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IFAC PapersOnLine 50-1 (2017) 6483–6488

Distributed implementation and design for state estimation

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Abstract: This paper presents an architecture for agent-based distributed state estimation of linear plants with distributed outputs. The estimation structure is based on an orthogonal decomposition of the local observable/unobservable subspaces associated to each set of locally accessible outputs. The design of the observers can be carried out in a distributed way, which might open the door to scalable designs when the number of agent grows. The proposed architecture is developed for a two-agents network, where we establish stability results for the error dynamics, but comments are given about the generalization to larger networks. Simulations are provided to illustrate the estimation scheme in such broader cases.

Keywords: Observers for linear systems, Decentralized Control, Decoupling Problems.

1. INTRODUCTION

When considering large-scale plants, such as factories, water irrigation channels or solar fields, the problem of state estimation becomes harder to solve than in small-size systems. The fact that the information from these systems is collected by many individual agents deployed in geographically remote locations complicates the design of estimators. Furthermore, these agents require to communicate with each other to achieve system-wide goals, which incurs in problems derived from network topology as well as communication drawbacks: delays, quantization, limited bandwidth, etc.

The problem of distributed state estimation has been tackled from different perspectives. Perhaps, the most well-known approach is the distributed Kalman filter, with very well-referenced results, as for instance the works of Olfati-Saber (2007, 2009). Most of the proposed distributed Kalman filter algorithms are based on a two-step strategy: first, a certain level of consensus between neighbors is reached, and, second, the distributed filters contribute to the stabilization of the observation errors. In Alriksson and Rantzer (2006), consensus and filtering work at the same rate, while local filter gains are chosen as a solution of a simple Riccati equation. The design of consensus gains requires to iteratively solve optimization problems off-line. In Carli et al. (2008), it is proven for a very simple system that when the number of iterations of the consensus steps is finite, the optimal gain differs from the one computed assuming that the consensus is reached.

Consensus is also used in the works of Ugrinovskii (see Ugrinovskii (2011), Ugrinovskii (2013), and Wu et al. (2015)), in which the author proposes iterative, distributed LMI-based designs of H_∞ filters, and in Acikmese et al. (2014), where the consensus and the measurement processes work at different rates. A similar framework is used in Shen et al. (2010) for the case of possible data dropouts. Both in Acikmese et al. (2014) and Shen et al. (2010), the observer gains are synthesized by means of LMI problems, which may be computationally hard to solve for large scale systems.

A different approach for the same problem is presented by Cattivelli and Sayed (2010b) and Cattivelli and Sayed (2010a), where a distributed estimation algorithm is proposed based on a sequence of Kalman iterations and data-aggregation. A moving horizon approach for the estimation of large-scale plants is studied in Farina et al. (2010), where the gains are computed by solving constrained optimization problems at each sampling time. Another line of research deals with low-communication observers, such as the one in Ribeiro et al. (2006) with one-bit messages, or reduced-order algorithms Orihuela et al. (2013).

From the point of view of information and communication topology, works on this topic typically consider the situation in which none of the agents is able to estimate the state of the system without collaborating with others, which implies that the so-called local observability does not hold. However, global observability is assumed, this meaning that all the agents have enough information to observe the state of the system if they exchange their local information across the network.

This paper proposes a novel structure for agent-based estimators based on an orthogonal decomposition of the

⋆ Research partially supported by grant DPI2013-44135-R funded by MCyT, by grant TEC2016-80242-P funded by AEI/FEDER, by grant OptHySYS funded by the University of Trento, and by grant PowerLyap funded by CaRiTRo.

10.1016/j.ifacol.2017.08.1045

local observable/unobservable subspaces. Each agent uses the system outputs locally measured to correct the local estimation errors in its observable subspace. Furthermore, information received from neighboring agents is projected onto its unobservable subspace, enabling then to estimate the whole state of the system. The separation principle that stems from the proposed structure allows the designer to tune the poles of the observable dynamics by designing local Luenberger-like observers. Then, the dynamics of the locally unobservable states are stabilized through consensus gains.

