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Eigenvalue analysis for plain-woven fabric structure using shell element and one smoothing cell in the smoothed finite element method

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Abstract. An efficient four-node quadrilateral (Q4) shell element based on the first-order shear deformation theory of plate (FSDT) and the strain smoothing technique in finite elements (referred as SFEM) was proposed for eigenvalue analysis of plain-woven fabric structure. A one smoothing domain (or cell) integration scheme in SFEM was proposed to evaluate the nodal train fields of Q4 shell elements. The numerical result of eigenvalue analysis, which was in the case of free vibration analysis, approximated to that one implemented in the finite element method (FEM) but gave a higher efficiency in computation in terms of central processing unit (CPU) time and numerical implementation.

1. Introduction and kinematics of shells

Finite element methods (FEM) have been developed and widely used for the analysis of eigenvalues in the context of buckling and natural vibration responses of plate/shell structures. The different stages of development of plate/shell finite elements can be found in the works of Yang and his coworkers. [1] Eigenvalue analysis using thin plate/shell finite element models in textile engineering can be referred to [2, 3]. The strain smoothing technique recently proposed by Liu and his coworkers [4], referred as SFEM, which improved the accuracy and convergence rate of the existing conventional finite element finite element method (FEM) of elastic solid mechanics problems [5-8]. This paper presents an efficient Q4 flat shell element based on the FSDT [9-11] and the cell-based smoothed finite element in SFEM [12, 13] for the eigenvalue analysis of thin and moderately thick plain-woven fabric structure. The summarized formulation of Q4 shell plate/shell elements and the smoothing cell integration scheme are respectively presented in the following sections. The numerical results indicated a better efficiency of numerical computation with accuracy results compared to the same Q4 shell elements without integrating cell-based models.

1.1. Kinematics of shells

Based on the FSDT, the displacement components $\boldsymbol{u} = [u, v, w]$ at a local coordinate system (x, y, z) within problem domain Ω bounded by Γ are defined as follows:

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$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\theta_x(x,y) \\
v(x,y,z) &= v_0(x,y) - z\theta_y(x,y) \\
w(x,y,z) &= w_0(x,y)
\end{aligned} \tag{1}
$$

where u_0, v_0 and w_0 be the translation displacements, θ_x and θ_y be the rotations about the yz and xz planes in the Cartesian coordinate system.

In terms of the mid-plane deformations, the strain vector $\boldsymbol{\varepsilon} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_{xy} \quad \varepsilon_{xz} \quad \varepsilon_{yz}\}^T$ can be written using Eq. (1), which gives:

$$
\boldsymbol{\varepsilon} = \begin{Bmatrix} \boldsymbol{\varepsilon}^m \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\boldsymbol{\varepsilon}^b \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \boldsymbol{\varepsilon}^s \end{Bmatrix} \tag{2}
$$

where superscripts m, b and s are respectively the membrane, bending and the shear terms. The generalized strain vector $\hat{\boldsymbol{\varepsilon}}$ can be written as

$$
\hat{\boldsymbol{\varepsilon}} = \begin{Bmatrix} \boldsymbol{\varepsilon}^m \\ \boldsymbol{\varepsilon}^b \\ \boldsymbol{\varepsilon}^s \end{Bmatrix}, \tag{3}
$$

in which

$$
\boldsymbol{\varepsilon}^m = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial x} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \boldsymbol{\varepsilon}^b = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y} \end{Bmatrix}, \boldsymbol{\varepsilon}^s = \begin{Bmatrix} \dfrac{\partial w_0}{\partial x} - \theta_x \\ \dfrac{\partial w_0}{\partial y} - \theta_y \end{Bmatrix}. \tag{4}
$$

