INVESTIGATION OF LEARNERS' WAYS OF WORKING WITH ALGEBRAIC GRAPHS IN HIGH-STAKES MATHEMATICS EXAMINATIONS.

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A thesis Submitted in fulfilment of the degree of M.Ed (Mathematics Education) in the School of Science and Mathematics Education (SSME), University of the Western Cape.

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DECLARATION

I declare that the investigation of learners' ways of working with algebraic graphs in high-stakes Mathematics examinations is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Paul Désiré Mutombo Lumbala   Date: November 2015

Signed……………………………..

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ABSTRACT

Algebraic graphs are a difficult topic for most secondary school mathematics learners. My experience as a Mathematics teacher in the Further Education and Training Phase (FET) is that learners solve problems involving graphs with difficulty. Consequently, the purpose of this research was to investigate learners' ways of working with algebraic graphs in high-stakes examinations including their errors and misconceptions in this respect. The investigation carried out to identify learners' errors and misconceptions is based on the analysis of 444 scripts from the 2012 grade 12 final Mathematics examination. More specifically, the study aimed to investigate the ways learners used to solve questions related to graphs in this examination. The focus of the study was the algebraic graphs tested in Paper 1 of the National Senior Certificate (NSC) examination with an emphasis on the identification of errors exhibited in the learners' scripts.

The study adopted a qualitative approach using documentary analysis methodology. As data, the study used the scripts of the final grade 12 Mathematics examinations of schools participating in a project for the improvement of Mathematics based at the University of the Western Cape (UWC).

The analysis of learners' scripts reveals that learners make many errors when they work with algebraic graphs. These errors that have been found in this investigation were coordinate, intercept, domain and range, asymptote, identification, drawing and function errors. Additional errors which were identified are transformation and inverse errors.

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KEY WORDS

Ways of working

Algebraic graphs

High stakes examinations

Misconceptions

Errors

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CHAPTER 1
INTRODUCTION

1.1 Background

Mathematics is important for the personal development of any learner. However, some researchers have discovered that the majority of learners consider Mathematics to be a challenging, annoying and tiring subject (Bekdemir, 2010; Nardi & Steward, 2003). Learners' proficiency in Mathematics is an important indicator of their ability to succeed in many scientific careers in an increasingly technological society (Sasman, 2011). Implicit in this statement is the notion that learners who have acquired a solid mathematical foundation should be able to show their understanding of crucial Mathematics topics such as algebraic graphs. Yet evidence suggests that many candidates who sit for the National Senior Certificate examinations in South Africa appear not to have a solid understanding of the characteristics of various families of graphs, and they are generally unable to sketch graphs (Sasman, 2011).

Furthermore, evidence suggests that the ability of learners from grade 6 (grade 7 in South African schools) to grade 12 to present graphs correlates with their capacity to reason logically (Berg & Philip, 1994; Roth & McGinn, 1997). It is generally expected of these learners that they can think, understand and be in possession of the skills required to solve problems that are related to graphs. For example, when learners are asked to draw a graph of the form $y = x^2 + 4x + 3$, they often fail to show the important components or characteristics of a graph, such as the coordinates of the turning point or the axis of the symmetry. It can therefore be hypothesized that failure to draw graphs correctly can suggest that learners are confused about the essential features of these graphs. In keeping with the preceding argument, one may state that learners with insufficient mastery and knowledge of the underlying mathematical ideas that would equip them to work on such algebraic graphs, are likely to obtain poor results in high-stakes Mathematics examinations. High-stakes Mathematics examinations are externally examined assessments for grade 12 learners. This is neither set by schools nor by people directly associated with individual schools.

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1.2 Motivation

The researcher's motivation to work on this subject stems from his eight years of experience as a Mathematics teacher in the Democratic Republic of the Congo, as well as his three years of experience as a Mathematics teacher in South Africa prior to undertaking this programme. During this time, the researcher noted that in both settings the majority of learners were not able to solve questions on functions, especially those pertaining to graphical representation. In view of this the present study investigates learners' ways of working with algebraic graphs in high-stakes Mathematics examinations. What is meant by 'ways of working' concerns the ways in which learners display their knowledge when solving questions relating to graphs. Knowing that algebraic graphs require mastery of all the characteristics, learners should be able to plot and extract the said characteristics and establish the differences that exist between them. Learners should also be able to construct a function from a given graph.

Evidence suggests that the majority of learners encounter a variety of problems in dealing with algebraic graphs (Hattikudur, Prather, Asquith, Alibali, Knuth & Nathan, 2012; Tairab & Al-Naqbi, 2004). It also suggests that learners do not have the ability to work with questions related to algebraic graphs such as those stipulated in the Curriculum and Assessment Policy Statement (Department of Basic Education, 2011), and that they cannot apply what they have learned in graphs in Mathematics. This dilemma indicates some difficulties relating to determining relationships between the graphs and equations (Mudaly & Rampersad, 2010).

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In South Africa the grade 12 learners sit for a final Mathematics examination. The examination itself is composed of two papers: Paper 1 and Paper 2. The algebraic graphs are found in Paper 1 only. A cursory investigation of the final grade 12 Mathematics Paper 1 reveals that in the 2008 and 2009 examinations a total of 40 out of 150 marks were allocated to the algebraic graphs, which in each case represents 26, 67% of the paper (Department of Basic Education, 2008; Department of Basic education, 2009). In 2010 the algebraic graphs were weighted at a total of 49 out of 150 marks, which is equivalent to 32, 67% (Department of Basic Education, 2010). In 2011 they were worth a total of 51 out of 150 marks, which represents 34% (Department of Basic Education, 2011) and in 2012 they accounted for 50 out of 150 marks which represents 33, 33% (Department of Basic Education, 2012). These incidental statistics suggest that the average mark allocated to algebraic graphs for Paper 1 is almost 30%.

In the light of this observation it appears that algebraic graphs constitute a very important topic for the final Mathematics Examination, specifically for Paper 1. It is this which has motivated me to investigate the ways learners work with algebraic graphs.

1.3 Aim of the study

The study aimed to investigate how grade 12 learners work with algebraic graphs and to identify the errors they make in the ways they approach this section of the work. In the light of this aim the present study is expected to contribute towards an understanding of learners' ways of working in a high-stakes Mathematics school examination on a topic which is of extreme importance for further studies in Mathematics and mathematics-based careers.

Before getting to the core of the problem it is necessary to make an observation about the relationship between functions and graphs. Functions and graphs are two interrelated concepts that cannot be separated as they relate to the construction and organisation of mathematical ideas. In functions and graphs two symbolic systems clarify each other. In other words, if graphs are used to explain functions then functions can be used to clarify the shape of the graph. It is on this interrelationship between graph and function that this study is situated. Therefore the study proposes to investigate this interrelationship between graph and functions based on learners' responses to three questions in the final grade 12 Mathematics Paper 1 Examination of November 2012 (Department of Basic Education, 2012).

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1.4 Scope

This investigation does not cover all the subjects in Mathematics – it deals only with the way learners work with algebraic graphs. The research excludes cubic graphs because these involve the use of calculus, which would add another dimension to the study. According to the Curriculum and Assessment Policy Statement (CAPS) of the Department of Basic Education (2011), the questions administered to the grade 12 learners in the Mathematics Examination Paper 1 for 2012 fall under Learning Outcome 2 which sets out that it is expected the learner is competent to:

- Demonstrate the ability to work with various types of functions and relations including the inverses;

- Demonstrate knowledge of the formal definition of function;
- Investigate and generate graphs of the inverse relation of functions and determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function;
- Identify the characteristics as listed below; and hence use applicable characteristics to sketch graphs of functions and the inverses of the functions listed above:

  - Domain and range;
  - Intercepts with the axes;
  - Turning points, minima and maxima;
  - Asymptotes;
  - Shape and symmetry;
  - Average gradient (average rate of change);
  - Intervals on which the function increases/decreases (Department of Basic Education, 2011).

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This investigation concerns Western Cape schools, particularly the schools participating in the Local Evidence-Driven Improvement of Mathematics Teaching And Learning Initiative (LEDIMTALI) Project.

1.5 Research question

The fundamental question for this research is the following:

What are the ways of working which grade 12 *learners used in solving algebraic graph questions in high-stakes Mathematics examinations in* 2012?

1.6 High-stakes examinations

To obtain evidence that learners have gained knowledge in an organized environment or from being taught, an examination is set to measure the unit of knowledge which has been covered in class. In South Africa, learners who have reached the final level of high school, grade 12, are required to sit for an examination referred to as a high-stakes examination because it has consequences for the learners. Known also as the matric examination, this high-stakes assessment is externally examined. This means that the examination is set neither by schools nor by people directly associated with individual schools. External examiners and moderators are used (Dreyer, 2008). The purpose is to examine learners' performance in all subjects that will enable them to get a certificate called the National Senior Certificate, which is seen as a bridge to higher education (Jacobs, Mhakura, Fray, Holtman & Julie, 20014). In the South African Education system the Minister of Education has sole mandate to publish the results and this is done through the media, including private newspapers and the main national broadcast media which is the South Africa Broadcasting Corporation (SABC).

In conclusion, high-stakes examinations remain an instrument that can help the government and the stake-holders to measure or to evaluate the educational system.

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1.7 Organisation of the study

This study is subdivided into five chapters:

Chapter 1 deals with the introduction to the investigation, the aim of this study, the research question and the research scope.

Chapter 2 deals with the literature review in which the following is presented: learners' treatment of graphs, the conceptual framework where researchers express their views on the concepts affected by misconceptions and errors, and the development of the conceptual framework.

Chapter 3 presents the research design where the research approach and the research method adopted to analyse the document are put forward. The chapter includes the manner in which data has been selected, the sampling procedure together with the data analysis. It also deals with the ethics statement pertinent to conducting the research.

Chapter 4 deals with results based on the major questions that learners have attempted during the examination.

Chapter 5 presents a discussion of the results and some recommendations for improvements in the teaching of algebraic graphs.

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CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

This chapter reviews literature relevant to errors and misconceptions about graphs. Four aspects constitute the frame of this section: learners' treatment of graphs, a conceptual framework providing a description of the concept of graphs, and also misconceptions and errors that have been identified. It ends with the development of a conceptual framework which exhibits the levels of errors learners make in different ways as these are identified and organized in a good way.

2.2 Learners' treatment of graphs

Graphical representations of functions form part of algebra and they are important for learners studying Mathematics (Hattikudur, Prather, Asquith, Alibali, Knuth & Nathan, 2012). Research suggests that a large number of middle and high-school learners do not have a sophisticated understanding of graphs (Dubinsky & Wilson, 2013). Some investigations have suggested that learners often have difficulties when they are asked to convey or extract information from graphs (Eraslan, 2008).

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There are many studies that have investigated the challenges learners experience in solving problems relating to graphs. These studies have essentially focused on learners' abilities to interpret graphs (Freil, Curcio, & Bright, 2001; Wainer, 1992) or on their ability to represent the algebraic graphs (Lloyd & Wilson, 1998). However, evidence also suggests that in order to understand graphs, it is not only interpretation skills that are needed but also construction skills (Hattikudur et al., 2012).

Studies addressing the issue of learners' comprehension of graphs have increased in recent years (Canham & Hegarty, 2010; Ratwani &Trafton, 2008). However, these studies have paid scant attention to how learners construct graphs by hand (Leinhardt, Zaslavsky & Stein, 1990). Furthermore, only very few studies have focused on graph construction (Kramarski & Mevarech, 1997; Mevarech & Kramarski, 1997). Still, certain investigations have been made into learners' reading and thinking about graphs (Berg & Smith, 1994; Norman, 2012).

One issue that has been investigated relates to how learners work with graphs. While some researchers link learners' specific mental structures to graphing difficulties, indicating that both content and cognitive development of the learners are factors that influence learners' abilities to interpret and construct graphs (Berg & Phillips, 1994). Other researchers are of the view that learners' ability to learn and develop graph-related skills is solely a matter of practice rather than cognition (Roth & McGinn, 1997).

While there seems to be consensus that suggests graph interpretation and construction capacities in learners are critical for scientific comprehension (Tairab & Al-Naqbi, 2004) researchers have found widespread deficiencies in learners' abilities in the zone of graph interpretation and construction (Berg & Smith, 1994; Leinhardt, Zaslavsky & Stein, 1990).

Observations from early research studies indicate that a large number of learners have difficulty in interpreting and transforming graphical information (Tairab & Al-Naqbi, 2004; Leinhardt, Zaslavsky & Stein, 1990; Roth & McGinn, 1997). Hence there has been a need to conduct additional studies in order to document the difficulties learners have with interpreting and constructing graphs (Roth & McGinn, 1997).

Padilla, McKenzie and Shaw (1986) explored learners' reading abilities with regard to graphs when determining the coordinates of a point: interpolating and extrapolating, describing connections or links between variables. Since it has been reported that learners need logical thinking abilities as necessary instruments to interpret and construct graphs (Berg & Phillips, 1994), Leinhardt, Zaslavsky and Stein's (1990) comment that learners need intuitions that arise most commonly from their everyday experience. It suggests that intuition exists prior to specific formal instruction (Leinhardt, Zaslavsky & Stein, 1990).

2.3 Framework of errors in graphing

In sum, this research has focussed on the way learners deal with graphs. It documents the errors and misconceptions exhibited by learners. This study was inspired by the ideas of Hattikudur, Prather, Asquith, Alibali, Knuth and Nathan (2012) who worked on learners' construction of graphs of linear functions by focusing on the difficulties of graphing. These researchers identified two components or characteristics of graphs namely the slope and y-intercept. Each of the errors was named with its particular characteristic. The types of errors which they analysed concerned the slope and y-intercept errors. Based on the nomenclature of their research this investigation has elaborated upon other types of errors such as coordinate errors, domain errors, drawing errors, etcetera.

This study has also taken from Kerslake (1981) the concept of three levels of understanding graphs which can be used to assess learners' comprehension. In relation to these three levels of understanding graphs, three levels of errors need to be identified. The three levels of errors established are level 1 errors, level 2 errors and level 3 errors.

These two perspectives have assisted the researcher in the construction of a conceptual framework to identify the errors committed by learners which in fact reflect investigations of graphs by other researchers such as Hattikudur et al. (2012) and Tairab and Al-Naqbi (2004). The errors are grouped into levels and organised with the goal of understanding learners' responses to graphs.

2.4 Conceptual framework

2.4.1 Concept of graphs

Graphs form a category of mathematical expression that is used as a tool in many different disciplines. They are seen as mathematical tools because communicating through graphical representations requires mathematical competencies such as visual perception, logical reasoning and the capacity to plot points from data or from a function rule. Knowledge of graphical representations enables the user to predict the movement of the line connecting points, to deduce the relationship between variables, etcetera (Uzun, Sezen, & Bulbul, 2012). Evidence suggests that graphs provide an invaluable aid in solving arithmetical and algebraic problems, in representing relationships between variables, and in displaying mathematical relationships that often cannot be easily recognised in numerical form (Ersoy, 2004).

Having briefly sketched what graphs are and the relationship that exists between variables, a mathematical expression of a graph is explained in a theoretical way.

Any graph – for school level Mathematics – consists of points in a two-dimensional plane. Each point consists of two coordinates or ordered pairs. The first coordinate, called the abscissa in the ordered pair is normally the x value and it is plotted horizontally. The second coordinate, called the ordinate, is normally the y value and is plotted vertically. This plane is called a Cartesian coordinate system and is named after the French mathematician René Descartes who lived between 1596 and 1650 (Kerslake, 1981).

Throughout this research, the focus will be on graphs of the following types:

- Straight line graphs of the form $y = mx + c$
- Parabolic graphs of the form $y = ax^2 + bx + c$
- Hyperbolic graphs of the form $y = \frac{a}{x+p} + q$
- Exponential graphs of the form $y = a.b^x + q$

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According to the Curriculum and Assessment Policy Statement (Department of Basic Education, 2011), grade 12 learners are taught the above kinds of graphs prior to their writing of the final examinations.

2.4.2 Concepts of misconceptions and errors

There are many ways in which researchers conceptualize misconceptions and errors. According to Smith III, Disessa and Roshelle (1994) a misconception is defined as learners' thinking that generates a systematic way of errors. As far as education is concerned, misconceptions affect all aspects of the sciences. Taylor and Kowalshi (2004) define misconceptions as opinions opposed to existing facts. It is worth emphasizing that many definitions of misconceptions have been found to exist amongst researchers. Özkan (2011) explains that misconceptions occur in knowledge which carries an obstacle to learning scientific facts; they are then performed by the learners within the context of learning experience. He argues that misconceptions are inappropriate thoughts or ideas that the learners deem correct and that they then proceed to practise regularly.

Similarly, Michael (2002) argues that misconceptions are the inconsistencies between the concept taught and the mental model that learners build in their minds. An error by contrast, is defined as a mistake, a wrong approach to calculating, and this domain has to do with systematic error (Muzangwa & Chifamba, 2012). On their part, Young and O' Shea (1981) define an error as a mistake which occurs mostly when working on mathematical problems by using certain procedures or steps. According to Godden, Mbekwa and Julie (2013) an error is correctable, but a misconception is not easy to eliminate from learners' minds.

The debate concerning misconceptions and errors is widespread. For constructivists, errors emerge from misconceptions. Their fundamental assertion is that a misconception begins with learners' structural thinking which hooks up with new ideas and affects new learning in the wrong way. From this constructivist stance, errors derive from misconceptions (Olivier, 1989). On their part, Ryan and William (2007) argue that errors emerge from an overgeneralization as they happen when generalizations make sense in some cases, but are then wrongly extended. In an attempt to distinguish errors from misconceptions, Spooner (2012) explains that an error is caused by a misconception and other factors such as carelessness and problems in reading or interpreting a question and a lack of number knowledge, whereas misconceptions are simply signs of poor understanding. Furthermore, Nesher (1987) is of the view that a misconception indicates a series of thoughts that produces some errors caused by wrong statements. He further notes that errors are systematic and recurrent mistakes displayed by the learner in certain circumstances.

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From a graph perspective some researchers have stated that learners' misconceptions have to do with their cognitive and developmental levels (Hershkowitz, 1989; Schwarz & Hershkowitz, 1999). In this regard Roth (2005) argues that errors and misconceptions are learners' lack of experience in having participated in repeated practices. Korner and Dietrich (2002) argue that a misconception is the result of insufficient knowledge, and they assert that a misconception is the knowledge that opposes presently admitted scientific teachings (Clement, 1993). Another view is that of Li (2006) who contends that learners' errors are a sign of failure to understand something while misconceptions are usually incorrect judgements built on what is true. In most cases, misconceptions result in incorrect problem solutions (Leinhardt, Zaslavsky & Stein, 1990).

The foregoing discussion suggests that misconceptions and errors have been differently conceptualized by researchers. In sum, a misconception is the inconsistency or the knowledge that contradicts certain established theories while an error is the product obtained or the result of misconception.

2.5 Developing a conceptual framework

There are many different errors that occur when learners respond to questions relating to graphs. Some of them will be explained later. Kerslake (1981) introduced the notion of levels of understanding based on items on graphs used for assessing learners' understanding. On the basis of Kerslake's (1981) taxonomy of levels of understanding, a framework of the different types of errors learners made with graphs is constructed. As noted in the previous sections, there are three levels of error referred to in this study namely, level 1 errors, level 2 errors and level 3 errors. These are further elaborated in the section that follows.

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2.5.1 Level 1 errors

These kinds of errors occur when the pre-requisite knowledge on topics such as solving linear equations, simultaneous equations, quadratic equations and inequalities are inappropriately applied by the learners. Likewise, these errors occur when learners do not accurately apply the pre-requisite knowledge on the determination of the domain and range. There are four types of errors that are included in this level namely, coordinate errors, intercept errors, domain and range errors.

2.5.1.1 Coordinate errors

Coordinate errors relate to an incorrect use of coordinates. Abdullah (2010) argues that learners have difficulty in reading the coordinates of Cartesian graphs: when they are asked to solve a mathematical problem by using the coordinates in a graphical representation, they make mistakes in using the coordinate values. He continues by stating that the misconception learners' display is that they often invert the order of the coordinates, that is to say they attribute the x-values to y

and vice-versa. When asked to determine the y-values from a given graph they sometimes make wrong decisions in attempting to get the corresponding y-values from the arbitrary x-values. Research also suggests that some learners often appear to have difficulty in reading the coordinates on the given graph when the axes are not scaled (Padilla, McKenzie & Shaw 1986).

2.5.1.2 Intercept errors

Intercept errors relate to an incorrect determination of the cutting points of the graph with the axes. Moschkovich (1998) analysed the results of the learners' assessment on the use of the x-intercept and found that learners were not able to apply their reasoning skills to the relationship of the line and x-intercepts. She further explained that the use of the x-intercept is not a slight error. Rather it is a failure to comply with the convention, a misconception. She then added that misreading the x-intercept can be seen as a misconception, an error, or otherwise a misunderstanding simply because the x-intercept cannot be directly visible in the function of the form $y = mx + a$. From this perspective, it even results in the graphs of the form $y = ax^2 + bx + c$, $y = \frac{a}{x+p} + q$ and $y = ab^x + q$ x- intercept(s) might not be directly visible.

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Errors in graphing the y-intercept are common amongst learners who have difficulty in graphing the y-intercept due to misconceptions. Hattikudur et al. (2012) argue that middle school learners in particular encounter difficulties when graphing the y-intercept. Overall, this study reveals that the understanding of this characteristic and graphing on this matter comes as the learner progresses through the grades. They argue that fundamental instructions should be given to learners to create opportunities that can enable them to understand the y-intercept in a more appropriate way.

Zaslavsky (1997) argues that the graph of a quadratic function can appear as if it is limited only to the visible area that is in reality drawn, while in fact it may cover an infinite domain. However, learners tend to take into consideration only the visible area of the graph of the function. If the y-intercept of a parabola does not show on the graph, learners tend to conclude easily that a point like that does not exist even though they have been taught that every quadratic function has a y-intercept.

Zaslavsky (1997) argues that when learners are asked to draw a parabola given a quadratic equation of the form $y = ax^2 + bx$, they tend to declare that this form of quadratic function does not have any y-intercept. Although c does not exist, it is well seen and learned that in such a given quadratic function c is representing y and is equal to zero. These instances confirm that learners hold a misconception about the y-intercept.

2.5.1.3 Domain and Range errors

Domain errors relate to an incorrect determination of the input found on the x-axis and range errors relate to an incorrect determination of the output found on the y-axis.

Learners find themselves in a situation where they seem not to distinguish the domain from the range. They are likely to think that the domain remains all the x-values and the range the y-values regardless of how the graphs are represented on the Cartesian plane. The observation made on this matter is that if learners are warned of their misconception throughout their years of Mathematics, a good conceptual understanding will be developed and established (Mudaly & Rampersad, 2010).

