THE ANNULLING OF THE SUDDEN APPEARANCE OF AN UNBALANCE IN ROTARY MACHINES BY USING ACTIVE MAGNETIC BEARINGS

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Abstract:

The application of magnetic bearings has become more frequent during the last 20 years and represents a significant aspect of improvements in the construction of machines with rotary motion. With the advancement of technology, the number of applications in which magnetic bearings have found their application is increasing. In this paper, it is shown how the effect of magnetic forces can annul the negative influence of unbalance, which suddenly appeared in a rotor supported in active magnetic bearings. Such cases may occur in operation due to breakage and rotor parts falling off (e.g., fan blades), which will lead to a sudden change in the mass balance of the rotor system and dislocation of the centre of mass in relation to the geometric centre of the rotor. In the paper, a mathematical model of the dynamic behaviour of a rigid rotor in active magnetic bearings was developed. The model is nonlinear and has five degrees of freedom and can only be solved numerically. The Newmark beta method and the Newton-Raphson method were used to solve the system of nonlinear differential equations. The results of the simulation showed the advantages of using active magnetic bearings for annulling sudden occurrences of unbalance in rotary machines.

1 Introduction

The bearings represent one of the most important elements in the construction of a rotary machine. They support rotating elements and have the task of ensuring relative movement and load transfer from moving elements to the machine construction, as well as ensuring the required accuracy between parts in relative movement [1]. Friction and mechanical vibrations are the main causes of problems that prevent bearings from performing their function effectively in machines [2-5]. Therefore, active magnetic bearings (AMB) are increasingly being used in applications for the support of rotating elements. Unlike conventional bearings, AMBs create magnetic forces that allow the rotor to float in a magnetic field, without contact between the bearing and the rotor [6]. The absence of physical contact avoids friction, which results in a reduction of power losses in the system. On the other hand, by controlling the magnetic forces that are generated on the poles of the electromagnet, it is possible to counteract the disruptive forces that cause the vibrations of the rotating system. In this way, vibrations are reduced and the accuracy of rotor rotation increases [7]. This is especially important for applications that require high rotor speeds because using AMB can significantly increase the number of revolutions compared to applications with conventional bearings [8].

The idea of supporting the rotor in magnetic bearings appeared immediately with the first patents for passive magnetic levitation [9,10], which is related to the period around 1842 and the British mathematician Earnshaw [11]. Over time, the conclusion was reached that it is necessary to control and adjust the forces produced by ferromagnetic materials for the body to achieve stable levitation [12, 13]. This led to the emergence and development of AMB. Higuchi et al. [14] applied frequency domain iterative learning control

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(FILC) to control AMBs. However, the foundations of the first decentralized control model for rotors supported in magnetic bearings were laid by Fan and Lee [15]. Knopse et al proposed an adaptive vibration control (AVC) model [16] similar to iterative learning control in the frequency domain. All these models use sensors and actuators to continuously monitor and control the response to external disturbances, thus improving the stability and precision of the rotation of the system itself. The system uses algorithms to identify the strength of vibrations and adjusts the control parameters accordingly, to soothe the rotor and reduce vibrations.

The most common control system used to control the AMB system in rigid rotors is PID control (Proportional-Integral-Derivative control) [17, 18, 19]. Muminović et al., for flexible rotors, propose PI-D control. In [20] they examine the performance of a flexible rotor/AMB system in the case of PID and PI-D control, both in combination with NOTCH filters. Using PID control, Štimac et al. have proposed a vibration-damping model using magnetic forces for a flexible rotor supported in an AMB [21]. Numanoy and Srisertpol also use PID controllers to reduce vibrations of the rotor/AMB system [7]. The vibration reduction benefits of AMB were exploited by Park et al. to increase the precision of rollers used in printing machines [22].

Srinivas et al. presented in [23] a detailed overview of the techniques used for vibration damping using AMB systems. Mass unbalance is singled out as the primary source of vibration in rotating machines. Several different factors are identified as causes of unbalance, such as [24]:

- Construction requirements - mainly refers to the impossibility of constructing symmetric rotors;
- Errors that exist in the structure of the rotor, i.e., inhomogeneity of the rotor material;
- Irregularities resulting from imprecise mechanical processing, i.e., rotor asymmetry;
- Improper and inadequate installation of rotating elements;
- Deformations that occur during exploitation as a consequence of the load;
- Breakages and damage of rotating elements, as well as damage resulting from wear and tear;
- Disturbances in the mass balance due to the appearance of impurities.

The research performed to reduce the intensity of vibrations that are a consequence of unbalance by using AMBs is mainly heading in two basic directions [25]:

- The automatic balancing method refers to the tendency for the rotor to turn around its axis of inertia (that is, the actual axis of rotation), and this can be achieved by annulling the synchronous electromagnetic forces. This method is used in cases where it is not necessary to achieve high rotation accuracy of the rotor, and the air gap that exists between the rotor and the stator in the AMB is large enough.
- The unbalance compensation method refers to the tendency for the rotor to rotate around its axis of symmetry, and this can be achieved by compensating the force that arises because of the unbalance. This method is more suitable for cases that require precise rotation of the rotary elements, but for the application of this method, a high-power amplifier, and attractive magnetic forces on the bearings due to the presence of a large residual unbalance are required. So back in 1996, Herzog et al. in [26] developed a model that uses a NOTCH filter to remove synchronous control currents and reduce vibrations resulting from unbalance. On the other hand, Schuhmann et al. [27] studied the improvement of rotor rotation accuracy using a Kalman filter to reduce the impact of unbalance vibrations on the system. Shafai et al. [28] managed to obtain the Fourier coefficients of vibrations that are a consequence of unbalance using the iterative algorithm through the method of automatic balancing.

More recently, Kejian et al. [29] proposed a method based on which data on the position of the unbalance mass is collected in real time. Cui et al. [30] identified the static mass unbalance by applying the unbalance detection method and achieved zero deviation magnetic field levitation control of the rotor. On the other hand, Hutterer et al. [31] applied a two-stage modulation approach to control the unbalance. In the works of Chen et al. [32], as well as Xu et al. [33], the damping of vibrations due to unbalance was controlled by applying the NOTCH filter, as developed much earlier by Herzog. Bian et al. [34] developed a model for the reduction of vibrations of the AMB system without the use of a rotation speed sensor by means of a nonlinear adaptive algorithm. The effectiveness of their model was proven experimentally. Most of the causes which are the consequence of the unbalance appearance can be corrected to a greater or lesser extent with adequate interventions before exploitation. However, unknown unbalances occur very often in the operation of machines, and this was analysed by Xu et al. in [35]. Sudden occurrences of unbalance during exploitation are a serious hazard that occurs in the case of breaking and/or tearing of individual parts of the rotary system, e.g., breakage of fan blades or turbine parts, etc. Often, the vibrations caused by such failures require an immediate stop of the machine, which can be extremely unfavourable for the rest of the production process, even though the damage is not great enough to require an immediate stop of the machine. As a rule, such cases cause, in addition, to repair and maintenance costs, enormous downtime costs because the system had to be stopped and production shut down at an unfavourable moment. Precisely AMB provides the possibility to annul this kind of sudden occurrence of unbalance without stopping the machines, immediately after it occurs during the actual exploitation. In this way, the need for an unplanned machine stop can often be postponed until a more favourable moment for production [36].