The proposed architecture has several positive features. First of all, the main goal of the paper is to move towards a distributed design of the observers, contrary to previous works from the authors of this note published in Orihuela et al. (2013), Millán et al. (2012), Millán et al. (2015), or by many others in the context of distributed estimation such as Alriksson and Rantzer (2006) or Ugrinovskii (2011), where the main bottleneck is that the observers must be designed altogether in a single centralized step. A distributed design scales better when the number of agents grows, and provides an adaptable observation structure that can adjust itself dynamically when the network topology or the measured outputs change.

The second interesting feature comes from the fact that the design method developed here provides freedom to tune the dynamics of the observation errors. In other previously mentioned works, as Olfati-Saber (2007) or Carli et al. (2008), a proportional relation is imposed between the gains that weight the corrections based on the measurements and on the information received from the neighboring agents.

This paper constitutes a preliminary work, considering a network with only two connected agents. The authors intention is to illustrate the core of the proposed formulation, setting up a solid mathematical basis to move forward to the next step: an estimation and control algorithm distributedly designed and implemented. As it has been shown in Millán et al. (2013); Orihuela et al. (2015, 2016), the joint problem of estimation and control for large-scale plants is very costly when the design of the controllers and observers is carried out in a centralized way.

The paper is organized as follows. Section 2 introduces some notation and mathematical preliminaries. The problem is formally stated in Section 3. The main results, concerning the design of the observers and the stability of the observation error dynamics, are presented in Section 4. A simulation example is given in Section 6. Finally, the main conclusions are drawn in Section 7.

2. NOTATION AND PRELIMINARIES

Consider a discrete-time linear autonomous system in the following state-space representation:

$$x^+ = Ax, \tag{1}$$

$$y = Cx, \tag{2}$$

where $x \in \mathbb{R}^n$ is the state of the system, $y \in \mathbb{R}^m$, with $m > 1$, is a vector containing all measured outputs of the overall system, and $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$ are known dynamic and output matrices, respectively.

For system (1)-(2), it is well known that it is possible to find a coordinate transformation matrix, $T \in \mathbb{R}^{n\times n}$, such that under the change of variables $\xi = Tx$, system (1)-(2) can be transformed into the observability staircase form:

$$\begin{aligned} \xi^+ &= \begin{bmatrix} \xi_{\bar{o}}^+ \\ \xi_o^+ \end{bmatrix} = TAT^\top \xi = \begin{bmatrix} A_{\bar{o}} & \tilde{A} \\ 0 & A_o \end{bmatrix} \xi, \\ y &= CT^\top \xi(k) = [0 \quad \tilde{C}] \begin{bmatrix} \xi_{\bar{o}} \\ \xi_o \end{bmatrix}, \end{aligned} \tag{3}$$

where $T^{-1} = T^\top$ holds. In (3), $\xi_{\bar{o}} \in \mathbb{R}^{n_{\bar{o}}}$ are the $n_{\bar{o}}$ unobservable components of the overall state ξ, and pair $(\tilde{C}, A_o)$ is observable and refers to the remaining component $\xi_o \in \mathbb{R}^{n_o}$ of ξ.

Let us write the coordinate transformation matrix T as $T = [V_{\bar{o}} \quad V_o]^\top$, where $V_{\bar{o}} \in \mathbb{R}^{n\times n_{\bar{o}}}$ is composed by $n_{\bar{o}}$ column vectors in $\mathbb{R}^n$ which form an orthonormal basis of the unobservable subspace of system (1)-(2). Correspondingly, $V_o \in \mathbb{R}^{n\times n_o}$ is some orthonormal basis of its orthogonal complement. Both bases are orthonormal and mutually orthogonal. For simplicity of exposition, we will denote by "unobservable subspace" the image space of $V_{\bar{o}}$ and by "observable subspace" the image space of V_o, even though this is a slight abuse of notation. From the above stated orthogonality, the following equations hold:

$$\begin{aligned} V_{\bar{o}}^\top V_{\bar{o}} &= I_{n_{\bar{o}}}, & V_o^\top V_o &= I_{n_o}, \\ V_{\bar{o}}^\top V_o &= 0_{n_{\bar{o}}\times n_o}, & V_o^\top V_{\bar{o}} &= 0_{n_o\times n_{\bar{o}}}. \end{aligned} \tag{4}$$

