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A trend-cycle decomposition with hysteresis

By: Javier G. Gómez-Pineda
Julián Roa-Rozo

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A trend-cycle decomposition with hysteresis

Javier G. Gómez-Pineda [1] and Julián Roa-Rozo [2] [3] [4]

Abstract

Business fluctuations can be estimated as the product of perturbations that do not need to be broken down into supply and demand shocks. Joint supply and demand (S&D) shocks can help estimate the cycle in the output gap as well as a cycle in trend output. The model is a univariate trend-cycle decomposition with hysteresis in trend output, that enables the estimation of the output gap and trend output in 81 economies in quarterly frequency, since 1995Q1; and 184 economies in yearly frequency, in several cases since 1950, and in a few cases since 1820. Volatility and dispersion, as well as the frequency of large joint trend-cycle shocks, were low during the Gilded Age period; high during the interwar period, even more so in advanced (AD) economies compared to emerging market and developing economies (EMDE); and low in AD economies and high in EMDE economies in the post WWII period. In contrast with other existing estimates of trend output, those from the trend-cycle decomposition with hysteresis do not evolve smoothly, do not result in an artificial boom before recessions and are less sensitive to new data.

Keywords: Hysteresis; Business cycles; Business Fluctuations; Univariate model; Trend-cycle decomposition; Trend output; Output gap; Potential output

JEL codes: E32; E50; O47; E58; E37

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[1]Banco de la República, Colombia, jgomezpi@banrep.gov.co

[2]Econometría Consultores and Centro de Estudios Manuel Ramírez, Bogotá, jroa@econometria.com

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Una descomposición tendencia-ciclo con histéresis

Javier G. Gómez Pineda[1] and Julián Roa-Rozo[23]

Resumen

Las fluctuaciones económicas pueden estimarse como producto de perturbaciones que no necesitan desagregarse entre choques de oferta y demanda. Choques conjuntos de oferta y demanda (S&D) pueden ayudar a estimar el ciclo de la brecha del producto, así como un ciclo en el producto tendencial. El modelo es una descomposición ciclo-tendencia univariada con histéresis en el producto tendencial, que permite la estimación de la brecha del producto y el producto tendencial en 81 economías en frecuencia trimestral desde 1995Q1 y en 184 economías en frecuencia anual desde 1975. La volatilidad, dispersión y frecuencia de choques conjuntos grandes fueron bajos durante el período de la Época Dorada; altos durante el período entre guerras, aún más en economías avanzadas (AD) en comparación con las emergentes y en desarrollo (EMDE); y bajo en las economías AD y alto en las economías EMDE en la segunda postguerra. En contraste con otros estimativos existentes del producto tendencial, los de la descomposición tendencia-ciclo con histéresis no evolucionan de forma suave, no resultan en un boom artificial antes de las recesiones y son menos sensibles a los datos nuevos.

Códigos JEL: E32; E50; O47; E58; E37

Palabras clave: Histéresis; Ciclo de los negocios; Fluctuaciones económicas; Modelo univariado; Descomposición tendencia ciclo; Producto tendencial; Brecha del producto; Producto potencial

[1]Banco de la República, Colombia, jgomezpi@banrep.gov.co

[2]Econometría, Bogotá, jroa@econometria.com

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Contents

List of acronyms

AD	Advanced (economies)
DDF	(Trend output) in deviation and difference form
EMDE	Emerging market and developing (economies)
HP	Hodrick-Prescott (filter)
LLT	Local linear trend (filter)
S&D	(Joint) supply and demand (shock)
S-level&D	(Joint) supply level and demand (shock)
S-growth&D	(Joint) supply growth and demand (shock)
WWII	Second world war

Executive summary

According to the traditional Keynesian view, business fluctuations are caused by shocks that affect the output gap while trend output evolves smoothly (see e.g. the account of this traditional view in Woodford, 2017, p. 273). In contrast with this traditional Keynesian view, Cerra and Saxena (2017), argue that "the traditional distinction between supply and demand shocks and the assumption that demand shocks have only transitory economic impact need to be revisited" (p. 25). In addition, Cerra and Saxena (2017) contend that, because shocks lead to shifts in the trend, "the business cycle is not a cycle" (p. 7). In a similar line of argumentation, Aguiar and Gopinath (2007) argue that in emerging market economies the cycle is the trend, or that the cycle is also in the trend.

We argue that, in both advanced (AD) and emerging market and developing (EMDE) economies, business fluctuations are the movement of output about the trend as well as the cycle in the trend itself; the trend exhibits cycles provided it is measured in stationary terms. Moreover, we estimate business fluctuations under a simplifying research strategy. The strategy consists of assuming that shocks to demand and supply are a unified, compound, joint supply and demand (S&D) shock. The assumption amounts to arguing that business fluctuations are not caused, as in the extant literature, by demand shocks whose effects on the output gap enter the supply equations. Instead, under the assumption, business fluctuations are the product of shocks that affect both trend output and the output gap. Then, trend output is not driven by pure supply shocks only, nor the output gap is driven by pure demand shocks exclusively; instead, they are both driven by joint S&D shocks and are part of the business cycle.

In the trend-cycle decomposition trend output has some important properties. The first one is that it does not evolve smoothly; indeed, it may collapse at the outbreak of a recession, examples are the drops in the estimated trend output during the COVID-19 recession in most countries. Second, the estimated trend output does not create an output gap boom before a recession, as it is the case when using filtering methods where trend output evolves smoothly; that is the case when using the local linear trend and Hodrick-Prescott filters. Third, the estimated output gap is less sensitive to incoming data, compared with said filters where trend output is smooth.

A trend that shifts along with joint S&D shocks may perform better in terms of prediction.

Thus, following Harvey (1989) we use the term trend as that level that contains information about the future of the series, without regard to whether it evolves smoothly or not.

The stationary measure of trend output is trend output in deviation from a long-term stochastic trend and in difference form. Trend output in deviation and difference form (DDF) is highly correlated with the output gap because, as has been said, both the output gap and trend output DDF are the business cycle, and they are both driven by joint S&D shocks.

The estimation of the business cycle is carried out using two samples, one in quarterly frequency and the other one in yearly frequency. In the quarterly database, during the 2008-2009 financial crisis and the COVID-19 recession, the business cycle is highly synchronized across countries. Nonetheless, the yearly database shows that such high synchronization was not the case before these recessions.

The results show that during the 2008-2009 financial crisis in AD economies trend output shows important hysteresis. This was not the case in EMDE economies. In contrast, during the COVID-19 recession trend output shows important hysteresis in both AD and EMDE economies.

The estimation of the posterior coefficients and the standard deviation of the shocks shows dispersion across countries. Therefore, the response of both trend output and the output gap to joint S&D shocks also has dispersion across countries; or, in other terms, is heterogeneous.

Based on the volatility and dispersion of the business cycle as well as on the frequency of extreme events, three historical periods were identified; a tranquil, 1870-1910, Gilded Age period; a turbulent, 1910-1950, war and interwar period, although somewhat less turbulent in EMDE economies; and the post WWII period, 1950-, tranquil in AD economies and turbulent in several EMDE economies. The long-term growth rate rose markedly during the post WWII economic expansion. Starting the 1970s, the long-term growth rate declined along with the productivity slowdown.

A comparison of the output gap estimates with other available public databases available in yearly frequency makes evident the advantages of the trend-cycle decomposition with hysteresis; namely, trend output is not smooth, no large booms necessarily appear before recessions and output gap revisions are smaller. These advantages are obtained at the cost of an output gap with a standard deviation that is somewhat smaller.

1 Introduction

In characterizing the current paradigm of the New Neoclassical Synthesis (NNS), Woodford (2009) points out that in “the traditional Keynesian view of business cycles, ... fluctuations are caused by a variety of types of real disturbances that affect economic activity solely through their effects on aggregate demand while aggregate supply evolves as a smooth trend.” In contrast with this traditional Keynesian view, Cerra et al. (2020) argue that “the traditional distinction between supply and demand shocks and the assumption that demand shocks have only transitory economic impact need to be revisited.” (p. 25). Furthermore, Cerra and Saxena (2017) contend that, because shocks lead to shifts in the trend, “the business cycle is not a cycle” (p. 7). In a similar line of argumentation, Aguiar and Gopinath (2007) argue that in emerging market economies the cycle is the trend or that the cycle is also in the trend.

We argue that, in both advanced (AD) economies as well as in emerging market and developing (EMDE) economies, business fluctuations are the movement of output about the trend as well as the cycle in the trend itself, provided the trend is defined in stationary terms. In addition, we estimate business fluctuations under a simplifying research strategy. The strategy consists of assuming that shocks to demand and supply are a unified, compound, joint supply and demand (SD) shock that affects both the trend and the cycle. The assumption amounts to arguing that business fluctuations are not caused, as in the extant literature, by demand shocks whose effects enter the supply equations. Instead, under the assumption, business fluctuations are caused by shocks that affect both trend output and the output gap. Then, trend output is not driven by pure supply shocks only, nor the output gap is driven by pure demand shocks exclusively; instead, they are both driven by joint S&D shocks and are part of the business cycle. Hence, the output gap and the stationary measure of trend output are correlated, both of them are parts of the business cycle.

In addition to the estimated trend output, we estimate a counterfactual trend output that is defined by shocks to the long-term rate of growth of trend output; it is not driven by joint S&D shocks or pure demand shocks. Trend output and counterfactual trend output both normally tilt upwards. The difference between them gives a non-stationary measure of trend output that we call trend output in deviation form and that is driven by joint S&D shocks and pure demand shocks. We use trend output in deviation form as our indicator of hysteresis.

Our approach that bundles the shocks that affect trend output and the output gap into a compound joint shock has similarities with two papers in the literature. First, Guerrieri et al. (2022) introduce "Keynesian supply shocks," or drops in trend output that turn into a drop in demand in other sectors; thus, in their set-up, supply shocks have a demand dimension. Second, Furlanetto et al. (2021) expand the supply-demand shock decomposition proposed by Blanchard and Quah (1989) with demand shocks that, potentially, can affect output permanently. These "permanent demand shocks" explain about half the variance of trend output growth.[5]

In contrast with these studies, we work in a univariate setting. The simpler setting allows the study of business fluctuations in a large number of economies and over long sample periods. With the univariate setting we obtain results similar to those presented by Furlanetto et al. (2021) in a multivariate setting, as they find that permanent demand shocks explain 50 percent of the fluctuations in long-run output growth while we find that joint S&D shocks explain about 60 per cent of the forecast error variance decomposition (FEVD) of trend output growth.

Other studies incorporate the output gap in the trend output equation, examples are DeLong et al. (2012), Jordà et al. (2020b), Ball and Onken (2021) and Kienzler and Schmid (2014). In like fashion; a related class of models incorporates permanent effects of the business cycle on the rate of growth; for instance, in the paper by Schmoller (2022) research and development investments depend on the state of the economy and those investments affect technology adoption and then TFP growth.

The interaction or correlation between the output gap and trend output under these approaches arises because the output gap enters the trend output equation or the potential output block of the model. In contrast, in the trend-cycle decomposition with hysteresis the output gap and trend output are correlated because both variables are driven by joint S&D shocks.

The approach that incorporates the output gap in the trend output equation imposes two restrictions or assumptions. The first one is that the shocks that cause hysteresis are shocks to demand; or, in other words, that hysteresis ultimately comes from demand shocks. Nonetheless, some shocks can hardly be seen as pure demand shocks, or as movements in trend output that are originated in pure-demand shocks, examples are the loss of lives during a civil

[5]For a paper with estimated separate demand and supply shocks see Maffei-Faccioli (2021).

war,[6] or the deterioration in institutional quality under populism.[7] The second restriction of this approach is that hysteresis could not possibly be originated in a pure-supply shock. Although a natural disaster; such as a hurricane, for example, can destroy the productive capacity of an economy; without a pure demand shock, the approach that incorporates the output gap in the trend output equation cannot explain hysteresis.

On its part, other methods incorporate an effect not of the extent of the output gap on trend output, but of the length of the expansions and recessions in the output gap, see Alichi et al. (2019). Our approach incorporates the length dimension with estimated joint S&D shocks that turn out to be successions or streams of joint S&D shocks. Still, other methods focus on the effect of monetary policy on trend output, see Jordà et al. (2020a). Monetary policy can indeed be a source of hysteresis, at times causing as much hysteresis as other factors.

Cerra et al. (2020) review the shift in theoretical paradigm towards models that incorporate hysteresis. They also present the evolution of the decomposition of output between trend and cycle, starting from deviations from a long-term deterministic trend and then covering theories where demand innovations have permanent long run effects. Then, they review the empirical evidence on the persistence of fluctuations, starting with the unit-root literature, and covering the literature on the long-term effects of financial crisis, pandemics and monetary and fiscal policies. The authors point out that monetary and fiscal policies can have permanent effects on output; they also note that these effects may work in both directions. As to macroprudential policy, they suggest that, by taming the amplitude of the cycle, macroprudential policy can help reduce hysteresis.

The different methodologies used to estimate trend or latent output correspond to different concepts and uses of the concept as well as different terminology. The output gaps that are estimated with inflation targeting models are endogenous to the transmission mechanisms of monetary policy, one of these transmission mechanisms is the effect of the output gap on inflation and the effect of interest rates on the output gap. In cases these transmission mechanisms are also endogenous to the relationship between the output gap and unemployment. The notion of the estimated latent output is that of non-inflationary output and is usually called potential output.[8] Other output gaps are obtained with the production func-

[6]For an illustration of the effect of civil war on potential output see Cerra and Saxena (2008).
[7]On this matter see Cerra and Saxena (2008) and Edwards (2019).
[8]Also in DSGE models latent output is normally called potential output.

tion method whereby Hodrick-Prescott (HP) filters are applied to the inputs of production and the technology factor. Such estimations are at times used in fiscal sustainability analysis and have been called, among other terms, trend output. In turn, output gaps that incorporate information about a financial cycle, such as the cycle in credit and property prices, can be used for financial-stability analysis and have been called finance-neutral output gaps. Under this approach, latent output has been called finance-neutral output and also sustainable output.

In the trend-cycle decomposition with hysteresis latent output is called trend output. As said above, in the traditional Keynesian view the cycle is caused solely by demand shocks while the trend evolves smoothly. Although a smooth trend does indicate some tendency; a non-smooth trend that shifts along with joint S&D shocks may be a better indicator in of the future of trend output. Hence, we use the term trend following Harvey (1989), p. 284 where he points out: "What is a trend?... Viewed in terms of prediction, the estimated trend is that part of the series which when extrapolated gives the clearest indication of the future long-term movements of the series... The definition makes no mention of smoothness and it is consistent with the idea of indicating 'general direction'".

Latent output could more properly be called potential output under a multivariate set up that would incorporate the trend-cycle decomposition with hysteresis into the real block of multivariate semistructural or DSGE models including Phillips curves, aggregate demand equations and interest rate policy functions. Joint S&D shocks capturing for instance civil wars, financial crisis or natural disasters, would be endogenous to the multivariate setting. Monetary policy could still affect demand and supply, as for example in the mechanism from policy interest rates to potential output mediated by investment.

The paper has seven sections including this introduction. Section 2 presents the trend-cycle decomposition with hysteresis. Section 3 presents the two databases used in the paper, the quarterly database and the yearly database. Section 4 deals with the estimation of the model. The empirical results are presented in Section 5. This section is divided into four subsections. The impulse responses to joint S&D shocks are explained in subsection 1. Using the quarterly database, the 2008-2009 financial crisis and the COVID-19 recession in the United States and the Euro Area are studied in subsection 2. Here, some of the features of the model are explained. The results of the entire quarterly database are studied in subsection 3. Using the yearly database, large joint S&D shocks in trend output, such as the Great Depression in the United States and the Depression of the Twenties in the United Kingdom are studied in subsection 4. The results of the entire yearly database are also studied in subsection 4. Section 6 deals with the robustness and limitations of the trend-cycle decomposition with hysteresis. Section 7 discusses some of the features of the trend-cycle decomposition with hysteresis by comparing the output gap estimates with other output gap estimates that are publicly available. Section 8 presents some conclusions. Appendix 1 presents the model in deviation form, as it is used in the codes to run the model that are available in the web page of the paper. Appendix 2 explains the statistics that are summarized in boxplots, as this illustration device is used throughout the paper. Finally, Appendix 3 presents the list of countries in the quarterly and yearly databases.

2 The trend-cycle decomposition with hysteresis

The trend-cycle decomposition with hysteresis augments an otherwise standard local linear trend (LLT) filter, Harvey and Jaeger (1993) and Durbin and Koopman (2012), in two dimensions. First, it adds joint S&D shocks to the output gap and trend output equations, broken down into supply-level *cum* demand (S-level&D) shocks and supply-growth *cum* demand (S-growth&D) shocks. Second, it incorporates another layer of trend output growth, a long-term trend output growth rate, in short, a long-term growth rate, that is driven by long-term trend output growth shocks and is not driven by joint S&D shocks.

In the trend-cycle decomposition with hysteresis, the long-term growth rate helps estimate trend output growth over long periods as well as in cases where trend output growth tilts. In addition, in the model for the counterfactual trend, the long-term growth rate helps obtain a counterfactual measure of trend output, the level of trend output that would have taken place in the absence of joint S&D shocks. Thus, trend output growth is driven by joint S&D shocks and converges to the long-term growth rate that we use to construct counterfactual trend output.

The long-term growth rate is modelled as a partial adjustment mechanism. The mechanism helps estimate trend output growth in cases where it does not lend itself to be modelled easily as a constant; not only where trend output growth tilts but also under wide changes in trend output growth, particularly in long samples.[9]

[9]The partial adjustment mechanism is important to maintain the step-like behavior of trend output in deviation form that we use as indicator of hysteresis; the step-like behavior is not preserved in the extreme case where the partial adjustment mechanism becomes a random walk.

On one hand, we use trend output in deviation form, in levels, to illustrate hysteresis. On the other, we use trend output DDF to show that trend output also undergoes business cycles that turn out to be correlated with those of the output gap. Hence, as mentioned above, business fluctuations are the cycle in both the output gap and trend output in deviation and difference form (DDF); they are both driven by joint S&D shocks.

2.1 The model for trend output

In the trend-cycle decomposition with hysteresis, the output gap is driven by joint S&D shocks as well as by pure demand shocks. In turn, trend output is driven by joint S&D shocks as well as pure supply shocks. Pure demand and joint S&D shocks do not enter the long-term growth rate equation; business fluctuations, explained by pure demand and joint S&D shocks, are orthogonal to the long-term growth rate, which could be said to be the subject matter for growth theory.[10]

Let y_t be output; $\hat{y}_t$, the output gap; $\bar{y}_t$, trend output; γ_t trend output growth; g_t, the long-term growth rate; g, the constant average trend output growth rate, in short, the constant average growth rate; ε_t^L, the joint S-level&D shocks; ε_t^G, the joint S-growth&D shocks; u, the share of S-level&D shocks that affects the trend output level equation; n, the share of joint S-growth&D shocks that enters the trend output growth equation; and $(1-u)$ and $(1-n)$, the shares of S-level&D and S-growth&D shocks that enter the output gap equation.

The trend-cycle decomposition with hysteresis and joint S&D shocks consists of the following equations:[11]

$$\hat{y}_t = \alpha \hat{y}_{t-1} + (1-u)\varepsilon_t^L + (1-n)\varepsilon_t^G + \varepsilon_t^{\hat{y}}, \quad (1)$$

$$\bar{y}_t = \bar{y}_{t-1} + \frac{1}{4}\gamma_t + u\varepsilon_t^L + \varepsilon_t^{\bar{y}}, \quad (2)$$

$$\gamma_t = \theta\gamma_{t-1} + (1-\theta)g_t + n\varepsilon_t^G + \varepsilon_t^{\gamma}, \quad (3)$$

$$g_t = \phi g_{t-1} + (1-\phi)g + \varepsilon_t^g, \quad (4)$$

and

$$y_t = \hat{y}_t + \bar{y}_t. \quad (5)$$

[10]In any case, as joint shocks shift the output level, they may affect output growth over periods of, say, 2, 5 or 10 years. Nonetheless, this effect on output growth is merely an arithmetic level effect, not the result of a change in the long-term growth rate.

[11]Fraction $\frac{1}{4}$ in equation (2) converts quarterly growth into annual terms. In yearly frequency equation (2) becomes $\bar{y}_t = \bar{y}_{t-1} + \gamma_t + u\varepsilon_t^L + \varepsilon_t^{\bar{y}}$. The remaining equations are unchanged.

The output gap equation (1) states that the output gap is driven by pure demand shocks as well as by both joint S-level&D and S-growth&D shocks, and that the output gap converges to zero at speed α.

In turn, the trend output level equation (2) states that trend output growth, $4(y_t^{\bar{y}} - y_{t-1}^{\bar{y}})$, is driven by trend output growth γ_t, joint S-level&D shocks ε_t^L, and pure trend output level shocks, $\varepsilon_t^{\bar{y}}$.

Next, the trend output growth equation (3) states that the trend output growth rate, γ_t, converges to the long-term growth rate g_t at the speed θ and is driven by joint S-growth&D shocks ε_t^G as well as by trend output growth shocks ε_t^{γ}.

Finally, the long-term growth equation (4) is a partial adjustment mechanism where the long-term growth rate is driven by long-term growth shocks, ε_t^g, and converges to the constant average long-term growth rate, g, at the rate ϕ.[12]

Note that the joint S-level&D shocks enter the output gap equation (1) as well as the trend output level equation (2) while the joint S&D-growth shocks enter the output gap equation (1) as well as the trend-output growth equation (3).

