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Composition of an Optimal Portfolio in the Capital Market - Elton & Gruber Model in Portugal's Capital Market

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Abstract: In order to maximize their utility function, investors select some assets over others by choosing the ideal portfolio that will maximize their wealth. Each asset is chosen taking into account the relationship between the risk of that particular investment (usually measured by variance)- and the return it can offer, as well as the risk between this and other assets (as measured by covariance).

The purpose of this work was to build an optimal portfolio using data on PSI-20's stock prices (2008-2016) where investors are aware of risk and want to minimize it. For this purpose, an optimal portfolio's comparison in the period between 2004-2007 was conducted. This period was referred to as the financial pre-crisis, compared to the optimal portfolio obtained in the period after the financial crisis (2008-2016).

The methodology used to estimate the expected profitability of each asset that makes up the PSI-20 was obtained by extracting the historical quotations from the Euronext Lisbon website. The Elton & Gruber model was used in order to determine the optimal portfolio, as well as the assets that should be part of it.

In the period after the financial crisis, it can be verified in the optimal portfolio's composition that, in the periods after the financial crisis and the financial crisis, there were no stocks to be included in the optimal portfolio, and an analysis in smaller periods was made. In the post financial crisis period actions were found with an attractiveness index superior to the cut-off point, which would lead them to be included in the optimal portfolio, and it was verified that the large distribution sector with (32.15%) has the greatest weight in the optimal portfolio, considering also the Oil and Gas (19.95%), Banking (11.84%) and Production (8.09%) sectors. While addressing shorter periods in pre financial crisis period, no asset was included in the optimal portfolio's constitution.

Key words: stock markets, portfolio, risk, profitability, financial crisis.

1. Introduction

The investment portfolio is considered as a set of assets, financial or not, allocated to a particular investor, that allow to compose a certain portfolio of investment.

In a context of uncertainty and with scarce resources, investors repeatedly have to select investment projects from a large investment opportunity offered by the capital market.

In the face of this situation, and taking into consideration the stock's market complexity, a high level of knowledge on the part of the financial analyst is required in order to obtain strategies that allow risk minimization and / or maximization of the investor's return, thereby achieving both high and low periods. These investors may have: (i) high risk aversion and are strongly concerned with their safety, requiring considerable increases in profitability in view of possible increases in risk, or (ii) an average aversion to risk, in which they aim to obtain increases in profitability compatible with any observed risk increases, and (iii) reduced risk aversion, where they tend to underestimate the possible counterparts due to increased risk.

This choice is called portfolio. The investor thus chooses the optimal portfolio taking only into account the average and variance of the return on assets, given that this notion of portfolio efficiency is based on the assumption that the individual's welfare increases with expected profitability and decreases with risk. Thus, its behaviour is conditioned by two components, the yield, which is the mean or expected value of the rate of return's probability distribution associated with a security or portfolio of securities corresponding to the potential profitability of the application, on the other hand, the risk, that is presented by the variance or standard deviation of the probability distribution of the rate of return associated with a security or securities portfolio corresponding to the risk. We can also say that a portfolio is only efficient if, for the same level of

risk, there is no other portfolio that allows a higher expected return.

2. Literature Review

Markowitz (1952) expanded the new financial theory's horizons by linking the valuation and stock selection problem to that of portfolio management, with its model being one of the most used. His approach to the selection of portfolios, which he called efficient, was based on the stocks' expected profitability and the volatility in obtaining these returns, in other words, the portfolio risk. The purpose of his model is to combine the stocks in a portfolio to reduce risk for the same level of profitability.

A rational investor wants to optimize the expected profitability and minimize the risk to which he is subject, in view of this, and given that investors have scarce resources, they will have to choose a composition of the investment portfolio to achieve their objectives. To do this, investors must select the assets in which they are to invest, as well as the proportions of the investment to be made in each of these assets.

Markowitz (1959) argues that for the investor, the expected return and the expected returns' volatility are the key aspects in trying to establish an optimal portfolio.

Martins and Fernandes (2003, page 221) show that "The construction of general equilibrium models allows a relevant risk measure to be obtained for each security, as well as the relation between expected profitability and risk for each asset when markets are in balance".

