Quantum Robotics, Neural Networks and the Quantum Force Interpretation

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Abstract

A future quantum technological infrastructure demands the development of quantum cyber-physical-cognitive systems, merging quantum artificial intelligence, quantum robotics and quantum information and communication technologies. To support such a development, the current work introduces a new interpretation of quantum mechanics, grounded on a link between quantum computer science, systems science and field-based computation. This new interpretation is applied to quantum artificial neural networks, with examples implemented experimentally on IBM's five qubit transmon bowtie chip, accessed via cloud using IBM Q Experience, illustrating how quantum neural computing can be implemented on actual quantum computers. A new form of quantum neural machine learning, based on a quantum optimization of a conditional utility function is also introduced and applied to quantum robotics, where a quantum robot, characterized by an interface and a multilayer quantum artificial neural network, interacts with a quantum target, changing the target's dynamics adaptively, based upon the quantum optimization dynamics, computing the optima for a performance measure and changing the target's dynamics accordingly.

Keywords: Quantum Robotics, Quantum Neural Machine Learning, Quantum Force Interpretation, Quantum Optimization

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1 Introduction

The current accelerating technological change is the result of the convergence between Information and Communication Technologies (ICTs) and the Cognitive Technologies (CogTech), namely, Artificial Intelligence (AI), powered by new machine learning algorithms, synergized with the Third Industrial Revolution's ICTs led to the cyber-physical-cognitive (CPC) systems' revolution[1] that, in turn, gave momentum to a Fourth Industrial Revolution (Ivancevic *et al.*, 2018; Schwab, 2017).

A CPC system is the result of the integration of CogTech in the ICTs. In today's accelerating transformations, CPC systems form the basis for a distributed intelligent artificial cognition and includes smart devices embedded in different physical systems, linked to the Internet (Schwab, 2017), such that the current technological transformation can be described by an acronym SDO (*Smart*, *Digital*, *Online*). However, quantum technologies, in particular quantum computers, threaten the current infrastructure of this technological basis, in the sense that quantum computers can solve cryptographic problems that are central for secure communications, making cryptographic *quantum resistance* one of the major problems to the SDO-based transformation (Bowmeester *et al.*, 2000; Chen, *et al.*, 2016; Cheng *et al.*, 2017).

Thus, as quantum computation becomes feasible, the ICT infrastructure will have to become quantum-based and dominated by research on quantum secure communications. However, quantum ICTs are not enough, the CogTech pillar will also have to be developed. As the Third Industrial Revolution's ICTs merged with CogTech, leading to the ground basis for the Fourth Industrial Revolution, so too will the new quantum ICTs have to merge with quantum CogTech in order to be able to support the further exponential expansion of the cyber-physical-cognitive revolution.

Quantum CPC systems, therefore, need to become a research branch of quantum technologies, linking quantum computation, quantum information science and quantum AI. The current work is aimed at such a branch of quantum technologies. Namely, we address the concept of a quantum robot

[1] We are using, within the technological context, the concept of cyber-physical-cognitive system worked in Ivancevic (*et al.*, 2018), which addresses the integration of the ICT's in physical systems (the cyber-physical technological transformation) and of the CogTech (cyber-physical-cognitive transformation). Since the Fourth Industrial Revolution involves the merge of the ICTs with the CogTech, the CogTech pillar is explicitly addressed in the acronym CPC (cyber-physical-cognitive).

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as a fundamental basis for the development quantum CPC systems (Tandon *et al.*, 2017; Ivancevic *et al.*, 2018).

Quantum robotics is not new, indeed, one can trace it back to early efforts of introducing alternative interpretations of quantum theory, in particular, to Everett's PhD Thesis (Everett, 1957; 1973), where the author used cybernetics and the concept of an automaton as a basis to model the observer and the observation act as a quantum interaction. While, in classical mechanics, the observation act does not affect the behavior of the observed system, in quantum mechanics observation involves a form of quantum interaction by which observer and observed become entangled, a main point in Everett's work (Everet, 1957; 1973), who explicitly addressed this entanglement dynamics by incorporating a general form of observer that is formulated, within the formalism of quantum theory, in terms of its cognitive record of the observed system.

To physically address the observer, within the formalism, Everett postulated an abstract model of an automaton, in order to strip down the observation act to its bare essentials, namely: an interaction by which the observer's memory contents become entangled with the target, in its interaction with the target. In this way, Everett established quantum measurement as a form of computation that can be addressed within the formalism of wave dynamics.

A basic automaton with a memory addressed as a quantum degree of freedom, obeying the rules of quantum dynamics, would, thus, constitute a model for any observer.

An expansion on the basic "observer" automaton was introduced by Benioff (1998a, 1998b), who proposed a concept of a quantum robot as an artificial agent that not only measures the environment but also carries out tasks, interacting with the environment to lead to particular changes on that environment, fulfilling specific goals. This introduces a different dynamics, in the sense that a quantum robot can function as a complex artificially intelligent measurement apparatus.

Dong *et al.* (2006) addressed the efficiency of a quantum robot over a classical robot, using an architecture for quantum robots based on three fundamental parts: multi quantum computing units, a quantum controller/actuator and sensory units.

A connection between quantum robotics, nanotechnology and quantum Artificial Life (ALife) research, was established in Tarasov (2009). Tarasov's work is particularly relevant for the issue of the development of quantum

CPC systems within the context of quantum nanotechnology, including the possibility of interaction and nanoscale management of quantum systems. More recent works have focused on machine learning and quantum robotics (Siomau, 2014; Paparo *et al.*, 2014; Abdulridha and Hassoun, 2017).

Quantum robotics is specific, within quantum technologies' research, in the sense that it deals with four main points that need to be dealt with, for an effective development quantum CPC systems solutions, as well as for the expansion of the scientific basis of quantum robotics, these main points are:

- Quantum CPC systems are not closed systems, rather, the interaction with the environment is a key feature of these systems;
- Entanglement is a key dynamics for the computation of quantum CPC systems to work;
- AI must be integrated in quantum CPC systems, so that the computation not only needs the entanglement to work but, also, it takes advantage of that entanglement to produce adaptive changes in the environment;
- Field-based cognitive science is needed to effectively address the computational basis of quantum AI.

In order to address these four main points, within the context of quantum robotics, however, we need an interpretation of quantum mechanics that has a fundamentally systemic basis, is consistently closer to cognitive science and where entanglement, nonlocality and the dynamics of quantum fields play a fundamental role.

This interpretation, which we call the quantum force interpretation (QFI), is introduced in section 2., the advantage of QFI is that it allows for a working basis that makes it effective in connecting fundamental concepts from cognitive science and computer science with the quantum formalism. A specific point of QFI is that it is a computational theory at its core, that is, it addresses the dynamics of quantum fields from a specific computational basis, furthermore, this computational basis works with a dynamics without states, which means that the input and output of a quantum computation is actually a transition from one dynamics of the field to another dynamics.

The QFI is addressed in section 2., working mainly with Quantum Artificial Neural Networks (QuANNs) as a basic example to illustrate the fundamental points of the interpretation, in this way, the link with QuANNs and quantum AI is already developed in that section, allowing us to simultaneously present the fundamental points of the interpretation and to link these to quantum AI, highlighting its connections with cognitive science, computer science and quantum neural computation. The provided examples of QuANNs are also implemented on a quantum computer, with access via cloud, using IBM Q Experience.

In section 3., we introduce a new form of quantum neural machine learning based on the optimization of a conditional utility function and apply this framework to quantum robotics, addressing a quantum adaptive dynamics where the robot, equipped with a multilayer QuANN, not only processes a target quantum system, but is also capable of conditionally selecting actions to adaptively change that target's dynamics, solving an optimization problem. In section 4., we conclude by expanding on main implications of QFI for quantum CPC systems and quantum technologies.

Regarding some notation, given the binary alphabet $\mathbb{A}_2 = \{0,1\}$, we denote by $\mathbb{A}_2^d$ the set of all d-length binary strings. Also, we denote Pauli's operators by $\hat{\sigma}_1$, $\hat{\sigma}_2$ and $\hat{\sigma}_3$, respectively defined, using Dirac's notation, as:

$$\hat{\sigma}_1 = |0\rangle\langle 1| + |1\rangle\langle 0| \tag{1}$$

$$\hat{\sigma}_2 = -i|0\rangle\langle 1| + i|1\rangle\langle 0| \tag{2}$$

$$\hat{\sigma}_3 = |0\rangle\langle 0| - |1\rangle\langle 1| \tag{3}$$

with the *ket* vectors $|0\rangle$ and $|1\rangle$ given by:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

and with the *bra* vectors given by $\langle 0| = |0\rangle^\dagger$, $\langle 1| = |1\rangle^\dagger$. We also define the unit operator on the two dimensional Hilbert space spanned by the basis $\{|0\rangle, |1\rangle\}$ as follows:

$$\hat{1}_2 = |0\rangle\langle 0| + |1\rangle\langle 1| \tag{5}$$

and use the notation $|\pm\rangle$ and $\langle\pm|$ for the *ket* and *bra* vectors:

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \tag{6}$$

$$\langle\pm| = \frac{\langle 0| \pm \langle 1|}{\sqrt{2}} \tag{7}$$

We now introduce the QFI interpretation and how quantum neural dynamics of QuANNs can be addressed within the context of this interpretation.

2 Quantum Neural Dynamics, Echoes and Quantum Force

At the most basic level, quantum theory can be considered as a theory that addresses quantized rhythms, this point can be made clear by considering how energy eigenvalues are related to the reduced Planck's constant $\hbar$, by way of a discrete multiple of a term given by the product of 2π by a frequency ν (expressed in Hertz) and by the reduced Planck constant, the product $2\pi\nu$ gives an angular frequency for a cyclical dynamics, while the reduced Planck constant $\hbar$ is equal to the action per radian.

The importance of rhythms in quantum theory is particularly relevant when one looks at the computational substratum of the theory as well, a substratum that becomes more evident when one explictly addresses models of quantum computing systems.

To make the point explicit, let us consider a QuANN, comprised of d two-level neural activity neurons, where the ground level corresponds to a nonfiring neural activity, and the excited level corresponds to a firing neural activity, such a network can be addressed, in terms of quantum field theory, as a field on a network, where the neural field, associated with the network, is described in terms of a Hilbert space $\mathcal{H}_d$, spanned by the basis:

$$\mathcal{B}_d = \left\{ \bigotimes_{k=1}^{d} |s_k\rangle : s_k \in \mathbb{A}_2 \right\} \tag{8}$$

The neural firing energy level is linked to the network's total neural firing frequency, so that we can introduce the neural firing frequency operator:

$$\hat{\nu} = \frac{\hat{1}_2 - \hat{\sigma}_3}{2}\nu \tag{9}$$

and the local Hamiltonian, for each neuron, can be defined as (Gonçalves, 2017):

$$\hat{H}_k = \hat{1}_2^{\otimes(k-1)} \otimes \hbar 2\pi\hat{\nu} \otimes \hat{1}_2^{\otimes(d-k)} \tag{10}$$

for $1 < k < d$, and

$$\hat{H}_1 = \hbar 2\pi\hat{\nu} \otimes \hat{1}_2^{\otimes(d-1)} \tag{11}$$

$$\hat{H}_d = \hat{1}_2^{\otimes(d-1)} \otimes \hbar 2\pi\hat{\nu} \tag{12}$$

The total Hamiltonian is, in turn, given by the sum (Gonçalves, 2017):

$$\hat{H}_{Net} = \sum_{k=1}^{d} \hat{H}_k \tag{13}$$

with the eigenvalue spectrum:

$$\hat{H}_{Net} |\mathbf{s}\rangle = \left(\sum_{k=1}^{d} s_k \right) 2\pi\nu\hbar |\mathbf{s}\rangle \tag{14}$$

where $\mathbf{s}$ is a d-length binary string $s_1 s_2 ... s_d$.

This Hamiltonian has a degenerate spectrum on the basis $\mathcal{B}_d$, since different firing patterns can lead to the same total final neural firing energy.

From to Eqs.(9) to (12), it follows that the local firing frequencies can, in turn, be obtained from the local Hamiltonians as:

$$\hat{\nu}_k = \frac{1}{2\pi\hbar} \hat{H}_k \tag{15}$$

So that the neural network's total firing frequency operator is given by:

$$\hat{\nu}_{Net} = \sum_{k=1}^{d} \hat{\nu}_k \tag{16}$$

which leads to the eigenvalue equation:

$$\hat{\nu}_{Net} |\mathbf{s}\rangle = \left(\sum_{k=1}^{d} s_k \right) \nu |\mathbf{s}\rangle \tag{17}$$

Each neural firing pattern corresponds to a dynamics of the neural field on the network, where each node (neuron) is either firing with the frequency of ν Hz (therefore with an energy of $2\pi\nu\hbar$ Joules) or not firing (the frequency, in this case, is 0 Hz and the energy is zero Joules) (Gonçalves, 2017). This allows us to consider the QuANN as a rhythmic system, where there is a basic rhythm linked to each neuron's firing activity, and the eigenvalue spectrum of the operator $\hat{\nu}_{Net}$ corresponds to the total rhythm of the neural network. The term rhythm comes from the Latin *rhythmus*, which, in turn, comes from the Greek ῥυθμός, synthesizing a sense of order/measure/proportion/symmetry in motion, a flowing motion (ῥέω) that expresses an order.

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The order, in this case, is measured in terms of the firing frequency, which is a measure of a dynamical activity in terms of cycles per second. In this sense, regarding the neural field, we cannot speak of states but rather of rhythmic dynamics, which means that the quanta of the neural field are not "objects" but rather rhythmic patterns expressed by the neural firing patterns, where the total frequency associated with the neural activity is the result of the sum of the firing frequencies of all the neurons.

This is a point that can be raised about quantum theory in general, which puts into question the assumption of a classical mechanical reasoning on dynamics in terms of a transition between states, as a basis for quantum mechanics. While this classical reasoning has been applied to the quantum description, it does not follow from the formalism itself, nor from the empirical level that this is the appropriate language for quantum dynamics, a point that was raised by Baugh, Finkelstein and Galiautdinov (2003).

