives an amount M^A (with a front end delay of 1 month with respect to the risk aversion task), while in Option B it receives an amount M^B with a T period delay with respect to Option A. Let the parameter λ define the number of periods over which the two monetary amounts are integrated with background consumption ω. Let the time t be the time period where the individual would receive the first payment from Option A. Then the stream of non-random incomes of the first option is $A=\left(\omega+\frac{M_t^A}{\lambda},\ldots,\omega+\frac{M_{t+\lambda-1}^A}{\lambda},\omega,\ldots,\omega\right)$ and the stream of non-random incomes of the second option is $B=\left(\omega,\ldots,\omega,\omega+\frac{M_{t+T}^B}{\lambda},\ldots,\omega+\frac{M_{t+T+\lambda-1}^B}{\lambda}\right)$. For this experiment $M_t^A=M^A$ and $M_t^B=M^B$ for all t. Furthermore, if $\lambda=1$, then the period of assumed consumption is the same of the received payment.

The discounted utility of Option A is then given by

$$PV^A=\sum_{i=t}^{t+\lambda-1}\frac{1}{(1+\delta)^{(i-t)}}U\left(\omega+M^A/\lambda\right)+\sum_{i=t+\lambda}^{t+T+\lambda-1}\frac{1}{(1+\delta)^{(i-t)}}U(\omega) \tag{20}$$

and the discounted utility of Option

$$B \tag{is}$$

$$PV^B=\sum_{i=t}^{t+T-1}\frac{1}{(1+\delta)^{(i-t)}}U(\omega)+\sum_{i=t+T}^{t+T+\lambda-1}\frac{1}{(1+\delta)^{(i-t)}}U\left(\omega+M^B/\lambda\right), \tag{21}$$

where the utility function is assumed to be stationary over time. The condition determining the indifference between Options A and B is, as before, given by the condition $\left(V_t^{EU}\right)^A=\left(V_t^{EU}\right)^B$ in (12) or, equivalently, $\left(V_t^{QP}\right)^A=\left(V_t^{QP}\right)^B$ in (14). Simple algebra shows that, for $\lambda=1$, this condition can be expressed as

$$U\left(\omega+\frac{M^A}{\lambda}\right)+\frac{1}{(1+\delta)^T}U(\omega)=U(\omega)+\frac{1}{(1+\delta)^T}U\left(\omega+\frac{M^B}{\lambda}\right), \tag{22}$$

where T denotes the time of the delayed payoff M^B of Option B with respect to Option A.

Now we illustrate the equality condition in (22), and relate it to the results on separability between EIS and risk attitude in Section 2.3. We assume a more familiar language and consider the income pairs $\left(\omega+\frac{M^A}{\lambda},\omega\right)$ and $\left(\omega,\omega+\frac{M^B}{\lambda}\right)$ to be the optimal intertemporal consumption choices of individuals under Options A and B, respectively.[14] Equation (22) shows that an individual is indifferent between consuming the pair $(c_t,c_{t+T})\equiv\left(\omega+\frac{M^A}{\lambda},\omega\right)$ and the pair $(c_t,c_{t+T})\equiv\left(\omega,\omega+\frac{M^B}{\lambda}\right)$. The equality condition in (22) implies that both consumption paths lie on the same indifference curve describing the individuals' optimal consumption choices at periods t and $t+T$. First, notice that the left hand side (or the right hand side) of Equation (22) is same as Equation (11) applied to deterministic prospects. Therefore, as discussed in Section 2.3, it follows that the EIS for a isoelastic utility function is $\frac{1}{\gamma}$. Second, the choice between Options A and B is driven by the individuals' willingness to substitute consumption across the two periods, which is determined from the first order conditions $U(c_t)\partial c_t+\beta^T U(c_{t+T})\partial c_{t+T}=0$ obtained from Equations (10) and (11), that are equal under no uncertainty, yielding the condition:

$$\frac{\partial c_{t+T}}{\partial c_t} = -\frac{1}{\beta^T}\frac{U'(c_t)}{U'(c_{t+T})}. \quad (23)$$

This marginal condition relates the curvature of the utility function, the discount rate $\beta = \frac{1}{1+\delta}$, and the differences between the short term and the long term, given by T, in the intertemporal experiment. For individuals with isoelastic preferences, as in (13), the curvature of the utility function is characterized by the parameter γ, hence, the willingness to substitute consumption across periods c_t and c_{t+T} is determined by the condition

$$\frac{\partial c_{t+T}}{\partial c_t} = -\frac{1}{\beta^T}\left(\frac{c_t}{c_{t+T}}\right)^{-\gamma}. \quad (24)$$

Replacing with the figures above, the willingness to substitute consumption across periods for individuals in the intertemporal experiment is $\frac{\partial c_{t+T}}{\partial c_t} = -\frac{1}{\beta^T}\left(\frac{\omega+\frac{M^A}{\lambda}}{\omega}\right)^{-\gamma}$ under Option A and $\frac{\partial c_{t+T}}{\partial c_t} = -\frac{1}{\beta^T}\left(\frac{\omega}{\omega+\frac{M^B}{\lambda}}\right)^{-\gamma}$ under Option B, which are functions of the EIS and discount factor.