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Some researchers argue that since the concepts of domain and range in Mathematics are introduced to the learners at the same time, this practice creates confusion in the minds of learners in the sense that the definition of both concepts is mixed up and misconceptions occur (Özkan & Ünal, 2009). Also, a problem related to graphs is that questions on them may be too many and multifaceted.

2.5.2 Level 2 errors

These errors occur when the learners do not apply certain formulae well. The formulae in question relate to certain aspects of a graph like the slope, asymptotes, symmetry and turning points. Accordingly, four types of errors are included at this level namely slope errors, asymptote errors, turning point and axes of symmetry errors.

2.5.2.1 Slope errors

These errors relate to an incorrect gradient of a line that describes its steepness, or incline. Slope is an important concept in Physics and Mathematics. In Mathematics, it is helpful in the sense that it is considered a prerequisite to the notion of derivative. In Physics, it is necessary as most of the physical quantities such as velocity and acceleration are defined as gradients and they are represented with line graphs (Planinic, Milin-Sipus, Katic, Susac & Ivanjek, 2012). Planinic, Milin-Sipus, Katic, Susac & Ivanjek (2012) reveal that learners encounter difficulties with this characteristic in the context of Mathematics and Physics. They also identify the problems learners display in both contexts. Further they argue that learners hardly recognize the difference between the slope and height of a graph and they cannot interpret the change in height and in slope. For instance Planinic et al. (2012) indicate that in Physics the common mistakes learners commit relate to confusing the meaning of the slope of a line with the height of a point on the line. Woolnough (2000) argues that some learners hold misconceptions when calculating the slope of the line, and they think that it is not appropriate to assign units to the slope due to their belief that slope is the characteristic of graphs or functions that can only be used in Mathematics. Furthermore, Leinhardt et al. (1990) argue that there is confusion in the minds of learners as to the understanding of Mathematical characteristics. They use the height of the graph for slope.

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Another contribution of Planinic et al. (2012) has been brought to light. They found that a great number of learners identify slope with the angle between the straight line and the x-axis, or they evaluate the sign of the slope according to the quadrant in which the line is drawn. These researchers have made an important discovery by saying that the same difficulty known in other studies as slope/height confusion is likely to be dominant in the context of Physics as it is in the context of Mathematics.

Hattikudur et al. (2010) have also elaborated on the notion of slope errors. They argue that learners often make arithmetical errors in the process of calculation especially when graphing slope. They build a line with a wrong slope magnitude by decreasing it from the existing line or by keeping it unchanged.

2.5.2.2 Asymptote errors

Asymptote errors relate to an incorrect reading, determining or showing of the line that a curve approaches but never reaches. Asymptote errors are one of the error types that have not gained much attention from researchers. One of the researchers who has studied asymptote errors, Zaslavsky (1997), is of the view that the graph of a quadratic function may appear as if it has vertical asymptote despite the fact that it does not have any asymptotes.

For information purposes, it is known that in pre-calculus or in calculus, there are three kinds of asymptotes of the line in the plane when one works with rational functions:

- Horizontal asymptote (H.A): straight line parallel to ox and with the equation $y = b \; (b \in R)$
- Vertical asymptote (V.A): straight line parallel to oy and with the equation $x = a \; (m \in R)$
- Slant (oblique) asymptote (S.A): slant line with the equation $y = mx + p \; (m \in R)$.

Furthermore, Yerushalmy (1997) points out two aspects of misconceptions that learners display. The first one is the way they conceptualize the geometric and numeric aspects of Mathematics and the second relates to the definition attributed to asymptotes.

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2.5.2.3 Turning point and Axes of symmetry errors

Turning point errors relate to an incorrect reading or miscalculation of the maximum or minimum point on a curve. Axis of symmetry errors are related to an incorrect reading or determination of a vertical line that divides a graph into two congruent halves. These two types of errors have been combined for the reason that the first coordinate of the turning points helps to extract or to draw the axis of symmetry.

Turning point and axis of symmetry errors have not been extensively investigated. Zaslavsky (1997) argues that the misconception learners have is that they tend to think the turning point is determined by one of its coordinates, although in fact one coordinate is not sufficient. Zaslavsky (1997) then explains that, when asked to decide whether or not two parabolas have the same vertex, learners based their responses only on the x coordinate of the vertex. It is at this stage that they decide erroneously, for example, that the parabolas of $y = ax^2 + bx + 3$ and

$y = ax^2 + bx + 5$ have the same vertex or the same coordinates of the turning point. Zaslavsky (1997) found further that the majority of the learners did not at all check the second value which is the y value or y_{vertex}.

Nevertheless, conceptual misunderstandings that could not be traced to prior formal instruction were also identified, most of which were related to the concept of line-symmetry. Learners were able to find the equation of the line of symmetry of a parabola, but they were not able to use implicit information related to the line of symmetry when not directly linked into focusing on symmetry.

2.5.3 Levels 3 errors

These are errors that relate to construction and the intuition of the understanding of algebraic graphs. According to Leinhardt, Zaslavsky and Stein (1990), construction refers to the act of generating something new. It concerns sketching of a graph or determining a function from a graph. Intuitions are features of a learner's knowledge that arise mostly from everyday experience. In general, intuitions are seen to occur prior to a specific formal instruction (Leinhardt, Zaslavsky & Stein, 1990). There are three types of errors that are included in this level namely identification errors, drawing errors and function errors.

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2.5.3.1 Identification errors

Identification errors relate to an inaccurate identification of graphs of function. Some of these errors concern how to determine whether a graph represents a function or not as most of them signal an inability to recognize that a graph is simply a way of representing the connection between variables (Planinic, et al., 2011). In this vein certain research insights suggest that learners exhibited some misconceptions regarding graphs (Cansiz, Küçük & İşleyen, 2011). They further state that because of these misconceptions learners prefer to do algebraic manipulations rather than to deal with graphs. This finding is corroborated by Tripathi (2008) who states that learners prefer algebraic expression, equations, inequalities and the determination of the decreasing or increasing values over given intervals. In the same line of thought Assiala, Cottril, Dubinsky and Schwingerdorf (1997) posited the idea that learners preferred the algebraic (formula) in place of the graphical form of graphs.

In his study conducted on learners working with graphs Jones (2006) explored learners' graphical representation and identification thereof. Jones (2006) found that learners had difficulty in identifying whether or not a graph represents a function. She found that learners are unable to determine whether a graphical expression of a discontinuous curve represents only one function, and that they rely on what they know. When given an example of a graph they have learnt, learners hesitate to agree, and consider it as a graph of function. Jones (2006) concluded that learners think a graphical representation of a discontinuous curve or a graph built from a piecewise defined function represents only one function.

The misconceptions that learners display with graphs and functions are various. Learners think that a function is represented solely by quadratic graphs and not by linear graphs (Abdullah, 2010). Vinner (1983) argues that learners think functions are represented by certain types of graphs; some seem unfamiliar and superficial. Markovits, Eylon and Bruckheimer (1986) argue that learners have a limited idea of what graphs of functions represent and cannot take into account some graphs as graphs of functions (constant or piecewise function).

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2.5.3.2 Drawing errors

Drawing errors relate to an incorrect graph drawn from a given function or incorrect sketching. Drawing graphs requires some competency and mathematical knowledge. In the Curriculum and Assessment Policy Statement (Department of Basic Education, 2011), it is indicated that to draw graphs learners require paper and pencil. They are called to present graphs through point by point plotting, to make and test conjectures, recognize characteristics such as the domain, range, intercepts with the axes, turning points, maximum and minima, asymptotes, shape, axes of symmetry and intervals on which increasing and decreasing functions may be found. Tripathi (2008) argues that learners require representational competency and the ability to translate from one representation to another. Tripathi (2008) further argues that the Cartesian graph describes a functional relationship that is generally obtained by tabulating values that display the equation first, plotting corresponding points and finally joining the points together by hand. Anderson

(2008) also reveals that the relation between the graph and the equation is known as the Cartesian connection and this often poses a difficulty to learners.

It has been observed that learners encounter difficulties when drawing graphs. They make errors that generally result from wrong graphical representations of functions. These difficulties are due to certain misconceptions. With regard to this statement, Mevarech and Kramarski (1997) have noticed that learners' understanding, interpretation and construction of graphs are problematic as they are confused in their application of these terms. They argue that learners do not realize that interpretation is called for when they read a graph and understand what it means, and construction is when they produce or create what is new; that is to say, starting from the equation of function to constructing a graph or plotting points from data obtained after tabulation. Also, learners encounter difficulties in dealing with the behaviours of graphs when they describe maximum, minimum, increasing and decreasing function. According to Körner (2005), learners' misconceptions about drawing graphs are also observed when representing graphs or translating the drawn graphs. When given a function to draw a graph, they often tend to draw linear graphs and they look forward to getting the correct, symmetrical and continuous graphs. They have a tendency to draw a $y = x$ graph although it is not adequate. They generally draw the graph from the origin since this is the important part.

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On the one hand, Wavering (1989) argues that learners unsuccessfully scale the axes of graphs and on the other hand, they are unable to establish variables to the right axis (Berg & Phillips, 1994; Tairab & Al-Naqbi, 2004).

2.5.3.3 Function errors

Function errors relate to an incorrect function obtained from a given graph. The concept of a mathematical function is a well-developed subject in Mathematics education research. Hence it is of great importance in the teaching and learning of Mathematics. Some researchers have tried to understand how learners deal with functions and mathematical graph problems. Doorman, Drijvers, Gravemeijer, Boon and Reed (2012) argue that function is one of the concepts not clearly understood, but nonetheless that forms part of school Mathematics. To Akkus, Hand and

Seymour (2008) and Ponce (2007) this notion is not an easy subject in the secondary school Mathematics curricula.

The aforementioned researchers are of the view that the study of function is a process which contains multiple steps that enable learners to solve problems relating a given function to a graph. Learners find it easier to respond to a graph question when the equation of the function is given. However, learners find it problematic to respond to questions based on graphs. The multiple steps that follow may lead learners to two types of misconceptions and errors. The first type of misconception arises when learners have to read and extract information from the graph, and the second misconception arises when learners have to do calculation. A function presents some features such as arrows, tables, graphs, formulas and phrases – each with a particular view on the same object. While learners must bear these features in mind, they can be regarded in various ways. Some learners find it difficult to get a function from a graph when the axes are not scaled (Padilla, McKenzie & Shaw, 1986). Literature has revealed that learners do not develop their intuitive beliefs at this grade level regarding misconceptions on the decision made about a certain type of function that they can obtain from a given graph (Fischbein, 1987).

Literature suggests three types of errors that need to be considered when learners are set to work on functions and the graphs thereof. These types include symbols $f(x)$, connecting $f(x)$ with graphs and what constitutes the definition of function. The understanding of the notion of function from the cognitive point of view requires the ability to establish some links between various representations of the notion (Abdullah, 2010). Some investigations have shown that a certain number of representations are not easier to express than others (Hitt, 1998).

However, solving graph problems necessitates intuitive minds that enable the learners to refresh what they know about graphs. Learners who are asked to extract information from a given graph must first of all have mastered all the basic characteristics or key components of each type of graph referring to a particular type of function. It seems that this matter generates confusion in the minds of the learners. This confusion emerges simply because learners have to do with their own intuition. On the contrary if the intuition is developed through exercises, they will present an excellent understanding of all the different characteristics of a function. Literature suggests that the reading of information from a graph can be a problem for learners at different school

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grade levels (Tairab & Al-Naqbi, 2004). Reading coordinates and writing the equation of the graph constituted two main obstacles that have been reported. As Vekiri (2002) and Planinic et al., (2012) found, learners with low prior knowledge and low spatial ability have difficulty in extracting information from graphs.

The foregoing discussion provides details of the errors encountered when reading learners' work on graphs and it categorises the errors into levels which are not necessarily hierarchical. From this a conceptual framework has been developed which has been used as an analytical and explanatory tool. This is represented diagrammatically below:

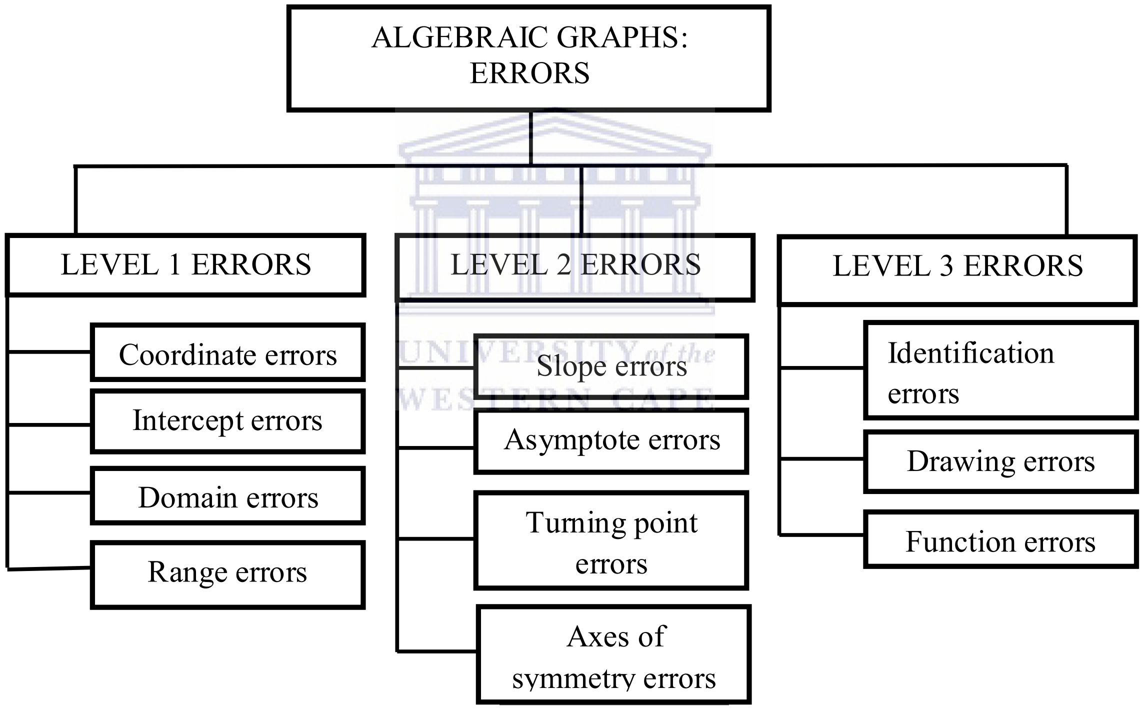

Figure 1: Diagrammatical representation of the error levels

2.6 Additional Errors

During the Inter-Rater agreement, it was agreed that those errors which did not conform to the framework discussed above be added as a new category of errors and be designated as additional errors. These additional errors are transformation and inverse errors and are discussed in the sections that follow.

2.6.1 Transformation errors

According to Eisenberg and Dreyfus (1994, p.58) transformation operates when we move from an initial state to a final state of the graph. The graph of a function *f(x)* on a Cartesian plane can undergo the following transformations: *f(ax)*, *f(-x)*, *f(x) + k* and *f(x – k)* correspond to a dilation, reflection, vertical translation and horizontal translation of *f(x)*, respectively.

In a study on horizontal translations of a function where learners were asked to sketch the graph of $y = x^2$ and the graph of $y = (x-3)^2$ on the same coordinate system, and to explain how they generate their sketch, Zaskis, Liljedahl and Gadowsky (2003) found that half the learners could not predict the spatial location of $y = (x-3)^2$ in the correct way. Instead, they found that half the learners did it to the left rather than to the right of the canonical parabola $y = x^2$. Learners who participated recognized the inconsistency of the function behaviour with their initial expectation.

Zaskis et al. (2003) extended their investigation by accepting that the direction of translation of parabola is inconsistent with intuitive expectations. The lack of harmony is due to the vertical translation that operates as expected, which means that the graph of the function $y = x^2 + 3$ is vertical upward translation by the three units of the graph of $y = x^2$. Learners then were confronted with counterintuitive behaviour of functions and inconsistency for the mere fact that some translations follow their expectations but others do not. This is to confirm what Eisenberg and Dreyfus (1994, p.58) reported that learners found difficulty in visualizing a horizontal translation. In addition, it appears as if transformations in the vertical direction were easier for the learners than those in the horizontal direction. This can happen because of the complexity of the statements themselves and because much more is involved when processing the transformation of $f(x)$ to $f(x+k)$ visually than in processing the transformation to $f(x)+k$ visually. Ninness, Barnes-Holmes, Rumph, McCuller, Ford, Payne, Ninness, Smith, Ward and Elliott (2006) further reveal that a great number of learners face difficulties in learning to recognize graphed representations displayed individually or in various combinations of horizontal and vertical shifts because for many of them, horizontal transformations run contrary to the expectation.

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Another interesting investigation is a case study of Borba and Confrey (1996). Their investigation covers the vertical and horizontal translations, reflections around vertical and horizontal lines, and vertical and horizontal stretches of functions. One of their foci was a sixteen-year-old Ron, who was considered by his teachers to be a good learner. This learner could perform tasks of various levels of complexity. Amongst the tasks assigned to the learner were transforming functions in a computer based multi-representational environment. Ron was instructed to establish the relationship occurring between charges in different graphs and changes in coefficients of their algebraic equations. At first this learner had difficulty in understanding the horizontal translation of a parabola, but he managed to work out some tasks with few difficulties. The study reveals that the learner could easily combine the graphs of two different parabolas using function probe, translate the reflected graph until the vertices of the two coincided, and eventually stretch the graph up to the point that the curves coincided. Ron was also instructed to complete the visualization approach by investigating the relationship between different changes in different graphs and changes in coefficients of their algebraic equations. In particular he was asked to choose a form by which to represent a quadratic in an algebraic way by demonstrating the relation of the coefficients with the translation, stretches and reflections. Borba and Confrey (1996) observed that Ron faced another problem: changes in the coefficients corresponding to stretches and contractions. Surprisingly he was successful.

2.6.2 Inverse errors

Not many investigations have been conducted on the notion of the inverse of graphs. Bayazit and Gray (2004) conducted a study on "understanding inverse functions: the relationship between teaching practice and students learning" while Even (1992) conducted a study on "the inverse function: Prospective teachers use of undoing". Here Even (1992) asserts that "undoing" is an informal expression for inverse function.

Bayazit and Gray (2004) conducted their investigation on the learning of inverse functions in liaison with two particular educators named Ahmet and Mehmet. Each had his own style of teaching, but nevertheless they both dealt with the same subject. Much of Ahmet's teaching was specifically based on the notion of undoing, whereas Mehmet stressed algorithmic and procedural skills. Bayazit & Gray (2004) observed that learners were evaluated in a pre-test and post-test so as to measure their level of understanding of inverse functions. The results indicate a remarkable difference after the learners were asked to work on the problems regarding the notion of the inverse functions. Ahmet, whose teaching was focused on the concept of “undoing” had attempted to get learners to understand the concept. According to him algebraic expressions, mainly linear ones, are not useful in explaining the essence of an inverse function. Unlike Ahmet, Mehmet’s teaching had centred on algorithmic skills and the learning of procedural rules. He seemed to focus on teaching learners to reverse an algebraic function. Cartesian graphs and sets of ordered pairs did not appear in his teaching. In general, on a practical level a small minority of learners seemed to possess cognitive control over the process.

The second phase of teaching entailed graphs on display to which questions were posed. The result of the study indicates that learners gave wrong answers, showing a degree of misunderstanding: for example in sketching the graph of a straight line, sketching the graph which is given as the graph of an inverse or else, reflecting the graph of the function given in the y-axis. Statistically three quarters of Mehmet’s learners did not respond correctly or there was no answer at all. Meanwhile, there were correct answers with two qualitatively different aspects. A point-wise aspect was displayed by the first team of learners. They could work certain points in the Cartesian space, draw a straight line through them or use the algebraic form of the function for transition from the given graph to the graph required.

By contrast, Ahmet’s class managed to sketch the graph of inverse function straight away without point by point referring. The ordinary method reflects the graph given in the line of $y = x$.

2.7 Conclusion

It has emerged from this literature review that most of the investigations suggest that a large number of learners experience difficulties when asked to solve problems regarding graphs. Another important factor is that the difficulties and inabilities learners confront when reading, interpreting and constructing graphs result from certain misconceptions which in turn generate errors that have been listed and explained in detail previously. The next chapter explains the research design.

CHAPTER 3
RESEARCH DESIGN

3.1 Introduction

In this chapter the design for conducting this study is described. This description outlines the steps by which the data was analysed and importantly, specifies the plan used in the investigation.

This chapter presents the research approach and the methodological approach. The description also relates to document analysis, data collection and the matter of sampling which will be developed in the following paragraph. It offers an overview of the LEDIMTALI project that made the data available for analysis. Finally the chapter presents issues of reliability and validity of the study and inter-Rater agreement before closing with an account on the ethical procedures that were followed.

3.2 Research approach

This research adopted a qualitative approach. Qualitative research uses procedures that produce findings which cannot be arrived at by means of statistical procedures (Strauss & Corbin, 1998, McMillan & Schumacher, 2010). The researcher's motivation in adopting a qualitative approach stems from the fact that the study uses actual data derived from learners' scripts gathered in naturally occurring situations. More specifically, the main procedure guiding this research design was that the learners' scripts were collected, presented and analysed as they were in their raw form.

In the light of what was envisaged through this approach the researcher believes, like Fraenkel and Wallen, (1990), that a qualitative approach facilitates an investigation in which the quality of relationships, activities, situations or materials may be probed. Also, there is a greater emphasis on holistic description, that is, on describing the detail of what occurs in a specific situation. In the natural setting of high-stakes examinations, learners' responses to questions concerning algebraic graphs were used to identify the errors they had committed.

3.3 Research method

This study adopted a document analysis method. Simply defined, document analysis is a systematic procedure that aims to review or evaluate documents (Bowen, 2009). It is worth mentioning that document analysis is associated with other analytical methods in qualitative research. It therefore necessitates that data be examined and interpreted in order to elicit meaning, to gain understanding and to construct new knowledge (Strauss & Corbin, 1998).

As a research method document analysis is particularly applicable to qualitative case studies – intensive studies producing rich descriptions of a single phenomenon, event, organisation, or program (Stake, 1995; Yin, 1994).

In this study, the documents analysed comprise the final examination scripts of grade 12 mathematics learners of 2012. The analysis aims to present the types of errors committed by respondents according to the conceptual framework elaborated in this study.

The use of document analysis methodology has advantages laid out by Yin (1994). These are outlined in the points that follow.