From a computer science perspective, this is a relevant point, in the sense that we are dealing with a different computational matrix from that which underlies classical mechanics. From a computational standpoint, classical mechanics can be approached in terms of finding dynamical rules that link states to states, so that given the state, at a given time, we can find out the state at a future time, such a state can be considered as the output of the system's computation given the past input.

This pressuposes a stability of a state at each point in time, namely, that the system is stable enough for there to be a state that fully characterizes it at each point in time. Even if a stochastic dynamics is assumed, within a classical setting, this sufficient stability is still being assumed, that is: that the system must be stable enough for it to have a state and that a state is an appropriate concept when addressing the system and its dynamics.

In the dynamics of quantum systems the stability needed for there to be a state is not present, this point can be raised by considering basic physical quantities like position, momentum and energy. Quantum particles are not classical particles with a position and momentum. Quantum particles are quanta of a field, in this context, a position is a positioning supported by the field's dynamics and it is not stable enough to be a state, since the momentum becomes less and less determined, as the position becomes more and more determined, as per Heisenberg's uncertainty principle.

An assumption of a fixed thing there at a specific position that moves to another position is not consistent with quantum mechanics. A quantum particle is not an autonomous entity like a classical mechanics' particle that is traveling, a point raised by Bohm (1997, [1957]) as a criticism to the pilot wave model.

There is a common misconception that Bohm defended the pilot wave model. Indeed, Bohm worked on the pilot wave model but just as a first approximation. In Bohm (1997, [1957]) the author introduced the model as a first approximation but then criticized it, in particular, in regards to the assumption of a particle being separate from the field, even more, Bohm defended that, *at a lower level*, the *particle does not move as a permanent existing entity, but is formed in a random way by suitable concentrations of the field's energy* (Bohm, 1997, [1957], p.121), we will return to this point further on.

In quantum mechanics, position, momenta and energy need to be addressed as computable properties that may characterize the field's activity and whose values result from the field's activity at each moment. Thus, for instance, in the case of a QuANN, an energy eigenvalue is supported by the neural field's rhythmic dynamics, as we saw above.

While eigenvalues cannot be considered states, one might still argue for a state associated with, for instance, a density operator, and consider the transition from input to output density operators as a dynamics of state transition. However, this is a misconception on the concept of vector and on what the density operator is representing.

As we show next, the density operator is expressing a dynamics of the field and not a state, and, when we consider the unitary propagation, we are actually addressing a unitary propagation of one dynamics to another. The dynamics expressed by density operators plays a functionality in the field's computation, that is, we can consider the density operators in terms of a field's computational dynamics.

2.1 Quantum Artificial Neural Networks, Density Operators and the Quantum Echo

The extraction of probabilities from the formalism of quantum theory, in its most general form, demands the use of a particular type of operator on the Hilbert space used to describe the relevant degree(s) of freedom, these are the density operators: self-adjoint positive trace-class operators with unit trace. Given a relevant Hilbert space $\mathcal{H}$ we, therefore, need to work with the space

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of density operators[2] $\mathcal{D}(\mathcal{H})$ (Giulini, 2003; Kupsch, 2003).

To address the basic quantum dynamics, let us consider a specific subset of $\mathcal{D}(\mathcal{H})$, $\mathcal{S}(\mathcal{H})$ which is comprised of the projectors:

$$|\psi\rangle\langle\psi| \tag{18}$$

where $|\psi\rangle$ is a normalized *ket* vector and $\langle\psi| = |\psi\rangle^{\dagger}$. A projector density is, thus, a projector on the quantum system's Hilbert space $\mathcal{H}$ and can be evaluated for different bases of the same Hilbert space. For instance, let us consider a Hermitian operator $\hat{A}$, obeying the eigenvalue equation, with a non-degenerate eigenvalue spectrum (not necessarily finite in the number of eigenvectors):

$$\hat{A}|a_k\rangle = a_k|a_k\rangle \tag{19}$$

with $k = 1, 2, ...$, where the eigenvectors span the space $\mathcal{H}$. Then, for any density projector $|\psi\rangle\langle\psi| \in \mathcal{S}(\mathcal{H})$, we can expand the corresponding operator in the basis $\mathcal{B} = \{|a_1\rangle, |a_2\rangle, ...\}$ by expanding the projector's *ket* and *bra* vectors in this basis, leading to:

$$\begin{aligned} |\psi\rangle\langle\psi| &= \sum_{k,k'} \langle a_k|\psi\rangle |a_k\rangle \langle a_{k'}| \langle\psi|a_{k'}\rangle = \\ &= \sum_{k,k'} \psi(a_k) |a_k\rangle \langle a_{k'}| \psi(a_{k'})^* \end{aligned} \tag{20}$$

To understand the terms that comprise the density in Eq.(20), we need to consider a systemic concept of vector. Vector, from the Latin term with the same name, means *carrier*. Each carrier, in this case, can be addressed as an *information carrier*, understanding information not in terms of act, fact or pattern but, instead, in terms of a dynamics *towards the act*, *towards the fact*, *towards the pattern* (Madeira and Gonçalves, 2013). The actualization, in turn, needs to be addressed in terms of the concept of *echo*. To address this concept we need to consider the *carriers* that appear in the density operator expansion.

Each term that comprises the sum in Eq.(20) is the product of two *carriers*, namely, a *probe carrier* $|a_k\rangle$ representing a motion proceeding *towards* the final eigenvalue a_k, with an associated amplitude $\psi(a_k)$ and a *response*

[2] For a QuANN, this space is $\mathcal{D}(\mathcal{H}_d)$, the space of self-adjoint positive trace-class operators with unit trace on the Hilbert space $\mathcal{H}_d$ used to geometrize the network's neural field dynamics.

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carrier $\langle a_{k'}|$, representing a motion proceeding from the final eigenvalue $a_{k'}$, with an associated amplitude $\psi(a_{k'})^*$.

Systemically, the density operator, expanded in the basis $\mathcal{B}$, expresses a dynamics aimed at solving a problem: that of selecting the final eigenvalue, this is the final computational output that the quantum system is working for.

The above approach recovers a similar framework to that introduced by the Transactional Interpretation (TI) of quantum mechanics (Cramer, 2016), where the *probe carrier* can be interpreted in the same terms as a *retarded wave* (also known, in TI, as *offer wave*), proceeding forward towards the output eigenvalue a_k, while the *response carrier* can be interpreted in similar terms to TI's *advanced wave* (also known, in TI, as *confirmation wave*) proceeding backwards from the final output eigenvalue $a_{k'}$.

Of the different terms that comprise the density operator, only the diagonal ones coincide with regards to the final eigenvalue, all the others exhibit a mismatch, where the *probing* is not met by a matching *response.* When the *probe carrier* is met by a matching *response carrier*, an *echo* is formed. In the off-diagonal terms we have failure to produce an *echo,* since the *probe dynamics* does not meet a matching *response dynamics.* The *echo* intensities are associated with the diagonal terms, which correspond to squared amplitude modulated projections:

$$\psi(a_k)\,\psi(a_k)^*\,|a_k\rangle\langle a_k| = |\psi(a_k)|^2\,|a_k\rangle\langle a_k| \tag{21}$$

Considering the case of a QuANN and the firing pattern selection problem, the density is given by:

$$|\psi\rangle\langle\psi| = \sum_{\mathbf{r},\mathbf{s}\in\mathbb{A}_2^d} \psi(\mathbf{r})\,|\mathbf{r}\rangle\langle\mathbf{s}|\,\psi(\mathbf{s})^* \tag{22}$$

Each component of the density operator $\psi(\mathbf{r})\,|\mathbf{r}\rangle\langle\mathbf{s}|\,\psi(\mathbf{s})^*$ is the result of a computational dynamics of the neural field. It is important to stress that probe and response dynamics are, both of them, simultaneous computational dynamics of the neural field, that play a role in the field's cognition around the problem of selecting a firing pattern.

Thus, when we consider each term $\psi(\mathbf{r})\,|\mathbf{r}\rangle\langle\mathbf{s}|\,\psi(\mathbf{s})^*$ in the sum (22) the neural field is simultaneously computing in the two directions, such that a probing of the firing pattern $\mathbf{r}$ is computed with amplitude $\psi(\mathbf{r})$ and met by a response for the firing pattern $\mathbf{s}$, with an amplitude $\psi(\mathbf{s})^*$.

An *echo* is formed when the *probe carrier* meets a matching *response carrier*, leading to an echo intensity $|\psi(\mathbf{s})|^2$. Systemically, an *echo*, in this context, can be addressed as a field projective signalizer of an order to be actualized, thus, in the context of quantum fields, it plays a role in the field's cognition, this actualization takes the form of a projection selection for an output eigenvalue. In this case, the squared amplitudes $|\psi(\mathbf{s})|^2$ are associated with a projective intensity for each firing pattern, so that, in a probabilistic description, it is assumed that the neural field evaluates, simultaneously, each alternative, discarding the cases that do not lead to an *echo*, and selecting the final projection, from the projected alternatives, with a probability equal to their respective *echo intensity*.

Returning to the general case, the above results apply to any density operator $\hat{\rho} \in \mathcal{D}(\mathcal{H})$ and not just to the operators in $\mathcal{S}(\mathcal{H})$, indeed, considering the general form:

$$\hat{\rho} = \sum_{k,k'} \rho_{k,k'} |a_k\rangle \langle a_{k'}| \tag{23}$$

the off-diagonal terms correspond to failed echoes, while the terms in the diagonal correspond to the cases where the *probe carrier* meets a matching *response carrier*, producing an echo with an intensity $\rho_{k,k}$. In the special case of a diagonal density, there are no failed echoes, the density is comprised of a sum over the projectors $|a_k\rangle \langle a_k|$:

$$\hat{\rho} = \sum_{k} \rho_{k,k} |a_k\rangle \langle a_k| \tag{24}$$

In this way, we get an account for both the actualization and the off-diagonal terms in the density operator, as well as to how the diagonal terms are linked to Born's probability rule. We do not have, however, a reason for the probabilistic dynamics to follow the *echo*, and not deviate from it. This is postulated in the theory, which has by implication, if taken as such, a fundamental stochastic selection between different alternatives. That is, when a quantum field has to determine the final order to be actualized it does so by way of a random selection.

Assuming the above setting, regarding the density operator, there is, however, another account that can be given for both the "selection" and the probability, this account can be provided by recovering Bohm's proposal, who addressed the squared amplitudes under the concept of a *quantum force*, as we now review.

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2.2 The Quantum Force Interpretation and Quantum Neural Computation

A common misconception regarding Bohm's proposal, as referred previously, is the statement that Bohm defends that there is a particle which is guided by a field in a pilot wave model where particle and field are separate entities, this is not the case, as reviewed previously. Indeed, Bohm (1997, [1957], p.117) considers the pilot wave model as a first approximation, criticizing the assumption of particle and field as being separate and stating that because wave and particle are never found separately, that this suggests that they must be different aspects of a same entity: a field. In the above case of QuANNs, the quanta of energy of the neural field correspond to spikes of neural activity.

Bohm (1997, [1957], p.119, 120) defends that, in what regards quantum mechanics, the fundamental entities are the fields, and that, even in vacuum, these fields are undergoing *violent and very rapid random fluctuations*, that are *not directly observable at the macroscopic level, because they average out* (Bohm, 1997, [1957], p.120), besides these turbulent fluctuations, there are small systematic oscillations that do not average out and that are detected at a higher level, these systematic oscillations lead to certain patterns that form basic discrete (quantized) properties of the fields at this higher level.

Accepting this, and recovering the previous subsection's results, we have a basic emergent statistical pattern in which the probabilities result from an average tendency of the field to follow the *lines of force* whose intensity coincides with the *echo* intensity. If we accept Bohm's proposal and bring it into this context, then, we need to consider a subquantum turbulence in the field's activity to make emerge a systematic pattern that, on average, follows the *lines of force*. In this proposal, then, the field's activity depends on a lower-level subquantum activity, which introduces a basic stochastic behavior in the field's dynamics, thus, explaining the probabilistic description.

Unlike Bohm, however, we do not consider the squared amplitudes under the concept of quantum force, but rather as representing the *echo intensity* for an order to be risen, in this case, the order associated with a specific field dynamics that is characterized by a specific eigenvalue. The concept of *quantum force* enters into the description, when we consider the rising of a specific order, in the sense that, ontologically, a specific eigenvalue can be considered as a pattern/order of a quantum field's dynamics that is risen, and, thus, takes place in act being supported by the field's activity, which means that, systemically, in a quantum field, forces mobilize to rise and support the actualized order.

In this sense, the actualization can follow one of different *lines of force*, leading to one of the specific eigenvalues, with a probability that coincides with the intensity of that *line of force*, an intensity that coincides, in turn, with the corresponding *echo* for that alternative.

A point raised by Bohm (1997, [1957]) is that the rule that states that probabilities are numerically equal to the squared amplitudes holds on average, as an emergent statistical pattern, resulting from a subquantum level turbulence, averaging out at the scale of the quantum descripion.

This means that the risen dynamical patterns result from the field's quantum and subquantum dynamics working in tandem to determine the specific alternative in each case, this field's dynamics tends to make emerge a dynamics that follows the *echoes*, which accounts for the stochastic dynamics associated with the probabilistic description, that is, even if each risen order is determined by the field's quantum and subquantum dynamics, which makes rise a specific dynamical order in each case, the resulting randomness in the variation of the final eigenvalues exhibits an average pattern that follows the *lines of force* whose intensity coincides with the *echo intensities*.

Rather than the fundamental stochastic choice leading to a random actualization for the same conditions, we will assume this later interpretation based on the concept of *quantum force*. We call this interpretation that combines elements of the Cramer's TI and Bohm's proposal: the quantum force interpretation (QFI). It is this interpretation that we now apply to QuANNs.