Also note that without joint S&D shocks; that is, if $\sigma_{\varepsilon_t^L} = 0$ and $\sigma_{\varepsilon_t^G} = 0$, and without a long-term growth rate $\sigma_{\varepsilon_t^g} = 0$, or in other words, without equation (4), the trend-cycle decomposition with hysteresis collapses to the LLT filter in Harvey and Jaeger (1993) and Durbin and Koopman (2012). In addition, note that if $\theta = 1$, $\sigma_{\varepsilon_t^{\bar{y}}} = 0$ and for an appropriate choice of relative standard deviations of the shocks, the model collapses to the HP filter.

We calibrate the standard deviation of long-term growth shocks $\sigma_{\varepsilon_t^g}$ so as to have the rates of growth of trend output $4(\bar{y}_t - \bar{y}_{t-1})$ and γ_t fluctuate around the long-term growth rate g_t as dictated by the joint S&D shocks, and also to let long-term growth rate g_t capture shifts in trend output growth over long samples.[13]

While this completes the trend-cycle decomposition with hysteresis, it is convenient to define some variables in counterfactual terms as well as in deviation from the counterfactual trend.

[12] In the limit, when $\phi = 1$, the long-term growth equation (4) becomes a random walk, a form akin to that used by Laubach and Williams (2003) in the estimation of the natural interest rate. We maintain $\phi < 1$ so as to preserve a step-like behavior of our measure of hysteresis, trend output in deviation form.

[13] In this sense, the long-term growth rate acts as a sort of core growth rate.

2.2 The model for counterfactual trend output

A comparison of trend output relative to counterfactual trend output gives a notion of the permanent loss of output due to a given recession, a notion of hysteresis. Accordingly, at the outbreak of a recession, the hypothetical counterfactual trend output continues growing at the long-term growth rate, g_t, while actual trend output drops at the rate $4(\bar{y}_t - \bar{y}_{t-1})$,[14] as dictated by the joint S&D shocks.

Let $\bar{y}_t^C$ be counterfactual trend output, γ_t^C the counterfactual trend output growth rate, and y_t^C be a measure of counterfactual trend output plus the output gap. The model for counterfactual trend output is given by equations[15]

$$\bar{y}_t^C = \bar{y}_{t-1}^C + \frac{1}{4}\gamma_t, \tag{6}$$

$$\gamma_t^C = \theta\gamma_{t-1}^C + (1-\theta)g_t, \tag{7}$$

and

$$y_t^C = \hat{y}_t + \bar{y}_t^C. \tag{8}$$

Note that in equation (6) counterfactual trend output $\bar{y}_t^C$ grows at a rate that is driven by long-term growth shocks ε_t^g; in addition, counterfactual trend output does not follow joint S&D shocks ε_t^L and ε_t^G or pure supply shocks $\varepsilon_t^{\bar{y}}$ or ε_t^{γ}.[16]

As we show below, we calibrate the standard deviation of long-term growth shocks, ε_t^g, so as to have the rates of trend output growth $4(y_t^{\bar{y}} - y_{t-1}^{\bar{y}})$ and γ_t fluctuate around g_t. Trend output growth is driven by joint S&D shocks ε_t^L and ε_t^G;[17] in turn, the long-term growth rate g_t is driven by long-term growth shocks ε_t^g. In addition, as said above, we estimate the standard deviation of long-term growth shocks, ε_t^g, within a narrow range that preserves the qualitative behavior of the model; that is, that maintains trend output growth $4(y_t^{\bar{y}} - y_{t-1}^{\bar{y}})$ and γ_t fluctuating around the long-term growth rate g_t.

2.3 The model in deviation form

Our purpose is to define a notion of trend output that is driven by joint S&D shocks, ε_t^L and ε_t^G, and that at the same time abstracts from long-term growth shocks, ε_t^g. In other terms,

[14]In yearly frequency the rate is $\bar{y}_t - \bar{y}_{t-1}$.

[15]In yearly frequency equation (6) becomes $\bar{y}_t^C = \bar{y}_{t-1}^C + \gamma_t^C$.

[16]Note that a g_t^C variable is not defined in a long-term growth equation analogous to equation (4). Instead, the long-term growth shocks ε_t^g enter the model for the counterfactual trend through variable g_t in equation (7).

[17]Trend output is also driven by pure supply shocks $\varepsilon_t^{\bar{y}}$ and ε_t^{γ}, of lesser importance.

we subtract counterfactual trend output from trend output and so obtain trend output in deviation form.

Counterfactual trend output is the level of trend output that would have taken place in the absence of joint S&D shocks; in turn, trend output in deviation form is the level of output that would have taken place if it were driven by S&D shocks and not by long-term growth shocks.

Let y_t^D be output in deviation form; $\bar{y}_t^D$, trend output in deviation form; and γ_t^D, trend output growth in deviation form. The model in deviation form is given by equations[18]

$$\bar{y}_t^D = \bar{y}_{t-1}^D + \frac{1}{4}\gamma_t^D + u\varepsilon_t^L + \varepsilon_t^{\bar{y}}, \tag{9}$$

$$\gamma_t^D = \theta\gamma_{t-1}^D + n\varepsilon_t^G + \varepsilon_t^\gamma \tag{10}$$

and

$$y_t^D = \hat{y}_t + \bar{y}_t^D. \tag{11}$$

Note that joint S&D shocks, ε_t^L and ε_t^G, pure supply level shocks, $\varepsilon_t^{\bar{y}}$, and pure supply growth shocks, ε_t^γ, affect trend output in deviation form; in contrast, long-term growth shocks ε_t^g do not affect trend output in deviation form.[19]

Trend output in deviation form is not robust to ε_t^g. Then, we use trend output in deviation form for illustration purposes; that is, for the purpose of illustrating hysteresis under the estimated posterior coefficients. Nonetheless, trend output DDF, $4(\bar{y}_t^D - \bar{y}_{t-1}^D)$, is stationary and correlated with the cycle in the output gap and hence regarded as another part of the business cycle.

3 The quarterly and yearly databases

We use one database in quarterly frequency and another one in yearly frequency. In quarterly frequency we use real GDP data from the IMF International Financial Statistics database (IFS), 65 countries from seasonally adjusted data and 30 countries from the not seasonally adjusted data, adjusted by us with the X11 method.[20] For two countries data are from FRED Federal Reserve Bank of Saint Louis.[21] Out of the 97 countries in the initial database the euro

[18] In yearly frequency equation (9) becomes $\bar{y}_t^D = \bar{y}_{t-1}^D + \gamma_t + u\varepsilon_t^L + \varepsilon_t^{\bar{y}}$.

[19] In the model in deviation form there is no equation for a g_t^D variable since it is zero by the definition of a model in deviation form.

[20] In a few cases, to obtain earlier data but still starting 1995, we used data from the country central bank and statistics departments.

[21] The countries are Georgia and Iceland.

area was excluded to avoid double counting; 6 countries where excluded because the available data covered less than 8 years; 1 country was excluded because the estimated posterior parameters did not converge, suggesting that the data was too variable to be captured by the model and the general calibration; and 6 countries were excluded because the estimated output gap had a strong tilt, suggesting that a tailored calibration is required in these countries. The number of countries in the final quarterly database is 81, of which 31 are AD economies and 50 are EMDE economies. The list of countries in the final quarterly database and the countries that were excluded, appears in Appendix 1.

In yearly frequency we use real GDP data from the World Bank Development Indicators (WBDI) database; the Maddison Project Database (MPD), version 2020, see Bolt and Van Zanden (2020); the World Economic Outlook database, April 2022; and the Penn World Tables (PWT), see Feenstra et al. (2015). We used the WBDI database as the default source; we updated this source with the WEO database and spliced backwards using the Maddision and the WEO databases. We also used the PWT to include two countries that were not in other databases[22] Out of the 222 countries in the initial database, 7 were excluded because they had fewer than 8 data points, 20 countries were excluded because the estimated posterior parameters did not converge, and 11 countries were excluded because the estimated output gap had a strong tilt. The number of countries in the final quarterly database is 184, of which 46 are AD economies and 138 are EMDE economies. The list of countries in the final yearly database, as well as the excluded countries, appears in Appendix 2.

An overview of the data is presented in Figure 1. The boxplots report the main features of the distribution of output growth, in yearly frequency and in logarithmic terms. They convey information about the median, interquartile range, maximum and minimum. Because we use boxplots to illustrate some results, Appendix 1 presents the boxplots as summaries of statistics and the information they convey.

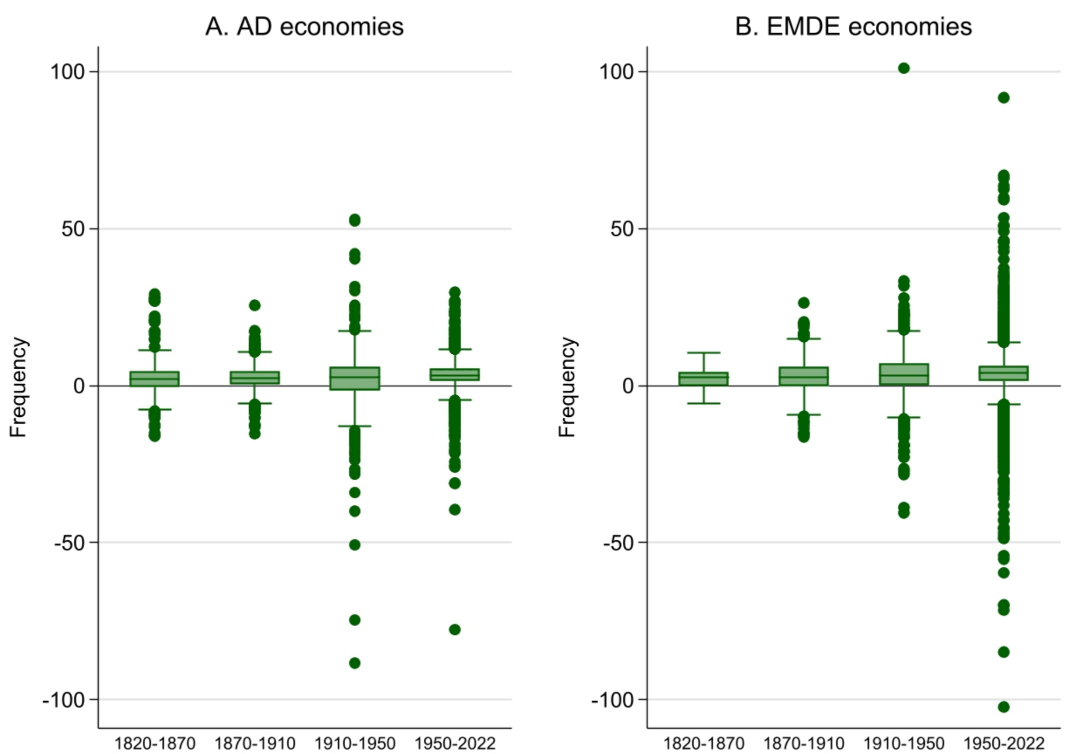
A look at the boxplots in Figure 1, for the first difference of log output,[23] conveys a story of normal business cycles; that is, those data points within the range; combined with extreme events; that is, those data points outside and far out the range.

The number of economies with available real GDP data increases over time, particularly in EMDE economies. An important jump in the number of countries with available data takes

[22] These countries are Angilla and Taiwan Province of China.

[23] The first difference is defined as $\Delta y_t = y_t - y_{t-1}$, where $y_t = 100 \log(Y_t)$, Y_t is real GDP and the logarithm is in base e.

Figure 1: A summary of the data – The yearly database



Note: standard rates of growth are within the interquartile range indicated by the box of the plot; large rates of growth are within the fences indicated by the length of the whyskers. Extreme rates of growth are beyond the fences indicated by the whyskers.
Source: WBDI; WEO database, April 2022; MPD, version 2020; and PWT.

place in 1950. Still, data are available for several AD economies before 1950 and for a few of them since 1820. In quarterly frequency (not reported) most data starts in 1995.

4 Estimation

In quarterly frequency the estimation period was 1995Q1-2022Q4. As said above, most quarterly GDP data are available for the bulk of countries starting in 1995Q1; in addition, the period ends with the latest data that are available at the time of writing the paper, for some countries in 2022Q4.

In yearly frequency the estimation period is 1975-2022. Data is available for most countries starting in 1950 when a period of relatively high long-term growth and low volatility starts in AD economies. However, a drop in long-term output growth takes place in 1975, see also Kose and Terrones (2015). Although the g_t equation can help capture the productivity slowdown well in most economies, in order to increase the number of economies with stable posterior estimated coefficients we chose to start the estimation period in 1975.[24]

The calibration and estimation of the model involved all model parameters except the standard deviation of pure demand shocks $\sigma_{\varepsilon_t^{\hat{y}}}$. This standard deviation was normalized to 1 so that the estimated standard deviations can be understood as relative standard deviations.

The estimation involved the speeds of adjustment, α, θ and ϕ; the standard deviation of joint S&D shocks, $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$; and the standard deviation of long-term growth shocks, $\sigma_{\varepsilon_t^g}$.

The model was fed with quarterly annualized output growth data, $4(y_t - y_{t-1})$.[25] Levels were pinned down by setting trend output and counterfactual trend output equal to 100 in a given base year; as well as trend output in deviation form equal to 0 in the base year.[26]

We estimate the model with Bayesian methods. We used uniform priors across countries and frequencies, with the exception, naturally, of the speeds of convergence, α, θ and ϕ, that differ across the quarterly and yearly frequencies. We maintained a uniform calibration across economies but left a number of economies out of the sample where the parameters did not converge. We regarded the lack of convergence of the parameters as indicative that the

[24] Filtration results using data starting in 1820 and 1870 run with parameters that are calibrated at the priors means.

[25] As mentioned above in another context, log output is defined as $y_t = 100 \log(Y_t)$, where Y_t is real GDP and the logarithm is in base e.

[26] In other terms, $\bar{y}_j = \bar{y}_j^C = 100$ and $\bar{y}_j^D = 0$, where j is the base year, defined depending on the database under analysis.

general calibration was not appropriate for these economies and that in these cases a granular calibration would be needed.[27]

For simplicity, parameters $\sigma_{\varepsilon_t^{\bar{y}}}$ and $\sigma_{\varepsilon_t^{\gamma}}$, that are of lesser importance, were calibrated.

4.1 The priors

Table 1: Bayesian priors means and median posterior estimates

Parameter	Quarterly		Yearly	
	Prior mean	Median posterior	Prior mean	Median posterior
α	0.8	0.775	0.5	0.530
θ	0.85	0.836	0.55	0.668
ϕ	0.95	0.942	0.8	0.848
u	0.5	0.496	0.5	0.472
n	0.5	0.455	0.5	0.745
$\sigma_{\varepsilon_t^L}$	1	1.158	1	1.220
$\sigma_{\varepsilon_t^G}$	1	1.072	1	1.312
$\sigma_{\varepsilon_t^g}$	0.2	0.210	0.2	0.289

Note: the reported median posterior estimates are the median across countries of the median of the estimated posterior distribution. Priors for the speed of adjustment α, θ and ϕ differ by frequency.
Source: see text.

Prior means for the speeds of adjustment α, θ and ϕ, in Table 4.1, were set so that the half life of a shock, $\log 0.5/\log\alpha$ were comparable across frequencies. The prior mean for the speed of adjustment α was set so that the half life of a pure demand shock, $\varepsilon_t^{\hat{y}}$, was in the range of 3 to 4 quarters in quarterly frequency and in a similar range in yearly frequency. The prior means for θ were set so that the half-life of a pure supply growth shock, ε_t^{γ}, was about 4.5 quarters in quarterly frequency and in a similar length in yearly frequency. Prior means for ϕ were set to that the half-life of a shock to the long-term growth rate ε_t^g was about 12 quarters in quarterly frequency and of a similar length in yearly frequency.[28] Priors means for u and n are 0.5, which amounts to assuming that, *a priori*, joint S-level&D and joint S-growth&D shocks have an equal effect on supply and demand; that is, on the output gap and trend output.

[27]The list of countries where the posterior parameters did not converge appears in Appendix 3.

[28]The prior mean for parameter ϕ was set as less than one so as to preserve the step-like behavior of trend output in deviation form. Nonetheless, parameter ϕ may be approximated as 1 without material difference in the results if the purpose of the analysis is the trend-cycle decomposition alone and not the model for counterfactual trend output or the model in deviation form.

The prior means for parameters $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$ were set equal to the calibrated standard deviation of pure demand shocks $\sigma_{\varepsilon_t^{\hat{y}}}$; that is, equal to 1. The assumption amounts to making joint S&D and pure demand shocks *a priori* equally important.

The prior mean for $\sigma_{\varepsilon_t^g}$ was set to obtain posterior estimates that would maintain a long-term growth rate g_t that is at the same time smooth in the short term and flexible in the long term.

Table 2: Bayesian prior distributions

Parameter	Quarterly		Yearly	
	Distribution	Interval	Distribution	Interval
α	N(0.8,0.1)	[0.75 - 0.85]	N(0.5,0.1)	[0.4 - 0.6]
θ	N(0.85,0.1)	[0.8 - 0.9]	N(0.55,0.1)	[0.45 - 0.65]
ϕ	N(0.95,0.1)	[0.925 - 0.975]	N(0.8,0.1)	[0.75 - 0.9]
u	N(0.5,0.1)	[0.25 - 0.75]	N(0.5,0.1)	[0.25 - 0.75]
n	N(0.5,0.1)	[0.25 - 0.75]	N(0.5,0.1)	[0.25 - 0.75]
$\sigma_{\varepsilon_t^L}$	N(1,0.1)	[0.5 - 1.5]	N(1,0.1)	[0.5 - 1.5]
$\sigma_{\varepsilon_t^G}$	N(1,0.1)	[0.5 - 1.5]	N(1,0.1)	[0.5 - 1.5]
$\sigma_{\varepsilon_t^g}$	N(0.2,0.1)	[0.1 - 0.3]	N(0.2,0.1)	[0.1 - 0.3]

Note: prior intervals and standard deviations were calibrated so as to maintaining both the qualitative behavior of the impulse responses and heterogeneity in the impulse responses.
Source: see text.

Regarding the prior distributions, in Table 4.1, we used normal distributions constrained to lie within given intervals. The standard deviations of the prior distribution of the parameters as well as the intervals enabled dispersion in the estimated posterior estimates and in the impulse response functions across economies. At the same time, the standard deviations and intervals constrain the posterior estimates so as to strike a balance between supply and demand shocks in the estimation of the output gap and trend output. On one hand, if the weight of supply is too large the output gap tends to close. On the other, if the weight of supply is to small the output gap tends to widen. Parameter coefficients outside the intervals were taken as conducive to trend-cycle decompositions with too little or to much of demand or supply.

The prior standard deviations of the prior distribution of the parameters were set at 0.1. This choice also intends to preserve the balance between supply and demand in the trend-cycle decomposition.

The intervals for the speeds of adjustment α, θ and ϕ were set so as to preserve some comparability of the half-life of the shocks across frequencies at the ends of the intervals. Another criterion was, for simplicity, to try to maintain these intervals in round numbers.

Some parameter intervals were set so as to preserve the balance between supply and demand shocks in the estimation of the output gap and trend output. As shown in Table 2; shares u, η were constrained to lie within the $0.25 - 0.75$ interval; standard deviations $\sigma_{\varepsilon_t^L}$, $\sigma_{\varepsilon_t^G}$ were constrained to lie within the $0.5 - 1.5$ interval; and standard deviation $\sigma_{\varepsilon_t^g}$ was constrained to lie within the $0.1 - 0.3$ interval.

All in all, the standard deviation of the prior distributions and the intervals allow some room for the data to speak and at the same time maintain a balance between supply and demand in the estimation of the output gap and trend output.

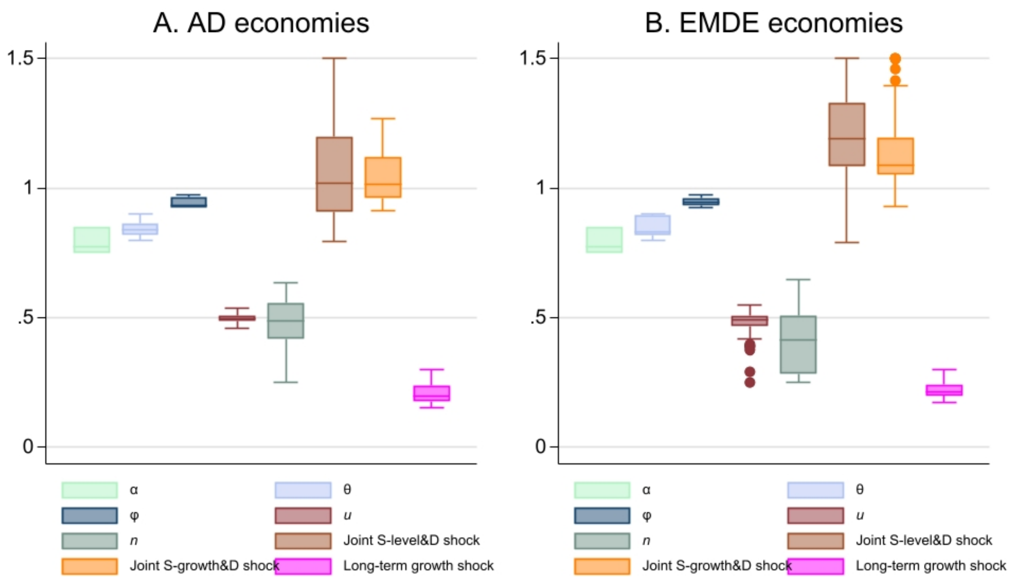
4.2 The posterior estimates

The distribution of the mode of the posterior distribution of the estimated parameters across countries is presented in Figures 2 and 3. As a consequence of confronting the model with the data, the distribution of the mode of the speeds of adjustment α, θ and ϕ as well as that of the share u and the standard deviation $\sigma_{\varepsilon_t^g}$ are relatively close to the prior distributions. In contrast, the distribution of the share n and the standard deviations $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$ lie at some distance from the prior distribution.