Although the present theory is anchored in Markowitz's portfolio theory and Sharpe's (1963) market model, the formalization of the CAPM, as we know it today, stems essentially from the autonomous contributions of Sharpe (1964), Lintner (1965) and Mossin (1966). These contributions were to a certain extent complementary, because although they are all focused on the way in which asset prices are determined, they contain different perspectives of analysis and, at the same time, different levels of mathematical complexity. The fundamental idea underlying the CAPM is that, in equilibrium, the market compensates investors according to the level of risk assumed in their investment. However, since part of an asset's total risk can be eliminated by diversification, only the non-disposable risk part is remunerated, with the risk prize of a particular security directly related to that security's contribution to an efficiently diversified portfolio.

In this model, the price of a financial asset is the result of the market's level of risk associated with that asset, taking into account that this risk allows determining the level of profitability desired by the investor. On the other hand, the concept of risk premium plays a major role in the CAPM, since the investor intends to be compensated by investing his savings in risky assets as an alternative to risk-free assets.

Thus, we obtain the following equation of the CAPM model:

$$E(r_i) = r_f + [E(r_m) - r_f]\beta_i$$

Equation 1: Expected profitability

In order to determine the optimal portfolio, taking into account the model presented by Elton and Gruber (2011), it becomes necessary to accept the single index model and the constant correlation model as a source of the covariance structure among the various assets.

The model presented by Markowitz (1952) is valued by Elton, Gruber and Padberg (1976), however, they point out the obstacles of operation at the time of its development and the unavailability of technological resources.

Elton et al. (1976) present some operational drawbacks in the development of the Markowitz model, such as some difficulty in estimating the data necessary for its implementation; too much time spent and associated costs, since the model should be obtained with the help of quadratic programming; and presenting difficulties in preparing and instructing professionals on the choice of portfolios in order to understand the importance of the relationship between risk and return, starting with return rates, covariance and standard deviation.

Samanez (2006) states that the Markowitz model (1952) requires estimates of each pair's correlations of securities that allow the constitution of a portfolio. This process requires that for this the analyst has a certain level of understanding in the construction as well as in the interpretation of the covariance matrix, increasing the number of assets involved the level of complexity is increased.

Elton and Gruber (1995) defend that the model of selection of optimal investment portfolios has the main advantage of greater easiness in the construction of the model. The model is presented as an appropriate method when considering the single index model, being the best way to present the covariance structure among the rates of returns of the assets.

According to Reilly and Norton (2008), covariance is a measure that can be affected by the variability between the two individual return indices. In view of this, it is verified that if the value is negative, we are facing the existence of a weak negative correlation between the indexes if the two rates are volatile.

Elton and Gruber (2011) state that Treynor's attractiveness index or single index model is the process by which it is used to identify the assets that will be selected for the optimal portfolio and has as main objective to obtain results similar to those obtained with quadratic programming. It is known, through the attractiveness index, the larger this indicator the greater the expected profitability per unit of systematic risk (Tosta de Sá, 1999).

According to Elton et al. (1976), after having determined the proportions to be invested in each security that will compose the optimal portfolio, the necessary calculations must be made to determine the expected return and risk of the portfolio. Thus, the expected return on the optimal portfolio is obtained as follows:

$$\bar{R}_p = \sum_{i=1}^{N} X_i \bar{R}_i$$

Equation 2: Expected profitability

And,

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{N} X_i \sigma_{ei}^2$$

Equation 3: Portfolio risk

The model presented (Elton, 2011), shows that the sum of the weights of the securities composing the portfolio should be equal to 1, that is,

$$\sum_{i=1}^{N} |X_i| = 1$$

Equation 4: Weight of assets

Some empirical studies have shown that in most cases the value of profitability estimated by the CAPM deviates considerably from the observed prices. In this sense, models that included more than one portfolio of assets were developed in order to measure market risk. These models are called multifactor models, taking into account that each of these portfolios is identified as a risk factor, the APT model can be understood as a multifactor model so far as it involves several risk factors. However, it is possible to individualize it, given that it is not formalized in the expected profitability / standard deviation space, nor does it require the prior identification of the market copy portfolio, which will happen in the contributions detailed in this point (Neves and Quelhas, 2013) .

Novy-Marx (2013) identified that companies with high profitability generate significantly higher average returns than those with low profitability, which leads to the conclusion that the profitability factor has a greater explanatory power for returns than for profits. Besides, Aharoni, Grundy and Zeng (2013) concluded, through the tests carried out, that there is a negative relationship between investments and return, that is, they verified the existence of the factor "Investment". In this way, Fama and French (2015) presented the five-factor model, since the average returns related to profitability and investment are left unexplained by Fama and French (1993) three factor model, which leads the authors to examine a model that aggregates profitability and investment factors to market factors, size and B/M to the three-factor model.

