

Coulomb effects in electronic transport

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Abstract

Charge transport in metals and semiconductors is often dominated by electron-impurity and electron-phonon scattering. Coulomb effects could be found in small corrections to the leading behavior, drag effects in specially fabricated samples, compensated semimetals, and hydrodynamic phenomena in ultra-pure materials. In contrast, electrical resistivity in strongly correlated systems is poorly understood. Understanding the fate of electron-phonon scattering in these materials may offer a route towards future advancements.

KEYWORDS

electrical resistivity, electronic hydrodynamics, electronic transport, strange metals

Coulomb's law describes one of the four fundamental interactions known to modern physics. Out of the four interactions, only Coulomb's forces act on length scales relevant to condensed matter. Yet, many phenomena observed in solids can be described by theories of “noninteracting” electrons^[1,2] as most readily illustrated by the traditional theory of electrical conduction.

Conventional conductors are typically described within the Landau Fermi-liquid paradigm.^[3,4] The central principle of the Landau theory is the concept of quasiparticles, namely, the statement that elementary excitations of an interacting Fermi system can be described similarly to those in a free Fermi gas. Assuming that global symmetries of the Fermi gas (particle number, energy, and momentum conservation, spin-rotation invariance, etc.) are preserved in the Fermi-liquid state, quasiparticles are described by the same set of quantum numbers as the free fermions. However, quasiparticles are not “free”: they do not correspond to exact eigenstates of the Hamiltonian. Quasiparticles interact with each other and have a finite “lifetime.” As a result, physical properties of the quasiparticles (mass, compressibility, spin susceptibility, etc.) differ from the properties of the free gas.

In its original form, the Fermi-liquid theory describes a physical liquid, ³He.^[4] Applying the same concept to electrons in solids brings about several significant changes.^[5] In contrast to helium atoms, electrons carry charge and interact by means of the long-ranged Coulomb potential; hence, screening effects need to be taken into account. Furthermore, electrons in solids exist in the environment created by a crystal lattice and can scatter off both lattice imperfections (or “disorder”) and lattice vibrations (phonons): the electronic momentum is not conserved. Assuming Galilean invariance, the momentum defines the electric current and the above scattering processes provide natural mechanisms of current relaxation leading to electrical resistance.

Resistance (and more generally, transport) measurements remain one of the most common experimental tools in condensed matter physics. Measured within linear response, the electrical resistance satisfies the Ohm's law.^[1,6] At an

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elementary level, the phenomenon can be described using the Drude theory.^[7] The resulting Drude formula for electrical resistivity,

$$\rho = \frac{m}{e^2 n \tau}, \quad (1)$$

is widely used to characterize conducting properties of materials^[8] (e is the electron charge, n and m are the carrier density and mass, and τ^{-1} is the “transport scattering rate”). The rate τ^{-1} does not have a fundamental, model-independent definition^[9] (unlike n that can be extracted from the Hall coefficient and m that can, in principle, be found from the specific heat) and can be taken as phenomenologically defined by Equation (1). Assuming temperature-independent n and m , the measured temperature dependence $\rho(T)$ can be assigned to τ .

In conventional conductors, the above procedure can be justified by using the kinetic (Boltzmann) equation approach.^[1] The scattering rate could then be inferred from the collision integral. Taking into account the above two mechanisms of momentum relaxation (disorder and electron–phonon scattering), one finds $\rho(T)$ [or $\tau(T)$] in a reasonably good agreement with experiment.

In copper (a conductor widely used in practice, e.g., for household wiring), the electrical resistivity exhibits three clearly identified regimes^[10]: (i) the low-temperature regime of roughly T -independent “residual resistance” (typically attributed to disorder scattering); (ii) the phonon-dominated regime with $\rho \sim T^5$; which at higher temperatures crosses over to (iii) the linear-in- T resistance, $\rho \sim T$. The latter behavior begins at approximately 100 K and continues up to the melting temperature, 1358 K. This behavior is typical for simple nonmagnetic metals^[11] (in magnetic metals electrons can also scatter on spin waves; in ordered alloys scattering on long-range fluctuation may be important).

