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Excess Asset Returns Predictability in an Emerging Economy: The Case of Colombia

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Abstract

We examine the extent in which the ratios of book-to-market and earnings-to-price predict excess asset returns in an emerging market economy like Colombia. We want to find the magnitude in which these ratios help to forecast excess returns and if there is any evidence that one of the ratios outperforms the other. In addition, we want to address the impact of the spread between the domestic and the foreign policy interest rate in the excess asset returns. Using Bayesian techniques, we find that the magnitude of the effect is similar for both ratios and that the impact is slightly higher in the case of firms with higher book-to-market ratios. Moreover, we find evidence that the spread of interest rates explains the excess returns in a way according to the Uncovered Interest Parity theory.

Keywords: Book-to-market, Earnings-to-price, Excess returns, Bayesian.

JEL classification: G12; G17; C11; C15

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Predictibilidad de los excesos de retornos en una economía emergente: El caso de Colombia[*]

Martha López **Eduardo Sarmiento G.**

Resumen

Analizamos en qué medida las relaciones de *book-to-market* y de ganancia-a-precio tienen poder predictivo de los excesos de retorno en una economía emergente como la colombiana. Queremos responder cuál es la magnitud en que estas relaciones ayudan a pronosticar los excesos de retorno y si hay alguna evidencia de que una sea mayor que la otra. Adicionalmente, queremos analizar cuál es el impacto del diferencial de tasas de interés de política doméstica y externa sobre los excesos de retorno. Mediante técnicas Bayesianas encontramos que la magnitud del efecto es similar para las dos relaciones y que el impacto es un poco mayor en el caso de firmas con razón de *book-to-market* más alta. Más aún, encontramos evidencia que el diferencial de tasas de interés explica los excesos de retornos en línea con las predicciones de la teoría de Paridad No-cubierta de Intereses.

Palabras clave: Book-to market, ganancia-a-precio, excesos de retorno, bayesiano.

Clasificación JEL: G12; G17; C11; C15

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1. Introduction

Forecasting stock returns has a long history. The evidence has shown that they are predictable. Asset allocation improves with the real-time forecast of stock returns which enhances investment performance. Recently, given that the world has faced an environment of high inflation rates and that this has come hand in hand with very volatile stock markets, the interest in the predictability of stock returns has revived. In countries with very developed stock markets, the analysis has been highly documented, nonetheless, in emerging markets countries the evidence is scarce. In these economies asset returns fluctuate more than in developed economies, which makes more difficult to find their economic determinants. We investigate if, like in advanced economies, the earnings-to-price ratio and book-to-market ratio remain the main drivers of excess asset returns, but in the context of an emerging market economy like Colombia. Moreover, we analyze if the evidence supports some findings that in advanced economies the book-to-market ratio outperforms other determinants as the earnings-to-price ratio. In light that Colombia is a small open economy, we also contribute to the empirical literature by studying if there is evidence regarding the impact of the spread between the domestic and the foreign policy interest rate as another factor, given its relationship with the expected exchange rate devaluation.

To answer these questions an empirical shortcoming is the lack of long time series data on the asset market variables for the case of Colombia. In addition, the Colombian financial system is mainly bank based. The 25 enterprises considered in our dataset are almost the total of firms that trade currently in the stock market. Given the lack of availability of long time series and cross-section data, our dataset contains annual information from 2015 to 2022. In addition, the Colombian stock market has frequent entry and exit of companies and some of them stay less than 10 years. This rotation means that the companies of the stock market differ if considered for a long period of time. To overcome this empirical shortcoming, we use Bayesian econometric techniques which are appropriate for panel data of small samples.

Our analysis yields the following robust results. First, even though excess asset returns fluctuate considerably in Colombia, in line with the findings of Kothari and Shanken (1997) for the United States, we find a positive and economically meaningful incidence of book-to-market ratio on excess asset returns in the Colombian economy. The evidence shows that a one standard deviation in the book-to-market ratio translates in an increase of the excess returns of nearly 1.5%. Second, our findings are that a one standard deviation increase in the earnings-to-price ratio increases excess asset returns also in about 1.5%. These two facts mean that the book-to-market ratio does not outperforms the earnings-to-price ratio in explaining excess asset returns in Colombia or the other way around. Finally, an important finding of this study is that in an emerging market economy like Colombia, the spread is another economic factor explaining excess asset returns; an increase of one p. p. (percentage point) in the spread translates in a decrease of about 1.9 p. p. in the expected excess returns.

Our results hold after a variety of robustness checks. We re-run the main specification for different subsamples, one for firms with high book-to-market ratio and one with low book-to-market ratio. We also modify the main specification by altering the choice of the prior distributions of the parameters of the book-to-market and earnings-to-price ratios: the standard deviation of the baseline gamma distribution and the choice of a normal distribution instead of a gamma distribution.

We organize the paper as follows. Section 2 discusses the related literature. Section 3 describes the data and econometric strategy. Section 4 presents the results. Section 5 presents three robustness exercises. Conclusions are presented in section 6.

2. Related literature

As mentioned in the introduction, forecasting stock returns has a long history. Its study goes back to Cowles (1933) and other seminal papers as the ones by Fama and Schwert (1977), and Campbell and Shiller (1988).

The main variables used to predict returns are book-to-market, dividend yields and earnings-to-price. The book-to-market incidence over excess returns is expected to be positive since it increases when stocks are cheap with respect to the book value. An expansion in dividend yields increase the value of the firm raising excess returns and earnings-to-price is an indicator used by investors to buy and sell stocks, buying when it is higher and selling when it becomes lower.