It is important to notice that this argument holds under both EU and QP theories. The willingness to substitute consumption/income across periods is determined by the combination of the discount rate and the marginal rate of substitution across periods as discussed in Frederick et al. (2002) and illustrated in (23) and (24).

5 | ESTIMATION METHODS

We estimate the parameters of interest for the QP model using structural estimation. As mentioned above, and following recent experimental studies (see, e.g. Andersen et al. (2008), Andreoni and Sprenger (2012a, 2012b) and Cheung (2015)), we assume an isoelastic utility function $U(x) = x^{1-\gamma}/(1-\gamma)$. The variable x is the monetary payoff and γ is the curvature of the utility function.

$$\Delta V^{AB}(\theta) = \frac{\left(V^B\right)^{1/\theta}}{\left(V^A\right)^{1/\theta} + \left(V^B\right)^{1/\theta}}. \quad (25)$$

For the risk aversion (RA) experiment, using Equation (25), we define the latent index

$$\Delta V^{AB}_{RA}(\tau,\mu) = \frac{Q_\tau[B]^{1/\mu}}{Q_\tau[A]^{1/\mu} + Q_\tau[B]^{1/\mu}}, \quad (26)$$

where μ is a structural noise parameter used to allow randomness in the model.[15] The index ΔV^{AB}_{RA} is in the form of a cumulative probability distribution function defined over $Q_\tau[\cdot]$ of the two lotteries and the noise parameter μ. For empirical estimation, instead of using the latent index in Equation (18), we directly employ the ratio form in (26), which is already in the form of a probability between 0 and 1. Hence, we avoid taking the probit or logit transformation. The risk aversion coefficient is characterizing Equation (26).

Similarly, for the second experiment, analysing the discount rate (DR) and the elasticity of intertemporal substitution (EIS), the latent index (25) together with Equations (20)–(21) is equivalent to

$$\Delta V^{AB}_{DR,EIS}(\gamma,\delta,\nu) = \frac{\left(PV^B\right)^{1/\nu}}{\left(PV^A\right)^{1/\nu} + \left(PV^B\right)^{1/\nu}}, \quad (27)$$

with PV defined above, and $\nu > \mu$ a structural noise parameter introducing randomness in the specification. As before, the index $\Delta V^{AB}_{DR,EIS}$ is a cumulative probability distribution function defined over $Q_\tau[\cdot]$ of the two options and the noise parameter ν. The parameters δ and γ are related through Equation (27).

We estimate the parameters by maximum likelihood from expressions (26) and (27)[16]:

$$\ln L(\gamma,\delta,\tau,\mu,\nu) = \ell_{RA}(\tau,\mu) + \ell_{DR,EIS}(\gamma,\delta,\nu), \quad (28)$$

where $\ell_{RA}(\tau,\mu)$ and $\ell_{DR,EIS}(\gamma,\delta,\nu)$ can be defined by the following general expression:

$$\begin{aligned}\ell_* = \sum_{i=1}^{n}\sum_{j=1}^{m_i^*}\Bigg[&1\left[y_{ij}=B\right]\ln\left(\Delta V^{AB}_{i,*}\right)\\ +\ &1\left[y_{ij}=A\right]\ln\left(1-\Delta V^{AB}_{i,*}\right)\\ +\ &1\left[y_{ij}=I\right]\left(\frac{1}{2}\ln\left(\Delta V^{AB}_{i,*}\right)+\frac{1}{2}\ln\left(1-\Delta V^{AB}_{i,*}\right)\right)\Bigg],\end{aligned} \quad (29)$$

where we should substitute $* = RA$ or $* = DR,EIS$ and have omitted the parameters (τ,μ) and (γ,δ,ν). In (29), y_{ij} is the choice variable of individual $i = 1,2,\ldots,n$ in task $j = 1,2,\ldots,m_i$. Each individual receives tasks in a random order, where each task may be of the RA or (DR,EIS) type, and within those, it has several variations. In both cases, the choices available for each task option are to choose either A, B or being indifferent, denoted by I. Let m_i^{RA} be the number of tasks of individual i of the RA type and $m_i^{DR,EIS}$ the number of tasks of individual i of the (DR,EIS) type, and $m_i = m_i^{RA} + m_i^{DR,EIS}$.

The MLE of τ is simply the value of τ that maximizes the log-likelihood function in Equation (29). The log-likelihood function $\ell_{DR,EIS}(\gamma,\delta,\nu)$ is the same of the expected utility (EU) case because that experiment does not involve randomness. It is important to note, however, that in contrast to the EU case, we can separate the estimation of the risk aversion coefficient that is obtained from $\ell_{RA}(\tau,\mu)$, from the estimation of γ and δ, that are obtained from $\ell_{DR,EIS}$. This implies that even if we use the same likelihood function for the EU and QP exercises for the DR-EIS component, the estimates of γ, δ and ν will be different. The reason for this is that γ is obtained from the *joint* likelihood for EU case and is obtained from the marginal likelihood $\ell_{DR,EIS}(\gamma,\delta,\nu)$ for the QP case.