The above phenomenology should be contrasted with the almost century-old discussion of the possible contribution of electron–electron scattering to resistivity, with $\rho \sim T^2$ being the most quoted expectation. Detailed experimental work targeting such effect was performed in Ref. [12]. The authors cited Ref. [13] as the source of the original suggestion that electron–electron interaction could contribute to resistivity, although that paper only reported that experimental data on platinum were not in agreement with any existing theory represented either by a single power law^[14–16] or a linear combination of several of them.^[17] The platinum data could be fitted to the T^2 behavior in the interval $T \approx 1 \div 4$ K and to T^4 at about 20 K. Ref. [12] reported an observation of a quadratic temperature dependence in aluminum and indium (in contrast to potassium and sodium where such behavior was not observed). The data (measured in the interval $T \approx 1 \div 4$ K) were analyzed using the phenomenological expression (suggested in Ref. [18])

$$\rho = \rho_0 + A_2 T^2 + A_5 T^5. \quad (2a)$$

A wider temperature range ($T \approx 6 \div 327$ K) was explored in Ref. [19]. Data in vanadium, platinum, and copper were analyzed using a more complex expression

$$\rho = \rho_0 + A_2 T^2 + A_3 (T_D/T) T^5 + A_5 (T_D/T) T^5. \quad (2b)$$

Fitting over the entire temperature range, the data did not support the inclusion of the quadratic term. Narrowing down to the smaller interval $T \approx 6 \div 12$ K, the authors did find an estimate for A_2 in platinum that was comparable to previous studies, while confirming $A_2 \approx 0$ in copper.

Theoretically, the T^2 temperature dependence of resistivity is known to arise due to electron–magnon processes that are beyond the scope of this paper or due to *Umklapp* scattering.^[20] The resulting expression is

$$\rho \approx \frac{m}{e^2 n} \frac{T^2}{E_F}, \quad (3)$$

where E_F is the Fermi energy, in most “simple” metals (except for transition metals) is subleading to the phonon contribution and could be observed only at low temperatures. This conclusion is supported by the above experimental data as well as by the numerical analysis. In particular, Ref. [21] reports that a T^2 contribution does not persist at high temperatures, despite accurately describing resistivity in palladium below $T = 5$ K (although experimental estimates of A_2 vary depending on sample purity^[22,23]). It is then not surprising that experimental observation of the T^2 contribution and especially its interpretation in terms of electron–electron scattering is difficult and at best inconclusive.

Comparing Equations (1) and (3), one may surmise the existence of “scattering rate” (in units with $k_B = 1$)

$$\tau^{-1} \sim \frac{T^2}{E_F},$$

related to the quasiparticle lifetime in the Fermi-liquid theory. It is important to stress that this is *not* the case. The quasiparticle lifetime has nothing to do with electrical resistivity. The contribution described by Equation (3) appears only in cases where the Fermi surface is open and crosses the Brillouin zone boundary. In that case, electrons can jump between the opposite parts of the Fermi surface losing momentum to the lattice as a whole. The resulting resistance is by no means universal and strongly depends on the details of the Fermi surface topology.

The brief historical overview presented so far was not intended to be comprehensive. It does however illustrate the following point. At the level of the leading approximation often assumed in literature, the electron–electron interaction does not contribute to electrical resistivity. In the kinetic theory approach, this can be seen as a consequence of additivity of the collision integral (which leads to the so-called Mathiessen’s rule^[11,24]). Indeed, if one restricts the scattering probability to the Golden Rule level, then contributions of different scattering mechanisms add up. A collision integral describing electron–electron interaction (without the possibility to transfer momentum to the lattice) does not contribute to ρ unless the electric current is not determined by the momentum density (as is the case in intrinsic graphene^[25–29]). This leaves the following possibilities to observe the effects of electron–electron interaction in transport measurements: (i) “corrections” to kinetic coefficients due to higher order processes^[5,30]; (ii) drag effects^[31]; (iii) materials with specific features of the Fermi surface allowing, for example, for *Umklapp* scattering; (iv) compensated semimetals^[25]; and (v) indirectly, electronic hydrodynamics.^[25,32–34]