The aim of this paper is to research the possibility of applying AMBs to annul the impact of the sudden appearance of rotor unbalance due to various damages that occur in rotary machines. In the paper a mathematical model that describes the above-mentioned problem that often occurs in practice is developed. The mathematical model serves for further development of the AMB control system in the event of such sudden failures. In the paper, the differential equations of the motion of the rigid rotor in the AMB were derived, considering the working and other loads that occur during the operation of the machine, as well as the disruptive forces caused by the sudden appearance of unbalance and attractive forces that occur on the poles of the electromagnet. By solving the differential equations, the control magnetic forces are obtained, which must be enacted to soothe the rotor and damp out the vibrations. The Newmark beta method and the Newton-Raphson method were used to solve differential equations.

2 Basic characteristics of active magnetic bearings

Magnetic bearings can be derived as passive magnetic bearings (PMB), active magnetic bearings (AMB), and a combination of these two types of performance (hybrid MB) [12]. Full suspension of the rotor by using passive bearings in all six degrees of freedom is not possible because there is always one unstable degree of freedom [11]. For the system to be completely stable, that is, to achieve support in all six degrees of freedom, it is necessary to act on the rotor with some external force, and this can be achieved by installing an electromagnet. By supplying the current to the electromagnets, a magnetic field is created that opposes the force of gravity and thus allows ferromagnetic bodies to levitate. Active magnetic bearings are machine elements that, in synergy with electronic components, form a single mechatronic system that can improve the support conditions of machines with linear or rotary motion.

Based on the standard configuration [37] of active magnetic bearings, it is possible to explain the basic principle of operation of a simple magnetic bearing system, which consists of a rotor supported in only one direction (Figure 1):

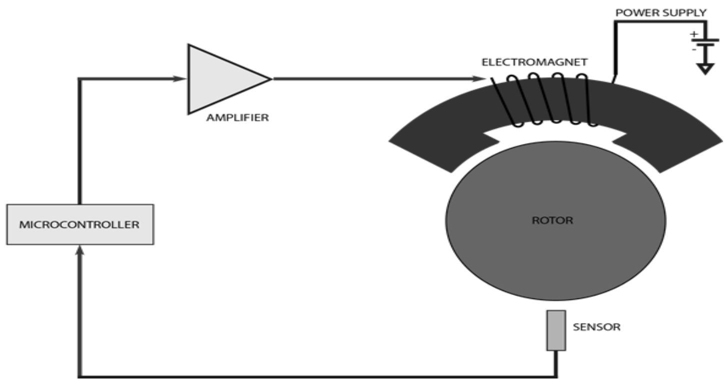


Figure 1. Schematic representation of the operation principle of a simple system of AMBs.

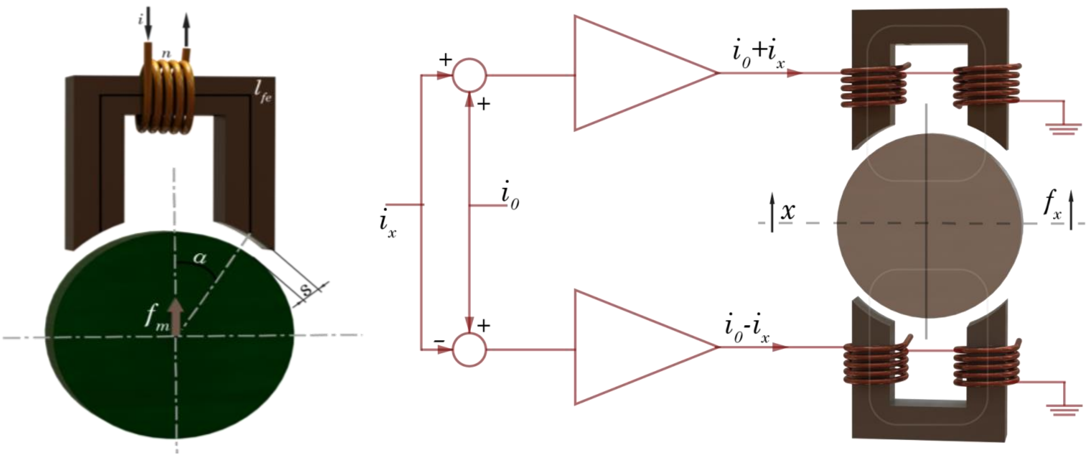
The sensor measures the displacement of the rotor from the reference position, and the micro-processor as a controller sends a control signal to the amplifier, acting on data from the sensor. The amplifier converts this control signal into a control current, and the control current generates a magnetic field in the active magnets. The resulting magnetic forces affect the rotor in such a way that it remains in its levitating position [12]. The rotor that levitates in the magnetic field of the stator, besides the magnetic force, is affected by the force of its weight, and other active and disruptive forces that occur during exploitation. In order for the rotor to be in a stable position, it is necessary for the system of forces acting on the rotor to be in balance, i.e., that the magnetic force is in balance with all the internal and external forces of the rotating system. If the current that flows through the electromagnets is not regulated, the magnetic force will not be in balance with the other forces acting on the rotor, which will lead to contact between the rotor and the stator. To avoid such situations, sensors are introduced into the magnetic bearing system that continuously measure the deviation of the rotor from the reference position.

2.1 The calculation of attractive magnetic force

To explain the attractive magnetic force created by an electromagnet more easily, it is necessary to start from a simple active magnetic bearing system, that is, from a radial single-pole active magnetic bearing (Figure 2 a). The equation for calculating the attractive magnetic force for a radial single-pole active magnetic bearing is as follows:

$$F = \frac{1}{4}\mu_0 \cdot A_0 \cdot n^2 \cdot \frac{i^2}{s^2} \cdot cos\alpha = k \cdot \frac{i^2}{s^2} \tag{1}$$

where μ_0 is the magnetic permeability of vacuum, A_0 is the cross-section of the air gap, n is the number of electromagnet windings, magnetic forces act on the rotor with an angle α, i is the current flowing through the windings, s is the air gap between electromagnet and rotor, and $k = \frac{1}{4}\mu_0 A_0 n^2 cos\alpha$ a constant that depends on the geometry of the electromagnets used for the AMB system [12].



(a) *(b)*

Figure 2. a) Radial single-pole active magnetic bearing; b) Differential control of one pair of poles of active magnetic bearing.

Considering that the mentioned case with only one pole of the electromagnet (Figure 2a) is not used in practice, by combining them and forming a pair of poles, a structural combination of electromagnets is obtained, which enables the control of the ferromagnetic rotor in several axes.

The forces with which electromagnets act on the rotor during operation can be both positive and negative at the same time. Active magnetic bearings work in differential operation mode (Figure 2b) [12]. This means that one electromagnet in a pair is powered by the sum of the stable current i_0, which maintains the rotor in the reference position at the air gap s_0, and the regulation current i_x. The total current on that magnet is $i_1 = i_0 + i_x$. The second electromagnet in the pair is powered by their difference ($i_2 = i_0 - i_x$). In accordance with the action of the electromagnet on the ferromagnetic rotor, the distances between the rotor and the stator are:

- for first pole:

$$s_1 = s_0 - x, \tag{2}$$

- for second pole:

$$s_2 = s_0 + x. \tag{3}$$

Based on the differential operating mode of the active magnetic bearing system and applying the equation for obtaining the attractive force (1), the equation for calculating the attractive force for one pair of poles (X or Y axis) of the electromagnet is obtained:

$$F = F_{+} + F_{-} = k \cdot \left[\frac{(i_0 + i_x)^2}{(s_0 - x)^2} - \frac{(i_0 - i_x)^2}{(s_0 + x)^2} \right] \tag{4}$$

