Performance Evaluation of Portfolio Stocks Selected with the EU–EV Risk Model

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Abstract. In this paper, the performance of portfolios consisting of stocks selected with the recently proposed expected utility, entropy and variance (EU–EV) risk model is analysed. The portfolios were constructed using data of the PSI 20 index, from January 2019 to December 2020, by reducing the number of stock components to the half with the EU–EV risk model. The efficiency of these portfolios in terms of the mean–variance model was shown to be approximately equal to the efficiency of portfolios obtained from the whole set of stocks. The aim is to evaluate the performance of the constructed portfolios, by comparing their in-sample and out-of-sample results with those of the benchmark. For that purpose, cumulative returns in the in-sample period from January 2019 to December 2020 and in the out-of-sample period from January 2021 to December 2022, considering both an one-year and a two-year time horizon, as well as different performance metrics, such as Sharpe ratio, Sortino ratio, Beta and Alpha, are analysed. The results reveal that the portfolios constructed with the EU–EV risk model outperform the benchmark portfolio in the given periods, where a better performance was obtained in the one-year out-of-sample period. These results suggest that the strategy of constructing portfolios using the best ranked stocks according to the EU–EV risk model can be useful for short-term investment objectives.

Keywords: EU–EV risk model · Stock selection · Portfolio performance evaluation.

1 Introduction

Classifying stock risks and the selection of efficient stocks is an important task for the construction of portfolios. The mean–variance model was proposed by Markowitz [11] to assess and construct portfolios by minimizing risk, expressed by variance, and maximizing the expected return. Several other stock selection models were proposed in the literature, where also entropy was used for measuring risk and combined with other measures, for example the mean–variance-entropy model [8], the expected utility and entropy (EU–E) model [16], the fuzzy cross-entropy model [14], or the expected utility, entropy and variance model (EU–EV) model [2]. Also machine learning methodologies were developed for stock selection and portfolio optimization. Huang [7] used support vector regression together with genetic algorithms, and Paiva et al. [12] applied the support vector machine method to compose optimal portfolios. In [5], extreme gradient boosting were used for preselecting stocks with higher potential returns before employing the mean–variance model. Other portfolio construction strategies depend on factor investing criteria (e.g. value, profitability, momentum) [1] or on environmental, social and governance investment criteria, see e.g. [15]. In several of the research works the proposed methodologies lead in certain applications to portfolios that can outperform the benchmark portfolios.

Recently, the expected utility, entropy and variance model (EU–EV model), developed in [3] and in [4], was applied to the selection of stocks for portfolio construction [2]. In the EU–EV risk model, entropy and variance are used as uncertainty risk factors, that are combined with expected utility, as preference factor, using a trade-off parameter. The model was applied in [2] to the PSI 20 index to form subsets with half the number of stocks with lower EU–EV risk. Using the mean–variance model, the efficiencies of the subsets' portfolios were compared with the efficiency of the whole stock set. The results revealed that the risk model selects the relevant stocks for an optimal portfolio construction.

The aim of the present work is now to evaluate the performance of portfolios constructed with the EU–EV risk model by analysing in-sample and also out-of-sample results of different performance indicators and comparing these results with those obtained with the benchmark portfolio, in order to further test the reasonability and adequacy of the EU–EV risk model for stock selection. In this study cumulative returns and perfomance metrics such as Sharpe ratio, Sortino ratio, Beta and Alpha were considered.

This paper is organized as follows. In section 2, the methodology of selecting stocks with the EU–EV risk model is explained and the application to data of the PSI 20 index in order to obtain sets with the best ranked stocks for the portfolio construction is presented. Section 3 deals with the performance evaluation of the portfolios, where cumulative returns, Sharpe and Sortino ratios, Beta and Alpha values of the portfolios are compared with those of the benchmark. Section 4 contains the conclusions of this work.