The list of observed phenomena that do not fit in the above simplistic picture is too long to be included here. The effects related to magnetic field (such as the fractional quantum Hall effect^[35]) are deliberately left outside the scope of this paper as are phenomena in mesoscopic (e.g., the Coulomb blockade)^[36] and (quasi-) one-dimensional systems.^[37] Transport properties of strongly correlated systems^[9,38–41] represent a major unsolved problem in modern physics. Over the last several decades, a lot of attention was devoted to compounds exhibiting linear-in- T resistivity.^[8,9,41] Despite certain progress achieved in understanding various models of non-Fermi-liquid (NFL) behavior, many aspects of the observed phenomenology remain puzzling. Experimentally, the linear behavior is observed only in a part of the (rather complicated) phase diagram. A seemingly small change in doping may lead to a change of the exponent $T \rightarrow T^\alpha$ with a sizable portion of the phase diagram exhibiting the quadratic behavior ($\alpha \approx 2$)^[42,43] associated not with the *Umklapp* processes, but rather with electron–phonon scattering (in the case of a cylindrical Fermi surface). Why would electron–phonon scattering cease to contribute or, alternatively, undergo a significant change yielding a different exponent at different doping levels is not completely understood. A recent attempt to explain the linear-in- T resistivity by electron–phonon scattering with unusually low Bloch-Grüneisen temperature^[44] does not account for the whole temperature range of the observed effect.^[41] A further puzzle is related to the nature of elementary excitations: angular-resolved photoemission (ARPES) measurements seem to be pointing towards a novel quantum state where excitations cannot be decomposed into individual quasiparticles,^[40] while angle-resolved magnetoresistance (ADMR) data^[45] are typically interpreted using the Boltzmann equation approach. It is safe to assume that the nature of electronic structure in NFL metals and any possible universality of its features have not been definitively established yet.

1 | QUANTUM INTERFERENCE CORRECTIONS

The T^2 behavior due to *Umklapp* scattering, Equation (3), is sometimes presented as the “Fermi-liquid” behavior (in contrast to, e.g., “non-Fermi-liquid” linear-in- T resistivity). It is therefore important to point out, that a standard disordered Fermi liquid in two dimensions (2D)^[5] is perfectly capable of exhibiting the linear temperature dependence,^[46] albeit in a relatively narrow temperature range.

A 2D electron gas (2DEG) is typically realized either in semiconductor heterostructures^[47,48] or at an interface between two oxide insulators.^[49–52] In these systems, the carrier density is typically low such that the Fermi surface occupies a relatively small volume in the center of the Brillouin zone. The low-temperature resistivity of a 2DEG strongly depends on the carrier density and exhibits an apparent metal–insulator transition^[47,48,52] (i.e., a change in the temperature behavior from metallic with $\partial\rho/\partial T > 0$ to insulating with $\partial\rho/\partial T < 0$). The metallic behavior is associated with the resistivity that is less than the “resistivity quantum,” h/e^2 . In that case, the dimensionless conductance of the 2DEG (defined

as the conductivity in units of e^2/h is large, $g = h/(e^2\rho) \gg 1$, and one can develop a perturbation theory with the small parameter $1/g$ ^[5,46] (in terms of the mean free path τ_{dis} , the dimensionless conductance may be expressed as $g \sim E_F\tau_{\text{dis}}/\hbar$).

To the “zeroth” order of the perturbative expansion in $1/g$, one finds the T -independent “residual” resistivity due to electron-impurity scattering. The leading temperature dependence arises due to quantum interference processes known as the weak localization and interaction (or Altshuler-Aronov) corrections (one typically calculates conductivity)

$$\sigma(T) = \sigma_0 + \sigma_{\text{WL}}(T) + \sigma_{ee}(T). \quad (4)$$

The term “corrections” reflects the fact that these contributions are small compared with the residual conductivity by the factor of $1/g$ (obtained in the “first” order of perturbation theory).