Our paper contributes to various strands of literature. First, to the literature on the most prominent financial factors that predict excess returns. Fama and French (1988), by using value and equal-weighted portfolios of New York Stock Exchange during 1927-1986, provide evidence of the explanatory power of the dividend yields on expected returns with a slope increasing with the time horizon from one month, one quarter, and one to four years. Campbell and Shiller (1988) present evidence indicating that data on earnings help to predict exact returns as well as discounted returns. These authors, using an extended annual data from 1871-1987 in the United States, find a significant relation of stock returns with lagged dividend-to-price, lagged earnings-to-price and a moving average of earnings-to-price of ten and thirty years. Our paper contributes to this strand of literature by showing that in emerging markets economies, variables of profitability such as earnings-to-price ratio also have predictive power on excess asset returns.

Second, more recently, the predictability of stock returns has been addressed mainly in the United States and in countries with well-developed stock markets. For example, Ang and

Bekaert (2007) provide evidence of predictability of stock returns for France, Germany, United Kingdom, and the United States. There are other studies for a group of developed countries, among them those by Henkel et al. (2011), Della Corte et al. (2010), Cooper and Priestly (2009) and Kellard et al. (2010). Meanwhile, determinants of asset returns in emerging markets economies remain a matter of study. In this sense, our paper contributes to filling that gap.

Third, stock returns literature uses both times series data and panel data. In the United States long-term time series data of excess returns, book-to-market, dividend-price and earnings-to-price are available from sources such as the Dow Jones Industrial Index, Standard & Poor Composite Stock Price Index and New York Stock Exchange from the Center for Research in Security Prices (CRSP). In Colombia stocks price information is available from the Bogota Exchange Index and Medellin Exchange Index, and from the Colcap Index for more recent data. The firm's information is available from the Value Stock Exchange of Colombia, but as mentioned in the introduction, the sample size in Colombia is very short. Addressing the small sample size bias of the OLS estimates, Kothari and Shanken (1997) use a Bayesian bootstrap procedure and find a significant relation of the real stock returns with lagged book-to-market and lagged dividend-to-price for annual data between 1926-91 in the United States. Our document also uses Bayesian techniques to overcome the small sample bias problem and contributes to this strand of literature by showing that lagged book-to-market and lagged earnings-to-price data have predictability power on stocks excess returns, but in the context of an emerging market economy like Colombia.

Fourth, there is a debate on the relative importance of book-to-market and profitability variables to contribute to the predictability of excess asset returns. On the one hand, for example, Kothari and Shanken (1997) evaluate the ability of an aggregate book-to-market ratio to track time series variations in expected market index returns and compare its forecast ability to that of dividend yields, finding that the first outperforms the second for the whole sample period, but for a subsample between 1941-91 the dividend ratio outperforms the book-to-market ratio. Consequently, they conclude that neither variable consistently

dominates the other. On the other hand, Pontiff and Schall (1998), using monthly and annual data during 1926-94, also for the United States, find that book-to-market predicts better market returns in the pre-period of 1960. Among the determinants of market returns they also consider dividend yields, with less explanatory significance. Our paper contributes to this debate by finding that in an emerging market economy the earnings-to-price ratio and the book-to-market ratio have similar predictive power over excess returns, but their economic significance is lower than in advanced economies.

Moreover, Fama and French (1993) present a three-factor model designed to capture the relationship between average return with size and book-to-market. Later, Fama and French (2015) expand it to a five-factor model for the United States with monthly data during 1963-2013, and besides the book-to-market ratio and size incidence on returns, the profitability and investment patterns are addressed. They find that excess returns increase in firms that are smaller in size, more profitable, higher in book-to-market and smaller in assets growth. With respect to the book-to-market ratio, they find that if parsimony is an issue, it is a redundant factor. "But if one is also interested in portfolio tilts toward size, value, profitability, and investment premiums, the five-factor model is the choice" (pp 19). Our paper contributes by showing that in Colombia the incidence of book-to-market on excess returns is more significant in firms with higher book-to-market ratio.

Finally, in recent empirical literature some other factors have been considered, but it is the book-to-market ratio, earnings-to-price and the dividend-to-price, which remain the most important for explaining assets returns. For example, Feng, Giglio and Xiu (2019) evaluate the determinants of asset returns from July 1976 to December 2017 in the United States, considering 150 factors and multiple portfolios, of which the most significant are the ones related with profitability. Our paper contributes to this new evidence by considering a factor that, to our knowledge, has not been used in the literature: the spread between the domestic and foreign policy interest rate. According to the Uncovered Interest Parity theory, this factor is important in a small open economy because it is related to expected devaluation of the exchange rate, which in turn affects expected excess returns.

3. Bayesian Estimation

3.1 Methodology

We apply Bayesian techniques, which are increasingly popular in the fields of macroeconomics and finance. We chose this because the Bayesian approach outperforms Generalized Method of Moments (GMM) and Maximum Likelihood in small samples, which is an important concern for our study. This empirical approach involves obtaining the posterior distribution of the model's parameters based on its log-linear state-space representation. The posterior distribution is obtained by the combination of the likelihood function for the observed data (obtained from the help of a Kalman filter) with the selected prior distributions for each of the parameters of the model. If conjugacy is obtained by this combination, then the posterior can then be analytically optimized with respect to the model parameters directly. Computational tools, like Monte-Carlo Markov-Chain (MCMC) sampling, should be used as we do in this study.