We remark that the log-likelihood function $\ln L(\gamma,\delta,\tau,\mu,\nu)$ in (28) is not smooth. Nevertheless, there is a large existing literature in econometrics establishing the asymptotic properties—consistency, asymptotic normality and bootstrap inference—for this class of semi-parametric estimators (as the MLE), where the criterion function does not obey standard smoothness conditions. The theories allow for non-smooth objective functions of finite-dimensional unknown parameters (e.g. Pakes and Pollard (1989) and Newey and McFadden (1994, section 7)) and both finite-dimensional and infinite-dimensional parameters (e.g. Chen et al. (2003)). In addition, Chen et al. (2003) show that bootstrapping for these methods provides asymptotically correct confidence regions for finite-dimensional parameters. Throughout the paper, we apply bootstrap procedures to compute the standard errors of the parameters of interest.

6 | EMPIRICAL RESULTS

In this section we report the maximum likelihood estimator (MLE) results for the QP approach. The discontinuity of the log-likelihood function $\ell_{RA}(\tau,\mu)$ in (29) with respect to τ implies that a closed-form solution cannot be obtained from the corresponding score function. Instead, we apply a numerical grid search method and maximize the likelihood over a fine grid of $\tau \in \{0.01,0.02,\ldots,0.99\}$.

6.1 | Estimating individual's risk aversion, the discount factor and the EIS

We compute estimates for pooled data, treating the entire sample as one 'representative' participant. The main results for the QP estimates are presented in Table 3 below. The table reports estimates along with the corresponding bootstrap standard errors and confidence intervals clustered by individuals.

TABLE 3 Estimates of risk, EIS and discount factor assuming exponential discounting. $\omega = 118$, $\lambda = 1$.

Parameter	Estimate	Std. Err.	Lower 95% CI	Upper 95% CI
τ	0.455	0.0059	0.443	0.467
γ	0.923	0.0287	0.866	0.979
EIS $(1/\gamma)$	1.083	0.0340	1.018	1.150
δ	0.075	0.0038	0.067	0.082
μ (for RA)	0.859	0.0200	0.821	0.898
ν (for DR)	0.005	0.0220	0.001	0.010

The point estimate for the risk aversion coefficient, τ, is 0.455 with standard error of 0.006, such that the estimate is statistically different from zero at standard levels of significance.[17] As discussed above, the risk parameter is obtained from the risk experiment. Moreover, the risk attitude in QP is relative, and a value of τ below the median can be considered, following Manski (1988), as risk aversion. Thus, the empirical evidence suggests a mild risk aversion of individuals participating in the experiment.

As discussed previously, estimation of the discount rate, δ and the EIS coefficient, $\frac{1}{\gamma}$, is achieved from the marginal likelihood function $\ell_{DR,EIS}(\gamma,\delta,\nu)$. This is an interesting feature of the QP model that is due to the separation between risk and time preferences. The estimate of the discount factor, δ, assuming background consumption $\omega = 118$ and $\lambda = 1$ as in Andersen et al. (2008) is $\widehat{\delta} = 0.075$ with corresponding standard error of 0.0038 (reported in Table 3). This discount rate estimate implies a discount factor, $\beta = 1/(1+\delta)$, of 0.93. These estimates suggest that individuals are patient and discount similarly income received in different instances into the future. The discount rate (factor) estimate obtained in this paper is slightly smaller (larger) than the estimate obtained in the related literature.

The results in Table 3 also include the EIS estimate. The QP model identifies the EIS coefficient, which is not related to risk aversion as it happens in the standard EU model. The model reports an estimate of the EIS coefficient of 1.083 with standard error of 0.034. The EIS is the reciprocal of the γ parameter, and its standard error is calculated using the delta method clustered by individuals. The results in this paper using experimental data provide empirical evidence that the EIS is slightly above one. To test the hypothesis that the instantaneous utility is linear, one simply tests the null that $\gamma = 0$ against $\gamma > 0$. The test statistic is 32.2 implying the rejection of the null hypothesis at standard levels of significance. Hence, the result suggests that the instantaneous utility is somewhat close to linear, but significantly concave. Similar results regarding concavity of the utility, using different experimental data, are documented by Andreoni and Sprenger (2012a) and Cheung (2020).

Consider now the estimates presented in Table 3 above in comparison with others in the literature. Our estimate of risk aversion is different from the estimate reported in Table III of Andersen et al. (2008). These authors find a relative risk aversion coefficient equal to 0.74 and a discount rate $\delta = 0.101$; Andersen et al. (2018) obtain a discount rate of 0.114, which implies a very low discount factor $(1/(1+\delta) = 0.898)$ and might be related to the dynamic inconsistency of the generalized expected utility preferences. In contrast, we find a δ of 0.075, which implies a discount factor of 0.93. Kapteyn and Teppa (2003), in contrast to Andersen et al. (2008), are able to separately estimate the discount factor and the EIS coefficient. These authors, based on information from direct questions about hypothetical intertemporal consumption choices rather than using revealed preference approaches, find a negative discount rate that ranges between -0.094 and -0.11, depending on whether individuals' preferences are modelled with a isoelastic utility function or an habit formation model. Finally, regarding the EIS coefficient, Kapteyn and Teppa (2003) provide estimates of the EIS coefficient in the range $(0.51, 0.57)$. Andersen et al. (2018) report an EIS estimate of 2.85, which is large relative to the existing estimates in the literature using aggregated and disaggregated data.