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- Efficient method: less time-consuming and more efficient. It requires data selection, instead of data collection.
- Availability: many documents are in the public domain, especially since the advent of the internet, and they are obtainable without the authors' permission. This makes document analysis an attractive option for qualitative researchers.
- Cost-effectiveness: document analysis is less costly than other research methods and is often the method of choice when the collection of new data is not feasible.
- Lack of obtrusiveness and reactivity: documents are unobtrusive, non-reactive and unaffected by the research process.
- Stability: documents are stable.
- Coverage: documents provide broad coverage; they cover a long span of time, many events, and many settings (Yin, 1994).

3.4 Data collection and sampling

In this section the main focus is not on the way data was collected but on the way data was selected. How the selection of the population and the sample is made is an important element that is explained in this section. A definition of population is provided as well as that of sample.

A population is a group of elements or cases, whether individuals, objects, or events, that follow the specific criteria, and that are said to generalize the results of the research (McMillan & Schumacher, 2010). The population for this study consists of all the schools of the Western Cape Education Department. A sample, by contrast, is a group of individuals or items selected from the population of interest. In this case, the 444 scripts which form the sample were selected from the population of all scripts of learners who wrote the particular high-stakes examination in the Western Cape.

For this study the selection of data was governed by the availability of data to a project named LEDIMTALI, a project involved in the development of Mathematics teaching. The schools selected by the Western Cape Education Department (WCED) were those identifiable as operational, which means the schools with at least one class doing Mathematics at grade 12 level. The few schools in the same district fulfilling this criterion were schools located in a radius of approximately 25 km from the University of the Western Cape (UWC). That is to say the schools from which the selection was made participated in the LEDIMTALI Project.

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Hence the sampling method used in this study can be classified as convenience or opportunity sampling. Convenience sampling, also called 'available sampling,' is a type of nonprobability sampling that involves the sample being drawn from the part of the population which is available (Wilson, 2009). This is to suggest that the sample of the study was selected because it was readily available and convenient. It is worth noting that the sample drawn by this method does not claim to be representative of the larger population nor does it claim generalizability. Although in the scope of this study this type of sample makes it difficult to generalize from the sample to any type of population (McMillan & Schumacher, 2010), it is believed that there might be other schools with similar socio-economic conditions to which the findings of the present study may apply.

3.5 Project from which data was selected: LEDIMTALI

LEDIMTALI is a project that groups mathematicians, Mathematics teachers, Mathematics educators as well as Mathematics curriculum advisors who work together in order to develop the teaching of Mathematics. These people form a Continuous Professional Development (CPD) unit and they collaborate in order to administer comprehensible teaching in the field of Mathematics. This initiative works on the assumption that all the aforementioned educators can successfully help learners in their efforts to reach the required level in Mathematics.

Generally speaking, the world of science requires a solid mathematical background. Engineering, architecture, geology, astronautics, actuarial sciences are some of the professions which seek to benefit from sound mathematical teaching and learning. In fact, running counter to this is the unfortunate reality that of the young people who participated in this project a great number cannot achieve their highest potential in Mathematics. The project has implemented some mechanisms by well-established stakeholders to collaborate and increase the numbers of learners registering for Mathematics at a Further Education and Training (FET) level. Consequently the task of the project is to increase the quality of the teaching of Mathematics at Further Education and Training to fulfil the required goals.

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Any project, before taking action, must have established some goals that will enable the organizations to pursue what is to be achieved. It works hard to promote a community of school Mathematics practitioners with the goal of creating some reflective opportunities based on the teaching of Mathematics. Members of the organized community must be creative so that the project attracts and interests teachers and that they adhere to it without any difficulty. Teaching Mathematics to some extent requires much creativity because it is always considered a difficult subject for learners.

Another goal of LEDIMTALI is the implementation of strategies in the sense of improving ways of teaching Mathematics. At this point, mathematicians, Mathematics teachers, Mathematics educators and Mathematics curriculum advisors collectively set up some strategies to help move the project forward without interruption, especially in the domain of Mathematics teaching. It is important that if educators want to make the project successful, they have to remain active and implement actions that are suggested. Learners who wish to further their education – especially in the world of science – need to have mastered Mathematics. The reason is that fields such as Physics, Architecture, Astronomy and others are mathematically-based. At this stage, inculcation is one of the keys to help learners to stir up their conscience so that they learn and master Mathematics. The project as an entity supplies the necessary material for developing the teaching and the learning of Mathematics. The project is meant to have qualified personnel who can cope with a difficult situation. The materials needed are both for the educators and the learners and must be available and ready for use when needed.

In order to develop mathematical comprehension in the teaching of Mathematics, the project works to empower teachers with skills and thereby, to create conditions at schools favourable for developing learners' logical thinking and mathematical proficiency. The project sets out to help teachers gain knowledge for understanding the world around them. A significant aspect to the work is the belief that if the management chain is strong, learners will never be ill-equipped as there will be harmony among Mathematics curriculum advisors, Mathematics educators, mathematicians and Mathematics teachers. In reality, the general aim is to enhance collaborative strategies that exist between all those who are called to the teaching of Mathematics. It is hoped that working together under one roof will contribute towards achieving the goals of the project.

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3.6 Data analysis

Mertler (2009) argues that the main goal of data analysis is to reduce immense amounts of data into smaller and more manageable sets of information. Mertler (2009) further argues that data analysis helps to organise, to provide structure and to elicit meaning. To Creswell (2009) the process of data analysis involves making sense of a text, preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data, representing the data and making an interpretation of the larger meaning of the data. In the scope of this investigation, the errors contained in the scripts fundamentally reflect algebraic graphs and they are classified according to the framework provided in this study. Each error is named and defined following the component or characteristic of algebraic graphs. Learners' errors were identified, labelled in detail with possible meanings according to the three levels described in Chapter 2. These are level 1 errors which comprise coordinate, intercept, domain and range errors; level 2 errors which contain slope, asymptote, turning point and axis of symmetry errors; and level 3 errors comprising identification, drawing errors and function errors.

The present study is guided by the view that qualitative data analysis is mainly an inductive process consisting of data collected and organized into categories so as to identify the main patterns and the relationships between the categories (McMillan & Schumacher, 2010; Mertler, 2009).

In this study, the data analysis involved the thorough reading and analysis of 444 individual scripts and the identification of questions on graphs where learners had given the incorrect answers.

In this respect all errors found in the scripts were identified and categorised in terms of the framework of levels 1, 2 and 3 errors as outlined in Chapter *2* or alternatively allocated to the category of additional errors described in the same chapter. Furthermore, the particular subcategory in which the error belongs was allocated to it.

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3.7 Reliability, Validity and inter-Rater agreement

Reliability and validity are two vital concepts that cannot be overlooked in any research. Reliability and validity ensure that the research is considered genuine, that it is believable and credible. In research, data is considered to the point that it describes or deals at once with the topic under consideration (Mertler & Charles, 2005). The reliability and validity in research are relevant as they make the research findings acceptable and convincing to the researcher and to anyone who is expected to read it.

In his comments regarding reliability Wilson (2009) argues that the concept of reliability implies the rigour, steadiness and specially the true value of the investigation. Basit (2010) in his turn explains that reliability denotes that the research process can be repeated at another time with similar participants in a similar context and yield more or less similar results. In the scope of this study, there is a belief that the results might be similar when the same research is repeated by another investigator in a similar context. Furthermore, data is reliable to the extent that it is consistent (Mertler & Charles, 2005). The observations conducted on the scripts are reliable to the extent that the results on the incorrect answers provided by the raters (observers) on the errors committed by learners are approximately the same.

As to validity, McMillan and Schumacher (2010) argue that validity is the degree of agreement between the explanations of the phenomena and the realities of the world. Therefore data is valid to the extent that it depicts or deals directly with the topic under consideration (Mertler & Charles, 2005). In the scope of this study, validity is achieved on the grounds that the observations on learners' responses have to do with the knowledge of algebraic graphs.

Inter-Rater agreement is the degree to which two or more raters using the same rating scale independently give the same rating to an identical observable situation (Graham, Milanowski & Miller, 2012). This is to suggest that inter-Rater agreement is a measurement of the consistency between the absolute value of the evaluators' ratings (Graham, Milanowski & Miller, 2012) Regarding this study, the inter-Rater agreement is essential in the sense that it is an evaluation in which judgements are made about the errors committed in each script as well as the types of errors expected in each question. It is expressed in percentages.

To achieve the inter-Rater agreement in this study, an estimated sample of 300 scripts was used. Out of the 300 scripts, ten percent was randomly selected representing a total of 30 scripts. These scripts were given sequentially to three different raters for analysis, including the researcher. Each rater identified from the 30 scripts the types of errors learners had made on the basis of the adopted framework in Chapter 2. The data was then put together and the number of times the raters agreed on ratings was calculated and divided by the total number of ratings in order to get the percentage of absolute agreement which had to be more than 75% (Graham, Milanowski & Miller, 2012). The application of this procedure resulted in an absolute agreement of 94, 33%. This meant that the rating agreement between the raters was absolute with near-perfect inter-Rater agreement.

The process of inter-Rater agreement identifies the questions reflecting the algebraic graphs. The questions are asked to determine either a function or maybe some characteristics of a function from a given graph (or learners were asked to draw a graph) by the respondent identifying some characteristics from a given function. As part of this process, after detecting the types of errors to each script and each question on algebraic graphs, the reader acquired an insight into the evaluation of learners' ways of working in response to the Mathematics Examination. The process also entailed the determination of the degree of trust so that the academic sphere may have confidence in what is being done. Similarly, the process of inter-Rater agreement used for the 30 scripts was used for the remaining scripts with the purpose of facilitating the counting of errors.

3.8 Ethics statement

Ethics is generally concerned with beliefs about what is right or wrong in the conduct of research (McMillan & Schumacher, 2010). For this research, permission to collect data was obtained from the Western Cape Education Department (WCED) and the University of the Western Cape (UWC) by the Local Evidence Driven Improvement of Mathematics Teaching and Learning Initiative (LEDIMTALI) Project.

In Ethics, privacy plays an important role. Privacy in ethical considerations simply denotes that the organizing institutions and the researcher must be free from any judicial pursuit. Privacy concerns the participants' rights and is a major concern as it remains a secret throughout the entire investigation and thereafter (Basit, 2010). Besides, those who participate in the research have the right to privacy and protection (McMillan & Schumacher 2010).

In scientific research, Ethics pertains to certain values that must be observed while an investigation is being conducted. The use of privacy in Ethics is often coordinated on the basis of three practices: anonymity, confidentiality and appropriate storing of data (McMillan & Schumacher, 2010).

According to McMillan and Schumacher (2010), anonymity means that there is no connection between the data and the participants, and that the researcher cannot identify the participants from information that has been gathered. In addition, data and the participants are not connected, and besides, there is no means by which the investigator can identify the participants from what he collected (McMillan & Schumacher, 2010). Regarding this study, since the examination scripts contained only the learners' examination numbers, the researcher and the raters had no knowledge of who the examinees were. Furthermore, the centres where the examination was conducted were indicated only by a centre number and the researcher and raters had no access to mechanisms by which to identify the centres using centre numbers. To further reinforce anonymity, candidates' examination numbers and examination centre numbers were not referred to in the research.

For this study confidentiality is linked to anonymity and the researcher and the raters had to sign a confidentiality agreement with the project leader who was legally responsible for the collection and ethical handling of the examination scripts. This stated that they could divulge no information about candidates or examination centres.

Concerning the data storage and security, the examination scripts were kept in a secure place by the project leader so that no person directly associated with this study could have access to them. This was intended to help preserve the content of the collected data (McMillan & Schumacher, 2010).

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3.9 Conclusion

This chapter has presented details concerning the approach and methods used in the study. It has also explained the process of data collection and how it was analysed. Further to this, the chapter has presented a discussion on the reliability and validity as well as on the ethical issues as these obtain. The next chapter presents the results.

CHAPTER 4
RESULTS

4.1Introduction

The previous chapter involved a discussion of the research design that includes the research method and the way the data was collected and analysed. This chapter presents the results in relation to the three questions and sub-questions on graphs that formed part of the Paper 1 of the 2012 final Mathematics examinations. The chapter begins with an outline of the relevant questions as they appeared in the paper. This is followed by a discussion of the errors committed by learners in each question as they responded to the questions, and the category into which each error falls. The chapter ends with a table which summarises the number of errors made by learners at each level of errors.

Pertinent to the analysis are Questions 1, 2 and 3 which were extracted from the 2012 Paper 1 grade 12 Mathematics examinations; their original order was 4, 5 and 6 respectively. It is in this order that the learners answers appear. Exemplars of errors and subsequent statistics of errors per question are provided.

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Question 1

1.1 Consider the function $f(x) = 3.2^x - 6$.

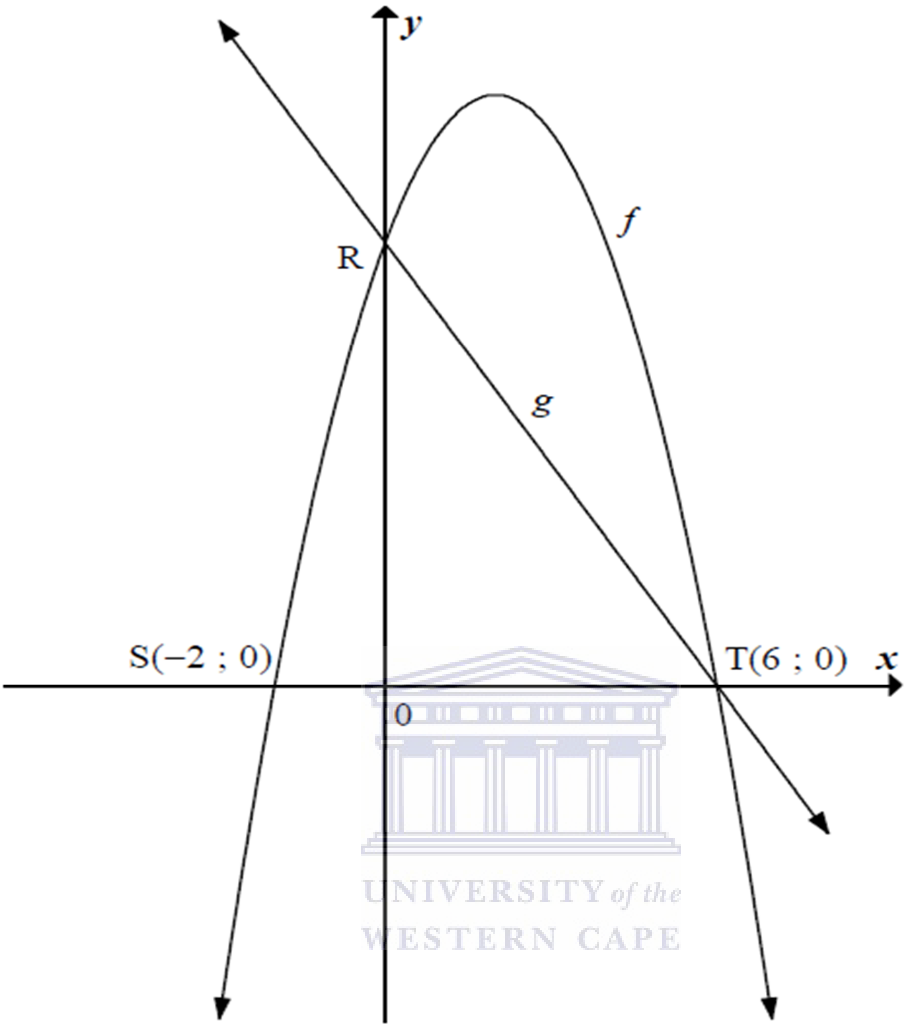
1.1.1 Calculate the coordinates of the y-intercept of the graph of f .

1.1.2 Calculate the coordinates of the x intercept of the graph of f .

1.1.3 Sketch the graph of f in your ANSWER BOOK. Clearly show ALL asymptotes and intercepts with the axes.

1.1.4 Write down the range of f .

1.2 $S(-2;0)$ and $T(6;0)$ are the x-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the y-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



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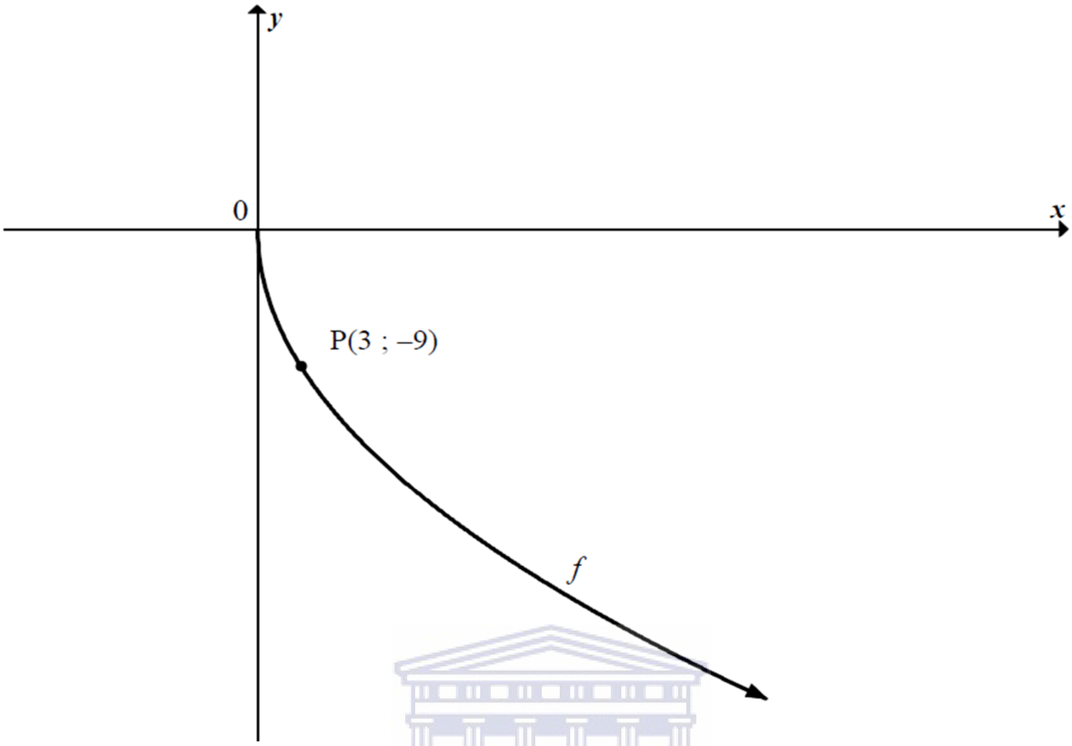
1.2.1 Determine the value of d .

1.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$.

Question 2

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below. The point P (3;-9) lies on the graph of f .



2.1 Use your graph to determine the values of x for which $f(x) \geq -9$.

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2.2 Sketch f^{-1}, the inverse of f, in your answer book. Indicate the intercept(s) with the axes and the coordinates of ONE other point.

2.3 Describe the transformation from f to $g(x) = \sqrt{27x}$, where $x \geq 0$.

Question 3

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in R, x \neq 5$
- Range: $y \in R, y \neq 1$
- Passes through the point $(2; 0)$

Determine $f(x)$. (Department of Basic Education/November 2012)

The analysis of learners' scripts was made with particular attention to their errors in the framework of the research question. Nevertheless, the statistics of the correct and the no responses are provided as well.

4.2 Analysis of learners' errors

In this section a description of each question is given, then the correct answer is proposed and lastly the learners' responses are provided as exemplars in the form of figures.

4.2.1 Level 1 errors

These types of errors occur when learners inappropriately apply the basics of the pre-requisite knowledge of algebra.

4.2.1.1 Coordinate errors

Coordinate errors are those in which learners use the coordinates incorrectly in plotting or extracting them. This is to suggest that by reversing the order the values are wrongly used. For instance, the x-values are attributed to y-values and the y-values to x-values.

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Firstly, the questions referring to the coordinates are those related to x-intercept and y-intercept because they are both the coordinates. In these types of questions, learners were being asked to calculate and to plot the coordinates as in questions 1.1.1, 1.1.2 and 1.1.3. The correct answers and the learners' responses are shown in the section concerning intercept errors.

Secondly, learners were asked to determine the coordinates of the inverse of the graph on questions 2.2. The correct answers and the learners' responses are indicated as level 3 errors related to drawing errors.

4.2.1.2 Intercept errors

Intercept errors occur when the cutting point with the axis is incorrectly determined or found. Learners were first asked to find the y-intercept from $f(x) = 3.2^x - 6$, which then, according to question (1.1.3) had to be plotted on the graph by showing the intercepts on the drawing.

The correct answer to this question (1.1.1) is:

$y = 3.2^0 - 6$

$y = 3 - 6$

$y = -3 \qquad (0;-3)$

The analysis of learners' responses is presented in Figure 2, Figure 3, Figure 4, and Figure 5.
As shown in Figure 2, it was found that some learners confused the dot which represents the multiplication sign with a decimal comma and treated 3.2^0 as $(3.2)^0$ which is equal to 1.

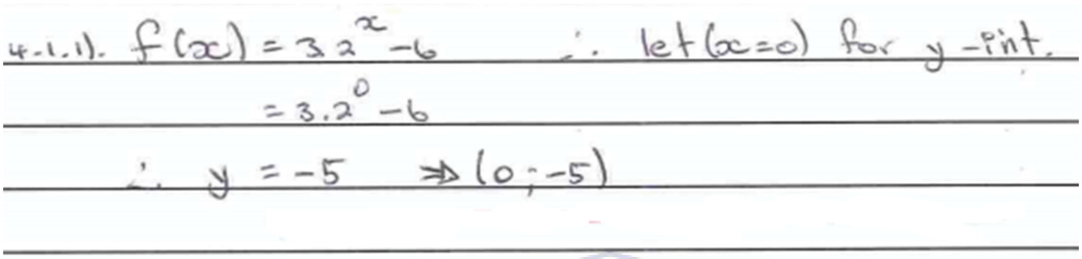

Figure 2: y -intercept error exemplar 1

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As shown in Figure 3, this is clearly shown in the following figure where the dot is replaced by a decimal comma.

