Linear or mixed integer programming in long-term energy systems modeling – A comparative analysis for a local expanding heating system

Energy 283 (2023) 129056

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Linear or mixed integer programming in long-term energy systems modeling – A comparative analysis for a local expanding heating system

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ARTICLE INFO

Handling Editor: Neven Duic

Keywords:
Housing
District heating
Local energy system modeling
TIMES
Mixed integer linear programming
Linear programming

ABSTRACT

Most computer models used in energy systems optimization modeling studies are formulated using linear equations. However, since linear formulations do not always well reflect real-world conditions, they may not always be adequate as policy and support tools. This is particularly the case for local system studies attempting to represent technologies at the individual scale, as in the case for local heating system modeling. Thus, the aim of this paper is to investigate differences in the resulting heating solutions and model solution times for a local expanding heating system. Three different investment cost structures for individual and district heating solutions for the heating of new housing are investigated using linear and mixed integer linear programming. The results show that the use of district heating is higher for the cost structures that use mixed integer linear programming than it is for the linear cost structures. This result is attributed mainly to the fact that individual air-to-water heat pumps benefit from the linear equation formulation due to its high coefficient of performance during summertime. This finding is important to consider when modeling local energy systems. The solution time is, however, significantly shorter for the linear formulations than for the mixed integer linear formulations.

1. Introduction

Energy systems are large and complex since they involve many types of plants and different energy carriers, the components involved often have long lifetimes; and the energy demand evolve with time. Computer models are, therefore, often used to study optimal investments and operation. Over the years, many different energy modeling frameworks have been used for such studies. Different computer tools available for studying renewable energy integration are investigated in Ref. [1] and trends in modeling of energy transitions in Ref. [2], in which it is concluded that it is not possible to develop at tool which can include all aspects affecting energy transitions. In recent years, there has also been a large increase in the use of urban and city energy system models [3]. Different kinds of energy models have different focuses [4] and are not equally well suited for all purposes. This results in a variety of models developed to address e.g. economics of the system, climate policy impacts or energy security concerns. Energy models cannot, however, take every aspect into account as this would make the models too large to be solvable. Thus, the implementation of simplifications is necessary. Accordingly, energy models results differ depending on the features included [5].

1.1. Programming methods in energy system models

Several different programming methods are available in energy systems modeling. In this section, linear programming (LP), mixed integer linear programming (MILP), and non-linear programming (NLP) will be introduced, as these are the most commonly used programming methods in energy systems studies.

In LP models, the most widely used model type for investigations of energy systems, both at larger scale and at a more-local scale [4], all constraints and equations are linear. Usually, they can be solved relatively rapidly compared to other model formulations, and it can be mathematically proven that the solution found is the optimal one. Solvers for LP problems are widely available, in both free-of-charge and commercial versions. However, linear models have some major drawbacks when it comes to assessing real energy systems, in that some aspects cannot be considered or are difficult to consider. Examples of such aspects are economies of scale, minimum sizes of investments in new technologies, and varying efficiencies for plants operating at different output levels. These aspects mean that it is crucial to consider which simplifications are used in the modeling of local systems in which representations of technologies at an individual scale are required.

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https://doi.org/10.1016/j.energy.2023.129056
Received 17 March 2023; Received in revised form 31 August 2023; Accepted 10 September 2023
Available online 15 September 2023

MILP has been used in energy system studies to allow for integration of technical aspects that LP cannot handle. Thus, in local energy systems, e.g. local heating systems, aspects such as minimum loads of power plants [6–9] and minimum cost and size levels of new technologies [10, 11] have been investigated using MILP. The use of MILP for commitment of district heating plants has been investigated in Ref. [12]. While commonly scenarios are used to handle uncertainties in models [13], in Ref. [14], stochastic methods was used for handling uncertainties in input data together with MILP for modeling of an electricity system. The computational power needed to solve MILP problems is a potential issue, since the same mathematical methods used to solve LP models cannot be used to solve MILP models.

Different methods have been used to cope with the problem of the heavy computational burdens of MILP models. In Ref. [7], a "rolling planning horizon approach" has been applied that divides the investigated period into several parts to reduce the computational time. A rolling horizon optimization approach has also been applied in Ref. [15]. Instead of solving the model in an optimal fashion, the relative optimality criteria has been set to >0 in another study [8]. Linearization of some parts of a MILP model has been performed by adding a "two-variable" approach [9], enabling a representation of the minimum load levels in power plants using linear variables.

Another option to handle this problem is to use machine learning [16], which was applied to pre-solved MILP optimization problems for wind power placements. It was found that machine learning could drastically shorten the solution times. However, still there is a need to solve some of the MILP problems beforehand, so the machine learning algorithm could use these as a training dataset. That machine learning can be significantly faster than other optimization techniques is pointed out in Ref. [17], but it is also noted that machine learning models are black-box models which cannot guarantee that a global optimum is found.

A more advanced alternative to MILP is to use NLP, which can take into account even more-advanced features than is the case with MILP. NLP can, however, take much longer to solve than MILP [18,19], although the results obtained from MILP and NLP may be similar. In Ref. [19], LP and MILP formulations for the dispatch of plants in an existing DH system were compared. The results showed clear differences between these two programming methods. In two other studies [20,21], various simplifications of computationally heavy NLP models have been investigated for the dispatch of hydropower. Different levels of linearization of a detailed nonconvex hydro power model is investigated in Ref. [20]. In Ref. [21], aggregated equivalent models of large hydro power systems are developed and investigated. In Ref. [22], a MILP formulation for dispatch of a heating system in Berlin is compared with a merit order-formulation, which is simpler and easier so solve computationally. These studies concluded that simplifications that decrease the computation time while preserving the quality of the results can be made to the original NLP models.

Although there have been studies that have had as explicit goals to investigate the impacts (in terms of results and computation times) of different programming methods and simplifications [19–22], none of the studies compare how different programming methods or model simplifications affect long-term local energy systems models where investments in new production technologies are necessary.