Overall, the results we provide in this paper are important for the ongoing debate on the value of EIS. Thimme (2017) discusses several recent advances of the theory and highlights challenges for the estimation. An estimate of the EIS slightly above 1 is in contrast with the evidence found in some areas of economics. For example, the evidence that emerges from microeconomic studies, as surveyed for instance in Attanasio (1999) and Attanasio and Weber (2010), which use an isoelastic specification of preferences (such as Attanasio and Weber (1993, 1995) and Blundell et al. (1994)) together with observational data, is that the EIS of consumption is just below 1. Early work (see, e.g. Hall (1988)) finds EIS close to zero. Although the general discussion still seems to be prevailed by Hall's early EIS estimates close to zero, the literature shows that several deviations from the time-additive constant relative risk aversion model speak in favour of considerably higher values. In macroeconomic studies, Lucas (1990), based on average consumption and interest rates in the United States, rules out an EIS below 0.5. Work by Kydland and Prescott (1982) set up equilibrium business cycle models and argue that an EIS between 0.8 and 1 gives the best fit to the data. More recently, a key assumption of the long-run risks asset pricing model developed by Bansal and Yaron (2004) is an EIS above 1. Standard choices in the long-run risks literature are 1.5 or 2, see also Ai (2010) and Drechsler and Yaron (2011). Ortu et al. (2013) obtain estimates of the EIS that range between 2.09 and 5.54 based on different samples. Importantly, these authors argue that using disaggregated consumption data is key to finding a value for the EIS greater than one. Our results using QP show that after separating risk from intertemporal substitution, there is empirical evidence from the experimental data on the Danish population that the EIS is slightly above 1.

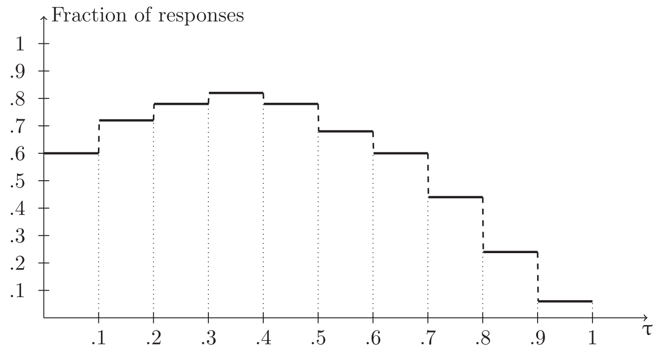

FIGURE 4 Fraction of individuals that choose the lottery predicted by quantile preferences across values of $\tau \in (0,1)$ in the risk experiment.

6.2 | Empirical evidence on the quantile choice

A simple way to assess if individuals' responses in the experiment are rational from a QP perspective is to compute the number of individuals that respond according to the QP model discussed above. Figure 4 reports, for each $\tau \in (0,1)$, the share of individuals' actual responses to the risk experiment that coincide with the responses provided by the QP model shown above. That is, for each τ, we calculate QP_τ^A and QP_τ^B for all lotteries available to choose and evaluate the fraction of responses where the individual chose A if $QP_\tau^A > QP_\tau^B$, B if $QP_\tau^A < QP_\tau^B$ and I if $QP_\tau^A = QP_\tau^B$.

The maximum value of the function in Figure 4 is achieved at $\tau \in [0.31, 0.40]$ with a fraction of correct responses around 80%. Using a bootstrap procedure clustered by individuals we find that the limits of the interval range between 0.31 and 0.33 for the minimum and between 0.38 and 0.40 for the maximum. This preliminary analysis suggests that individuals' responses are most consistent with a τ−QP model, with τ in this range. The figure also shows that the lotteries used in the RA exercise do not have enough granularity to identify τ

TABLE 4 Robustness exercises varying ω.

Parameter	Estimate	Std. Err.	Lower 95% CI	Upper 95% CI
$\omega = 118, \lambda = 7$				
τ	0.455	0.0059	0.443	0.467
γ	0.864	0.0388	0.788	0.940
EIS ($1/\gamma$)	1.157	0.0520	1.056	1.259
δ	0.129	0.0047	0.120	0.138
μ (for RA)	0.864	0.0200	0.825	0.903
ν (for DR)	0.0076	0.0023	0.003	0.012
$\omega = 50, \lambda = 1$				
τ	0.455	0.0064	0.441	0.466
γ	0.915	0.0052	0.905	0.925
EIS ($1/\gamma$)	1.093	0.0062	1.081	1.105
δ	0.064	0.0041	0.056	0.072
μ (for RA)	0.977	0.0220	0.933	1.021
ν (for DR)	0.006	0.0005	0.005	0.007
$\omega = 200, \lambda = 1$				
τ	0.455	0.0072	0.443	0.470
γ	0.789	0.0080	0.773	0.804
EIS ($1/\gamma$)	1.268	0.0129	1.242	1.293
δ	0.103	0.0058	0.092	0.114
μ (for RA)	0.750	0.0160	0.718	0.782
ν (for DR)	0.017	0.0009	0.149	0.018

other than within those coarse intervals. Note that this value is different from the value of τ that maximizes the likelihood function that lies in the interval [0.41,0.50], and for which we take the average $\tau = 0.455$ as the representative value. The differences arise because of the specific form of the likelihood, and the form of weighting the differences between the two lotteries, that is, the logit form. The estimate τ is robust to different likelihood implementations. Although not reported, the same value is achieved if we use a Gaussian model as in Harrison et al. (2002).