3. PROBLEM FORMULATION

Let us consider the problem of state estimation for a multi-output autonomous linear system observed by two agents indexed by subscripts 1 and 2. The system is described by the following equations:

$$x^+ = Ax, \tag{5}$$

$$y_1 = C_1 x, \tag{6}$$

$$y_2 = C_2 x, \tag{7}$$

where $x \in \mathbb{R}^n$ is the state of the system, and agents 1 and 2 have access to generally distinct system outputs $y_1 \in \mathbb{R}^{m_1}$ and $y_2 \in \mathbb{R}^{m_2}$, respectively.

System (5)-(7) is not necessarily stable, nor it is assumed that either agent 1 or 2 can locally observe the system state from its measured outputs. Thus, it is not required that pairs (C_1, A) and (C_2, A) are observable. However, the following assumption involving collective observability will be needed:

Assumption 1. System (5)-(7) is collectively observable. That is, pair (C, A) is observable, where $C := [C_1^\top \; C_2^\top]^\top$.

Remark 1. Assumption 1 corresponds to requiring that agents 1 and 2 can locally observe a sufficient number of linearly independent system modes. In particular, denoting by $n_{o,k}$ the dimension of the observable subspace from (C_k, A), for each $k \in \{1, 2\}$, Assumption 1 implies that $n_{o,1} + n_{o,2} \geq n$.

This paper aims at proposing a new type of consensus-based observer wherein each agent computes (and exchanges information about) a local estimate of the overall state x, in such a way that:

i) The local estimates of both agents tend exponentially to the actual plant state.

ii) The observers implemented in each agent can be designed in a distributed fashion. That is, each agent can design its own observer independently.

4. STRUCTURE OF THE PROPOSED OBSERVER

As explained in the previous section, the state of system (5) must be estimated by two agents that measure, at each sampling time $k \in \mathbb{Z}$, the system outputs (6) and (7), and share information about their respective estimates $\hat{x}_1(k)$ and $\hat{x}_2(k)$.

The following observer structure is proposed, for $(i, j) \in \{(1, 2), \ (2, 1)\}$:

$$\begin{aligned} \hat{x}_i^+ &= A\hat{x}_i + V_{o,i}L_i(y_i - \hat{y}_i) + V_{\bar{o},i}u_i(\hat{x}_i, \hat{x}_j), \\ \hat{y}_i &= C_i\hat{x}_i \end{aligned} \tag{8}$$

where $\hat{x}_i$ is the state estimation carried out by agent i, $\hat{y}_i$ is the corresponding estimate of the plant output y_i, and $V_{o,i}$ and $V_{\bar{o},i}$ form the orthogonal basis defined in Section 2, which satisfy (4). Finally, L_i and $u_i(\hat{x}_i, \hat{x}_j)$ are, respectively, a local observer gain, and a consensus-based correction, both of them to be designed.

The observation structure proposed in (8) decomposes the observer dynamics in three different terms:

- The first one, $A\hat{x}_i$, is the classical model-based open-loop prediction.
- The second term, containing $L_i(y_i - \hat{y}_i)$, is a local Luenberger-like output injection term, intended to correct the previous prediction with the difference between the locally measured and predicted outputs, y_i and $\hat{y}_i$. It is worth noting that this term is pre-multiplied by $V_{o,i}$, which implies that the elements in the correction vector $L_i(y_i - \hat{y}_i)$ are actually used as weights to perform linear combinations of the column vectors forming $V_{o,i}$. Thus, these corrections only affect the observable subspace of agent i, which makes full sense, as the locally available output y_i only contains information about this subspace. [1]
- Lastly, consider the correction term $u_i(\hat{x}_i, \hat{x}_j)$, to be designed. This last term corrects the estimates $\hat{x}_i$ with information received from agent j. Since it is multiplied by $V_{\bar{o},i}$, this information impacts the local unobservable subspace of agent i.