1.2. Local heating systems modeling

Heat can be supplied through either communal or individual solutions. Communal solutions, often in the form of district heating (DH), consist of central power plants that produce heat, which is then distributed by the transportation of hot water through underground pipes. Individual solutions consist of technologies that are installed in each individual building and that can only provide heat to that specific building. As individual DH plants may be sufficiently large to cover a large share of the DH demand, the representation of individual plants is of importance for local heating systems. In general, DH is used in more-densely populated areas whereas individual solutions are more prevalent in less-densely populated areas. As every local heating system has distinctive building density characteristics, it is important to consider the local conditions when investigating the development of such systems.

The role of DH in decarbonizing energy systems has been investigated for the EU [23], for Sweden [24] and for Denmark [25]. Although DH can be highly efficient and contribute to decarbonization, its climate impact can depend heavily on the size of the DH system, and individual heating solutions may have a lower climate impact than DH solutions [11]. This further highlights the importance of taking local conditions into consideration when investigating local heating systems.

Current heating systems are closely connected with the electricity sector, as heating system can both consume and produce electricity. Various aspects of interactions of the electricity sector with the local heating system have been covered by Refs. [6,7,26,27]; e.g. the impacts of a limited electricity connection to a city [26] and how the optimal operation of a combined heat and power (CHP) plant depends if the operation shall be optimized on regional or city level [27]. However, in these studies the DH demand was set beforehand without consideration of other heating solutions. By considering other heating solutions in systems modeling, as in Refs. [10,11], the interactions of different heating options with other parts of the energy system can be better understood. Considering both the demand and supply of heat in a local system has also been done with the DH supply in focus [28] and with the resulting heating solution of new housing in focus [29].

Recently, there has been a dramatic increase in the use of air-to-water heat pumps (HPs) in Sweden [30], as well as in the EU as a whole [31]. This increase is attributed to decreased cost, improved efficiency, and the fact that HPs are relatively easy to install. Air-to-water HPs differ from other heating solutions due to its seasonally dependent coefficient of performance (COP). The technical characteristics and performance of air-to-water HPs have been investigated in several previous studies [32–35] but the use of such HPs has not been investigated previously on a systems level. Thus, this aspect is included as a novelty in the present study.

1.3. Research gap and questions

When investigating large energy systems, such as national or international systems, the use of certain simplifications in the modeling can be acceptable, and is often necessary, as the finer details within the system are of minor importance. However, when investigating local systems, the types of simplifications that it is reasonable to use in larger systems can have severe effects on the results for the local systems in which individual technologies are of high importance. Studies comparing the different programming methods and how the resulting energy system solution and solution time are affected, are lacking. This can be problematic when investigating local systems such as individual or small communities of end-users, for which there can be limitations as to how new technologies can be installed and used, which means that such systems cannot be investigated in detail. This is of significant interest e.g. due to the steadily increasing use of air-to-water HPs, with their inherent seasonal dependent COP, as individual heating solution as this type of technology has not been investigated on a systems level.

Based on the identified research gaps, the main aim of this paper is to investigate how using LP or MILP affects the resulting system solution for a system where investments into new technologies are necessary. A local heating system developing over time in which both large-scale power plants connected to distribution grids and individual heating solutions are available is considered in this paper. Since it has been identified that MILP implies a computational burden, the solution time will also be addressed. The following questions are thus guiding this paper:

- How do technology investment cost and size restrictions affect the long-term cost-optimized solutions of a local heating system?
- How do LP and MILP model formulations, corresponding to cost and size restrictions, influence the resulting modeled heating solutions and the model solution time?

The paper attempts to contribute with knowledge that actually may guide the choice of modelling formulations in long-term studies of local energy systems. Thus, the paper is focusing on the type of models commonly used for these types of studies even if there are other, more advanced model formulations that potentially could be used. The commonly used LP and MILP methods are therefore tailored in this study for being able to investigate local energy systems over long time period where investments into new heating technologies are necessary. As LP formulations encourages mixing of technologies compared to MILP, this aspect is of high importance to consider for local energy systems, which has previously not been investigated in long term studies for these types of systems.

2. Method

The methodology presentation is divided into five sections (technical data together with assumptions are presented in a separate chapter, Chapter 3). First, in Section 2.1, ways in which investments can be made in new technologies, termed *investment cost structures*, corresponding to the use of LP and MILP formulations, are presented. Then, Section 2.2 presents how the modeling is carried out, as well as the optimal solution criteria and solution times. An overview of the modeled heating system is presented in Section 2.3. Section 2.4 presents the technical details of how the air-to-water HPs have been modeled, and, lastly, Section 2.5 presents a sensitivity analysis.

2.1. Investment cost structures and investment cost structure schemes

How investments can be made in technologies in real systems due to different cost allocations between equipment cost and installation cost are reflected in this paper by the term *investment cost structures*. Three different investment cost structures are used in this paper:

- *Linear investment.* This cost structure implies a linearly increasing cost with increasing capacity. There is no requirement regarding minimum size for any heating solution.
- *Starting cost.* In the starting cost structure, there is a fixed starting cost for any size of investment in the specific technology. The starting cost is assessed as the percentage of the investment cost that is attributed to installation costs in the dataset of the Danish Energy Agency [36].
- *All-or-nothing.* In the all-or-nothing cost structure, the specific technology can only be installed at one size, which is sufficient to cover the full heat demand throughout the year.

The cost to install a certain technology that is sufficiently large to supply all the required heat at all times is the same for all three cost structures (Fig. 1).

Formulation of a model in which the investment cost of installing a technology increases linearly with the installed capacity can be achieved using LP. However, as stated in the *Introduction*, there may be reasons why, in real systems, the installation sizes of new technologies cannot be increased with a linearly increasing cost with no restriction on the minimum installation size. To reflect this aspect in a model, it is possible to use MILP, which makes it possible to restrict the minimum sizes of new installations. *Linear investments* is implemented using LP, while *starting cost* and *all-or-nothing*, are implemented using MILP.