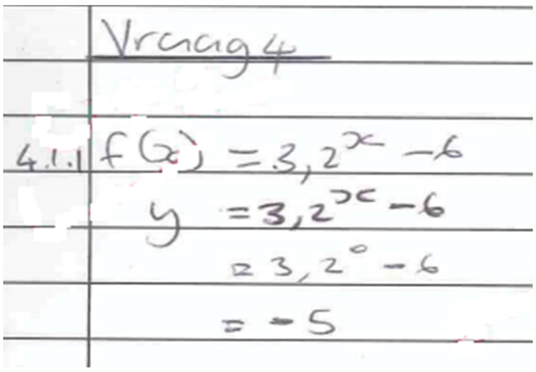
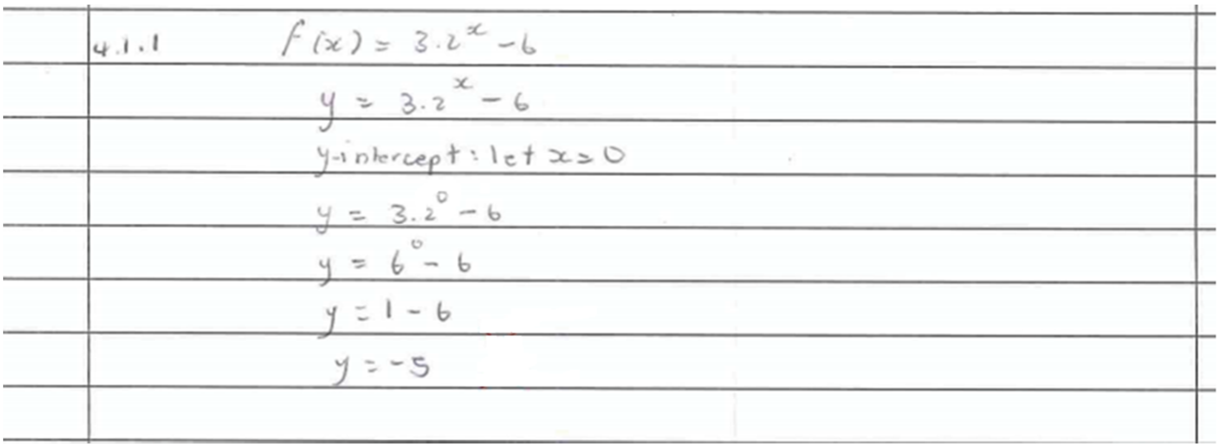


Figure 3: y -intercept error exemplar 2

As displayed in Figure 4, it was found that the learner first applies multiplication to the bases and then raises the product to power 0.


4.1.1 $f(x) = 3.2^x - 6$

$y = 3.2^x - 6$

y-intercept : let $x = 0$

$y = 3.2^0 - 6$

$y = 6^0 - 6$

$y = 1 - 6$

$y = -5$

Figure 4: y -intercept error exemplar 3

Another question (1.2.1) in this regard was to determine d which corresponds to R on the graph, bearing in mind that d and R share the same point. R is the y -intercept on the curve of the graph of $f(x) = ax^2 + bx + c$ and d is also y -intercept of the straight line of $g(x) = -2x + d$. This might bring confusion in the learners' minds. But according to the function $g(x) = -2x + d$, this is visibly the cutting the point.

The correct answer to this question is:

$y = -2x + d$

$0 = (-2)(6) + d$

$d = 12$

Or

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 6)$

$y = -2x + 12$

$d = 12$

Or

Since $m = 2$ and $m = \frac{-d}{6}$

$$-2 = \frac{-d}{6}$$

$$d = 12$$

As illustrated in Figure 5, it was found that the learner wrongly extracts the coordinate from the graph with the opposite sign to calculate the y-intercept.

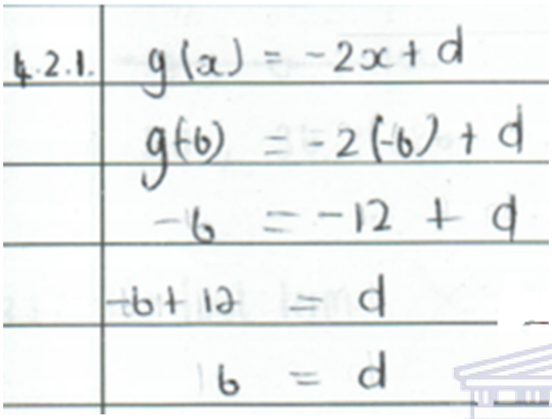

4.2.1. $g(x) = -2x + d$
$g(-6) = -2(-6) + d$
$-6 = -12 + d$
$-6 + 12 = d$
$6 = d$

Figure 5: y-intercept error exemplar 4

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As presented in Figure 6, it was found that the learner interchanges the values of the coordinates, that is the x-value to y-value (the abscissa becomes the ordinate) and vice versa.

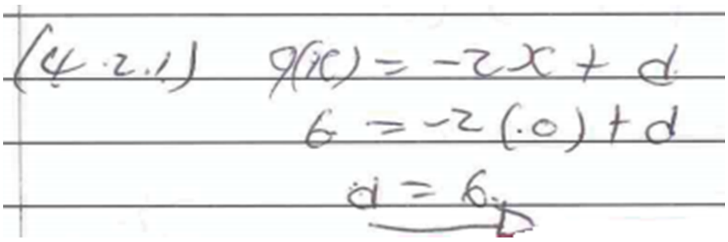
(4.2.1) $g(x) = -2x + d$
$6 = -2(.0) + d$
$d = 6$

Figure 6: y-intercept error exemplar 5

As presented in Figure 7, it was found that the learner extracts the wrong coordinate from the graph to calculate the intercept.

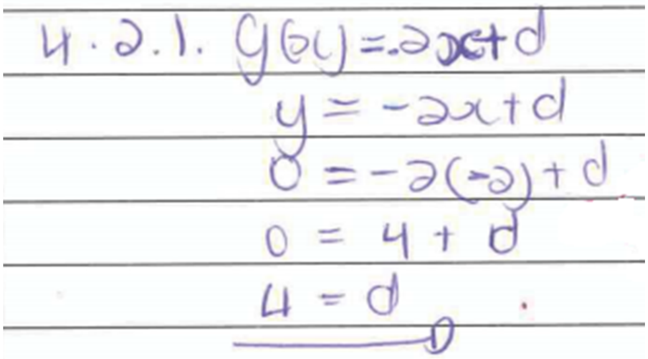

Figure 7: y -Intercept error exemplar 6

Lastly, learners were asked to find in question (1.1.2) the x -intercept from $f(x) = 3.2^{x} - 6$, which had then to be plotted later as asked in question (1.1.3) by showing the intercepts on the drawing. The correct answer to this question is:

$0 = 3.2^{x} - 6$

$3.2^{x} = 6$

$2^{x} = 2^{1}$

$x = 1$ (1;0)

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The information displayed in Figure 8 shows that the learner multiplies directly without considering 2^{x} as an exponential form, and then applies interchanges x and y .

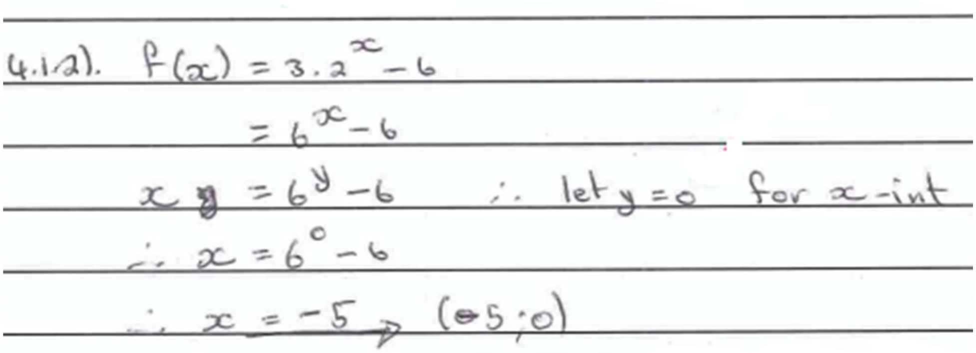
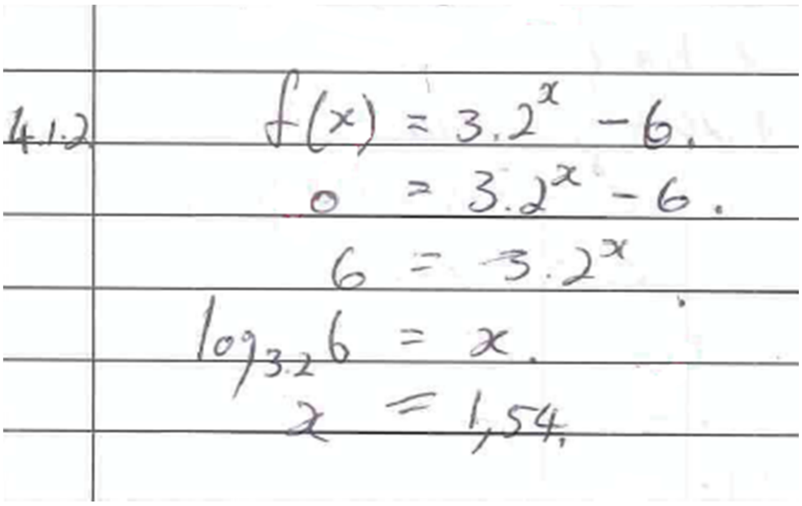


Figure 8: x -Intercept error exemplar 1

As shown in the Figure 9, the learner wrongly introduces the algorithm in their calculation of the x-intercept.



4.1.2 $f(x) = 3.2^x - 6.$

$0 = 3.2^x - 6.$

$6 = 3.2^x$

$\log_{3.2} 6 = x.$

$x = 1{,}54.$

Figure 9: x-intercept error exemplar 2

As shown in Figure 10, the learner used the wrong answer for the y-intercept obtained in Figure 2. This value was substituted for y in the equation $f(x) = 3.2^x - 6$. The learners then proceeded to solve for x.

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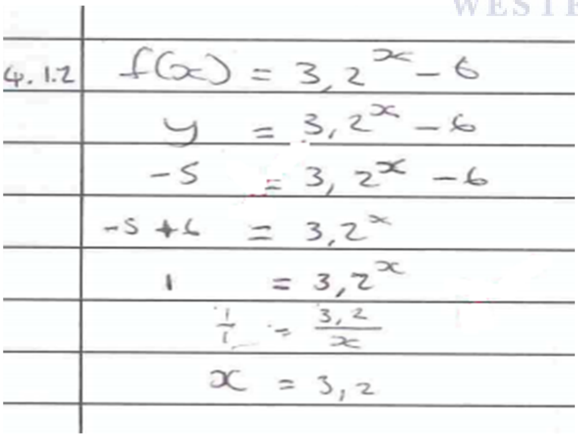

4.1.2 $f(x) = 3{,}2^x - 6$

$y = 3{,}2^x - 6$

$-5 = 3{,}2^x - 6$

$-5 + 6 = 3{,}2^x$

$1 = 3{,}2^x$

$\frac{1}{1} = \frac{3{,}2}{x}$

$x = 3{,}2$

Figure 10: x-intercept error exemplar 3

4.2.1.3 Domain and Range errors

This is the type of question which confused learners most. The inputs on the x-axis and the outputs on the y-axis are not determined correctly. The notion of ordering in the intervals causes learners difficulty as they are unable to write real numbers in ascending order or vice versa.

Domain errors

In question 2.1 the learners were asked to use the graph in order to determine the values of x for which $f(x) \geq -9$. More specifically, the objective of the question was to determine a part of the domain which satisfies the given restriction. The correct answer to this question is:

$0 \leq x \leq 3$ or [0; 3].

The information displayed in Figure 11 and Figure 12 shows that learners obtain an incorrect domain, as illustrated in the following two exemplars.

As shown in Figure 11, the learner commits an error in not considering the graph as it is represented by obtaining a wrong domain. For this learner, the domain is all values greater or equal to 3.

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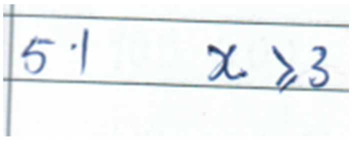

Figure 11: Domain error exemplar 1

As shown in Figure 12, the learner takes the given restriction as the required domain.

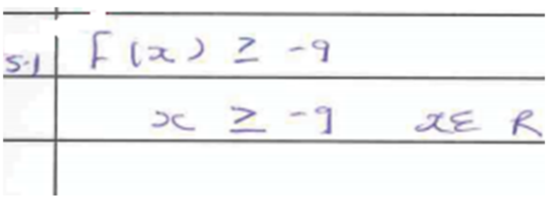

Figure 12: Domain error exemplar 2

On the basis of the result presented in Figure 13, it can be said that the learner fails to rearrange the values of the interval notations. By simply looking at the way he/she represents the second interval it is clear that this learner does not show that he/she has any knowledge of the interval notation.

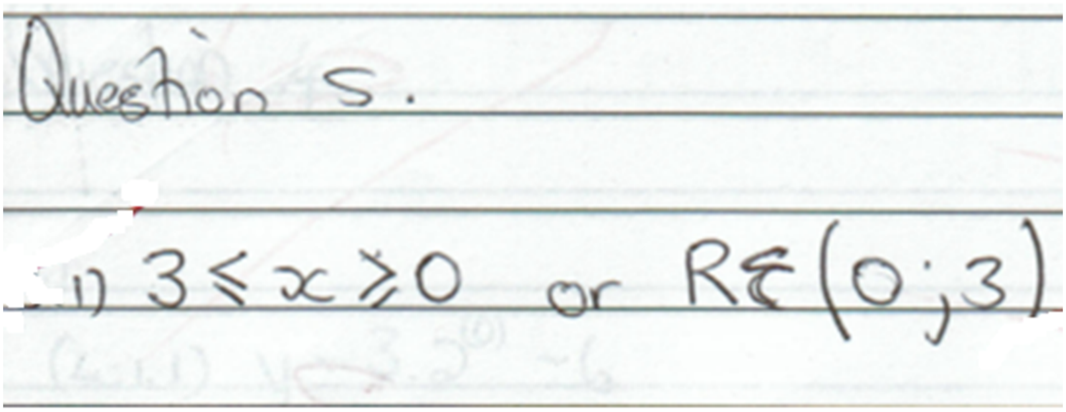

Figure 13: Domain error exemplar 3

Range errors

The question at this level was to determine the range of f. To correctly answer this question, the learners should either determine the range from the given function which is $f(x) = 3.2^x - 6$ or from the graph drawn (sketched) on 1.1.4 The correct answer to this question is: y >-6 or (-6 ;∞).

The information presented in Figure 14 shows the incorrect rearrangement of the lower and upper bounds of the intervals.

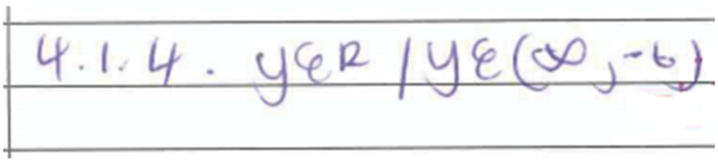

Figure 14: Range error exemplar 1

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In view of the information displayed in Figure 15, it was found that the learners obtained the incorrect range.

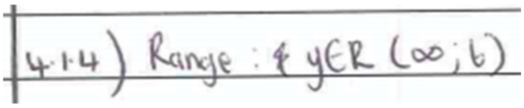

Figure 15: Range error exemplar 2

Table 1 summarizes the level 1 errors in terms of their types, the number of errors, the correct and no responses.

Table 1: Level 1 errors

Characteristics	Errors	Percent	Correct	Percent	No Responses	Percent	Total
Coordinates	794	59,61	387	29,05	151	11,34	1332
Y-Intercept	208	46,85	223	50,22	13	2,93	444
X-Intercept	305	68,7	119	26,80	20	4,5	444
Intercept d	157	35,36	244	54,96	43	9,68	444
Domain	334	75,22	28	6,31	82	18,47	444
Range	342	77,03	19	4,28	83	18,69	444
Total	**2140**	**-**	**1020**	**-**	**392**	**-**	**-**

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4.2.2 Level 2 errors

4.2.2.1 Asymptote errors

In this set of questions, there were no open questions on level 2 errors according to the set of questions used in this investigation by which to assess learners. In other words, there were no questions directly or indirectly related to graphs. However, learners were asked in one of the sub-questions (1.1.3) to show all asymptotes and to plot them. In this case learners were asked to show one horizontal asymptote only. The correct answers and the learners' responses are shown on level 3 on the drawing errors again.

Table 2 summarizes the number of asymptote errors, the correct and the no responses. It is the only type of error that has been found in this level.

Table 2: Level 2 errors

Characteristics	Errors	Percent	Correct	Percent	No Responses	Percent	Total
Asymptote	272	61,26	121	27,25	51	11,49	444

4.2.3 Level 3 errors

4.2.3.1 Identification errors

Identification error relates to an inaccurate identification that the learners make of the graph of function when the graph or the function is given respectively. In the scope of this investigation, an identification error is displayed in questions 1.1.3 and 3, where few learners could not identify the graph properly. The following figures present the learners' errors as analysed in their responses.

A look at Figure 16 reveals that the learner uses an inaccurate standard form of the quadratic function instead of the hyperbolic form.

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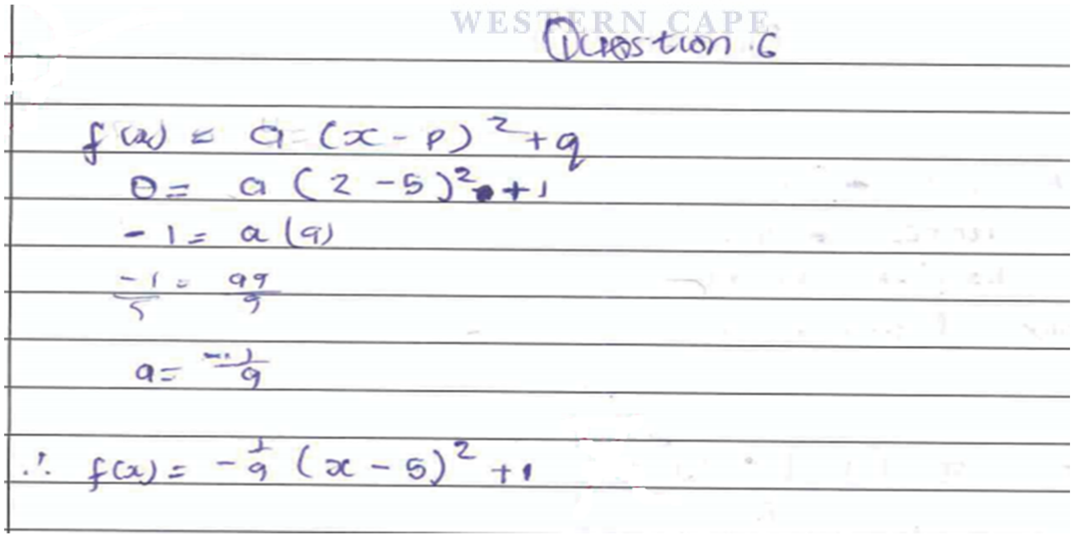

Figure 16: Identification error exemplar 1

By examining Figure 17 and Figure 18 it emerges that the learners draw the hyperbolic and linear graph respectively instead of the exponential graph.

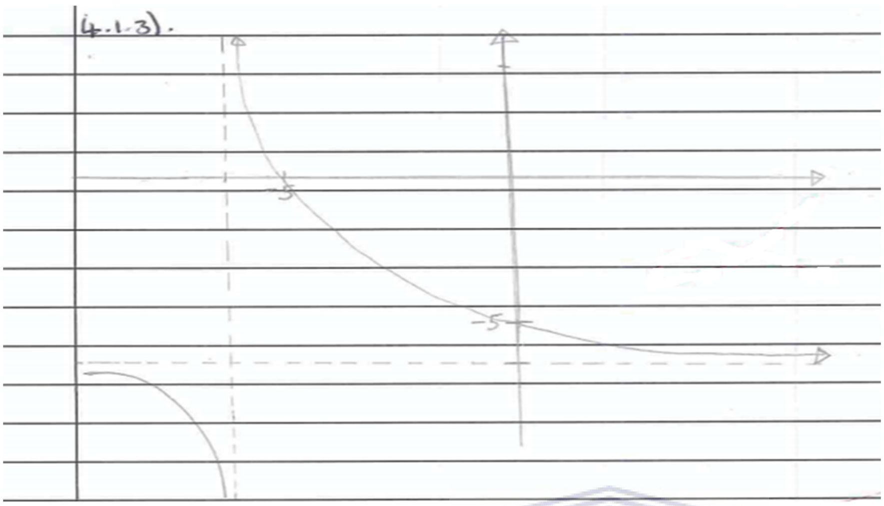

Figure 17: Identification error exemplar 2

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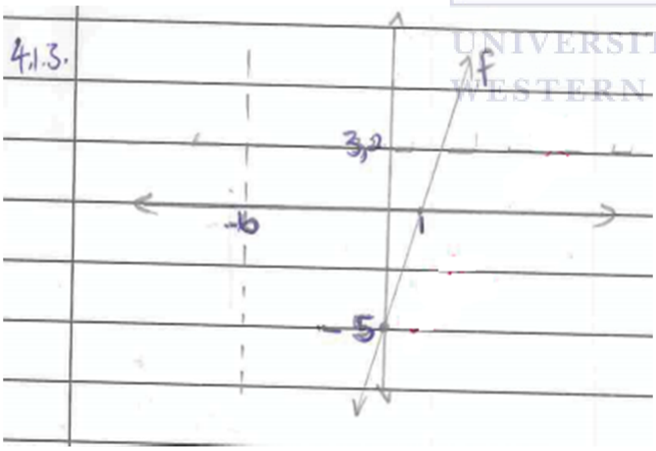

Figure 18: Identification error exemplar 3

As indicated in Figure 19, it was discovered that the learner draws a parabola instead of a hyperbolic graph.