2 Stock selection using the EU–EV risk model

2.1 EU–EV risk model

The EU–EV model for classifying stock risks is defined as follows. Consider a set of stocks $S = \{S_1, \ldots, S_I\}$ and the action space $A = \{a_1, \ldots, a_I\}$, where $a_i = (x_{i1}, p_{i1}; x_{i2}, p_{i2}; \ldots; x_{iN}, p_{iN}) \in A$ is the action of selecting stock S_i, $i = 1, \ldots, I$, yielding the frequency distribution of stock returns, where x_{in} are the outcomes occurring with probabilities p_{in}, $n = 1, \ldots, N$, that are represented by the discrete random variable X_i. The EU–EV risk for the action a_i is defined by (see [2])

$$R(a_i) = \frac{\lambda}{2}\left[H(X_i) + \frac{\mathrm{Var}[X_i]}{\max\limits_{a_i\in A}\{\mathrm{Var}[X_i]\}}\right] - (1-\lambda)\frac{\mathbb{E}[u(X_i)]}{\max\limits_{a_i\in A}\{|\mathbb{E}[u(X_i)]|\}}, \qquad (1)$$

where $0 \le \lambda \le 1$, $u(\cdot)$ is the utility function and $H(X_i) = -\sum_{n=1}^{N} p_{in} \ln p_{in}$ is the entropy. If $\lambda = 0$, then the risk measure depends only on the expected utility and if $\lambda = 1$ the risk measure uses only the uncertainty factors entropy and variance to assess risk. For $\lambda \in (0, 1)$, the effect of the expected utility on the risk measure is bigger if $\lambda < 0.5$, for $\lambda > 0.5$ the risk measure is more influenced by the uncertainty than by the expected utility and if $\lambda = 0.5$, it is equally influenced by both factors. The stocks are ranked according to the EU–EV risk, where given two stocks S_{i_1} and S_{i_2}, $i_1, i_2 \in \{1, \ldots, I\}$, if $R(a_{i_1}) < R(a_{i_2})$, then the optimal stock is S_{i_1}.

2.2 Data and portfolio formation

The PSI 20 index consists, from January 2019 to December 2020, of 18 component stocks of companies denoted by $S = \{S_1, \ldots, S_{18}\}$ (see [2] for more details). These stocks were classified, using the daily returns' frequency distributions, according to the EU–EV risk (1) with utility function

$$u(x) = \begin{cases} \ln(1+x), \, x \ge 0, \\ -\ln(1-x), \, x < 0. \end{cases}$$

The daily returns were calculated from the daily closing prices, collected from Yahoo Finance. The best 9 stocks with lower risk were selected for different ranges of λ to construct portfolios. The following five stock subsets were obtained:

$$\begin{aligned}
Q_1 &= \{S_1, S_3, S_5, S_6, S_8, S_9, S_{11}, S_{16}, S_{18}\}, && \lambda \in [0, 0.1260),\\
Q_2 &= \{S_1, S_3, S_4, S_5, S_6, S_9, S_{11}, S_{16}, S_{18}\}, && \lambda \in [0.1260, 0.4685),\\
Q_3 &= \{S_1, S_3, S_5, S_6, S_9, S_{11}, S_{16}, S_{17}, S_{18}\}, && \lambda \in [0.4685, 0.5311),\\
Q_4 &= \{S_3, S_5, S_6, S_9, S_{11}, S_{13}, S_{16}, S_{17}, S_{18}\}, && \lambda \in [0.5311, 0.7771),\\
Q_5 &= \{S_3, S_5, S_6, S_9, S_{12}, S_{13}, S_{16}, S_{17}, S_{18}\}, && \lambda \in [0.7771, 1].
\end{aligned}$$

The mean–variance optimization problem was applied in [2] to the whole set of stocks S and to subsets $Q_1, \ldots, Q_5$. A comparison of the efficient frontiers of S with those of the five subsets revealed that the performance of the sets $Q_1, \ldots, Q_4$ corresponding to $\lambda \in [0, 0.7771)$ was similar to those of S. As for Q_5, with λ close to 1 and therefore privileging stocks with lower uncertainty and almost ignoring expected utility, it performed less well than S, considering the mean–variance performance.

In the following analysis, we will consider also the following sets:

$$\begin{aligned}
Q_6 &= \{S_2, S_4, S_7, S_8, S_{10}, S_{12}, S_{13}, S_{14}, S_{15}\},\\
Q_7 &= \{S_3, S_4, S_{10}, S_{11}, S_{12}, S_{14}, S_{15}, S_{16}, S_{17}\},\\
Q_8 &= \{S_1, S_2, S_4, S_6, S_8, S_{14}, S_{15}, S_{16}, S_{18}\}.
\end{aligned}$$

Q_6 consists of the worst ranked stocks by the EU–EV risk (stocks that were mostly left out by the EU–EV selection) and Q_7 and Q_8 contain randomly picked stocks that were presented in [2] and shown to be less efficient than sets $Q_1, \ldots, Q_5$.