5. OBSERVERS DESIGN

In this section, a method to design the observers in (8) for system (5)-(7) is proposed. The design method satisfies objectives i) and ii) defined in Section 3. First, the structure of the consensus-based correction terms $u_i(\hat{x}_i, \hat{x}_j)$ in (8) is chosen as follows:

$$u_i(\hat{x}_i, \hat{x}_j) := N_{ij}V_{o,j}^\top(\hat{x}_j - \hat{x}_i), \tag{9}$$

where N_{12} and N_{21} are gains to be designed. Then, the transformations in (3) are made for each agent i, which makes it possible to obtain the corresponding $A_{o,i}$, $A_{\bar{o},i}$, $\tilde{C}_i$,$V_{o,i}$, and $V_{\bar{o},i}$. Finally, the observer gains are designed to fulfill the following property (see Proposition 1 below for the feasibility of this property).

Property 1. The local observation gains L_i, L_j and the consensus gains N_{ij}, N_{ji} are designed in such a way that, for $(i, j) \in \{(1, 2), \ (2, 1)\}$, matrices

$$\left[A_{o,i} - L_i\tilde{C}_i\right] \text{ and } \left[A_{\bar{o},i} - N_{ij}V_{o,j}^\top V_{\bar{o},i}\right], \tag{10}$$

are both Schur stable.

Based on this property, the following result can be stated.

Theorem 1. Consider plant (5) observed by two agents, 1 and 2, that can measure local outputs (6) and (7), respectively, and that implement local observers (8)-(9). If the observer gains satisfy Property 1, then the estimates of both agents tend exponentially to the actual plant state. In particular, defining the observation errors $e_1 := \hat{x}_1 - x$, $e_2 := \hat{x}_2 - x$, the origin of the corresponding error dynamics is globally exponentially stable.

Proof. Let us write the dynamics of the observation errors (e_1, e_2). According to equations (5)-(8), it corresponds to:

$$\begin{aligned} e_i^+ &= Ax - A\hat{x}_i - V_{o,i}L_iC_i(x - \hat{x}_i) - V_{\bar{o},i}u_i \\ &= (A - V_{o,i}L_iC_i)e_i - V_{\bar{o},i}u_i, \end{aligned}$$

where, for brevity, $u_i(\hat{x}_i, \hat{x}_j)$ in (9) has been denoted as u_i.

Next, consider the transformation in Section 2 to write the estimation error of agent i in its local observability staircase form:

$$\varepsilon_i := \begin{bmatrix} \varepsilon_{\bar{o},i} \\ \varepsilon_{o,i} \end{bmatrix} := T_ie_i := \begin{bmatrix} V_{\bar{o},i}^\top \\ V_{o,i}^\top \end{bmatrix} e_i, \tag{11}$$

where $\varepsilon_{\bar{o},i} = V_{\bar{o},i}^\top e_i$ and $\varepsilon_{o,i}(k) = V_{o,i}^\top e_i$ are, respectively, the unobservable and observable modes of the estimation error for agent i. Under this transformation, the dynamics of the observation error can be written as:

$$\begin{aligned} \varepsilon_i^+ &= T_i(A - V_{o,i}L_iC_i)T_i^\top\varepsilon_i - T_iV_{\bar{o},i}u_i \\ &= (T_iAT_i^\top - T_iV_{o,i}L_iC_iT_i^\top)\varepsilon_i - T_iV_{\bar{o},i}u_i \\ &= (T_iAT_i^\top - \begin{bmatrix} V_{\bar{o},i}^\top \\ V_{o,i}^\top \end{bmatrix} V_{o,i}L_i[0 \quad \tilde{C}_i])\varepsilon_i - T_iV_{\bar{o},i}u_i \\ &= \begin{bmatrix} A_{\bar{o},i} & \tilde{A}_i \\ 0 & A_{o,i} - L_i\tilde{C}_i \end{bmatrix}\varepsilon_i - \begin{bmatrix} V_{\bar{o},i}^\top \\ V_{o,i}^\top \end{bmatrix} V_{\bar{o},i}u_i \\ &= \begin{bmatrix} A_{\bar{o},i} & \tilde{A}_i \\ 0 & A_{o,i} - L_i\tilde{C}_i \end{bmatrix}\varepsilon_i - \begin{bmatrix} I \\ 0 \end{bmatrix} u_i, \end{aligned} \tag{12}$$

where it has been used that $C_iT_i^\top = [0 \quad \tilde{C}_i]$, $V_{\bar{o},i}^\top V_{\bar{o},i} = I_{n_{\bar{o},i}}$, and $V_{o,i}^\top V_{\bar{o},i} = 0_{n_{o,i}\times n_{\bar{o},i}}$ (see Section 2 for details), and where matrix $\tilde{A}_i$ denotes some "don't care" matrix appearing in the error dynamics.