By observing the active magnetic bearing system and by its linearization, i.e., presuming that the rotor does not deviate from the reference position and that $x \ll s_0$, the equation for the ideal model for the calculation of the attractive magnetic force can be obtained:

$$F = \frac{4 \cdot k \cdot i_0}{s_0^2} \cdot cos\alpha \cdot i_x + \frac{4 \cdot k \cdot i_0^2}{s_0^3} \cdot cos\alpha \cdot x = k_i \cdot i_x + k_s \cdot x \tag{5}$$

where k_i is force – current factor:

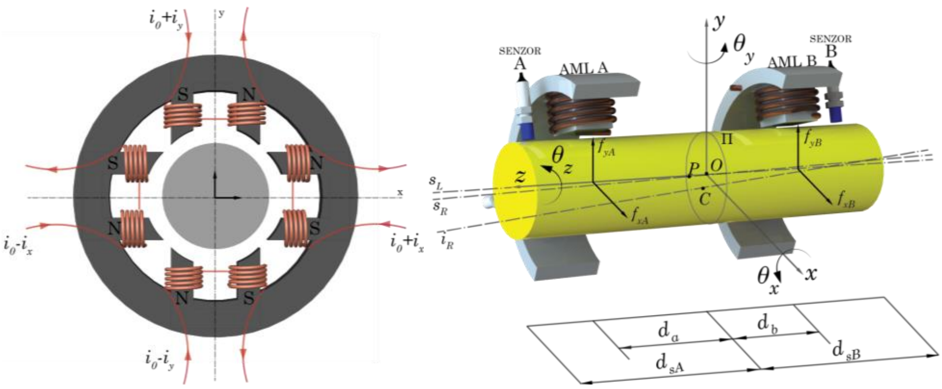
$$k_i = \frac{4 \cdot k \cdot i_0}{s_0^2} \cdot cos\alpha \tag{6}$$

and k_s is force – displacement factor:

$$k_s = -\frac{4 \cdot k \cdot i_0^2}{s_0^3} \cdot cos\alpha \tag{7}$$

3 Mathematical model of the dynamic behaviour of a rigid rotor in active magnetic bearings due to the influence of unbalance

Active magnetic bearings provide the possibility of active control of rotor vibrations that may be the result of various causes. One of the most common causes is the sudden appearance of unbalance during machine operation, due to breakage of fan blades, accumulation of dirt on the rollers of the rotary mill, etc. These phenomena will cause sudden and very strong vibrations due to an unbalance in rotary machines. Because of attractive magnetic forces, it is possible to continuously position the axis of inertia in relation to the axis of rotation, which can significantly annul the impact of unbalance, and thus reduce the intensity of vibrations that are a consequence of sudden unbalance. The mathematical model of the active influence of radial magnetic bearings on the sudden occurrence of unbalance was derived for the rotor system with two AMBs, which is shown in Figure 3.



(a) (b)

Figure 3. (a) Structural configuration of an active magnetic bearing with two pairs of poles; (b) Structural model of the rotor system supported on two radial active magnetic bearings.

The system consists of two radial active magnetic bearings and has 5 degrees of freedom (two degrees of freedom for translational movement in the radial directions x-y, two for rotational movement around the x-y axis, and one degree of freedom for rotational movement in the direction of the z axis). To monitor the position of the rotor axis, two displacement sensors are placed next to the radial bearings. The coordinate system is placed in the centre of the rotor system (point O), that is, it is obtained by the intersection of the plane Π (the plane in which the centre of mass of the rotor is located, and which is normal to the axis z - the axis of rotation) and the axis of symmetry of the magnetic bearings A and B (s_L). Point P represents the intersection of the axis of symmetry of the rotor (s_R) and the plane Π, and point C is obtained by the intersection of the axis of inertia of the rotor (i_R) with the plane Π and represents the new position of the centre of mass of the rotor.

The angles of rotation around the axes of the coordinate system *x*, *y*, and *z* are denoted by θ_x, θ_y, and θ_z. The distances of the radial active magnetic bearings (bearing A and bearing B) from the coordinate origin O are represented by d_A and d_B, while the sensor distances from the coordinate centre O are indicated by d_{sA} and d_{sB}. The attractive magnetic forces used to position the rotor are marked with: f_{xA} and f_{yA} (magnetic bearing A) and f_{xB} and f_{yB} (magnetic bearing B). For the structural model of the rotor system shown in Figure 3b, the equation of the dynamic movement of the rotor supported on radial active magnetic bearings according to Newton's law of motion (Newton's second law) and the equations for the rigid rotor model with five degrees of freedom of movement can be written as:

$$\begin{cases} m\ddot{x}_i = f_{xA} + f_{xB} + me\dot{\theta}_z^2 cos\dot{\theta}_z t \\ J_x\ddot{\theta}_{xi} + J_z\dot{\theta}_z\dot{\theta}_{yi} = f_{yA}d_A - f_{yB}d_B - \chi \cdot \dot{\theta}_z^2 \cdot (J_x - J_z) \cdot \cos\dot{\theta}_z t \\ m\ddot{y}_i = f_{yA} + f_{yB} - mg + me\dot{\theta}_z^2 sin\dot{\theta}_z t \\ J_y\ddot{\theta}_{yi} - J_z\dot{\theta}_z\dot{\theta}_{xi} = -f_{xA}d_A + f_{xB}d_B + \chi \cdot \dot{\theta}_z^2 \cdot (J_y - J_z) \cdot sin\,\dot{\theta}_z t \end{cases} \tag{8}$$

where is m rotor mass, J_x and J_y transverse moments of inertia, J_z represents the polar moment of inertia of a rigid rotor, $mg = F_g$ is the force arising from the weight of the rotor, $me\dot{\theta}_z^2 = F_c$ and $\chi\dot{\theta}_z^2(J_r - J_z) = F_c$ centrifugal forces resulting from unbalance.

If the transverse moments of inertia are greater than the polar moment of inertia (J_x,$J_y \gg J_z$) a rigid rotor can be viewed as a long rod. Considering that the excitation (static) current is supplied to the electromagnets i_0 which creates a magnetic field in which the rotor levitates, the influence of the weight force of the rigid rotor can be ignored.

According to the equation for linearization of the attractive magnetic force (5), the following relations are obtained for the attractive forces of electromagnets of radial bearings A and B:

$$\begin{array}{ll} f_{xA}^+ = k_s x_A + k_i i_1 & f_{xA}^- = -k_s x_A + k_i i_3 \\ f_{yA}^+ = k_s y_A + k_i i_2 & f_{yA}^- = -k_s y_A + k_i i_4 \\ f_{xB}^+ = k_s x_B + k_i i_5 & f_{xB}^- = -k_s x_B + k_i i_7 \\ f_{yB}^+ = k_s y_B + k_i i_6 & f_{xB}^- = -k_s x_B + k_i i_7 \end{array} \tag{9}$$

Because the displacement of the rotor from the reference position is observed in radial active magnetic bearings, it is necessary to translate the displacement from the coordinate system into the bearing positions. The displacement translation is represented by the following relations:

$$\begin{cases} x_A = x_i - d_A \cdot \theta_{yi} \\ y_A = y_i + d_A \cdot \theta_{xi} \\ x_B = x_i + d_B \cdot \theta_{yi} \\ y_B = y_i - d_B \cdot \theta_{xi} \end{cases} \tag{10}$$