The weak localization correction^[53,54] is always negative (“insulating”) and is generally considered to be the precursor of Anderson localization.^[55,56] In contrast, the interaction correction may change sign depending on the value of the Fermi-liquid constant (Landau parameter) in the triplet channel^[46,57,58] explaining the observed metallic ($\partial\rho/\partial T > 0$) behavior. The explicit expression for the correction,

$$\begin{aligned} \delta\sigma_{ee}(T) = & -\frac{e^2}{\pi h} \left[4 - \frac{3 \ln(1 + F_0^\sigma)}{F_0^\sigma} \right] \ln \frac{\hbar}{T\tau_{\text{dis}}} \\ & + \frac{2e^2}{h} \left[1 + \frac{3F_0^\sigma}{1 + F_0^\sigma} \right] \frac{T\tau_{\text{dis}}}{\hbar} - \frac{e^2}{h} K(T\tau_{\text{dis}}; F_0^\sigma), \end{aligned} \quad (5)$$

contains the logarithmic term dominating in the so-called “diffusive” regime ($T\tau_{\text{dis}} \ll \hbar$), the linear term which is the hallmark of the “ballistic” regime ($T\tau_{\text{dis}} \gg \hbar$), and the crossover function. The two limiting expressions are in a good agreement with experiment.^[47,59]

The linear temperature dependence in the ballistic limit can be established at the level of a simple quantum mechanical scattering problem, where one finds a correction to the scattering amplitude due to coherent scattering off Friedel oscillations.^[46] The scattering amplitude defines the transport scattering rate; hence, the obtained result is a correction to the latter, the point that is not as clear within the diagrammatic approach. In fact, the notion of the “scattering rate” lacks a precise (i.e., free from any approximation and interpretation) definition within any Green’s function formalism (this problem becomes apparent when trying to specify the so-called Planckian scale^[9,41]), although no such issue arises within the quantum kinetic equation approach.^[46]

The narrow temperature range where the linear dependence shown in Equation (5) is observed^[47,59] is limited from below by the crossover to the diffusive regime and from above by the onset of higher order corrections exhibiting higher powers of temperature. At low carrier densities the dominant contribution is quadratic in T and comes from the so-called drag effect.^[31]

2 | COULOMB DRAG

Drag effects appear in systems comprising two distinct types of carriers or interacting particles. Scattering between particles of different types leads to energy and momentum transfer between the two subsystems. In case one of them is carrying the electric current, the other is being “dragged” along once it acquires the transferred momentum.

The earliest phenomenon of that type to be studied is the “phonon drag,”^[60] which provides a correction to the standard transport theory typically observable in thermoelectric measurements (although affecting charge transport as well). A “cleaner” observation of drag is possible in artificially fabricated samples comprising two closely spaced, but electrically isolated, conductors (often referred to as “layers”). Passing a current through the “active layer” induces a voltage (in an open circuit) in the “passive layer.” The effect is sometimes referred to as “mutual friction,” “frictional drag,” or (especially in semiconductor heterostructures) “Coulomb drag.” The ratio of the induced voltage to the passed current determines the “transresistivity” or the “drag coefficient” which can be described by the Drude-like formula with the “drag scattering rate” τ_D^{-1} . If both layers are in the Fermi-liquid state, the usual phase-space argument yields the quadratic temperature dependence $\tau_D^{-1} \propto T^2$.^[61]

The key physical feature of electronic systems in both layers required for drag is the electron–hole asymmetry. When one passes a current through the active layer, the oppositely charged carriers, “electrons” and “holes,” move in opposite directions. The active layer can then be characterized by a nonzero total momentum only if there is some asymmetry

between them. Similarly, the momentum is transferred equally to electrons and holes in the passive layer and hence the resulting state can carry the current only in the case of electron–hole asymmetry.

In the Fermi-liquid state, the two types of carriers are the “electron-like” excitations with energies above the Fermi energy $\varepsilon > E_F$ (i.e., the occupied states outside the Fermi surface) and “hole-like” excitations with $\varepsilon < E_F$. The electron–hole asymmetry appears due to curvature of the conduction band spectrum (leading to the energy dependence of the density of states [DOS] and/or diffusion coefficient). In the Fermi-liquid theory, the electron–hole asymmetry can be expressed^[62] as a derivative of the single-layer conductivity with respect to the chemical potential (assuming either a constant impurity scattering time or diffusive transport). The simple estimate $\partial\sigma/\partial\mu \sim \sigma/\mu$ then explains the typical smallness of the effect. As a result, the drag effect in a single conductor is usually subleading, which is the reason why one needs to fabricate double-layer samples to observe drag. Here, there are no competing contributions and the whole effect is due to electron–electron interaction. The above smallness manifests itself in strong mesoscopic fluctuations at low temperatures.^[62–64]