It is worth pointing out that, according to (12), the proposed structure for the estimators decomposes the influence of the observation gain L_i, which only affects the errors in the locally observable modes $\varepsilon_{o,i}$, from the influence of the consensus term u_i, which has an effect on the locally unobservable modes $\varepsilon_{\bar{o},i}$. In particular, dynamics (12) reveals a cascaded structure of the error

[1] In Luenberger observers and Kalman filters, this correction only affects the observable subspace as well. Even though this is not commonly explicitly indicated in classical observers, using the coordinate transformation matrix T described in Section 2, the resulting gain L can always be rewritten as $\left[0 \ \ L_o^\top\right]^\top$, only affecting the observable subspace.

dynamics, where subsystem $(\varepsilon_{o,1}, \varepsilon_{o,2})$ drives subsystem $(\varepsilon_{\bar{o},1}, \varepsilon_{\bar{o},2})$. From Property 1, it follows that the upper dynamics $(\varepsilon_{o,1}, \varepsilon_{o,2})$ is globally exponentially stable. Then, from cascaded results of linear systems (the cascade of two exponentially stable LTI systems is globally exponentially stable), to prove the result it is enough to show exponential stability of the following lower subsystem:

$$\varepsilon^+_{\bar{o},i} = A_{\bar{o},i}\varepsilon_{\bar{o},i} - u_i, \quad i \in \{1, 2\}, \tag{13}$$

arising from (12) evaluated with $(\varepsilon_{o,1}, \varepsilon_{o,2}) = (0, 0)$. By substituting (9) in (13), we obtain for each $(i, j) \in \{(1, 2), (2, 1)\}$:

$$\begin{aligned} \varepsilon^+_{\bar{o},i} &= A_{\bar{o},i}\varepsilon_{\bar{o},i} - N_{ij}V^\top_{o,j}(\hat{x}_j - \hat{x}_i) \\ &= A_{\bar{o},i}\varepsilon_{\bar{o},i} - N_{ij}V^\top_{o,j}(e_i - e_j) \\ &= A_{\bar{o},i}\varepsilon_{\bar{o},i} - N_{ij}V^\top_{o,j}(V_{\bar{o},i}\varepsilon_{\bar{o},i} - V_{\bar{o},j}\varepsilon_{\bar{o},j}) \\ &= (A_{\bar{o},i} - N_{ij}V^\top_{o,j}V_{\bar{o},i})\varepsilon_{\bar{o},i}, \end{aligned} \tag{14}$$

where it has been used that $V_{\bar{o},j}$ and $V_{o,j}$ are orthogonal (from (4)). From Property 1, we obtain that (14) is exponentially stable, thus completing the proof. ♢

It is well-known that, once a system has been transformed into its observability staircase form, it is always possible to find an observer L_i to stabilize the observable dynamics. However, it remains to be shown that, under the assumption of collective observability (Assumption 1), it is always possible to find consensus gains N_{ij} satisfying Property 1. This is proven in the next feasibility result.

Proposition 1. Under Assumption 1, Property 1 is always feasible. Therefore, there exist local Luenberger gains L_i, $i \in \{1, 2\}$, such that the left matrix in (10) is Schur stable, and there exist consensus gains N_{ij}, $(i, j) \in \{(1, 2), (2, 1)\}$, such that the right matrix in (10) is Schur stable.

Proof. The existence of the gains L_i is straightforward from the observability of pair $(\tilde{C}_i, A_{o,i})$ in the observable decomposition (3).

To prove the existence of gains N_{ij} stabilizing the right matrix in (10), for each $(i, j) \in \{(1, 2), (2, 1)\}$, it is enough to show that under Assumption 1, pair $(V^\top_{o,j}V_{\bar{o},i}, A_{\bar{o},i})$ is observable, which is a sufficient condition for the stabilizability of (14) through the consensus gain N_{ij}.