A broader view of the posterior estimates is obtained by looking at the distribution of the posterior mode of the parameters across countries, in Figures 2 and 3. The estimated parameters are broadly similar across type of countries; that is, across AD and EMDE economies, but naturally differ across the quarterly and yearly frequencies, one reason is that the two databases run over different sample periods. Indeed, the distribution of the mode of the posterior distributions does not differ much across AD and EMDE economies; that is, across Panels A and B in either Figure 2 or 3. In contrast, setting aside the speeds of adjustment α, θ and ϕ, the estimated coefficients do differ when comparing the estimates obtained with the quarterly and yearly frequencies; that is, across Figures 2 and 3 in either Panel A or B, particularly in the case of EMDE economies.

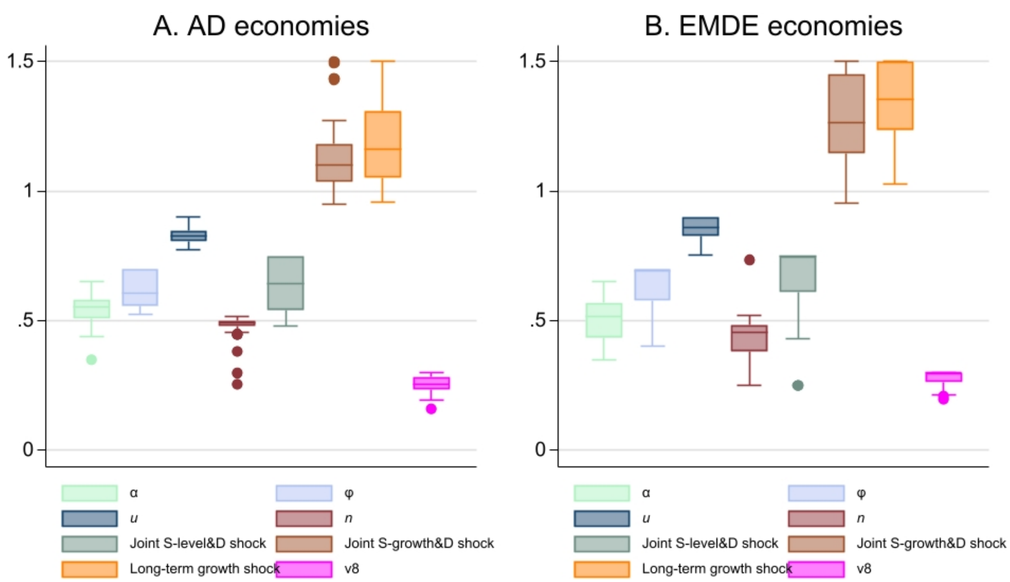
In order to asses the convergence of the posterior distribution of the parameters to a stationary distribution, the Heidelberger and Welch (1981) and Heidelberger and Welch (1983) convergence test was performed. The test consists of comparing the distribution of early batches of the Markov chain to that of batches at the end the of the chain. If the null that

Figure 2: Posterior estimates – The quarterly database



Note: posterior estimates for α, u and $\sigma_{\varepsilon_t^g}$ are relatively close to the priors means, posterior estimates for θ, n, $\sigma_{\varepsilon_t^y}$, $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$ are further apart and show dispersion across countries.
Source: authors' estimates and the quarterly database.

Figure 3: Posterior estimates – The yearly database



Note: posterior estimates for α, u and $\sigma_{\varepsilon_t^g}$ are relatively close to the priors, posterior estimates for θ, n, $\sigma_{\varepsilon_t^y}$, $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$ are further apart and show dispersion across countries.
Source: authors' estimates and the yearly database.

the distribution of the early and later batches is not different is not rejected, the conclusion is that the posterior distribution of the estimated parameter is stationary.

In the quarterly data, 50 thousand replications were used with a burn-in of 50%.[29] In 74 out of the 80 economies, the posterior distribution of the parameters converged to a stationary distribution given that the p-value of the Cramer-Von-Mises test lied above the 0.05 significance level (Figure 4, Panel A). In 4 out of the 80 economies, the posterior distribution of at least one of the parameters did not converge to a stationary distribution.[30] Overall, in most economies the posterior distribution of most parameters converged to a stationary distribution.

As to the yearly data, 100 thousand replications were used with a burn-in of 50%. In 179 out of the 214 economies, the posterior distribution of each one of the parameters converged to a stationary distribution (Figure 5, Panel A). In 25 out of the 214 economies the posterior distribution of at least one parameter did not converge to a stationary distribution. In turn, in 10 of the economies the posterior distribution of no parameter converged to a stationary distribution.[31] Overall, in most economies the posterior distribution of each one of the parameters converged to a stationary distribution.

To asses whether the length of the Markov Chain allows for a precise estimation of the mean of the posterior distribution, the ratio between the half-width of the 95% confidence interval (CI) and the mean was calculated. Intuitively, the lower the ratio, the lower the dispersion of the data and the higher the precision in the estimation of the mean. Normally, a ratio below 0.1 is deemed as sufficiently precise.

In the cases were the Markov chain converged to a stationary distribution, the length of the chain allowed for a precise estimation of the mean of the posterior distribution, as shown by the ratio between the half-width of the 95% confidence interval (CI) and the mean. This ratio was below 0.1, see Figure (4, Panel B).

4.3 The calibrated parameters

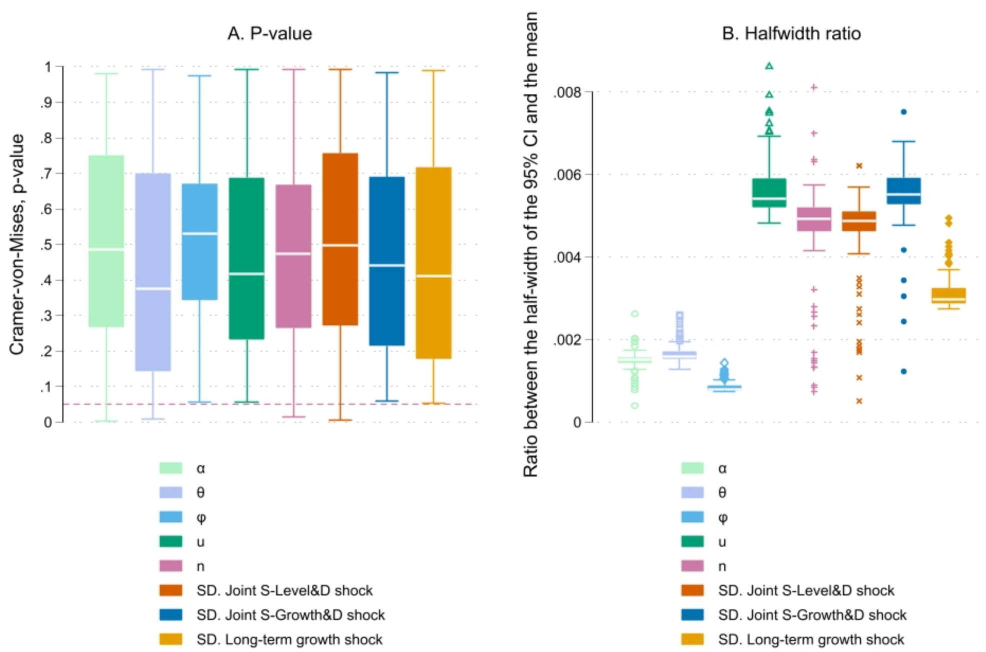
For simplicity, the remaining parameters were calibrated, these are parameters $\sigma_{\varepsilon_t^{\bar{y}}}$ and $\sigma_{\varepsilon_t^{\gamma}}$ that were kept in the model for comparability with the LLT filter. Nonetheless, these pa-

[29] Each economy was estimated separately.

[30] These economies are YYY, with extreme output volatility.

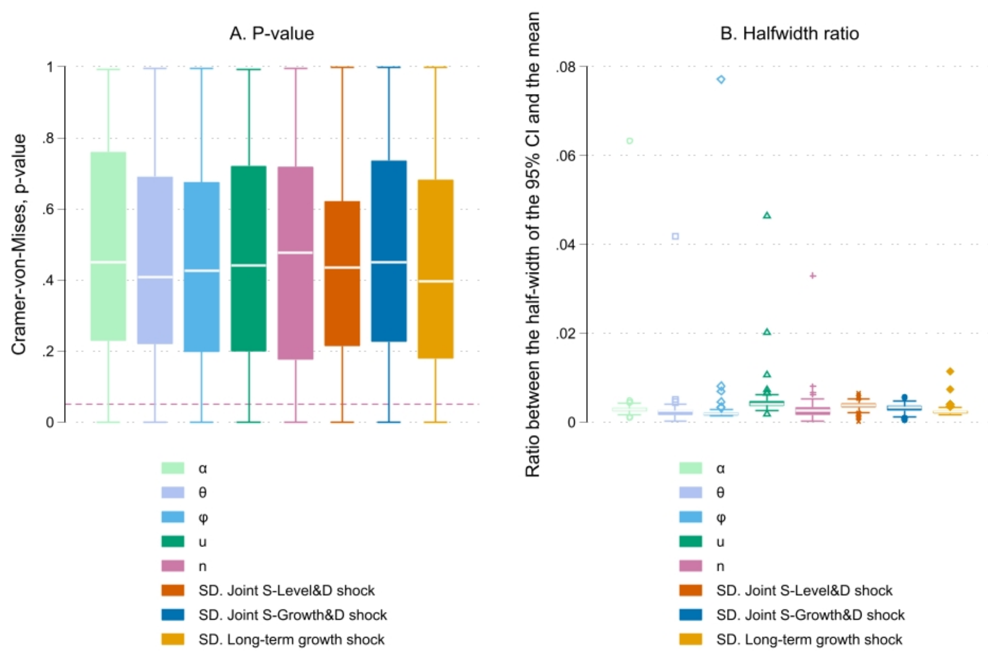
[31] These economies are Macao, Azerbaijan, Iraq, Kuwait and Liberia, all of them also with extreme output volatility.

Figure 4: Convergence test – The quarterly database



Note: in Panel A, the p-value of the Cramer-von-Mises test is arranged for each of the parameters within a boxplot across economies. As the p-value is greater than 0.05, the null hypothesis of convergence to a stationary distribution is not rejected for most parameters and economies. In Panel B, the mean of the posterior distribution is estimated with precision for all parameters as the ratio between the half-width of the 95% CI and the mean is well bellow 0.1.
Source: authors' estimates and the quarterly database.

Figure 5: Convergence test – The yearly database



Note: in Panel A, the null hypothesis of convergence to a stationary distribution is not rejected for most parameters and economies. In Panel B, the mean of the posterior distribution is estimated with precision, as the ratio between the half-width of the 95% CI and the mean is well bellow 0.1.
Source: authors' estimates and the yearly database.

rameters are not regarded as relevant as they do not involve supply and demand at the same time, see Table 3.

As for the constant rate of growth of counterfactual trend output, g, it was set at the historical average of output growth over each sample under study.

Table 3: Calibrated parameters

Parameter	Quarterly	Yearly
$\sigma_{\varepsilon_t^{\bar{y}}}$	0.05	0.05
$\sigma_{\varepsilon_t^{\gamma}}$	0.05	0.05

Note: the parameters are invariant across frequencies.
Source: see text.

5 Results

5.1 Response to joint S&D shocks

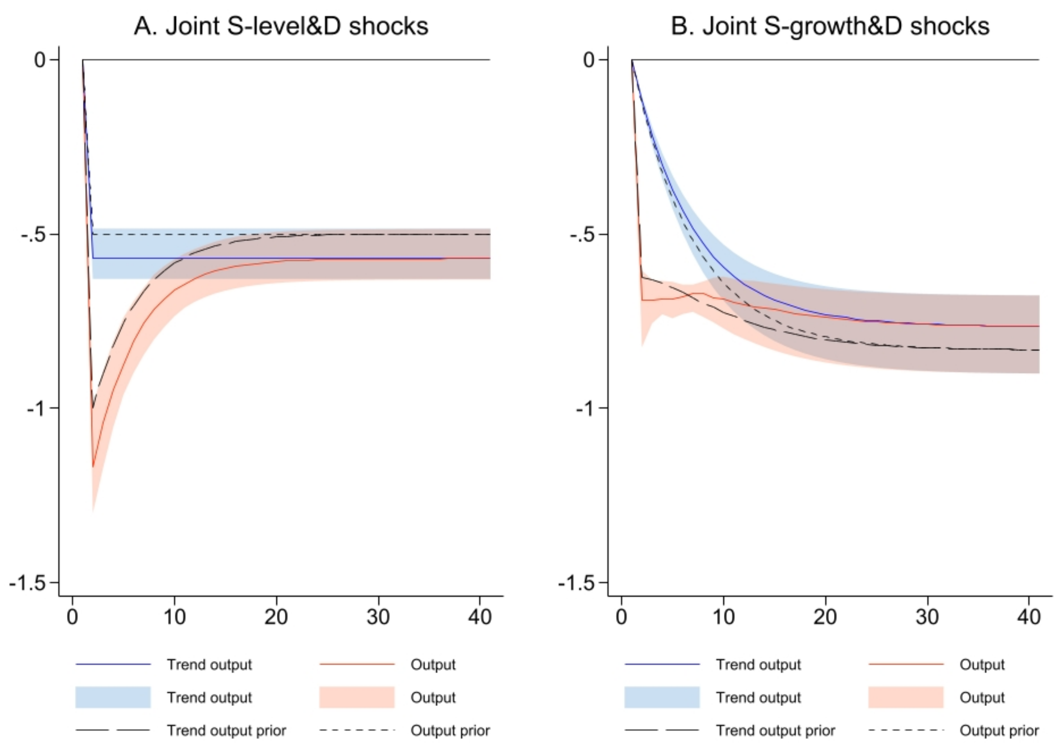
Figures 6 and 7 show the response to joint S&D-level and S&D-growth shocks.[32] The response of the non-stationary variables, output and trend output, appears in Figure 6; in turn, the response of the stationary variables, the output gap and trend output in difference form, is presented in Figure 7. Using the prior coefficients, the model responses are presented in black lines. Using the posterior estimates, the interquartile ranges of the impulse responses produced with the mode of the posterior estimates are presented in color. The responses are to a one standard deviation shock; that is, to a shock equal in size to the posterior estimate of the standard deviation of the shock.

A look at the response of the non-stationary variables, in Figure 6, reveals that under a joint S-level&D shock trend output undergoes an L-shaped recession. On its part, output drops further and then bounces back towards trend output. In comparison, under a joint S-growth&D shock, trend output drops gradually while output drops further and then converges to the final hysteresis effect, suggesting a broadly L-shaped recession.

As to the response of the stationary variables, in Figure 7, both joint S-level&D and S-growth&D shocks lead to a correlated response of both the output gap and trend output

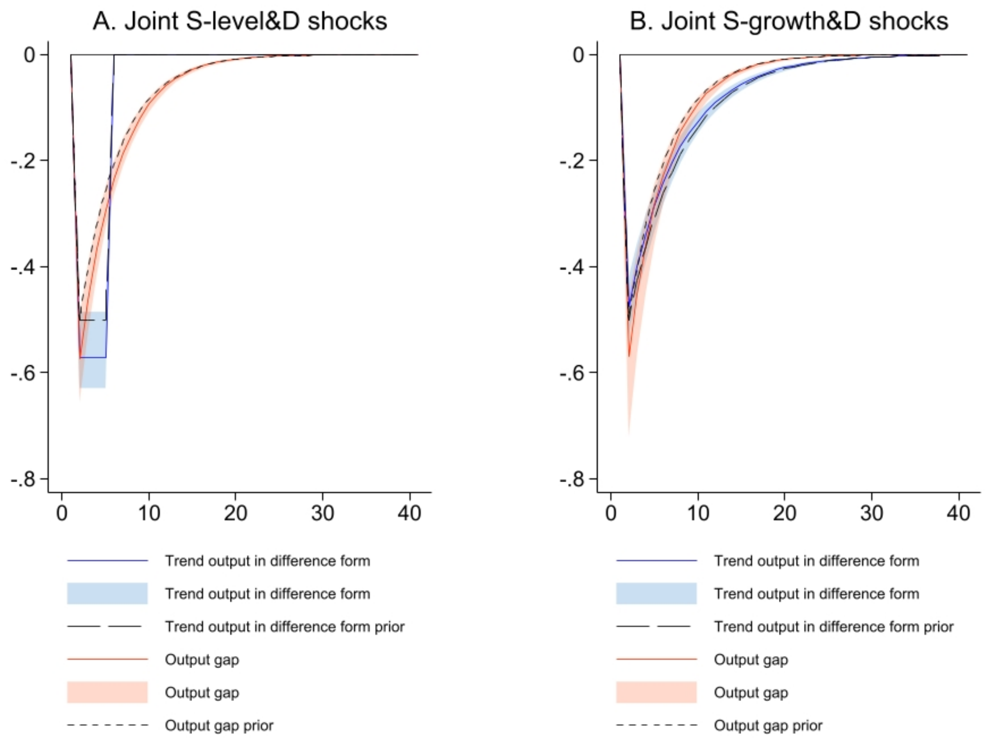
[32]The figures show the responses in quarterly frequency, the responses in yearly frequency are available on request.

Figure 6: Response of output and trend output to joint S&D shocks



Note: the figure shows the interquartile range of the distribution of the impulse responses across countries using the median of each of the estimated parameters in each country. A joint S&D-level shock leads to a broadly V-shaped recession in output and a L-shaped recession in trend output; in turn, a joint S&D-growth shock leads to a broadly L-shaped recession in output and a convex-shaped recession in trend output. Responses are for the model in quarterly frequency.
Source: author's estimates and the quarterly database.

Figure 7: Response of the output gap
and trend output in difference form to joint S&D shocks



Note: the figure shows the interquartile range of the distribution of the impulse responses across countries using the median of each of the estimated parameters in each country. A joint S&D shock leads to a correlated response of both the output gap and trend output DDF. Responses are for the model in quarterly frequency. First differences are shown over 4 quarters in Panel A and over 1 quarter in Panel B.
Source: authors' estimates and the quarterly database.

in difference form. Thus, business fluctuations are also present in trend output in difference form.

The responses in Figure 6 show some dispersion across countries. The interquartile range for the responses shows heterogeneity in the extent of hysteresis.

The impulse responses can be analyzed not only for the interquartile range but also for the complete range of responses in the economies in the database.[33] Figure 8 shows the response of trend output to joint S-level&D and S-growth&D shocks. Above all, the complete range of responses shows diversity in the response to the shocks. The response to a joint S-level&D shock is somewhat asymmetrical while the response to a joint S-growth&D shock is clearly asymmetrical. The response of hysteresis to a S-growth&D shock is more heterogeneous, within a range that is about twice as large that of the S-level&D shock.

Turning to the range of responses of the output gap, Figure 9 shows that the response to both joint S&D-level and S&D-growth shocks is about symmetrical. On impact, the range of the response is similar across type of shock.

5.2 Some features of the model - The United States and the Euro Area in quarterly frequency starting in 1995

5.2.1 Filtration results for the business cycle

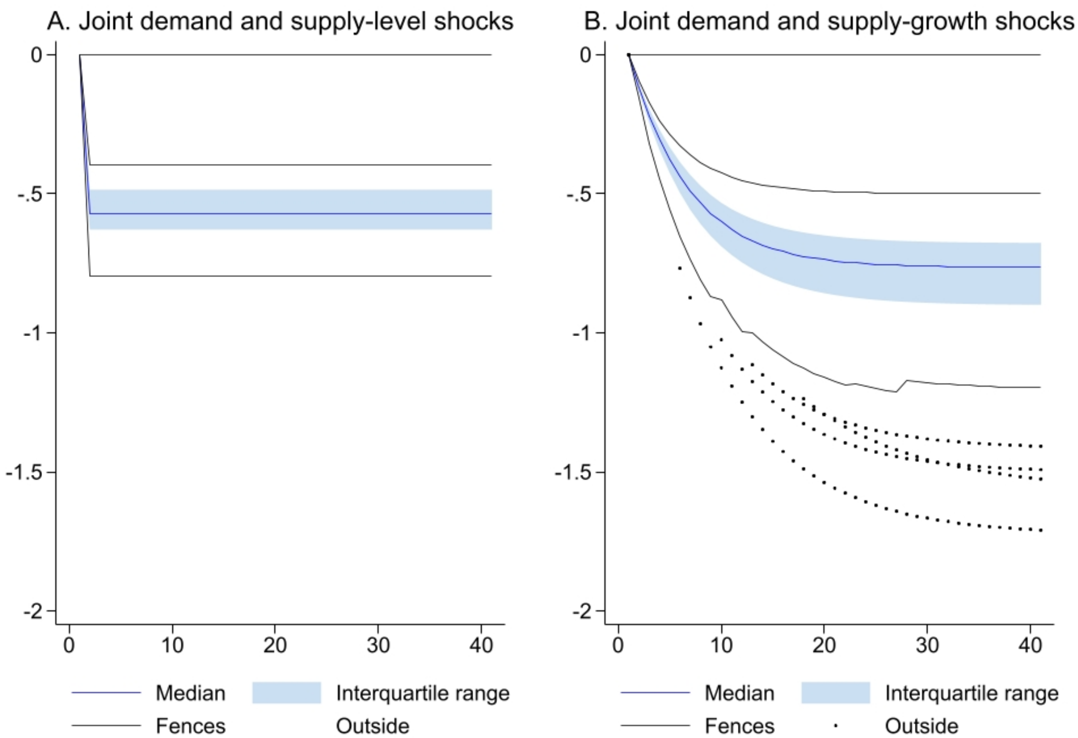
For explanatory purposes, we first consider the filtration results for the United States and the Euro Area in quarterly frequency starting in 1995Q1. Figures 10 and 11 compare the trend-cycle decomposition with hysteresis with the LLT filter.[34].