3 | ELECTRONIC HYDRODYNAMICS

Interaction effects discussed so far can be accounted for either within the kinetic approach or (equivalently) within a diagrammatic perturbation theory. This method may fail in “strongly interacting” or “strongly correlated” systems. A universal description of strong electron–electron interactions in the long-time, long-wavelength limit may be provided by hydrodynamics.^[25] Indeed, several authors have suggested that the linear-in- T resistivity has hydrodynamic origin.^[65–67]

Conventional hydrodynamics relies on momentum conservation. In solids, electronic momentum is never truly conserved (due to scattering off impurities, phonons, etc.). However, in ultra-pure materials,^[68–72] it may be possible to find an intermediate temperature range where electron–electron interaction is the dominant scattering process reflected in a hierarchy of typical length scales $\ell_{ee} \ll \ell_{\text{dis}}, \ell_{e-ph}, \dots$ (in self-evident notations). At high temperatures, ℓ_{ee} is expected to decrease and is especially short in “bad” metals,^[73,74] making hydrodynamics potentially relevant for transport beyond the Mott-Ioffe-Regel (MIR) limit.^[75,76] Recently it was argued that hydrodynamics leads to a rigorous upper bound on resistivity of an electronic fluid^[67] if the system possesses a nonthermal diffusion mode, that is, the imbalance between different bands.

Generalizing the hydrodynamic approach beyond conventional fluids, one may consider it in a broader sense of a long-wavelength theory of small perturbations relative to an equilibrium state.^[77] Then viscous flow and diffusion (with Ohmic dissipation) could be discussed on equal footing. The two types of dissipative dynamics can be combined in a single Gurzhi equation,^[78,79] their relative strength being controlled by the Gurzhi length, $\ell_G = \sqrt{\nu\tau_{\text{dis}}}$ (ν is the kinematic viscosity).

What can one expect from a hydrodynamic approach? Derived from the kinetic theory, the Gurzhi equation^[78,79] has two terms coming from the collision integral: (i) viscosity (due to electron–electron interaction) and (ii) Ohmic decay due to “momentum non-conserving” scattering (electron-impurity or electron–phonon) as described by two “kinetic coefficients”: the shear viscosity and conductivity. Even including quantum corrections,^[30,46] the latter remains a constant describing a diffusive flow that may be inhomogeneous in confined geometries.^[80,81] Viscous terms describe a different type of inhomogeneity in the current flow (due to sample geometry or close to sample edges).^[79,81–83] However, the kinetic coefficients (in particular, their temperature dependence) are not determined by hydrodynamics. Instead, they have to be established on the basis of the underlying microscopic theory.^[84–86] Without microscopic input, a phenomenological hydrodynamic approach cannot fully account for the observed temperature dependence of transport coefficients. What it can describe is the inhomogeneity of the current density and magnetotransport^[87–90] (the non-quantizing magnetic field affects electronic transport via the Lorentz force, while the collision integral remains field-independent).

The hydrodynamic regime is apparently easiest to achieve in graphene.^[25,32,33] This material can host two drastically different types of hydrodynamic behavior: (i) “conventional” in the Fermi-liquid regime^[32,78,79] and (ii) “unconventional” at charge neutrality.^[25] In graphene, the electronic momentum density defines the energy (not charge!) current. In “doped graphene,” all currents are equivalent, leading to a Navier–Stokes-like equation for the electric current.^[78,79] At charge neutrality ($B = 0$), the energy and electric currents decouple.^[91,92] The energy current may become hydrodynamic,^[93,94] while the electric current remains Ohmic,^[92] leading to a strong violation of the Wiedemann-Franz law.^[95] The diffusive charge flow is nonuniform in confined geometries^[94] exhibiting the anti-Poiseuille profile in a weak magnetic field.

Intrinsic graphene is neither a strongly correlated system nor a Fermi liquid. The only energy scale in the system is T . All timescales are proportional to T^{-1} with a factor depending on the interaction strength^[26,29,92] (unlike the Planckian limit^[41]). In the clean limit, the resistivity (T -independent up to renormalizations) is due to electron–electron scattering.^[26–29] Optical conductivity has nearly the Drude-like shape.^[85,96] Phonons also contribute to resistivity showing the linear T dependence at low-enough temperatures.^[97] In confined geometries, magnetoresistance of neutral graphene is linear in B (in “classically strong” fields).^[87,91] Finally, the system possesses an additional diffusive mode^[93] due to quasiparticle imbalance^[98] (leading to the linear magnetoresistance).