Formally, defining Θ as the parameter space, we wish to estimate the model parameters, denoted by $\theta \in \Theta$. Given a prior $p(\theta)$, the posterior density of the model parameters, θ, is given by.

$$p(\theta|Y^T) = \frac{L(\theta|Y^T)p(\theta)}{\int L(\theta|Y^T)P(\theta)d\theta} \quad (1)$$

where $L(\theta|Y^T)$ is the likelihood conditional on observed data, Y^T. The posterior distribution is typically characterized by measures of central location, such as the mode or the mean, and measures of dispersion, such as the standard deviation or the highest posterior density (HPD).

For the MCMC sampling we use Gibbs sampler updates in the cases when the parameters prior distributions allowed them, and we made the estimation in blocks.

3.2 Data and summary of statistics

Based on the literature preview, and our observations about the relationship about the exchange rate depreciation and the real excess returns, we aim to estimate the following equation for the Colombian stock market.

$$(2)\ R_{it+1} = \alpha + \beta BM_{it} + \gamma EP_{it} + \theta Spread_t + \varepsilon_i + e_{it} \quad e_{it} \backsim i.i.d\ N(0, \sigma_0)$$
$$\varepsilon_i \backsim i.i.d\ N(\beta_0, \sigma_{id})$$

BM is the ratio of the book-to-market per share at the end of year t of the stocks in Colombian market, divided by its standard deviation. R is the annual real excess return in year t+1 inclusive of prices and dividends discounted by the annual interest rate of the banks CDT´s. EP is the ratio of earnings in year t with respect the prices at the end of year t, divided by its standard deviation[1]. In the Colombian stock market, there is an observed relationship between the exchange rate devaluation and the excess returns of stocks, therefore we include, as another factor in the model, the $Spread$ between the domestic and the United States policy interest rate. The spread is a variable that explains expected exchange rate depreciation as predicted by the Uncovered Interest Parity -UIP- theory. Table 1 presents the descriptive statistics of the variables in the model.

During 2015-22 the excess returns in Colombia had considerable fluctuations. The highest was in 2019 with 26.1% and the lowest in 2018 with -22.5%.

The book-to-market variations and price fluctuations were hand in hand. The book-to-market increased in 2015, 2018, 2020 and 2021, and decreased in 2019. In the period the lowest book-to-market was presented by a firm with 0.3 and the highest by a firm with 8.3. When this ratio is high the next year's excess returns becomes higher because of the readjustment of prices since the stocks are cheap.

[1] In this way, the slopes of the regression can be interpreted as the incidence of a one standard deviation change in book-to-market and earnings-to-price. This makes easier the formulation of prior beliefs and the interpretation of the economic significance of the parameters.

The earnings-to-price mean during 2014-21 was 7.9%. The earnings-to-price depends on the profitability of the firms and the prices fluctuations. In the Colombian stock market and in other countries earnings-to-price ratio depends on the trend of price variations during the past years. Firms that have a positive trend have lower earnings-price below 5%, and firms without a positive trend have earnings-price close to 10%. An increase in earnings-to-price increases the next year's excess returns through the higher prices.

In the United States the earnings-to-price ratio of dynamic high-tech stocks can be close to 2%, meanwhile stocks without a positive trend in prices can have a ratio of 10%. The same fact is true in Colombia depending on the stock.

The spread between the policy interest rate of Colombia and the one of the United States determines expected exchange rate devaluation according to the UIP theory. During 2014-2021 the mean spread was 3.7%, with standard deviation of 1.7%, maximum of 7.1% and minimum of 1.7%.

3.3 Priors

We assign the corresponding priors according to previous studies. We use gamma distributions for the variables that are restricted to be positive and normal distributions for the variables for which we do not have any previous information. We use inverse-gamma distributions for the variances of the model as is common in the Bayesian literature.

For the parameter β we use previous estimates by Kothari and Shanken (1997). For γ we have the estimates of Campbell and Shiller (1988), but we adjust the mean in order to have the same distribution of β because we want to asses if the ratio of book-to-market outperforms the earnings-to-price ratio or the other way around. For the parameter of the

spread of the policy interest rate, θ, we use our own estimations and assign a normal distribution with a high variance. For σ_0^2 and σ_{id}^2 we use our own estimations. Table 2 presents the corresponding prior distributions for the parameters of the model. We use 250,000 MCMC iterations and discard the first 125,000. We block the estimation of the parameters and use Gibbs sampling to improve their efficiency when the prior distribution allowed us to do that. The model was estimated using Stata.

4 Results

First, we proceed by analyzing the properties of the Bayesian estimation of the model, that is, the convergence of the parameter values and the goodness of fit of the model.

Table 3 presents the acceptance rate of the model, 0.483, and the average efficiency, 0.1173. Both are very good diagnostics of the convergence of an MH MCMC chain according to Gelman, Gilks and Roberts (1997). The convergence diagnostics of the individual model parameters are also very well behaved as can be observed in Figures 1 - 6. First, for all the parameters in the model, the diagnostic of the trace[2] does not show any trend that could suggest an autocorrelation of the MCMC chain. The last is confirmed by the very low autocorrelation diagnostics. As for the density functions and histograms, they are unimodal. For the parameters β and γ we present also the graph of the prior distributions in order to show that the data provides a considerable information moving priors to posteriors.

For the evaluation of the model, regarding their in-sample predictions, it is expected that the posterior predictive p-values of the probability that the mean of the simulated estimations is greater or equal to the expected value of the observed outcome variable is far from cero or from 1. For our model this is the case for most of the simulations which indicates very good

[2] The trace of a parameter plots the simulated values versus the iteration number. The trace plot of a well-mixing parameter should traverse the posterior domain rapidly and should have nearly constant mean and variance.

fit[3], Figure 7. Moreover, with respect to the model residuals[4], Tables 4 and 5 present the posterior predictive p-values of their mean and variance, respectively, which are close to 0.5 which is the ideal. The posterior predicted p-values of the normality test are presented in Table 6 for the skewness and the kurtosis, which are also far from 0 or 1.