If the model is the LLT filter, in Panels A of Figures 10 and 11, trend output evolves smoothly and the output gap, represented by the difference between output and trend output, suggests a large boom before the 2008-2009 financial crisis as well as before the COVID-19 recession. In contrast, if the model is the trend-cycle decomposition with hysteresis, in Panels B of Figures 10 and 11, trend output drops during the recessions and the output gap does not suggest large booms before recessions.

[33] That is, 81 responses in quarterly frequency and 184 in yearly frequency.

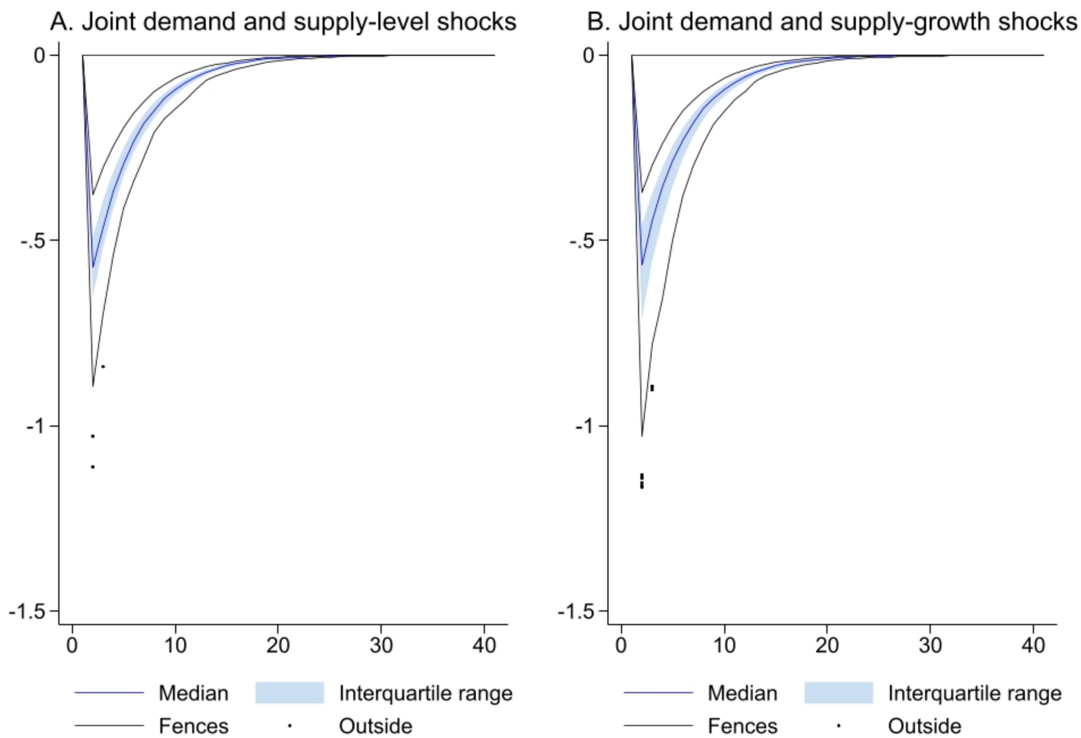
[34] Maintaining the standard deviation of pure demand shocks in both models as $\sigma_{\varepsilon_t^{\hat{y}}} = 1$, in the LLT filter the standard deviation of supply shocks was calibrated as $\sigma_{\varepsilon_t^{\bar{y}}} = 0.09$ and $\sigma_{\varepsilon_t^{\gamma}} = 0.21$ so that the sum of the standard deviations of the supply shocks adds up to 0.3. In turn, in the trend-cycle decomposition with hysteresis the standard deviation of pure supply shocks plus that of long-term growth shocks, as reported in Table 4.1 and 3, also adds up to 0.3

Figure 8: Response of trend output to joint S&D shocks:
the complete range of responses



Note: the figure shows the median, interquartile range, range and outliers of the distribution of the impulse responses across countries using the median of each of the estimated parameters in each country. The figure shows a disperse and asymmetrical response of trend output to S&D shocks. The range indicated by the fences may not evolve smoothly owing to a change in country in the impulse response that is closest and inside the interval defined by = 1.5, see Appendix 2.
Source: authors' estimates and the quarterly database.

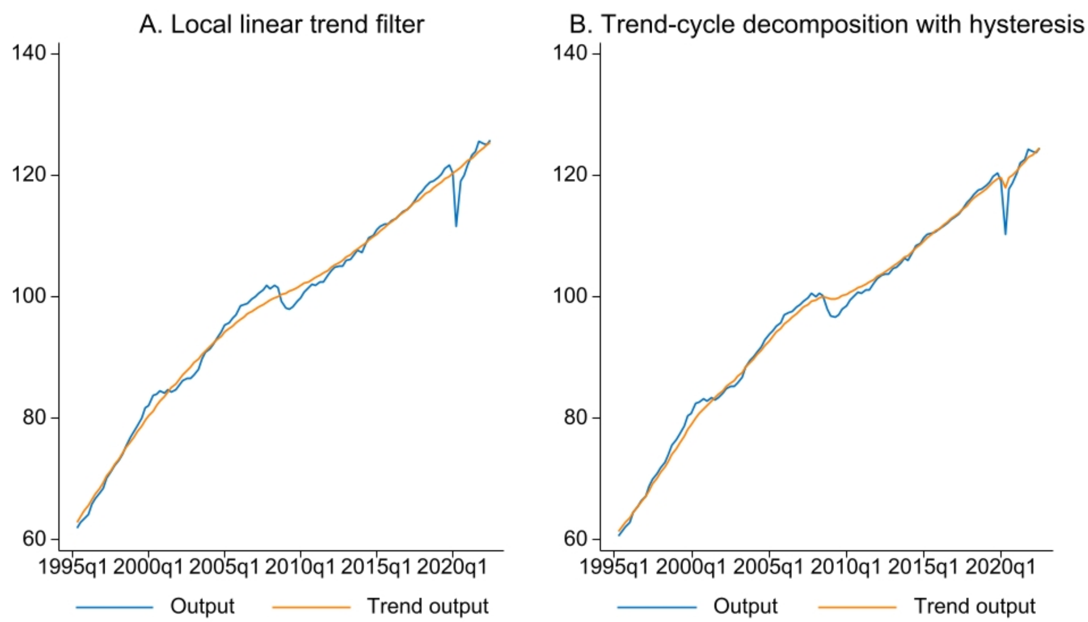
Figure 9: Response of the output gap to joint S&D shocks:
the complete range of responses



Note: the figure shows the median, interquartile range, range and outliers of the distribution of the impulse responses across countries using the median of each of the estimated parameters in each country. The figure shows a dispersed and about symmetrical response of the output gap to S&D shocks.

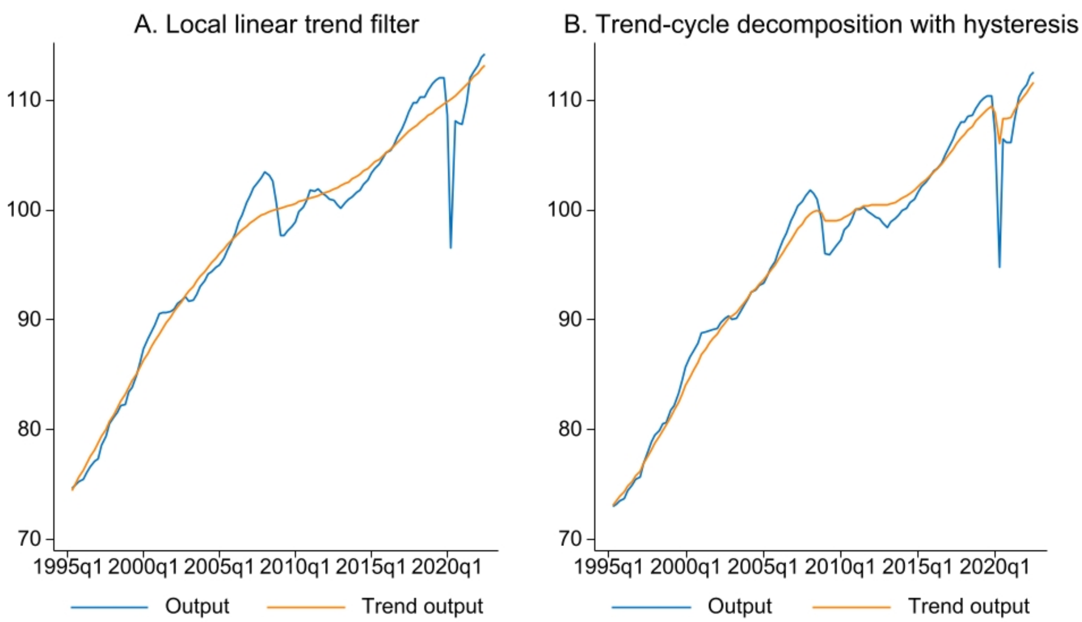
Source: authors' estimates and the quarterly database.

Figure 10: Comparison of the LLT filter
and the trend-cycle decomposition with hysteresis – The United States



Note: in the trend-cycle decomposition with hysteresis trend output does not necessarily evolve smoothly and does not create a boom before recessions. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

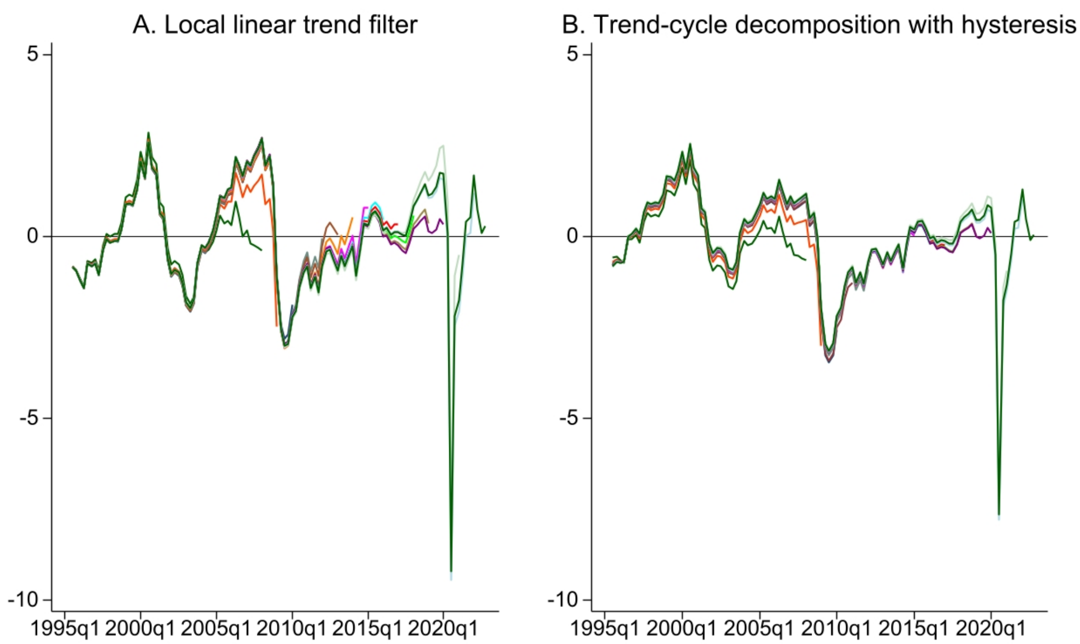
Figure 11: Comparison of the LLT filter
and the trend-cycle decomposition with hysteresis – The Euro Area



Note: in the trend-cycle decomposition with hysteresis trend output does not necessarily evolve smoothly and does not create a boom before recessions. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

Another feature of the trend-cycle decomposition with hysteresis is that, compared with the LLT filter, the output gap is less sensitive to new data. Figures 12 and 13 show the estimated output gap for the United States and the Euro Area over progressively increasing estimation periods. Using the LLT filter output gap revisions are large. For instance, the output gap estimated with data up to 2007 was about zero; in contrast, using longer samples large booms are estimated before the 2008-2009 financial crisis and the COVID-19 recession in both the United States and the Euro Area. The relative stability of the output gap estimates obtained with the trend-cycle decomposition with hysteresis is clearer in the Euro Area, as the LLT filter not only overstates the peak of the boom prior the 2008-2009 financial crisis but also overstates the trough of the recession.

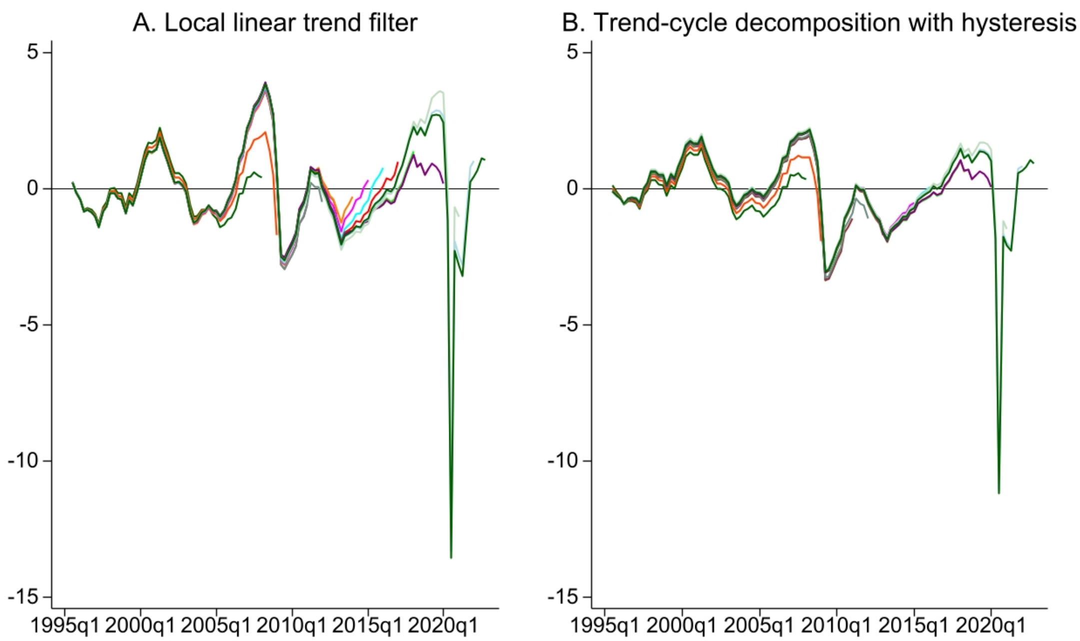
Figure 12: Output gap revisions in the LLT filter
and the trend-cycle decomposition with hysteresis – The United States



Note: the trend-cycle decomposition with hysteresis results in smaller output gap revisions. The figure shows the output gap estimates over progressively increasing estimation periods. The first estimation period is 1995Q1-2007Q4; thereafter, the estimation period includes 4 additional quarters until the estimation runs over the complete available sample. Data are in quarterly frequency for the period 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

Turning to counterfactual trend output, it grows at the long-term growth rate, g_t, and so provides a model projection that serves as benchmark to illustrate hysteresis. Counterfactual

Figure 13: Output gap revisions in the LLT filter
and trend-cycle decomposition with hysteresis – The Euro Area

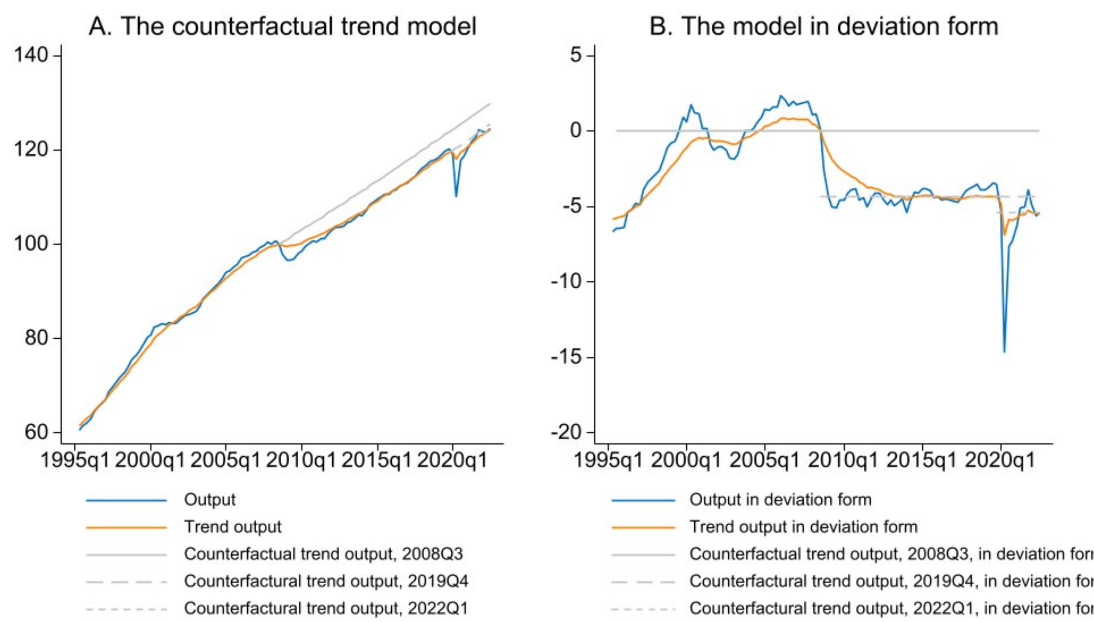


Note: the trend-cycle decomposition with hysteresis results in smaller output gap revisions. The graph shows the output gap estimates over progressively increasing estimation periods. The first estimation period is 1995Q1-2007Q4; thereafter, the estimation period includes 4 additional quarters until the estimation runs over the complete available sample. Data are in quarterly frequency over 1995Q1-2022Q1.

Source: authors' estimates and the quarterly database.

trend output, in Panels A of Figures 14 and 15, is equal to trend output before 2008Q3; that is, before the sequence of large negative realizations of joint S&D shocks during the 2008-2009 financial crisis. Thereafter, counterfactual trend output grows at the long-term growth rate, g_t, the rate that had taken place had no large negative realizations of joint S&D shocks taken place.

Figure 14: The model for the counterfactual trend and the model in deviation form – The United States



Note: output and trend output both tilt upwards; without tilt, trend output in deviation form serves as measure of hysteresis. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

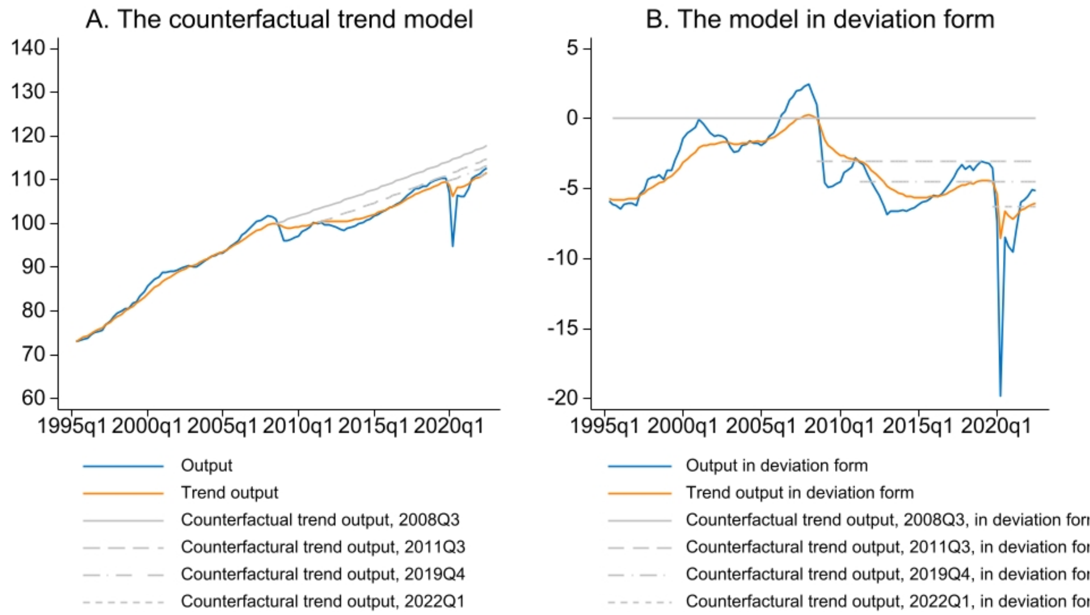
We now turn to trend output in deviation form, the variable that we use to illustrate hysteresis. A look at the model in deviation form, equations (9) and (10), makes clear that our measure of hysteresis is the outcome of the joint S&D shocks.[35]

In the United States and the Euro Area, the 2008-2009 financial crisis and the COVID-19 recession had important consequences in terms of hysteresis, as illustrated in Panels B of Figures 14 and 15. Conditional on the estimated posterior standard deviations of long-term growth shocks, $\sigma_{\varepsilon_t^g}$, in the United States the hysteresis effect of the 2008-2009 financial crisis was about 5 per cent. In the Euro Area, the hysteresis effect of both the 2008-2009 financial

[35]It is also a function of the less important pure supply shocks.

crisis and the Euro Area Crisis was also about 5 percent. In both the United States and the Euro Area, the COVID-19 recession appears to have added to hysteresis an additional 2 percentage points.