Since heating investments consist of a combination of technologies, each with different investment cost structure characteristics, the heating technology alternatives are combined into different sets, here referred to as *investment cost structure* schemes. In each investment cost structure scheme, each new heating technology follows a specific investment cost structure. Six different such investment cost structure schemes are investigated, as presented below and in Table 1.

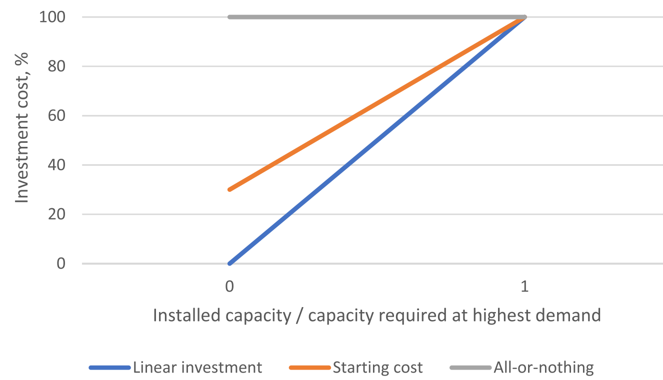

Fig. 1. Investment cost structures for investments in heating solutions for new housing. The cost of installing a specific technology depends on the size of the investment in the linear investment and starting cost investment structures, while in the all-or-nothing cost structure the technology can only be installed at a size that can always cover the full demand. The starting cost starting value differs between technologies. The value for each technology is presented in Table 2 in Chapter 3.

Two kinds of heating options are available for new housing: individual heating, and communal heating. For individual heating, there are five different technologies available: biomass boilers, electric boilers, air-to-water HPs, ground source HPs and ventilation HPs. The communal technology, DH, requires both substations and piping. Thus, each investment cost structure scheme consists of the three components: individual heating technologies, DH substation and DH piping.

In Scheme 1, all heating technologies can be installed according to the linear investment cost structure. For DH piping, most of the investment cost is associated with the burying of the piping. DH piping is, therefore, only investigated using linear (Scheme 1) or all-or-nothing investment cost structures (Schemes 2–6).

The individual heating technologies are widely available on the market in different sizes, while DH substations do not have the same range of available sizes. Furthermore, a DH substation has a higher share of the investment costs associated with the installation cost, relative to the equipment cost. This difference is investigated in Schemes 3 and 4, in which the individual heating technologies use the linear investment cost structure, while the DH substations follow the starting cost structure or all-or-nothing cost structure. In Schemes 5 and 6, both the individual heating technologies and the DH substations follow the same cost structures.

The schemes have been numbered according to the assumed solution time, where a higher number indicates an assumed longer solving time as the model becomes increasingly non-linear due to additional binary

Table 1
Investment cost structure schemes with their corresponding cost structures for each heating technology used for new housing.

Scheme number	Individual heating technologies	DH substation	DH piping
1	Linear	Linear	Linear
2	Linear	Linear	All-or-nothing
3	Linear	Starting cost	All-or-nothing
4	Linear	All-or-nothing	All-or-nothing
5	Starting cost	Starting cost	All-or-nothing
6	All-or-nothing	All-or-nothing	All-or-nothing

variables. Schemes 3 and 4 have the same number of binary variables, as do Schemes 5 and 6.

2.2. Modeling framework and optimal solution criteria

This study uses a cost-optimizing, bottom up-model of a type often used for energy and heating system studies (the TIMES modeling framework). Supply and demand are treated simultaneously and together throughout the entire modeling period. This dynamic systems approach, as previously used in Refs. [28,29], has major value when investigating developing systems in which the system components have long lifetimes. This paper investigates the system over a long time-scale, in this case up until Year 2050. In TIMES, a cost-optimizing model, the total cost of the system for the entire investigated period is minimized, while fulfilling the demand during all time periods [37].

The developed TIMES model is solved using the CPLEX optimization software, developed by IBM [38]. The solver options, with the exception of *optcr*, and their values used in this paper are presented in Appendix A. The *optcr* option states how close the best solution found must be to the best bound before the solver stops. The *optcr* option is computed in CPLEX according to the following equation:

$$optcr = \frac{|bestbound - bestinteger|}{10^{-10} + |bestinteger|}$$

This means that *optcr* is decreased either by improving *bestbound* or by improving the solution *bestinteger*. Improving the solution or the best bound may require extensive computational power and time. Consequently, the equation gives that even though different *optcr* criteria are used for different runs with the same input data, the solution found may not necessarily be improved by applying additional computational time, since with that extra time, the computer may be able to improve *bestbound* but not *bestinteger*. The running times required to fulfill three different *optcr* solution criteria, 0.1%, 0.01% and 0% respectively, are investigated in this paper. The maximum solution time was set to 6 h. The hardware and software specifications of the computer used are listed in Appendix A.

By providing the developed model with input data, the output of the model is the cost-minimized heating solution over the whole modeling horizon, including investments in and dispatch of the different available technologies. This includes the heating solution for the new housing and how investments are made in the DH supply side.

2.3. Modeled heating system

In the model, instead of having one total heat demand, the total heat demand is disaggregated into multiple demands, where each demand corresponds to one housing area, consisting of one housing type, built in a specific year. This separation of housing types ensures that investments in technologies made in previous years cannot be used for the new housing, but instead the heating solution must be found for each housing area. Therefore, it becomes clear what investments are made in different heating solutions in different years for the same type of housing, depending on when it is built. DH and the individual heating options presented in Section 2.1.2 are available for all housing types.

The buildings already present in the heating system that use DH from the beginning are assumed to continue to use DH also in the future, without any change in the heat demand.

Housing constructed in a specific year has the same heat demand throughout the modeling period. However, for each year, the housing built in that specific year has a slightly decreased annual heat demand compared to the housing built the year before to represent energy efficiency improvements (only space heating, not hot tap-water). This has the effect that in addition to decreasing the total heat demand, the heat demand profile becomes slightly flatter each year since the hot tap water demand is constant throughout the year.