Having analyzed the diagnostics of the parameters, the model itself and its goodness of fit, we can proceed to interpret the parameters. Table 3 also presents our estimates of the parameters. The one for the book-to-market ratio has a mean of 1.458 with a credible interval between 0.746 and 2.304. The parameter of the earnings-to-price has a mean of 1.537 with a credible interval between 0.708 and 2.634. This implies that for an increase of one standard deviation in both, the book-to-market and earnings-to-price, the excess returns increase in nearly 1.5%. These estimates imply that, contrary to some findings for the United States, the ratio of book-to-market does not outperform the earnings-to-price ratio in explaining excess returns. In our estimations, the 95% credible interval for the two variables overlaps. In section 5 we present robustness exercises for this result. However, the estimated posterior mean of the book-to-market ratio is close to 1 while for studies for the United States it is usually higher, close to 4.

A very interesting result is that the parameter of the spread of the policy interest rates, θ, is negative. This means that, as expected by the UIP theory, a higher spread in time t corresponds to an expected devaluation of the exchange rate in t+1 and therefore to lower expected excess returns. An increase of 1 pp in the spread translates into a fall in expected excess returns of about 1.946 pp, with a 95% credible interval between -3.362 pp and -0.560 pp.

[3] A PPP (or a Bayesian p-value or a Bayesian predictive p-value) is then defined as the probability that a test quantity for the replicated data could be as or more extreme than for the observed data. You can think of a PPP as a classical p-value averaged over the posterior distribution (Meng (1994)). For a well-fitting model, the PPP should, ideally, be close to 0.5, although values between 0.05 and 0.95 are often considered acceptable in the literature (Gelman et al. (2014), 150; Congdon (2010), sec. 2.5.2).

[4] Consider simulated outcome values y_{it}^{sim} for an observation it = 1, 2, . . . , n, where $y_{it}^{sim} = (y_{it}^{sim,1}, y_{it}^{sim,2}, y_{it}^{sim,3}, y_{it}^{sim,4}, y_{it}^{sim,5} \dots y_{it}^{sim,M})$. Let $\hat{\mu}_{it} = (\hat{\mu}_{it,}^{1}\ \hat{\mu}_{ti,}^{2}\ \hat{\mu}_{it,}^{3}\ \hat{\mu}_{ti,}^{4} \dots \hat{\mu}_{it,}^{M})$, where $\hat{\mu}_{it,}^{1} = Ey_{it}|x_{it,}, \theta^{1}$ is the estimated expected value of y_{it} given covariate vector x_{it} and simulated parameters θ^{m} m = 1, 2, . . . , M. Let $r_{it}^{sim} = (r_{it}^{sim,1}, r_{it}^{sim,2}, r_{it}^{sim,3}, r_{it}^{sim,4}, r_{it}^{sim,5} \dots r_{it}^{sim,M})$ be simulated residuals for an observation it. Simulated residuals are then defined as $r_{it}^{sim} = y_{it}^{sim} - \hat{\mu}_{it}$.

5 Robustness

In order to check the robustness of the results we perform three exercises.

5.1 Priors with a higher standard deviation

We increase the standard deviation for the prior gamma distribution of the β and γ coefficients of the regressions. We increase this deviation from 0.85 to 2.3 and the results are presented in Table 7. The mean for these parameters is about 1%, which is a value that is initially contained in the 95% confidence interval when we used the baseline estimation with a prior with standard deviation of 0.85 (see Table 3)[5]. This also means that none of them is higher than the other.

5.2. Firms with high book-to-market ratio versus firms with low book-to-market ratio.

We do not have enough data to be able to perform out-of-sample forecasting, however, we can assess the robustness of the model using different samples that have enough data. In this section we run the same regression as in equation (2) but for two subsamples.

We divide the firms between the 13 with higher book-to-market ratio and the 12 with lower book-to-market ratio in the spirit of Fama and French (2015).

Our results are as follows. The posterior mean of the parameters for book-to-market and earnings-to-price are statistically similar between the two subsamples and the whole sample

[5] The diagnostic graphs for the estimation are provided under request.

(considering the 95% confidence intervals), Tables 8 and 9. However, according to their mean, the impact is of about 2.0% in the case of firms with high book-to-market ratios. In the case of the spread, their impact is also bigger for high book-to-market firms.

These robust results imply that with both high and low book-to-market, the excess returns can be forecasted regardless of the differences of the characteristics of the stocks, and that the determinants of book-to-market, earnings-price and spread are significant[6].

5.3 Normal distribution and test for nonnegative expected returns

One concern in the literature on assets returns is to address its nonnegative result with respect to book-to-market and earnings-to-price ratios. One way to tackle this issue in our analysis, and to check if the results remain robust to the selection of priors, is to re-run our regressions with a normal prior distribution for the parameters β and γ. We use a normal distribution with mean 2 and standard deviation of 2.3 and compute the probability that the parameters are between 0 and 2.5 for β and between 0 and 3.3 for γ. The results are presented in Tables 10 and 11. They show that the posterior mean of the parameter β is 1.407 with a standard deviation of 0.599 and an equal-tailed 95% credible interval between 0.194 and 2.569. The results for the parameter γ are a posterior mean of 1.412 with a standard deviation of 0.958 and an equal-tailed 95% credible interval between -0.506 and 3.277. With respect to the nonnegativity of the results, the mean of the probability that β is between 0 and 2.5 is 0.956 with standard deviation of 0.205 and Monte Carlo standard errors of 0.004. The probability that γ is between 0 and 3.3 is 0.903 with standard deviation of 0.295 and Monte Carlo standard errors of 0.004. These results support the ones of the baseline model that the

[6] The diagnostics statistics and model predictive performance in sample are available upon request.

incidence of the book-to-market-ratio and the earnings-to-price ratio is similar and close to 1.5%[7].

