STOCHASTIC PACKETIZED MODEL PREDICTIVE CONTROL FOR NETWORKED CONTROL SYSTEMS SUBJECTS TO TIME-DELAYS AND DROPOUTS

I. JURADO, D. E. QUEVEDO, P. MILLÁN, AND F. R. RUBIO*

Key words. Stochastic, Model Predictive Control, Networked Control Systems

AMS subject classifications. 93E03, 93C41, 60G25

Introduccion. Networked Control Systems (NCS) are systems in which serial communication networks are used to exchange system information and control signals between various physical components of the systems that may be physically distributed. Major advantages of NCS include low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. Nonetheless, closing a control loop on a shared communication network introduces additional dynamics and constraints in the control problem. In addition to being bit-rate limited [1], [2], practical communitacion channels are commonly affected by packet dropouts and time delays, mainly due to transmission errors and waiting times to access the medium; see, e.g., [3]-[5] and the many references therein.

To overcome these problems, it has been proposed to send from the controller a sequence of control signals that, appropriately buffered and scheduled at the actuator end, become a safeguard in case of delays or eventual packet dropouts. This concept naturally fits the model predictive control (MPC) paradigm. Although a significant body of research has developed different strategies combining MPC and buffering strategies there is still room for further research and improvements. On the one hand in works such as [6] or [7] neglect the effect of the network induced delays focusing the attention on the problem of packet dropouts, while in [8] only delays are considered. Further, in many works on MPC for NCS a deterministic approach is considered, yielding a worst-case approach.

The present work considers both packet dropouts and random delays. We adopt a stochastic approach which allows to improve the control performance provided that the statistical distribution of the delays are known.

Notation. We write $\mathbb{R}$ for the real numbers, $\mathbb{N}$ for $\{1,2,\ldots\}$, and $\mathbb{N}_0$ for $\mathbb{N}\cup\{0\}$. The $p\times p$ identity matrix is denoted via I_p. For the column vector in $\mathbb{R}^p$ containing only ones we write $\mathbf{1}_p$, whereas $\mathbf{0}_p = 0\cdot\mathbf{1}_p$. The norm of a vector x is denoted $|x|$. To denote the probability of an event Ω, we write $\mathbf{Prob}\{\Omega\}$. The conditional probability of Ω given Γ is denoted via $\mathbf{Prob}\{\Omega\,|\,\Gamma\}$. The expected value of a random variable ν given Γ, is denoted by $\mathbb{E}\{\nu\,|\,\Gamma\}$, whereas for the unconditional expectation we will write $\mathbb{E}\{\nu\}$. We use the same notation for random variables and their realizations.

Problem formulation. Consider the following discrete linear system:

$$x(k+1) = Ax(k)+Bu(k)+B_\omega\omega(k), \qquad (0.1)$$

$$x(0) = x_0. \qquad (0.2)$$

*I. Jurado, P. Millán and F. R. Rubio are with Dpto. de Ingeniería de Sistemas y Automática, Escuela Técnica Superior de Ingenieros, Universidad de Sevilla. D. Quevedo is with the School of Electrical Engineering and Computer Science, The University of Newcastle. The authors would like to acknowledge MCyT (Grant DPI2010-19154.), and the European Commission(EC) (FeedNetBack Project, grant agreement 223866), for funding this work. And also the Australian Research Council's Discovery Projects funding scheme (project number DP0988601).
ijurado,pmillan@cartuja.us.es, rubio@us.es, dquevedo@ieee.org

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and are the state vector and control input vector respectively and $\omega(k) \in \mathbb{R}^w$ is an exogenous disturbance affecting to the plant.

In our setup, the plant and controller are assumed to be linked through a communication network.The relevant phenomena to consider in this paper are transmission delays and packet dropouts, which can degrade the control performance or even destabilize the plant. The random nature of both effects in real-time communication networks motivates the stochastic approach taken in this work. Delays and dropouts are assumed to be stochastic i.i.d. processes with known statistical distributions.

To summarize, for the proposed control scheme to work, all elements in the control loop are assumed to behave in a time-driven manner, with the following elements:

1. Sensors periodically sample the plant state $x(k)$ and send it to the controller.
2. A stochastic predictive controller computes a control sequence $U^*(k) = [u^*(k|k)\; u^*(k+1|k) \ldots u^*(k+N|k)]$ at each sampling time and sends it through the network.
3. At the actuator side, control inputs are applied to the plant according to the last signal stored in the buffer. The buffer is updated discarding old control sequences whenever a newer one arrives.
4. Network is affected by i.i.d. dropouts and i.i.d delays $\tau(k)$. Where

$$\tau(k) = \begin{cases} i & \text{if } U^*(k) \text{ is received at time } k+i \\ & \quad \text{at the actuator node,} \\ \infty & \text{if } U^*(k) \text{ is lost} \end{cases} \tag{0.3}$$

Assumption 1: The process $\{\tau(k)\}_{k\in\mathbb{N}_0}$ is i.i.d., with delay distribution,

$$\mathbf{Prob}\{\tau(k) = i\} = p_i, \quad i \in \mathbb{N}_0, \tag{0.4}$$

and dropout probability $\mathbf{Prob}\{\tau(k) = \infty\} = p_\infty$.