First, from the Popov-Belevitch-Hautus test (see, e.g., (Hespanha, 2009, Thm 15.9)) system (14) is observable if and only if

$$\operatorname{rank}\begin{bmatrix} A_{\bar{o},i} - \lambda I \\ V^\top_{o,j}V_{\bar{o},i} \end{bmatrix} = n_{\bar{o},i}, \quad \forall \lambda \in \sigma(A_{\bar{o},i}). \tag{15}$$

To establish (15), we introduce the matrix $V_{o,ij} := [V_{o,j}\ V_{o,i}]$ and we note that:

$$V^\top_{o,ij}V_{\bar{o},i} = \begin{bmatrix} V^\top_{o,j}V_{\bar{o},i} \\ V^\top_{o,i}V_{\bar{o},i} \end{bmatrix} = \begin{bmatrix} V^\top_{o,j}V_{\bar{o},i} \\ 0 \end{bmatrix}.$$

Therefore we complete the proof by proving the rank condition:

$$\operatorname{rank}(V^\top_{o,ij}V_{\bar{o},i}) = \operatorname{rank}(V^\top_{o,j}V_{\bar{o},i}) = n_{\bar{o},i}, \tag{16}$$

which clearly implies (15).

To show (16), note that, according to Assumption 1, $rank(V_{o,ij}) = n$, because $V_{o,ij}$ contains the basis of both the observable subspaces from outputs y_1 and y_2. Therefore, it is possible to find a selection matrix S_{ij} such that

$$M_i := V_{o,ij}\overline{S}_{ij} := V_{o,ij}\begin{bmatrix} S_{ij} & 0 \\ 0 & I_{n_{o,i}} \end{bmatrix} = [V_{o,j}S_{ij}\ \ V_{\bar{o},i}] \in \mathbb{R}^{n\times n}$$

is nonsingular. Consider then:

$$\operatorname{rank}\big(M_i^{-\top}\underbrace{\overline{S}^\top_{ij}V^\top_{o,ij}}_{=M_i^\top}V_{\bar{o},i}\big) = \operatorname{rank}\big(V_{\bar{o},i}\big) = n_{\bar{o},i},$$

which, using $\operatorname{rank}(AB) \le \min\{\operatorname{rank}(A), \operatorname{rank}(B)\}$, clearly implies that $\operatorname{rank}\big(V^\top_{o,j}V_{\bar{o},i}\big) \ge n_{\bar{o},i}$. However, following the same reasoning again, we get $\operatorname{rank}\big(V^\top_{o,j}V_{\bar{o},i}\big) \le n_{\bar{o},i}$, thus establishing (16) and completing the proof. ♢

6. SIMULATION EXAMPLES

In this section, two simulation examples are presented in order to show the effectiveness of the proposed observers.

Example 1. Aiming at a straightforward interpretation of the proposed technique, we choose a simple system with only three states. The first state, x_1, has an unstable dynamic and it is decoupled from the last two states, x_2 and x_3, which correspond to a pair of conjugated imaginary poles:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^+ = \begin{bmatrix} 1.05 & 0 & 0 \\ 0 & 0.9954 & -0.08757 \\ 0 & 0.1248 & 0.9945 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The two agents, 1 and 2, have access to plant outputs $y_1 = x_1$ and $y_2 = x_3$, respectively. It is clear from the chosen system structure that agent 1 can locally observe the first state, while agent 2 can locally observe x_2 and x_3. However, neither of them can estimate the whole plant state without communicating with the other.

The basis vectors of the observable and unobservable subspaces of agents 1 and 2 can be easily obtained as:

$$\text{agent 1:} \quad V_{o,1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad V_{\bar{o},1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\text{agent 2:} \quad V_{o,2} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad V_{\bar{o},2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

For this system, it is also straightforward to see that $V^\top_{o,1}V_{\bar{o},1} = 0$ and $V^\top_{o,2}V_{o,2} = 0$ hold.