Chan, Hamao and Lakonishok (1991), Fama and French (1998, 2012), Griffin (2002), Hou, Karolyi and Kho (2011), identify d and B / M patterns in international stock returns. In order to study how international returns relate to profitability and investment, Titman, Wei and Xie (2013) show that high investment is accompanied by low average returns in many markets. Sun, Wei and Xie (2013) and Watanabe, Yu, Yao and Yu (2013) confirm this result and show that higher profitability is associated with higher future returns.

Chordia, Goyal and Shanken (2015) argue that there is evidence of positive beta prizes on Fama and French (2015) profitability and investment factors, a negative prize on the size factor and a positive prize in the market.

Fama and French (2016) argue that the five-factor model significantly improves performance for average return anomaly patterns. However, different regions present different types of anomalies, which implies that the importance of a particular Factor differs in different regions, such as the factors value, profitability and investment are strong for North America, Europe and Asia-Pacific, but for Japan there is little relation of average returns with profitability and investment (Fama and French, 2012, 2016).

Chordia, Goyal and Shanken (2015) have demonstrated that by testing for a sample of NYSE, AMEX and NASDAQ stocks during the period 1963-2013, which results in evidence of a positive beta premium on profitability (RMW) and investment factors (CMA), a negative prize on the dimension factor (SMB) and a less robust positive prize in the market, however no evidence was found for the free market factor (HML) or the momentum (MOM), on the other hand, they found that the rates estimated to zero beta exceeded the risk-free rate by at least 6 percentage points.

Fama and French (2015) find that the GRS (Gamma-ray spectra) test rejects all the models considered, meaning that it indicates that all models presented are incomplete descriptions of the expected returns. However, Fama and French (2015) observed that the five-factor model, when compared to the three-factor model, does not only present smaller GRS statistics, but also the average of the intercepts is smaller, which indicates that the five-factor model is more adequate to explain the stocks' return.

The five-factor model of Fama and French (2015) is very important to explain the equities return, since it can explain between 71% and 94% of these returns verified on the NYSE, so we can say that the main variables are already found in the model and the introduction of new variables would only have marginal gains.

3. Methodology

In order to verify the characteristics in terms of efficiency in the Portuguese market, it was attempted to analyse the listed companies in PSI-20, in order to verify the portfolio's efficiency.

The methodology used was the Elton & Gruber model and followed two main steps: (1) identification of the market portfolio proxies: for this purpose, the PSI-20 index was observed, in particular the quotations of the companies that compose this index, between May 2008 and May 2016, using the weekly data that were extracted. For this purpose, historical data was used in order to estimate the returns and expected risk for each of these companies that make up the PSI-20, by the fact that there is a great subjectivity and inherent difficulties in its prediction, based on the assumption that these historical data are relevant, which leads us to believe that these data correspond to a reasonable representation of what may occur without a future.

4. Analysis and Results Discussion

4.1. Post-financial crisis period

At the time of analysis, the PSI-20 index was quoted by only 18 companies. However, for our analysis, three companies were withdrawn for not being provided for some of the periods concerned, as is the case of CTT, EDP Renováveis and Montepio, which will not be the subject of our analysis; (2) Determination, by the model and Elton & Gruber: for this point we intend to build the minimum variance portfolio. To do so, and in order to comply with the assumptions of the model, the variance and expected returns of the securities that are represented in the index under analysis, the beta of each asset and the respective attractiveness index were calculated, and finally, the cut-off point for each of the assets.

The database refers to the quotations of the companies that make up the PSI-20 in May 2016:

Table 1

Descrição empresas PSI-20			
Simbolo	Nome	Simbolo	Nome
ALTR.LS	ALTRI	JMT.LS	JERONIMO MARTINS
BCP.LS	BCP R	MPIO.LS	CEMG
BPI.LS	BANCO BPI R	NOS.LS	NOS
COR.LS	CORTICEIRA AMORIM	PHR.LS	PHAROL
CTT.LS	CTT	RENE.LS	REN
EDP.LS	EDP-ENERGIAS R	SEM.LS	SEMAPA R
EDPR.LS	EDP RENOVAVEIS	SON.LS	SONAE R
EGL.LS	MOTA ENGIL	SONC.LS	SONAE CAPITAL
GALP.LS	GALP ENERGIA B	NVG.LS	NAVIGATOR COMPANY

The PSI-20 index is based on the 20 largest listed companies on Euronext-Lisbon. The data base consists of the weekly returns during the period from May 26th, 2008 to May 23rd, 2016. The data presented will exclude companies that do not present stock prices for a period of more than 8 years. Having been removed from this analysis, CTT, EDP Renováveis and Montepio, as stated previously.