The aim is to analyse the performance of portfolios formed with the best ranked stocks, that is, with stocks of Q_i, $i = 1, \ldots, 5$, and compare it with the benchmark S portfolio's performance using the in-sample and also out-of-sample data. We will also investigate if the portfolios constructed with stocks of Q_6, Q_7, Q_8 underperform the benchmark portfolio with respect to the performance indicators, since these were formed with less well classified stocks by the EU–EV risk model and one would therefore expect a poorer performance.

We will denote the five portfolios, favourite in terms of the EU–EV risk, by $Q_1, \ldots, Q_5$, where each portfolio is formed as an equally weighted combination of the stocks of each corresponding set Q_i, $i = 1, \ldots, 5$. The other three portfolios will be denoted by Q_6, Q_7, Q_8 and are built in an analogous way. The stragtegy of using equal weights (in this case 1/9) is chosen, since it has been reported in the literature that equal-weighted portfolios outperform value-weighted strategies (see e.g [6], [9]). The portfolios contain thus half the number of stocks than the benchmark porfolio PSI 20 index, here represented by S.

3 Performance evaluation of the portfolios

In order to analyse the perfomance of the portfolios, different performance indicators and metrics will be determined, using in-sample data from January 2019 to December 2020 and out-of-sample data from January 2021 to December 2022. The performance evaluation will be conducted considering a time horizon of one year and a time horizon of two years and comparing the portfolios' performances with those of the benchmark portfolio PSI 20 index S.

3.1 Cumulative returns and performance metrics

As a first performance indicator, the cumulative returns, obtained from the daily returns, are calculated for the five portfolios $Q_1, \ldots, Q_5$ and for the benchmark index. The cumulative returns are presented in Figure 1 for the in-sample data. Figures 2 and 3 contain the cumulative returns corresponding to the out-of-sample data for the one-year and two-year period, respectively. Observing the evolution of the cumulative returns in Figure 1, all portfolios, $Q_1, \ldots, Q_5$, outperform the benchmark portfolio S in the given time horizon, where Q_5 underperforms Q_1, Q_2, Q_3, Q_4, and Q_2 seems to outperform the other portfolios in the second half of 2020. These results indicate that the portfolios containing stocks selected with the EU–EV risk model, by weighting more the expected utility than the variance and entropy components, achieve also higher cumulative returns in the considered time interval, whereas Q_5, constructed with stocks weighting more the variance and entropy component than the expected utility component, is the worst performing portfolio among the five. Note that, in the

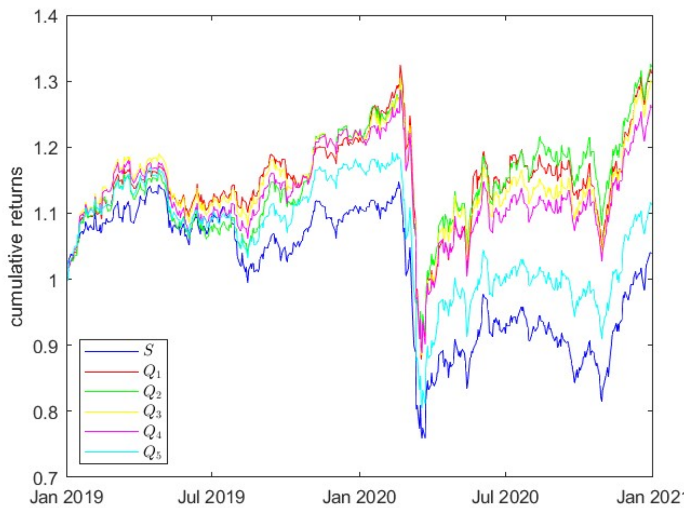

Fig. 1. Cumulative returns of S and $Q_1, \ldots, Q_5$ from January 2019 to December 2020.

mean–variance efficiency analysis, Q_5 also performed less well than Q_1, Q_2, Q_3 and Q_4.