By further arranging the obtained relations (10), the final form of the rotor movement is obtained as a function of the movement in the magnetic bearings A and B:

$$\begin{aligned} x_i &= \frac{d_B x_A + d_A x_B}{d} & y_i &= \frac{d_B y_A + d_A y_B}{d} \\ \theta_{xi} \approx tg\theta_{xi} &= \frac{y_A - y_B}{d} & \theta_{yi} \approx tg\theta_{yi} &= \frac{x_B - x_A}{d} \end{aligned} \tag{11}$$

where d is total length between two magnetic bearings.
By applying the first and second derivatives to equations (11) and inserting them and equations (9) into (8), the final equations of the dynamic movement of the rotor in the positions of the magnetic bearings A and B are obtained:

$$\begin{aligned} \ddot{x}_A + \frac{J_z\dot{\theta}_z d_A}{J_y d}(\dot{y}_A - \dot{y}_B) - x_A\left(\frac{2k_s}{m} + \frac{2k_s d_A^2}{J_y}\right) - x_B\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_y}\right) \\ = i_{xA}\left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_y}\right) + i_{xB}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_y}\right) + e\dot{\theta}_z^2 cos\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \\ \cdot \frac{(J_y - J_z) \cdot d_A}{J_y} \cdot sin\,\dot{\theta}_z t \end{aligned} \tag{12}$$

$$\begin{aligned} \ddot{x}_B - \frac{J_z\dot{\theta}_z d_B}{J_y d}(\dot{y}_A - \dot{y}_B) - x_A\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_y}\right) - x_B\left(\frac{2k_s}{m} + \frac{2k_s d_B^2}{J_y}\right) \\ = i_{xA}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_y}\right) + i_{xB}\left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_y}\right) + e\dot{\theta}_z^2 cos\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \\ \cdot \frac{(J_y - J_z) \cdot d_B}{J_y} \cdot sin\,\dot{\theta}_z t \end{aligned} \tag{13}$$

$$\begin{aligned} \ddot{y}_A + \frac{J_z\dot{\theta}_z d_A}{J_x d}(\dot{x}_B - \dot{x}_A) - y_A\left(\frac{2k_s}{m} + \frac{2k_s d_A^2}{J_x}\right) - y_B\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_x}\right) \\ = i_{yA}\left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_x}\right) + i_{yB}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_x}\right) + e\dot{\theta}_z^2 sin\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \\ \cdot \frac{(J_x - J_z) \cdot d_A}{J_x} \cdot cos\,\dot{\theta}_z t \end{aligned} \tag{14}$$

$$\begin{aligned} \ddot{y}_B - \frac{J_z\dot{\theta}_z d_B}{J_x d}(\dot{x}_B - \dot{x}_A) - y_A\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_x}\right) - y_B\left(\frac{2k_s}{m} + \frac{2k_s d_B^2}{J_x}\right) \\ = i_{yA}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_x}\right) + i_{yB}\left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_x}\right) + e\dot{\theta}_z^2 sin\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \\ \cdot \frac{(J_x - J_z) \cdot d_B}{J_x} \cdot cos\,\dot{\theta}_z t \end{aligned} \tag{15}$$

4 Solution of the mathematical model

In the previous chapter, a mathematical model of the dynamic behaviour of a rigid rotor supported on two radial active magnetic bearings was derived. The model has five degrees of freedom of movement. The derived model considers the case when there are known static and dynamic unbalances in the rotor structure. For a simpler calculation, the elastic properties of the rotor structure, magnetic nonlinearities that can occur in the active magnetic bearing system, the influence of eddy currents, the dissipation of magnetic flux, the influence of the current signal amplifier, and the power supply that exists in the system are ignored. The resulting mathematical model of the dynamic motion of the rigid rotor cannot be solved analytically, so to obtain the

solutions of the derived differential equations, the problem must be solved using numerical methods. To solve and approximate the final dynamic equations the Newmark beta method was used [38].
By applying Taylor's series and Lagrange's theorem, the first order and second order derivatives of the Newmark beta method in the direction of the x-y axis can be obtained, which read [39]:

$$\dot{x}_{i+1} = \frac{dx_i}{dt} = a_1 \cdot (x_{i+1} - x_i) - a_4 \cdot \dot{x}_i - a_5 \cdot \ddot{x}_i \tag{16}$$

$$\dot{y}_{i+1} = \frac{dy_i}{dt} = a_1 \cdot (y_{i+1} - y_i) - a_4 \cdot \dot{y}_i - a_5 \cdot \ddot{y}_i \tag{17}$$

$$\ddot{x}_{i+1} = \frac{d^2x_i}{dt^2} = a_0 \cdot (x_{i+1} - x_i) - a_2 \cdot \dot{x}_i - a_3 \cdot \ddot{x}_i \tag{18}$$

$$\ddot{y}_{i+1} = \frac{d^2y_i}{dt^2} = a_0 \cdot (y_{i+1} - y_i) - a_2 \cdot \dot{y}_i - a_3 \cdot \ddot{y}_i \tag{19}$$

where the coefficients a_0 - a_5 represent the coefficients of the Newmark beta method, which are calculated according to the following relations:

$$\begin{aligned} a_0 &= \frac{1}{\beta \cdot t^2} & a_3 &= \tfrac{1}{2\cdot\beta} - 1 \\ a_1 &= \frac{\alpha}{\beta \cdot t} & a_4 &= \tfrac{\alpha}{\beta} - 1 \\ a_2 &= \frac{1}{\beta \cdot t} & a_5 &= \tfrac{t}{2} \cdot (\tfrac{\alpha}{\beta} - 2) \end{aligned} \tag{20}$$

The coefficients of the Newmark beta method depend on the coefficients α and β, and on the time step size t. The solution of the motion equations is determined within the time interval $0 \leq t \leq t_N$. This time interval is split into N equal subintervals using the set of nodes t_i, $i = 0...N$. These nodes form the grid on the interval $[0, t_N]$, where the distance between the nodes represents the grid step which can be obtained by:

$$\Delta t = \frac{t_N}{N} \tag{21}$$

The generalized coordinates x_i and y_i depend on the location of the nodes t_i, and they are determined by the following equations [39]:

$$\begin{gathered} t_i = i \cdot \Delta t, \qquad i = 0,..,N \\ x_i = x(t_i) \; and \; y_i = y(t_i) \end{gathered} \tag{22}$$

To obtain the most accurate solutions using the Newmark beta method, the trapezoidal rule is used, that is, the assumption where $\alpha = 1/2$ and $\beta = 1/4$, i.e., when the acceleration varies on average during the time interval [39]. The time step size is inverse to the sampling frequency, and the higher the sampling frequency, the more accurate will be the solutions of the equations using numerical methods.
By substituting equations (16) - (19) into the final equations of the dynamic movement of the rotor (12) - (15), the following relations for the time interval are obtained $t_i, i = 0..n$:

$$\begin{aligned} f_{xA} = {} & a_0 \cdot \left(x_{A(i+1)} - x_{Ai}\right) - a_2 \cdot \dot{x}_{Ai} - a_3 \cdot \ddot{x}_{Ai} \\ & + \frac{J_z \dot{\theta}_z d_A}{J_y d}\left(a_1 \cdot \left(y_{A(i+1)} - y_{Ai}\right) - a_4 \cdot \dot{y}_{Ai} - a_5 \cdot \ddot{y}_{Ai} - \left(a_1 \cdot \left(y_{B(i+1)} - y_{Bi}\right)\right.\right. \\ & \left.\left. - a_4 \cdot \dot{y}_{Bi} - a_5 \cdot \ddot{y}_{Bi}\right)\right) - x_A\left(\frac{2k_s}{m} + \frac{2k_s d_A^2}{J_y}\right) - x_B\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_y}\right) \\ & - i_{xA}\left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_y}\right) - i_{xB}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_y}\right) - e\dot{\theta}_z^2 cos\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \\ & \cdot \frac{\left(J_y - J_z\right) \cdot d_A}{J_y} \cdot sin\,\dot{\theta}_z t = 0 \end{aligned} \tag{23}$$