6. Conclusions

Using Bayesian techniques, we find that for the stock market of an emerging market economy like Colombia, the ratios of book-to-market and earnings-to-price have predictability power during the period 2015-2022. One standard deviation in the ratios of book-to-market and earnings-to-price increases expected excess asset returns in about 1.5%. This finding is in line with some for the United States that none of them outperforms the other. The spread is another economic factor explaining excess asset returns; an increase of one p. p. (percentage point) in the spread translates in a decrease in about 1.9 p. p. in the excess returns. The later result is consistent with the UIP theory that predicts that when there is a positive spread, there are expectations of devaluation of the exchange rate and therefore the predicted excess asset returns falls.

The results are robust in terms of estimations based in two subsamples of firms, one with higher and another with lower book-to-market ratios. But even though, both, the earnings-to-price ratio and the book-to-market ratio have a similar impact between one another, their impact is of about 2.0% in the case of firms with high book-to-market ratios.

The results are also robust to an increase in the standard deviation of the prior distributions of the parameters of book-to-market and earnings-to-price and to the assumption of normal prior distributions of these parameters.

[7] The acceptance rates and other models diagnostics are provided upon request.

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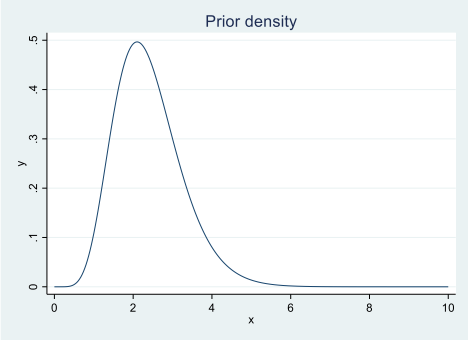
Figures

Figure 1

Book-to-market



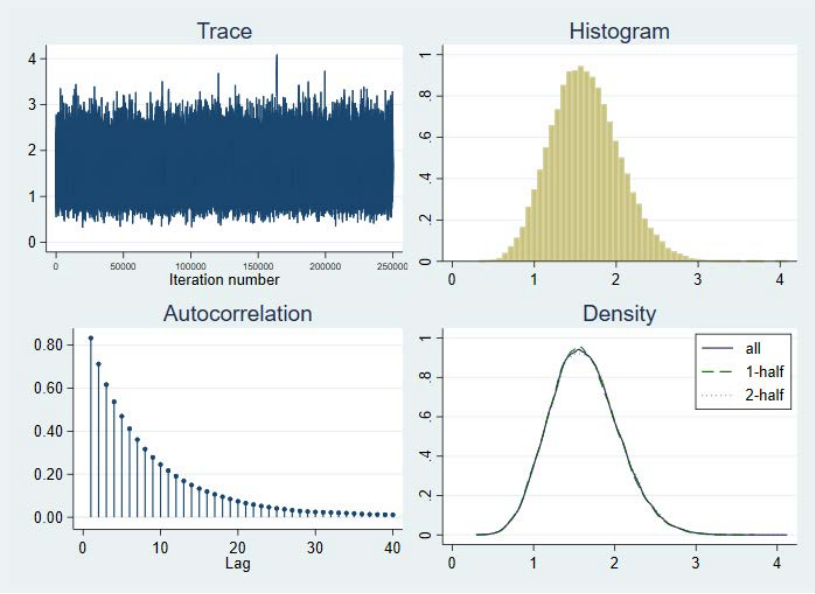
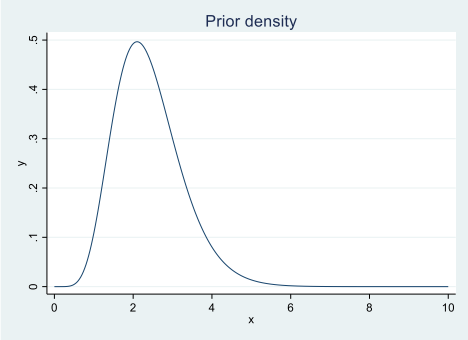