Considering the one-year out-of-sample period, the portfolios $Q_1, \ldots, Q_5$ continue exhibiting higher cumulative returns than the PSI index S (see Figure 2), and also in the two-year period, however with an exception in the last quarter of 2022, where Q_1 underperforms S (see Figure 3). Q_2 is the best performing portfolio in the one-year period and over a larger time interval in the two-year period and Q_1 the worst. But notable is the strong performance of Q_4 and Q_5 in 2022 and, in particular, that of Q_5 in the last quarter of 2022. The portfolios Q_4 and Q_5 contain the best ranked stocks that were selected by the EU–EV risk model weighting more the variance and entropy component than the expected utility, and a higher variance can lead to higher returns, which may explain the higher cumulative returns obtained by these portfolios in further time intervals in the out-of-sample period. However, in general, the portfolios formed with the best ranked stocks according to the EU–EV risk lead also in the out-of-sample periods to higher cumulative returns, when compared to the benchmark returns.

As for portfolio Q_6, constructed with the worst ranked stocks by the EU–EV risk model, and for the portfolios Q_7 and Q_8, that were shown to be less efficient using the mean–variance model, the evolution of the cumulative returns in the in-sample period is illustrated in Figure 4 and in the out-of-sample periods in Figures 5 and 6. In the in-sample period, these portfolios underperform the benchmark portfolio, as it would be expected, with a slight exception during the first quarter of 2019, where Q_7 surpasses S. And, the portfolio that leads in

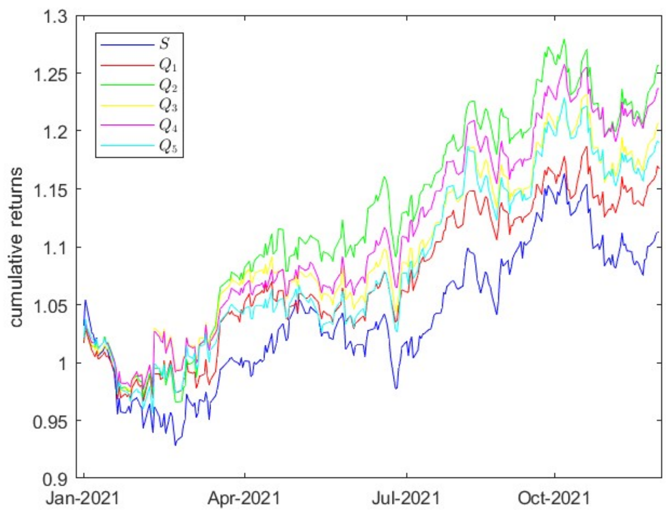

Fig. 2. Cumulative returns of S and $Q_1, \ldots, Q_5$ from January 2021 to December 2021.

the in-sample period over a wider time range to the lowest cumulative returns is in fact Q_6. Regarding the out-of-sample periods, the cumulative returns of these portfolios exceed the cumulative returns of the benchmark portfolio for several months in 2021 and 2022. Afterwards, in the second half of 2022, the cumulative returns of the portfolios tend to approach and there are periods where S outperforms again the other three portfolios.

For the evaluation of the portfolios' performances we will also consider the following metrics (see e.g. [10],[13]). The Sharpe ratio measures the excess return (the return of the portfolio less the risk-free rate of interest) per unit of total risk of the portfolio (the standard deviation of the porfolio's returns) and is defined by

$$\text{Sharpe} = \frac{r_{\mathrm{P}} - r_{\mathrm{f}}}{\sigma_{\mathrm{P}}},$$

where r_{P} represents the expected return of the portfolio, r_{f} the risk-free rate, and σ_{P} is the standard deviation of the portfolio returns. Here we will consider a zero risk-free rate $r_{\mathrm{f}} = 0$ and the Sharpe ratio quantifies in this case the relation between the expected returns and the standard deviation of the returns of the portfolio. Portfolios with higher Sharpe ratios perfom better according to this measure.