Figure 15: The model for the counterfactual trend and the model in deviation form – The Euro Area



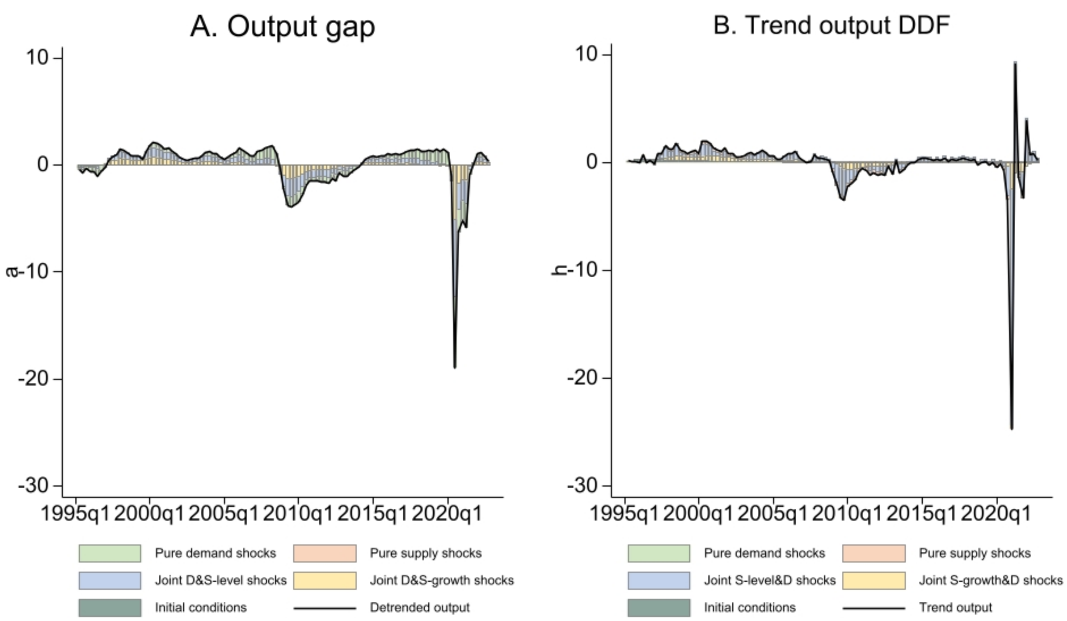
Note: output and trend output both tilt upwards; without tilt, trend output in deviation form serves as measure of hysteresis. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

5.2.2 The role of joint S&D shocks in the business cycle

Joint S&D shocks have a primary role in explaining the business cycle. Figures 16 and 17 present the historical shock decomposition of the output gap and trend output DDF in the United States and the Euro Area, respectively.[36] The historical decomposition (HD) shows the primary role of joint S&D shocks in explaining the demand and supply components of the business cycle. The upturns and downturns in the output gap are lead not only by pure demand shocks but to a much greater extent by joint S&D shocks. On its part, the upturns and downturns in trend output DDF are led by joint S&D shocks, without a meaningful role for pure supply shocks.

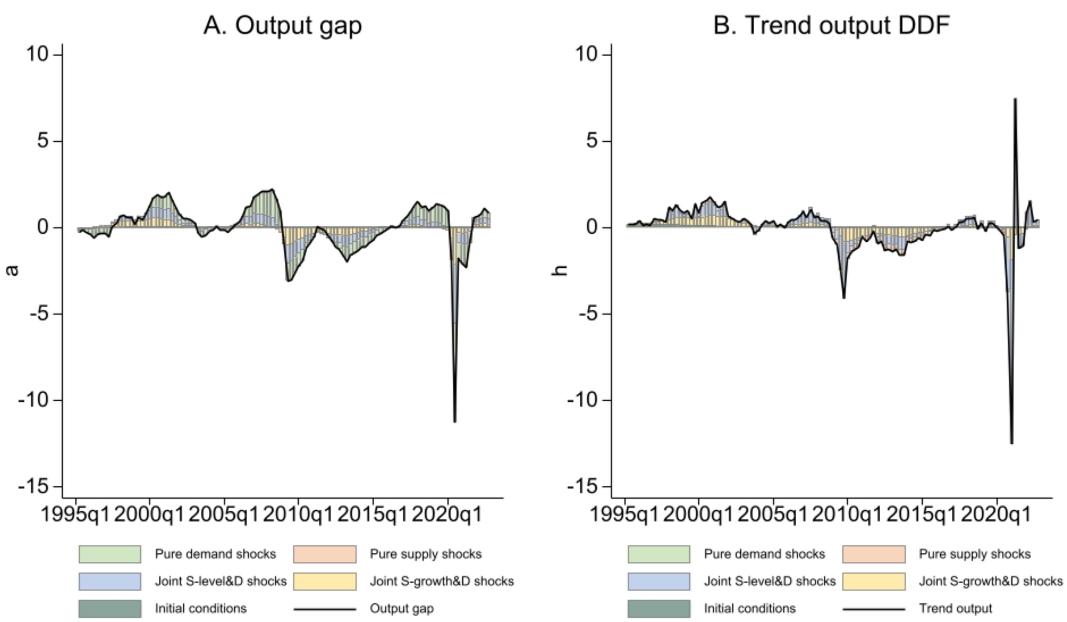
[36]In Figures 16 and 17, trend output in deviation form enters the HD in 4-quarter differences.

Figure 16: The role of joint S&D shocks:
HD for the United States



Note: joint S&D shocks are important in the HD of the business cycle. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

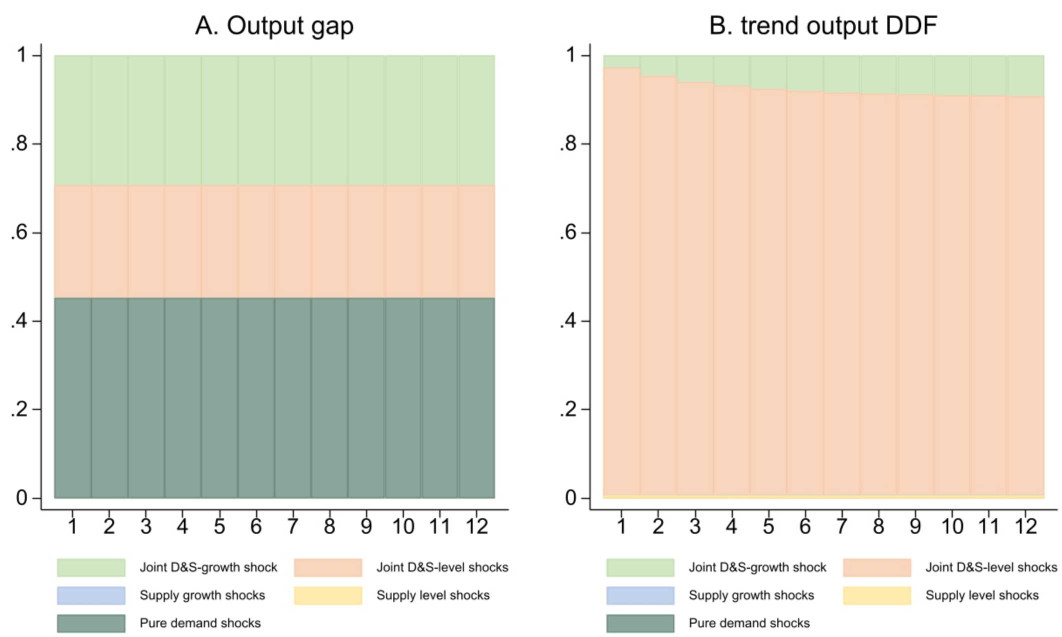
Figure 17: The role of joint S&D shocks:
HD for the Euro Area



Note: joint S&D shocks are important in the HD of the business cycle. The cycles in the output gap and trended output in deviation and difference form are highly correlated. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

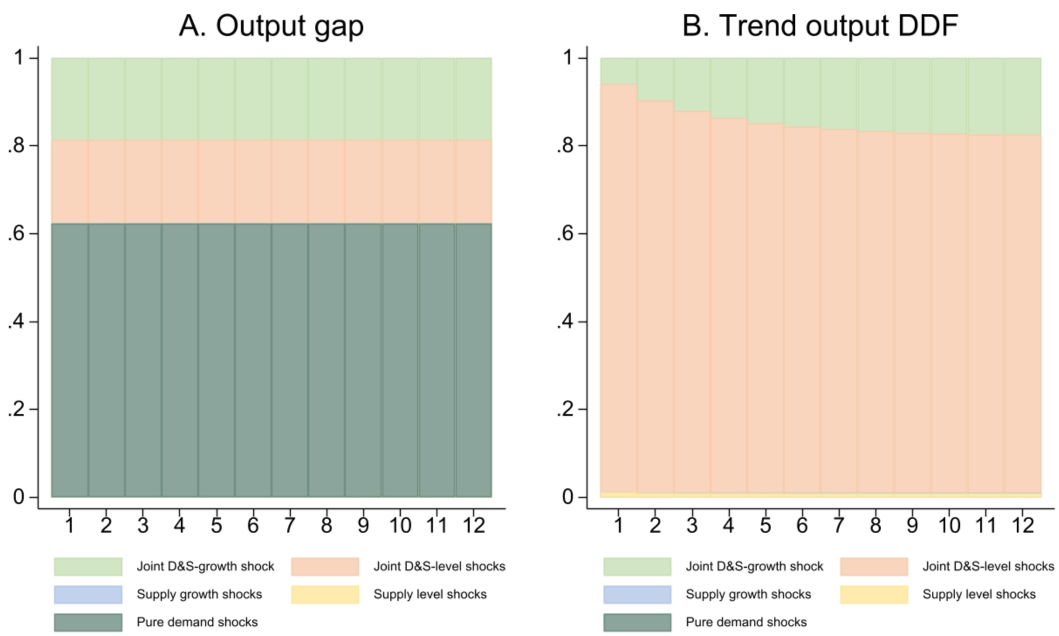
Another assessment of the role of joint S&D shocks in the business cycle is obtained by looking at the FEVD of the variables that fluctuate with the business cycle. Figures 18 and 19 show the FEVD of the output gap, on one hand; and trend output DDF, in the other, in the United States and the Euro Area, respectively. As to the output gap, important shares of the FEVD are explained by joint S&D shocks, about a quarter in the Untied States and a third in the Euro Area, in Panels A in Figures 18 and 19. A large share of the FEVD of the output gap is explained by pure demand shocks. As for trend output in deviation and difference from it is the joint S&D shocks that explain all the FEVD, with a negligible share explained by pure supply shocks (Panels B in Figures 18 and 19).

Figure 18: The role of joint S&D shocks:
FEVD for the United States



Note: joint S&D shocks are important in the of the business cycle. Data are in quarterly frequency over 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

Figure 19: The role of joint S&D shocks:
FEVD for the Euro Area



Note: joint S&D shocks are important in the of the business cycle. Data are in quarterly frequency for the period 1995Q1-2022Q1.
Source: authors' estimates and the quarterly database.

5.3 The 2008-2009 financial crisis and the COVID-19 recession in AD and EMDE economies

A more general view of the 2008-2009 financial crisis and the COVID-19 recession can be studied using the interquartile range and the median of the business cycle estimated with the quarterly database. Figures 20 to 23 show the interquartile range and the median of the non-stationary variables, output and trend output, as well as the interquartile range and median of the variables that are stationary, the output gap and trend output DDF.

As said above, the non-stationary variables, in Panels A, may not be robust to the choice of the prior standard deviation of long-term growth shocks, $\sigma_{\varepsilon_t^g}$, as well as the corresponding posterior estimates, so they are reported for illustration purposes. In contrast, the stationary variables, in Panels B, are robust to the choice of the prior for this standard deviation, $\sigma_{\varepsilon_t^g}$, and the posterior estimates.

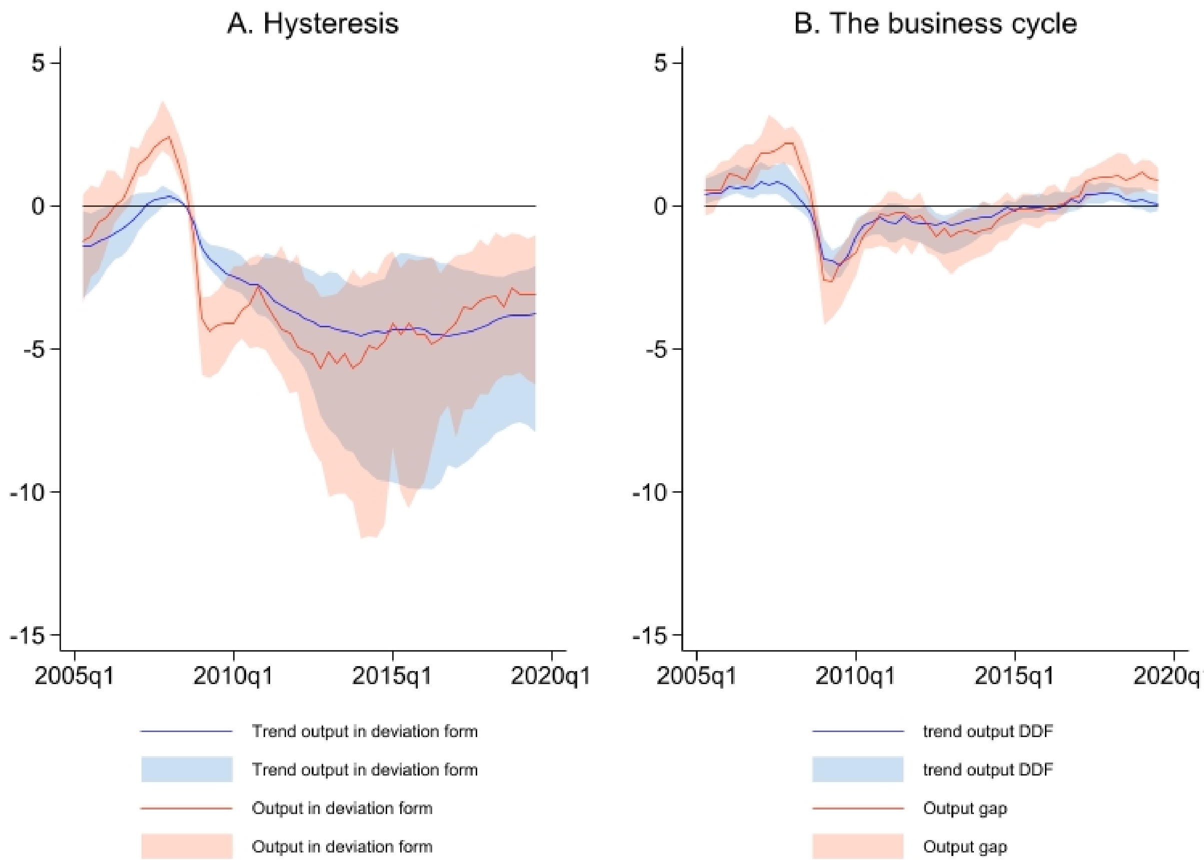
Figure 20 shows the 2008-2009 financial crisis. In AD economies, in Panel A, trend output in deviation form indicates hysteresis. On its part, output in deviation form undergoes a double-dip recession. Thereafter, it approaches trend output in deviation form. In Panel B, both the output gap and trend output DDF undergo a complete cycle. It is the sluggish recovery in trend output DDF in Panel B that explains the hysteresis effect on trend output in deviation form in Panel A.

Figure 21 shows the 2008-2009 financial crisis in EMDE economies. In contrast to the case of AD economies, trend output in deviation form, in Panel A, does not appear to represent much evidence of hysteresis. In addition, output in deviation form rapidly returns to trend output in deviation form; or, in other terms, the output gap closes rapidly. In Panel B, the output gap and trend output DDF also exhibit a complete cycle. But in contrast with AD economies, it is the the fast recovery in trend output DDF in Panel B that explains the absence of hysteresis in trend output in deviation form in Panel A.

Figures 22 and 23 show the COVID-19 recession in AD and EMDE economies, respectively. In both, AD and EMDE economies trend output in deviation form suggests sizable hysteresis. Correspondingly, the recovery in trend output DDF in both AD and EMDE economies is somewhat sluggish, although not as much as the one that took place in AD economies during the 2008-2009 financial crisis.

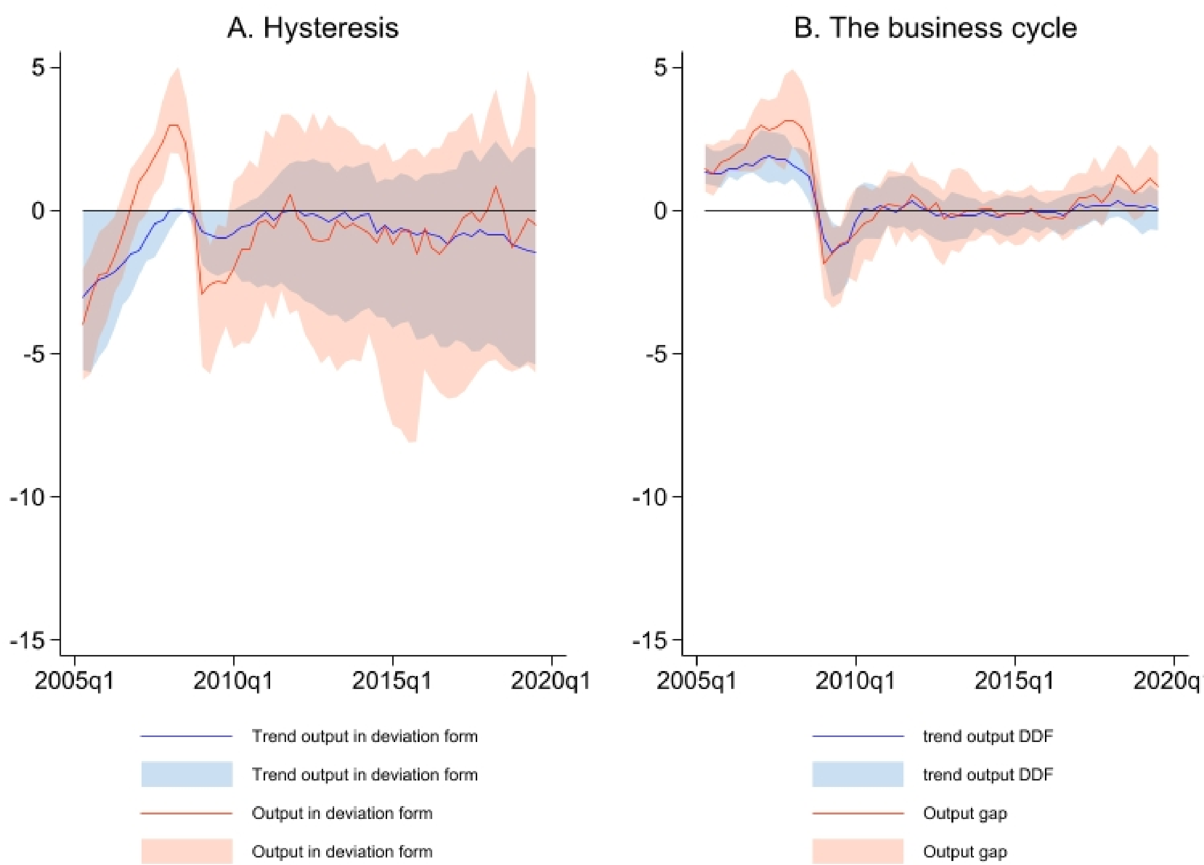
While the interquartile range accounts for business fluctuations in standard economies, a

Figure 20: The 2008-2009 financial crisis in AD economies



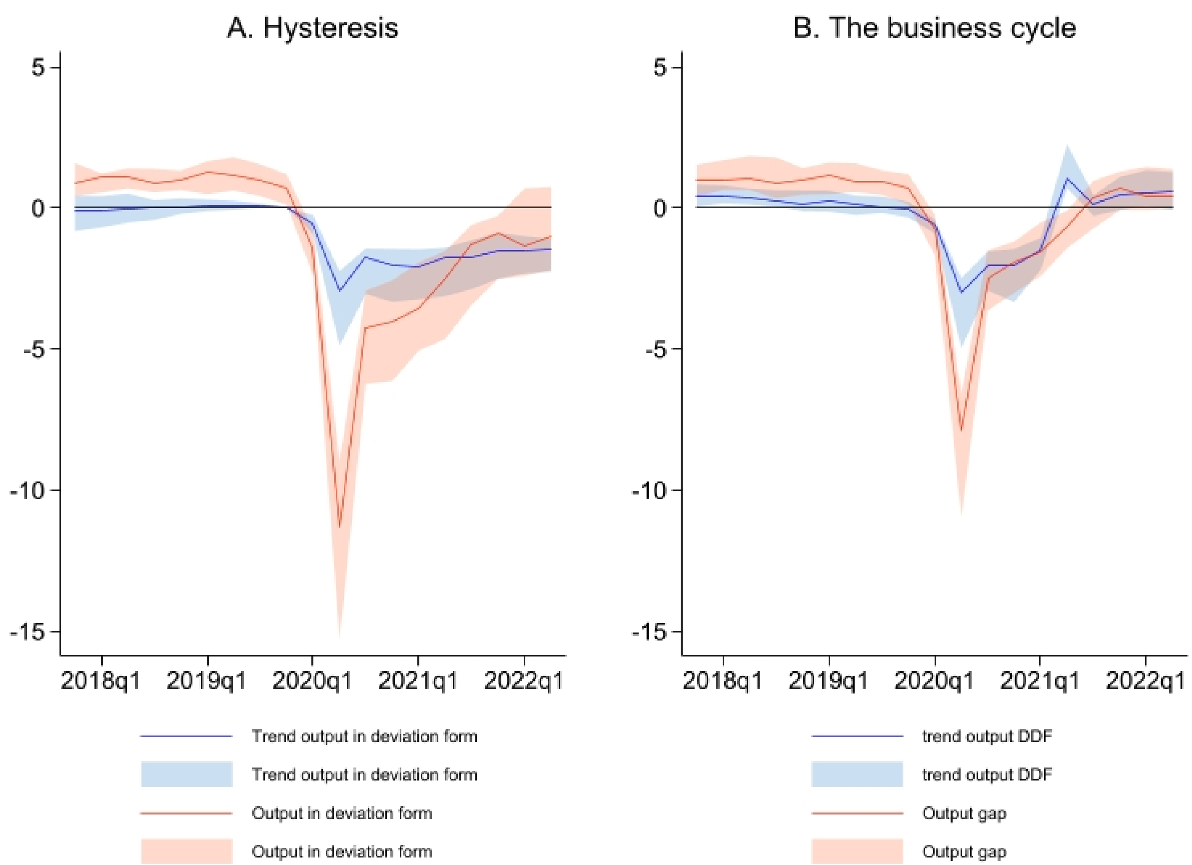
Note: output and trend output are in deviation form. Shades show the interquartile range, lines show the median. Trend output in deviation form is normalized to zero in 2008Q3. The 2008-2009 financial crisis led to hysteresis in AD economies.
Source: authors' estimates and the quarterly database.