Following the proposed design method, the local and consensus observers are synthesized to stabilize the corresponding matrices in Theorem 1. This can be performed individually for each agent, for instant through pole placement methods. The stabilizing solution used in the simulations is given by:

$$\text{Agent 1:} \quad L_1 = 0.2423, \qquad N_{12} = \begin{bmatrix} 0.1248 & 0.1464 \\ 0.1069 & -0.0876 \end{bmatrix},$$

$$\text{Agent 2:} \quad L_2 = \begin{bmatrix} 0.7853 \\ 0.7073 \end{bmatrix}, \quad N_{21} = 0.2035.$$

	$i = 1, j = 2$	$i = 2, j = 1$
$eig\left(\left[A_{o,i} - L_i\tilde{C}_i\right]\right)$	0.8077	$\begin{bmatrix} 0.7696 \\ 0.5130 \end{bmatrix}$
$eig\left(\left[A_{\bar{o},i} - N_{ij}V^\top_{o,j}V_{\bar{o},i}\right]\right)$	$\begin{bmatrix} 0.8481 \\ 0.8850 \end{bmatrix}$	0.8465

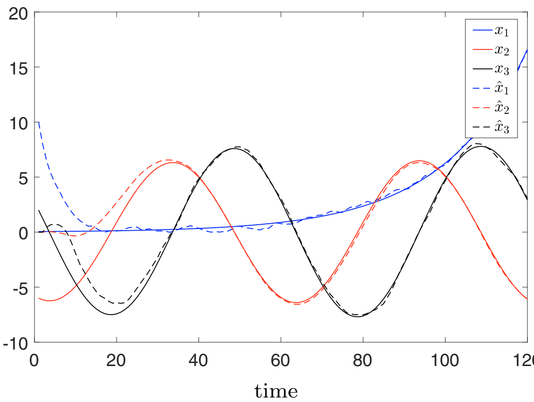

Fig. 1. Example 1. Evolution of the plant states and estimates of Agent 1 (in dashed lines).

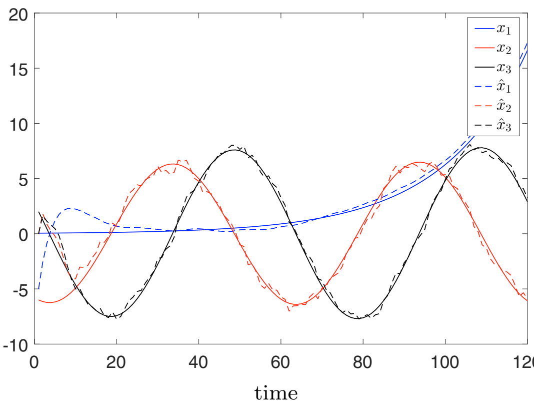

Fig. 2. Example 1. Evolution of the plant states and estimations of Agent 2 (in dashed lines).

Figures 1 and 2 show the estimation performance of both agents. It can be seen that each agent is able to estimate the overall plant states using the designed observation structure. In the simulations, the estimates of each agent are randomly initialized, and the measurements are affected by random noises in order to check the robustness of the observers.

Example 2. Consider the following system:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^+ = \begin{bmatrix} 0.95 & 0 & 0 & 0 \\ 0 & 0.9954 & -0.08757 & 0 \\ 0 & 0.1248 & 0.9945 & 0 \\ 0 & 0 & 0 & 1.0015 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

The system is observed by a set of agents with the network topology defined in Figure 3. Now, each agent receives information from more than one neighbour, so the proposed architecture cannot be directly applied. However, it is possible to use a direct extension in such a way that the estimated states are computed as:

$$\begin{aligned} \hat{x}_i^+ &= A\hat{x}_i + V_{o,i}L_i(y_i - \hat{y}_i) + V_{\bar{o},i} \sum_{j \in \mathcal{N}_i} N_{ij} V_{o,j}^\top (\hat{x}_j - \hat{x}_i), \\ \hat{y}_i &= C_i \hat{x}_i \end{aligned}$$

where $\mathcal{N}_i$ denotes the neighborhood of the agent i. This additive structure incorporates information from every agent belonging to the neighbourhood, provided that collective observability is fulfilled. As it is shown in the simulations, the observers are able to estimate the system states, even though Property 1 is not satisfied for each pair of agents.