2.2.1 Quantum Neural Computation, Backpropagation and Holographic Dynamics

Let us, now, consider a sequence of computational gates $\left\{\hat{U}_k\right\}_{k=1}^{N}$ on the Hilbert space $\mathcal{H}_d$ for a QuANN's neural field. Computationally, the problem we will address first regards the transition to a final neural firing pattern, from an initial neural firing pattern, under the sequence of neural computational gates $\left\{\hat{U}_k\right\}_{k=1}^{N}$. In this case, we begin by expressing the quantum computational circuit for the sequence $\left\{\hat{U}_k\right\}_{k=1}^{N}$ by the product of unitary operators that represent the computational circuit:

$$\hat{K} = \hat{U}_N \hat{U}_{N-1} ... \hat{U}_1 = \prod_{k=N}^{1} \hat{U}_k \tag{25}$$

The conjugate transpose of this circuit leads to the reverse order chain:

$$\hat{K}^\dagger = \hat{U}_1^\dagger ... \hat{U}_{N-1}^\dagger \hat{U}_N^\dagger = \prod_{k=1}^{N} \hat{U}_k^\dagger \tag{26}$$

Thus, while $\hat{K}$ propagates forward in the computational circuit, its conjugate transpose $\hat{K}^\dagger$ propagates backward in the computational circuit.

Let us, now, assume that the neural network exhibits, intially, a neural firing pattern corresponding to the projector $|\mathbf{s}_0\rangle \langle \mathbf{s}_0|$, and the problem that the neural network is solving is that of selecting a final neural firing pattern. This means that the density operator for this problem is obtained by applying the two quantum circuits to the projector $|\mathbf{s}_0\rangle \langle \mathbf{s}_0|$ and then expanding in the neural firing basis, leading to the operator structure:

$$\hat{K} |\mathbf{s}_0\rangle \langle \mathbf{s}_0| \hat{K}^\dagger = \sum_{\mathbf{r},\mathbf{s} \in \mathbb{A}_2^d} \left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{s}_0 \right\rangle |\mathbf{r}\rangle \langle \mathbf{s}| \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^\dagger \right| \mathbf{s} \right\rangle \tag{27}$$

Each term that comprises this density operator is the product of the probe carrier $\left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{s}_0 \right\rangle |\mathbf{r}\rangle$ by the response carrier $\langle \mathbf{s}| \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^\dagger \right| \mathbf{s} \right\rangle$. The first carrier's amplitude results from the forward propagation in the quantum circuit from the initial firing pattern $\mathbf{s}_0$ to the final firing pattern $\mathbf{r}$, so that the first carrier probes the firing pattern $\mathbf{r}$ with an amplitude $\left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{s}_0 \right\rangle$. The second carrier's amplitude results from the backpropagation from the final firing pattern $\mathbf{s}$ to the initial firing pattern $\mathbf{s}_0$, that is, it follows the quantum circuit backward from the end to the beginning, which leads to the response carrier $\langle \mathbf{s}| \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^\dagger \right| \mathbf{s} \right\rangle$.

The probe, thus, proceeds from the beginning to the end of the quantum computation, while the response comes from the end of the quantum computation to the beginning. Again, the off-diagonal terms do not form an echo, while the diagonal terms form an echo, with an intensity given by the products:

$$\left\langle \mathbf{s} \left| \hat{K} \right| \mathbf{s}_0 \right\rangle \left\langle \mathbf{s}_0 \left| \hat{K}^\dagger \right| \mathbf{s} \right\rangle = \left\langle \mathbf{s} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{s}_0 \right\rangle \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^\dagger \right| \mathbf{s} \right\rangle \tag{28}$$

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The intensity of each echo is thus the result of the "encounter" of two propagations: one from the input firing pattern to the output, which is associated with the probe, and a second one from the output firing pattern to the input firing pattern, which is associated with the response. The response is a form of backpropagation, such that the intensity of the echoes result from the product of the forward and backpropagating amplitudes for each alternative final neural firing pattern.

In the context of QFI, this introduces a specific form of quantum computation, namely: in the computational system, the forward propagating probe and the backpropagating response play a cognitive role in producing an echo for each final neural firing pattern, this echo plays, in turn, a field projective signalizer role in the neural field's cognition, so that the neural field's quantum and subquantum dynamics make emerge, at the quantum level, a tendecy to follow, on average, the lines of force leading from one neural firing pattern to another one. Statistically, the emergent quantum dynamics, then, follows one of the lines of force with a probability equal to the intensity of that line of force which, in turn, coincides with the respective echo intensity.

As examples of this dynamics, let us consider that the network has three neurons and that the initial density operator is given by the projector $\hat{\rho}_{in} = |000\rangle \langle 000|$, let the neural network structure be comprised of the chain of synaptic connections:

$$n_1 \rightarrow n_2 \rightarrow n_3 \tag{29}$$

with the connection between the neurons n_1 and n_2 and between the neurons n_2 and n_3 being expressed, respectively, by the operators:

$$\hat{U}_{1,2} = |0\rangle \langle 0| \otimes \hat{1}_2 \otimes \hat{1}_2 + |1\rangle \langle 1| \otimes \hat{\sigma}_1 \otimes \hat{1}_2 \tag{30}$$

$$\hat{U}_{2,3} = \hat{1}_2 \otimes |0\rangle \langle 0| \otimes \hat{1}_2 + \hat{1}_2 \otimes |1\rangle \langle 1| \otimes \hat{\sigma}_1 \tag{31}$$

and let us consider the computational chain given by $\hat{U}_{1,2}\hat{U}_{2,3}\hat{U}_0$, where the operator $\hat{U}_0$ is given by:

$$\hat{U}_0 = \hat{U}_{WH} \otimes \hat{U}_{WH} \otimes \hat{1}_2 \tag{32}$$

where $\hat{U}_{WH}$ is the Walsh-Haddamard gate:

$$\hat{U}_{WH} = \frac{\hat{\sigma}_1 + \hat{\sigma}_3}{\sqrt{2}} \tag{33}$$

As an example of entanglement dynamics in the quantum neural computation, it is relevant to consider how the density changes as each gate in the computational chain is applied. Initially, the echo for the neural field is only for a nonfiring neural dynamics. As the first operator is applied, we get for the probe and response carriers:

$$\hat{U}_0 |000\rangle = |+\rangle \otimes |+\rangle \otimes |0\rangle \tag{34}$$

$$\langle 000| \hat{U}_0^\dagger = \langle +| \otimes \langle +| \otimes \langle 0| \tag{35}$$

thus, after the first computational gate, the neural field is probing the firing and nonfiring dynamics with equal weight for neurons n_1 and n_2, likewise, there is a response from a firing and nonfiring dynamics associated with neurons n_1 and n_2, with equal amplitude. The third neuron's carriers have not yet changed. Now, the first synaptic connection to be activated is $n_2 \rightarrow n_3$ (it is important to notice that the quantum circuit does to follow, in this example, the feedforward order). Then, we get the probe and response dynamics:

$$\hat{U}_{2,3}\hat{U}_0 |000\rangle = |+\rangle \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \tag{36}$$

$$\langle 000| \hat{U}_0^\dagger \hat{U}_{2,3}^\dagger = \langle +| \otimes \frac{\langle 00| + \langle 11|}{\sqrt{2}} \tag{37}$$

The second and third neurons now have an entangled dynamics, where the neural field's probing and response dynamics is for a synchronized neural activity of the second and third neurons. Now, the activation of the first synaptic connection leads to the final probe and response dynamics:

$$\hat{U}_{1,2}\hat{U}_{2,3}\hat{U}_0 |000\rangle = \frac{|0\rangle \otimes (|00\rangle + |11\rangle)}{2} + \frac{|1\rangle \otimes (|01\rangle + |10\rangle)}{2} \tag{38}$$

$$\langle 000| \hat{U}_0^\dagger \hat{U}_{2,3}^\dagger \hat{U}_{1,2}^\dagger = \frac{\langle 0| \otimes (\langle 00| + \langle 11|)}{2} + \frac{\langle 1| \otimes (\langle 01| + \langle 10|)}{2} \tag{39}$$

Therefore, although the first neuron is only synaptically connected to the second neuron, since the second neuron already has an entangled dynamics with the third neuron, the conditional unitary propagation associated with the synaptic connection $n_1 \rightarrow n_2$ changes the entangled dynamics of the second and third neurons, so that, if the neural field probes, for the first neuron, a nonfiring dynamics, then, the probing dynamics for the other two neurons is still for a synchronized neural activity.

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On the other hand, if the probe dynamics for the first neuron is for a firing dynamics, then, the probing dynamics for the second and third neurons is for a out-of-sync quantum correlation, that is, when one is firing the other is not firing and vice-versa. A similar argument can be produced for the response dynamics, so that, in the resulting lines of force, the activation of the first neuron leads to an inhibitory response at the level of the entangled dynamics of the second and third neurons.

There are four resulting lines of force, each with an intensity equal to 1/4, two of these lines have the projector structure $|0\rangle \langle 0| \otimes |ss\rangle \langle ss|$ and the other two the structure $|1\rangle \langle 1| \otimes |s1-s\rangle \langle s1-s|$. We can implement experimentally this neural network on a quantum computer using IBM's five qubit transmon bowtie chip 3 (ibmqx4), accessed via cloud using IBM Q Experience[3]. Figure 1 shows the quantum circuit used and table 1 shows IBM's quantum simulator's results and the experiment's results.

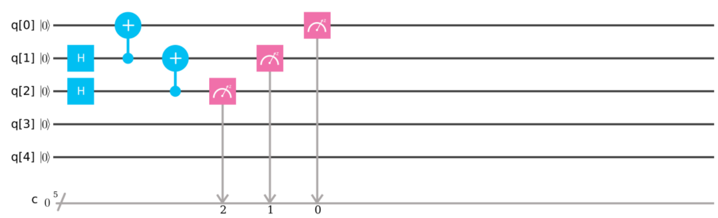

Figure 1: Quantum circuit for the neural network example using the IBM Q Experience framework. The registers q[3] and q[4] are not used, since the implemented network only has three neurons.

[3]Information and access to IBM's quantum computing resources can be found at the website https://www.research.ibm.com/ibm-q/.

Firing pattern	Simulator	Experiment
000	0.250	0.227
011	0.245	0.206
101	0.250	0.217
110	0.256	0.205
001	0	0.040
010	0	0.039
100	0	0.053
111	0	0.012

Table 1: Simulator and experiment's results from implementation on IBM's five qubit transmon bowtie chip 3 (ibmqx4) (IBM Q 5 Tenerife), in both cases, 8192 shots were used.

If we consider the experiment's results we can see that the dominant frequencies are those that match the final results of the theoretical model, there are also some residual cases that correspond to solutions for the other neural firing dynamics[4].

Now, there are two relevant of entangled dynamics that occur in the above example and that play a relevant role in quantum neurocomputation. The first is the excitatory dynamics, where the echoes are for the alternatives where two neurons exhibit a positively correlated activity (either they are both nonfiring or they are both firing), and the second is the inhibitory dynamics where the echoes are for the alternatives where the two neurons exhibit a negatively correlated activity (if one is firing the other is nonfiring and vice-versa). The first dynamics corresponds to an excitatory relation, while the second to an inhibitory relation.

These two basic dynamics occur for the most elementary networks (two neurons feedforward networks). For instance, considering the quantum circuit $\hat{U}_{1,2}\hat{U}_0$ with the quantum gates given by:

$$\hat{U}_0 = \hat{U}_{WH} \otimes \hat{1}_2 \tag{40}$$

$$\hat{U}_{1,2} = |0\rangle \langle 0| \otimes \hat{1}_2 + |1\rangle \langle 1| \otimes \hat{\sigma}_1 \tag{41}$$

then, if $\hat{\rho}_{in} = |00\rangle \langle 00|$, the resulting dynamics leads to an excitatory entanglement, while, if $\hat{\rho}_{in} = |11\rangle \langle 11|$, the resulting dynamics leads to an

[4]These deviations are to be expected in the context of actual physical implementation of quantum computation.

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inhibitory entanglement. Results from the quantum simulator for these two circuits and from experiments run on IBM's chip are shown in the following table. Again we see that the dominant frequencies are those associated with the circuit's entangled dynamics, with the non-dominant frequencies corresponding to deviations from the theoretical output.

Firing pattern	$\hat{\rho}_{in} = \|00\rangle\langle 00\|$		$\hat{\rho}_{in} = \|11\rangle\langle 11\|$	
	Simulator	Experiment	Simulator	Experiment
01	0	0.072	0.495	0.458
10	0	0.062	0.505	0.422
00	0.509	0.465	0	0.108
11	0.491	0.401	0	0.012

Table 2: Simulator and experiment's results from implementation on IBM's five qubit transmon bowtie chip 3 (ibmqx4) (IBM Q 5 Tenerife), with $\hat{\rho}_{in} = |00\rangle\langle 00|$ and $\hat{\rho}_{in} = |11\rangle\langle 11|$, in each case, 8192 shots were used.