Control strategy. In order to achieve an appropriate performance level, this work proposes the use of a stochastic predictive controller framework. That way, the controller will try to minimize the expected value of the following cost function:

$$V(x(k), \mathscr{U}_d(k), \mathscr{T}(k), U^*(k)) = \sum_{i=k}^{k+N-1} \ell(x'(i), b'(i)) + F(x'(k+N)) \tag{0.5}$$

where N is the prediction horizon, $x(k)$ is the measured state of the plant in k, $\mathscr{U}_d(k)$ is the set of optimal control sequences sent between $k-1$ and $k-\tau^{max}$, $\mathscr{T}(k) \triangleq \tau(i)$, $\forall i \in [k, k-1, \ldots, k-\tau^{max}]$ is the set of possible delays of those control sequences, $U^*(k)$ is the new control sequence to be computed by the controller at time k, $\ell(\cdot)$ denotes the *stage cost* and $F(\cdot)$ is the *terminal cost*. Moreover, $x'(i)$ and $b'(i)$ are state and control input open-loop predictions according to the buffer policy and the delay and dropouts statistical distribution:

$$\text{Open loop predictions} \begin{cases} x'(k) = x(k), \\ x'(k+1) = Ax(k) + Bu'(k), \\ x'(k+2) = Ax(k+1) + Bu'(k+1), \\ \qquad \vdots \end{cases}$$

where $u'(k)$, $u'(k+1)$, ... is the predicted control sequence.

When random time-varying delays and dropouts are taken into account, one of the main difficulties is the impossibility of predicting the system trajectory in a deterministic way, as the inputs actually applied to the plant are unknown by the controller. Different approaches, including min-max or worst-case approaches can be taken to deal with this difficulty.

In this work we exploit the fact that the statistics of time delays and dropouts can be measured or estimated to improve the control performance. That way, the open-loop predictions described above depend on future delay and dropout realizations, so that the control inputs applied to the plant can be predicted by explicit enumeration of the realizations. For instance, when considering the case $\tau(k)=t$, then $u'(k+t)=u^*(k+t|k)$, $u'(k+t+1)=u^*(k+t+1|k)$ and so on.

The actual control inputs applied to the plant depends on the arrival of the control sequences sent by the controller and the buffer policy. The latter corresponds to the intuitively appealing idea of *"Use the most recent control sequence if available. If not, use predictions from the buffer."*

Let us represent the state of the buffer at a given time instant k as $\beta(k)\in\mathbb{R}^{mN}$ and denote

$$\hat{k}=max\{k-l:\tau(k-l)=l\}$$

It easy to see that $\tau(k-l)=l$ indicates that the optimal control sequence computed in $k-l$, that is $U^*(k-l)$, arrives at time k to the buffer. Then, the dynamics of the buffer can be expressed as the recursive rule:

$$\beta(k)=\alpha(\mathscr{T}(k))U^*(\hat{k})+(1-\alpha(\mathscr{T}(k))S\beta(k-1) \tag{0.6}$$

where $S\in\mathbb{R}^{mN\times mN}$ is a shift matrix defined as the block matrix:

$$S_{i,j}=\delta_{i+1,j}\cdot I_m;\quad i,j=1,\ldots,N$$

In (0.6), $\alpha(\mathscr{T}(k))\in\{0,1\}$ is a signal accounting for reception of control sequences at the buffer computed by the controller subsequent to those received before, such that:

$$\alpha(k)=\begin{cases}1 & \text{if}\quad \hat{k}\in\{k,k-1,\ldots,k-\tau^{max}\}\\ 0 & \text{if}\quad \hat{k}\equiv\emptyset\end{cases}$$

With this description the control action $u(k)$ provided by the buffer at instant k can be expressed as

$$u(k)=\begin{bmatrix} I_m & 0_m & \ldots & 0_m\end{bmatrix}\beta(k) \tag{0.7}$$

From equations (0.1)-(0.2) and (0.7) one can easily see that the state of the buffer is involved in the state of the NCS. However, the controller does not have access to the state of the buffer at any time k entailing a non standard MPC problem. Every sampling time, the controller has access to the plant states $x(k)$ and finds a finite horizon optimal control sequence $U^*(k)\in\mathbb{R}^{mN}$ by solving the following optimization problem:

$$\min_{U^*(k)\in\mathbb{R}^{mN}}\mathbb{E}\{V(x(k),\mathscr{U}_d(k),\mathscr{T}(k),U^*(k)|x(k),\mathscr{U}_d(k),\mathscr{T}(k))\} \tag{0.8}$$

where expectation is taken with respect to the discrete distribution of $\mathscr{T}(k)$. This can be done by explicit enumeration of the realization of $\mathscr{T}$ weighting all these realization with the corresponding probability.