In the model, there is a DH system already in place that consists of DH supply plants, a DH grid, and connections to already existing buildings. All plants are dismantled when they reach their respective end of technical lifetime, but investments into new plants can be made at any point.

2.4. Integrating air-to-water heat pumps into heating systems modeling

The relatively low installation cost and high seasonal coefficient of performance (SCOP) make air-to-water HPs a viable heating solution for future housing, although the seasonally dependent COP of air-to-water HPs should be taken into account as the COP is lower during colder seasons when the heating demand is higher. The precise COP at different ambient temperatures of an air-to-water HP is, however, seldom stated in the technical documentation of the specific HP; instead, it is the SCOP that is most often the parameter that is presented. Due to the seasonal variability of the COP, the SCOP needs to be broken down into seasonally dependent COPs to allow investigations of how the varying COP values affect the heating solution.

The procedure employed in this study for computing the COP for the air-to-water HPs is inspired by the EN 14825 standard and is presented in detail in Appendix B. Briefly, measured data related to how a real air-to-water HP performs under different temperatures are used. If it is assumed that that this HP is used to cover the full heat demand for the whole year, the amount of electricity that is used can be estimated, and thus the SCOP can be calculated by dividing the annual heat demand by the amount of electricity used. In this way, a SCOP can be mapped to a seasonally dependent COP.

2.5. Sensitivity analysis

For every investment cost scheme, two parts of the input data are varied: the heat source availability for the DH HPs; and the electricity price. As shown by Ref. [28], it can be cost-efficient to use DH HPs, although the heat source for such HPs may have limited availability, since sewage water is often used for such HPs. In addition, the development of the electricity price affects the technologies in different ways, with potentially significant impacts on the resulting heating system.

3. Data and assumptions

The input data and assumptions, such as the heat demand distribution, electricity and fuel prices, and available heating technologies, are presented in this section.

Initially, the heating demand side, which consists of several different types of housing, is presented. This is followed by a presentation of the available heating technologies for new housing. Then, the DH supply system is outlined, including two cases of heat source availability for DH HPs. Finally, the electricity price cases, heat demand profiles and fuel prices are presented.

All investment cost structure schemes are investigated for all three electricity price cases and for the two DH HPs heat source availability levels.

3.1. Heating demand side

The total heat demand is disaggregated into multiple demands, where each demand corresponds to one specific housing area, built in a certain year, in which one type of new housing is built. Six new housing types are used:

- Two types of apartment buildings of different sizes, but with the same annual heat demand per m^2;
- Two types of single-family housing of different sizes, with a high heat demand per m^2; and
- Two types of single-family housing of different sizes, with a low heat demand per m^2.

In total, there are 181 housing areas: 30 each for the six new housing types (one new area for each housing type is added annually from Year 2021 up to Year 2050) and one area for the already existing housing.

3.2. Heating technologies for new housing

The available heating technologies for new housing are summarized in Table 2. The calculation of the specific investment costs for the technologies follows the same procedure as described previously [28, 29]. In short, the specific costs for individual heating technologies are calculated by taking the total cost for installing a specific technology and dividing it by the capacity stated in the data from the Danish Energy Agency [36].

The same procedure is carried out for the installation of a DH substation and piping, although the total cost of installing a DH connection is the same for all types of single-family housing (even though they have different peak heat demands), while the total cost is the same for all types of apartment housing. This procedure is the same as that in Refs. [28,29], except that the piping and substation in this paper are separated into two different investments with different costs and technical lifetimes. As a consequence, the cost per kW of installing DH is different for the different types of housing.

3.3. District heating supply side

The DH supply side is based on the current DH system of the City of Gothenburg. The supply mix includes several types of supply plants, including those involving industrial excess heat (EH), waste incineration CHP, biomass CHP, biomass heat-only boilers, HPs and electric boilers. It is assumed that waste incineration and industrial EH will continue to be available throughout the modeling period, although the installed capacity cannot be increased.

No climate gas emissions are allowed after Year 2025, except for the waste incineration. The use of electricity and biomass is treated as carbon-neutral.

The cost data used for making investments in and the running of the DH supply plants have been acquired from the Danish Energy Agency [40].

The HPs in the current DH system of Gothenburg use sewage water as their heat source. As this heat source may have limits regarding availability, two different availability levels are investigated in this paper. The availability is set either as unlimited, indicating that other similar heat sources are locally available, or as the maximum output of the DH HPs installed in Year 2020.

No ramp up or ramp down times, startup or shut down costs, or minimum load levels are considered for any of the supply plants in this study. This is due to the limited time resolution of the model. Further, the CHP plants are assumed to have constant α-values (ratio between produced heat and electricity).

Table 2
Summary of the available heating technologies for new housing. The installation costs are used in investment cost Schemes 3 and 5, in which there are starting costs for installing new technologies (see Fig. 1 and Table 1).

	Notes	Technical lifetime	Installation cost of total investment cost
DH	Requires both piping and substation on the housing side. Requires plants on the supply side.	Piping: 50 years Substation: 25 years	Piping: 100% Substation: 30% for apartments 52% for single-family housing
Ventilation HP	Maximum recovery limit of 40% for apartment buildings[a]	15 years	25%
Biomass boilers	Electricity price-independent	20 years	22%
HP ground source	COP assumed to be independent of season	20 years	33% for apartments 25% for single-family housing
HP air-to-water	COP dependent upon season	16 years	25%
Electric boilers	–	30 years	30%

[a] The limit on apartment ventilation HPs was inspired by Ref. [39], but has increased somewhat due to new apartment buildings having improved energy efficiencies.

3.4. Electricity price profiles, heat demand profile and fuel prices

Three different electricity price cases are investigated in this paper: high, low and varying. All price cases start with the same price profile in Year 2020 but is changed each year by a certain amount after Year 2020 for each price case. The price profile in Year 2020 has been acquired from Ref. [28]. The price profiles are shown in Fig. 2.

The high price case increases the price by 50% in Year 2050 compared to Year 2020, for all seasons.