Let us now return to the general case and consider that the initial density is a projector of the form $|\psi\rangle\langle\psi|$, then, the propagation in both directions of a computational chain leads to:

$$\hat{K}|\psi\rangle\langle\psi|\hat{K}^{\dagger} = \sum_{\mathbf{r},\mathbf{s}\in\mathbb{A}_2^d} \left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \psi \right\rangle |\mathbf{r}\rangle\langle\mathbf{s}| \left\langle \psi \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle \tag{42}$$

The echo intensities, in this case, are given by:

$$\left\langle \mathbf{s} \left| \hat{K} \right| \psi \right\rangle \left\langle \psi \left| \hat{K}^{\dagger} \right| \mathbf{s} \right\rangle = \left\langle \mathbf{s} \left| \prod_{k=N}^{1} \hat{U}_k \right| \psi \right\rangle \left\langle \psi \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle \tag{43}$$

Now, since we have the relations:

$$\left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \psi \right\rangle = \sum_{\mathbf{s}_0\in\mathbb{A}_2^d} \left\langle \mathbf{r} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{s}_0 \right\rangle \psi(\mathbf{s}_0) \tag{44}$$

$$\left\langle \psi \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle = \sum_{\mathbf{s}_0\in\mathbb{A}_2^d} \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle \psi(\mathbf{s}_0)^* \tag{45}$$

then, we get for Eq.(43):

$$\left\langle \mathbf{s} \left| \hat{K} \right| \psi \right\rangle \left\langle \psi \left| \hat{K}^{\dagger} \right| \mathbf{s} \right\rangle =$$
$$= \sum_{\mathbf{r}_0, \mathbf{s}_0 \in \mathbb{A}_2^d} \left\langle \mathbf{s} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{r}_0 \right\rangle \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle \psi\left(\mathbf{r}_0\right) \psi\left(\mathbf{s}_0\right)^* \tag{46}$$

Introducing the amplitudes $\Psi_{\mathbf{r}_0,\mathbf{s}}$ and $\Psi^*_{\mathbf{s},\mathbf{s}_0}$, respectively as:

$$\Psi_{\mathbf{r}_0,\mathbf{s}} = \left\langle \mathbf{s} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{r}_0 \right\rangle \psi\left(\mathbf{r}_0\right) \tag{47}$$

$$\Psi^*_{\mathbf{s},\mathbf{s}_0} = \left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^{\dagger} \right| \mathbf{s} \right\rangle \psi\left(\mathbf{s}_0\right)^* \tag{48}$$

the echo intensities result from the sum of products:

$$\left\langle \mathbf{s} \left| \hat{K} \right| \psi \right\rangle \left\langle \psi \left| \hat{K}^{\dagger} \right| \mathbf{s} \right\rangle = \sum_{\mathbf{r}_0, \mathbf{s}_0 \in \mathbb{A}_2^d} \Psi_{\mathbf{r}_0,\mathbf{s}} \Psi^*_{\mathbf{s},\mathbf{s}_0} \tag{49}$$

The term $\Psi_{\mathbf{r}_0,\mathbf{s}}$ is a probe quantum amplitude linking an initial (*input*) neural firing pattern described by the string $\mathbf{r}_0$ to the final (*output*) neural firing pattern described by the string $\mathbf{s}$. The term $\Psi^*_{\mathbf{s},\mathbf{s}_0}$ is, in turn, a response quantum amplitude linking the final (*output*) neural firing pattern described by the string $\mathbf{s}$ to an initial (*input*) neural firing pattern described by the string $\mathbf{s}_0$.

The two amplitudes are linking the same output to different inputs, thus, the field's computation is effectively propagating like a stream in both directions of the computational chain, through each alternative initial firing pattern and multiplying the corresponding amplitudes. The input activity does not need to match, indeed, the field takes into account the initial echoes and the initial failed echoes, in accordance with the dynamics associated with the initial density $|\psi\rangle\langle\psi|$, since the initial failed echoes can end up contributing to a final echo as long as the output is the same.

In the field's dynamics of probe and response, as described above, we have a form of holographic computation, in such a way that each echo's intensity results from the sum of all of the propagations that proceed from each component of $|\psi\rangle\langle\psi|$ propagated in both directions of the computational chain,

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simultaneously. Each component of the projector $|\psi\rangle\langle\psi|$, propagated in both directions of the computational chain, therefore, enters in the computation of each echo, either with a zero amplitude (if there is no path leading to that output), or non-zero amplitude (if there is a path leading to that output). In this way, we get quantum wave-like interference.

The neural field's computation, thus, flows like a stream in both directions of the computational chain simultaneously, using the whole initial projective dynamics $|\psi\rangle\langle\psi|$ to get a probe and response across the entire computational chain, for each final output neural activity, summing all the products of probe and responses that are associated with the same final output neural activity.

This flowing of the probing and response that add up with respect to the final output, explains the quantum interference, a point already raised by Bohm and Hiley (1993, p.365, 366) regarding the algebraic properties of the quantum dynamics associated with a specific quantum holographic dynamics, when one works with projectors. We use the term *holoflowing*, for this holographic flowing dynamics, which expresses the dynamical activity described above, taken as concept.

Bohm and Hiley (1993) used the term *holomovement*, however, we prefer the term *holoflowing* due to the nature of the wave dynamics. From a computer science perspective, the concept of *holoflowing* can be defined as a holographic dynamics where the computation can be addressed like a stream flowing in both directions from input to output (forward propagation), which is Cramer's offer wave dynamics, and from output to input (backpropagation), which is Cramer's confirmation wave dynamics, computing, in this way, each path towards the final projection, which is added to form the final amplitudes associated, respectively, with the computational output's probe and response carriers, this explains the dynamics leading to the final echoes.

Under the above concept of *holoflowing*, the echo formation dynamics is such that the neural field's computation needs to be considered in its whole, that is, we cannot consider just a few neurons or even just a few pairing of probe and response dynamics, the field's computation needs to account for all of the initial pairings of probe and response in order to compute the final echoes, since, as long as these dynamics lead to the same final neural activity, they must be added up, contributing to the final echo intensity associated with a corresponding projected neural activity.

The *lines of force* related to the final echoes thus reflect the result of this holographic dynamics, which explains how the probabilities associated with the quantum neural activity exhibit wave-like interference.

In the field's computational dynamics, the *holoflowing* leads to an echo rematching, in the sense that the product $\Psi_{\mathbf{r}_0,\mathbf{s}}\Psi^*_{\mathbf{s},\mathbf{s}_0}$, with $\mathbf{r}_0 \neq \mathbf{s}_0$, expresses a transition from an initial failed echo to a final dynamics where probe and response carriers agree, thus, forming an echo. In this sense, the modulated projection associated with each final neural firing activity is given by the sum:

$$\left\langle \mathbf{s} \left| \hat{K} \right| \psi \right\rangle \left\langle \psi \left| \hat{K}^\dagger \right| \mathbf{s} \right\rangle |\mathbf{s}\rangle \langle \mathbf{s}| = \sum_{\mathbf{r}_0,\mathbf{s}_0 \in \mathbb{A}_2^d} \Psi_{\mathbf{r}_0,\mathbf{s}}\Psi^*_{\mathbf{s},\mathbf{s}_0} |\mathbf{s}\rangle \langle \mathbf{s}| \tag{50}$$

where each product $\Psi_{\mathbf{r}_0,\mathbf{s}}\Psi^*_{\mathbf{s},\mathbf{s}_0}$ with $\mathbf{r}_0 \neq \mathbf{s}_0$ corresponds to the transition from an initial failure of the response carrier to respond to the probe, to a final output dynamics where the response matches the probe. The degree to which initial mismatches are propagated to final matchings depends on the structure of the computational chain, namely, on the forward propagating amplitudes $\left\langle \mathbf{s} \left| \prod_{k=N}^{1} \hat{U}_k \right| \mathbf{r}_0 \right\rangle$ and the backpropagating amplitudes $\left\langle \mathbf{s}_0 \left| \prod_{k=1}^{N} \hat{U}_k^\dagger \right| \mathbf{s} \right\rangle$, as long as the product of these amplitudes is non-null for a pair $\mathbf{r}_0 \neq \mathbf{s}_0$, then, we get an echo rematching for $\psi(\mathbf{r}_0) \neq 0$ and $\psi(\mathbf{s}_0) \neq 0$. The sum over the final matchings explains the workings of the holographic dynamics in its relation with the quantum neural computation dynamics implemented for these QuANNs. We call these echo rematchings *echo correction dynamics*.

The *holoflowing* is a nonlocal dynamics involving the whole quantum field, including quantum and subquantum levels. Indeed, in the above context of QuANNs, we cannot locate this dynamics at the level of a specific neuron, but, instead, the dynamics involves the whole neural field. A point that is specifically seen, within the formalism, in the sums $\sum_{\mathbf{r}_0,\mathbf{s}_0 \in \mathbb{A}_2^d} \Psi_{\mathbf{r}_0,\mathbf{s}}\Psi^*_{\mathbf{s},\mathbf{s}_0}$, which are being summed over the entire neural activity patterns. It is, thus, relevant to consider the concept of nonlocality in this context.

2.2.2 Nonlocality and the Quantum Force Interpretation

In quantum mechanics, nonlocality is usually discussed in connection with quantum correlated dynamics that violate the signal-based communication limit postulated, in special relativity, to be the speed of light in vaccuum. That is, in a quantum field, correlated dynamics, with regards to specific field properties' eigenvalues, evaluated at different points in space, occur beyond the limits of classical signal-based communication, that is, we have a dynamics where not even a signal (classically) propagating at light speed can link the two points.

Different interpretations of quantum mechanics deal differently with nonlocality. In Bohm's proposal, which assumes a subquantum dynamics, nonlocality is not directly related to superluminal quantum correlations, rather, these superluminal quantum correlations are, in Bohm's proposal, a consequence of nonlocal dynamics of a quantum field, but even when we are dealing with a single degree of freedom, and not with what quantum theory calls multipartite entanglement, nonlocality still plays a role. Also in Cramer's TI nonlocality plays a role, not only being linked to quantum correlations but also in linking emitters in the present with future absobers (Cramer, 2016).

Our proposal, the QFI, combines elements of the TI and elements of Bohm's proposal with systems science. In systems science, nonlocality is linked to the systemic identity and unity, namely, considering that any system is not reducible to the sum of its parts, we need to consider the parts and their relation to the whole, in this way, we can speak of a diversity of parts and, simultaneously, of a unity that sustains the systemic coherence, thus, the systemic nonlocality needs to be considered as a dynamics of the systemic unity that plays a role in the system's sustainability, coherence and identity (Madeira, 2013). In a system, nonlocality involves a holographic projective cognitive dynamics that works at all scales of the system, towards the system's integrity, coherence, sustainability and identity.

In a system's dynamics, the conservation of systemic integrity, coherence and identity necessarily mean that coordinated synchronized activity needs to be permanently assured, so that one part cannot go one way and another one go another way, leading to a systemic disaggregation. This means that the system will necessarily exhibit correlated activity that cannot be dependent upon classical finite signal speed propagation, otherwise the system would collapse. Entanglement means, in this case, that: entangled dynamics between the parts imply a connection that must be independent of finite speed propagation signal-based communication.

One might assume that the subquantum level is supporting finite superluminal propagation of signals, also assuming that nonlocality comes from such a fundamental dynamics, but that signal propagation speed cannot be finite.

Since nonlocality is necessarily a dynamics of the systemic unity towards the system's sustainability, integrity and identity (Madeira, 2013), otherwise we would not have a system in the proper sense of the term, the system's nonlocality is necessarily *that* system's nonlocality, and the system's nonlocality cannot, thus, depend upon a signal-based communication with a finite

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speed of signal propagation, be it luminal or superluminal, otherwise, the system would lose coherence and disaggregate, because, for each of the system's dynamics, all of the parts must always be coordinated in keeping the system's integrity and coherence.

The loss of systemic coherence, in this last case, would be due to a lag always present, related to the passage of a signal at finite speed, even a superluminal one. Entangled dynamics between different parts of the system must always work either without the need for a passage of signal, or through the instaneous passage of signal. In the first case, Einstein's special theory of relativity can still be assumed to hold for signal-based communication, in the second case, the postulate of the speed of light limit breaks down at the quantum level (Bohm and Hiley, 1993).

Regarding the first case, in biophysics, from experiments of communication with *megaptera* (Beamish, 2004), later on generalized for different animal species (Beamish, 2011), a new concept of communication, different from signal-based communication, was found pertinent and robust, being researched by Beamish, who proposed the (bio)physical concepts of *'Rhythm Based Communication'* (RBC), *'Rhythm Based Time'* (RBT) and *'Rhythm Based Information'* (RBI). A connection of these concepts with quantum mechanics was researched by Kitada (2004). Future research on this area may provide for a relevant basis for new computational physics and technologies, with possible links to quantum technoscience, in general, and quantum robotics in particular, since these concepts work with a communication dynamics that is different from signal-based communication, an added point regarding the role of RBC and RBT in quantum mechanics is the relevance of rhythms in quantum systems, as reviewed in the previous subsection.

If we consider the echoes resulting from correlated probe and response carriers in entangled dynamics coming from conditional unitary propagation, then, the final probe and response dynamics are correlated. One may argue, however, that such a correlation cannot be sustained without the passage of a signal at the level of the quantum field's nonlocal dynamics, in that case, the signal has to pass at an instanteous speed, supported by the quantum and subquantum dynamics.

Accepting this last approach, means that the chronological temporal connection is, in this case, between dynamics of a field that computes nonlocally at more than a superluminal scale, it computes at an instantaneous scale (beyond any finite signal propagation postulate). If one accepts this last framework, then the point, which also holds for the dynamics of multiple systems

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with where their respective fields exhibit entangled dynamics, is that the dynamics, at the level of the nonlocal connections, are not chronometrizable, therefore, neither relativity, nor any classical theory of finite signal propagation speed apply as an explanatory framework (Bohm and Hiley, 1993).

Nonlocal signaling is, in this last case, a process different from any assumed in a classical computational framework, where dynamics is thought in terms of transitions between states mediated by causally finite chains. Some confusion arises in physics regarding this signaling, it is important to stress that: quantum entanglement is not about cause and effect, but about connection sustained by a quantum field's nonlocal dynamics.

The no-signaling theorem does not regard an instantaneous passage of signal associated with quantum entanglement, rather, it is about the statement impossibility of signal-based communication between localities (two observers) using the nonlocal field dynamics just by making measurements on the entangled parties (Cramer, 2016; Walleczek and Grössing, 2016). Thus, it is a way to make quantum mechanics less problematic for relativity[5] (Walleczek and Grössing, 2016). However, relativity is a classical mechanical theory that does not deal with nonlocal dynamics (Bohm and Hiley, 1993), including, for instance, the case of two fields' probe and response dynamics that exhibit nolocal correlations, which implies that, if we accept the instantaneous signal passage, that passage occurs between nonlocalities, not involving propagation in a "space-time".

Conceptual categories, coming from classical physics, such as space-time, causality and classical signal-based communication are not applicable at this level, where we are dealing with different computational dynamics, and where "neighborhood" is a "neighboring" (verb) determined by connection, which is dynamical and non-metric, rather than by a geometrically determined metrizable background space.