The Sortino ratio is a modification of the Sharpe ratio, where only the downside deviation is taken into account, and it is expressed by

$$\text{Sortino} = \frac{r_{\mathrm{P}} - r_{\mathrm{f}}}{\sigma_{\mathrm{P}}^{-}},$$

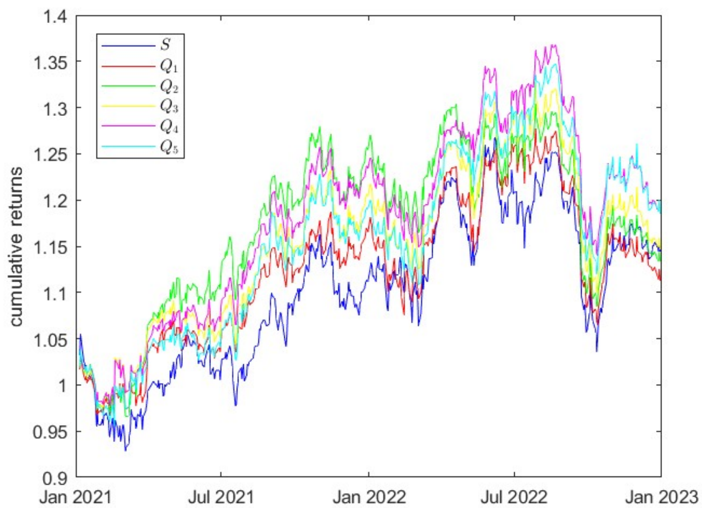

Fig. 3. Cumulative returns of S and $Q_1, \ldots, Q_5$ from January 2021 to December 2022.

where σ_{P}^{-} denotes the standard deviation of the negative portfolio returns. Here we will consider a zero risk-free rate $r_{\mathrm{f}} = 0$, as in the determination of the Sharpe ratio.

The risk metric Beta quantifies the risk or volatility of a portfolio compared to the market and is given by

$$\mathrm{Beta} = \frac{\mathrm{Cov}(r_{\mathrm{P}}, r_S)}{\sigma_S^2},$$

where $\mathrm{Cov}(r_{\mathrm{P}}, r_S)$ is the covariance between the expected return of the portfolio and the expected market return r_S of the benchmark S, and σ_S^2 is the variance of the market returns. Portfolios having Beta> 1 can be interpreted to be more volatile or riskier than the benchmark. In that case the portfolio is also said to be less sensitive to the benchmark volatility. If Beta< 1 the portfolio is less volatile than the benchmark and if Beta$= 1$, it has the same volatility as the benchmark.

Jensen's Alpha is a performance metric that measures the portfolio return relative to the market return and represents the amount by which the average return of the portfolio deviates from the expected return given by the Capital Asset Pricing Model. The metric is defined by

$$\mathrm{Alpha} = r_{\mathrm{P}} - [r_{\mathrm{f}} + \mathrm{Beta}(r_S - r_{\mathrm{f}})],$$

with r_{P}, r_{f}, r_S and Beta defined above. Here, again, we set $r_{\mathrm{f}} = 0$. A value of Alpha greater than zero indicates that the portfolio has performed better

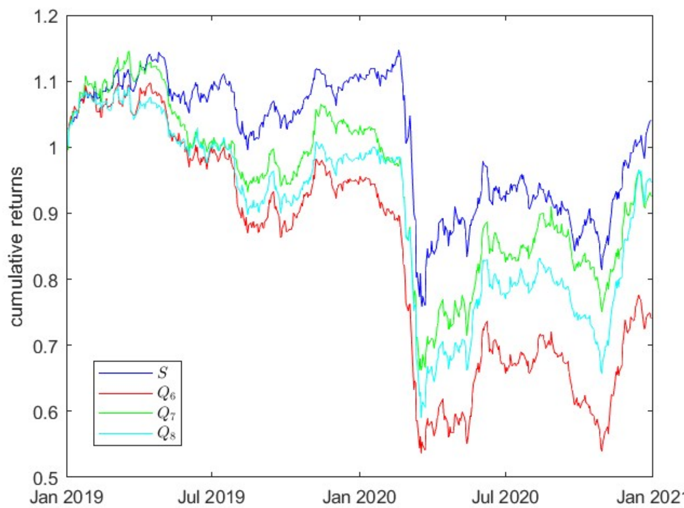

Fig. 4. Cumulative returns of S and Q_6, Q_7, Q_8 from January 2019 to December 2020.

than the market index, a negative value, that the portfolio underperformed the market index and a zero value means that the portfolio's performance is in line with that of the market.

The results of the metrics for the portfolios Q_i, $i = 1, \dots, 5$, and for the benchmark portfolio S in the in-sample period are listed in Table 1. The Sharpe and Sortino ratios of Q_i, $i = 1, \dots, 5$, are higher than those of S, as expected, where Q_2 is the best performing portfolio and Q_1, Q_2 and Q_3 perform better than Q_4 and Q_5. The portfolio Q_5 attains the lower ratios. The results of the Alpha values, indicating a slight excess return with respect to the market, are in agreement with these conclusions. Since Beta$<$1 for all portfolios, one can conclude that the portfolios are less volatile than the benchmark.