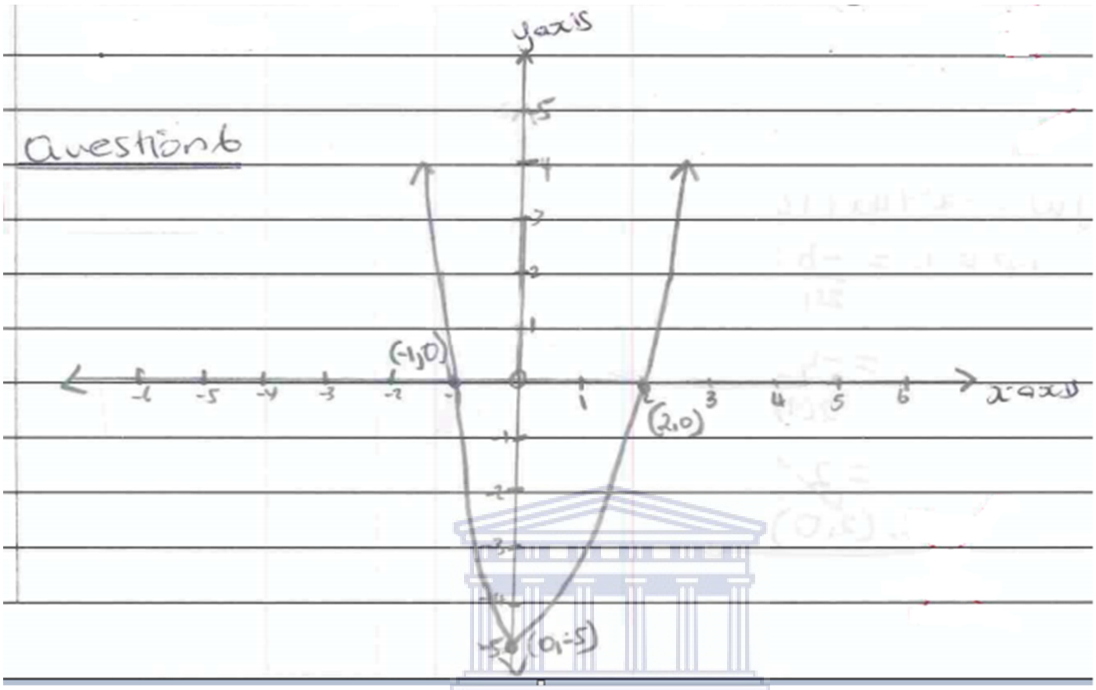

Figure 19: identification error exemplar 4

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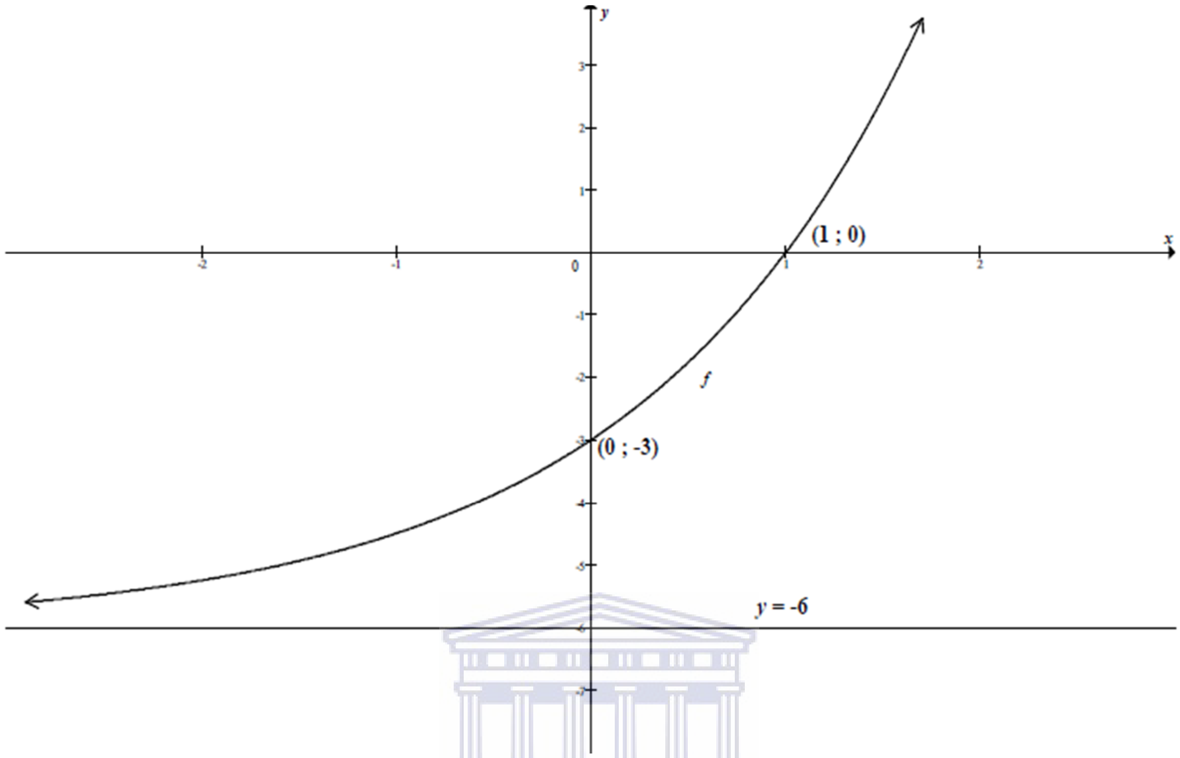
4.2.3.2 Drawing errors

A drawing error is the incorrect manner in which learners represent a function graphically due to errors relating to the algebraic representation. In other words, learners make a good choice of the type of graph to draw but the algebraic manipulations cause a problem. In this investigation, Question 1.1.3 was asked. First learners were to sketch the graph of the function $f(x) = 3.2^x - 6$ and then they were asked to clearly show all asymptotes and intercepts with the axes. The correct answer to the question is:



An analysis of learners' responses reveals the following errors as presented in Figure 20 and Figure 21.

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A look at Figure 20 shows that the learner draws the graph wrongly and the asymptote is presented as if it is horizontal shift at the x-negative values or to the left.

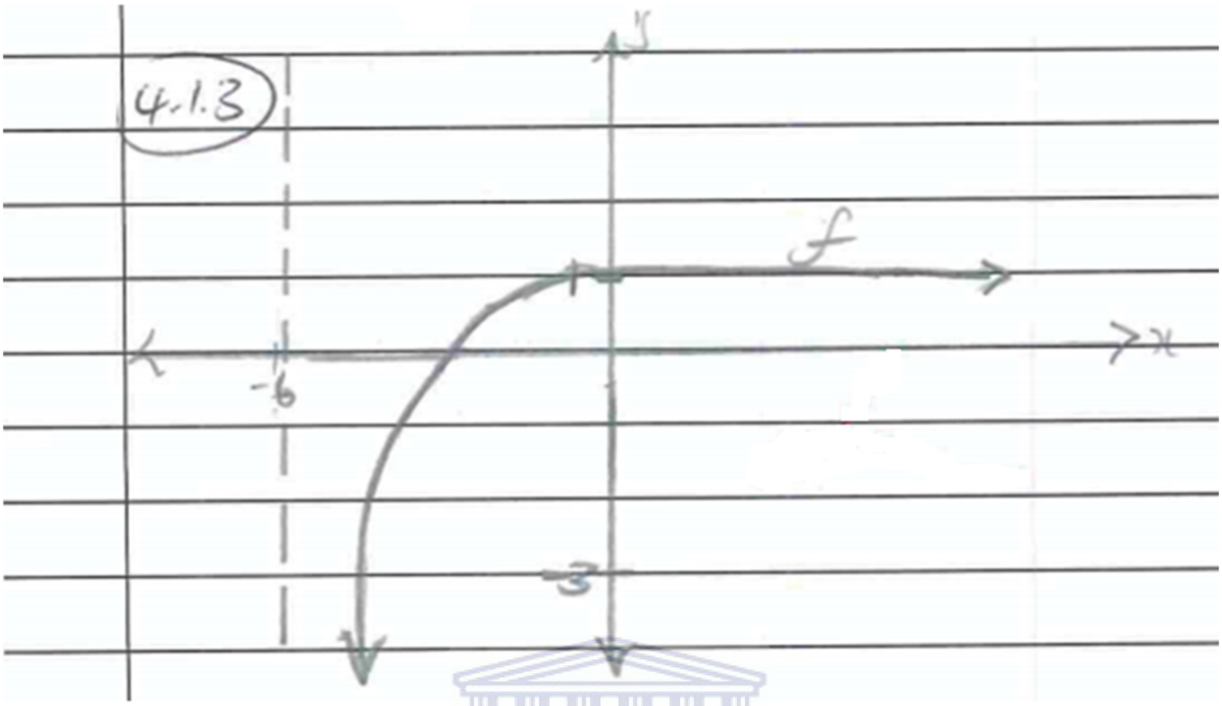

Figure 20: drawing and asymptote errors exemplar 1

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A look at Figure 21 shows that the learner draws the graph wrongly and the asymptote is presented as if it is horizontal shift at the x-positive values or to the right.

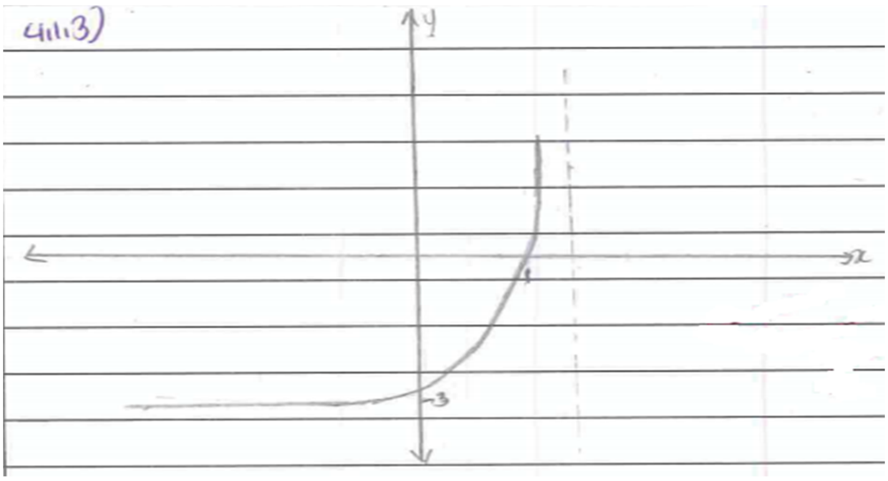
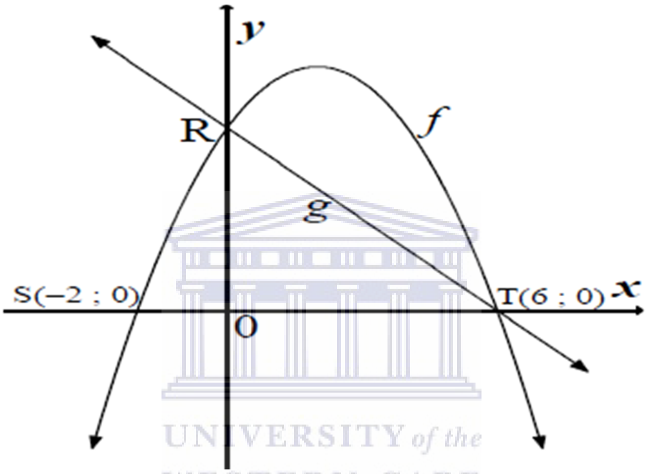


Figure 21: drawing and asymptote errors exemplar 2

The second question on the drawing or the sketching of the graph will be described in the inverse errors.

4.2.3.3 Function errors

A function error occurs when an incorrect function is obtained from a given graph. Learners were asked to determine the equation of f in the form of $f(x) = ax^2 + bx + c$ from the graph given below. They then had to construct a quadratic function with respect to the characteristics on the given graph. The correct answer to this question is:



$y = a(x-6)(x+2)$

$12 = a(0-6)(0+2)$

$a = -1$

$y = -(x^2 - 4x - 12)$

$\quad = -x^2 + 4x + 12$

Or

$y = ax^2 + bx + 12$

$0 = a(-2)^2 + b(-2) + 12$ i.e. $0 = 4a - 2b + 12$

$0 = a(6)^2 + b(6) + 12$ i.e. $\underline{0 = 36a + 6b + 12}$

$0 = 24b - 96$

$b = 4$

$0 = 4a - 2(4) + 12$

$a = -1$

$y = -x^2 + 4x + 12$

Or

$y = a(x-2)^2 + q$

$0 = a(-2-2)^2 + q$ Or $0 = a(6-2)^2 + q$ i.e. $0 = 16a + q$

$12 = a(0-2)^2 + q$ i.e. $12 = 4a + q$

$$12 = -12a$$

$$a = -1$$

$$q = 16$$

$$\begin{aligned} y &= -(x-2)^2 + 16 \\ &= -(x^2 - 4x + 4) + 16 \\ &= -x^2 + 4x + 12 \end{aligned}$$

Or

$$\begin{aligned} y &= a(x-6)(x+2) \\ &= a(x^2 - 4x - 12) \\ &= -(x^2 - 4x - 12) \\ &= -x^2 + 4x + 12 \end{aligned}$$

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The analysis of learners' responses reveals the following errors as presented in Figure 22, Figure 23 and Figure 24.

A look at Figure 22 shows that the learner does not first obtain the coefficient before finding the required function.

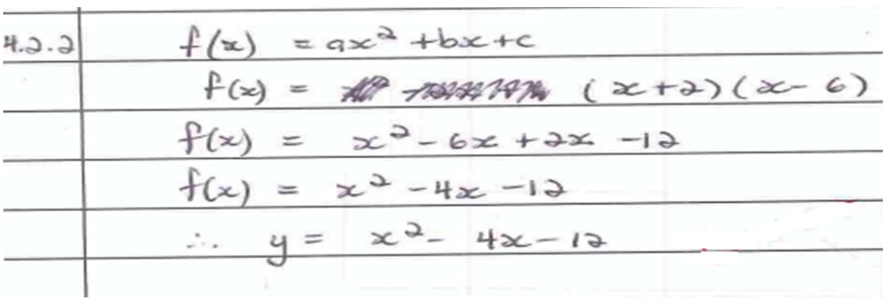
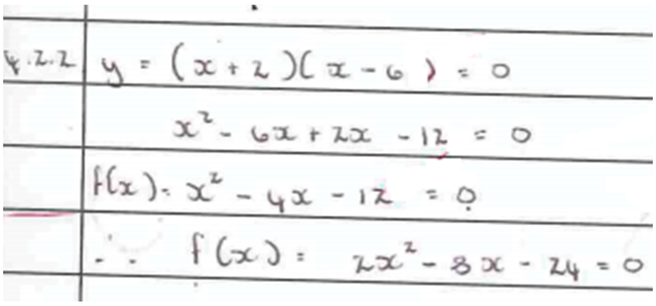


Figure 22: Function error exemplar 1

A look at Figure 23 shows that the learner does not know the distinction between the function and the equation. Also, the learner does not find the value of the coefficient a .


4.2.2 $y = (x+2)(x-6) = 0$

$x^2 - 6x + 2x - 12 = 0$

$f(x) = x^2 - 4x - 12 = 0$

$\therefore f(x) = 2x^2 - 8x - 24 = 0$

Figure 23: Function error exemplar 2

A look at Figure 24 shows that the learner has difficulty in applying the basic knowledge of exponents and algebraic calculation. Although the learner succeeds the substitution of the first x -intercept $S(-2,0)$ into the equation of the function, he or she is confused about the substitution of the second x -intercept $T(6,0)$.

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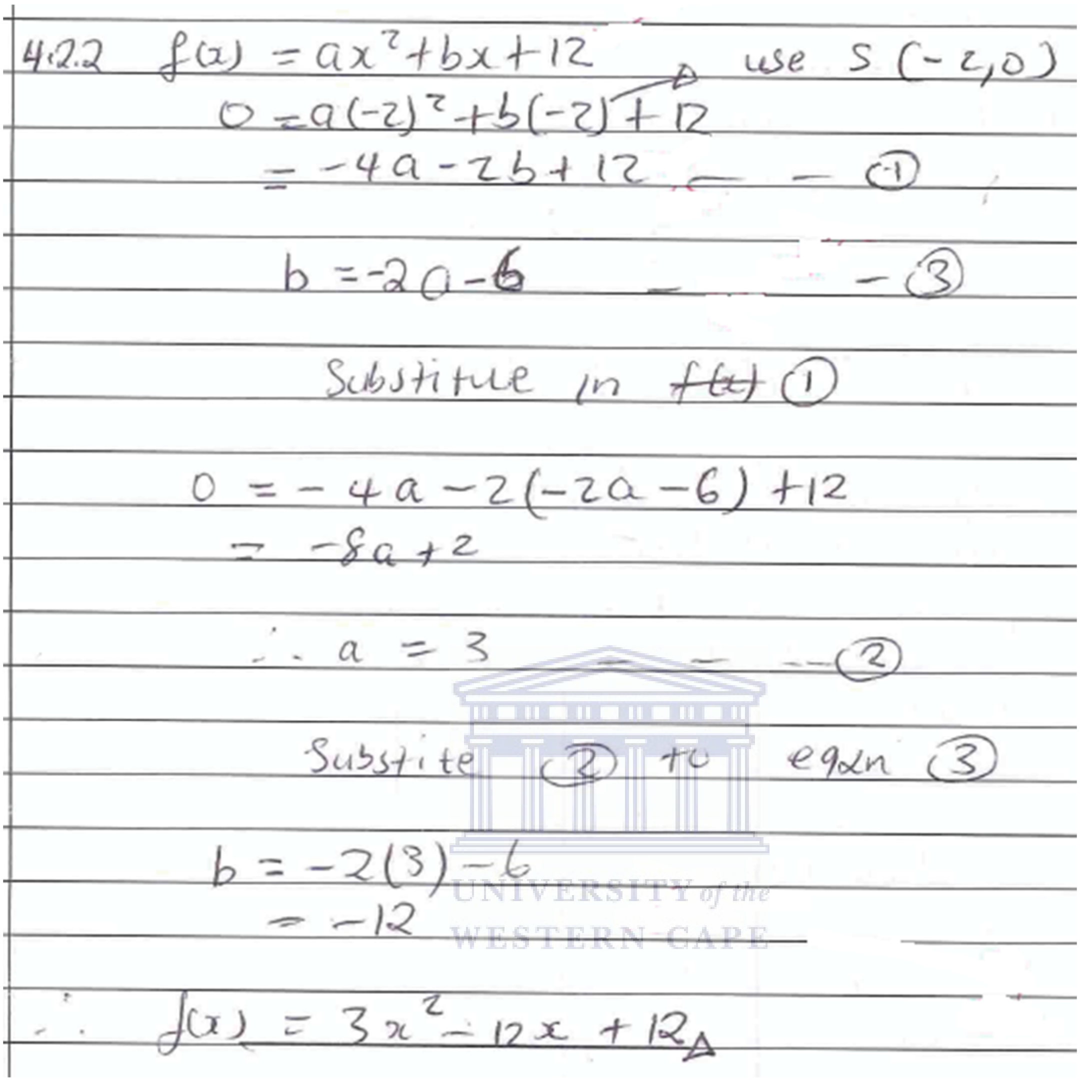

4.2.2 $f(x) = ax^2 + bx + 12$ use $S(-2,0)$

$0 = a(-2)^2 + b(-2) + 12$

$= -4a - 2b + 12$ — ①

$b = -2a - 6$ — ③

Substitute in $\cancel{f(x)}$ ①

$0 = -4a - 2(-2a - 6) + 12$

$= -8a + 2$

$\therefore a = 3$ — ②

Substite ② to eqn ③

$b = -2(3) - 6$

$= -12$

$\therefore f(x) = 3x^2 - 12x + 12$

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Figure 24: function error exemplar 3

In addition learners were asked to construct a hyperbolic function of the form $f(x) = \frac{a}{x+p} + q$ from the given properties of the graph of this kind. A great number of learners constructed the function without sketching the graph from the given properties, but few of them managed to first sketch the graph and eventually construct the function easily. The correct answer to this question is:

$$f(x) = \frac{a}{x-5} + 1$$

$$0 = \frac{a}{(2) - 5} + 1$$

$$-1 = \frac{a}{-3}$$

$$a = 3$$

$$f(x) = \frac{3}{x-5} + 1$$

Or

$$(x-5)(y-1) = k$$

$$(2-5)(0-1) = k$$

$$a = 3$$

$$(x-5)(y-1) = 3$$

$$y = \frac{3}{x-5} + 1$$

Note: $f(x) = \dfrac{x-2}{x-5}$ as an alternative simplified form.

A look at Figure 25 shows that the learner does not first determine the coefficient a of the standard form and changes the signs of p and q to negative instead of positive.

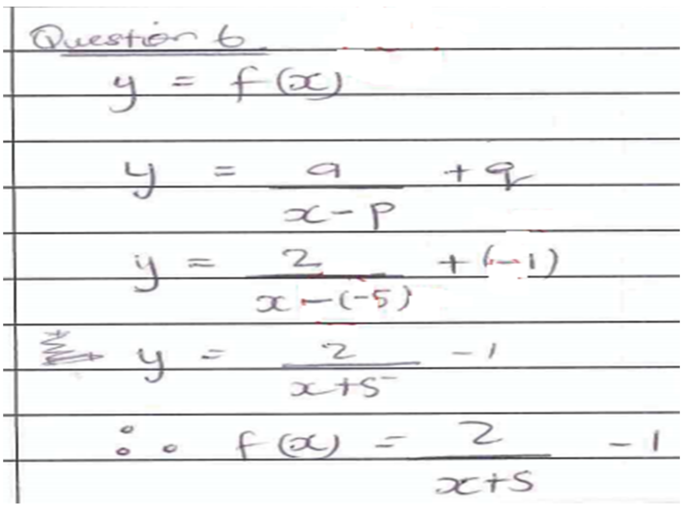

Figure 25: Function error exemplar 4

In view of the information displayed in Figure 26, it can be said that the learner drew the right graph but did not know how to find the coefficient a and to substitute the values of p and q.