$$\begin{aligned} f_{xB} = {} & a_0 \cdot \left(x_{B(i+1)} - x_{Bi}\right) - a_2 \cdot \dot{x}_{Bi} - a_3 \cdot \ddot{x}_{Bi} \\ & - \frac{J_z \dot{\theta}_z d_B}{J_y d}\left(a_1 \cdot \left(y_{A(i+1)} - y_{Ai}\right) - a_4 \cdot \dot{y}_{Ai} - a_5 \cdot \ddot{y}_{Ai} - \left(a_1 \cdot \left(y_{B(i+1)} - y_{Bi}\right)\right.\right. \\ & \left.\left. - a_4 \cdot \dot{y}_{Bi} - a_5 \cdot \ddot{y}_{Bi}\right)\right) - x_A\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_y}\right) - x_B\left(\frac{2k_s}{m} + \frac{2k_s d_B^2}{J_y}\right) \\ & - i_{xA}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_y}\right) - i_{xB}\left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_y}\right) - e\dot{\theta}_z^2 cos\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \\ & \cdot \frac{\left(J_y - J_z\right) \cdot d_B}{J_y} \cdot sin\,\dot{\theta}_z t = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} f_{yA} = {} & a_0 \cdot \left(y_{A(i+1)} - y_{Ai}\right) - a_2 \cdot \dot{y}_{Ai} - a_3 \cdot \ddot{y}_{Ai} \\ & + \frac{J_z \dot{\theta}_z d_A}{J_x d}\left(a_1 \cdot \left(x_{B(i+1)} - x_{Bi}\right) - a_4 \cdot \dot{x}_{Bi} - a_5 \cdot \ddot{x}_{Bi} - \left(a_1 \cdot \left(x_{A(i+1)} - x_{Ai}\right)\right.\right. \\ & \left.\left. - a_4 \cdot \dot{x}_{Ai} - a_5 \cdot \ddot{x}_{Ai}\right)\right) - y_A\left(\frac{2k_s}{m} + \frac{2k_s d_A^2}{J_x}\right) - y_B\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_x}\right) \\ & - i_{yA}\left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_x}\right) - i_{yB}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_x}\right) - e\dot{\theta}_z^2 sin\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \\ & \cdot \frac{\left(J_x - J_z\right) \cdot d_A}{J_x} \cdot cos\,\dot{\theta}_z t = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} f_{yB} = {} & a_0 \cdot \left(y_{B(i+1)} - y_{Bi}\right) - a_2 \cdot \dot{y}_{Bi} - a_3 \cdot \ddot{y}_{Bi} \\ & - \frac{J_z \dot{\theta}_z d_B}{J_x d}\left(a_1 \cdot \left(x_{B(i+1)} - x_{Bi}\right) - a_4 \cdot \dot{x}_{Bi} - a_5 \cdot \ddot{x}_{Bi} - \left(a_1 \cdot \left(x_{A(i+1)} - x_{Ai}\right)\right.\right. \\ & \left.\left. - a_4 \cdot \dot{x}_{Ai} - a_5 \cdot \ddot{x}_{Ai}\right)\right) - y_A\left(\frac{2k_s}{m} - \frac{2k_s d_A d_B}{J_x}\right) - y_B\left(\frac{2k_s}{m} + \frac{2k_s d_B^2}{J_x}\right) \\ & - i_{yA}\left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_x}\right) - i_{yB}\left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_x}\right) - e\dot{\theta}_z^2 sin\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \\ & \cdot \frac{\left(J_x - J_z\right) \cdot d_B}{J_x} \cdot cos\,\dot{\theta}_z t = 0 \end{aligned} \tag{26}$$

To solve the system of equations (23) – (26) the Newton-Raphson method for solving the system of differential equations was used. The Jacobian matrix of the system (23) – (26), which consists of partial derivatives, is as follows:

$$J=\begin{bmatrix}\frac{\partial f_{xA}}{\partial x_A} & \frac{\partial f_{xA}}{\partial y_A} & \frac{\partial f_{xA}}{\partial x_B} & \frac{\partial f_{xA}}{\partial y_B}\\ \frac{\partial f_{yA}}{\partial x_A} & \frac{\partial f_{yA}}{\partial y_A} & \frac{\partial f_{yA}}{\partial x_B} & \frac{\partial f_{yA}}{\partial y_B}\\ \frac{\partial f_{xB}}{\partial x_A} & \frac{\partial f_{xB}}{\partial y_A} & \frac{\partial f_{xB}}{\partial x_B} & \frac{\partial f_{xB}}{\partial y_B}\\ \frac{\partial f_{yB}}{\partial x_A} & \frac{\partial f_{yB}}{\partial y_A} & \frac{\partial f_{yB}}{\partial x_B} & \frac{\partial f_{yB}}{\partial y_B}\end{bmatrix} \tag{27}$$

If solving for the time interval t_i, $i = 0\ldots n$, the relation $(x_{A(i+1)}, y_{A(i+1)}, x_{B(i+1)}, y_{B(i+1)}) = (x_{A0}, y_{A0}, x_{B0}, y_{B0})$ represents the first solution of the system of equations (23) – (26), then the following relation applies to the other members of the given sequence in the time interval:

$$\begin{bmatrix}x_{A(i+1),k+1}\\ y_{A(i+1),k+1}\\ x_{B(i+1),k+1}\\ y_{B(i+1),k+1}\end{bmatrix}=\begin{bmatrix}x_{A(i+1),k}\\ y_{A(i+1),k}\\ x_{B(i+1),k}\\ y_{B(i+1),k}\end{bmatrix}-J^{-1}\cdot\begin{bmatrix}f_{xA}\left(x_{A(i+1),k}, y_{A(i+1),k}, x_{B(i+1),k}, y_{B(i+1),k}\right)\\ f_{yA}\left(x_{A(i+1),k}, y_{A(i+1),k}, x_{B(i+1),k}, y_{B(i+1),k}\right)\\ f_{xB}\left(x_{A(i+1),k}, y_{A(i+1),k}, x_{B(i+1),k}, y_{B(i+1),k}\right)\\ f_{yB}\left(x_{A(i+1),k}, y_{A(i+1),k}, x_{B(i+1),k}, y_{B(i+1),k}\right)\end{bmatrix} \tag{28}$$

Centrifugal forces which tend to throw the rotor out of balance act on a rigid rotor with an unbalance in its structure. For a rotor supported on two radial AMBs to remain in a state of equilibrium during exploitation, it is necessary to act on it by attractive magnetic forces. That is why the movement of the rotor must be continuously monitored, and attractive magnetic forces must be generated to match the strength of the centrifugal forces. The attractive magnetic forces solely depend on the strength of the control currents, by means of which, in addition to the stable currents that maintain the rotor in a hovering position, perform the positioning of the rotor via pairs of electromagnets depending on the direction of displacement.