Table 1. Performance metrics for S, $Q_1, \dots, Q_5$ from January 2019 to December 2020.

	S	Q_1	Q_2	Q_3	Q_4	Q_5
Sharpe	0.1938	0.7860	0.8078	0.7532	0.6867	0.3737
Sortino	0.2289	0.8748	0.9306	0.8626	0.7924	0.4331
Beta	1	0.8215	0.8650	0.8703	0.8562	0.8683
Alpha	0	0.1272	0.1298	0.1194	0.1027	0.0359

Considering the out-of-sample period of one year, the Sharpe and Sortino ratios of the five portfolios remain higher than those of the benchmark (see

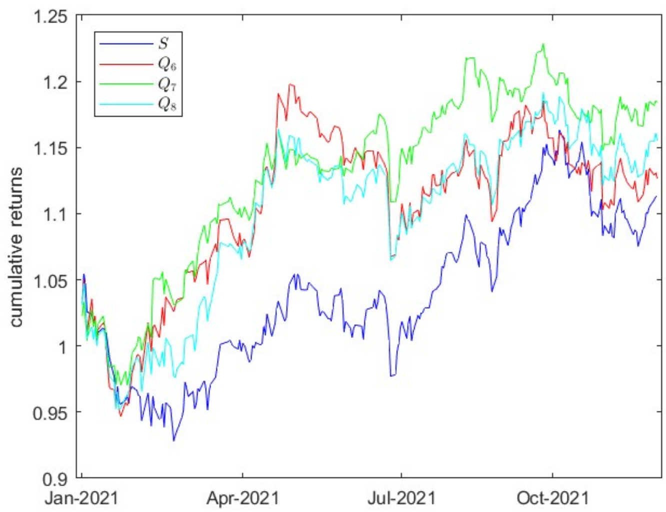

Fig. 5. Cumulative returns of S and Q_6, Q_7, Q_8 from January 2021 to December 2021.

Table 2), where Q_2 has again the best Sharpe ratio, however the best Sortino ratio is now associated with Q_4 and the second best with Q_2. The lowest ratios are now associated with Q_1. The Alpha values replicate the observed behaviour. According to the Beta values, the portfolios are less volatile than S.

Table 2. Performance metrics for S, $Q_1, \ldots, Q_5$ from January 2021 to December 2021.

	S	Q_1	Q_2	Q_3	Q_4	Q_5
Sharpe	0.7724	1.2771	1.6830	1.4251	1.6312	1.3202
Sortino	1.1478	1.8703	2.3443	2.2183	2.5713	2.1677
Beta	1	0.7504	0.8443	0.8087	0.7891	0.8290
Alpha	0	0.0792	0.1528	0.1098	0.1403	0.0914

The results for the out-of-sample period of two years in Table 3 reveal that now Q_4 and Q_5 provide the best Sharpe and Sortino ratios, where the highest Sharpe ratio is observed for Q_4 and the highest Sortino ratio, for Q_5. The Sharpe and Sortino ratios of S are closer to those of the five portfolios. Indeed, the benchmark performs better than Q_1 and Q_2. The positive Alpha values can be considered approximately equal to zero, which means that the portfolios' performances are in line with the benchmark performance. A closer look reveals that Q_4 attains the highest Alpha, followed by Q_5, whereas Q_1 has the lowest

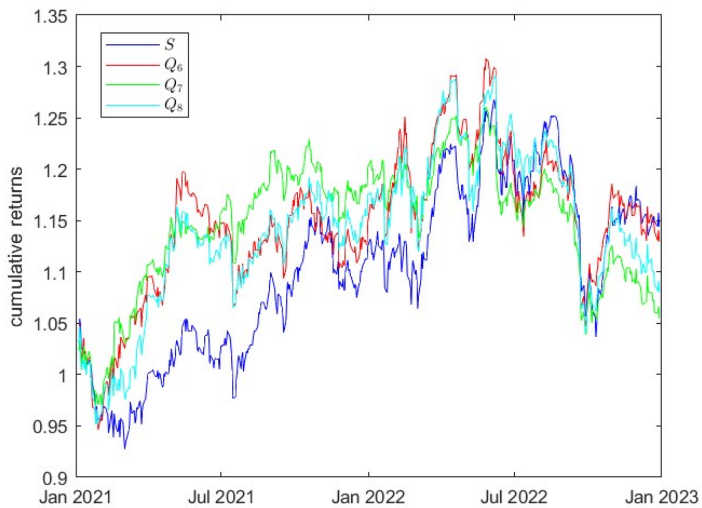

Fig. 6. Cumulative returns of S and Q_6, Q_7, Q_8 from January 2021 to December 2022.