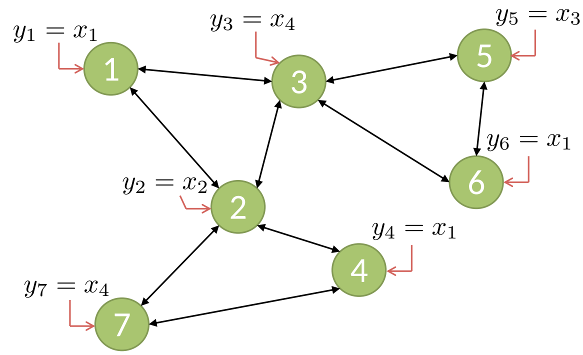

Fig. 3. Network topology in Example 2. The output of the agents are: $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_4$, $y_4 = x_1$, $y_5 = x_3$, $y_6 = x_1$ and $y_7 = x_4$.

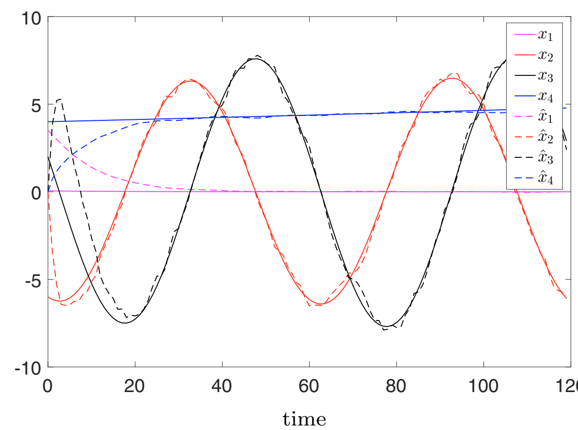

Fig. 4. Example 2. Evolution of the plant states and estimations of agent 2 (in dashed lines).

Figure 4 shows the estimates of agent 2, who has 4 neighbours (most favourable case), and Figure 5 shows the estimates of agent 5, with only two neighbours. In both cases, the agents are able to observe the states of the plant with a good estimation performance.

This last example encourages the authors to work towards an estimation structure for plants in which a large number of agents are connected in arbitrary topologies, in pursuing equivalent feasibility and stability results as the ones presented for two agents. However, in the general case, the problem of building the observable subspaces without global information remains open.

7. CONCLUSIONS

A novel structure for agent-based distributed estimation has been presented. By decomposing observation errors in locally observable and unobservable subspaces, a distributed design method for the observers has been developed. It has been shown that, for a pair of agents and under mild assumptions, involving collective observability of the system, it is always possible to carry out a distributed de-

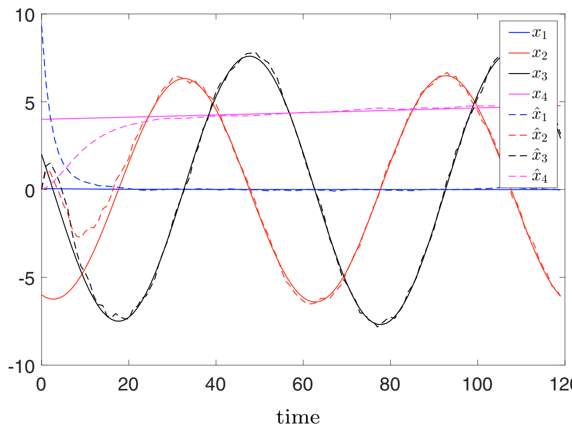

Fig. 5. Example 2. Evolution of the plant states and estimations of agent 5 (in dashed lines).

sign of the observers. This work establishes a preliminary result and the mathematical basis to move forward to more complex problems, considering multiple agents, arbitrary communication topologies, and control.

ACKNOWLEDGEMENTS

The authors would like to thank to the Lifelong Learning Programme of the European Commission for partially funding this work under the Erasmus+ Programme. P. Millán and L. Orihuela acknowledge MEyC (Grant DPI2013-44135-R) for funding this work. L. Zaccarian acknowledges grants OptHySYS funded by the University of Trento, and grant PowerLyap funded by CaRiTRo.

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