4 | PUZZLING TRANSPORT PROPERTIES OF STRONGLY CORRELATED SYSTEMS

Unconventional transport properties remain the definitive feature of NFL that can be roughly defined as metallic systems with a phenomenology incompatible with the predictions of the Fermi-liquid theory. Observed signatures of the NFL behavior include^[9,40,41] (i) single particle scattering rates as extracted from ARPES data^[40] deviating from standard Fermi liquid results (e.g., $\tau^{-1} \sim \max(\omega, T)$ ^[99,100]); (ii) an anomalous frequency dependence of the optical conductivity often accompanied by ω/T scaling with $\sigma(\omega) \sim \omega^{-\gamma} F(\omega/T)$ ^[101] (inconsistent with the usual Drude form); (iii) “bad” metallic behavior with resistivity above the MIR limit^[74]; and (iv) linear-in- T resistivity, $\rho = \rho_0 + AT$,^[8,38,39,102] (although other powers T^α have also been observed^[103,104]).

Linear-in- T resistivity may be related to a “transport scattering rate” using the Drude formula Equation (1),^[8] leading to the Planckian limit (provided the effective mass and carrier concentration are T -independent). Elastic disorder scattering leads to a nonzero rate at $T = 0$ (cf. the residual resistivity), so the concept of the Planckian dissipation should only be applied to inelastic dynamics^[41] (this is supported by ADMR data^[45,105]). T -dependent optical timescale may also have the Planckian form.^[106] The Planckian limit has been argued to apply to the electron–phonon scattering rate^[107] (based on the thermal diffusivity measurements^[108,109]).

The available experimental data (on a wide variety of strongly correlated materials) do not paint a clear physical picture. Even combined with decades of theoretical research, several issues remain controversial hindering our understanding of these systems. Arguably the two most discussed points are the temperature dependence of resistivity and the nature of elementary excitations.

The linear-in- T resistivity is often quoted as the defining property of NFL metals.^[9,41] Yet, many different theoretical models yield this behavior. Moreover, the linear resistivity is often contrasted with the “expected” quadratic behavior.^[9] Looking back at the known temperature dependence of simple metals,^[10] the origin of this expectation remains unclear.

Significant efforts were devoted to develop theories involving a quantum-critical boson coupled with the electronic Fermi surface.^[110,111] The boson may represent either an order parameter (breaking the point group, time reversal, or spin rotation symmetry) or a transverse component of an emergent gauge field. Within that framework, it was established early on^[112] that the electronic self-energy at the Fermi surface exhibits a sublinear frequency dependence $\text{Im}\Sigma(\omega) \sim \omega^{2/3}$. The Fermi surface remains well defined (in momentum space), but quasiparticle excitations do not. However, where does the boson come from? At least in some cases, it was argued to originate with higher-energy electronic modes.^[113] This means that the electron-boson coupling ultimately represents electron–electron interaction which conserves momentum. Momentum conservation implies that the conductivity of a disorder-free theory is unaffected by the anomalous self-energy.^[114,115] Optical conductivity has the Drude form for clean systems, which can be traced to the exact cancellation between the self-energy and vertex diagrams.^[116,117] Non-zero resistance can be introduced by taking into account Umklapp scattering,^[114,118] potential disorder,^[116,119,120] or random fluctuations of the fermion-boson coupling^[115] inspired by the Sachdev-Ye-Kitaev (SYK) model.^[9,121]

The absence of quasiparticle excitations is supported by several types of experiments. ARPES measurements provide evidence for angular anisotropy in the quasiparticle scattering rate and suggest that the electron self-energy is linear in the frequency for $\omega > T$.^[40,122] Optical conductivity studies show a shift of the spectral weight^[106] towards frequencies exceeding the scattering rate consistent with dc transport measurements. Resistivity exceeding the MIR limit^[73,74] can be interpreted in terms of a mean free path smaller than the Fermi length scale (or interatomic distance). The Hall angle data are inconsistent with the conventional Boltzmann theory^[123] ($\cot \theta_H \sim T^2$ in contrast with the expectation of the same temperature dependence of $\cot \theta_H$ and ρ). These results are often considered as evidence for a novel quantum state where excitations cannot be decomposed into individual quasiparticles,^[9] supported by experiments showing the lack of well-defined plasmon modes.^[124,125]