The requirement that is required to obtain the values of the control currents, which must be brought to the electromagnets to counteract the centrifugal forces due to the existence of unbalance, is the following:

- for control currents in the x axis direction for magnetic bearings A and B:

$$\begin{gathered} i_{xA}\cdot v_{1iA}+i_{xB}\cdot v_{1iB}+w_1=0\\ i_{xA}\cdot v_{2iA}+i_{xB}\cdot v_{2iB}+w_2=0 \end{gathered} \tag{29}$$

- for control currents in the x axis direction for magnetic bearings A and B:

$$\begin{gathered} i_{yA}\cdot v_{3iA}+i_{yB}\cdot v_{3iB}+w_3=0\\ i_{yA}\cdot v_{4iA}+i_{yB}\cdot v_{4iB}+w_4=0 \end{gathered} \tag{30}$$

The requirement for obtaining control currents represents a system of four equations with four unknowns (control currents in the x and y directions for magnetic bearings A and B). By solving the mentioned system of equations, the values of the control currents are obtained:

- for control currents in the x-axis direction for magnetic bearings A and B:

$$\begin{gathered} i_{xA}=\frac{w_1\cdot\frac{v_{2iB}}{v_{1iB}}-w_2}{v_{2iA}-\frac{v_{1iA}\cdot v_{2iB}}{v_{1iB}}}\\ i_{xB}=-\frac{w_1+i_{xA}\cdot v_{1iA}}{v_{1iB}} \end{gathered} \tag{31}$$

- for control currents in the x-axis direction for magnetic bearings A and B:

$$
\begin{aligned}
i_{yA} &= \frac{w_3 \cdot \frac{v_{4iB}}{v_{3iB}} - w_4}{v_{4iA} - \frac{v_{3iA} \cdot v_{4iB}}{v_{3iB}}} \\
i_{yB} &= -\frac{w_3 + i_{yA} \cdot v_{3iA}}{v_{3iB}}
\end{aligned}
\tag{32}
$$

Where:

$$
\begin{aligned}
w_1 &= e\dot{\theta}_z^2 cos\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \cdot \frac{\left(J_y - J_z\right) \cdot d_A}{J_y} \cdot sin\,\dot{\theta}_z t \\
w_2 &= e\dot{\theta}_z^2 cos\dot{\theta}_z t + \chi \cdot \dot{\theta}_z^2 \cdot \frac{\left(J_y - J_z\right) \cdot d_B}{J_y} \cdot sin\,\dot{\theta}_z t \\
w_3 &= e\dot{\theta}_z^2 sin\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \cdot \frac{\left(J_x - J_z\right) \cdot d_A}{J_x} \cdot cos\,\dot{\theta}_z t \\
w_4 &= e\dot{\theta}_z^2 sin\dot{\theta}_z t - \chi \cdot \dot{\theta}_z^2 \cdot \frac{\left(J_x - J_z\right) \cdot d_B}{J_x} \cdot cos\,\dot{\theta}_z t
\end{aligned}
\tag{33}
$$

$$
v_{1iA} = \left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_y}\right); v_{1iB} = v_{2iA} = \left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_y}\right); v_{2iB} = \left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_y}\right)
$$

$$
v_{3iA} = \left(\frac{k_i}{m} + \frac{k_i d_A^2}{J_x}\right); v_{3iB} = v_{4iA} = \left(\frac{k_i}{m} - \frac{k_i d_A d_B}{J_x}\right); v_{4iB} = \left(\frac{k_i}{m} + \frac{k_i d_B^2}{J_x}\right)
$$

A program was created in Matlab for solving the system of equations and managing the position of the centre of mass of the rotor in relation to the axis of rotation, based on the mathematical model developed above. The developed program simulates the behaviour of a real rotor system with active magnetic bearings during exploitation. As a result, a simulation of the operation of the dynamic system is obtained, at the moment of unbalance and after the corrective action of the system of active magnetic bearings.

The parameters of the rigid rotor system supported on two radial active magnetic bearings required to obtain the simulation results are shown in Table 1.

At the initial moment of time $t = 0$ for the rigid rotor system supported in two radial AMBs A and B, the following assumptions were made:

- The air gap is fixed (s_0), and is the same for both active magnetic bearings, as is the excitation stable current (i_0) that maintains the rotor in a hovering position, meaning that the rotor is in the reference position, and there is no deviation at the initial moment of time. In addition, it was assumed that the velocities and accelerations of the system in the direction of the coordinate axes at the initial moment are equal to zero.
- It is assumed that the eccentricity and angular eccentricity of the centre of mass are known, so that the centrifugal force acting on the system due to static and dynamic unbalance is known.

After entering the parameters and setting the initial conditions for the rigid rotor model, it is necessary to choose an adequate sampling frequency and time step size. The simulation was performed for a sampling frequency of f_s = 1000 [Hz].

Table 1. Parameter values of the rigid rotor system supported on magnetic bearings.

Parameter	Magnitude	Unit
Rotor mass – m	2000	[g]
Rotor radius – r	25	[mm]
Rotor length – h	500	[mm]
Polar moment of inertia – J_z	625·103	[gmm^2]
Transverse moment of inertia (x – osa) – J_x	4.1979·107	[gmm^2]
Transverse moment of inertia (y – osa) – J_y	4.1979·107	[gmm^2]
Rotor angular velocity – $\dot{\boldsymbol{\theta}}_{\boldsymbol{z}}$	21π	[1/s]
Rotor rpm – n_r	630	[1/min]
Distance from bearing A to the coordinate origin – d_A	260	[mm]
Distance from bearing B to the coordinate origin – d_B	200	[mm]
Total distance between two bearings – d	460	[mm]
Magnetic permeability – μ	$12.5663706 \cdot 10^{-7}$	[N/A^2]
Cross section of the magnet core – A_0	625	[mm^2]
Number of electromagnet windings – N	220	/
Inclination angle of the magnetic pole of the core – γ	22.5	[°]
Exciting stable current ¬ i_0	1.75	[A]
Initial air gap – s_0	1	[mm]
Factor force – current – k_i	61.4594	[N/A]
Factor force – displacement - k_s	-107.554	[N/mm]
Eccentricity of the center of mass – e	1	[mm]
Angular eccentricity of the center of mass – χ	0.05	[rad]

4.1 AMB system control model

The mathematical modelling so far aimed to develop methods for the calculation of control parameters (control currents) for the control model that would be used to automatically balance the rigid rotor in the magnetic bearing. This subsection presents the basic concept for automatic balancing of a rigid rotor in an AMB. As a typical mechatronic system, the system of AMBs uses a control unit that defines the control signals based on which the electromagnetic actuators control the rigid rotor by analysing data supplied by the displacement sensor. The control unit, sensors, and electromagnetic actuators constitute the control system of the AMB (Figure 4). To achieve the positioning of the rigid rotor around the equilibrium position, it is necessary to apply appropriate control algorithms. In AMB systems, PID controllers are widely used, because they are characterized by good stability, high precision, and very suitable modulation of parameters.

The AMB system is based on MIMO (Multiple Input Multiple Output) control architecture, but with the use of PID control models, the system can be easily observed and modelled through a suitable number of SISO (Single Input Single Output) control systems. In this way, PID controllers position the rigid rotor around the reference position by calculating the error between the measured and desired position values for each position.

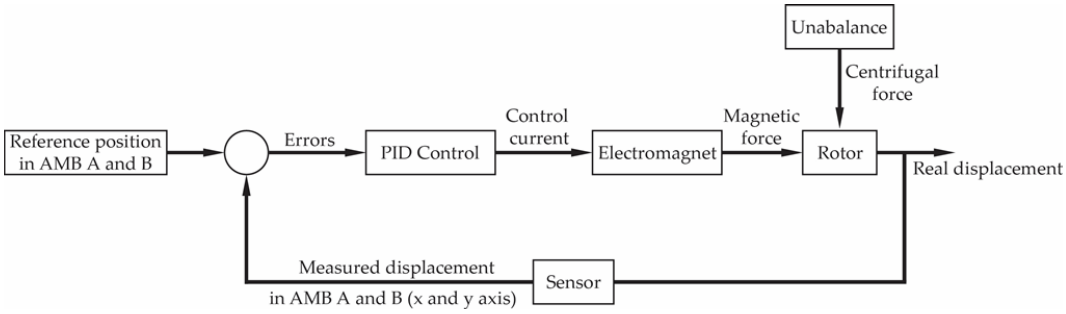


Figure 4. Block diagram of the AMB system control model.