In order to obtain the composition of the portfolio that minimizes risk, each assets quotations data that make up the PSI-20 were obtained using data obtained from Yahoo Finance (finance.yahoo.com), in which the expected returns for each of the assets comprising the PSI-20 were determined by using the weekly quotes in continuous returns using the formula:

$$t = ln\left(\frac{x_t}{x_{t-1}}\right)$$

Equation 5: Determination of expected profitability

Based on the closing prices of each Friday for a period between May 26th 2008 and May 23rd 2016.

Considering the following returns which are presented in table 2:

Table 2

Average profitability of listed companies in PSI-20	
Listed Companies	**Average Profitability**
ALTRI-SGPS	-0.09%
BCP	-0.70%
BPI	-0.23%
CORTICEIRA	0.36%
EDP	-0.08%
GALP ENERGIA	-0.08%
JERONIMO MARTINS	0.26%
MOTA ENGIL	-0.30%
NOS	-0.02%
PHAROL	-0.99%
REN	-0.04%
SEMAPA	0.05%
SONAE	-0.04%
SONAE CAPITAL	-0.21%
NAVIGATOR	0.05%

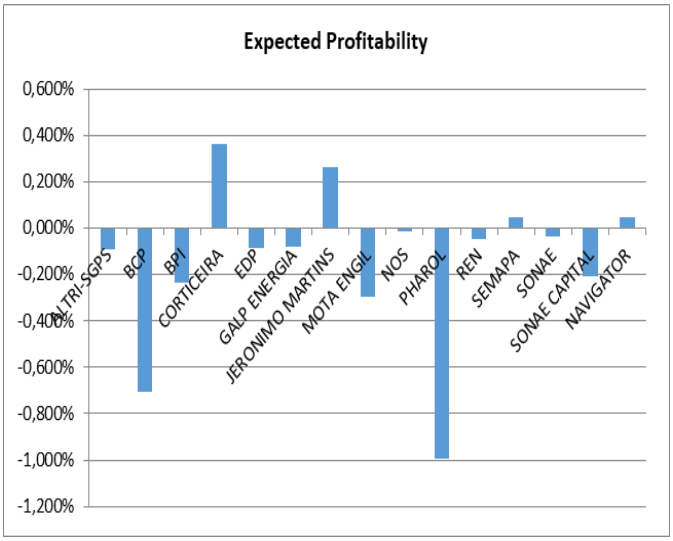
From the above table, we can see that of the 15 companies listed in the Portuguese PSI-20 index, only four of these had a positive average return in the period analysed, which we will now refer in descending order of their respective returns, Corticeira which had the highest average profitability in this period, about 0.36%, JERONIMO MARTINS with a profitability of 0.26%, with the remaining two companies presenting a positive profitability, however very close to zero, SEMAPA, with around 0.047% and NAVIGATOR, with an expected return of 0.046%.

It should be noted that these companies, with average returns for the period under consideration, are from quite different sectors.

By exploring the data presented in Table 2, we will determine the minimum variance portfolio.

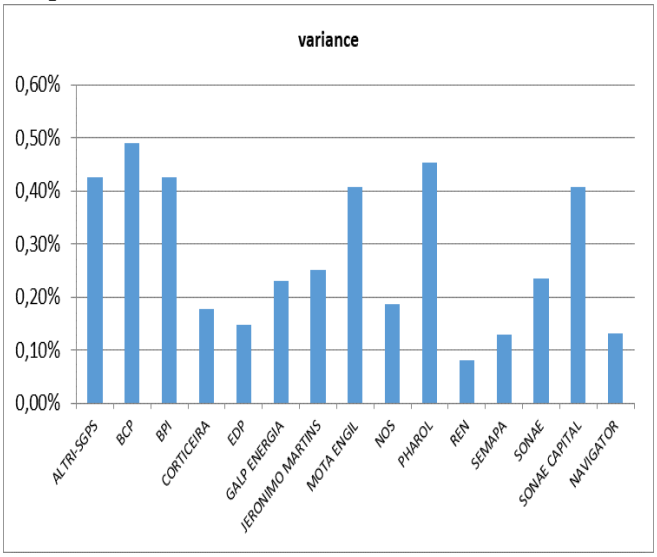
Graphics 1 and 2 introduce the analysis of securities assigned to PSI-20, in terms of expected return and risk, measured by variance. The graphics are presented for the period from May 2008 to May 2016. On the other hand, we can also see in graphic 2 the risk for each of these companies.