On the other hand, ADMR measurements have been interpreted on the basis of the Boltzmann transport approach^[45,126]; observations of quantum oscillations appear to be in agreement with the Lifshitz-Kosevich formula^[127–129]; low-temperature thermal transport measurements^[130,131] were interpreted as evidence for conventional fermionic excitations carrying charge and heat; while resistivity data in lightly doped cuprates^[132] show saturation at high temperatures. Recent ARPES data^[133] point towards the existence of a T -independent boundary between a strange metal (at lower doping) and a conventional metal (at higher doping) in cuprates, a viewpoint that is inconsistent with the idea of the underlying quantum critical point. Finally, magnetoresistance measurements^[134] suggest coexistence of two charge sectors within the single-band cuprate strange metal: one with coherent quasiparticles and another exhibiting Planckian dissipation.

5 | PERSPECTIVES IN ELECTRONIC TRANSPORT

Neither conventional methods nor theories based on the idea of quantum criticality underlying the unusual features of strange metals can account for the full NFL phenomenology.^[9,39,41] In the absence of a comprehensive theory,^[39] experimental data are often interpreted using Drude-type expressions for transport coefficients^[8,66,135] (a controversial approach^[136] since the Drude formula involves an “effective mass,” which is not known a priori; at low temperatures, it appears to diverge if extracted from specific heat^[137]; on general grounds, it is unclear what is the meaning of “mass” if there are no well-defined quasiparticles) and the generalized Einstein relation for conductivity defining “diffusivity.”^[66,107] The results can be expressed in terms of general bounds on the diffusivity, conductivity, and their scaling with temperature.^[66,138]

The role of the electron–phonon scattering in strongly correlated systems remains poorly understood. Recently, it was suggested that the linear-in- T resistivity could arise entirely due to electron–phonon scattering (under the assumption of a “dilute metal” with unusually low Bloch–Grüneisen temperature).^[44] In fact, thermal diffusivity data^[107] support the role of electron–phonon scattering in establishing the Planckian scale at high temperatures and draws parallel with heat transport in insulators, where a Planckian bound can be found.^[139] However, it remains unclear how to explain the linear resistivity data at even lower temperatures.^[41] Neither the problem of coexistence of two different scattering mechanisms (which somehow manage to yield the same slope in the temperature dependence of ρ) nor the possibility of electron–phonon scattering not contributing to transport have been studied in sufficient detail.^[41] It seems that electron–phonon scattering represents a true puzzle behind the strange metal phenomenology. It seems unlikely that a complete theory of NFL metals can be constructed without understanding the fate of the electron–phonon scattering.

Another parallel exists with the conjectured lower bound on the shear viscosity to entropy density ratio^[140] bringing about interesting connections to “holographic metals”^[141] (a direct calculation^[142] of the optical conductivity in holographic metals support the Drude-type phenomenology) and the hydrodynamic approach to transport. Indeed, several authors have suggested that the linear-in- T resistivity has hydrodynamic origin.^[65–67,143]

To conclude, electronic transport phenomena are far from being well understood. Simple models where a single scattering mechanism is responsible for the observed behavior may describe “simple” (or “conventional”) metals but seem to fail in more complicated compounds. Even in simple cases, temperature dependence of resistivity is too crude of a measure: similar dependences could be achieved in multiple models, while specific results for particular mechanisms may be strongly affected by material properties including the form of the Fermi surface.

A hydrodynamic approach to electronic transport may be a way forward. Long-wavelength physical properties are the most likely to be disentangled from short-distance complications of material structure (complexity of the unit cell, etc.). Adding an additional scattering mechanism (such as the electron–phonon interaction without which any theory of NFL metals will be incomplete) is relatively straightforward. The concept of a “relaxation rate” is well defined and is independent of microscopic features, such as the effective mass. At the same time, the temperature dependence of relaxation rates and hence the resulting resistivity has to come from an underlying microscopic theory. But it seems that developing such a theory is easier with the macroscopic approach serving as a guiding hand.

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Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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