The low price case decreases the price by 50% in Year 2050 compared to Year 2020, for all seasons.

The varying price case decreases the price by 50% during the summer months, increases it by 100% during spring and fall, increases it by 150% during winter and increases it by 200% during the peak season.

The different electricity price cases are not based on any specific outlook or assumptions but are included because the electricity price development is deemed to be one of the most-important aspects affecting the development of the heating sector.

The heat demand profile is shown in Fig. 2. The heat demand is separated into space heating and hot tap-water demands. For new housing, built in the future, with lower heat demands due to improvements in energy efficiency, the space heating demand is decreased, while the hot tap-water demand remains unchanged. Thus, this results in a flatter heat demand profile for future housing. The heat profile has been acquired from real measurements of an area with housing that has low heat demands. The highest peak is calculated by assuming that there is an extra cold period during which the demand is the highest. The heat demand during the highest peak is set at 50% higher than the second coldest period.

The costs for available fuels have been acquired from Ref. [28]. It is

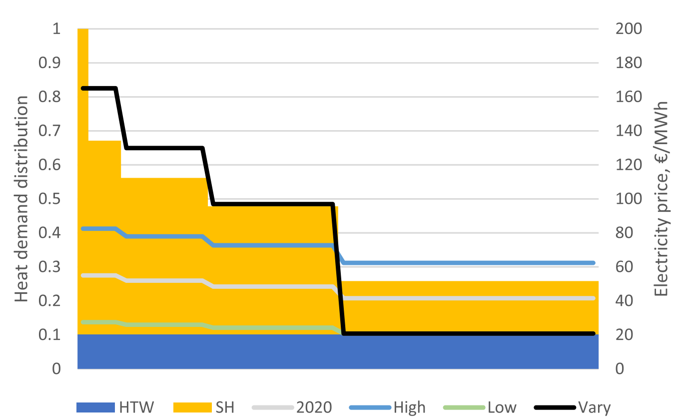

Fig. 2. Heat demand profiles and electricity price profiles. The heat demand profile in this figure is for a house with a specific heat demand of 60 kWh/m^2. For houses with other specific heat demands, only the space heating demand (SH) is changed per m^2. The hot tap-water demand (HTW) remains unchanged per m^2. This results in a higher relative heat use during the colder seasons for a house with a higher heat demand. All electricity price cases use the Year 2020 electricity profile as the starting electricity price in Year 2020, although the price is gradually changed annually for each season for each electricity price case.

important to note that the fuel costs and electricity prices are based on data and calculations made before the beginning of Year 2022, at which time-point the prices for many fuels increased significantly in Europe [41].

4. Results

In this section, the cost-optimized modeling results for the different investment schemes are presented initially. Even though the investment schemes directly affect only investments in technologies on the demand side, the DH supply side can be affected because the supply and demand are connected. For this reason, the results for the DH supply side are also presented.

The solution times for all the cases are presented for the different schemes in the last part of this section.

For both the resulting heating solutions and the solution times, the focus here is on the differences between the investment cost structure schemes, thus reflecting the different programming methods.

4.1. Differences in the modeling results with respect to the heating solutions for the different investment schemes

The modeling results for the different investment schemes for the heating solutions are presented in this section. The heating solutions for large apartment buildings built in Year 2020 are presented in Section 4.1.1, as this housing type shows the largest differences between the investment schemes.

As the modeling results for the high and low electricity price cases are similar for most of the schemes, the results for the low electricity price case are not presented in this section.

The heating solutions for the remainder of the housing are presented together in Section 4.1.2.

The DH solution supply mix is presented for each electricity price in Section 4.1.3.

For both the heating solution for new housing, as well as for the DH supply mix the results are very similar in Scheme 1 and Schemes 2–4, while Scheme 5 is very similar to Scheme 6 for each electricity price case. Thus, the results presented here focus on the results for Schemes 1 and 6.

4.1.1. Large apartment buildings

The heating solutions for large apartment buildings built in Year 2020 are shown in Fig. 3. It is clear that the share of DH is higher for Scheme 6 than for Scheme 1 in all the cases. The greater use of DH mainly corresponds to a lower use of air-to-water HPs.

The increase in the number of air-to-water HPs in Scheme 1 compared to Scheme 6 stems from the fact that the COP of the air-to-water HPs is very high during summer, which is also the time period during which the distribution losses of DH are highest. This makes it economically beneficial to install and run small air-to-water HPs. In Scheme 6, the installation of small air-to-water HPs is not allowed, with the result that the combination of ventilation HP and DH is more beneficial economically.

For the low electricity price case, there is greater use of air-to-water HPs when the heat source availability is limited, although the trend whereby Scheme 6 has a greater use of DH than Scheme 1 is also present in the low electricity price case (not shown).

4.1.2. Small apartment buildings and single-family housing

In general, for all housing types, the use of DH is lower for housing built after Year 2030 for the high and low electricity price cases. These results are linked to the facts that: 1) the DH supply plants available at the beginning of the investigated period have reached their end of technical lifetime in Year 2030; and 2) it is more expensive to make investment in both DH connections and supply plants for buildings that are constructed later on.

A noteworthy finding is that for the housing built after Year 2030, there is an increase in the use of DH in Scheme 1 compared to Schemes 2–4 for the high and low electricity price cases (not shown). This reflects that being able to invest in laying down only a small pipe underground is cost-beneficial, whereas paying the full cost is too expensive. Laying down the piping is arguably associated with a cost that is difficult to reduce by having smaller pipes. Thus, a fully linear model could over-estimate the use of DH compared to requiring a certain price to lay down the piping.

The results are, however, different in the varying electricity price case, where DH is an economically feasible solution for all types of housing, except for single-family housing with low heat demands. There is, however, investment in air-to-water HPs in Schemes 1–4, which decreases the use of DH compared to Schemes 5 and 6.