In the context of quantum mechanics, under the QFI, we consider that a quantum field works towards the rising of an actualized order, which is risen, at the quantum level, as a discrete (quantized) pattern. In computational terms, the *holoflowing* can be considered as a reflexive holographic nonlocal dynamics that involves the whole field towards the rising of an order, a rising that is determined by the lines of force which depend upon the itensity of

[5]However, as Cramer (2016) argues, even if it were shown to be possible to use the nonlocal dynamics for observer-to-observer communication, that communication can still be argued to be consistent with respect to Lorenz invariance.

the respective echoes that result from the *holoflowing*.

In this sense, the quantum dynamics, described above, works in a way that involves the whole field, including quantum and subquantum dynamics, where the rising of an actualized order depends upon the intensity of the echoes resulting from the *holoflowing* (the reflexive holographic flowing) of the field and the intensity of the lines of force that are the result of the quantum and subquantum activity that supports, in the field, the mobilization of the forces needed to determine and make rise an actualized order, lines of force that coincide with the reflexive echoes.

The coincidence between echo intensities and the intensity of the lines of force is a fundamental computational result of the field's activity around a problem, and it involves both quantum and subquantum levels working in tandem towards the rising of an order, a work that involves a reflexive dynamics, that connects beginning and end of the computation, which leads to the *echoes* that signalize the forces to be mobilized. In this way, the *lines of force* are formed in direct correspondence with the *echoes*, since these are the *lines of force* that are specifically directed towards the solution of the problem in question.

In this sense, in terms of computational physics, the quantum dynamics corresponds to a basic computational dynamics that involves a specific form of learning about a problem, involving a forward and backpropagating wave-like dynamics, leading to the formation of a specific configuration of lines of force that allow the field to make emerge a specific order, using these lines of force that connect the beginning to the endpoint of the computational problem and, thus, solve it. Such a computational dynamics involves the whole field working, supported by both quantum and subquantum levels.

In this computational dynamics, in the case of QuANNs, entanglement results from the local Hamiltonian structure associated with each neural link. In terms of machine learning, this computational dynamics allows the development of a new form of quantum neural machine learning based on quantum optimization, which we will introduce in the next section and apply to a quantum robotics problem.

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3 The Quantum Force Interpretation and Quantum Robotics

Following Russel and Norvig's (2014) definition of agent, a robot can be defined as an artificial agent capable of *perceiving* the environment through a interface and changing that environment's dynamics through *actuators*. Within a classical context, the *perception* process can be addressed as a mapping of the sensory data and does not, by itself, change the environment, in the quantum context, however, this is no longer the case, as made explicit by Everett (1957).

Everett (1957) applied the formalism of quantum wave dynamics to both a target quantum system and the observer, whose *cognitive dynamical space* is addressed within the Hilbert space formalism and assumed to follow the quantum mechanical rules just as the target system. The *cognitive dynamical space* is assumed by Everett in terms of memory patterns that obey the rules of quantum wave dynamics, in this way, the observation act involves a quantum computation by which the observed system and the observer's cognitive dynamics become correlated, so that we get an entangled dynamics of observer agent (Everett's automaton) and the observed system.

While Everett worked with the concept of a quantum dynamics in terms of state transitions and worked on relative states, we are dealing with an interpretation of quantum mechanics without states, working from a projective dynamics resulting from the "encounters" of *probe carriers* and *response carriers*, so that we can no longer speak of a state vector but rather, each *ket vector* corresponds, in the mathematical formalism, to a probing dynamics of a quantum field and each *bra vector* corresponds to a response dynamics.

The computation of the field is, thus, like a search for a solution to a problem, where each probed alternative also gets a response, with the field's lines of force being formed along the (cognitive) echo resulting from the encounters of matching probe and response carriers and with a force intensity that matches the echo's intensity. This whole dynamics involves the field's computation at both quantum and subquantum levels. The quantum field, in this case, is the field associated with the robot's neural network.

While Everett's automaton model is specifically built to address the measurement and observation of a quantum system, the immediate extension of this model, that arises from classical robotics, is to consider the ability to change the target system's dynamics through actuators, with the actions de-

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pending upon the robot's quantum cognitive dynamics. A point that was raised by Benioff (1998) as well as by Dong *et al.* (2006).

The robot architecture we will work with follows a similar approach as that of Dong *et al.* (2006) regarding the *multi quantum computing units*, quantum actuator and sensory units, with some differences, namely: the quantum actuator and sensory units are associated with a single unit, which is the robot's interface through which the robot interacts with a target quantum system and the *multi quantum computing units* will take the form a feedforward multilayer quantum neural network. The sensory processing involves the interface and the first layer of neurons, and the action involves the interface and the neural network, which means that the actuator operator must be a conditional unitary operator on the target that depends upon the interface and neural network.

The context of quantum neural machine learning assumed here is expanded from Gonçalves (2017), where, instead of an iterative scheme, the convergence of the network occurs within a continuous time framework, such that the QuANN converges to an optimized solution for a conditional utility performance measure, as the neural processing time tends to a learning period, as we now review.

3.1 Quantum Neural Machine Learning and Quantum Optimization

Let us consider the single neuron general Hamiltonian structure (Gonçalves, 2017):

$$\hat{H}_x = -\frac{\hbar}{2}\frac{\omega(x)}{\triangle t_0}\hat{1}_2 + \frac{\theta(x)}{\triangle t_0}\sum_{j=1}^{3} u_j(x)\frac{\hbar}{2}\hat{\sigma}_j \tag{51}$$

where x is a parameter assuming values in the domain $\mathbf{F}$ (which can be any set), $\omega(x)$ and $\theta(x)$ are angles measured in radians, $\triangle t_0$ is the learning period and $u_j(x)$ are the components of a real unit vector. The unitary operator for such a Hamiltonian structure is, thus, given by the U(2) operator:

$$\begin{aligned} &\exp\left(-\frac{i}{\hbar}\triangle t\hat{H}_x\right) = \\ &= e^{i\frac{\omega(x)\triangle t}{2\triangle t_0}}\left[\cos\left(\frac{\theta(x)\triangle t}{2\triangle t_0}\right)\hat{1}_2 - i\sin\left(\frac{\theta(x)\triangle t}{2\triangle t_0}\right)\sum_{j=1}^{3} u_j(x)\hat{\sigma}_j\right] \end{aligned} \tag{52}$$

If we let $\omega(x)$, $\theta(x)$ and $u_j(x)$ be defined such that:

$$\omega(x) = (1 - f(x))\,\pi \tag{53}$$

$$\theta(x) = \frac{2 - f(x)}{2}\pi \tag{54}$$

$$u_1(x) = u_3(x) = \frac{1 - f(x)}{\sqrt{2}} \tag{55}$$

$$u_2(x) = f(x) \tag{56}$$

where f is a map $f : \mathbf{F} \mapsto \mathbb{A}_2$, then, as shown in Gonçalves (2017), as $\triangle t \to \triangle t_0$ we have the convergence for the probe and response carriers $|+\rangle$ and $\langle +|$:

$$e^{-\frac{i}{\hbar}\triangle t \hat{H}_x} |+\rangle \to |f(x)\rangle \tag{57}$$

$$\langle +| \, e^{\frac{i}{\hbar}\triangle t \hat{H}_x} \to \langle f(x)| \tag{58}$$

which means that an initial density $|+\rangle\langle +|$ will converge to the final density $|f(x)\rangle\langle f(x)|$, therefore, while, initially, we have two echoes, one for the alternative firing pattern $|0\rangle\langle 0|$ and another for the alternative firing pattern $|1\rangle\langle 1|$, each with equal echo intensity, after the unitary evolution, the neural field dynamics converges to a single echo either for $|0\rangle\langle 0|$, if $f(x) = 0$, or for $|1\rangle\langle 1|$, if $f(x) = 1$, this convergence is addressed, in connection to quantum neural machine learning in Gonçalves (2017), allowing one to introduce a form of quantum optimization in the basic quantum neural computation of a feedforward neural network, where the learning takes place as a form of convergence of the probe and response carriers to the optimum.

Indeed, let us consider the most elementary case of a QuANN with two fully connected layers. Where the input layer has d_{in} neurons and the output layer has d_{out} neurons, then, let $v : \mathbb{A}_2^{d_{in}} \times \mathbb{A}_2^{d_{out}} \mapsto \mathbb{R}$, be a conditional utility function that assigns a real number to each pair in $\mathbb{A}_2^{d_{in}} \times \mathbb{A}_2^{d_{out}}$, given the pair of strings $(\mathbf{r}, \mathbf{s})$, with $\mathbf{r} \in \mathbb{A}_2^{d_{in}}$ and $\mathbf{s} \in \mathbb{A}_2^{d_{out}}$, then, we write $v[\mathbf{s}|\mathbf{r}] = v(\mathbf{r}, \mathbf{s})$ for the utility of the output binary string $\mathbf{s}$ given the input binary string $\mathbf{r}$. Let $\mathcal{V}$ be the space of such utility functions, with the added condition that, for each $\mathbf{r} \in \mathbb{A}_2^{d_{in}}$, there is a unique string in $\mathbb{A}_2^{d_{out}}$ that maximizes the utility conditional on $\mathbf{r}$. Then, we have a single maximum utility output string for each input string $\mathbf{r}$, this string is the solution to the equation:

$$\mathbf{s}_{\mathbf{r}} = \arg\max_{\mathbf{s}} v[\mathbf{s}|\mathbf{r}] \tag{59}$$

We denote by $\mathbf{s}_{\mathbf{r},k}$ the k-th bit in the string $\mathbf{s}_\mathbf{r}$ and define:

$$\mathbf{s}_{\mathbf{r},k} = \arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}] \tag{60}$$

where the notation on the right hand side is interpreted as the value of the argument that maximizes the conditional utility, on the input pattern $\mathbf{r}$, evaluated at the k-th position.

Now, let us consider that the neural network, at the beginning of the computation, is characterized by the projector $|\Psi(t_0)\rangle \langle\Psi(t_0)|$ where the probe and response carriers are given by:

$$|\Psi(t_0)\rangle = |+\rangle^{\otimes d_{in}} \otimes |+\rangle^{\otimes d_{out}} \tag{61}$$

$$\langle\Psi(t_0)| = \langle +|^{\otimes d_{in}} \otimes \langle +|^{\otimes d_{out}} \tag{62}$$

And let us assume that each neuron on the output layer is connected to all the neurons of the input layer. Then, we have the following conditional unitary operator:

$$\hat{U}_{\triangle t} = \sum_{\mathbf{r}\in\mathbb{A}_2^{d_{in}}} |\mathbf{r}\rangle \langle\mathbf{r}| \bigotimes_{k=1}^{d_{out}} \exp\left(-\frac{i}{\hbar}\triangle t \hat{H}_{k,\mathbf{r}}\right) \tag{63}$$

where the Hamiltonians for each output layer's neuron $\hat{H}_{k,\mathbf{r}}$ incorporate the conditional utility optimization problem as follows:

$$\begin{gathered}\hat{H}_{k,\mathbf{r}} = \\ = -\frac{\hbar}{2}\frac{\left(1-\arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}]\right)\pi}{\triangle t_0}\hat{1}_2 + \\ + \frac{\left(2-\arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}]\right)\pi}{2\triangle t_0}\sum_{j=1}^{3} u_{j,k}(\mathbf{r})\frac{\hbar}{2}\hat{\sigma}_j\end{gathered} \tag{64}$$

$$u_{1,k}(\mathbf{r}) = u_{3,k}(\mathbf{r}) = \frac{1-\arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}]}{\sqrt{2}} \tag{65}$$

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$$u_{2,k}(\mathbf{r}) = \arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}] \tag{66}$$

Then, the probe carrier, after a neural processing period of $\triangle t$, is given by the superposition:

$$\begin{aligned} \hat{U}_{\triangle t} \left|\Psi(t_0)\right\rangle = \left|\Psi(t_0 + \triangle t)\right\rangle = \\ = \sum_{\mathbf{r}\in\mathbb{A}_2^{d_{in}}} \frac{|\mathbf{r}\rangle}{\left(\sqrt{2}\right)^{d_{in}}} \bigotimes_{k=1}^{d_{out}} \exp\left(-\frac{i}{\hbar}\triangle t \hat{H}_{k,\mathbf{r}}\right)|+\rangle = \\ = \sum_{\mathbf{r}\in\mathbb{A}_2^{d_{in}}} \frac{|\mathbf{r}\rangle}{\left(\sqrt{2}\right)^{d_{in}}} \bigotimes_{k=1}^{d_{out}} \left|\psi_{k,\mathbf{r}}\left(t_0 + \triangle t\right)\right\rangle \end{aligned} \tag{67}$$

with $\left|\psi_{k,\mathbf{r}}\left(t_0 + \triangle t\right)\right\rangle$ given by:

$$\begin{aligned} \left|\psi_{k,\mathbf{r}}\left(t_0 + \triangle t\right)\right\rangle = \\ = e^{i\frac{\left(1-\mathbf{s}_{\mathbf{r},k}\right)\pi\triangle t}{2\triangle t_0}} \cos\left(\frac{\left(2-\mathbf{s}_{\mathbf{r},k}\right)\pi\triangle t}{4\triangle t_0}\right)|+\rangle - \\ -ie^{i\frac{\left(1-\mathbf{s}_{\mathbf{r},k}\right)\pi\triangle t}{2\triangle t_0}} \sin\left(\frac{\left(2-\mathbf{s}_{\mathbf{r},k}\right)\pi\triangle t}{4\triangle t_0}\right)\left[\left(1-\mathbf{s}_{\mathbf{r}.k}\right)|0\rangle - i\mathbf{s}_{\mathbf{r},k}|-\rangle\right] \end{aligned} \tag{68}$$

The formula for the response carriers can be obtained from the forward carriers by applying the conjugate transposition$\langle\Psi(t_0 + \triangle t)| = |\Psi(t_0 + \triangle t)\rangle^\dagger$, $\left\langle\psi_{k,\mathbf{r}}\left(t_0 + \triangle t\right)\right| = \left|\psi_{k,\mathbf{r}}\left(t_0 + \triangle t\right)\right\rangle^\dagger$.