The constitutive equations can be expressed as

$$
\hat{\boldsymbol{\sigma}} = \widehat{\boldsymbol{D}}\hat{\boldsymbol{\varepsilon}}, \tag{5}
$$

where

$$
\hat{\boldsymbol{\sigma}} = \begin{Bmatrix} \widehat{\boldsymbol{N}} \\ \widehat{\boldsymbol{M}} \\ \widehat{\boldsymbol{Q}} \end{Bmatrix}, \widehat{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{D}^m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{D}^b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{D}^s \end{bmatrix}, \tag{6}
$$

in which $\widehat{\boldsymbol{N}} = \{N_x \quad N_y \quad N_{xy}\}^T$ is the vector of membrane force, $\widehat{\boldsymbol{M}} = \{M_x \quad M_y \quad M_{xy}\}^T$ is the vector of bending moment, $\widehat{\boldsymbol{Q}} = \{Q_x \quad Q_y\}^T$ is the vector of transverse shear force and $\boldsymbol{D}$ are stiffness constitutive coefficients matrix, which defined as

$$
\begin{aligned}
\boldsymbol{D}^m &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \dfrac{E_{warp}}{1 - v_{warp}v_{weft}} & \dfrac{v_{weft}E_{warp}}{1 - v_{weft}v_{warp}} & 0 \\ \dfrac{v_{warp}E_{weft}}{1 - v_{warp}v_{weft}} & \dfrac{E_2}{1 - v_{weft}v_{warp}} & 0 \\ 0 & 0 & G \end{bmatrix} dz, \\
\boldsymbol{D}^b &= \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \begin{bmatrix} B_{warp} & 0 & 0 \\ 0 & B_{weft} & 0 \\ 0 & 0 & H \end{bmatrix} dz, \boldsymbol{D}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{5}{6} G \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} dz.
\end{aligned} \tag{7}
$$

where h is the thickness of the plate or shell, G is shear modulus, E is Young's modulus and ν is Poisson's ratio corresponding to the warp and weft direction of yarns, B and H stand for flexural moduli and torsional rigidity.

The problem domain Ω is discretized into a set of four-node quadrilateral flat shell elements Ω^e with boundary Γ^e, the generalized mid-plane displacement vector $\hat{\boldsymbol{u}}$ can be then defined as

$$\hat{\boldsymbol{u}} = \sum_{I=1}^{4} \begin{bmatrix} N_I(\mathbf{x}) & 0 & 0 & 0 & 0 \\ 0 & N_I(\mathbf{x}) & 0 & 0 & 0 \\ 0 & 0 & N_I(\mathbf{x}) & 0 & 0 \\ 0 & 0 & 0 & N_I(\mathbf{x}) & 0 \\ 0 & 0 & 0 & 0 & N_I(\mathbf{x}) \end{bmatrix} \begin{Bmatrix} u_I \\ v_I \\ w_I \\ \theta_{xI} \\ \theta_{yI} \end{Bmatrix} = \sum_{I=1}^{4} \boldsymbol{N}_I(\mathbf{x})\boldsymbol{d}_I, \tag{8}$$

where $N_I(\mathbf{x})$ is the basis function associated to node I of a four-node quadrilateral shell element.

The eigenvalue equation for free vibration analysis can be expressed through the direct application of variational principles and using Eq. (1 to 9), which is given as

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\boldsymbol{d} = 0, \tag{9}$$

where ω is the natural frequency, $\boldsymbol{K}$ and $\boldsymbol{M}$ are the global stiffness matrix and the global mass matrix, respectively, which given in [12, 14].

The shear locking phenomenon may appear due to incorrect transverse forces under bending, or in the case of the thickness of the plate tends to zero. To overcome the shear locking phenomena, the approximation of the shear strain $\boldsymbol{\varepsilon}^s$ in Eq. (4) can be formulated with MITC4 as in [9].

1.2. A one smoothing cell integration scheme in SFEM

This technique avoids evaluating derivatives of mesh-free shape functions at nodes and therefore eliminates defective modes. The major techniques used in smoothed finite element methods appear summarized in references [13, 15, 16].