4.1.3. District heating supply side

The DH supply mix is shown in Fig. 4. The total DH heat production is slightly higher for Scheme 6 than for Scheme 1 for all electricity price

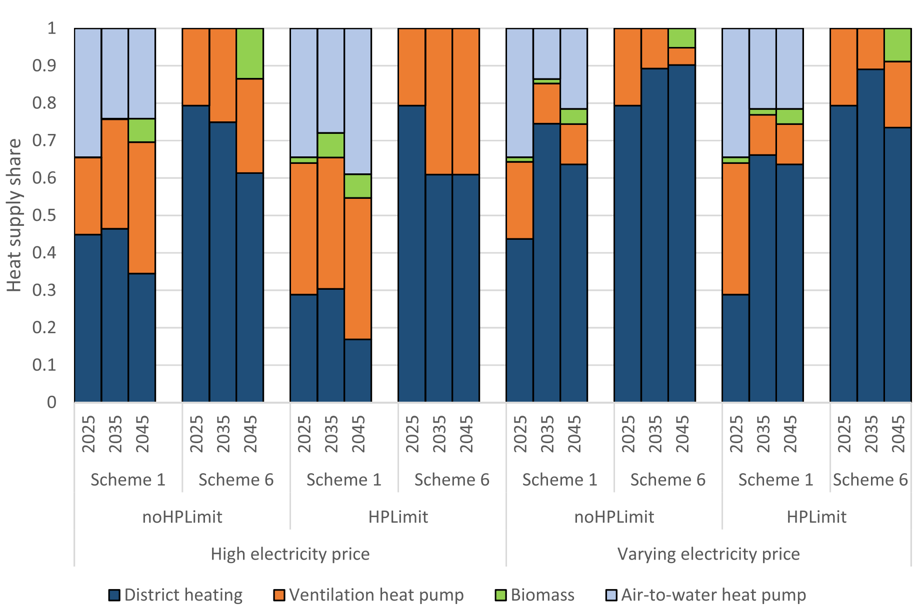

Fig. 3. Modeling results for heating solutions in Years 2025, 2035 and 2045 for large apartment buildings built in Year 2020 for Schemes 1 and 6. Schemes 1–4 show similar results, while Schemes 5 and 6 show similar results; Schemes 2–5 are therefore left out of in the figure. The designation *noHPLimit* indicates schemes in which the heat source availability for HPs in the DH network is unlimited, whereas *HPLimit* denotes schemes in which the heat source capacity for HPs is limited to the Year 2020 level.

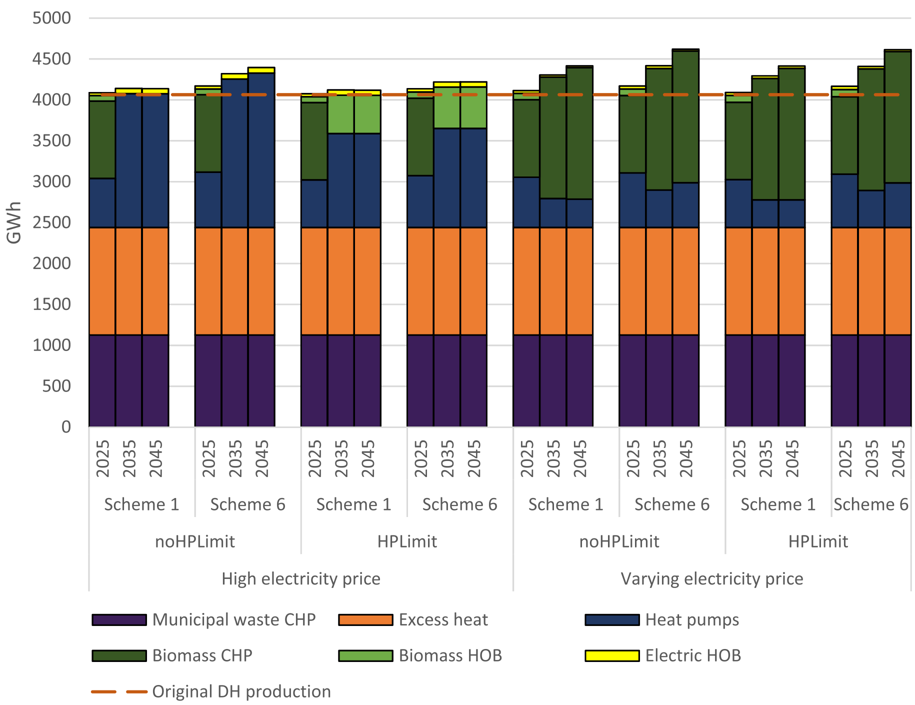

Fig. 4. Modeling results for the DH supply in Years 2025, 2035 and 2045 for Schemes 1 and 6. Schemes 1–4 show similar results, while Schemes 5 and 6 show similar results; Schemes 2–5 are therefore left out in the figure. The *noHPLimit* designation indicates schemes in which the heat source availability for HPs in the DH network is unlimited, whereas *HPLimit* denotes schemes in which the heat source capacity for HPs is limited to the Year 2020 level. The original DH production level indicates the DH production level that is required to supply the existing DH system with heat. Any increase above this level means that new housing has been connected to the DH system.

cases, as most of the DH production is used for heating the system already in place at the start of the modeling period. There is, however, an increase in the total DH production, indicating that new housing is connected to the DH system. This increase is significantly smaller for the high electricity price if the heat source availability for HPs is limited. For the varying electricity price, Scheme 6 has a higher level of DH heat production regardless of the DH HP heat source availability, as biomass CHP plants are instead built and used to supply heat, leaving the DH HP heat source with an untapped potential in most of the seasons.

4.2. Modeling solution time

In Table 3, the solution times for the different electricity price cases and schemes are presented. The solution time generally increases when the model becomes more non-linear. The number of binary variables increases from Scheme 1 up to Scheme 6, whereas the number of binary variables is the same for Schemes 3 and 4, and Schemes 5 and 6 have the same number of binary variables.

Some trends are evident for the solution times. The most-linear model can be solved with an optimal solution within 1 h for all the electricity price cases. For the models with more binary variables, the high electricity price cases are more often solved more rapidly than the low electricity price cases if there is no heat source limit. If there is a heat source limit, the converse is true.