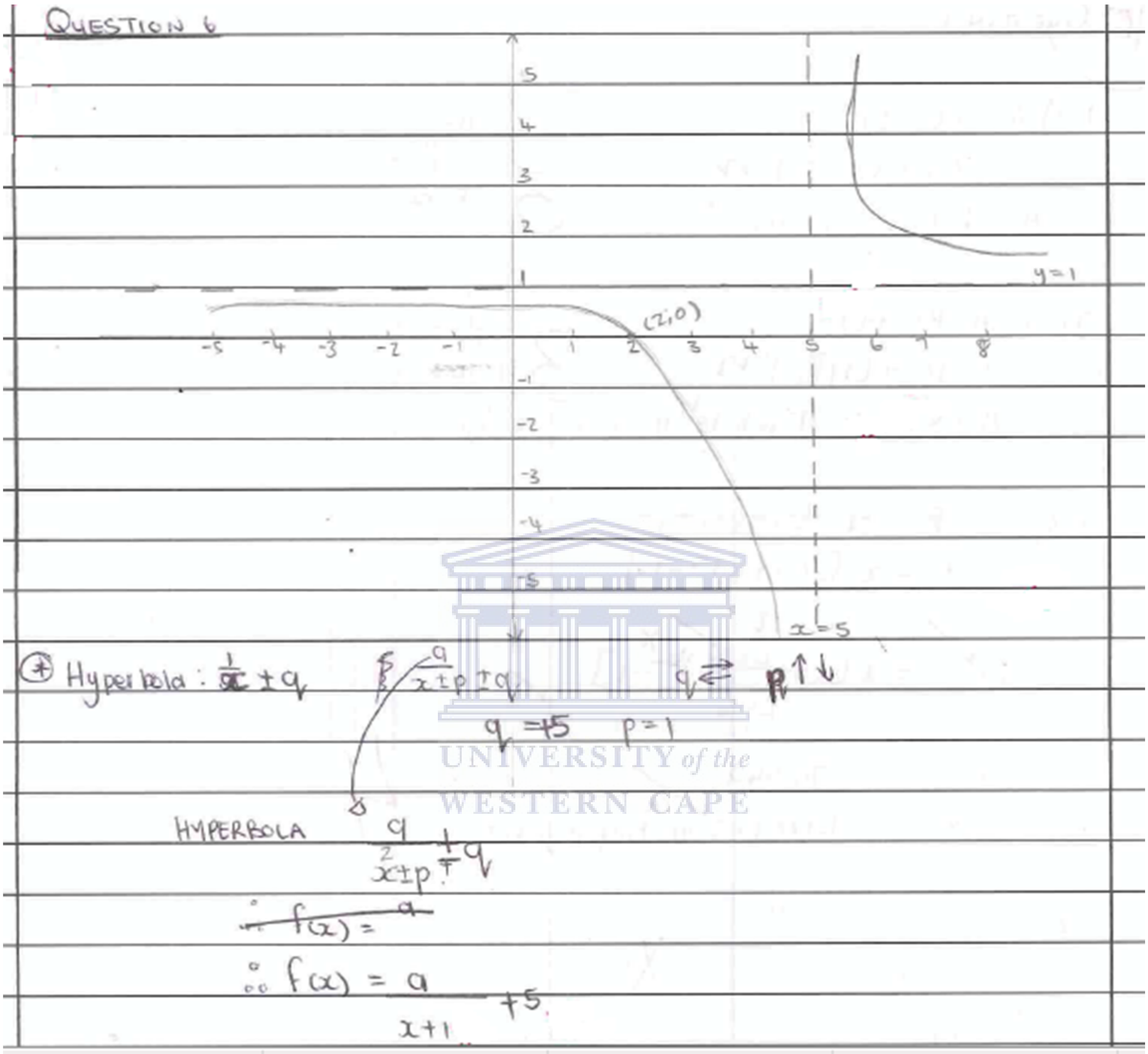

Figure 26: function error exemplar 5

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Table 3 summarizes the level 3 errors in terms of their types, the number of errors, the correct and the no responses.

Table 3: Level 3 errors

Characteristics	Errors	Percent	Correct	Percent	No Responses	Percent	Total
Identification	112						
Drawing	279	62,84	112	25,22	53	11,94	444
Quadratic Function	298	67,12	79	17,79	67	15,09	444
Hyperbolic Function	229	51,58	90	20,27	125	28,15	444
Total	**918**	**-**	**281**	**-**	**245**	**-**	**-**

4.2.4 Additional errors

These errors were not part of the initial conceptual framework. They were identified during the inter-Rater agreement as explained in Chapter 2.

4.2.4.1 Transformation errors

Transformation error is an incorrect way of moving the graph of a function into a required one through dilation, reflection, vertical or horizontal translation. Regarding this question on transformation, learners were asked to describe the transformation of one function $f(x) = -\sqrt{27x}$ where $x \geq 0$ to $g(x) = \sqrt{27x}$ yet where $x \geq 0$. The correct answer to this question is:

Reflection about the x-axis or $(x; y) \to (x; -y)$; $x \geq 0$.

The analysis of learners' responses reveals the confusion which the learners had about reflections. The errors are presented in Figure 27 and Figure 28.

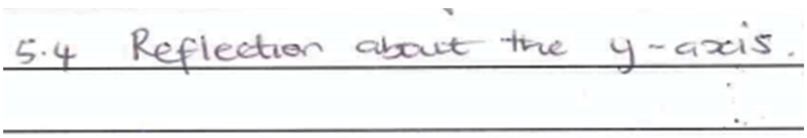

Figure 27: Transformation error exemplar 1

A look at Figure 28 shows that the learner pays attention to neither the restriction nor for which values of x to determine the transformation.

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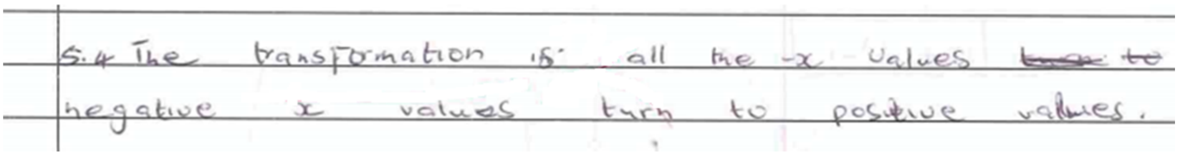
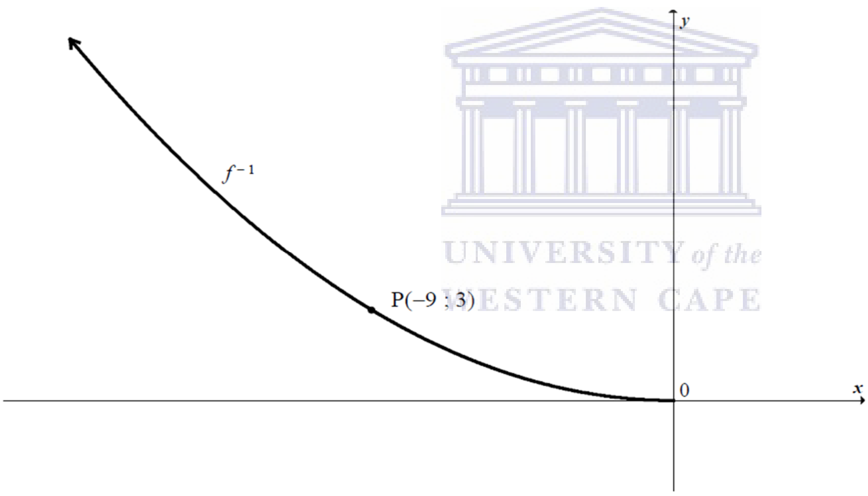


Figure 28: transformation error exemplar 2

4.2.4.2 Inverse errors

Concerning this question, learners were asked to sketch the inverse of f , with the function given graphically. In the same question, they had to indicate the intercepts with the axes and the coordinates of one other point. The correct answer to this question is:



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The analysis of learners’ responses reveals the following errors as presented in Figure 29 and Figure 30.

A look at Figure 29 shows that the learner does not know for which values of x the graph can be drawn.

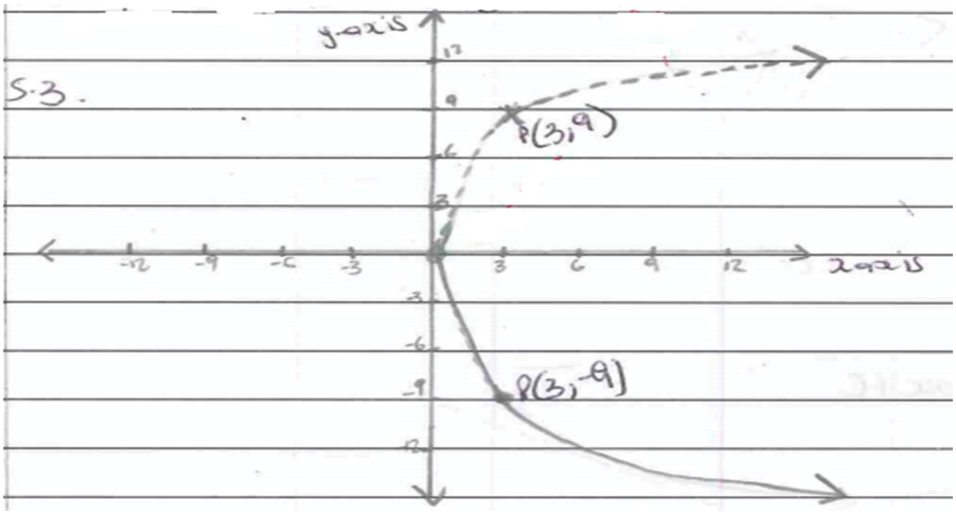

Figure 29: inverse error exemplar 1

A look at Figure 30 shows that the learner does not obtain the correct inverse and hence does not draw the correct graph. For this learner, it is the reflection about the y -axis.

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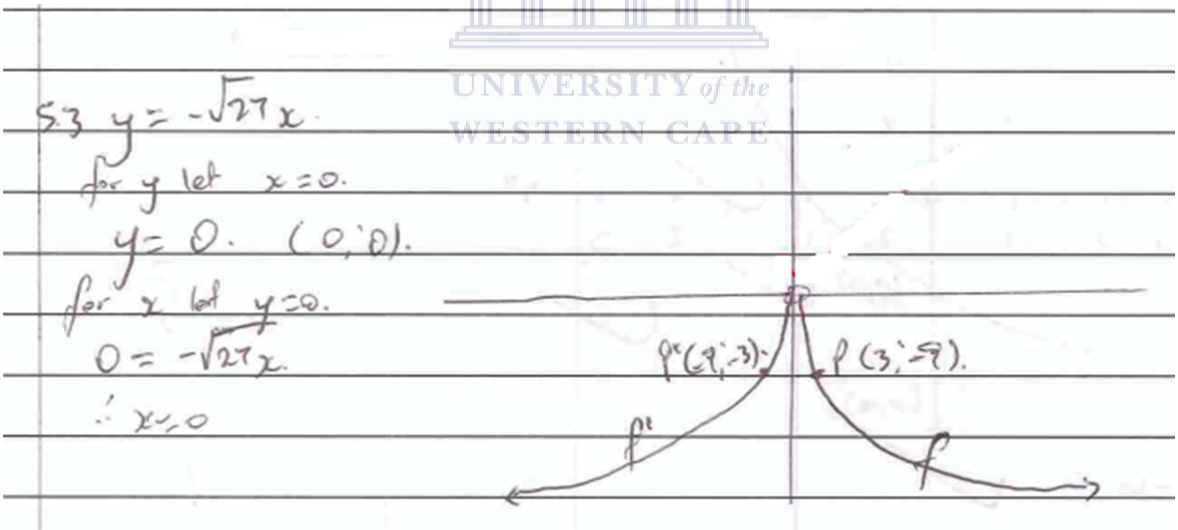

Figure 30: Inverse error exemplar 2

Table 4 below summarizes the additional errors in terms of their types, the number of errors, the correct and the no responses.

Table 4: Additional errors

Characteristics	Errors	Percent	Correct	Percent	No Responses	Percent	Total
Transformation	257	57,90	60	13,50	127	28,60	444
Inverse	241	54,28	82	18,47	121	27,25	444
Total	**498**	**-**	**142**	**-**	**248**	**-**	**-**

4.3 Conclusion

Learners' responses exhibited a considerable number of errors when they were asked to work with algebraic graphs. The errors on which this investigation was founded were coordinate, intercept, domain and range, asymptote, identification, drawing and function errors. Besides these aforementioned errors there were some additional errors such as transformation and inverse errors that were found in many scripts. Table 5 summarises the total number of errors identified in the scripts analysed, the correct and the no responses.

Table 5: Total number of errors

Characteristics	Errors	Percent	Correct	Percent	No Responses	Percent	Total
Coordinates	794	59,61	387	29,05	151	11,34	1332
Y-intercept	208	46,85	223	50,22	13	2,93	444
X-intercept	305	68,7	119	26,80	20	4,5	444
Intercept d	157	35,36	244	54,96	43	9,68	444
Domain	334	75,22	28	6,31	82	18,47	444
Range	342	77,03	19	4,28	83	18,69	444
Asymptote	272	61,26	121	27,25	51	11,49	444
Identification	112						
Drawing	279	62,84	112	25,22	53	11,94	444
Quadratic Function	298	67,12	79	17,79	67	15,09	444
Hyperbolic Function	229	51,58	90	20,27	125	28,15	444
Transformation	257	57,90	60	13,50	127	28,60	444
Inverse	241	54,28	82	18,47	121	27,25	444
Total	**3828**	**-**	**1564**	**-**	**936**	**-**	**-**

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Based on their frequency of occurrence, the highest and the lowest frequency of errors ranged as follows: coordinate, domain and range, x-intercept, function (the construction of quadratic function), drawing, asymptote, coordinate, intercept on graphing, transformation and identification, inverse and y-intercepts errors. Concerning the coordinate errors, the main difficulties learners faced were the failure to correctly compute the coordinate and to substitute the values of the coordinates in the building of the function. Furthermore, as to the range errors, the results showed that learners performed poorly in questions regarding the range because they did not have enough knowledge. The errors on the domain were predominant. Learners did not attempt to get the domain by using the restriction following the graph correctly. They had the same difficulties as those presented concerning the range. Unlike the domain and range, learners made enormous errors on the x-intercept. Ignoring the exponent, they failed to recognize the multiplication and therefore had difficulties in relation to the determination of both x and y-intercepts. Another area in which problems were manifest by learners was function error. It was found here that the majority of them could not use the correct steps necessary to construct the quadratic function properly.

Concerning the drawing errors, most of the learners could not intuitively draw and show the asymptote and the intercepts. In line with drawing errors, learners could not get the asymptote; and therefore representing it on the graph constituted a big challenge. The errors learners made on identification were the lack of Mathematics intuition on algebraic graphs to correctly identify a parabolic graph or a hyperbolic graph. Other errors that learners made were transformation and inverse errors. On transformation, many learners had little knowledge on this topic, but in comparison to inverse error, they were unable to get the mirror graph.

The next chapter of this study presents the interpretations and a discussion of the findings with some recommendations and a conclusion.

CHAPTER 5

DISCUSSION, RECOMMENDATIONS AND CONCLUSION

5.1 Introduction

As the purpose of this investigation was to analyse the ways in which learners deal with algebraic graphs in the final grade 12 Mathematics examinations, and more importantly to detect the errors they had committed, this chapter is a discussion of the results presented in Chapter 4. The chapter connects the research aim and the results generated by the analysis of learners' errors, as it engages these efforts with the body of literature on errors in graphs. The chapter further relates the results to the diagnostic report on learner performance issued by the Department of Basic Education 2012 and it provides some recommendations for improvements in the teaching and learning of graphs.

5.2 Discussion of results

5.2.1 Level 1 errors

5.2.1.1 Coordinate errors

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About the coordinate errors it is worth noting that in the collected data, there were no questions as such on this characteristic. Nevertheless, there was one particular question coded as question 2.3 that is scrutinized. At first the learners were asked to find the coordinates of the y -intercept and x-intercept. The results presented in Chapter 4 show that out of the 1332 responses expected from the learners, 59, 61% failed to get the correct coordinates. This was due to their difficulties with algebraic manipulation and hence affected their capacity to find the intercepts that were to be plotted on the graph.

Learners were also asked to use the coordinates in order to determine the intercept d (which will be better explained under the intercept errors) of the function $g(x) = -2x + d$. The results showed that the learners failed because of the incorrect manipulation of the coordinates as they did not take into account the correct coordinates to substitute into the equation of the function. Instead, this suggests that learners interchanged the values as the abscissa became the ordinate and the ordinate became the abscissa. For instance the results showed that for the coordinate (6,

0) extracted from the given graph, learners used 6 as $g(x)$ which is the y-value and 0 as x-value in the determination of d. An interchange of values like this could lead learners to make coordinate errors. This confirms the learners' misconception, as Abdullah (2010) reported when he found learners attributing the x-values to y and vice-versa.

As was indicated at the beginning of Chapter 4, the only direct question on coordinates was question 2.3 where learners were required to find the coordinates of the other point. Learners' responses to the question 2.3 reveal that most of them did not attempt to find the coordinates because of the difficulties they had in manipulating the inverses and in taking the restriction into consideration.

5.2.1.2 Intercept errors

Errors related to y-intercept

The observation made of learners working on the question concerning the y-intercept is that most of them mistook the multiplication sign – which is in this case the dot – for a comma of the decimal number. In other words, they indicated 3.2^0 as equal to 1. By doing so an error occurred in their calculation as they attributed the exponent 0 to the product of 3 and 2. In view of this result, it can be said that the learners failed to first proceed by finding that 2^0 is equal to 1 and then to multiply the 1 by 3.

Furthermore, some learners proceeded with erroneous multiplication like 3.2^0 is equal to 6. They ignored the exponent on top of 2 which should be the first operation to be done; therefore the multiplication was wrongly applied. This process led these learners to commit the calculation error.

In a few other cases, learners considered 3.2^0 as equal to 0. This group of learners neglected the exponent and the multiplication; and this also resulted in an error.

The last group of learners on this error type did not have a proper way of working with questions involving the y-intercept. From beginning to end they did not use any basic mathematical pattern to solve the question. Instead, they proceeded according to their own unique ways of working and this led them to make errors.

In sum, the result obtained from the analysis of 444 scripts shows that 46, 85% of the learners failed to get the correct y-intercept.

The diagnostic report (Department of Basic Education, 2012) indicates that by looking at the way learners were processing their responses, there were some algebraic errors that occurred like the one below:

$3.2^x = 6$

$6^x = 6$

$x = 1$

Some learners did not follow any mathematical rule to get $x = 1$ even if it was the right answer. This situation shows how much learners have mastered in earlier grades and more importantly that facility in algebraic manipulation and understanding of algebraic properties is key if they are to succeed in answering questions in all subjects in Mathematics (Department of Basic Education, 2012).

Concerning the intercept, another question was for the candidates to determine d, the y-intercept of $g(x)$. The analysis of learners' responses shows that 35, 36% of learners did not have adequate knowledge of which x-intercept should be involved in the determination of d, since the parabola $f(x)$ crossing the x-axis on $S(-2,0)$ and $T(6,0)$, and $g(x)$ crossing the x-axis on $T(6,0)$ were on the same system of axes.

However when learners extracted the x-intercept involved in the determination of d, there were three routes leading to the errors that emerged. The first appears in Figure 2 as shown in Chapter 4 when the learners extracted the abscissa (x-intercept) with an opposite sign and attributed it to x and to $g(x)$ in this way:

$g(x) = -2(-6) + d$

$-6 = -12 + d$

The second appears in Figure 6 as shown in Chapter 4. Here the learners switched the abscissa and the ordinate: the abscissa became the ordinate and the ordinate became the abscissa as seen in this way:

$g(x) = -2x + d$

$6 = -2(0) + d$

In the third instance learners began by extracting and substituting the correct values but they used the wrong procedure in their calculation and this led to intercept error too.

The y-intercept error happened again when other learners extracted and used the x-intercept which is not involved in the determination of d . This is to suggest that these learners could choose the x-intercept $S(-2,0)$ on which the straight line $g(x)$ did not cross the x-axis as illustrated in this way:

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$g(x) \quad y = -2x + d$

$0 = -2(-2) + d$

In this case learners made errors in their choice of the x-intercepts to be taken into account.

Errors related to x-intercept

The way in which learners approached this question is compared to what they did in dealing with the y-intercept. From the analysis, 68, 70% of learners wrongly obtained the x-intercept. For instance, some learners took the dot of the multiplication for a comma of the decimal number, and this was not correct. In question 1.1.2, some learners wrote 3.2 as the base of the logarithm to solve the equation, ignoring the fact that such an inaccurate process would lead them astray. In other words, learners who laboured under this confusion acquired an incorrect value that would then be plotted on the graph.

Other learners proceeded with erroneous multiplication like 3.2^x is equal to 6^x; ignoring the exponent on top of 2. These learners could first have transferred 3.2^x to the left hand side of the equal sign, which they did not do. This way of working led them to make an error.

Concerning this matter, some learners did not adopt the right approach to the question. There was a lack of basic mathematical pattern from the start. The wrong answer they obtained resulted from the algebraic manipulations.

There is much to say about this case in the sense that some learners were unable to get either the y-intercept or the x-intercept through solving the equations. Nevertheless, the discrepancy between x-intercept and y-intercept in terms of analysis is that learners experienced more difficulties in obtaining the x-intercept than the y-intercept due to the calculation involved in the determination of x-intercept. To make things clearer, the x-intercept is obtained by solving the equation once $y = 0$. Getting the y-intercept seemed easier, which implies that it is sufficient to say that $x = 0$ to get the cutting point with the y-axis which is the independent value.

This part of the investigation does however suggest learners might have found it easier when plotting the x-intercept and the y-intercept on the graph. Once the two characteristics (x-intercept and y-intercept) are found, the plotting will not cause any problem. By contrast, if the two characteristics are wrong, then the plotting will also be wrong. This is to confirm the argument put forward by Hattikudur et al. (2010) that once the learner obtains the correct intercept, the plotting on the graph becomes easier. Grade 12 Mathematics learners need to master the algebraic calculation or the manipulation for the x-intercept and the y-intercept.

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5.2.1.3 Domain and Range errors

Domain errors

There was only one question concerning the domain and this was question 2.1. In this question, learners were asked to use the graph to determine the appropriate x-values on condition that $f(x) \geq -9$. The results presented in Chapter 4 show that the majority of learners did not use the graph properly when determining the values that fulfilled the requirement for obtaining the domain. Instead, they preferred to obtain the domain by using the algebraic methods. In this way it was discovered that learners presented a lack of knowledge because they did not know how to solve the irrational equation. In fact, out of 444 responses expected 75, 22% of learners failed to get the correct answer. According to the diagnostic report, it was also found that many learners did not have enough knowledge concerning the domain (Department of Basic Education, 2012).

With reference to Figure 11 of Chapter 4, the difficulty that some learners encountered was to consider the restriction and apply the notion of interval properly. They had to observe the values on the graph that should be greater or equal to zero and then less or equal to 3. But these learners gave one part of the answer wrongly, which is $x \geq 3$.

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In Figure 12 of Chapter 4, learners did not show that they understood what they should get; for them the domain was the restriction. They wrote: $F(x) \geq -9$

$$x \geq -9 \; x \in R \, .$$

Instead they should have used the graph to get the correct domain.

In Figure 13, it was observed that some learners possessed the knowledge of what the domain was and eventually found how to get it. These learners also recognized the part of the domain that should satisfy the restriction. However, the problem was that they wrote the domain with an error and got $3 \leq x \geq 0$. By doing so, it can be concluded, these learners demonstrated that they had not mastered the notion of interval particularly that of rearranging the interval.

Range errors

In this case, learners should know what is involved in the determination of the range of exponential graph $f(x) = 3.2^x - 6$. The result presented in the previous chapter shows that some learners first considered that the range comprised all the values greater than -6 on the y-axis. These learners had an idea of what the range should look like. But the misunderstanding of the notion of the interval and how to arrange an interval led them to make an error. Consequently these learners obtained an incorrect range like (∞,-6). This means that they did not know how to arrange the interval. A tentative explanation is that these learners think that ∞ is smaller than -6. Yet, they should know that -6 is smaller than ∞.