The smoothing operation performed over an element Ω^e bounded by Γ^e, this can be referred as smoothing domain (or cell) Ω_k^s, which is expressed as

$$\bar{\nabla}\boldsymbol{u}(x_k) = \int_{\Omega_i^s} \nabla\boldsymbol{u}(x)\Phi(x - x_k)d\Omega, \tag{10}$$

which has to satisfy the basic conditions:

$$\Phi \geq 0 \; and \int_{\Omega_i^s} \Phi d\Omega = 1, \Phi(x - x_k) = \begin{cases} \frac{1}{A_k^s}, & x \in \Omega_k^s \\ 0, & x \notin \Omega_k^s \end{cases}, \tag{11}$$

where $\nabla\boldsymbol{u}(\mathbf{x})$ is equivalently $\boldsymbol{\varepsilon}(\mathbf{x})$, and Φ is a smoothing or weight function in Ω_k^s, $A_k = \int_{\Omega_k^s} d\Omega$ is the area of smoothing domain.

The strain in smoothing domain Ω_k^s can be further assumed to be a constant and equals $\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k)$, which gives:

$$\bar{\boldsymbol{\varepsilon}}_k = \bar{\boldsymbol{\varepsilon}}_k(x) = \bar{\boldsymbol{\varepsilon}}(x_k) = \frac{1}{A_k^s} \int_{\Omega_k^s} \boldsymbol{\varepsilon}(x)d\Omega. \tag{12}$$

By substituting smoothing function Φ into Eq. (1), the smoothed gradient of displacement can be defined as

$$\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k) = \frac{1}{A_k^s} \int_{\Gamma_k^s} \boldsymbol{n}(\mathbf{x}).\hat{\boldsymbol{u}}(\mathbf{x})d\Gamma, \tag{13}$$

where $\boldsymbol{n}(\mathrm{x})$ is the outward unit normal matrix containing the components of the outward unit normal vector to the boundary Γ_k^s.

Eq. (10 to 13) can apply to evaluate the membrane and bending trains of a Q4 shell element being formulated in the previous section using the following shape functions:

$$N_1 = (1,0,0,0), N_2 = (0,1,0,0), N_3 = (0,0,1,0), N_4 = (0,0,0,1). \tag{14}$$

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2. Numerical implementation and results

Mechanical and physical parameters of a plain-woven fabric sample were measured with Kawabata evaluation system for fabrics (KES-FB) [17, 18] and derived as: elastic modulus [gf/cm], $E_1 = 3823.7993$, $E_2 = 14092.4464$ and $E_{12} = 6896.5517$, Poisson's ratio $v_1 = 0.0211$ and $v_2 = 0.0778$, bending rigidity $[gf.cm^2/cm]$ of $B_1 = 0.1237$, $B_2 = 0.1333$ and $B_{12} = 0.0880$, transverse shear modulus $[gf.cm^2]$ of $G = 217.3100$.