The varying electricity price case seems more difficult to solve than either the high or low electricity price cases. This seems to stem from the fact that in the varying electricity price case, there are investments in new biomass CHP plants for which the restrictions related to minimum plant size make it more difficult to solve, as both the supply and demand sides require a high number of binary variables, thereby significantly increasing the solution time. Moreover, the percentage of housing that uses DH is higher in this electricity price case than in the other two cases.

The bolded values are those not solved for a gap of maximum 0.1%, but instead showing the achieved relative gap after 6 h.

Table 3
Solution times, in seconds, for the modeling of the different electricity price cases and schemes.

Scheme	1	2	3	4	5	6
High electricity price						
No HPlimit	2399	2379	4745	3874	3715[b]	5677[b]
HP limit	651	2495	2981	4302	**0.17%**	**0.35%**
Low electricity price						
No HP limit	909	3531	2462[a]	5781[a]	**0.12%**	2375[b]
HP limit	702	702	1221	782	**0.11%**	10,411[b]
Varying electricity price						
No HP limit	1229	9627[a]	9572[b]	16,226[a]	6096[b]	7831[b]
HP limit	2034	15,435[a]	**0.16%**	15,847[b]	**1.73%**	**1.86%**

[a] Solution gap of maximum 0.01%.
[b] Solution gap of maximum 0.1%.

5. Analysis and discussion

The results clearly show that the choice of programming method used for the modeling is important for the studied system. Heating solution results are shown to be sensitive to whether LP or MILP is used, as LP implementation encourages mixes of technologies, in contrast to MILP implementation. This finding is of significance because the mixing of technologies within the same building is uncommon; there may be technical reasons why mixes are not viable. Furthermore, when there is a starting cost associated with installing new individual technologies in new housing, this may discourage mixing for economic reasons alone. This further supports the conclusion of [19], in which it is concluded that too simple representations can result in solutions infeasible in real systems. Previous studies have not fully investigated how different programming methods affect local energy systems. The scientific contribution of this article is thus the systematic investigation of how the different programming methods affect the long-term cost-optimal technology choices showing that there are significant differences resulting from the application of the LP and MILP formulations. This highlights the need to consider which programming method to use for long-term investigations of cost-efficient local energy systems. With a LP formulation, there is a higher risk, compared to a MILP formulation, that the model results show an optimal technology mix that is actually infeasible in real systems.

When all new heating solutions have a starting installation cost or can only be installed in an all-or-nothing manner (Schemes 5 and 6, both of which use MILP), there is an increase in the use of DH solutions for all electricity prices. The extents to which the LP and MILP formulations reflect how investments can be made in housing in real systems are of high importance to consider. This is evident in the results of this study, in that the individual heating technology mainly benefiting from a decrease in DH when using LP (compared to MILP) are air-to-water HPs. This is because, in the model, small installations can be installed and utilized during the summertime when the COP is highest. Air-to-water HPs were added as a novelty to this study, and the results highlights the importance of considering this technology in future local energy system studies. In the schemes where MILP formulations are used also for individual heating technologies, the starting cost or full installation cost is too high to be economically feasible. Therefore, the combination of low installation cost and high COP benefits air-to-water HPs while decreasing the use of DH in the model.

Limiting the heat source availability for DH HPs does not seem to affect significantly the resulting heating solution for the varying electricity price case, in which there are investments in new CHP plants. For the high and low electricity price cases, there is a difference, especially if the heating solutions can only be invested in, as in Schemes 5 and 6.

The solution time for the model varies substantially depending on the electricity price profile. The varying electricity price case has, in general, a longer solution time than the other two electricity price cases. One of the main differences between the varying price case and the high and low price cases is that there are investments in new CHP plants only in the varying case. Another important difference is that more heat is produced and distributed by DH in the varying electricity price case. Since supply and demand are treated together in this paper, it seems that the combination of the result that new CHP plants are economical, which in turn makes it more economical to use DH for new housing, which makes the model more difficult to solve. In the model used, CHP plants have restrictions regarding minimum size. As a consequence, binary variables must be used in the model, which can increase the solution time. It is important to note here, however, that CHP plants generally have a higher power-to-heat ratio when they are large and benefit from economies of scale. Simplifying the investments in CHP plants into fully linear equations could, therefore, be a too-large simplification of how investments can be made in real systems. Moreover, CHP plants often have ramp times and minimum load levels which have not been included in this study. Adding these aspects could further increase the computational time due to the associated binary variables needed to consider such aspects, as shown in Ref. [19]. Although those aspects are arguably mostly of importance in models with a high time resolution.

When a DH HP limit is applied, the solution times are higher in Schemes 5 and 6 for all electricity price cases. This is also the case for the varying electricity price case for which the solution is unaffected by the DH HP limit. To improve the solution time, it may be worthwhile to rule out heating technologies that are not feasible, technically or economically, instead of just providing the input data to the model and hoping for the best. However, answering the question as to which technologies are feasible (or not feasible) beforehand is not a simple task, as one of the goals of using energy system modeling is to try to answer such questions in the first place. Without making any simplifications, models would become too hard to solve but simpler models can give similar results as more complex models while having a dramatically reduced model solving time [20–22].

From the results, it is clear that disaggregating the housing into separate demands provides insights into which types of housing use which forms of heating solution. However, a drawback is that the solution time increases with the heat demand separation, as much more data need to be supplied to the model and more binary variables are needed if any of Schemes 2–6 is used. Determining where best to draw the line regarding how high the resolution needs to be for this type of modeling is challenging. Nevertheless, by categorizing the housing built in different years it becomes clear that the competitiveness levels of different heating solutions may be affected with respect to future housing. The results do, however, show that new single-family housing with low heat demands never uses DH as the heating solution. Therefore, omitting this type of housing when investigating whether to use DH or other options for new housing could reduce the solution time, as less data and fewer binary variables would need to be included in the model.