In Figure 14 and Figure 15, learners ignored the sign and the notion of the interval. They were taught that in the interval, the lower bound must be the small value and the upper bound the big value. But some learners displayed confusion in their use of the interval to represent the range.

The easiest route to getting the range is by a way of using the graph. Attention must be focused on the y-axis on the graph. In the case of exponential graphs of the previous kind, the range should be $y \in (-6; \infty)$ or $y > -6$. Some learners looked at the y-axis instead of looking at x-axis. Their answer was $x > -6$. In view of this finding, it can be argued that the difficulty these learners had with the range is attributable to insufficient knowledge of the notion of the interval; also, they had an incorrect approach to extracting the values on the graph. They confused the axes to be used in each case of these two notions. In so doing they clearly demonstrated their confusion regarding the domain and the range.

The results also show that certain learners had a vague understanding of the range. After working on the question, they got $y \neq 6$ and $y \in R$; $y \leq -6$ and $y \leq 6$ as the answer. The reason for making such an error may lie their not having used the graph properly as they lacked knowledge about the notion of the intervals. This situation has also been conveyed in the diagnostic report where it is indicated that because of the difficulties learners experienced with graphic interpretation, the majority of them were unable to interpret the range from the graph (Department of Basic Education, 2012).

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In fact, the result presented in Chapter 4 shows that 77, 03% of the learners failed to get the correct range.

5.2.2 Level 2 errors

5.2.2.1 Asymptote errors

In the sub-question of question 1.1.3, learners had to show the asymptote on the graph. The asymptote should be drawn from the function $f(x) = 3.2^x - 6$. This is the function of the form $f(x) = a.b^x + q$. In such cases the value of q determines the vertical shift of the graph (up or down). In fact, the equation of the horizontal asymptote is $y = q$.

In this question most learners did not represent the asymptote in their drawing. This sub-question was left empty.

It was noted that learners worked differently with the characteristics. Normally they should have got the answer intuitively from the given function; then there would have been no problem when plotting the asymptote on the graph. The value of q represents the horizontal asymptote which is equal to $y = -6$. The value of -6 determines the vertical shift of the graph (upward or downward). Therefore learners should know that an exponential graph has only a horizontal asymptote but never a vertical asymptote. This value should be plotted on the negative values of y-axes. The results presented in the previous chapter show that most learners plotted $y = -6$ as if it were intended to determine the horizontal shift either at the negative values of the x-axis with $x = -6$ or at the positive value of the x-axis with $x = 6$. This action employs the incorrect side to represent the asymptote. The result showed that learners were confused about representing the asymptote on the graph and that they had not properly mastered the process of plotting.

However some learners recognized that q determines the vertical shift, which means that it is really a horizontal asymptote. As a result few learners successfully extracted the value, but by changing its sign. These learners plotted the positive value on y-axis. The correct answer for

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this asymptote is not $y = 6$ but $y = -6$ which should be plotted on the y-axis at the negative values.

For the asymptote, the results presented in Chapter 4 show that of 444 responses, 61, 26% of the learners gave the wrong answer.

5.2.3 Level 3 errors

5.2.3.1 Identification errors

The errors in identification as noted in the previous chapter concern question 1.1.3 and 3 only. In question 1.1.3, learners were asked to draw an exponential graph from a given function. In question 3, they were asked to find the hyperbolic function from the properties or characteristics of the graph of a hyperbola.

The results presented in Chapter 4 show that some learners could recognize neither the graph of a function nor the characteristics (properties) of a certain graph given to determine a certain function.

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Concerning question 1.1.3, the results show that some learners did not associate the function to its appropriate graph. They were asked to sketch the exponential graph from the function $f(x) = 3.2^x - 6$. The answer to this question could be an exponential graph (correct or incorrect) according to characteristics obtained. But it was found that some learners followed their own imagination. They drew a linear graph, or a parabola or else a hyperbolic graph and the graph. This situation confirms exactly Jones' (2006) finding that it is with difficulty that learners come to identify functions.

In question 3, learners were given some properties of hyperbolic graphs as follows: the domain $x \in R$; $x \neq 5$ and the range $y \in R$; $y \neq 1$ passing through the point (2, 0), and then they were asked to determine the function. However the results show that some learners constructed a linear function while others did the quadratic function in place of a hyperbolic function.

In the case of the identification error, the result presented in the previous chapter showed that 112 errors were identified from all the scripts considered for the study.

5.2.3.2 Drawing errors

The question relating to the drawing is 1.1.3, but question 2.2 also concerns the drawing of the inverse and it will be discussed under the inverse error. In the question 1.1.3, learners were asked to draw an exponential graph, and they were required to show all asymptotes and intercepts previously discussed concerning the axes. The function $f(x) = 3.2^x - 6$ that should be drawn is the form $f(x) = a.b^x + q$. Similarly, by comparing the terms to the given function it follows that $a = 3$, $b = 2$ and q is -6. Therefore these terms produce a situation where $a > 0$; $b > 1$ and $q < 0$. Intuitively, the graph should be an increasing graph with an asymptote on y-axis negative.

Regarding the results presented in this chapter, it appears that some learners did not have an idea of what the exponential graph is or when an exponential graph can increase or decrease. In fact an average of 62, 84% did not sketch the correct exponential graph.

Learners were asked to determine the general trend of the form of the graph. In other words, they were also requested to indicate the intercepts of the graph with the axes. The results indicate that only a few learners developed their mathematical intuition; in other words, knowing the visible characteristics of the form of the function, they determined the general trend of the kind of exponential graph. In fact, any learner who could find the horizontal asymptote and the intercepts correctly could determine the curve of the exponential graph easily.

In Figure 15, one learner failed to draw and show all other characteristics such as the asymptote and the intercepts the way they should be. For instance in this question, this learner drew a graph as if it were a case where $a < 0$; $0 < b > 1$ and $q > 0$. He/she ended up obtaining the intercepts with the wrong value and sketched the graph wrongly, displaying some confusion about working with exponential graphs. This situation is in accordance with Tripathi's (2008) view that learners need representational competency in their sketching or drawing of the graph.

In Figure 17, it seems that when solving the question, some learners contemplated a>0; b>1 and thought that the curve or the graph is therefore decreasing. This is another point of confusion that brought some learners to a drawing error.

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5.2.3.3 Function errors

In this investigation, there are two questions concerning the construction of the functions: question 1.2.2 and question 3. In question 1.2.2, learners were asked to construct a quadratic function from a given graph, and in question 3, they had to construct a hyperbolic function from certain properties of a graph.

The first case to be dealt with in this investigation is the question relating to the construction of the quadratic function. Learners were given a graph with its characteristics and they should have worked as follows in order to get the correct answer:

Consider $y = a(x - x_1)(x - x_2)$ x_1 and x_2 as the first and the second x-intercepts. Since the given graph is a parabola, the learners should substitute x_1 (first x-intercept) and x_2 (second x-intercept) extracted from the graph and introduce them in $y = a(x - x_1)(x - x_2)$ with the opposite sign. It was said that d and R share the same point on the graph; this means that respectively they represent the y-intercept for the straight line g and the curve f. In fact, this can enable learners to substitute y with 12 in order to get the value of a; a will then help to determine whether the parabola opens upwards or downwards. In the case of this graph, learners should intuitively be sure that the coefficient a should be negative because the parabola that has been drawn in the question paper is downward.

In Figure 22, some learners managed to extract the values of x-intercepts by substituting an opposite sign in the form $y = a(x - x_1)(x - x_2)$, but the error they committed is that they forgot to find the value of a – which many learners failed to determine. This situation led them to get the incorrect quadratic function.

A second method that learners should have used to get the quadratic function of the form $f(x) = ax^2 + bx + c$ is to substitute c as the y-intercept of the curve f which is equal to 12, and the R on the graph to obtain $y = ax^2 + bx + 12$ first. Next, substitute $S(-2,0)$ into (1) $y = ax^2 + bx + 12(1)$ and $T(6,0)$ into $y = ax^2 + bx + 12(2)$ to get two equations with two unknowns called simultaneous equations. The solution will give the values of a and b. The values obtained would then be substituted into $f(x) = ax^2 + bx + c$ to get $f(x) = -x^2 + 4x + 12$, the required quadratic function.

In Figure 24, some learners proceeded to get the quadratic function in their own way as in the case below. They first wrote the general form of quadratic function and they managed to substitute c with 12, which is the y-intercept of the curve on the graph. They easily substituted the first x-intercept $S(-2,0)$. But confusion emerged when these learners had to substitute the second x-intercept $T(6,0)$. They committed an error by obtaining first b from the first substitution; simultaneously they substituted b into the same equation to be solved.

In the case of the building of quadratic function, the result showed that out of 444 responses, 67, 12% of the learners failed to construct the required function correctly. This situation confirms what is suggested in the literature that the reading of information from a graph is a problem for most learners of different ages (Tairab & Al-Naqbi, 2004). Vekiri (2002) also reported that reading coordinates and writing equations of the graph were two main obstacles; and Planinic et al. (2012) found that learners with low prior knowledge and low spatial ability had difficulties extracting information from graphs.

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In the second construction that was given, learners were to construct a hyperbola function of this form $f(x) = \dfrac{a}{x+p} + q$ (3) from certain properties or characteristics. The given properties were: the domain $x \in R; x \neq 5$; the range $y \in R$; $y \neq 1$ and a certain point $(2,0)$ by which the curve passes.

To get the required form, learners should have done the following: firstly they should substitute p and q by their given values in the function $f(x) = \dfrac{a}{x+p} + q$ and remember that p should take the opposite value of $x \neq 5$ and q should keep the sign of $y \neq 1$. Secondly, they should find the value a, the constant by substituting $f(x) = y = 0$ and x by 2. Creative ways of manipulating the principles of algebra would indeed lead them to the correct answer.

The results presented in the previous chapter indicate that from some old mathematics classroom textbooks, the form that was given to the learners is $f(x)=\dfrac{a}{x-p}+q$ (4) (Laridon, Jawrek, Kitto, Pike, Myburgh, Rhodes-Houghton, Sasman, Scheiber, Sigabi & Van Rooyen, 2006 p.104). In fact they should have been given $f(x)=\dfrac{a}{x+p}+q$ (Department of Basic Education, 2011). The two formulae (3) and (4) are not different from each other but they each depend on the way learners manipulate the variables. It was further observed that teachers had taught this form of $f(x)=\dfrac{a}{x-p}+q$ in schools, so learners had been using this form.

According to this form of hyperbolic function, learners should use the domain $x \in R; x \neq 5$ as p, that means p is equal to 5 and the range $y \in R;\ y \neq 1$ as q, that means q is equal to 1. They should also substitute $f(x) = y = 0$ and x by 2 and then seek to determine the coefficient a. This coefficient has to be substituted into the hyperbola form to get the required function.

In Figure 25, some learners wrote the correct form of the hyperbolic function viz. $f(x)=\dfrac{a}{x-p}+q$ but they could not manage to substitute the values of p and q correctly. They substituted the values of p and q with the opposite sign. They took the abscissa or x-coordinate of the point (2, 0) and attributed it to the coefficient a. That means for them a is equal to 2, which gives a wrong hyperbolic function $f(x)=\dfrac{2}{x+5}-1$.

Indeed, some learners first drew the correct hyperbolic graph from the given properties even though it was not part of the answer. In their process they swapped the values. They considered the value of p as the value of q and vice versa and they did not think to seek the value of the constant a. They were unable to distinguish the values of p and q, that is, which values have to be considered as domain and range. This confusion about the domain and the range led them to get the incorrect hyperbolic function $f(x)=\dfrac{a}{x+1}+5$.

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Furthermore, the results presented in Chapter 4 Figure 26 show that some learners could have drawn a wrong graph and mistakenly exchanged the values of p and q. In other words, they took the values of p for q and vice versa but could not think to seek the constant a. Such tendencies amongst the learners have also been reported in the diagnostic report where it is indicated that instead of obtaining its equation, they drew the curve. The diagnostic report also observes that some learners confuse the values of p and q in $y = \frac{a}{x-p} + q$ and they do not understand the characteristics of the hyperbola (Department of Basic Education, 2012).

As a result, an average of 51, 58% of learners failed to construct the correct graph of a hyperbolic function from the given properties or characteristics.

5.2.4 Additional errors

5.2.4.1 Transformation errors

There was only one question that produced transformation errors and it focused on reflection. Concerning this question, learners were asked to describe the transformation of $f(x) = -\sqrt{27x}$ when $x \geq 0$ to $g(x) = \sqrt{27x}$ where $x \geq 0$. They were taught that when a function is multiplied by -1 that means $y = f(x)$ becomes $y = -f(x)$, a graph of $y = f(x)$ is then reflected across the x-axis called a vertical reflection (in the x-axis). Also, if the variable is multiplied by -1, that means $y = f(x)$ becomes $y = f(-x)$; a graph of $y = f(x)$ is then reflected across the y-axis called a horizontal reflection (in the y-axis). Regarding the question about the description of the above function, it is clear that $f(x)$ has been multiplied by -1 to give $g(x) = \sqrt{27x}$. Within this paradigm learners have learnt that it is a reflection about the x-axis or $(x; y) \rightarrow (x; -y)$ with $x \geq 0$.

After analysing the learners' scripts, it was found that there were different answers such as the one in Figure 27 "reflection about the y-axis". It was also found that learners were confused about the reflection. For instance some learners did not realize that to get $g(x) = \sqrt{27x}$ you have

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to multiply the function $f(x) = -\sqrt{27x}$ by -1. The fact that they did not realize that the function $g(x)$ is the product of $f(x)$ by -1 led them to the transformation error.

In Figure 28, some learners gave the following answer: "the transformation is all the x values negative x values turn to positive values". In this case, some learners had difficulty in using the correct words of transformation, leading to profound confusion. This is clear from the fact that learners did not pay attention to the restriction that shows for which value of x values to determine the transformation. The diagnostic report confirms the same idea: that many examinees do not understand the transformation (Department of Basic Education, 2012). In fact, 57, 90% of learners failed to get the correct transformation.

5.2.4.2 Inverse errors

As to the question relating to the inverse error, learners were given a graph of function $f(x) = -\sqrt{27x}$ for $x \geq 0$ as the restriction. In addition, they were given a certain point (3; -9) on that graph of $f(x)$. The question instructed them to sketch $f^{-1}(x)$ to indicate the interception with the axes and the coordinates of one other point.

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Learners were taught that an inverse is a function which means "undo". Besides, Even (1992) believes that the meaning of inverse function is undoing. If $f(x)$ is an invertible function, that means $f(x)$ has an inverse function. Then $f^{-1}(x)$ can be easily drawn. The domain of a function $f(x)$ is the range of $f^{-1}(x)$ and the range of $f(x)$ is the domain of $f^{-1}(x)$ (Stewart, Redlin &Watson, 2013 p. 229).

Confrey and Smith (1991) believe that the majority of secondary school learners know that the inverse of a graph of f^{-1} is obtained by reflecting the graph of f in the straight line $y = x$ called the line of symmetry. This is to suggest that the graph of a function and its inverse are mirror images of each other.

According to the pattern designed in the ways learners had worked, it was found that some learners could not apply the knowledge they possessed on the inverse. They may probably have thought that if f and g are inverses of each other they would have graphed them in the same representation knowing that the graph of g is the reflection of the graph of f in the line $y = x$.

In Figure 29, some learners reflected the graph about the x-axis. For them, the line of x-axis is the line of symmetry. These learners found the correct intercepts or the end at origin but most of them failed to obtain any other point on the graph.

In Figure 30, the learners drew the graph as if it were the case of reflection about the y-axis in the y-axis negative. For these learners, the line of y-axis is the line of symmetry. In contrast to Confrey and Smith's (1991) view, these learners did not reflect the graph given in the line of $y = x$. They had an idea about how the point $(3,-9)$ will appear on the inverse graph but they incorrectly obtained any other point on the graph which for them is the point $(-9,-3)$.

These findings unequivocally confirm what has been reported in the diagnostic report where it was noted that many learners did not draw a graph, and that the majority of them drew it in the wrong direction and wrong quadrant. The report referred to shows that the examinees did not properly understand reflections in the graphic context (Department of Basic Education, 2012). In the same line of thought, the result of this study has shown that out of 444 responses expected, 54, 28% learner participants failed to get the correct inverse graph.

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5.3 Recommendations

5.3.1 Coordinate errors

The analysis of data has shown that learners displayed two things when they were dealing with the coordinates. The plotting was easier for them and there was no confusion when they had to find the axes. In their investigation Padilla et al. (1986) found that many high school learners could determine x and y coordinates of a point and they were able to plot them. But the difficulty occurred when they had to extract the coordinates to construct the quadratic function. Many learners displayed confusion by attributing the x-values to the y-values and vice versa. This is what Tairab & Al-Naqbai (2004) found when they reported that many learners could not extract the information from a graph. Furthermore, the challenging question was the sub-question

2.3 where learners were asked to obtain the coordinates of the inverse. The learners could not apply the knowledge correctly in this case. The majority of them failed to get the coordinates of any other point.

In the light of these revelations and the confirmation they present, it is recommended that teachers place greater emphasis on and attention to exercises, as advised in the Curriculum and Assessment Policy Statement (Department of Basic Education, 2011). They should give practice exercises with more manipulations of the coordinates, and give learners the meaning of the coordinates. Learners at this level should know that the graphs of f and f^{-1} are related to each other in the following way: if the point (a,b) lies on the graph of f, then the point (b,a) eventually lies on the graph of f^{-1} and vice versa. About the coordinates of f inverse (f^{-1}), many learners did not do well.

5.3.2 Intercept errors

The analysis of learners' scripts revealed that there were many algebraic errors in the determination of the x-intercept and y-intercept. More specifically, the learners could not differentiate between these two characteristics especially as regards how to calculate them. Nevertheless they could successfully find the y-intercept more readily than the x-intercept. But they had some difficulty in plotting both. In the case of this particular function $f(x) = 3.2^x - 6$, getting the y-intercept is a matter of substituting the x value in the function by zero. There is not as much involved in the calculation of the y-intercept as there is in the x-intercept when the y-value is equal to zero.

In such cases teachers should help learners by organizing more routine exercises so that they can master the topics such as equations, inequalities, the simultaneous equations, because these exercises are likely to enable them to distinguish the y-intercept from the x-intercept when extracting them from the graph.

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5.3.3 Domain and range errors

The results have shown that learners made many errors when dealing with the domain and range because they did not master these points properly. For this reason, teachers should emphasize the notion of intervals in their teaching of graphs.

Although these two characteristics are introduced at the same time (Özkan & Ünal, 2009), teachers are urged to teach them systematically. At first, they might start with the domain by giving more and more exercises and then conclude with the range, in such a way to help learners understand and to distinguish them easily.

5.3.4 Asymptote errors

The sub-question concerning the asymptote was a challenging one. Many learners did not get the right answer. They could not figure out the equation of the asymptote and what it would look like; they were confused about the sign of its value. In the light of this confusion, learners should know that the exponential function has only one horizontal asymptote. For this particular question it is indeed a horizontal asymptote that learners should draw at the y -negative values and not at the y -positive values, or else not the vertical asymptote as was noted in the case of many learners. Mathematics is a domain that needs more and more practice on a regular basis. For this situation, teachers should find a remedy to improve learners' understanding by giving more exercises repeatedly.

5.3.5 Identification errors

In question 1.1.3, learners experienced confusion about the choice of graph to adopt. They were drawing any type of graph according to their own understanding. At this stage, teachers should provide more diverse types of functions and graphs in such a way that they help learners distinguish and determine all characteristics for each type of graph and attribute them to an adequate function, and vice versa. The right choice of graph becomes possible only when teachers contribute towards it by giving learners more types of algebraic representation to work on. As learners become more adept at identification, their success will make the work increasingly attractive to them. Through more practice exercises teachers may emphasize the description of the general form of functions with their characteristics and their respective graphs.

Another question in which learners showed insufficient knowledge in the way they proceeded with identification, was one relating to the construction of the hyperbolic function from some given properties of the graph of the same kind. In this instance learners constructed any type of function according to their own mistaken understanding based on the incorrect form of function adopted. Some learners constructed linear function, while others constructed the quadratic or exponential function instead of the hyperbolic one. Owing to the difficulties learners face, teachers are encouraged to give more diverse exercises in this regard in order to help them. They should give the properties of any algebraic graph so as to enable learners to build a function and vice versa. It is believed that the identification will be successful only if teachers help learners to associate each type of function with the correct type of graph, and each type of graph with the appropriate type of function together with their respective characteristics.

5.3.6 Drawing errors

The question on the drawing also turned out to be a challenging one. It was about the drawing of exponential graphs from an exponential function. In general, learners failed to obtain the characteristics such as the x-intercept, y-intercept, et cetera. Once the characteristics obtained by means of algebraic ways are correct, the graph plotted is also correct. It was discovered that Grade 12 learners encountered difficulties based on the characteristics to obtain which constitute the preliminary stage for the construction of the graph. But they did not have any problem with plotting the characteristics obtained on the graph.

For this reason, teachers are urged to emphasize the algebraic calculation to help learners get the correct characteristics which are to be correctly plotted on the graph later. Concurring with the diagnostic report this study confirms that learners do indeed have to solve equations in the context of graphs on regular basis (Department of Basic Education, 2012).

5.3.7 Function errors

The results have shown that learners had difficulties in constructing a quadratic function. The values extracted from the graph were not properly manipulated. The signs were incorrectly used in the extraction of the coordinates. Normally, these coordinates should accordingly be used in

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the construction of the general form. Learners were erroneously using the coordinates by interchanging them or attributing them to a wrong sign in the algebraic expression. They failed to get the coefficient a for the quadratic function and they could not arrange the order of the steps to follow for the construction of a function. For this reason, teachers are urged to care about this problem by giving more exercises on a regular basis to help learners avoid these kinds of errors. The diagnostic report also suggests that teachers should play their part by focusing on the characteristics and behaviours of functions, including interval (Department of Basic Education, 2012).

Furthermore, in other questions, learners had difficulties handling the hyperbolic function. There was huge confusion – especially when substituting the domain and the range. It also happened that the learners could not correctly decide what sign of the x-value or y-value had to be introduced in the general form of function. What learners should remember is that when it is about the domain one should first look at the denominator. Then, they should not interchange the domain and the range in their substitution. This is to suggest that the learners should not manipulate these two characteristics in their own way; they should seek to determine the value of the coefficient a. As for the teachers, they need to be more insistent on the correct steps that have to be followed when building a function. They should be giving learners a variety of exercises. In addition, they should draw learners' attention to the meaning of the domain.