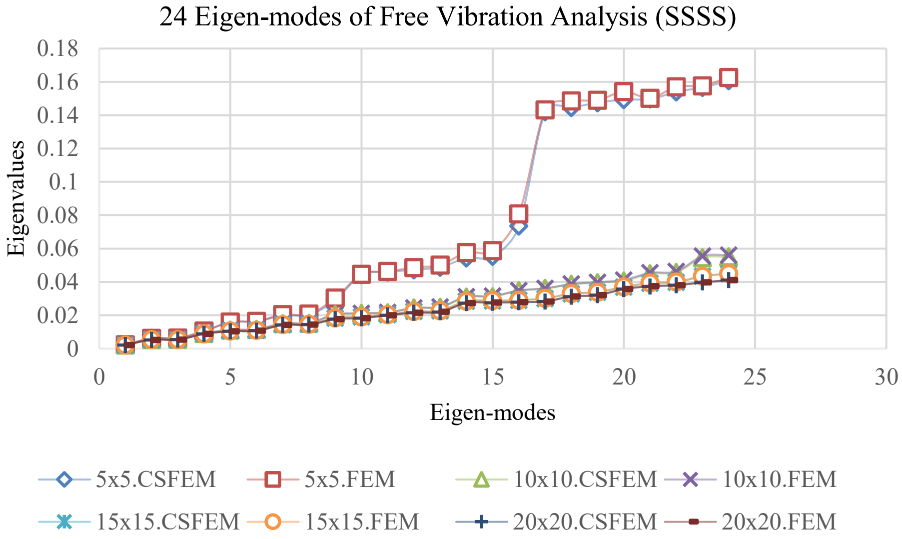

Figure 1. Eigenvalues of free vibration analysis of a SSSS plain-woven fabric sheet with different mesh density subjected natural frequency using 1 smoothing cell

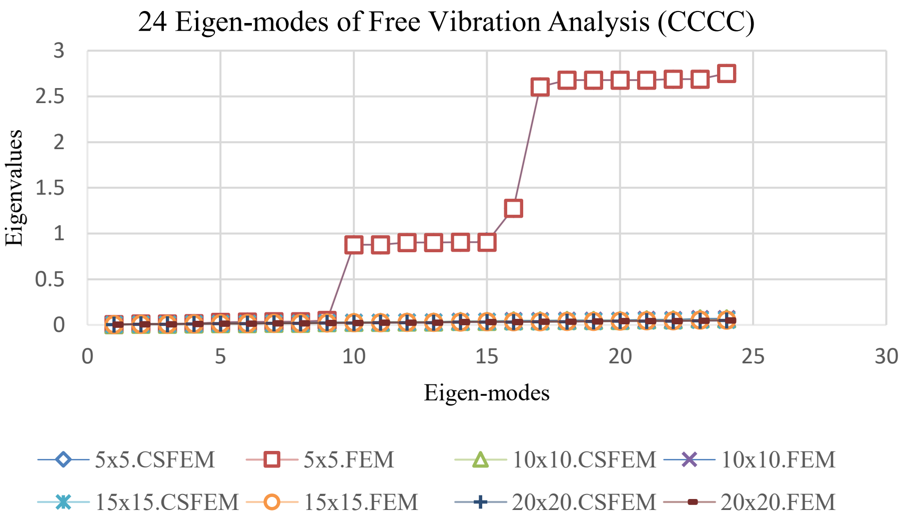

Figure 2. Eigenvalues of free vibration analysis of a CCCC plain-woven fabric sheet with different mesh density subjected natural frequency using 1 smoothing cell

The span-to-thickness ratio l/h of a square woven fabric sample was taken to be 23.5849 in the numerical example. Two types of boundary conditions, simply supported (S) and clamped (C) edges applied for meshes including 5x5, 10x10, 15x15 and 20x20. Both the Q4 shell elements with and without being integrated smoothing cells were programmed.

Figures 1 and 2 indicated that the eigenvalues of 24 eigen-modes extracted from free vibration analysis computed by CSFEM model is approximate and coincided with that one of FEM on the same boundary conditions and mesh configurations. The linear shape functions in SFEM model are constants as presented in Eq. 14. However, the corresponding shape functions of a Q4 element in FEM are bilinear shape functions interpolated in natural coordinates (ξ, η, ζ), that are needed to be transformed into Cartesian coordinates (x, y, z) to evaluate the strain gradient matrices. Thus, the strain smoothing technique reduces the numerical implementation and computation time in terms of the central processing unit time.

3. Conclusions

The one smoothing domain integration scheme in SFEM for a Q4 element, which based on the FSDT, gave an effective computation in terms of CPU time for the eigenvalue analysis, e.g. in the case of vibration behavior of plain-woven fabric structure. One smoothing cell for membrane and bending strain energy integration achieved accurate results and also reduced tasks in numerical implementation compared with FEM.

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