5 Analysis and discussion of simulation results

Based on the data shown in Table 1, a simulation of operation was performed in a situation where the rotor is exposed to centrifugal force due to unbalance, (e.g., parts of the rotor falling off), and is not actively controlled, as well as in the situation, in which the rigid rotor is actively controlled and thereby neutralizes the centrifugal force that results from unbalance. Due to the impetuous occurrence of unbalance, a centrifugal force is generated that pushes the rotor out of its equilibrium position.

The occurrence of unbalance in the system causes an increase in the amplitude of vibrations in magnetic bearings A and B, which is shown in the left half of the graphics in Figure 5.

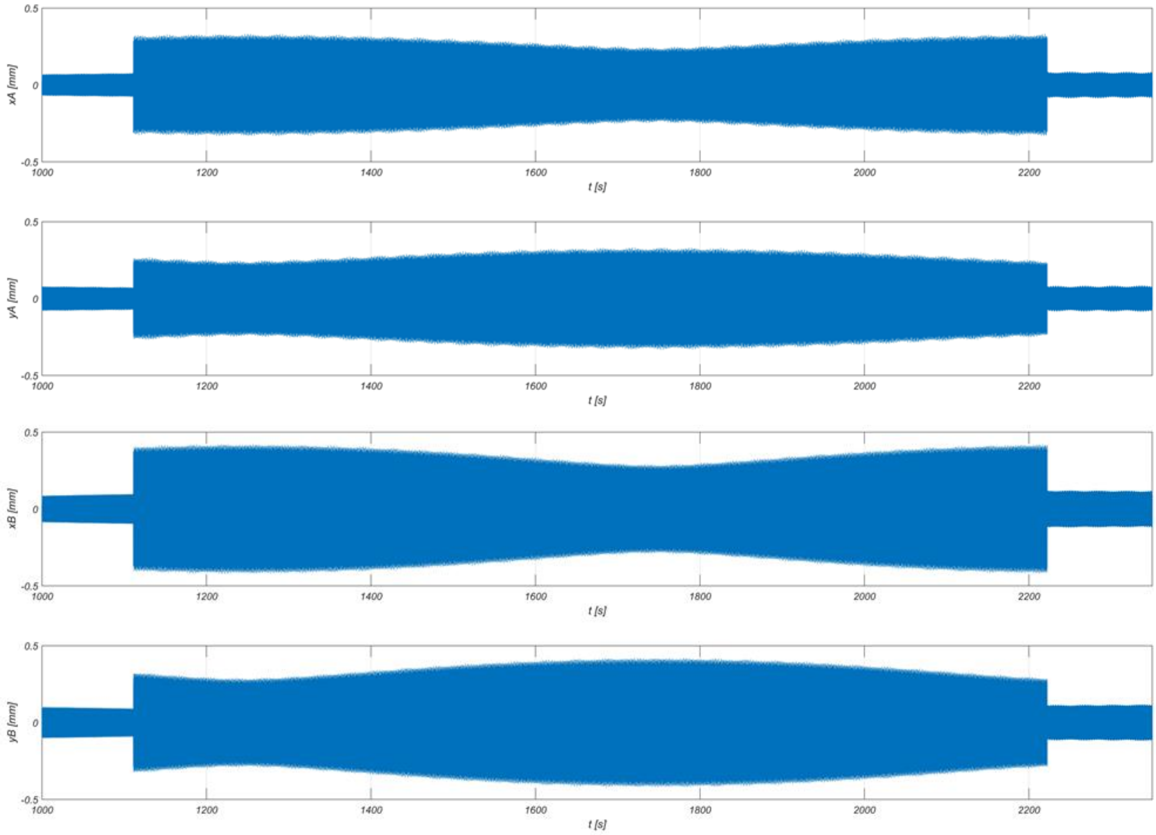


Figure 5. Graphic representation of rotor displacement along the x and y axes for AMBs A and B when during exploitation comes to the active control of the rigid rotor.

By analysis of the left half of the graphics in Figure 5, it can be observed that the rigid rotor during exploitation deviates from the reference position due to the presence of centrifugal force. That displacement occurs within the limits of the air gap that exists between the rigid rotor and the magnetic bearings. Even though the displacement is contained within the limits of the air gap, the increase in the vibration amplitudes of the displacement is not good, and must be returned to an adequate level, by application of active attractive forces to the rigid rotor to return it to its normal state. Because of active control currents and their resulting magnetic forces, the rotor calms down and the vibration amplitudes become significantly smaller (right half of the graph in Figure 5). In that way, active magnetic forces return the rotor to a normal state and enable the smooth continuation of the exploitation of the rotor. This is best shown by the 3D graphics in Figure 6.

The vibration amplitude peak for magnetic bearing A in the case when there is no active control of the rigid rotor in the system is $x_A = 0.2968$ mm, while in the situation when the rotor is actively controlled it is $x_A = 0.051$ mm (Figure 5). For magnetic bearing B without active rotor control, the vibration amplitude reaches its peak at a value of $x_B = 0.3695$ mm, while in the case where there is active control in the system, the amplitude value is $x_B = 0.0683$ mm (Figure 5). By comparing these values, it can be determined that by applying active control through active magnetic bearings there is a reduction of vibrations intensity by approximately 6 times.

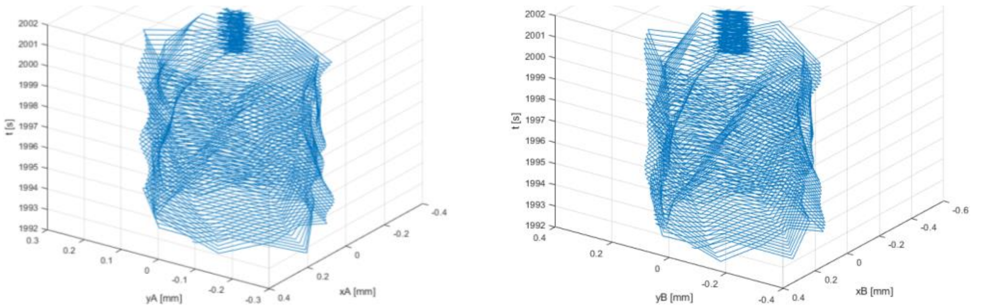
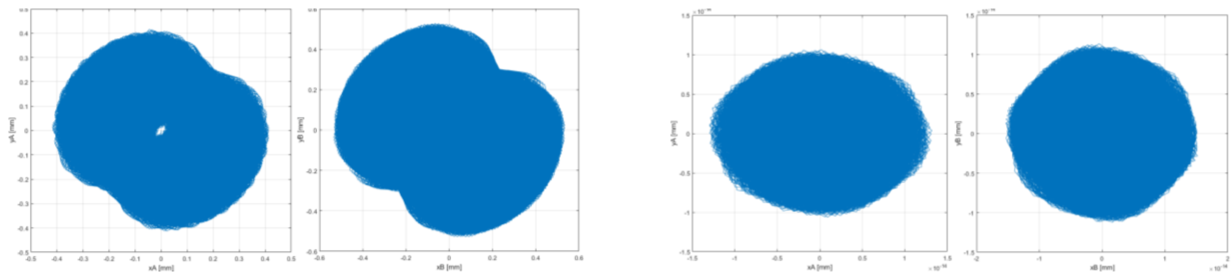


Figure 6. Graphic representation of rotor displacement along the x and y axes for active magnetic bearings A and B when during exploitation comes to the active control of the rigid rotor.