Figure 21: The 2008-2009 financial crisis in EMDE economies



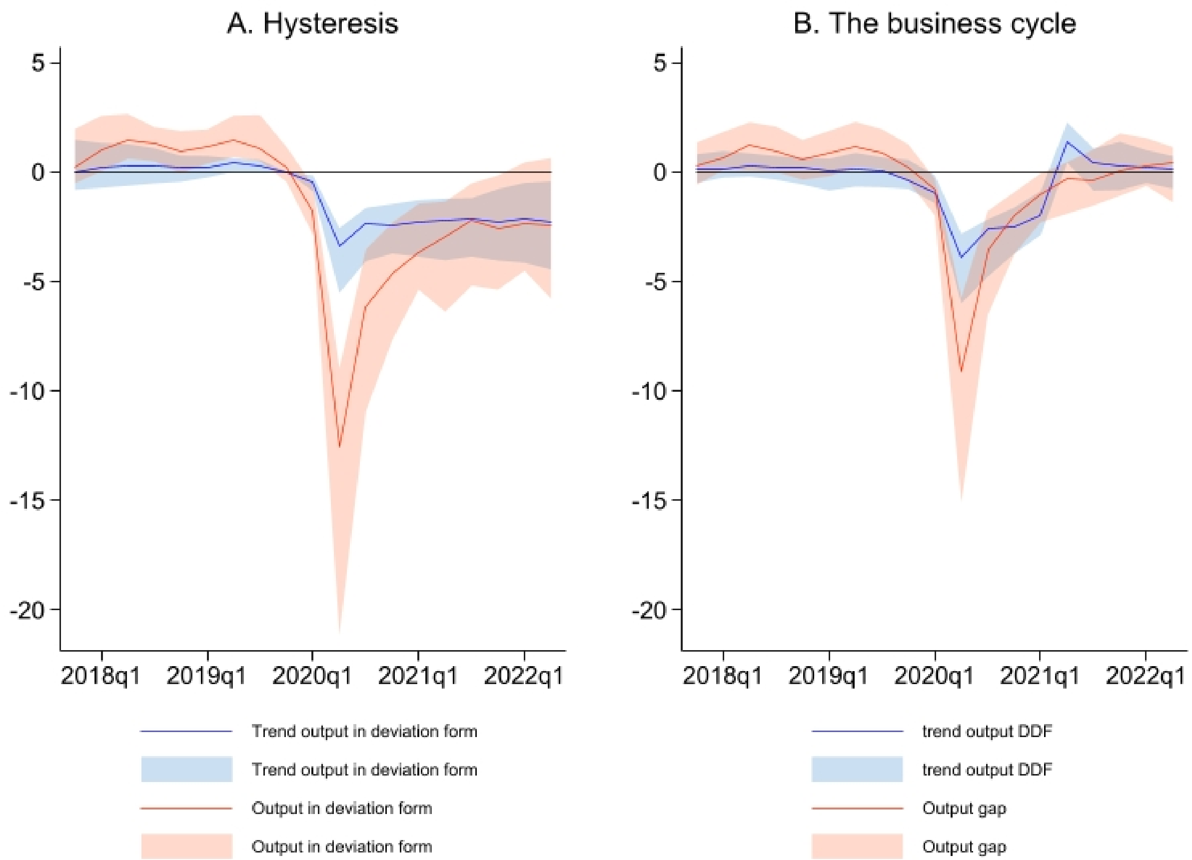
Note: shades show the interquartile range, lines show the median. The business cycle is the cycle in both the output gap and trend output DDF. In EMDE economies the 2008-2009 financial crisis did not lead to hysteresis. Differences are over 4 quarters.
Source: authors' estimates and the quarterly database.

Figure 22: The COVID recession: – AD economies



Note: shades show the interquartile range, lines show the median. Output and trend output are in detrended terms. Trend output in deviation form is normalized to zero in 2019Q4. In AD economies the COVID-19 recession led to hysteresis.
Source: authors' estimates and the quarterly database.

Figure 23: The COVID recession: – EMDE economies

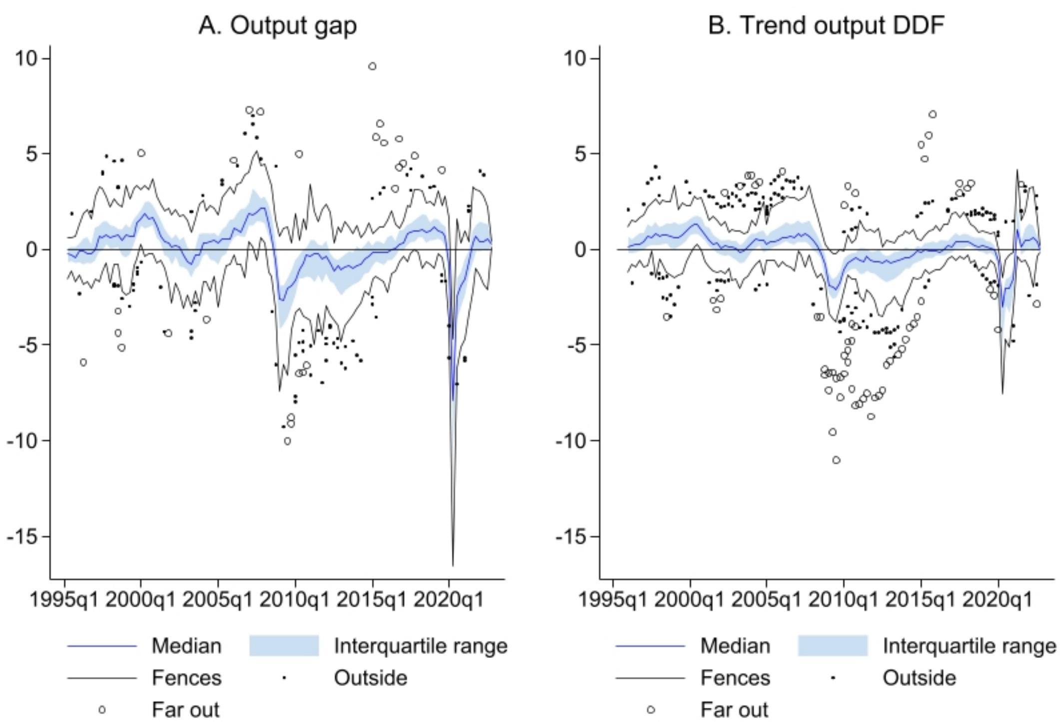


Note: output and trend output are in detrended terms. Shades show the interquartile range, lines show the median. Trend output in deviation form is normalized to zero in 2019Q4. In EMDE economies the COVID-19 recession led to hysteresis.
Source: authors' estimates and the quarterly database.

broader view of the 2008-2009 financial crisis and the COVID-19 recession can be studied by taking a look at a more complete distribution of the estimated business cycle.

Figure 24 presents a dynamic boxplot of the 2008-2009 financial crisis and the COVID-19 recession in AD economies.[37] The figure depicts the evolution of the central tendency, dispersion and volatility of the output gap, in Panel A, and trend output DDF, in Panel B. The central tendency is indicated by the median. The dispersion is indicated by the interquartile range, the range and the outliers. The volatility is indicated, among other features of the boxplot, by the evolution overtime of the median.

Figure 24: An overview of the 2008-2009 financial crisis and the COVID-19 recessions – AD economies

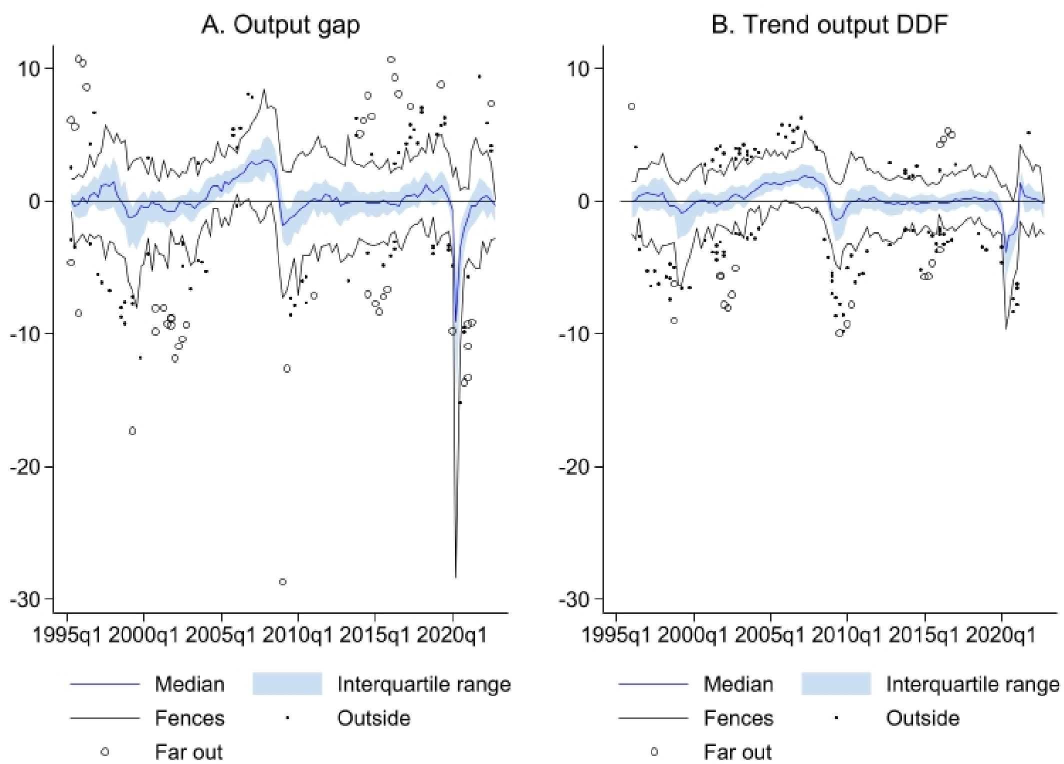


Note: the figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF. The solid line indicates the range, calculated using Tukey's fences with $k = 1.5$; dots indicate outside the range; circles indicate far out or outside a range defined by $k = 3$.
Source: authors' estimates and the quarterly database.

Surprisingly, over the last two business cycles dispersion and volatility are not clearly

[37]The figure shows the median, interquartile range, range and the extreme events. The whyskers of standard boxplots become solid lines that indicate the range. Outside the range, the points indicate the events that are outside while the circles indicate the events that are far out the range.

Figure 25: An overview of the 2008-2009 financial crisis and the COVID-19 recessions – EMDE economies



Note: the figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF. The solid line indicates the range, calculated using Tukey's fences with $k = 1.5$; dots indicate outside the range; circles indicate far out; that is, outside a range defined by $k = 3$.
Source: authors' estimates and the quarterly database.

higher in EMDE economies as is it would be expected. Instead, as reported in Figure 25, dispersion and volatility are comparable across the two groups of economies.

As to the outliers, they tend to be above the range before the outbreak of recessions and below the range before recovery. The outliers take place in AD economies mostly in the run up to the 2008-2009 financial crisis and also during its aftermath, including during the euro area Crisis. In EMDE economies, several extreme events take place by the 1997-1999 financial crisis and at the time of the tapper tantrum in 2015, an event that took place along with a revision of growth in China and a drop in oil prices. In a more general view, as shown by Cerra and Saxena (2008), Reinhart and Rogoff (2014), Von Peter et al. (2012) and Sufi and Taylor (2021) extreme events in the business cycle take place as a consequence of financial crisis, civil war, civil war *cum* weak institutions, and natural disasters.

5.4 The business cycle in historical perspective

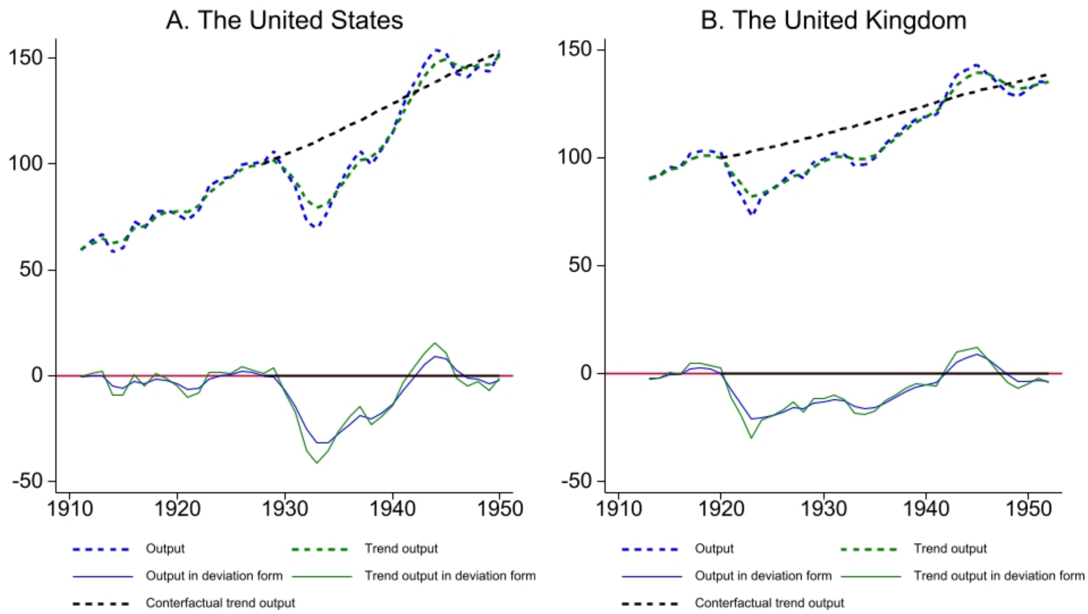
5.4.1 The United States and the the United Kingdom, 1910-1950

The trend-cycle decomposition can help shed light on hysteresis in some salient historical episodes, with the help of the broader historical coverage of the yearly database. Figure 26 shows extreme hysteresis events in the United States during the Great Depression and in the United Kingdom during the Depression of the Twenties.

In the United Kingdom, trend output collapsed in events related to deflation and the return to the gold standard at an overvalued exchange rate, as documented by Eichengreen (2008). In addition, as has been shown by Crafts (2018), trend output also dropped following reduced international trade. Crafts (2018) estimates that the drop in output due to deflation, reduced trade and an increased burden of public debt was about 11.3 percent, this figure is comparable to the drop in trend output in deviation form shown in Figure 26.

In the United States, trend output dropped relentlessly during the Great Depression in response to a succession of joint S&D shocks related with the financial crisis, see Figure 26. A recovery in the hysteresis indicator took place in the forties in both the United States and the United Kingdom owing to war-time-related government expenditure.

Figure 26: The trend-cycle decomposition with hysteresis during depression and war



Note: the figure shows the Great Depression in the United States, the Depression of the Twenties in the United Kingdom, and recovery during WWII in both countries. Trend output in deviation form is zero in the United States in 1928 and in the United Kingdom in 1918. Data are in yearly frequency. Source: authors' estimates based on data from the Maddison Project database, version 2020.

5.4.2 Filtration results for the business cycle

Also using the yearly database, a more general view of the business cycle can be studied by incorporating the entire distribution of both the output gap and trend output DDF. The long historical sample enables comparisons over three time spans; the Gilded Age period, 1870-1910; the war and interwar period, 1910-1950; and the post WWII period, 1950-2022. Business fluctuations are assessed on two types of measures. The first one gives and idea of volatility and dispersion, it is given by the median, interquartile range, range and outliers of the output gap and trend output DDF, Figures 27 and 28. The second one gives an idea of skew and kurtosis, it is the frequency of extreme events in trend output DDF, shown in Figures 29 and 29. The frequency of extreme events is defined as the realizations of trend output DDF that lie outside Tukey's (1977) fences for $\kappa = 1.5$, as explained in Appendix 2.

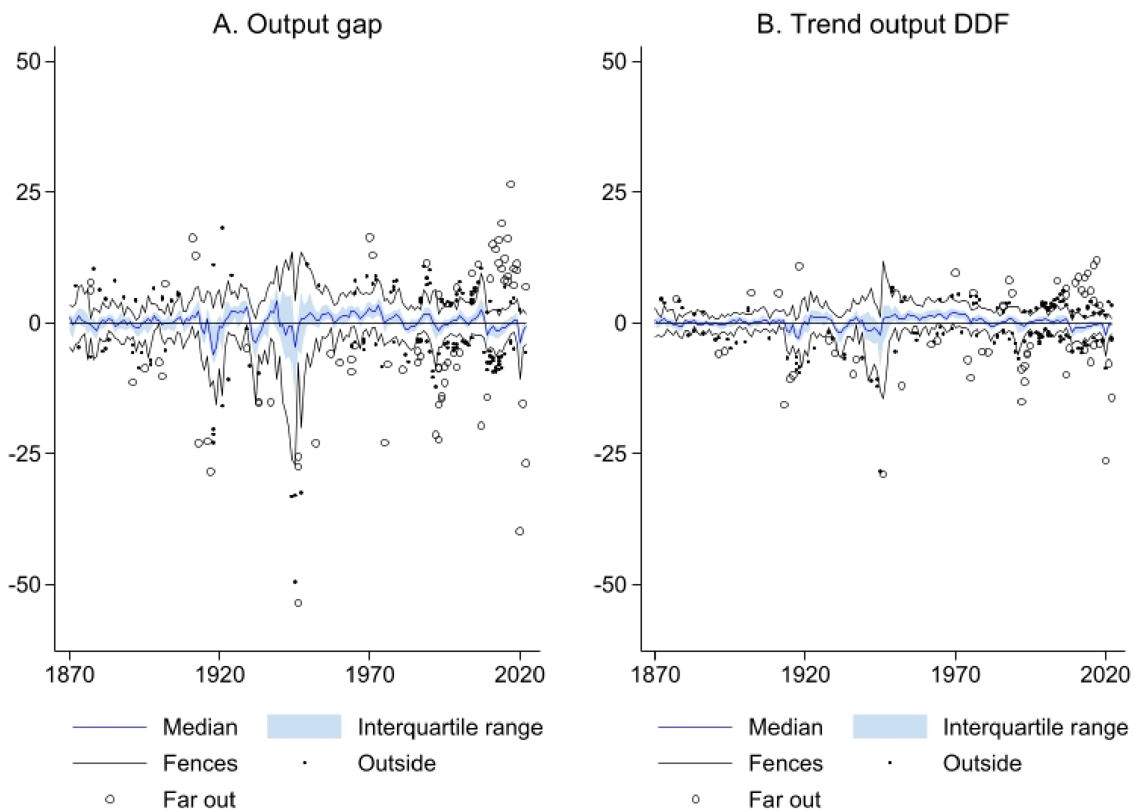
The Gilded Age period was tranquil in terms of volatility and dispersion in both AD and EMDE economies. Dispersion and volatility, assesed by the interquartile range, range and outliers, was the thinnest, see Figures 27 and 28.

The interwar period was turbulent, particularly in AD economies. Volatilty and dispersion are the highest across periods, as shown by the broadened interquartile range, range and outliers, in Figures 27 and 28. In addition, in AD economies the business cycle was asymmetric, as indicated by the higher frequency of negative extreme events compared with the frequency of positive extreme events in Figure 29.

The post WWII period has been relatively tranquil in the bulk of AD economies and turbulent in EMDE economies. In AD economies, with the exeption of the 2008-2009 financial crisis and the COVID-19 recession, volatility and dispersion are remarkably low (see Figure 27. At the same time, the frequency of extreme events increased, as indicated by an increase in the median frequency of events in Panel B of Figure 29. In addition, the frequency of extreme events became symmetric, as indicated by the comparable frequency of positive and negative extreme events in Panel B of Figure 29. Furthermore, as shown in Figure 27, there is an increase in the reach of the outliers in comparison to the range.