Alpha. The Beta values indicate again that the portfolios are less volatile than S.

Table 3. Performance metrics for S, $Q_1, \ldots, Q_5$ from January 2021 to December 2022.

	S	Q_1	Q_2	Q_3	Q_4	Q_5
Sharpe	0.4863	0.4416	0.4790	0.5292	0.6712	0.6534
Sortino	0.7364	0.6322	0.6741	0.7782	1.0359	1.0459
Beta	1	0.7594	0.8457	0.8040	0.7571	0.8048
Alpha	0	0.0022	0.0060	0.0144	0.0344	0.0310

Examining the results of the performance metrics obtained for the portfolios Q_6, Q_7 and Q_8 in the in-sample period (see Table 4), one can confirm that, in fact, these portfolios underperform the benchmark, not only in terms of lower cumulative returns, as seen before, but also taking into account the Sharpe ratios, the Sortino ratios and the Alpha values. The ratios are all negative, indicating that it is probable to get negative expected returns (losses) with these portfolios. In particular, the ratios of Q_6 are the worst ones, which is consistent with the graphical conclusion of Q_6 yielding the lowest cumulative returns (cf. Figure 4). Note that Q_6 contains the worst classified stocks by the EU–EV risk model. The negative Alpha values also indicate that the portfolios underperform the benchmark, where Q_6 is the worst classified portfolio according to this indicator.

As for the Beta values, one can observe that these portfolios (except Q_7) have higher values than the other portfolios Q_i, $i = 1, \ldots, 5$. The portfolio Q_6 can be considered more volatile than the benchmark, since Beta> 1.

Table 4. Performance metrics for S, Q_6, Q_7, Q_8 from January 2019 to December 2020.

	S	Q_6	Q_7	Q_8
Sharpe	0.1938	−0.4683	−0.0840	−0.0123
Sortino	0.2289	−0.5909	−0.1025	−0.0144
Beta	1	1.0415	0.8421	0.9336
Alpha	0	−0.1471	−0.0498	−0.0393

In contrast, in the one year out-of-sample period, all three portfolios exhibit higher Sharpe and Sortino ratios and a higher Alpha than the benchmark (see Table 5), with the ratios and Alpha of portfolio Q_7 being the highest ones. In fact, this portfolio achieved in this period the highest cumulative returns over a wider time range (cf. Figure 5). The Beta values are all less than 1.

Table 5. Performance metrics for S, Q_6, Q_7, Q_8 from January 2021 to December 2021.

	S	Q_6	Q_7	Q_8
Sharpe	0.7724	0.8251	1.3583	1.0230
Sortino	1.1478	1.2870	2.0375	1.5203
Beta	1	0.8107	0.6402	0.8756
Alpha	0	0.0375	0.1091	0.0545

In the two-year out-of-sample period, the results in Table 6 indicate that the benchmark outperforms again the portfolios Q_6, Q_7 and Q_8 in terms of the Sharpe and Sortino ratios. Considering the Alpha values, the negative values of Q_7 and Q_8 express the outperformance of the benchmark over these portfolios and the positive value of Q_6 indicates that this portfolio surpasses the benchmark. However the Alpha values are close to zero, suggesting that the differences between the portfolios' and benchmark's returns may be small.

Table 6. Performance metrics for S, Q_6, Q_7, Q_8 from January 2021 to December 2022.