Figure 2

Earning to price



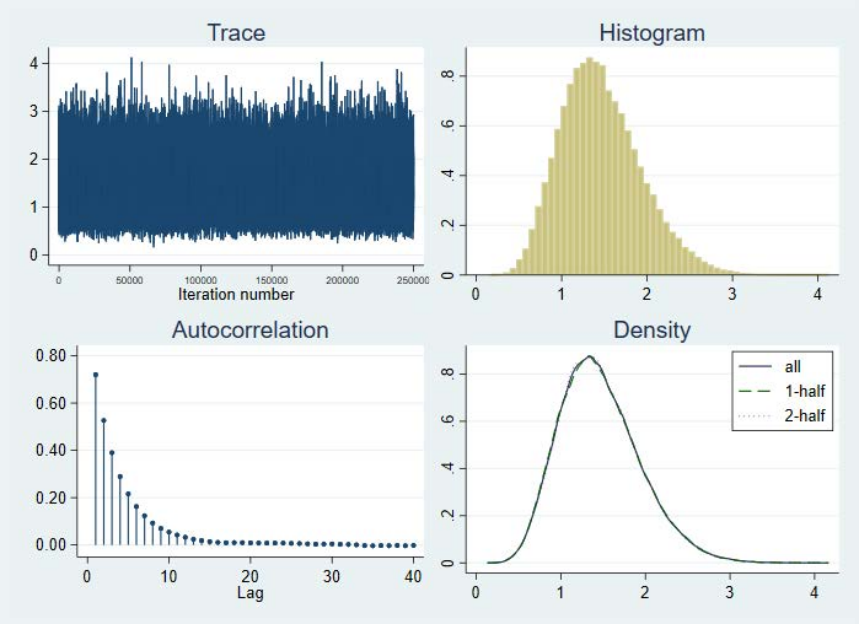


Figure 3

Interest rate spread

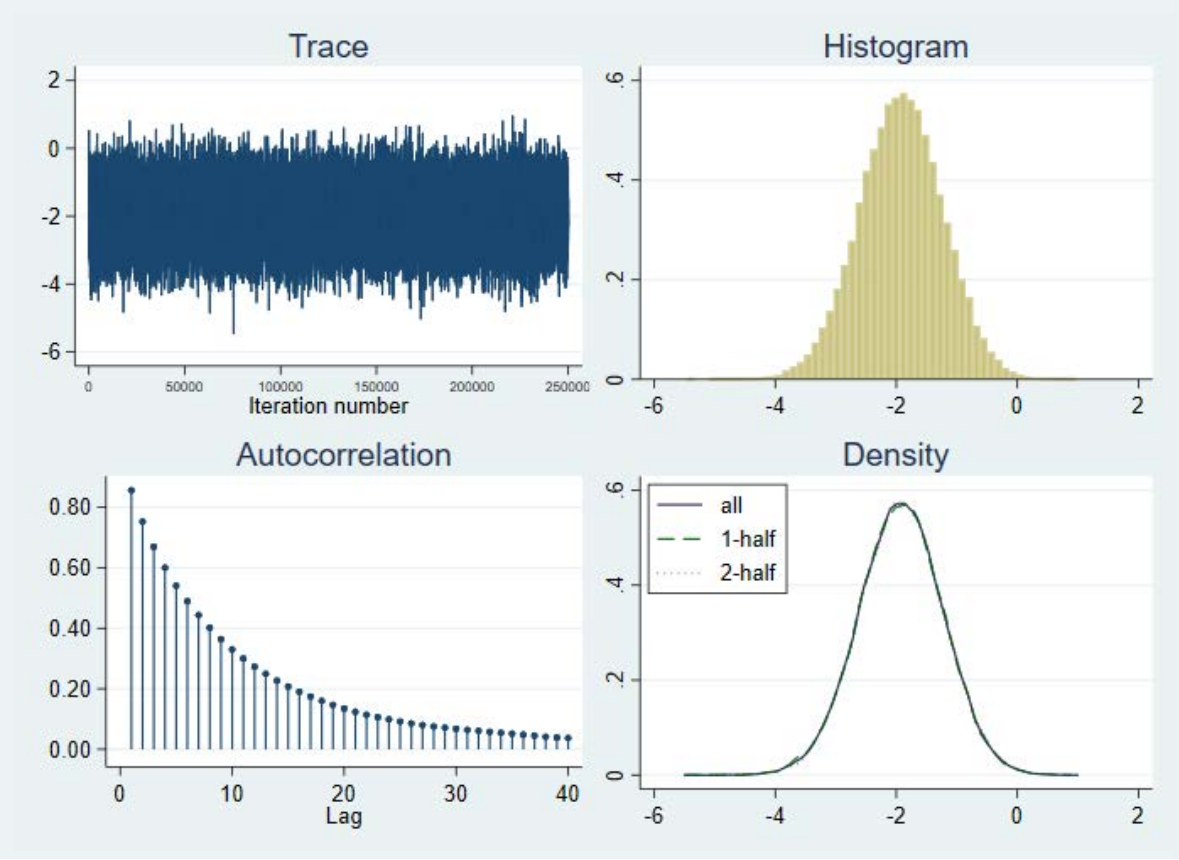


Figure 4

Model residuals variance, σ_0^2

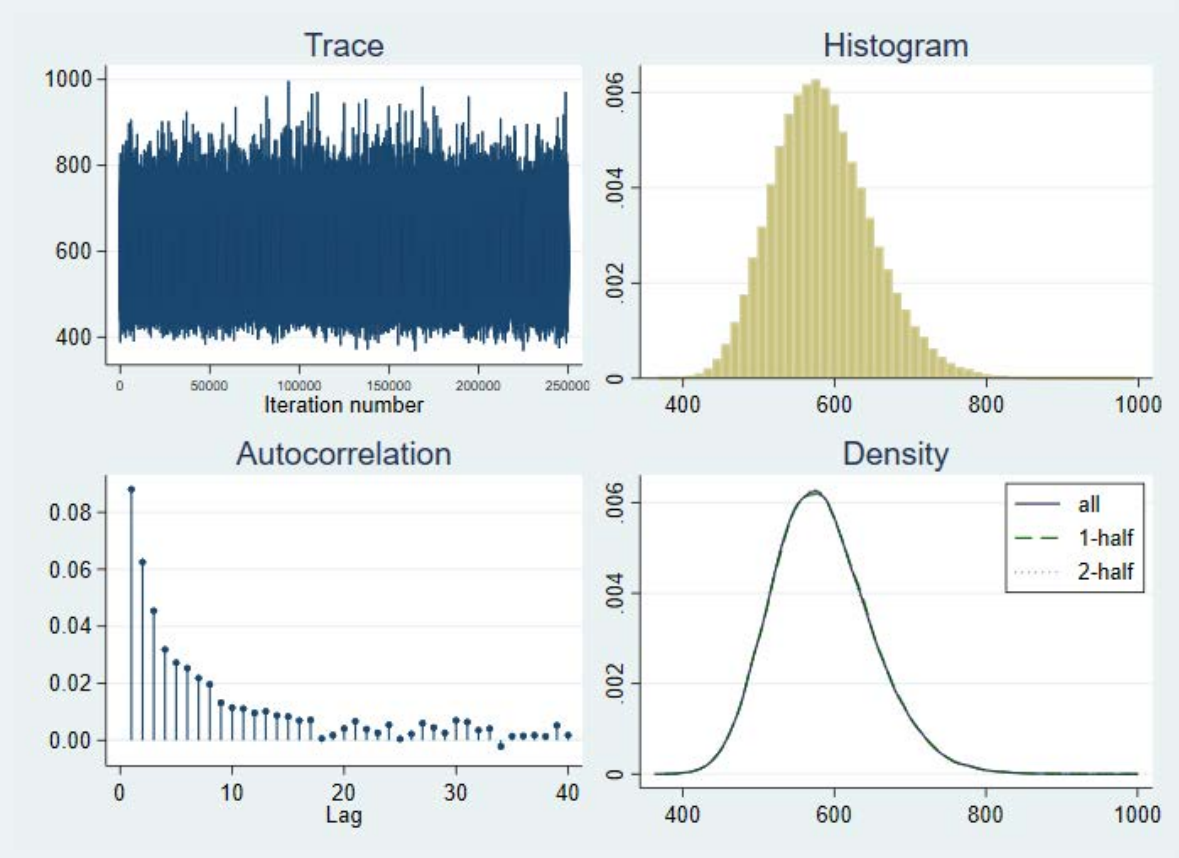


Figure 5

Firm's residuals variance, σ_{id}^2

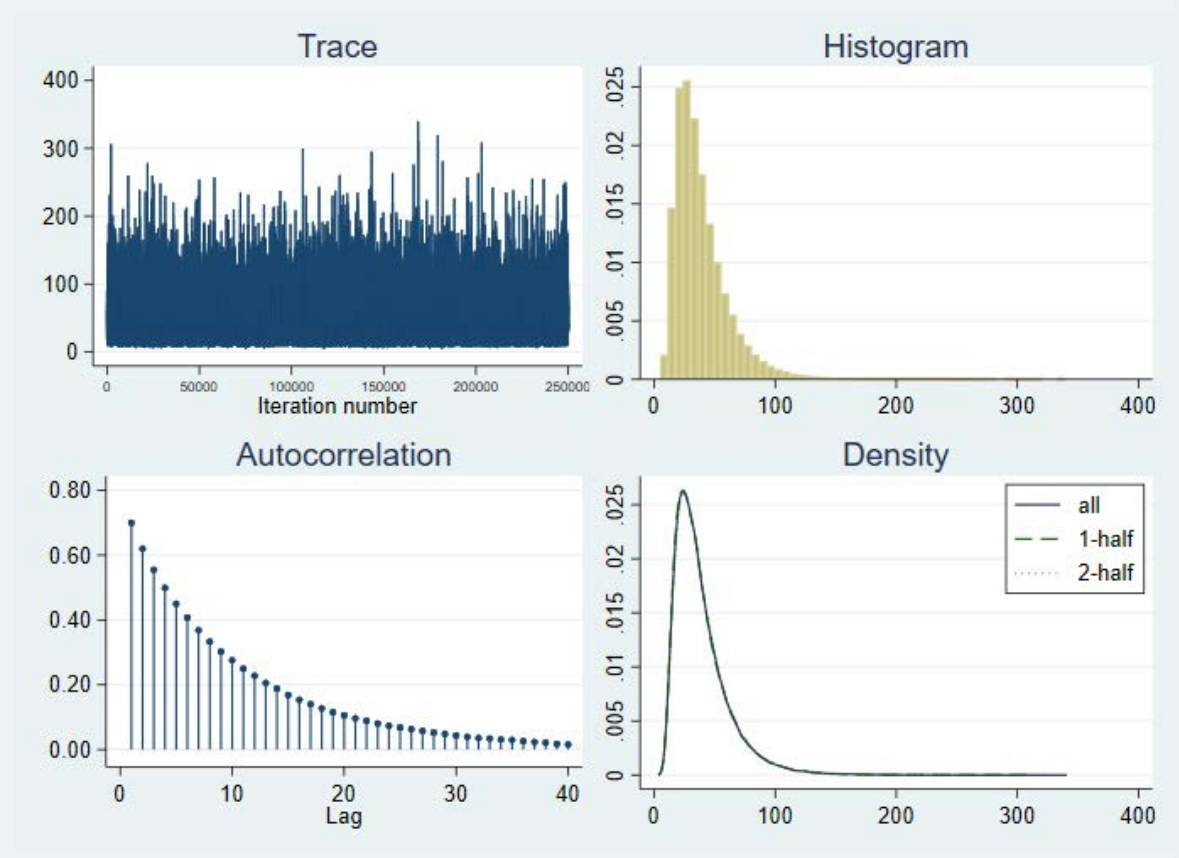


Figure 6

Constant