Graphic 1



As explained in the previous chart, we can verify the expected profitability for each of the assets that make up the PSI-20.

Graphic 2



The portfolio was optimized in order to obtain the minimum variance portfolio, determining first the market's expected return and market return in order to decide whether to buy or sell the stock. Thus, those actions that present a return expected by the market that is superior to the market return the decision will be to buy the stock, as shown in Table 3.

Table 3: Market Return

Stock	Market Return	Market's expected return	Decision
	(R_m)	$R_i = R_f + (R_m - R_f)\beta$	
BCP	-0.001820183	-0.00531507	Buy
BPI	-0.001820183	-0.004367774	Buy
MOTA ENGIL	-0.001820183	-0.00468829	Buy
PHAROL	-0.001820183	-0.002559403	Buy
SONAE	-0.001820183	-0.002711336	Buy

After the decision to buy or sell a particular stock, the stocks attractiveness index represented in PSI-20, which can be found in Table 4, was determined.

Table 4: Attractiveness Index

Stock	Attractiveness Index
	$(\bar{R}_i - R_f)/\beta_i$
JERÓNIMO MARTINS	-0.004052106
SONAE	-0.005889572
BPI	-0.006465415
CORTICEIRA	-0.006520681
MOTA ENGIL	-0.006746238
GALP ENERGIA	-0.007016576
NOS	-0.007031792
EDP	-0.008110859
SONAE CAPITAL	-0.00885161
BCP	-0.009207466
THE NAVIGATOR	-0.009350876
SEMAPA	-0.009693313
REN	-0.01270122
PHAROL	-0.014757409
ALTRI	-0.027844468

After obtaining the index of attractiveness, in which the ordering and determination of the attractiveness index technique was used, in order to be submitted to the model of Elton et al. (1978) to effectively determine the stocks forming the portfolio.

Thus, the first stage is completed, which consists in placing the stocks that will be included in the optimal portfolio in descending order of attractiveness index.

For the period of analysis between 2008-2016, the attractiveness index is negative for all actions, and this phenomenon is explained by the financial crisis of 2008.

The second stage is to invest in actions in which the attractiveness index is higher than the cut level. For this, the cut-off point was determined, that is, the point that identifies the actions that should compose the optimal portfolio.

(Elton et al., 1978) show that for an action to incorporate the optimal portfolio it should have an attractiveness index higher than the cut-off point, demonstrating that those actions whose attractiveness index is below this point should be excluded.

For the 2008-2016 period, all PSI-20 stocks have an attractiveness index inferior to the less than cut-off, which through the Elton & Gruber model (2008) leads us to reject the inclusion of any stock in the optimal portfolio.

4.2. Pre-crisis financial period

In order to compare the optimal portfolios data before the 2008 financial crisis, we optimized our portfolio to verify that before the crisis there were actions that could be included in the optimal portfolio, actions that had an index of attractiveness higher than the cut-off point.

The portfolio was optimized in order to obtain the minimum variance portfolio determining, at first, the expected market return and the market return, to decide whether to buy or sell the stock, as shown in the Table 5.

Table 5: Market Return

Stock	Market Return	Market's expected return	Decision
	(R_m)	$R_i = R_f + (R_m - R_f)\beta$	
BCP	0.003088	-0.001429368	Comprar
MOTA ENGIL	0.003088	-0.001585809	Comprar
SONAE	0.003088	-0.007160395	Comprar

After the decision to buy or sell a particular stock, the attractiveness index of the stocks represented in PSI-20, which can be found in Table 6, was determined.

Table 6: Attractiveness Index

Stock	Attractiveness Index
	$(\bar{R}_i - R_f)/\beta_i$
SONAE	-0.014547869
MOTA ENGIL	-0.01791648
BCP	-0.021105979
EDP	-0.025323946
SEMAPA	-0.02606064
THE NAVIGATOR	-0.030923963
PHAROL	-0.03162381
JERÓNIMO MARTINS	-0.032556144
BPI	-0.039236347
NOS	-0.039808485
CORTICEIRA	-0.052789619

This way, the first stage, which consists in placing the stocks that will be included in the optimal portfolio, in descending order of the attractiveness index, is completed.

For the 2004-2007 analysis period, the attractiveness index is negative for all actions, and this result is surprising compared to what would have been expected in the period before the financial crisis.