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5.3.8 Transformation errors

The results indicated that learners displayed confusion and difficulty in working with the reflection of the graph. They lack the appropriate words for the description about the transformation. For this situation teachers should create a classroom environment where learners find the lesson attractive and more specifically, where they are taught to avoid the use of inappropriate words about transformation. With the help of teachers, learners should exercise more so as to be able to distinguish all sorts of transformations. The diagnostic report suggested that transformations are integrated with graphs from time to time and teachers should let learners identify them, through regular practice (Department of Basic Education, 2012).

5.3.9 Inverse errors

The results have shown that the question on the drawing of the inverse of $f(x) = -\sqrt{27x}$ for $x \geq 0$ as the restriction for which the graph has been given was difficult for the majority of the learners. For such cases, teachers are encouraged to emphasize the meaning of the inverse and the purpose of the axis of symmetry. Through repeated practice, teachers need to help learners understand that if the given function is drawn in a certain quadrant; they have to determine the quadrant of the graph where the inverse is drawn precisely. Teachers must also help learners recapitulate what is required in order to determine the restriction using the graph. This will enable learners to arrive at the coordinates of one other point on the graph. Teachers need to embark on this field with determination and help learners identify the types of errors they make when working with the inverse. Also, they need to help learners distinguish the notion of inverse and that of transformation because it was found in this investigation that learners confuse them. The diagnostic report also suggests that teachers must observe that inverses of graphs are taught thoroughly not only in exponential function, but with more emphasis on inverses of quadratic functions, as recommended in the curriculum (Department of Basic Education, 2012). It further points out that teachers should focus on the restriction of the inverse function meaning that the inverse function should be made explicit to learners. Also, teachers should let learners know that inverses and reflections are related, and according to the curriculum, inverses are reflections along the straight line $y = x$ called the axis of symmetry while reflections are always along the same straight line (this could also be the x -axis and y -axis).

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5.4 Recommendations for future research

It is recommended that a more in-depth study be conducted to complement this research. It is suggested that learners be interviewed in the hope of casting more light on their engagement with algebraic graphs. It is believed that such interviews can add necessary insight into understanding why learners commit errors and misconceptions in this area.

Another way of handling the problem would be to conduct respective studies on linear graphs, quadratic graphs, hyperbolic graphs, exponential graphs, inverse, including the transformation of graphs with their respective functions. Also necessary is an initiative in which learners' insights and reflections are obtained through interviews.

As an extension of this study there is a need to supplement its objectives with an investigation into teachers' knowledge of course-content including their approach to working with the algebraic graphs as this can augment our understanding of learner's difficulties in working with graphs.

5.5 Conclusion

By relying on document analysis methodology, this study has provided an understanding of ways in which learners approach algebraic graphs in the context of a high-stakes grade 12 Mathematics examinations. The focus of the research concerned coordinate errors. The study discovered that the majority of learners had failed to find correct coordinates due to incorrect manipulation in interchanging the values so that the abscissa became the ordinate and vice versa. Regarding domain, it was observed that learners did not use the graph to determine or to obtain the domain but instead, wrongly preferred to use the algebraic methods, ignoring the restriction. The study also revealed that learners found it unduly challenging to identify the correct kind of graph, which should be the exponential graph and the correct function, which should be the hyperbolic graph.

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Other types of errors made by learners included transformation errors, drawing errors and function errors. In the case of transformation errors, learners had difficulty in using the correct words of transformation such as reflection about x-axis. This was because they failed to pay attention to the restriction that shows for which value of x-values to determine the transformation. The study also showed that the majority of learners had difficulty in correctly applying the restriction as they did not follow the correct procedure which says the domain of a function $f(x)$ is the range of $f^{-1}(x)$ and the range of $f(x)$ is the domain of $f^{-1}(x)$.

On the whole, the study found that learners displayed a lack of understanding, a lack of mathematical intuition, an inability to rearrange the interval and a lack of respect for the required steps when constructing a quadratic function, hyperbolic function from a graph or certain given properties amongst others.

Consequently, it is proposed that educators– including curriculum advisors, teachers of Mathematics and other relevant stakeholders – work together in order to strive for good teaching of Mathematics with the purpose of increasing the pass rate of learners who sit for their Mathematics examination. Efforts in this regard may include a special intervention that specifically addresses some of the concerns raised in this study. It is envisaged that the recommendations outlined here will be given the necessary attention to ensure that the teaching of Mathematics is improved within the Western Cape Province.

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APPENDICES

Appendix A: Inter-Rater Agreements

	QUESTION 1					QUESTION 2				QUESTION 3	TOTAL
	QUESTION 1.1			QUESTION 1.2			2.1	2.2	2.3		
RATERS	1.1.1	1.1.2	1.1.3	1.1.4	1.2.1	1.2.2					
RATER 1	y-intercept E. Coord. E Level 1	**y ok** x-intercept E. coord. E. Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	**OK**	**Empty**	Dom. E Level 1	**Empty**	**Empty**	**Empty**	
RATER 2	y-Intercept errors Coordinate errors Level 1	ok	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	OK	Empty	Dom. E Level 1	Empty	Empty	Empty	
RATER 3	Coord. E. y-Intercept E. Lev. 1 E	ok	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	ok	Empty	Dom. E Level 1	Empty	Empty	Empty	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9
RATER 1	**OK**	**y ok** x-intercept E. coord. E. Level 1	Intercept E. Lev. 1 E	Range E. Level 1 E.	**OK**	**OK**	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Ok	ok	Intercept errors Level 1	Range errors Level 1 E.	Ok	Ok	Dom. E level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation Error	Function E. Level 3 E	
RATER 3	ok	ok	Intercept E. Lev. 1 E	Range E. Lev 1 E	ok	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9
RATER 1	**OK**	**OK**	Drawing E. Asymptote E. **intercepts ok** Level 2 & 3 E	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	**OK**	**OK**	**Empty**	
RATER 2	Ok	Ok	Slope errors	Range errors Level 1 E.	Ok	Function errors Level 3 E.	Dom. E Level 1	Ok	Ok	Empty	
RATER 3	ok	ok	Asymptote E. Lev. 2 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	ok	ok	Empty	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9
RATER 1	y-intercept E. Coord. E Level 1	**OK**	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	**OK**	(3 out of 4)	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors	Drawing errors Asymptote errors Intercept errors Level 1,2 & 3 E	Range E. Level 1 E.	Ok	0k	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transform E.	Function E. Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	ok	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	0	1	1	1	0	1	1	1	1	8

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RATER 1	y-intercept E. Coord. E Level 1	**OK**	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	**Empty**	
RATER 2	Intercept errors Coordinate errors Level 1	Ok	Drawing errors intercept E Asymptote errors Level 1,2 & 3 E	Range errors Level 1 E.	Ok	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation error	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	ok	Drawing E. Asymptote E. intercept E Lev.1, 2 & 3 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	**OK**	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	**Empty**	Transformation Error	**Empty**	
RATER 2	ok	Intercept errors Coordinate errors Level 1	Drawing errors Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Intercept error	Function errors Level 3 E.	Dom. E Level 1	Empty	transformation Error	Empty	
RATER 3	ok	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Empty	Transformation Error	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	**OK**	**OK**	**Drawing ok** Asymptote E. **intercepts ok** Level 2 & 3 E	Range E. Level 1 E.	**OK**	**OK**	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	**OK**	**OK**	
RATER 2	Ok	Ok	Drawing errors Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Ok	Ok	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Ok	Ok	
RATER 3	ok	ok	Drawing E. Asymptote E. Lev. 2 & 3 E.	Range E. Lev 1 E	ok	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	ok	ok	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9
RATER 1	**OK**	**OK**	Drawing E. Asymptote E. Lev 1,2 & 3 E	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	**OK**	Function E. Level 3 E	
RATER 2	OK	OK	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors	Intercept error	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Ok	Function E. Level 3 E	
RATER 3	ok	ok	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	ok	Function E Lev 3 E	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9
RATER 1	**OK**	**OK**	**OK**	Range E. Level 1 E.	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	**OK**	**OK**	Function E. Level 3 E	
RATER 2	OK	OK	ok	Range errors Level 1	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Ok	Ok	Function E Level 3	
RATER 3	ok	ok	ok	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	ok	ok	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10

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RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	**Empty**	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Empty	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transform. E	Function E Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Empty	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	**OK**	**OK**	Drawing ok Asymptote E. intercepts ok Level 2 E.	Range E. Level 1 E.	Intercept E Level 1E.	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	**OK**	Function E. Level 3 E	
RATER 2	Ok	Ok	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Intercept error	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Ok	Function E. Level 3 E	
RATER 3	ok	ok	Drawing E. Asymptote E. Intercept Lev. 2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	ok	Function E. Level 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	**OK**	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1E.	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	**OK**	Function E. Level 3 E	
RATER 2	Ok	Intercept errors coord. E Level 1	Drawing Asymptote errors Level 2 & 3 E	Range errors Level 1 E.	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Ok	Function Error Level 3 E	
RATER 3	ok	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	ok	Function E Lev 3 E	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9
RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Level 1,2 & 3 E	Range E. Level 1 E.	Empty	Function E. Level 3 E.	Dom. E Level 1	Empty	Empty	Empty	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Empty	Function errors Level 3 E.	Dom. E Level 1	Empty	Empty	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Empty	Function E Lev 3 E	Dom. E Level 1	Empty	Empty	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation Error	Function E. Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9

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RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 3 E.	Function E. Level 3 E.	Dom. E Level 1	Empty	Transformation Error Lev 3 E	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Empty	Function errors	Function E Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Empty	Transform E.	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Empty	**OK**	Function E. Level 3 E.	Dom. E Level 1	coordinate E.	Empty	Empty	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Domain errors	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Empty	Ok	Function errors Level 3 E.	Dom. E Level 1	Coordinate errors	Function errors	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Empty	ok	Function E Lev 3 E	Dom. E Level 1	Coord. E.	Empty	Empty	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9
RATER 1	**OK**	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Empty	Intercept E	Function E. Level 3 E.	Dom. E Level 1	coordinate E.	Empty	Function E. Level 3 E	
RATER 2	ok	Intercept errors Coord. E Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Empty	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Coordinate errors	Empty	Function E	
RATER 3	ok	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Empty	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Coordinates E	Empty	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	transforamtion E	Function E Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E. Lev 3 E	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	**OK**	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Empty	Empty	Dom. E Level 1	Empty	Empty	Empty	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Empty	Function errors Level 3 E.	Dom. E Level 1	Empty	Empty	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Empty	Empty	Dom. E Level 1	Empty	Empty	Empty	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9

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RATER 1	**OK**	**OK**	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Empty	**OK**	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.) Coord. E.	**OK**	Empty	
RATER 2	Ok	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Empty	Ok	Function errors Level 3 E.	Dom. E Level 1	Drawing errors Coordinate errors Inverse E.	ok	Empty	
RATER 3	ok	Coord. E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Empty	ok	Function E Lev 3 E	Dom. E Level 1	Drawing E. Coord. E. Inverse E.	ok	Empty	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9
RATER 1	**OK**	**OK**	Asymptote E. Level 2	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	**OK**	**OK**	Function E Level 3	
RATER 2	Ok	Ok	Asymptote E Level 2	Range errors Level 1 E.	Ok	Function errors Level 3 E.	Dom. E Level 1	Ok	Ok	Function E Level 3	
RATER 3	ok	ok	Asymptote E. L. 2 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	ok	ok	Function E Lev. 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	**OK**	**OK**	**OK**	**OK**	**OK**	Function E Lev 3 E	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E Level 3	
RATER 2	Ok	Ok	Ok	Ok	Ok	Ok	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation	Function E Level 3	
RATER 3	ok	ok	ok	ok	ok	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev. 3 E	
AGREEMENT	1	1	1	1	1	0	1	1	1	1	9
RATER 1	**OK**	**OK**	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1E.	ok	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Ok	Ok	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1	Intercept error	Ok	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation	Function Error Level 3	
RATER 3	ok	ok	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. Coord. E Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	intercept E Level 1E.	Function E. Level 3 E.	Dom. E Level 1	Empty	Empty	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Empty	Transformation	Function E Level 3 E	
RATER 3	Coord. E. Intercept E. Lev. 1 E	X-intercept E Coord. E Level 1	Asymptote E. L. 2 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E. Level 3 E.	Dom. E Level 1	Empty	Empty	Function E Lev. 3 E	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9

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RATER 1	y-intercept E. Coord. E Level 1	x-intercept E. coord. E. Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	Intercept E Level 1 E.	Function E. Level 3 E.	Dom. E Level 1	Empty	Transformation Error	Function E. Level 3 E	
RATER 2	Intercept errors Coordinate errors	Intercept errors coord. E. Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1	Intercept error Level 1	Function errors Level 3 E.	Dom. E Level 1	Empty	Transformation E	Coordinate errors	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	Intercept E Lev 1 E	Function E Lev 3 E	Dom. E Level 1	Empty	Transform E.	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	0	9
RATER 1	y-intercept E. Coord. E Level 1	**y ok** x-intercept E. coord. E. Level 1	Empty	Empty	**OK**	Empty	Dom. E Level 1	Empty	Empty	Empty	
RATER 2	Intercept errors Coordinate errors Level 1	Coord. E. Intercept E. Lev. 1 E	Empty	Empty	Intercept error Level 1	Empty	Dom. E Level 1	Empty	Empty	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Intercept E. Lev. 1 E	Empty	Empty	ok	Empty	Dom. E Level 1	Empty	Empty	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
RATER 1	y-intercept E. Coord. E Level 1	**OK**	**OK**	Range E. Level 1 E.	intercept E Level 1E.	**OK**	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Empty	Empty	
RATER 2	Intercept errors Coordinate errors Level 1	Intercept errors Coordinate errors Domain errors	Ok	Range errors Level 1	Intercept error Level 1	Ok	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Empty	Empty	
RATER 3	Coord. E. Intercept E. Lev. 1 E	Coord. E. Lev. 1 E	ok	Empty	Intercept E Lev 1 E	ok	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Empty	Empty	
AGREEMENT	1	0	1	1	1	1	1	1	1	1	9
RATER 1	**OK**	**OK**	Drawing E. Asymptote E. **intercepts ok** Lev 2 & 3 E	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	Drawing E.(inver se E.),intercepts E. coordinate E.	Transformation Error	Function E. Level 3 E	
RATER 2	Ok	Ok	Slope error Asymptote errors	Range errors Level 1 E.	Ok	Ok	Dom. E Level 1	Drawing errors Coordinate errors Intercept errors Inverse E.	Transformation E	Function E. Level 3 E	
RATER 3	ok	ok	Asymptote E. L. 2 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	Drawing E. Intercept E. Coord. E. Inverse E.	Transform E.	Function E Lev 3 E	
AGREEMENT	1	1	0	1	1	1	1	1	1	1	9
RATER 1	**OK**	**y ok** x-intercept E. coord. E. Level 1	Drawing E. Asymptote E. intercepts E. Lev 1,2 & 3 E	Range E. Level 1 E.	**OK**	Function E. Level 3 E.	Dom. E Level 1	coordinate E.	Transformation Error	Empty	
RATER 2	Ok	Intercept errors Coord. E Level 1	Drawing Asymptote errors Intercept errors Level 1,2 & 3 E	Range errors Level 1 E.	Ok	Function errors Level 3 E.	Dom. E Level 1	Coordinate errors	Transformation Error	Empty	
RATER 3	ok	Coord. E. Intercept E. Lev. 1 E	Drawing E. Asymptote E. Intercept E. L. 1,2 & 3 E.	Range E. Lev 1 E	ok	Function E Lev 3 E	Dom. E Level 1	Coord. E.	Transform E.	Empty	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10

		x-intercept E.	Drawing E.	Range E.		Function E.	Dom. E	Drawing E.(inver	Transformation	Function E.	
RATER 1	**OK**	Coord. E	Asymptote E.		**OK**			se E.),intercepts E.	Error		
			intercepts E.					coordinate E.			
		Level 1	Lev 1,2 & 3 E	Level 1 E.		Level 3 E.	Level 1			Level 3 E	
	Ok	Intercept errors	Drawing	Range errors	Ok	Function errors	Dom. E	Drawing errors	Transformation	Function E	
RATER 2		Coordinate errors	Asymptote errors					Coordinate errors	Error		
			Intercept errors					Intercept errors			
		Level 1	Level 1,2 & 3 E	Level 1 E.		Level 3 E.	Level 1	Inverse E.		Level 3 E	
	ok	Coord. E.	Drawing E.	Range E.	ok	Function E	Dom. E	Drawing E.	Transform E.	Function E	
RATER 3		Intercept E.	Asymptote E.					Intercept E.			
			Intercept E.					Coord. E.			
		Lev. 1 E	L. 1,2 & 3 E.	Lev 1 E		Lev 3 E	Level 1	Inverse E.		Lev 3 E	
AGREEMENT	1	1	1	1	1	1	1	1	1	1	10
											94,33%

UNIVERSITY of the
WESTERN CAPE

Appendix B: Counting of errors

	COORDINATE1			COORDINATE2			X-INTERCEPT			Y-INTERCEPT			Y-INTERCEPT d			DOMAIN			RANGE			ASYMPTOTE			IDENTIFICATION			DRAWING			QUADR. FUNCTION			HYPER. FUNCTION			INVERSE			TRANSFORMATION		
No	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N	E	R	N
1	2	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1	H		1	0	0	1	0	0	0	0	1	0	0	1	0	0	1
2	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0			1	0	0	0	1	0	1	0	0	1	0	0	1	0	0
3	0	2	0	0	1	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	0	1	0	0	1	0
4	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	0	1	0	1	0	0	1	0	0	1	0	0
5	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0
6	0	2	0	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	0	0	1	0	0	1	1	0	0
7	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			0	1	0	0	1	0	0	1	0	1	0	0	0	1	0
8	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	H		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
9	0	2	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0			0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
10	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
11	0	2	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			0	1	0	1	0	0	1	0	0	1	0	0	0	1	0
12	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	L		1	0	0	1	0	0	1	0	0	1	0	0	0	1	0
13	2	0	0	0	0	1	1	0	0	1	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	0	0	1	0	0	1
14	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
15	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0			0	0	1	1	0	0	1	0	0	0	0	1	0	0	1
16	2	0	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0	0	1	1	0	0	0			0	0	1	1	0	0	0	0	1	0	1	0	0	0	1
17	1	1	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	1	1	0	0	1	P		1	0	0	1	0	0	1	0	0	0	1	0	0	0	1
18	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	0	1	0	0	1
19	1	1	0	1	0	0	0	1	0	1	0	0	0	0	1	0	0	1	1	0	0	1	0	0	1	P		1	0	0	0	0	1	0	0	1	0	0	1	0	0	1
20	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	0	1	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	0	1	0
21	0	2	0	0	1	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			0	1	0	1	0	0	0	1	0	0	1	0	0	1	0
22	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	0	1	0	0	1	0	0			0	1	0	0	1	0	0	1	0	1	0	0	1	0	0
23	0	2	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
24	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	0	1	0	0	1
25	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	H		1	0	0	1	0	0	1	0	0	0	0	1	1	0	0
26	2	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0			0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
27	1	1	0	1	0	0	0	1	0	1	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0			0	1	0	0	1	0	0	0	1	1	0	0	0	1	0
28	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
29	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	1	0	1	0	0
30	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
31	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0
32	2	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0	1	0	0	1	0			0	0	1	0	1	0	1	0	0	1	0	0	1	0	0
33	1	1	0	0	0	1	1	0	0	0	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	0	1	0	0	1
34	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0

35	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
36	2	0	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0
37	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
38	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
39	0	2	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			0	1	0	1	0	0	0	1	0	0	1	0	1	0	0
40	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
41	1	1	0	0	0	1	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	0	0	1	0	0	1	0	0	1	0	0	1
42	1	1	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1	0	0	0			1	0	0	0	0	1	1	0	0	0	1	0	1	0	0
43	2	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	1	0	0	0	1	0	1	0	0
44	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0	0	1	0	1	P		0	1	0	0	1	0	1	0	0	1	0	0	1	0	0
45	1	1	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1	0	0	1	E		1	0	0	0	0	1	0	0	1	1	0	0	1	0	0
46	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0			0	1	0	1	0	0	1	0	0	1	0	0	0	1	0
47	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
48	1	1	0	0	0	1	1	0	0	0	1	0	1	0	0	0	0	1	0	0	1	1	0	0	0			1	0	0	0	1	0	0	0	1	0	0	1	0	0	1
49	1	1	0	0	0	1	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	0	1	0	0	1
50	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	0	1	0
51	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0
52	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
53	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
54	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
55	2	0	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0			0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
56	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
57	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	0	1	0	0	1	0	0			0	1	0	0	1	0	1	0	0	1	0	0	0	1	0
58	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0			0	1	0	1	0	0	1	0	0	1	0	0	1	0	0
59	1	1	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1	0	0	0			1	0	0	0	0	1	1	0	0	1	0	0	0	0	1
60	1	1	0	0	0	1	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	1	0			0	0	1	0	0	1	0	0	1	0	0	1	1	0	0
61	2	0	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	0	1	0	1	0	0
62	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	L		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
63	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	0	1	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
64	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0			1	0	0	0	1	0	1	0	0	0	0	1	0	0	1
65	2	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	0	1	0	0	1
66	2	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	0	0	1
67	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	H		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
68	0	2	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	P		1	0	0	0	1	0	1	0	0	1	0	0	1	0	0
69	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	0	1	0	1	0	0
70	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0	1	0	0			0	1	0	0	1	0	0	1	0	1	0	0	0	1	0
71	0	2	0	0	0	1	0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0			0	1	0	1	0	0	0	0	1	0	0	1	0	0	1
72	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0	1	0	0			0	1	0	1	0	0	1	0	0	1	0	0	1	0	0
73	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
74	1	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	0	1	1	0	0	1	P		1	0	0	1	0	0	0	0	1	0	1	0	1	0	0

75	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
76	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
77	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0			0	0	1	1	0	0	1	0	0	1	0	0	1	0	0
78	2	0	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0			0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
79	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	S		1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
80	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0	0	0	0	1	1	0	0	1	H		1	0	0	0	0	1	0	0	1	1	0	0	0	0	1
81	1	1	0	0	0	1	1	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0			0	0	1	1	0	0	0	0	1	0	0	1	0	0	1
82	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	0	1	1	0	0	1	0	0
83	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
84	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0			1	0	0	1	0	0	0	1	0	1	0	0	1	0	0
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UNIVERSITY of the WESTERN CAPE

355	2	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0		1	0	0	0	1	0	0	0	1	1	0	0	0	0	1
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Total	513	342	33	281	45	118	305	119	20	208	223	13	157	244	43	334	28	82	342	19	83	272	121	51	112			279	112	53	298	79	67	229	90	125	241	82	121	257	60	127

UNIVERSITY of the
WESTERN CAPE