6.3 | Robustness exercises

Now we implement different exercises to assess the robustness of the estimates in the previous section. In particular, we consider variation in background consumption (ω) and number of periods for integrating payments into consumption (λ), as well as heterogeneity of preferences across individuals' observable characteristics. The analysis of the other robustness measures—such as heterogeneity of preferences and unobservable characteristics, as well as a statistical specification that allows each observation to potentially be generated by more than one latent data-generating process—is similar and omitted for space constraints.

Table 4 reports the estimates of risk aversion, EIS and discount factor for different values of the number of periods receiving income (λ) and background consumption (ω). In the QP setting, the risk aversion coefficient is estimated from the marginal likelihood $\ell_{RA}(\tau,\mu)$, hence, this parameter is not affected by changes in λ. It could potentially be affected by ω but we note that different values of background consumption produce the same τ estimates, not affecting the quantile interval where the maximum is achieved. In contrast, the discount factor and the EIS exhibit some slight variation in the estimates across values of ω and λ, with δ varying between 0.064 and 0.129, and the EIS varying between 1.093 and 1.268. These results are similar to the estimates obtained under the benchmark model and confirm empirically the quantities obtained under the exponential discounting model.

Table 5 reports the estimates of the robustness exercise that considers heterogeneity of preferences and individuals' observable characteristics. We estimate the model separately for men and women, and for individuals under 50 years old ('Under 50' group) and above or

TABLE 5 Robustness exercises for different sample restrictions.

Parameter	Estimate	Std. Err.	Lower 95% CI	Upper 95% CI
Men				
τ	0.455	0.0078	0.440	0.467
γ	0.923	0.0091	0.905	0.940
EIS ($1/\gamma$)	1.084	0.0110	1.006	1.105
δ	0.074	0.0040	0.066	0.082
μ (for RA)	0.785	0.0250	0.737	0.833
ν (for DR)	0.005	0.0008	0.004	0.007
Women				
τ	0.455	0.0217	0.431	0.467
γ	0.924	0.0380	0.850	0.999
EIS ($1/\gamma$)	1.082	0.0450	0.995	1.169
δ	0.075	0.0050	0.065	0.085
μ (for RA)	0.919	0.0300	0.861	0.977
ν (for DR)	0.005	0.0029	0.000	0.011
Under 50 years old				
τ	0.355	0.0420	0.342	0.455
γ	0.874	0.0090	0.855	0.892
EIS ($1/\gamma$)	1.144	0.0123	1.120	1.169
δ	0.076	0.0039	0.068	0.083
μ (for RA)	0.845	0.0240	0.801	0.894
ν (for DR)	0.010	0.0009	0.008	0.012
Above 50 years old				
τ	0.455	0.0119	0.444	0.468
γ	0.920	0.0090	0.902	0.937
EIS ($1/\gamma$)	1.087	0.0106	1.066	1.108
δ	0.082	0.0045	0.073	0.091
μ (for RA)	0.952	0.0350	0.882	1.021
ν (for DR)	0.005	0.0007	0.004	0.007

equal to 50 years old ('Above 50' group). In both exercises the full sample is split into two subsamples of approximately equal size. Except for the subsample covering individuals under 50 years old the other groups report a risk aversion coefficient τ of 0.455. This is an interesting result that suggests that under QP the 'Under 50' group is more risk averse than the 'Above 50' group independently of gender. In particular, the parameter τ for the 'Under 50' group lies in the range [0.31,0.40]. This result, despite appearing counter-intuitive at a first glance, reflects individuals' attitude towards the monetary payments of the risky options. Younger individuals may assign a larger value to the payoffs of both strategies than older individuals, hence, the larger risk aversion of this group with respect to the 'Above 50' group of individuals. The estimation of the EIS coefficient also reveals differences in terms of age but not of gender. Thus, for the group of men the EIS is 1.084 and for women the EIS coefficient is 1.082. In contrast, for the group of individuals under 50 years old the EIS estimate is 1.144 and for the 'Above 50' group the EIS is 1.087, suggesting that 'Above 50' individuals are (slightly) less willing to shift income (proxying consumption) across periods under small changes in interest rates. Finally, the analysis of the discount rate shows similar findings. The parameter δ is the same across men (0.074) and women (0.075), and is also very similar for the group of individuals under 50 years old. We observe a slightly higher discount rate (0.082) for the 'Above 50' group suggesting more impatience with regards to future income by these individuals.