In order to complete the second stage, the cut-off point was determined to verify if the attractiveness index in this period would be higher than it, which would lead to these actions being incorporated in the optimal portfolio. As the attractiveness index is lower than the cut-off point, the incorporation of these stocks into the optimal portfolio was rejected.

It was therefore considered appropriate to include a section where we intend to verify if in the last year before the financial crisis, compared to the year 2016, that corresponds to our last year of analysis, there were actions that could compose the optimal portfolio, using the model of Elton & Gruber (2008).

4.3. Portfolio analysis in shorter periods

4.3.1. Period after financial crisis

In order to verify the actions to be included in the optimal portfolio, it was found in the post-financial crisis period, what actions should be included in the portfolio, if we were to analyse the data for an annual period, so that we could verify if there were still no assets to be included in the optimal portfolio.

Thus, for the year 2016, it was analysed whether the attractiveness index is higher than the cut-off point, which would lead to the inclusion of these stocks in the portfolio. After this step, we determined the weight of each asset in the portfolio, the return of the portfolio, beta of the portfolio and the risk of the portfolio for those stocks whose attractiveness index was higher than the cut-off point. Table 7 verifies the above mentioned data:

Table 7: Optimal Portfolio composition

Stock	$X_i = \left(\frac{Z_i}{\sum Z_i}\right) * 100$	Optimum Portfolio Return	Optimal portfolio beta	Optimal Portfolio Risk
		$\bar{R}_p$	β_p	σ^2_p
CORTICEIRA	8.09%	0.06%	0.025	0.016%
SONAE CAPITAL	27.97%	0.28%	0.236	0.068%
JERÓNIMO MARTINS	32.15%	0.24%	0.273	0.060%
GALP ENERGIA	19.95%	0.08%	0.195	0.043%
BPI	11.84%	0.04%	0.102	0.025%
Portfolio	100%	0.69%	0.831	0.291%

4.3.2. Pre-crisis financial period

In order to compare the optimal portfolio obtained in the pre-crisis financial period (2004-2007), we analysed for 2007 if there were assets to be included in the optimal portfolio, since in the period analysed (2004-2007) there were no assets with an index of attractiveness higher than the cut-off point.

The first step culminated in the determination of the attractiveness index, as proposed by Elton & Gruber (2008). Obtaining the attractiveness index for each action organized in descending order, as presented previously (Table 8).

Table 8: Attractiveness Index

Stock	Attractiveness Index
	$(\bar{R}_i - R_f)/\beta_i$
SONAE	-0.020943959
MOTA ENGIL	-0.028963454
BCP	-0.031835507
SEMAPA	-0.035912942
THE NAVIGATOR	-0.043944955
JERÓMINO MARTINS	-0.044282429
EDP	-0.046202494
BPI	-0.077417687
NOS	-0.080499773
PHAROL	-0.082274954
CORTICEIRA	-0.130914962

In the second step, the cut-off point was determined, and the attractiveness index was lower than the cut-off point, which led us to reject the inclusion of all the stocks in the optimal portfolio.

5. Conclusions

For this study, the Elton & Gruber Model applied to the PSI-20 index was used, with the main purpose of analysing the stocks that had a high attractiveness index, so that they could be included in the optimal portfolio.

After verifying the inclusion of shares in the optimal portfolio, the portfolio's profitability and risk were analysed, previously determining the weight of each of these assets, whose attractiveness index was higher than the cut-off point.

Analysing the portfolio it was concluded that no action would be included in the optimal portfolio, since the attractiveness index was lower than the cut-off point for any one of the shares in the period 2008-2016. In the period 2004-2007, the pre-crisis financial period, the portfolio was analysed and the same conclusion was reached.

Finally, in order to verify if we analysed the periods separately, that is, analyse the license only to the last year of each period, we verified that by the year 2007 there was no action to be included in the optimal portfolio, however, in the year 2016, the last year of the period after the financial crisis, five assets were obtained to be included in the optimal portfolio, that is, with an index of attractiveness higher than the cut-off point. The portfolio was composed of the following companies: Jerónimo Martins (32.15%), Sonae Capital (27.97%), Galp Energia (19.95%), BPI (11.84%) and Corticeira).

It should be noted that the large distribution sector has the greatest weight in the optimal portfolio, also considering the Oil and Gas, Banking and Production sectors.

For future studies related to this topic it is suggested that another model to be implemented in order to test these hypotheses, and to verify the optimal structure for the portfolio or, on the other hand, to perform a comparison of the PSI-20 index with other indexes of the European market to verify market efficiency.

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