Now, at $t_0 = 0$, the neural field exhibits an independent probe and response dynamics for each neuron's activity, with the firing and nonfiring alternatives symmetrically weighted. As $\triangle t \to \triangle t_0$, however, the probe and response dynamics are no longer independent, since for the k-th second layer neuron, with $k = 1, 2, ..., d_{out}$, the respective probe and response carriers converge on the firing pattern that leads to the conditional utility maximizing solution evaluated at the position k.

This is a form of quantum search in continuous time, where the initial dynamics has a probe an response dynamics that are effectively evaluating all alternative solution, but, then, as $\triangle t \to \triangle t_0$, there takes place a convergence to the optimum. To see how this convergence works, let us consider the probe and response carriers for each second layer neuron. If $\mathbf{s}_{\mathbf{r},k} = \arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}] =$

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0, then, we get, the probe and response carriers:

$$\begin{gathered}\left|\psi_{k,\mathbf{r}}\left(t_0+\triangle t\right)\right\rangle = \\ e^{i\frac{\pi\triangle t}{2\triangle t_0}}\left(\cos\left(\frac{\pi\triangle t}{2\triangle t_0}\right)\left|+\right\rangle - i\sin\left(\frac{\pi\triangle t}{2\triangle t_0}\right)\left|0\right\rangle\right)\end{gathered} \tag{69}$$

$$\begin{gathered}\left\langle\psi_{k,\mathbf{r}}\left(t_0+\triangle t\right)\right| = \\ \left(\left\langle+\right|\cos\left(\frac{\pi\triangle t}{2\triangle t_0}\right) + \left\langle 0\right| i\sin\left(\frac{\pi\triangle t}{2\triangle t_0}\right)\right)e^{-i\frac{\pi\triangle t}{2\triangle t_0}}\end{gathered} \tag{70}$$

When $\triangle t$ is close to zero, the dominant probe and response carriers are, respectively, $\left|+\right\rangle$ and $\left\langle+\right|$, which means that the neural field is still close to the initial dynamics which explores each firing pattern with the same weight, however, as $\triangle t \to \triangle t_0$ these two carriers lose weight, and the final carriers converge to $\left|\psi_{k,\mathbf{r}}\left(t_0+\triangle t_0\right)\right\rangle = \left|0\right\rangle$ and $\left\langle\psi_{k,\mathbf{r}}\left(t_0+\triangle t_0\right)\right| = \left\langle 0\right|$, which matches the optimum, evaluated at position k, $\arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}]$. Likewise if $\mathbf{s}_{\mathbf{r},k} = \arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}] = 1$, then, we get:

$$\begin{gathered}\left|\psi_k\left(t_0+\triangle t\right)\right\rangle = \\ \cos\left(\frac{\pi\triangle t}{4\triangle t_0}\right)\left|+\right\rangle - \sin\left(\frac{\pi\triangle t}{4\triangle t_0}\right)\left|-\right\rangle\end{gathered} \tag{71}$$

$$\begin{gathered}\left\langle\psi_k\left(t_0+\triangle t\right)\right| = \\ \left\langle+\right|\cos\left(\frac{\pi\triangle t}{4\triangle t_0}\right) - \left\langle-\right|\sin\left(\frac{\pi\triangle t}{4\triangle t_0}\right)\end{gathered} \tag{72}$$

Again, when $\triangle t$ is close to zero, the dominant probe carrier and response carriers are, respectively, $\left|+\right\rangle$ and $\left\langle+\right|$, as $\triangle t \to \triangle t_0$, however, the weight of the second carrier in the superposition starts to increase and that of the first carrier to decrease, approximating each other. When $\triangle t = \triangle t_0$ the two carriers have a weight of $1/\sqrt{2}$, in this case, there occurs a wavelike destructive interference for the nonfiring alternative, with the corresponding amplitude becoming zero, while there is a constructive interference for the firing alternative, with the corresponding alternative becoming equal to one, which means that the carriers converge to $\left|\psi_{k,\mathbf{r}}\left(t_0+\triangle t\right)\right\rangle = \left|1\right\rangle$ and $\left\langle\psi_{k,\mathbf{r}}\left(t_0+\triangle t\right)\right| = \left\langle 1\right|$, respectively.

The neural field, thus, takes advantage of quantum interference in order to find the optimum. Since, when $\triangle t = \triangle t_0$, the carriers, for each second layer each neuron, coincide with $\arg\max_{\mathbf{s},k} v[\mathbf{s}|\mathbf{r}]$, the second layer's carriers coincide with the optimum given each alternative, which means that the density for $\triangle t = \triangle t_0$ is given by the entangled dynamics:

$$|\Psi(t_0 + \triangle t_0)\rangle \langle \Psi(t_0 + \triangle t_0)| = \sum_{\mathbf{r},\mathbf{r}' \in \mathbb{A}_2^{d_{in}}} \frac{|\mathbf{r}, \mathbf{s}_\mathbf{r}\rangle \langle \mathbf{r}', \mathbf{s}_{\mathbf{r}'}|}{2^{d_{in}}} \tag{73}$$

The echoes are, thus, only formed for pairs where the second layer exhibits the maximum conditional utility firing pattern given the first layer's firing pattern. This means that the lines of force with nonzero intensity are only those associated with the optimal dynamics of the second layer given the first layer's firing pattern.

This approach to quantum neural machine learning, based on quantum optimization, can be extended to multiple layers and has applications in quantum robotics, furthermore, these applications help illustrate major points regarding the quantum neural cognition involved in quantum neural computation, under the QFI, a point that we now address.

3.2 Quantum Optimizer Robot

Let us consider a target described by a Hilbert space $\mathcal{H}_{Target}$ spanned by the basis $\{|o_n\rangle : n = 1, 2, ..., N\}$, such that the basis *ket* vectors satisfy the eigenvalue equation for some observable:

$$\hat{O}_{Target} |o_n\rangle = o_n |o_n\rangle \tag{74}$$

where the spectrum is, in this case, assumed to be non-degenerate. We denote the set of target's eigenvalues by $O = \{o_1, o_2, ..., o_n\}$.

The quantum robot is defined as an artificial agent that has an interface that interacts with the target, working as both a measurement apparatus, that performs a von Neumann measurement of the observable $\hat{O}_{Target}$, and an actuator. Now, linked to the robot's interface is a feedforward QuANN comprised of two fully connected layers of neurons, plus a third layer comprised of a final neuron which is fully connected to the previous two layers.

Each layer, along with the interface, has a specific functionality in the robot's cognitive processing. The first layer maps the sensory data to binary firing patterns, thus, processing the sensory data coming from the interface, the second layer maps out a desired pattern in accordance with a conditional performance measure for the robot, the third layer, which is comprised of the last neuron, is an action triggering layer, that is, if the neuron fires, then an action is implemented, while if the neuron does not fire, an action is not implemented. Figure 1 shows the basic architecture for the robot plus interaction with the target.

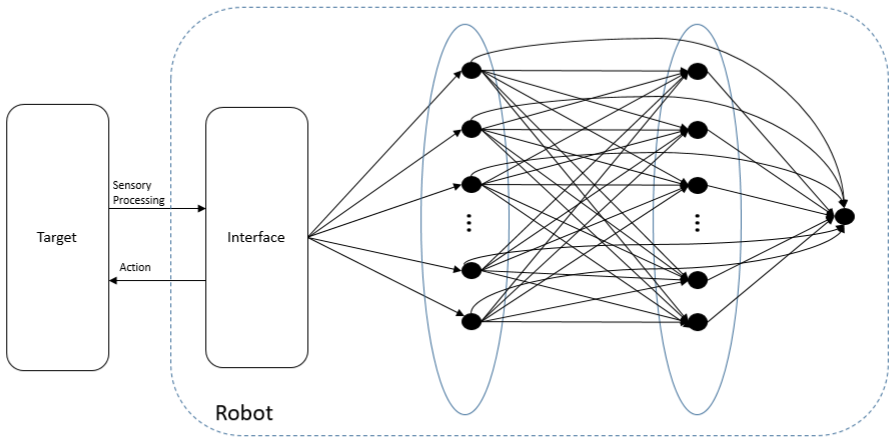

Figure 2: Robot's architecture and interaction with target.

Formally, the robot's interface with the target is described by the Hilbert space $\mathcal{H}_{Int}$ spanned by the basis $\{|a_n\rangle : n = 1, 2, ..., N\}$ of the Hermitian operator satisfying the eigenvalue equation:

$$\hat{A}_{int} |a_n\rangle = a_n |a_n\rangle \tag{75}$$

where a_n are real-valued quantum numbers that characterize the interface's field dynamics, the spectrum is also assumed to be non-degenerate. We denote the set of interface eigenvalues by $A = \{a_1, a_2, ..., a_n\}$.

The Hilbert space to describe the robot's cognition is, thus, given by the tensor product of the interface's Hilbert space plus the neural network's Hilbert space:

$$\mathcal{H}_{Robot} = \mathcal{H}_{Int} \otimes \mathcal{H}_{Net} \tag{76}$$

The neural network's Hilbert space is, in turn, given by the tensor product:

$$\mathcal{H}_{Net} = \mathcal{H}_2^{\otimes d_N} \otimes \mathcal{H}_2^{\otimes d_N} \otimes \mathcal{H}_2 \tag{77}$$

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where $d_N = \min\{d \in \mathbb{N} : d \geq \log_2 N\}$ is the width of the first and second layers, needed for the digital dynamical mapping of the N patterns associated with the interface's field dynamics.

In what follows, we define the unit operator for the target and interface as respectively:

$$\hat{1}_T = \sum_{n=1}^{N} |o_n\rangle \langle o_n| \tag{78}$$

$$\hat{1}_{Int} = \sum_{n=1}^{N} |a_n\rangle \langle a_n| \tag{79}$$

and, as before, we use the notation $\hat{1}_2$ for the unit operator on the single neuron Hilbert space, as per Eq.(5). Then, the first stage of the robot's computation is the sensory processing stage, by which the robot interacts with the target, so that the interface's field becomes entangled with the target's field, in such a way that there is a matching correlation between the probing and response dynamics of both target and interface's fields.

The sensory processing is implemented by way of the unitary operator:

$$\hat{U}_{Sensory} = \sum_{n=1}^{N} |o_n\rangle \langle o_n| \otimes \hat{U}_n \otimes \hat{1}_2^{\otimes(2d_N+1)} \tag{80}$$

with $\hat{U}_1 = \hat{1}_{Int}$ and, for $n > 1$, $\hat{U}_n$ has the following structure:

$$\hat{U}_n = |a_1\rangle \langle a_n| + |a_n\rangle \langle a_1| + \sum_{a \in A \setminus \{a_1, a_n\}} |a\rangle \langle a| \tag{81}$$

to illustrate the dynamics of this operator, let us consider that the initial density operator is given by a projector $|\Psi\rangle \langle\Psi|$, where the probe and response carriers are, respectively, given by the superpositions:

$$|\Psi\rangle = \sum_{n=1}^{N} \psi_n |o_n\rangle \otimes |a_1\rangle \otimes |+\rangle^{\otimes(2d_N+1)} \tag{82}$$

$$\langle\Psi| = \sum_{n=1}^{N} \psi_n^* \langle o_n| \otimes \langle a_1| \otimes \langle +|^{\otimes(2d_N+1)} \tag{83}$$

Before the interaction with the target, the probe and response dynamics of the interface's field is independent of the target's field, the neural field's probe and response dynamics is in turn independent for each neuron and separable from the interface, being characterized by a symmetric superposition of the nonfiring and firing carriers.

Thus, before interacting with the target, the robot and target have non-entangled dynamics. After interacting with the target, under the sensory unitary propagation, we get, respectively, the following results for the probe and response carriers:

$$\hat{U}_{Sensory} |\Psi\rangle = \sum_{n=1}^{N} \psi_n |o_n\rangle \otimes |a_n\rangle \otimes |+\rangle^{\otimes(2d_N+1)} \tag{84}$$

$$\langle\Psi| \hat{U}^{\dagger}_{Sensory} = \sum_{n=1}^{N} \psi_n^* \langle o_n| \otimes \langle a_n| \otimes \langle +|^{\otimes(2d_N+1)} \tag{85}$$

Thus, when the target's field probes the alternative eigenvalue o_n, the robot's interface's field probes the alternative eigenvalue a_n, so that the probing dynamics of target and interface's fields are entangled. Similarly, when the target's field response dynamics is associated with the eigenvalue o_n, the robot's interface's field response dynamics is associated with the eigenvalue a_n, therefore, the response dynamics is also entangled. The neural field dynamics, however, is not yet entangled, with the interface's field dynamics. The neural field is still probing each neural activity, for each neuron, symmetrically and independently, the same holds for the neural field's response dynamics.

This changes with the first layer's neural processing, which works to produce a neural mapping of the sensory data in the form of specific neural firing dynamics, functioning as another form of quantum measurement apparatus, in this case, as a measurement apparatus of the interface.

To address the first layer's neural processing, let $\mathbb{D}_2^{d_N} \subseteq \mathbb{A}_2^{d_N}$ be a set of binary strings such that $\mathbb{D}_2^{d_N}$ contains the first N binary strings, following the order in the integer expansion:

$$m = \sum_{k=0}^{d_N-1} s_k 2^{d_N-1-k} \tag{86}$$

in this case, we use the notation $\mathbf{s}[m]$ for the d_N-length binary string $s_0 s_1 ... s_{d_N-1}$ that represents m. The first element in $\mathbb{D}_2^{d_N}$ is, thus, $\mathbf{s}[0]$ and the last is

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$\mathbf{s}[N-1]$. When N is a power of 2, $d_N = \log_2 N$ and $\mathbb{D}_2^{d_N} = \mathbb{A}_2^{d_N}$, in the other cases, $\mathbb{D}_2^{d_N}$ is a proper subset of $\mathbb{A}_2^{d_N}$.