7 | CONCLUSION

Risk aversion, time preferences and elasticity of intertemporal substitution (EIS) are intertwined implying that identifying each parameter separately is a difficult task in dynamic settings. This paper elicits these parameters that characterize the three dimensions of intertemporal utility functions using dynamic QP theory. Under this approach, individuals exhibit preferences such that their risk attitude coefficient is driven by the quantile parameter $\tau \in (0,1)$. This distinct feature of the QP model of individual's behaviour allows for the separation between the three parameters of interest.

Using experimental data from Harrison et al. (2005), we have estimated these parameters. Our results provide interesting insights that reveal similarities and differences with the standard expected utility paradigm. We find a slightly lower discount rate than in most of the related literature. These estimates suggest that individuals are patient and discount similarly income received in different instances into the future. The risk aversion coefficient, which is captured by the quantile τ, reveals mild risk aversion of individuals participating in the experiment given by a quantile $\tau = 0.45$. The estimate of the EIS is slightly larger than one, suggesting that the curvature of individual's utility over time is somewhat close to linear, but statistically significantly concave, as shown in recent experimental studies. This estimate for the elasticity of intertemporal substitution is in line with recent empirical literature on long-risks asset pricing models, but contrasts with values reported by studies using observational disaggregated data, which in general, find an elasticity smaller than one—see, e.g. Thimme (2017) for a survey of the literature on the EIS coefficient.

The empirical results in this paper highlight the importance of considering QP as an alternative paradigm to the EU model in dynamic settings. These empirical findings obtained from applying QP to a widely explored experimental study provide further insight into individuals' discount factors and the EIS in models in which the latter parameter is not mechanically linked to the concavity of the utility function.

Of course, the QP model needs more investigation by the empirical community. To this date, only de Castro, Galvao, Kim, Montes-Rojas and Olmo (2022) and de Castro, Galvao, Noussair, and Qiao (2022) study QP in a lab setting. It would be desirable to make a more comprehensive analysis of these preferences using experimental procedures, especially comparing them to other preferences like rank-dependent utility (RDU) and prospect theory (PT). We leave those comparisons for future research.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in Econometrica at https://www.econometricsociety.org/publications/econometrica/2008/05/01/eliciting-risk-and-time-preferences.

ENDNOTES

[1] Indeed, those preferences are monotonic only in special cases—see details on Bommier et al. (2017). These authors also discuss a particular class of monotonic recursive utility models given by the risk-sensitive preferences introduced in Hansen and Sargent (1995). An early strategy to differentiate between risk aversion and EIS is found in Selden (1978). This author introduces the idea of ordinal certainty equivalent that departs from the EU. This strategy is, however, criticized by Epstein and Zin (1989) by noting that this approach is not dynamically consistent and the estimated equations are applicable only to a naive consumer who continually ignores the fact that plans formulated at any given time will generally not be carried out in the future.

[2] See de Castro and Galvao (2019, 2021) for a discussion of dynamic QP models. Manski (1988) was the first to study the properties of QP, which were later axiomatized by Chambers (2009), Rostek (2010) and de Castro and Galvao (2021). Recently, models of QP have been attracting attention, see e.g. Bhattacharya (2009) and Giovannetti (2013), Baruník and Čech (2021), Long et al. (2021), and Chen et al. (2021), Baruník and Nevrla (2022). From an experimental point of view, de Castro, Galvao, Noussair, and Qiao (2022) show evidence that the behaviour of between 30% and 50% of the individuals in their experiment can be better described with QP rather than EU.

[3] Independence is assumed here just for simplicity of exposition, but not required in the general model below.

[4] Virtually, all analyses of intertemporal choice assume both diminishing marginal utility function and positive time preference. These two assumptions create opposing forces in intertemporal choice: diminishing marginal utility motivates spreading consumption over time, while positive time preference motivates concentrating consumption in the present. Since economic agents do, in fact, spread consumption over time, the assumption of diminishing marginal utility seems strongly justified. This assumption is equivalent to considering an instantaneous utility function that is concave. Unfortunately, in standard EU models, an instantaneous concave utility function also reflects the degree of individuals' risk aversion in an atemporal setting.

[5] This experiment considers binary choices characterized by small sooner/large later payoffs and does not allow subjects to express any preference for mixtures of strategies. This approach yields corner solutions where utility and discounting are maximally confounded, hence our interest in this experimental setup. Andreoni and Sprenger (2012a) propose an alternative methodology that allows the selection of mixtures of choices and, in turn, the estimation of the curvature of the utility function reflecting intertemporal substitution.

[6] Variation in interest rates and time horizons for identification of the discount factor and the EIS is also considered in early work by Barsky et al. (1997).

[7] The summary of published discount rates reported in Table 1 of Frederick et al. (2002) suggests that an estimate of the discount rate of 0.075 can be considered as indicative of a patient attitude and is well below the majority of estimates found in the literature using experimental and field data.

[8] Such large rates are often attributed to present-biased discounting and are at odds with aggregate models of discounting. The latter models imply much lower annual discount rates (Cagetti, 2003; Gourinchas & Parker, 2002).

[9] The variables x_t and r_t are, respectively, the state and the shock in period t, both of which are known by the decision maker at the beginning of the period.