As a consequence of *Assumption 1*, the minimization problem becomes:

$$\min_{U^*(k)\in\mathbb{R}^{m}}\sum_{i\in\mathbb{N}_0}^{\infty}p_iV(x(k),\mathscr{U}_d(k-i),i,U^*(k)) \tag{0.9}$$

Assuming this setup, we will next illustrate how this stochastic predictive controller combined with a buffer provides robustness to packet delays and dropouts.

Simulation results. In this section the control strategy described above is applied to the following unstable system:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \omega(k)$$

Delays are discrete uniformly distributed between 0 and 4, while the disturbance are random bounded disturbances with $|\omega(k)| < 0.5$.

The results obtained applying the proposed method in this paper will be compared with the results from the method described in [6], assuming no quantization issues.

In figures 0.1 and 0.2 are shown the values of function V_t, which is defined in the following: $V_t = \sum_{i=0}^{ts} l(x(i), u(i))$, where $t_S = 100s$ is the simulation time. This funtion is represented with differents values of the control horizon and the initial value of x. In both figures it is possible to see how the value of V_t decreases with larger control horizons, as well when $x(0)$ is decreased.

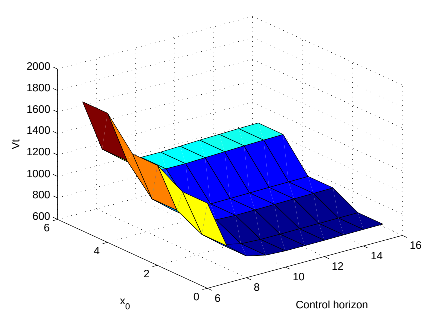

FIG. 0.1. *V_t evolution with the proposed control method.*

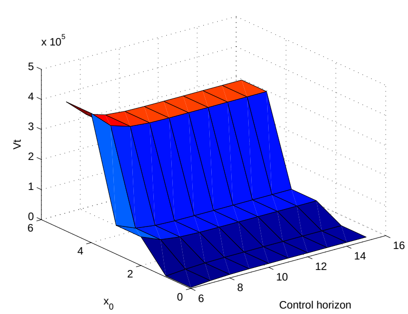

FIG. 0.2. *V_t evolution with the controller in [6]*

REFERENCES

[1] G. N. Nair, F. Fagnani, S. Zampieri, and R. J. Evans ”Feedback control under data rate constraints: An overview”, *Proc. IEEE*, 2007, vol. 95, pp. 108137.

[2] D. E. Quevedo, E. I. Silva, and G. C. Goodwin ”Subband coding for networked control systems”, *Int. J. Robust Nonlin. Contr.*, 2009, vol. 19, no. 16, pp. 18171836, 2009.

[3] J. Chen, K. H. Johansson, S. Olariu, I. C. Paschalidis, and Stokmenovic ”Guest editorial special issue on wireless sensor and actuator networks”, *IEEE Trans. Automat. Contr.*, 2011, vol. 56, pp. 22442246.

[4] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu ”A survey of recent results in networked control systems”, *Proc. IEEE*, 2007, vol. 1, pp. 138162.

[5] A. S. Matveev, and A. V. Savkin ”Estimation and Control over Communication Networks”, *Birkäusser*, 2009.

[6] D. Quevedo, J. Ostergaard, and D. Nešić ”Packetized predictive control of stochastic systems over bit-rate limited channels with packet loss”, *IEEE Transactions on Automatic Control*, 2011, vol. 56(11).

[7] D. Muñoz de la Peña, P.D Christofides, ”Lyapunov-based model predictive control of nonlinear systems subject to data losses”, *IEEE Transactions on Automatic Control*, 2008, vol. 53(9), pp. 2076-2089.

[8] G.P. Liu, J.X. Mu, D.Rees, S.C. Chai, ”Design and stability analysis of networked control systems with random communication time delay using the modified MPC”, *International Journal of Control*, 2006, vol. 79(9), pp. 288-297.

[9] D. E. Quevedo and E. I. Silva and G. C. Goodwin ”Control over unreliable networks affected by packet erasures and variable transmission delays”, *IEEE J. Select. Areas Commun.*, 2008, vol. 26(4).