The connection between the interface and the first layer of neurons is such that each neuron processes the input from the interface, this means that we have a unitary operator for the first layer's neural processing given by:

$$\hat{U}_{L_1,\triangle t} = \hat{1}_T \otimes \sum_{n=1}^{N} |a_n\rangle \langle a_n| \bigotimes_{k=1}^{d_N} e^{-\frac{i}{\hbar}\triangle t \hat{H}_{k,n}} \otimes \hat{1}_2^{\otimes d_N} \otimes \hat{1}_2 \tag{87}$$

thus, while the last two layers remain unchanged, each neuron's dynamics in the first layer is transformed accordingly with a conditional unitary operator, with the conditional Hamiltonians having the general structure of Eqs.(64) to (66) but such that the conditional utility optimization is conditional on the interface's eigenvalues, the Hamiltonians for each neuron are, thus, given by:

$$\begin{aligned} \hat{H}_{k,n} = \\ = -\frac{\hbar}{2}\frac{\left(1 - \arg\max_{\mathbf{s},k} v_1[\mathbf{s}|a_n]\right)\pi}{\triangle t_0}\hat{1}_2 + \\ + \frac{\left(2 - \arg\max_{\mathbf{s},k} v_1[\mathbf{s}|a_n]\right)\pi}{2\triangle t_0}\sum_{j=1}^{3} u_{j,k}(a_n)\frac{\hbar}{2}\hat{\sigma}_j \end{aligned} \tag{88}$$

$$u_{1,k}(a_n) = u_{3,k}(a_n) = \frac{1 - \arg\max_{\mathbf{s},k} v_1[\mathbf{s}|a_n]}{\sqrt{2}} \tag{89}$$

$$u_{2,k}(a_n) = \arg\max_{\mathbf{s},k} v_1[\mathbf{s}|a_n] \tag{90}$$

where v_1 is the conditional utility function for the first layer. Now, during the neural prodcessing period $\triangle t$, the probe carrier undergoes the conditional unitary propagation:

$$\begin{aligned} \hat{U}_{L_1,\triangle t}\hat{U}_{Sensory}|\Psi\rangle = \\ = \left(\sum_{n=1}^{N} \psi_n |o_n\rangle \otimes |a_n\rangle \bigotimes_{k=1}^{d_N} e^{-\frac{i}{\hbar}\triangle t \hat{H}_{k,n}} |+\rangle\right) \otimes |+\rangle^{d_N+1} \end{aligned} \tag{91}$$

so that the neural field no longer has a separable dynamics, with the first layer of neurons exhibiting a conditional propagation that depends upon the interface's field's probed alternatives. Likewise, for the response carrier, we get the correlated propagation:

$$\begin{gathered}\langle\Psi|\,\hat{U}^{\dagger}_{Sensory}\hat{U}^{\dagger}_{L_1,\triangle t} = \\ = \left(\sum_{n=1}^{N}\psi_n^*\,\langle o_n|\otimes\langle a_n|\bigotimes_{k=1}^{d_N}\langle +|\,e^{\frac{i}{\hbar}\triangle t\hat{H}_{k,n}}\right)\otimes\langle +|^{d_N+1}\end{gathered} \tag{92}$$

As $\triangle t \rightarrow \triangle t_0$ the probe and response carriers converge to an entangled probing and response dynamics with the interface. Assuming that:

$$\mathbf{s}[n-1] = \arg\max_{\mathbf{s}}\, v_1[\mathbf{s}|a_n] \tag{93}$$

we get the following probe and response carriers for the first layer's optimum:

$$\begin{gathered}\hat{U}_{L_1,\triangle t_0}\hat{U}_{Sensory}\,|\Psi\rangle = \\ = \left(\sum_{n=1}^{N}\psi_n\,|o_n\rangle\otimes|a_n\rangle\otimes|\mathbf{s}[n-1]\rangle\right)\otimes|+\rangle^{d_N+1}\end{gathered} \tag{94}$$

$$\begin{gathered}\langle\Psi|\,\hat{U}^{\dagger}_{Sensory}\hat{U}^{\dagger}_{L_1,\triangle t_0} = \\ = \left(\sum_{n=1}^{N}\psi_n^*\,\langle o_n|\otimes\langle a_n|\otimes\langle\mathbf{s}[n-1]|\right)\otimes\langle +|^{d_N+1}\end{gathered} \tag{95}$$

The probe and response dynamics of the neural field now exhibits an entangled dynamics with the probe and response dynamics of the interface's field and the target's field. Indeed, when the target's field probes the alternative o_n, the interface's field probes the alternative a_n and the neural field probes, for the first layer of neurons, the neural firing pattern matching the binary string $\mathbf{s}[n-1]$. Likewise, for the response dynamics we also get a similar correlation.

At this point, the robot is still working similarly to Everett's proposal, in the sense that Everett (1973) considered the possibility of addressing the observer's degree of freedom in terms of a (quantum) neurodynamical mapping of the target. In this case, we addressed the architecture of a complex measurement apparatus that includes a interface plus a neural network.

The importance of the neural network comes from the fact that we do not wish to just address the quantum measurement, that is, to just have a measurement automaton. The main point is that the robot's neural network must function as a cognitive basis for observation and decision, in this case, how to change the target's quantum dynamics.

The next step of the computation takes us beyond the standard quantum measurement, in the sense that the second layer of neurons produces an internal cognitive evaluation of what is a desirable dynamical pattern for the target, given the observed dynamical pattern.

The next stage of the quantum neural computation, thus, addresses the optimization of a performance measure incorporated in the second layer's conditional utility function, so that, in terms of reflexivity, the first stage of learning, reflexively, plays an adaptive role that can be described as the target's diagnosis, while the second stage implements the evaluation of what is "desirable" for the robot. This means that the conditional unitary propagation has to incorporate a performance optimization dependent upon the first layer's firing patterns.

Let us, then, define the conditional utility for the second layer such that if $\mathbf{r} \notin \mathbb{D}_2^{d_N}$, then, $\arg\max_{\mathbf{s}} v_2[\mathbf{s}|\mathbf{r}] = \mathbf{r}$, while, for $\mathbf{r} \in \mathbb{D}_2^{d_N}$, then, $\arg\max_{\mathbf{s}} v_2[\mathbf{s}|\mathbf{r}]$ corresponds to an optimal performance that may not coincide with $\mathbf{r}$. Then, the unitary operators for the second layer have the conditional structure:

$$\hat{U}_{L_2,\triangle t} = \hat{1}_T \otimes \hat{1}_{Int} \otimes \sum_{\mathbf{r}\in\mathbb{A}_2^{d_N}} |\mathbf{r}\rangle \langle\mathbf{r}| \bigotimes_{k=1}^{d_N} e^{-\frac{i}{\hbar}\triangle t \hat{H}_{k,\mathbf{r}}} \otimes \hat{1}_2 \tag{96}$$

with the local Hamiltonians given by:

$$\begin{aligned} \hat{H}_{k,\mathbf{r}} &= \\ &= -\frac{\hbar}{2}\frac{\left(1-\arg\max_{\mathbf{s},k} v_2[\mathbf{s}|\mathbf{r}]\right)\pi}{\triangle t_0}\hat{1}_2 + \\ &+\frac{\left(2-\arg\max_{\mathbf{s},k} v_2[\mathbf{s}|\mathbf{r}]\right)\pi}{2\triangle t_0}\sum_{j=1}^{3} u_{j,k}(\mathbf{r})\frac{\hbar}{2}\hat{\sigma}_j \end{aligned} \tag{97}$$

$$u_{1,k}(\mathbf{r}) = u_{3,k}(\mathbf{r}) = \frac{1-\arg\max_{\mathbf{s},k} v_2[\mathbf{s}|\mathbf{r}]}{\sqrt{2}} \tag{98}$$

$$u_{2,k}(\mathbf{r}) = \arg\max_{\mathbf{s},k} v_2[\mathbf{s}|\mathbf{r}] \tag{99}$$

Using the following notation:

$$\mathbf{s}_{max}[n-1] = \arg\max_{\mathbf{s}} v_2[\mathbf{s}|\mathbf{s}[n-1]] \tag{100}$$

as $\triangle t \to \triangle t_0$, the probe and response carriers converge to the following superpositions:

$$\begin{gathered}\hat{U}_{L_2,\triangle t_0}\hat{U}_{L_1,\triangle t_0}\hat{U}_{Sensory}\left|\Psi\right\rangle = \\ = \left(\sum_{n=1}^{N}\psi_n\left|o_n\right\rangle \otimes \left|a_n\right\rangle \otimes \left|\mathbf{s}[n-1];\mathbf{s}_{max}[n-1]\right\rangle\right) \otimes \left|+\right\rangle\end{gathered} \tag{101}$$

$$\begin{gathered}\left\langle\Psi\right|\hat{U}^{\dagger}_{Sensory}\hat{U}^{\dagger}_{L_1,\triangle t_0}\hat{U}^{\dagger}_{L_2,\triangle t_0} = \\ = \left(\sum_{n=1}^{N}\psi^*_n\left\langle o_n\right| \otimes \left\langle a_n\right| \otimes \left\langle\mathbf{s}[n-1];\mathbf{s}_{max}[n-1]\right|\right) \otimes \left\langle+\right|\end{gathered} \tag{102}$$

so that, as $\triangle t \to \triangle t_0$, at the level of the probe carrier, for the second layer, we actually have a convergence from a symmetric equally weighted superposition of the different firing patterns to an entangled dynamics with a probe carrier for the second layer that always probes the optimum performance alternative. A similar argument holds for the response carriers.

Now the third and last layer plays a role at the action stage level. The unitary operator for this layer is:

$$\hat{U}_{L_3,\triangle t} = \hat{1}_T \otimes \hat{1}_{Int} \otimes \sum_{\mathbf{r},\mathbf{s}\in\mathbb{A}_2^{d_N}}\left|\mathbf{r};\mathbf{s}\right\rangle\left\langle\mathbf{r};\mathbf{s}\right| \otimes e^{-\frac{i}{\hbar}\triangle t\hat{H}_{\mathbf{r},\mathbf{s}}} \tag{103}$$

with the local Hamiltonians given by:

$$\begin{gathered}\hat{H}_{\mathbf{r},\mathbf{s}} = \\ = -\frac{\hbar}{2}\frac{\left(1-\arg\max_{s} v_3[s|\mathbf{r},\mathbf{s}]\right)\pi}{\triangle t_0}\hat{1}_2 + \\ + \frac{\left(2-\arg\max_{s} v_3[s|\mathbf{r},\mathbf{s}]\right)\pi}{2\triangle t_0}\sum_{j=1}^{3}u_j(\mathbf{r},\mathbf{s})\frac{\hbar}{2}\hat{\sigma}_j\end{gathered} \tag{104}$$

$$u_1(\mathbf{r},\mathbf{s}) = u_3(\mathbf{r},\mathbf{s}) = \frac{1-\arg\max_{s} v_3[s|\mathbf{r},\mathbf{s}]}{\sqrt{2}} \tag{105}$$

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$$u_2(\mathbf{r}, \mathbf{s}) = \arg\max_{s} v_3[s|\mathbf{r}, \mathbf{s}] \tag{106}$$

Using the indicator function $1_{<condition>}$ which evaluates to 1, if the $< condition >$ holds, and to 0, if the $< condition >$ does not hold, we are, thus, assuming that the optima for the third layer are such that:

$$\arg\max_{s} v_3[s|\mathbf{r}, \mathbf{s}] = 1_{\mathbf{r} \neq \mathbf{s}} \tag{107}$$

Again as $\triangle t \to \triangle t_0$, the probe and response carriers converge to the following final superpositions:

$$\begin{aligned} &\hat{U}_{L3,\triangle t_0} \hat{U}_{L2,\triangle t_0} \hat{U}_{L1,\triangle t_0} \hat{U}_{Sensory} |\Psi\rangle = \\ &= \sum_{n=1}^{N} \psi_n |o_n\rangle \otimes \left| a_n; \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]; 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]} \right\rangle \end{aligned} \tag{108}$$

$$\begin{aligned} &\langle \Psi | \hat{U}^{\dagger}_{Sensory} \hat{U}^{\dagger}_{L1,\triangle t_0} \hat{U}^{\dagger}_{L2,\triangle t_0} \hat{U}^{\dagger}_{L3,\triangle t_0} = \\ &= \sum_{n=1}^{N} \psi^*_n \langle o_n | \otimes \left\langle a_n; \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]); 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]} \right| \end{aligned} \tag{109}$$

In the above equations the carriers for the last neuron are such that the neuron does not fire when $\mathbf{s}[n-1] = \mathbf{s}_{max}[n-1]$ and fires otherwise. This last neuron triggers the action stage.

The action stage proceeds in two steps, in the first step, we have a feedback dynamics from the neural field to the interface, in the second step, the actuator is applied where the feedback is now from the interface's field to the target's field. The operator for the first step is defined as:

$$\hat{U}_{Int} = \hat{1}_T \otimes \sum_{\mathbf{r}, \mathbf{s} \in \mathbb{A}_2^{d_N}} \sum_{r \in \mathbb{A}_2} \hat{G}(\mathbf{r}, \mathbf{s}, r) \otimes |\mathbf{r}; \mathbf{s}; r\rangle \langle \mathbf{r}; \mathbf{s}; r| \tag{110}$$

where the operator $\hat{G}(\mathbf{r}, \mathbf{s}, r)$ is given by:

$$\hat{G}(\mathbf{r}, \mathbf{s}, r) = \begin{cases} \hat{1}_{Int}, & r = 0 \vee \mathbf{r} \notin \mathbb{D}_2^{d_N} \\ \hat{U}^{int}_{\mathbf{r},\mathbf{s}}, & \text{otherwise} \end{cases} \tag{111}$$

and $\hat{U}^{int}_{\mathbf{r},\mathbf{s}}$ is given by:

$$
\begin{gathered}
\hat{U}^{int}_{\mathbf{r},\mathbf{s}} = \\
\left|g(\mathbf{s})\right\rangle \left\langle g(\mathbf{r})\right| + \left|g(\mathbf{r})\right\rangle \left\langle g(\mathbf{s})\right| + \\
+ \sum_{a \in A \setminus \{g(\mathbf{r}), g(\mathbf{s})\}} \left|a\right\rangle \left\langle a\right|
\end{gathered}
\tag{112}
$$

where g is a one-to-one and map from $\mathbb{D}_2^{d_N}$ onto the set of quantum numbers $A = \{a_1, a_2, ..., a_N\}$, such that, for $n = 1, ..., N$:

$$
g(\mathbf{s}[n-1]) = a_n \tag{113}
$$

Thus, under the interface's conditional propagation, we get for the probe and response carriers:

$$
\begin{aligned}
&\hat{U}_{Int}\hat{U}_{L_3,\triangle t_0}\hat{U}_{L_2,\triangle t_0}\hat{U}_{L_1,\triangle t_0}\hat{U}_{Sensory}\left|\Psi\right\rangle = \\
&= \sum_{n=1}^{N} \psi_n \left|o_n\right\rangle \otimes \\
&\otimes \left|g\left(\mathbf{s}_{max}[n-1]\right); \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]; 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]}\right\rangle
\end{aligned}
\tag{114}
$$

$$
\begin{aligned}
&\left\langle\Psi\right| \hat{U}^{\dagger}_{Sensory}\hat{U}^{\dagger}_{L_1,\triangle t_0}\hat{U}^{\dagger}_{L_2,\triangle t_0}\hat{U}^{\dagger}_{L_3,\triangle t_0}\hat{U}^{\dagger}_{Int} = \\
&= \sum_{n=1}^{N} \psi^*_n \left\langle o_n\right| \otimes \\
&\otimes \left\langle g\left(\mathbf{s}_{max}[n-1]\right); \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]; 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]}\right|
\end{aligned}
\tag{115}
$$

At this point, the interface's field probe and response dynamics are towards the optimal response pattern expressed in terms of the interface eigenvalue $g\left(\mathbf{s}_{max}[n-1]\right)$.