6. Conclusions

In the present study, different mathematical programming methods, corresponding to different investment cost schemes, have been applied to the long-term, cost-optimization modeling of an expanding local heating system. The modeling results differ clearly depending on whether the LP formulation or MILP formulation is used for investments in new capacity of heating technologies in new housing. The resulting amount of DH used in new buildings is higher if MILP is used for all the investments, corresponding to a case in which all the technologies have a starting cost or can only be installed at a size that can cover the full heat demand. If LP is used for investments in individual heating solutions, the use of DH is decreased while the use of air-to-water HPs is increased. The result that air-to-water HPs is a prominent heating solution in future housing highlights the importance to include this technology, which has been added as a novelty in this study, in further studies focusing on local heating systems.

The increased use of DH for the cases in which MILP is used for all technologies indicates that fully linear models potentially underestimate the economic viability of DH, whereas they overestimate the viability of individual HPs. This finding is important to consider when deciding on whether to use LP, MILP or some other programming method in energy system modeling, especially for local systems where the representation of individual technologies is of high importance. Due to the local nature of heating systems, together with the fact that different heating technologies are seldom mixed within individual buildings, there is significant risk that using LP models for investigating cost-efficient options for expanding heating systems can result in heating solutions which do not properly reflect the costs and best options of real systems.

There are significant differences in the model solution time depending on whether LP or MILP is used for new installations. The use of MILP increases the number of binary variables, which corresponds to an increased solution time. Most of this time seems, however, to be used to prove that the solution is within a certain deviation from the optimal value, rather than to enhance the solution itself. This leads to the conclusion that very strict optimal criteria do not affect the solution in any significant way, as compared to a slightly relaxed solution criterion.

CRediT authorship contribution statement

Karl Vilén: Conceptualization, Methodology, Investigation, Writing – original draft, Writing – review & editing, Visualization. **Erik O. Ahlgren:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

This study was funded by the research program TERMO – heating and cooling for the future energy system of the Swedish Energy Agency

(Project nr. 45990-1).

Appendix A

Cplex solver options:
rerun yes
iis yes
lpmethod 0
baralg 1
barcrossalg 0
barorder 3
epgap 0.
THREADS = 7
probe 3
mipdisplay 4
cuts 3
covers 3
cliques 3
disjcuts 2
liftprojcuts 3
localimplied 2
memoryemphasis 1
names no.
Computer and software specifications:
CPU: Intel Core i7-8650U (4 cores, 1.90 GHz, 8 logical processors).
RAM: 16 GB DDR4 2400 MHz.
CPLEX version: 22.1.0.0.
Veda 2.0, application version 1.248.1.2.
TIMES version: 4.6.0.

Appendix B

The COP computation procedure presented in this appendix is inspired by the EN 14825 standard in which the HPs shall provide a certain amount of heat with different ambient air temperatures and for different amounts of time. This is used to calculate the SCOP, which provides a better representation of how efficient the HP is over a full year.

According to Ref. [35], the COP of a real air-to-water HP can be calculated approximately as:

$$COP = 0.0023 \cdot T_{increase}^2 - 0.2851 \cdot T_{increase} + 10.677$$

when the target temperature is 55 °C and $T_{increase}$ is between 35 °C and 65 °C.

In the model, there are five temperature periods, and to calculate the COP during the different periods, the outside temperature is needed. It is assumed that the outside temperature is proportional to the space heating demand.

Using data from SMHI, the mean outside temperature is calculated to be 15 °C during the summer months. Since the heat demands and lengths of all the seasons are known, this can be used to calculate the mean outside temperatures of the other seasons as follows:

$T_{summer} = 15$ °C.
$T_{spring/fall} = 6.6$ °C.
$T_{winter} = 3.4$ °C.
$T_{peak\_low} = -4.1$ °C.
$T_{peak\_high} = -16.7$ °C.

All these temperatures, except for T_{peak_high}, are within the defined range of −10 °C–20 °C, where the above formula can be used to calculate the COP for each season.

For the T_{peak_high}, the temperature lies too far outside the defined range to be deemed reliable. This value has instead been assessed by looking at various models available on the market and their specific COPs for low-temperature values (see below when discussing SCOP).

The acquired COPs of the different seasons is with this method computed as:

$COP_{summer} = 2.95$.
$COP_{spring/fall} = 2.25$.
$COP_{winter} = 2.10$.
$COP_{peak\_low} = 1.85$.
$COP_{peak\_high} = 1.40$.With the COP for each season, *s*, the amount of electricity needed in each season can be calculated, and thereby compute the total amount of electricity needed for a whole year. By dividing the total heat demand by the total electricity demand, a SCOP of 2.26 is derived.

Lastly, the SCOP defined in the technical specification for an air-to-water HP in a certain year is used to scale the COP for each season. Thus, if the SCOP is specified as 3 for a HP, the COP for each season is multiplied by 1.33 (=3/2.26).

$$\mathrm{SCOP} = \frac{\sum_s \Delta \mathrm{T}_s \cdot \mathrm{length}_s}{\sum_s \frac{\Delta \mathrm{T}_s \cdot \mathrm{length}_s}{\mathrm{COP}_s}}$$

$$\Delta T_s = \mathrm{T}_{\mathrm{Indoor}} - \mathrm{T}_{\mathrm{Outdoor}}$$

The resulting COPs for each season for an air-to-water HP with a SCOP of 3 therefore become:

$\mathrm{COP}_{\mathrm{summer}} = 3.9$.

$\mathrm{COP}_{\mathrm{spring/fall}} = 3$.

$\mathrm{COP}_{\mathrm{winter}} = 2.8$.

$\mathrm{COP}_{\mathrm{peak\_low}} = 2.5$.

$\mathrm{COP}_{\mathrm{peak\_high}} = 1.85$.

The value for $\mathrm{COP}_{\mathrm{peak\_high}}$ is close to the values found in the technical specifications of air-to-water HPs with a SCOP of around 3.

It is assumed that improvements in the SCOPs of future installations will improve all seasons equivalently.

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