	S	Q_6	Q_7	Q_8
Sharpe	0.4863	0.4401	0.2646	0.3131
Sortino	0.7364	0.6252	0.3787	0.4445
Beta	1	0.7990	0.5870	0.8766
Alpha	0	0.0106	−0.0120	−0.0182

3.2 Summary analysis

The previous obtained results and analysis can be summarized as follows. The results of the performance evaluation show that the portfolios built with the best classified stocks according to the EU–EV risk model outperform the benchmark in the in-sample period. Among these portfolios, it is Q_5 that provides the worst performance indicators. These results are in accordance to the results obtained in the mean–variance efficiency analysis, where the stocks of Q_1, Q_2, Q_3 and Q_4 led to approximately equal efficient portfolios than S and the stocks of Q_5 led to less effcicient portfolios. On the contrary, the portfolios containing the worst and less well classified stocks underperform the benchmark in the same period, where the worst results were obtained with Q_6, containing the lowest ranked stocks according to the EU–EV risk model.

Considering the out-of-sample period, the results of the performance indicators also confirm that the portfolios Q_i, $i = 1, \ldots, 5$, can beat the benchmark, especially in the one-year time horizon. In the two-year time horizon, two of the five portfolios performed less well than the benchmark considering the Sharpe and Sortino ratios. The Alpha values are very close to zero, however, positive and indicating therefore that the portfolios outperform the benchmark. Surprisingly, the portfolios Q_6, Q_7, Q_8 reach higher cumulative returns, better Sharpe ratios and Sortino ratios (except Q_6, which attains a lower Sortino ratio) and better Alpha values than S in the one-year period. However, in the two-year period these portfolios present again a lower performance than the benchmark, except the cumulative returns, that only in the second half of 2022 decay below the benchmark returns, and the positive Alpha of Q_6, this being however approximately equal to zero.

Based on the obtained results, one can conclude that the selection of stocks with the EU–EV risk model provides portfolios (with half the number of stocks than the benchmark) that are not only efficient in terms of the mean–variance model when compared with the benchmark, but can lead also in a short term horizon to higher cumulative returns and perform better than the benchmark with respect to the measures Sharpe and Sortino ratios and Alpha. The Beta values indicate that the portfolios are less volatile than the benchmark. The positive performance of the portfolios can be explained due to the fact that the EU–EV risk model ranks stocks taking into account the expected utility, the variance and the entropy of the stock returns and these factors play an important role in the evolution, at least in the short term, of the cumulative returns and in the determination of performance factors, such as e.g. the Sharpe ratio, Sortino ratio or Alpha.

4 Conclusions

In this work we have analysed the performance of portfolios, formed with equally weighted stocks that were previously selected with the expected utility, entropy and variance (EU–EV) risk model. The portfolios were constructed using data of the PSI 20 index, from January 2019 to December 2020, and were formed with half the number of stocks than the index portfolio. In order to evaluate the performance, indicators and metrics of the portfolios were compared with those of the benchmark portfolio. Cumulative returns, Sharpe ratios, Sortino ratios, Beta and Alpha values were calculated for the in-sample period and for two out-of-sample periods: a one-year period ranging from January 2021 to December 2021 and a two-year period ranging from January 2021 to December 2022.

In the in-sample period, the portfolios formed with the best ranked stocks outperform the benchmark, as expected, in all the considered performance evaluation indicators, where the Beta values indicate that the portfolios are less volatile than the benchmark. In contrast, examples of other three portfolios, one of them constructed with the worst ranked stocks by the EU–EV risk, underperform the benchmark in the in-sample period. In the one-year out-of-sample period, the results show again that all favourite portfolios outperform the benchmark. In the two-year out-of-sample period, in general, the portfolios again perform better than the benchmark. Only two portfolios have slightly lower Sharpe and Sortino ratios than the benchmark and the cumulative returns of these two portfolios are exceeded by those of the benchmark in a time interval contained in the last quarter of 2022.

The results indicate that for short-term investments the strategy of constructing portfolios using the EU–EV risk model can be profitable. The EU–EV risk measure captures the relevant characteristics of stocks, such as expected utility, variance and entropy of stock returns, that have influence on the evolution of the cumulative returns and on the other considered performance indicators.

In the future, we will perform this analysis considering different in-sample periods and a wider time horizon for the out-of-sample period. We will also investigate the performance of portfolios, constructed with selected stocks using the EU–EV risk model, for markets containing more stocks.

Acknowledgements IB thanks support from FCT ("Fundação para a Ciência e a Tecnologia") through the Projects UIDB/00013/2020 and UIDP/00013/2020.

GJM acknowledges the financial support from FCT through the Project UIDB/04650/2020.

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