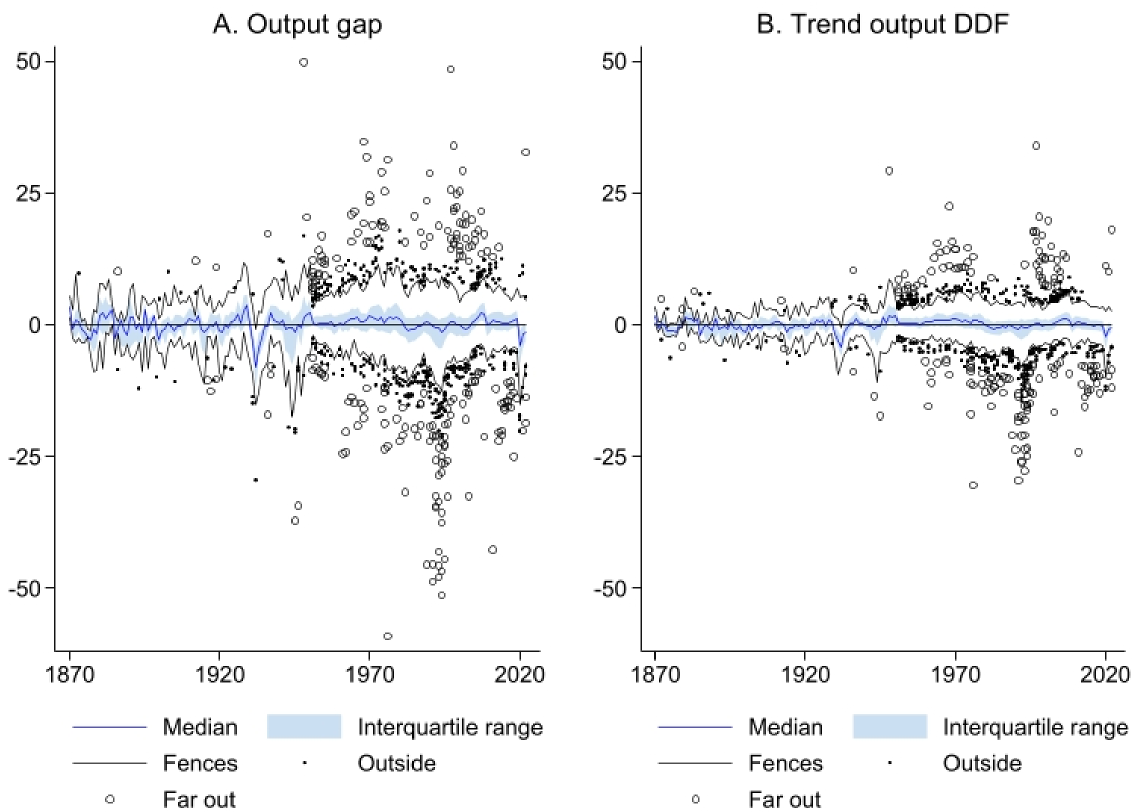
In sharp contrast with AD economies, volatility and dispersion surged in EMDE economies, indicating that several economies have experienced sharp fluctuations; this is seen in the remarkable increase in the interquartile range as well as in the reach of extreme events in Figure 28.

Figure 27: A historical perspective of the business cycle – AD economies



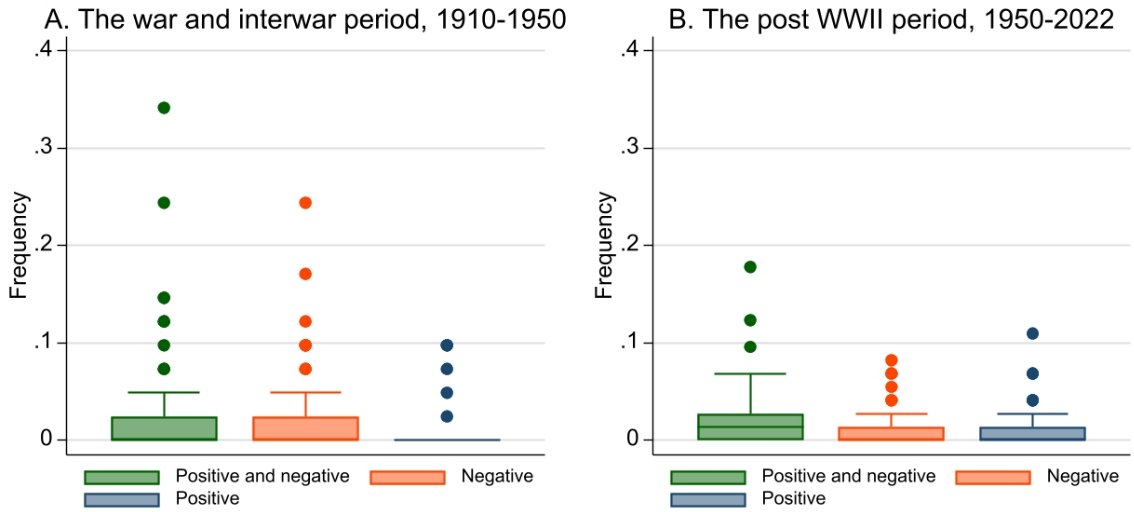
Note: the figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF. The solid line indicates the range, calculated using Tukey's fences with $k = 1.5$; dots indicate outside the range; circles indicate far out or outside a range defined by $k = 3$.
Source: authors' estimates and the yearly database.

Figure 28: A historical perspective of the business cycle – EMDE economies



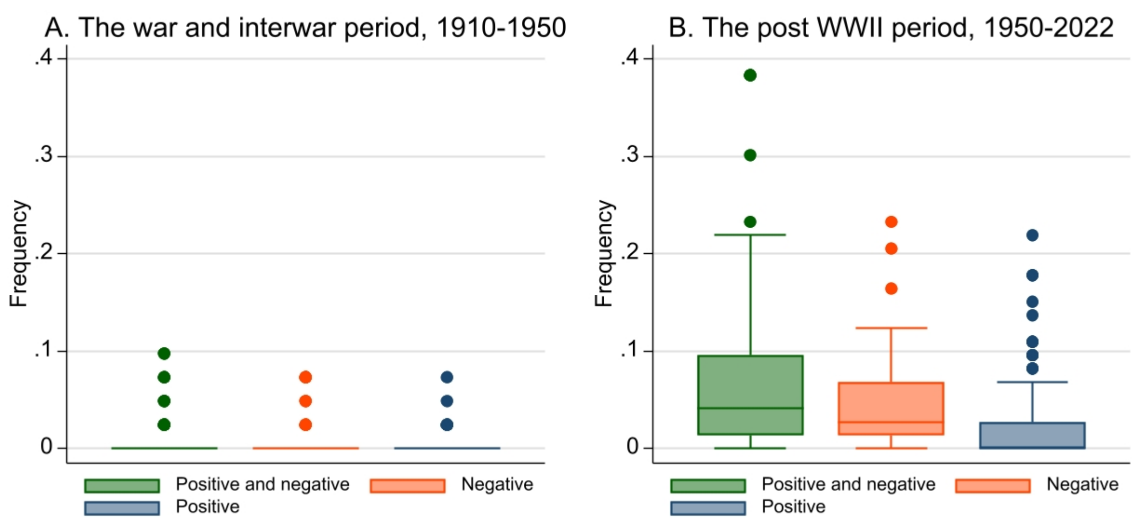
Note: the figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF. The solid line indicates the range, calculated using Tukey's fences with $k = 1.5$; dots indicate outside the range; circles indicate far out or outside a range defined by $k = 3$.
Source: authors' estimates and the yearly database.

Figure 29: The frequency of extreme events in trend output – AD economies



Note: the figure shows the frequency of extreme events in trend output DDF. During the interwar period the frequency is comparable to that of EMDE economies during the post WWII period. During the post WWII period the frequency drops.
Source: authors' estimates and the yearly database.

Figure 30: The frequency of extreme events in trend output – EMDE economies



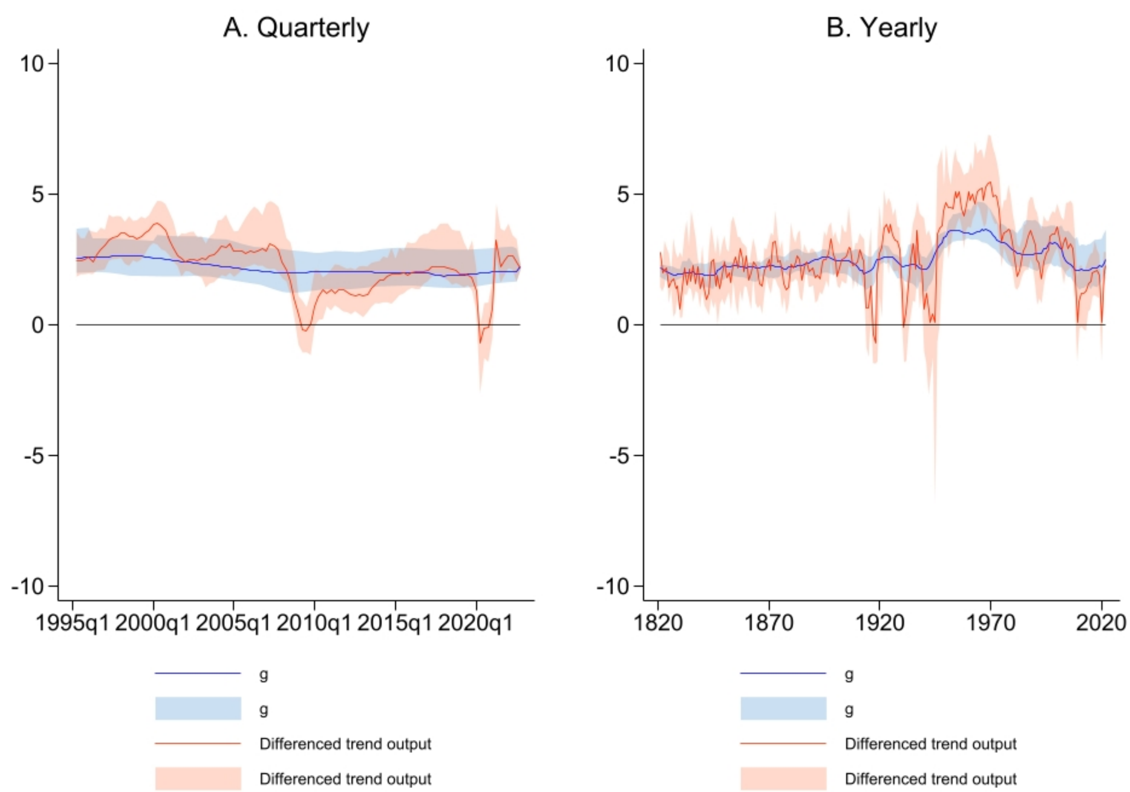
Note: the figure shows the frequency of extreme events in trend output DDF. During the interwar period only in extreme cases the frequency is high. During the post WWII period the frequency is the largest across economies and time periods.
Source: authors' estimates and the yearly database.

5.4.3 The business cycle in trend output growth

Driven by joint S&D shocks, the trend output growth rates $\bar{y}_t - \bar{y}_{t-1}$ and γ_t fluctuate around the long-term growth rate g_t. As mentioned above, in this sense the long-term growth rate, g_t, is a sort of core growth rate.

The long-term growth rate g_t evolves smoothly overtime shifting around periods of higher and lower long-term growth.[38] In both AD and EMDE economies, Figures 31 and 32, the long-term growth rate rose markedly during the post WWII economic expansion. Starting in the seventies, the long-term growth rate declined with the productivity slowdown. In addition, Figure 32 shows that the long-term growth rate has consistently been higher in EMDE economies.

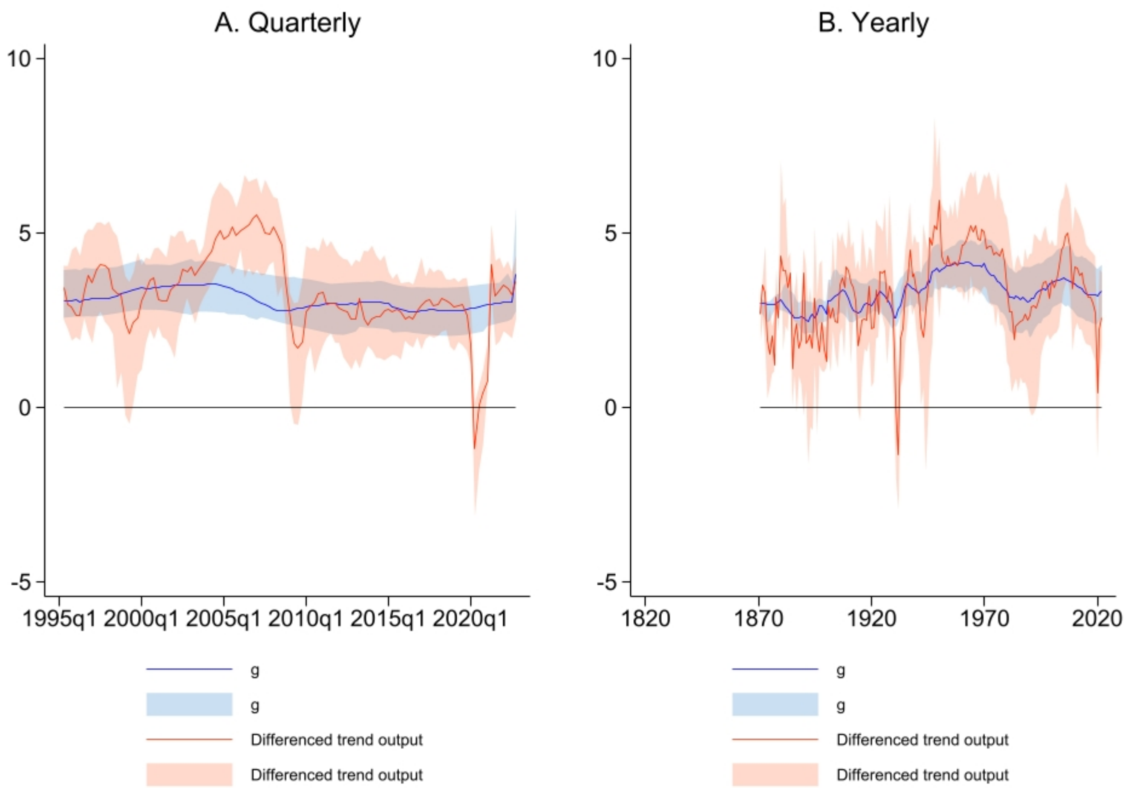
Figure 31: Trend output growth and the long-term growth rate – AD economies



Note: the figure shows the interquartile range, in shade, and the median, in line, of the rates of growth $y^{\bar{y}}_t - y^{\bar{y}}_{t-4}$ and g_t. The former rate of growth fluctuates around the latter.
Source: authors' estimates and the quarterly and yearly databases.

[38]As explained above, in the trend-cycle decomposition with hysteresis, the long-term growth rate is variable; in contrast, in the LLT filter the long-term growth rate is a constant.

Figure 32: Trend output growth and the long-term growth rate – EMDE economies



Note: the figure shows the interquartile range, in shade, and the median, in line, of the rates of growth $y_t^{\bar{y}} - y_{t-4}^{\bar{y}}$ and g_t. The former rate of growth fluctuates around the latter. The median of g_t may change discretely due to a change in the median country.
Source: authors' estimates and the quarterly and yearly databases.

6 Robustness

The model maintains at least two assumptions. First, before the estimation the treatment of expansions and contractions is symmetric. Before the estimation, the deviations of trend output about counter-factual trend output are *a priori* symmetrical, as the prior distribution of the joint shocks is normal. In contrast, after the estimation, hysteresis can arise in one direction or the other to the extent that the estimated joint shocks turn out not to be exactly symmetric.

Second, the standard deviation of the shocks is invariant overtime. For simplicity, for the moment the model was kept with a constant standard deviation. A variable standard deviation is a viable specification that, for the moment, is left for future research.

We also made at least two assumptions in the estimation. First, it is tempting to try to let the data speak more loudly in those coefficients that are important in defining the relative strength of supply and demand shocks in the estimation of the output gap and potential output. In other terms, it seems interesting to widen the standard deviations and intervals of the prior distributions of u, η, $\sigma_{\varepsilon_t^L}$ and $\sigma_{\varepsilon_t^G}$. However, the estimation needs to run with constrained standard deviation and intervals, otherwise the relative weight of supply and demand shocks can increase and the output gap can either close or widen. In other terms, it seems that the estimation makes economic sense under the constrained standard deviations and intervals. Still, in countries where the estimated parameters did not converge these countries were taken out from the sample.

Second, the priors for the coefficients are uniform across countries and time periods. Heterogeneity was obtained with the estimated posterior coefficients. The range for the impulse responses showed that the data did have some role in informing the qualitative behavior of the model across countries. Tailored prior distributions for countries with highly stylized behavior of real GDP can help improve the estimation of the output gap and trend output in those countries. We have left out of the sample those countries where the estimated parameters did not converge.

In the longer historical sample, starting in 1870 in most cases, we maintained the parameters invariant across economies and time periods as an analogy to parameters that are kept constant in other approaches alluded to in the introduction, namely, DeLong et al. (2012), Jordà et al. (2020b), Ball and Onken (2021). It is also usual practice to consider the λ coefficient in the HP filter as a fixed parameter across economies and sample periods.[39]

Finally, our hysteresis measure, trend output in deviation form, is not robust to the choice of the prior for the standard deviation of long-term growth shocks, $\sigma_{\varepsilon_t^g}$, or the corresponding posterior estimate; thus, the caveat is that the numbers should be read with caution as they do not pretend to be estimations but only illustrations of hysteresis. In contrast, our supply measure of the business cycle, trend output DDF, is a variable that we regard component of the business cycle that, for different settings of the prior for the standard deviation of long-

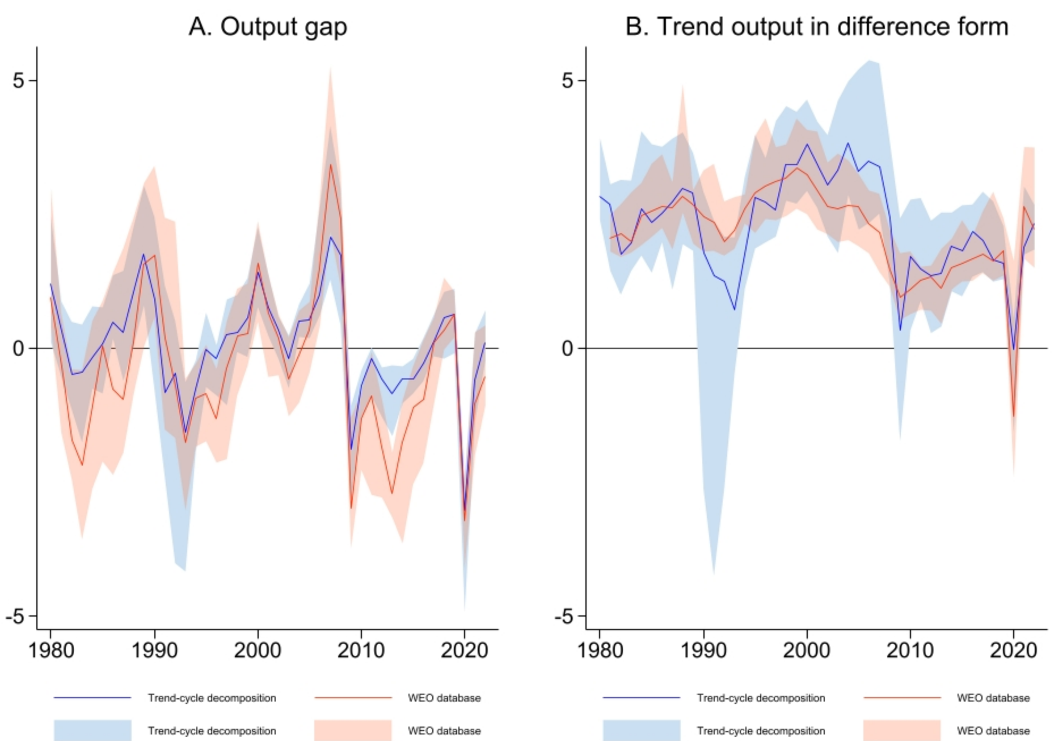
[39]We kept the parameters calibrated when the analysis covered long historical samples, for example since 1870. In contrast, we estimated the parameters in the quarterly sample starting in 1995, in the yearly sample starting in 1950 and in the particular cases of the Great Depression in the United States and the Depression of the Twenties in the United Kingdom.

term growth shocks, $\sigma_{\varepsilon_t^g}$, and the corresponding posterior estimate, maintains the correlation between the output gap $\hat{y}_t$ and trend output in difference form $y_t^{\bar{y}} - y_{t-4}^{\bar{y}}$ and so is robust to the standard deviation of long-term growth shocks, $\sigma_{\varepsilon_t^g}$.