The only final step now is for the robot to interact again with the target. In this, case we have the actuator operator:

$$
\hat{U}_{act} = \sum_{n=1}^{N} \sum_{\mathbf{s} \in \mathbb{A}_2^{d_N}} \hat{U}^{act}_{\mathbf{s};n} \otimes \left|a_n\right\rangle \left\langle a_n\right| \otimes \left|\mathbf{s}\right\rangle \left\langle \mathbf{s}\right| \otimes \hat{1}_2^{\otimes(d_N+1)} \tag{116}
$$

with the action unitary operator defined as:

$$
\hat{U}^{act}_{\mathbf{s};n} = \begin{cases} \hat{1}_T, & g(\mathbf{s}) = a_n \vee \mathbf{s} \notin \mathbb{D}_2^N \\ \hat{L}_{\mathbf{s};n}, & \text{otherwise} \end{cases} \tag{117}
$$

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$$
\begin{aligned}
&\hat{L}_{\mathbf{s};n} = \\
&= \left|o_n\right\rangle \left\langle f \circ g(\mathbf{s})\right| + \left|f \circ g(\mathbf{s})\right\rangle \left\langle o_n\right| + \\
&+ \sum_{o \in O \setminus \{o_n, f \circ g(\mathbf{s})\}} \left|o\right\rangle \left\langle o\right|
\end{aligned}
\tag{118}
$$

where f is a one-to-one map from the interface's quantum numbers set A to the observable's quantum numbers set O, such that:

$$
f(a_n) = o_n \tag{119}
$$

Under this actuator we get, for the probe and response carriers, the final dynamics:

$$
\begin{aligned}
&\hat{U}_{act}\hat{U}_{Int}\hat{U}_{L3,\triangle t_0}\hat{U}_{L2,\triangle t_0}\hat{U}_{L1,\triangle t_0}\hat{U}_{Sensory}\left|\Psi\right\rangle = \\
&= \sum_{n=1}^{N} \psi_n \left|f \circ g\left(\mathbf{s}_{max}[n-1]\right)\right\rangle \otimes \\
&\otimes \left|g\left(\mathbf{s}_{max}[n-1]\right); \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]; 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]}\right\rangle
\end{aligned}
\tag{120}
$$

$$
\begin{aligned}
&\left\langle\Psi\right| \hat{U}^{\dagger}_{Sensory}\hat{U}^{\dagger}_{L1,\triangle t_0}\hat{U}^{\dagger}_{L2,\triangle t_0}\hat{U}^{\dagger}_{L3,\triangle t_0}\hat{U}^{\dagger}_{Int}\hat{U}^{\dagger}_{act} = \\
&= \sum_{n=1}^{N} \psi_n^{*} \left\langle f \circ g\left(\mathbf{s}_{max}[n-1]\right)\right| \\
&\otimes \left\langle g\left(\mathbf{s}_{max}[n-1]\right); \mathbf{s}[n-1]; \mathbf{s}_{max}[n-1]; 1_{\mathbf{s}[n-1] \neq \mathbf{s}_{max}[n-1]}\right|
\end{aligned}
\tag{121}
$$

At the level of the target, the echoes are formed so that the initial echo dynamics transitions to:

$$
\left|\psi_n\right|^2 \left|o_n\right\rangle \left\langle o_n\right| \rightarrow \left|\psi_n\right|^2 \left|o_n^{max}\right\rangle \left\langle o_n^{max}\right| \tag{122}
$$

where o_n^{max} is the eigenvalue for the target that is the best response in terms of conditional utility for the agent, that is, following the full optimization of the performance measure, and given the definition of f and g we get:

$$
o_n^{max} = f \circ g\left(\arg\max_{\mathbf{s}} v_2\left[\mathbf{s}|g^{-1} \circ f^{-1}\left(o_n\right)\right]\right) \tag{123}
$$

the sequence $g^{-1} \circ f^{-1}\left(o_n\right)$ is the path taken by the robot's cognitive dynamics from initial interaction with the target $f^{-1}\left(o_n\right)$ to the neural processing of the sensory data by the neural field, $g^{-1} \circ f^{-1}\left(o_n\right)$, so that, if the

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target's field probes the alternative o_n, the interface's field probes the alternative $f^{-1}(o_n)$ and the first layer of neurons probes the alternative $g^{-1} \circ f^{-1}(o_n)$, then, the second layer of neurons probes the maximum conditional utility alternative corresponding to the optimum of the performance measure. From there on, the dynamics is such that the interface field no longer probes the alternative $f^{-1}(o_n)$ but, due to a feedback from the neural field, now probes the matching pattern, with respect to the interface eigenvalues, for the maximum conditional utility firing pattern, this is the sequence $g\left(\arg\max_{\mathbf{s}} v_2\left[\mathbf{s}|g^{-1} \circ f^{-1}(o_n)\right]\right)$, and, finally, through the actuator, the target no longer probes o_n but, rather, the pattern that matches the maximum conditional utility firing pattern under the sequence $f \circ g\left(\arg\max_{\mathbf{s}} v_2\left[\mathbf{s}|g^{-1} \circ f^{-1}(o_n)\right]\right)$. A similar dynamics can be described for the response dynamics.

Given the changes in the echoes, the robot, through its interaction with the target, assures that the initial line of force associated with each alternative $|o_n\rangle\langle o_n|$ changes its aim from that line of force $|o_n\rangle\langle o_n|$ to the conditional utility maximizing alternative $|o_n^{max}\rangle\langle o_n^{max}|$, as long as the two are different (if the two are not different, then the third neuron does not fire and the robot does nothing).

In the final dynamics, the target's field probing and response dynamics and the robot's interface field and neural field probing and response dynamics are entangled, in a specific way: there is a direct matching between the target, the interface and neural field at the level of the second layer's firing pattern, since that, if the target's field probes the n-th target eigenvalue or there is a response associated with that eigenvalue, then, the interface's probe/response are, respectively, also towards the n-th interface eigenvalue and the second layer's probe/response are, respectively, towards the matching firing pattern $\mathbf{s}[n-1]$.

The first layer's probe and response dynamics, in turn, correspond to the neural mapping of the initial dynamics of the target by way of the quantum neural processing of the interface, thus, it works as a memory layer. The relation of the first and second layer is that it represents the mapping defined by the robot's performance optimization, where the first layer's dynamics always addresses the starting point, while the second layer always addresses the maximum utility response leading to the optimum of the performance measure.

The maximization is incorporated in the conditional Hamiltonians for the neural connections between the two layers, so that, as shown above, the network converges to the optimum during the neural processing period as it converges to the learning period. Finally, the third layer's single neuron probes a nonfiring dynamics if the first two layer's probed firing patterns match and probes a firing dynamics otherwise. A similar dynamics applies to the response carriers.

A relevant point is that if the optimum is always the same for each alternative, then, the target is placed in a projective dynamics for that optimum. Also, even if the number N is very large, still the neural network is always able to "find" the optimum performance due to the conditional Hamiltonians linking the first and second layers. The optimization is incorporated in the Hamiltonian associated with the neural connections, in accordance with the quantum neural machine learning framework defined in the previous subsection.

4 Conclusion

Quantum mechanics has a different computational substratum from that of classical mechanics. Computationally, classical mechanics can be approached in terms of a computation where states are the object of the computation and the transition between states is expressed by a mechanical rule within the mathematical framework of calculus, quantum mechanics is different.

While, for calculations, and predictions in physical experiments it does not bring severe problems that one is using the wrong computational framework, when one starts to deal with next generation quantum technologies, in particular, those that merge ICTs with CogTech, such as quantum AI, machine learning, robotics and other applications of quantum nanoscience, then, the wrong computational framework does make a difference.

Cramer's TI was the first interpretation of quantum mechanics to consistently expose the main dynamics, namely, if one considers Cramer's work, one has to look at quantum mechanics in terms of a dynamics that is closer to a communication dynamics that links present and future. Instead of a single mathematical object (the wave function), Cramer's proposal raises the point that we need to consider the two wave dynamics: the *offer waves* that propagate forward towards the future and the *confirmation waves* that come from the future.

The quantities produced by the encounter of these two wave dynamics,

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one present to future bound wave going from the emitter to the absorber (the *offer waves*) and the other future to present bound wave coming from the absorber to the emitter (the *confirmation wave*), are, in Cramer (2016), the strengths of the echoes arriving back at the *site of emission*, so that a transaction for a final exchange of energy and momentum depends probabilistically on the strengths of these echoes.

This is a basic communication circuit that is not present in classical mechanics. However, despite other interpretations of quantum mechanics, that work with states, this circuit has always been represented, in the formalism, in one particular object that is used to extract probabilities: the density operator.

The density operator expresses a specific quantum computational dynamics, and points to a framework that is consistent with Cramer's proposal. Namely, by recovering the primitive concept of vector as *carrier*, the density operator, rather than expressing a state, is actually expressing a dynamics, where the off-diagonal terms are failed echoes and the main diagonal terms are the echoes resulting from the matching of a probe and response dynamics.

The QFI works with the basis from TI, linking it to the density operator, namely, the density is addressed, within the QFI, as a dynamics similar to that of echolocation, namely, we have a *probe carrier* and a *response carrier* for each alternative, for which an echo is formed, with the final alternative taking place with a probability that is equal to the echo intensity. If we were to remain with this approach, then, we would have a variant of Cramer's TI, working from the density operator, rather than the wave function as primitive, and we would have an account for the physical meaning of the off-diagonal terms as failed echoes as well as for Born's rule and why it works with the diagonal terms of the density.

However, from a physical and computational perspective a problem remains, which is: the reason for the quantum field to follow each alternative in accordance with the echo intensity. It is at this point that the concept of quantum force is introduced, working from Bohm's proposal. In this case, we are moving from what is basically a TI-variant to a theory of the source of the quantum dynamics.

While the TI-variant essentially implies, computationally, that we have a field-based cognition that works with a probing and response dynamics to probabilistically select a final solution to a quantum problem, the quantum force concept offers a systemic basis for this cognition, in terms of the field activity at the quantum level, supported by a subquantum level dynamics.

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Both quantum and subquantum levels are working in the field's dynamics towards an order to be risen, in this way, lines of force are risen that are consistent with the field's quantum cognition with respect to the problem, such that the intensity of each line coincides with the echo intensity, the field, then, tends to follow the lines of force with a probability that coincides with the force intensity. In terms of system's dynamics, we have something like a bifurcation where the system will follow, stochastically, one of the branches with a probability that coincides with the force intensity associated with each branch.

This is the key basis for the QFI. Thus, the QFI works with elements from Cramer's TI and Bohm's interpretation, combining them with computer science and cognitive science, providing for a substratum that is more consistent with the new generation of quantum technologies.

In the case of quantum CPC systems, this is particularly useful since we have a basis for addressing, computationally, the dynamics of open systems in interation with the environment and that are capable of adaptively acting on that environment. The example of the quantum robot, that was addressed, provides for a key point, namely: quantum entanglement, in the case of the robot, is a dynamics between fields by which a field's probing and response dynamics becomes correlated with the other field's probing and response dynamics, in this way, the interaction is a form of conditional transformation of the dynamics of the involved systems by which the echoes and lines of force become correlated, which means that from that point on the fields are nonlocally connected.

These nonlocal connections can be taken advantage of computationally, so that a quantum CPC system, equipped with an appropriate cognitive architecture, is capable of adapting to a target using the interactions and the entanglement to affect the target's field so that the target's field dynamics, namely, the target's lines of force point towards the alternatives that are consistent with the quantum CPC system's goals (given goals since we are dealing with technological constructions).

Quantum robots, as artificially intelligent measurement systems that are capable not only of measuring target quantum systems but, also, of acting on those targets based on the measured results, can thus be used to manage quantum physical systems, leading to an adaptive dynamics and faster optimization of target quantum systems. In particular, for nanosystems' energy manipulations this may provide a road for future quantum-based applications.

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This was already illustrated in the example, where the quantum robot was capable of processing the target using an interface and a QuANN that solves an optimization problem, finding the utility maximizing solution for each initial quantum dynamics of the target's field, and interacting with the target so that the target's lines of force are only directed towards the conditional utility maximizing solutions.

The optimization is like finding a *needle in a haystack* when the target's observable has a high number of alternatives. Indeed, as the dimensionality of the target's Hilbert space grows with N, we have an increase in the number of alternatives for each case, however, due to the conditional Hamiltonians associated with the neural conections, the QuANN is always capable of converging to the right solution during the learning period.

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