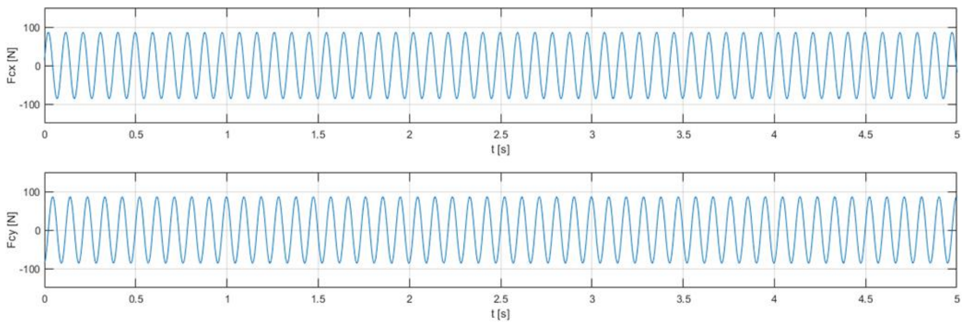
In the case when there is no active control in the system, the rotor deviates from the equilibrium position during exploitation and due to the unbalanced rotates around the new axis of rotation (that is, the inertia axis of the rotor) (Figure 7a).
In the case where there is active control the influence of the unbalance is annulled so that the inertial axis closely coincides with the actual axis of rotation. This is manifested by a significant reduction in the orbit of vibrations, which can be seen in Figure 7 b.



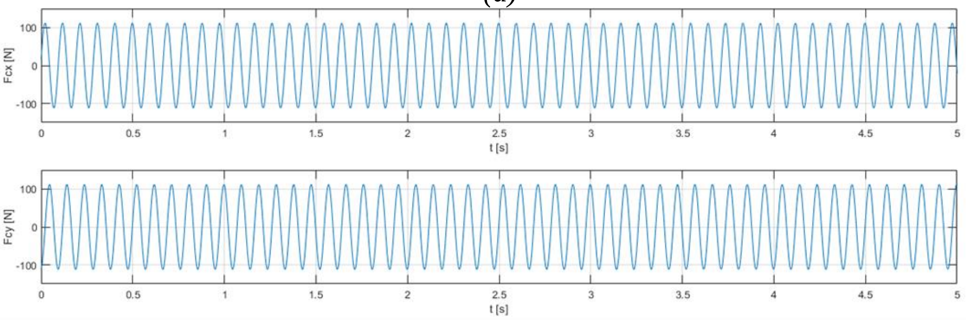
(a) *(b)*

Figure 7. Graphic representation of the orbit - positions of the rotor axis in active magnetic bearings A and B; (a) without active control; (b) with active control.

Figure 8 shows the graphs of the centrifugal force in magnetic bearings A and B. By looking at the graphs, it can be noticed the difference in the strength of the centrifugal force, i.e., that the centrifugal force has a greater influence in the plane of the magnetic bearing B.
As already stated, the reason for this difference is the unequal distance of the bearings from the center of mass of the rigid rotor.



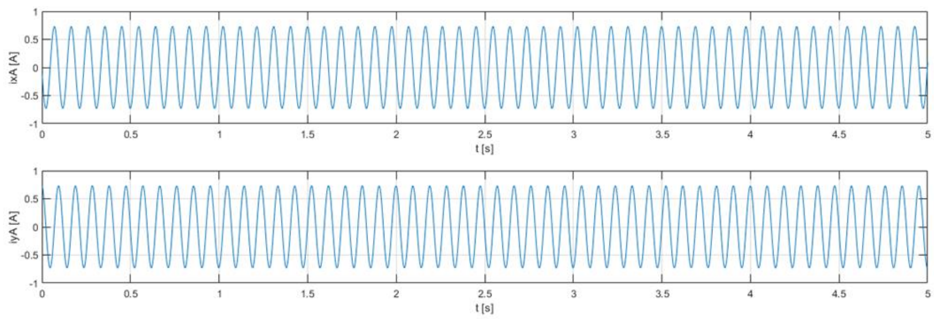
(a)



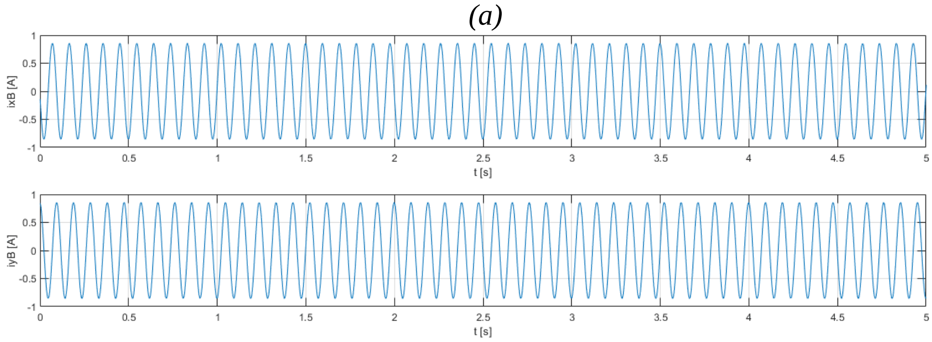
(b)

Figure 8. Graphic representation of the centrifugal force caused by the unbalance in the x and y planes of the rigid rotor structure: (a) in the magnetic bearing A; (b) in magnetic bearing B.

Active magnetic bearings act on the rotor with attractive magnetic forces to cancel the influence of centrifugal forces. The strength of the attractive magnetic forces depends solely on the amount of control current supplied to the electromagnets. Figure 9. shows graphs of control currents for magnetic bearings A and B. Analysis of the graphs for centrifugal force and control currents shows that a higher value of control current is needed to cancel out a larger centrifugal force.



(a)



(b)

Figure 9. Graphical representation of the control currents required to position the rotor in the equilibrium position: (a) in the magnetic bearing A; (b) in magnetic bearing B.

6 Conclusion

Modern mechatronic systems whose work is based on the rotary movement of working elements during exploitation can be greatly influenced by external factors that tend to throw the system out of balance. Mass unbalance in the structure of the rigid rotor has been identified as one of the main sources that leads the system to a chaotic state. AMBs are imposed as a realistic and possible solution to reduce the amplitude of vibrations caused by the occurrence of unbalance. This solution for the supporting of rotating parts of the mechatronic system enables constant monitoring of the vibration level and continuous positioning of the rotor. Continuous positioning means that the rigid rotor is maintained in an equilibrium position regardless of the disruptive forces that occur in the system during exploitation. Since AMBs represent a classic mechatronic system, the research for the possibilities of application of these bearings to reduce the impact of vibrations caused by unbalance required a special approach and the application of theoretical, analytical, numerical, and programming research. Based on the developed mathematical model that describes the dynamic behaviour of a rigid rotor supported by two radial AMBs, a program was created that simulates the operation of such a system.

The case in which an unbalance appears in the rotor structure was considered when considering the research problem. The following conclusions may be made based on the results obtained for the general case:

- By applying AMBs, the negative influence of centrifugal force can be successfully and adequately annulled by means of attractive magnetic forces. In this way, the increased level of vibrations caused by the impetuous and sudden appearance of unbalance is largely reduced. This leads to the successful return of the rigid rotor to its normal operating state, which enables the uninterrupted operation of the machine system.
- AMBs provide certain advantages when supporting rotating parts compared to conventional bearings, so their use allows for the possibility of active vibration control, which significantly improves the working performance of rotating machines.

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