7 Discussion

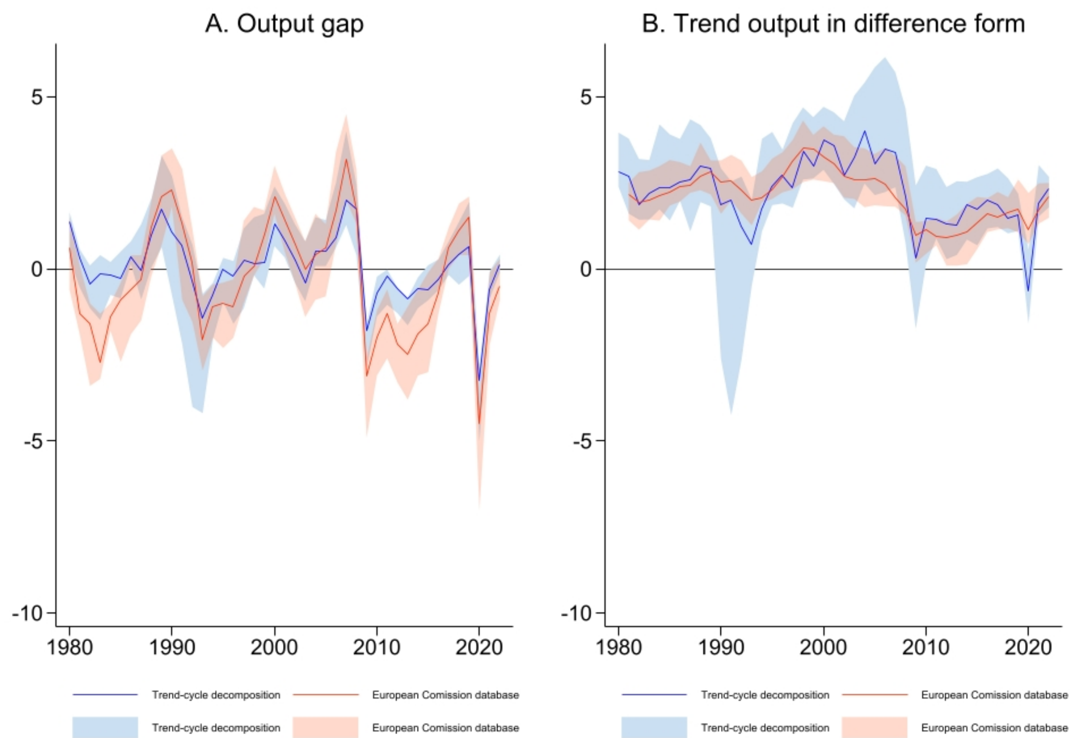
As we have estimated output gaps for a large number of countries, a question arises as to how do these estimated output gaps compare with other estimates. Currently, publicly available output gap estimates are available from three sources: the World Economic Outlook (WEO) database of the IMF, the macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs, and the OECD Economic Outlook Online Database. All currently available estimates are in yearly frequency.

Figure 33: Comparison of the estimates of the WEO database
and the trend-cycle decomposition with hysteresis



Note: in the trend-cycle decomposition with hysteresis, trend output does not evolve smoothly, the output gap booms that arise before recessions are smaller and the standard deviation of the output gap is smaller by about a fifth.
Source: authors' estimates, the yearly database and the WEO April 2022 database.

Figure 34: Comparison of the estimates of the European Commission and the trend-cycle decomposition with hysteresis

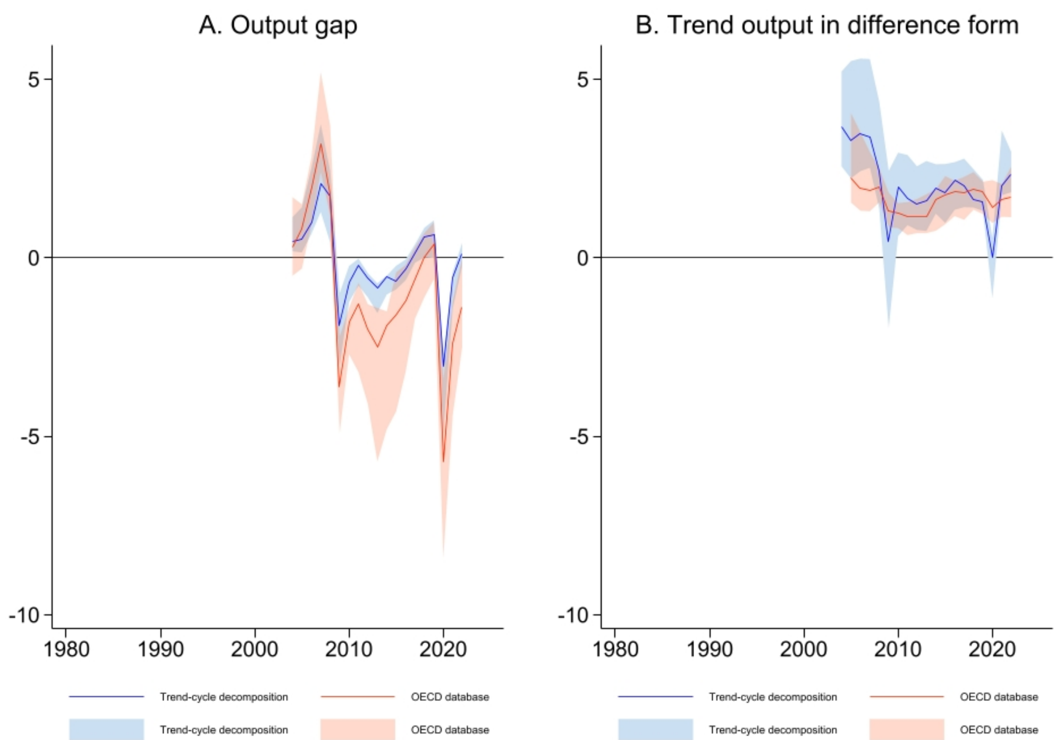


Note: in the trend-cycle decomposition with hysteresis, trend output does not evolve smoothly, the output gap booms that arise before recessions are smaller and the standard deviation of the output gap is smaller by about a fifth.
Source: authors' estimates, the yearly database and the macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs.

The first difference that arises is that the trend-cycle decomposition with hysteresis provides quarterly estimates for a large number of countries. Currently, no publicly available database includes quarterly output gap estimates. Quarterly estimates incorporate more information and so enable a more thorough estimation of the business cycle, for the time spans with available data. As for the yearly frequency, the trend-cycle decomposition with hysteresis enables the estimation of the output gap and trend output with a larger coverage and a broader historical perspective.

In any way, in comparing output gaps it is important to bear in mind that they are estimated with different methodologies, concepts of latent output and also with different analytical purposes. A comparison of the estimated output gaps with other publicly available estimates may be in order, keeping this caveat in mind and with the purpose of illustrating

Figure 35: Comparison of the estimates of the OECD
and the trend-cycle decomposition with hysteresis



Note: in the trend-cycle decomposition with hysteresis, trend output does not evolve smoothly, the output gap booms that arise before recessions are smaller and the standard deviation of the output gap is smaller by about a fifth.
Source: authors' estimates, the yearly database and the OECD Economic Outlook Online Database.

the features of the trend-cycle decomposition with hysteresis.

Figures 33, 34 and 35 compare the output gap estimates of each of the currently publicly available databases against the the output gap estimates of the trend-cycle decomposition with hysteresis. The figures also compare the trend output growth that is implicit in the output gap estimates.[40] The comparison makes evident the advantages of the trend-cycle decomposition with hysteresis; namely, trend output does not evolve smoothly and large booms do not necessarily appear before recessions; as a consequence, output gap revisions are smaller. Another difference that arises is that, because trend output does not evolve smoothly, the standard deviation of the estimated output gaps is somewhat smaller compared to that calculated with output gap estimates available from other sources.

8 Conclusions

Business fluctuations are the movement of output about trend output as well as the cycle in trend output in stationary form. In addition, we have estimated business fluctuations under the assumption that they are caused by joint S&D shocks that affect both the output gap and trend output.

We developed a trend-cycle decomposition with hysteresis in trend output. The model is versatile as it is univariate and so enables the estimation of the output gap and trend output in a large number of countries and over long historical periods.

Compared with the LLT and Hodrick Prescott filters, where trend output does not evolve smoothly, the estimated trend output obtained with the trend-cycle decomposition with hysteresis has some convenient features: it does not evolve smoothly, it does not cause large output gap booms before recessions and it is less sensitive to incoming data.

We showed that joint S&D shocks have a primary role in explaining the business cycle. They drive both trend output DDF and the output gap. During the last two recessions, output fluctuations were highly synchronized, such pervasive synchronization was not the case before.

The Bayesian estimation used relatively tight priors so as to preserve the qualitative

[40]Each comparison includes the set of countries with available output gap estimates in each of the publicly available databases. The WEO database has output gap estimates for 27 AD economies starting in most cases in 1980. The European Commission has estimates for 30 economies, 25 AD and 5 EMDE, with some estimates starting in 1965 and most estimates starting in 1995. In turn, the OECD database has estimates for 38 economies, 31 AD and 7 EMDE economies, the estimates start in 2004.

behavior of the impulse responses. Yet, the obtained dispersion in the estimated posterior coefficients, including the estimated relative standard deviations of the shocks, resulted in some heterogeneity in the response to joint S&D shocks.

Using the quarterly database we showed that during the 2008-2009 financial crisis hysteresis ensued in AD economies, as indicated by trend output in deviation form. In turn, during the COVID-19 recession, hysteresis ensued in both AD and EMDE economies.

Based on the volatility and dispersion of the business cycle, as well as on the frequency of extreme events, three historical periods were identified; the Gilded Age period, the interwar period and the post WWII period. The first period was relatively tranquil; the second one was turbulent, particularly in AD economies; the third one was tranquil in AD economies and turbulent in several EMDE economies.

Compared with other available output gap estimates, those produced by the trend-cycle decomposition with hysteresis are available in quarterly frequency and then allow for a more thorough estimation of business fluctuations, as they incorporate more information. As for the estimation in yearly frequency, the estimated output gaps enable larger coverage in terms of number of countries and estimation periods and so they allow for a broader historical perspective.

A comparison of the trend output estimates of the trend-cycle decomposition with hysteresis with those implicit in publicly available estimations of the output gap shows that the estimated trend output obtained with the trend-cycle decomposition with hysteresis does not evolve smoothly and does not lead to large booms before recessions. As a consequence, output gap revisions can be smaller.

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9 Appendix 1. The trend-cycle decomposition in stationary form

The trend cycle decomposition can be run at quarterly and annual frequencies in codes available on the article page. In the codes, the model runs in stationary form. The blocks and equations of the trend-cycle decomposition in stationary form are described below.

9.1 The model for trend output

$$\hat{y}_t = \alpha \hat{y}_{t-1} + (1-u)\varepsilon_t^L + (1-n)\varepsilon_t^G + \varepsilon_t^{\hat{y}}, \tag{12}$$

$$\bar{y}_t^{\Delta} = \gamma_t + 4u\varepsilon_t^L + 4\varepsilon_t^{\bar{y}}, \tag{13}$$

$$\gamma_t = \theta\gamma_{t-1} + (1-\theta)g_t + \eta\varepsilon_t^G + \varepsilon_t^{\gamma}, \tag{14}$$

$$g_t = \phi g_{t-1} + (1-\phi)g + \varepsilon_t^g, \tag{15}$$

$$y_t^{\Delta} = 4(\hat{y}_t - \hat{y}_{t-1}) + y_t^{\Delta}, \tag{16}$$

where $y_t^{\Delta} = 4(y_t - y_{t-1})$ and $\bar{y}_t^{\Delta} = 4(\bar{y}_t - \bar{y}_{t-1})$.

9.2 The model for counterfactual trend output

$$y_t^{C,\Delta} = \gamma_t^C, \tag{17}$$

$$\gamma_t^C = \theta\gamma_{t-1}^C + (1-\theta)g_t + \eta\varepsilon_t^G + \varepsilon_t^{\gamma^C}, \tag{18}$$

where $y_t^{C,\Delta} = 4(\bar{y}_t^C - \bar{y}_{t-1}^C)$.

9.3 The model in deviation form

$$y_t^{D,\Delta} = \gamma_t^D + 4u\varepsilon_t^L + 4\varepsilon_t^{\gamma^D}, \tag{19}$$

$$\gamma_t^D = \theta\gamma_{t-1}^D + \eta\varepsilon_t^G + \varepsilon_t^{\gamma^D}, \tag{20}$$

where $\bar{y}_t^{D,\Delta} = 4(\bar{y}_t^D - \bar{y}_{t-1}^D)$.

10 Appendix 2. The boxplot as a summary of statistics

The boxplot conveys information about 5 numbers in a distribution, the median, the lower fourth or Q_1, the upper fourth or Q_3, the lower extreme and the upper extreme. The boxplot summarizes these numbers with the line across the box, the upper hinge (the ceiling of the box), the lower hinge (the floor of the box), the upper whysker (the upper vertical line), and the lower whysker (the lower vertical line), respectively. The boxplot also illustrates the interquartile range, that is, the height of the box; the range, or the interval between the upper and lower whyskers; the first quartile, or the length of the lower whysker; the fourth quartile, or the upper whysker; and the outliers; that is, the values outside the range.

The whyskers are defined using Tukey's (1977) fences. Let the interval $[E_L, E_U]$ be defined as

$$[E_L, E_U] = [Q_1 - \kappa(Q_2 - Q_1), Q_2 + \kappa(Q_2 - Q_1)], \tag{21}$$

The range is $[F_L, F_U]$, where F_L is the observation that is closest to E_L and inside interval $[E_L, E_U]$ and F_U is the observation that is closest to E_U and inside interval $[E_L, E_U]$. The range is different from the interval because the ends of the range are observations while the ends of the interval may not necessarily be observed data. The whysker is not a percentile, so the number of outliers above and below the whyskers can be asymmetric.

In turn, the outliers or extreme events, ω_t, in variable v_t are defined as

$$\omega_t = v_t \tag{22}$$

if $v_t < F_L$ or $v_t > F_U$.

Also following Tukey (1977), events that are beyond the range defined by $k = 1.5$ are said to be outside while events that are outside the range defined by $k = 3$ are considered far out.

11 Appendix 3. List of countries in the quarterly and yearly databases

The countries in the quarterly and yearly databases are the following:[41][42]

Quarterly database

AD economies

Australia (1959Q1), Austria (1995Q1), Belgium (1995Q1), Canada (1961Q1), China P.R.: Hong Kong (1990Q1), China, P.R.: Macao (2001Q1), Cyprus (1995Q1), Czech Republic (1996Q1), Denmark (1995Q1), Estonia (1995Q1), Finland (1990Q1), France (1980Q1), Germany (1991Q1), Greece (1995Q1), Iceland (1995Q1), Ireland (1995Q1), Israel (1995Q1), Italy (1996Q1), Japan (1994Q1), Luxembourg (1995Q1), Malta (2000Q1), Netherlands (1996Q1), New Zealand (1987Q1), Norway (1978Q1), Poland (1995Q1), Portugal (1995Q1), Singapore (1975Q1), Spain (1995Q1), Sweden (1993Q1), Switzerland (1980Q1), United Kingdom (1955Q1), United States (1950Q1).

EMDE economies

Albania (2009Q1), Argentina (1995Q1), Bahamas (2015Q1) , Bosnia and Herzegovina(2000Q1), Bostwana (2006Q1), Brazil (1995Q1), Brunei Darussalam (2014Q3), Bulgaria (1995Q1), Cabo Verde (2007Q1), Chile (1995Q1), China P.R.: Mainland (1995Q1), Colombia (1995Q1), Costa Rica (1995Q1), Croatia (1995Q1), Dominican Republic (1995Q1), Ecuador (2000Q1) El Salvador (2000Q1), Georgia (2005Q1), Guatemala (2013Q1), Honduras (2000Q1), Hungary (1995Q1), India (1996Q1), Indonesia (1995Q1), Iran (2011Q2), Jamaica (1996Q1), Jordan (1995Q1), Kazakhstan (1995Q1), Kenya (2009Q1), Kosovo (2010Q1), Latvia (1995Q1), Lithuania (1995Q1), Mexico (1993Q1), Moldova (2010Q1), Nigeria (2010Q1), North Macedonia (2000Q1), Paraguay (1995Q1), Peru (2007Q1), Philippines (2013Q1), Romania (1995Q1), Russia (1995Q1), Serbia (1995Q1), Seychelles (2014Q1), Slovak Republic (1995Q1), Slovenia (1995Q1), South Africa (1993Q1), Santa Lucia (2006Q2), Thailand (2003Q1) Turkey (1995Q1),

[41]The number in parentheses indicates the date from which continuous data are available.

[42]For the yearly frequency, in order to preserve the times series as long as possible, one or up to two years of data where obtained by interpolation in the very few cases where this interpolation was feasible.

Ukraine (2010Q1), West Bank and Gaza (2011Q1).

Yearly database

AD economies

American Samoa (2003),Andorra (1971), Australia (1820), Austria (1870), Belgium (1846), Bermuda (1960), Canada (1870), Cayman Islands (2006), Cyprus (1951), Czech Republic (1990), Denmark (1820), Estonia (1980), Finland (1860), France (1820), French Polynesia (1965), Germany (1850), Greece (1833), Greenland (1970), Iceland (1950), Ireland (1921), Isle of Man (1984), Israel (1950), Italy (1820), Liechtenstein (1970), Lithuania (1995), Luxembourg (1950), Malta (1950), Netherlands (1820), New Caledonia (1965), New Zealand (1870), Northern Mariana Islands (2002), Norway (1830), Portugal (1820), Puerto Rico (1950), San Marino (1997), Singapore (1950), Slovakia (1992), Slovenia (1952), Spain (1850), Sweden (1820), Switzerland (1851), Turks and Caicos Islands (2011), Taiwan (1948), United Kingdom (1820), United States (1820). Virgin Islands (2002)

EMDE economies

Afghanistan (1950),Albania (1950), Algeria (1950), Antigua and Barbuda (1977), Argentina (1900), Aruba (1986), Azerbaijan (1980), Bahamas (1960), Bahrain (1980), Barbados (1950), Belarus(1980), Belize(1960), Benin (1950), Bhutan (1980), Bolivia (1900), Botswana (1950), Brazil (1870), Brunei (1974), Bulgaria (1920), Burkina Faso (1950), Burundi (1950), Cameroon (1950), Central African Republic (1950), Chile (1820), China P.R.: Mainland (1950), Colombia (1900), Comoros (1950), Congo (1950), Costa Rica (1920), Côte d'Ivoire (1950), Crotia (1952), Cuba (1902), Djibuti (1950), Dominica (1950), Dominican Republic (1950), Ecuador (1900), El Salvador (1920), Eswatini(1950), Ethiopia (1950), Fiji (1960), Gabon (1950), Gambia (1950), Ghana (1950), Grenada (1977), Guatemala (1920), Guinea (1950), Guinea-Bissau (1950), Haiti (1945), Honduras (1920), Hungary (1946), India (1884), Indonesia (1820), Iran (1950), Jamaica (1950), Jordan (1950), Kazakhstan (1980), Kenya (1950), Kiribati (1970), Kosovo (2009), Kurgyz Republic (1980), Laos (1950), Lesotho (1950), Liberia (1950), Libya (1950), Madagascar (1950), Malawi (1950), Malaysia (1947), Maldives (1995), Mali (1950), Marshall Islands (1982), Mauritania (1950), Mauritius (1950), Mexico (1895), Micronesia (1986), Moldova (1995), Mongolia (1950), Montenegro (1952), Montserrat (1997), Morocco (1950), Mozambique (1950), Myanmar (1950), Namibia (1950), Nepal (1950), Nicaragua (1920), Niger (1950), Nigeria (1950), North Macedonia (1952), Oman (1950), Pakistan (1950), Palau (2000), Panama (1906), Papua New Guinea (1960), Paraguay (1939), Peru (1896), Philippines (1902), Poland (1929), Romania (1920), Russia (1989), Samoa (1982), San Marino (1997), São Tomé and Príncipe (1950), Saudi Arabia (1950), Senegal (1950), Serbia (1952), Seychelles (1950), Sierra Leone (1950), Solomon Islands (1980), South Africa (1950), Sri Lanka (1870), St. Kitts and Nevis (1977), St. Lucia (1950), St. Vincent and Grenadine (1960), Sudan (1950), Suriname (1960), Syria (1950), Tanzania (1950), Thailand (1950), Timor-Leste (2000), Togo (1950), Tonga (1981), Trinidad and Tobago (1950), Tunisia (1950), Turkey (1923), Turkmenistan (1980), Tuvalu (1990), Uganda (1950), Ukraine (1980), United Arab Emirates (1950), Uruguay (1870), Uzbekistan (1980), Vanuatu (1979), Venezuela (1870), Vietnam (1950), West Bank and Gaza (1950), Yemen (1950), Zambia (1950), Zimbabwe (1950), Anguila (1946), Czechoslovakia (1950), Former Yugoslavia (1947)

Out of the 97 countries in the quarterly database the euro area was excluded to avoid double counting; 6 countries where excluded because the available data covered less than 8 years,[43] 1 country was excluded because the estimated posterior parameters did not converge[44]; and 8 countries were excluded because the estimated output gap had a strong tilt.[45]

Out of the 222 countries in the yearly database, 7 were excluded because they had fewer than 8 data points,[46], 20 countries were excluded because the estimated posterior parameters did not converge,[47] and 11 countries were excluded because the estimated output gap tilted.[48]

[43]These countries were Belarus, Egypt, Mauritius, Mongolia, Nicaragua and Uruguay.

[44]This country was Macao.

[45]The countries are Korea, Lesotho, Montenegro, Saudi Arabia, Senegal, Samoa, Rwanda and Sri Lanka.

[46]The countries are the following: British Virgin Islands, Channel Islands, Faroe Islands, Gibraltar, D.P.R. of Korea, St. Martin (French part) and Former USSR.

[47]The countries are Angola, Armenia, Bosnia and Herzegovina, Cabo Verde, Cambodia, Chad, Republic of Congo, Equatorial Guinea, Georgia, Guyana, Iraq, Kuwait, Latvia, Lebanon, Macao SAR, Nauru, Qatar, Rwanda.

[48]These countries are Bangladesh, Curaçao, Egypt, Eritrea, Guam, Hong Kong SAR, Japan, Korea, Sint Maarten (Dutch part), Somalia, Tajikistan. Equatorial Guinea was excluded becasuse the parameterse did not converge and the output gap had tilt


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