[10] As discussed in Section 2.2, the attitude towards risk is captured by τ in the quantile model.

[11] Another interpretation of the EIS is that it measures the sensitivity of consumption growth to changes in the interest rate (the return of investment opportunities).

[12] The data used in this paper is available at https://www.econometricsociety.org/publications/econometrica/2008/05/01/eliciting-risk-and-time-preferences.

[13] Data are collected in the field in Denmark to obtain a sample that offers a wider range of individual sociodemographic characteristics than usually found in subject pools recruited in colleges, as well as a sample that can be used to make inferences about the preferences of the adult population of Denmark.

[14] Cohen et al. (2020) note the differences between considering consumption and income as arguments of the period utility function. Nevertheless, these authors acknowledge that because of the methodological simplicity of using monetary outcomes, the latter framework remains the most widely used paradigm for estimating time preferences.

[15] For the first experiment in Andersen et al. (2008), which analyses static risk aversion for the EU case, this expression is equivalent to $\Delta V_{AB}^{RA}(\gamma,\mu) = \frac{(EU^B)^{1/\mu}}{(EU^A)^{1/\mu} + (EU^B)^{1/\mu}}$.

[16] Notice that, for the time discount factor and EIS, the relevant expression to obtain the likelihood function is (27) in the QP do not affect the PV problem where there is no uncertainty in the options. The joint likelihood function is obtained from the sum of the likelihoods as

[17] Since the objective function is non-smooth, the value of τ that maximizes the likelihood function lies in the interval $[0.41, 0.50]$, we report the middle point (mean or median) of the interval as a point estimate.

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APPENDIX A: On the generalized expected utility model.

In this appendix, we discuss in more detail the model used by Andersen et al. (2018), which is named generalized expected utility model by DeJarnet et al. (2020). In order to describe their model in a simple setting, let us consider consumption c_t and $c_{t'}$ in the dates t and $t' > t$. This consumption is evaluated by a utility function given by:

$$U(c_t, c_{t'}) \equiv \mathbb{T}[\varphi(D_t u(c_t) + D_{t'} u(c_{t'}))], \tag{30}$$

where $\mathbb{T}[\cdot]$ is an operator (further discussed below), D_t is the discounting factor for time t and $\varphi : \mathbb{R} \to \mathbb{R}$ and $u : \mathbb{R} \to \mathbb{R}$ are functions.

In most of their paper, $D_t = \delta^t$, for some constant discounting rate $\delta > 0$, that is, they adopt the standard exponential discounting. However, in their Section 5.4 they discuss other forms of discounting, including hyperbolic. Similarly, most of the paper considers $\mathbb{T}[\cdot]$ as the expectation, but in Section 5.5, they discuss $\mathbb{T}[\cdot]$ as an operator distorting probabilities in such a way that (30) becomes a variation of rank dependent utility. This is a variation of the rank dependent utility model because the function $\varphi : \mathbb{R} \to \mathbb{R}$ is not necessarily the identity (or a linear function). Indeed, (30) does not lead to expected utility or rank-dependent utility if φ is not linear. However, (30) is dynamically consistent only if φ is linear, as we show in section 8.1 below. On the other hand, if φ is linear, then (30) cannot separate risk aversion from the elasticity of intertemporal substitution (EIS). In other words, (30) does not allow dynamic consistency *and* the separation of risk aversion and EIS at the same time.

The shortcomings of the model (30) is not restricted to dynamic consistency. In the more familiar case in which $\mathbb{T}$ is just the expectation, Epstein and Zin (1989, p. 951–2) discuss several problems with these preferences. For instance, the preference ordering at time t' would depend upon past consumption $t < t'$ in such a way that 'the dependence is *greater* as the past becomes more *distant*' (emphasis in the original). These problems were used by Epstein and Zin (1989) to support the advantages of their model.

We make those observations to warn the experimental community that the study of the three parameters of interest (discounting rate, risk aversion and elasticity of intertemporal substitution) can be made with either the Epstein and Zin (1989) model or the de Castro and Galvao (2019) model, but the problems discussed above suggest that it is better to refrain from using model (30) for this task.

A.1. | Dynamic consistency

In this section, we provide a direct proof that model (30) is dynamically consistent only if φ is linear. For simplicity, we focus on the case that $\mathbb{T}$ is just the standard expectation.

Let $\mathscr{F} = \{\mathscr{F}_t\}_{t=0,1,\ldots,T}$ be a filtration in Ω, with $\mathscr{F}_0 = \{\emptyset, \Omega\}$, and let $\mathscr{F}_t(\omega) \subset \Omega$ denote the element of $\mathscr{F}_t$ containing ω. Let $C \subset \mathbb{R}$ be a nontrivial interval, representing the set of relevant consumption. Let $\mathcal{C}$ denote the set of random consumption streams $c : \Omega \to C^T$, which are adapted to the filtration $\mathscr{F}$. Let $\mathbb{T} = \{1, \ldots, T\}$. Consider a set of preferences over $\mathcal{C}$ denoted by $\succeq_{t,\omega}$, for each $t \in \mathbb{T}$ and $\omega \in \Omega$. The following is the standard definition of dynamic consistency, as can be found in Epstein and Schneider (2003) and Maccheroni et al. (2006):

> **Definition A1.** (Dynamic consistency). *We say that $\{\succeq_{t,\omega}\}_{(t,\omega) \in \mathbb{T} \times \Omega}$ is dynamically consistent if the following property holds: for any $t < T$, $\overline{\omega} \in \Omega$, and $c, c' \in \mathcal{C}$, if (i) $c_s(\omega) = c_{s'}(\omega)$ for all $s \leqslant t$ and $\omega \in \Omega$; and (ii) $c \succeq_{t+1,\omega} c', \forall \omega \in \mathscr{F}(\overline{\omega})$, then $c \succeq_{t,\overline{\omega}} c'$.*

Dynamic consistency is a very desirable property of models of intertemporal utility. Under time inconsistent preferences the ranking between two consumption bundles c and c' may vary over time as these bundles are evaluated at different points of the filtration. See, among others, Epstein and Le Breton (1993), Epstein and Schneider (2003). Its failure can lead to a number of problems highlighted in the behavioural economics literature such as the presence of hyperbolic discounting, ambiguity aversion, see, for instance, Al-Najjar and Weinstein (2009), or present bias. These empirical phenomena are usually modelled using alternatives to the specification of the intertemporal utility function (30) such as hyperbolic and geometric discounting functions. In what follows, we show that the class of GEDU functions also describes time inconsistent preferences under general settings, in contrast to what the recent literature seems to suggest, see, for example, Andersen et al. (2018).

For simplicity, let Ω be a finite set and $\varphi : \mathbb{R}_+ \to \mathbb{R}$ a real-valued function defined as $\varphi(x) = x^\alpha$, for some $\alpha \neq 0$. Furthermore, for each $c \in \mathcal{C}$, define the version of the intertemporal utility function (30) conditional on the filtration $\mathscr{F}_{t-1}(\omega)$. Then,

$$V_{t,\omega}(c) \equiv E\left[\varphi\left(\sum_{s=t}^{T} \frac{1}{(1+\delta)^{s-t}} u(c_s)\right) | \mathscr{F}_{t-1}(\omega)\right], \tag{31}$$

where $u: C \to \mathbb{R}_+$ is a utility function such that $[0,m] \subset u(C)$ for some $m > 0$. Under these conditions, we obtain the following result.

Proposition A2. *Assume that $\mathcal{F}$ is not trivial, that is, there exists $p \in (0,1)$ and $A \in \mathcal{F}_T$ such that $\Pr(A) = p$. Define $\{\succeq_{t,\omega}\}_{(t,\omega) \in \{1,\ldots,T\} \times \Omega}$ by*

$$c \succeq_{t,\omega} c' \Leftrightarrow V_{t,\omega}(c) \geqslant V_{t,\omega}(c'). \tag{32}$$

Then, $\{\succeq_{t,\omega}\}$ is dynamically consistent if and only if $\alpha = 1$, that is, φ is the identity.

Proof. If $\alpha = 1$, then (31) is an expected utility model and is, therefore, dynamically consistent. Conversely, assume that $\{\succeq_{t,\omega}\}_{(t,\omega) \in \{1,\ldots,T\} \times \Omega}$ defined by (32) is dynamically consistent. For simplicity, we will assume that $T = 2$; alternatively, consider below only the last and penultimate periods.

For each $y \in [0,m] \subset u(C)$, consider the lottery L_y that pays y utils if the event A occurs and 0 otherwise. Let $e(y)$ denote the certainty equivalent of this lottery, that is,

$$\varphi(e(y)) = p\varphi(y) + (1-p)\varphi(0)$$
$$\Rightarrow e(y) = (py^{\alpha})^{\frac{1}{\alpha}} = p^{\frac{1}{\alpha}} y.$$

Let $x \in [0,m] \subset u(C)$ and consider the lottery that pays x utils in the first period and the lottery L_y in the second period. Dynamic consistency implies that the utility of this lottery must be equal to the utility of the lottery that pays x in the first period and $e(y)$ for sure in the second period. That is, for any $x \in u(C)$,

$$\varphi\left(x + \frac{e(y)}{1+\delta}\right) = p\varphi\left(x + \frac{y}{1+\delta}\right) + (1-p)\varphi\left(x + \frac{0}{1+\delta}\right)$$
$$\Rightarrow \left(x + \frac{p^{\frac{1}{\alpha}} y}{1+\delta}\right)^{\alpha} = p\left(x + \frac{y}{1+\delta}\right)^{\alpha} + (1-p)(x)^{\alpha}.$$

Since x and y were arbitrary members of $[0,m] \subset u(C)$, we can take $x = \frac{ky}{1+\delta}$ for any $k \in \mathbb{R}_+$ and obtain:

$$\left(k + p^{\frac{1}{\alpha}}\right)^{\alpha} = p(k+1)^{\alpha} + (1-p)k^{\alpha}. \tag{33}$$

It is easy to see that (33) can be satisfied for all $k \in \mathbb{R}_+$ if and only if $\alpha = 1$.