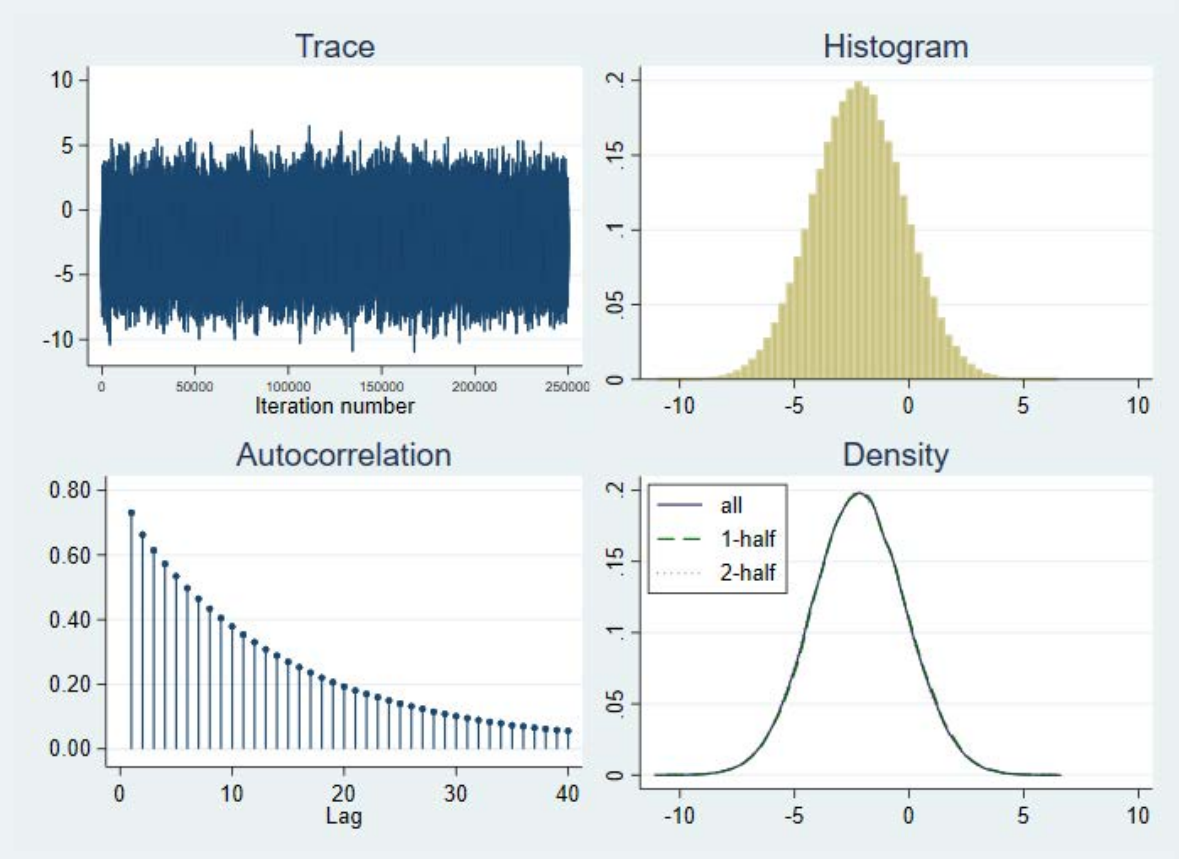
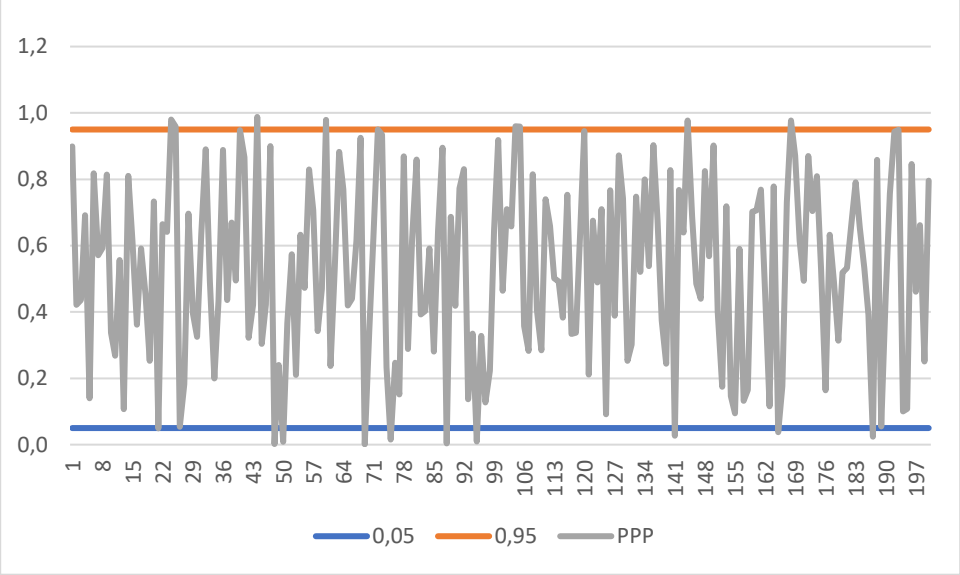


Figure 7

Estimated posterior predictive p-value. Probability that the mean of the simulation of the excess returns is greater or equal to the observed excess returns



Tables

Table 1

Data and descriptive statistics

Excess returns (%)				
Year	Mean	Std. dvn.	Min	Max
2015	-19.0	21.1	-54.2	22.0
2016	8.6	14.9	-15.5	53.7
2017	6.1	20.4	-28.6	51.7
2018	-22.5	19.8	-68.1	17.4
2019	26.1	23.3	-2.1	87.8
2020	-5.0	16.8	-29.9	29.0
2021	-2.8	21.7	-40.2	68.4
2022	-11.6	36.0	-50.1	73.1
2015-22	-2.5	26.7	-68.1	87.8
Book/market				
Year				
2014	1.0	1.1	0.3	5.4
2015	1.2	0.9	0.4	5.1
2016	1.2	1.2	0.3	6.6
2017	1.2	1.4	0.3	7.2
2018	1.7	1.7	0.5	8.3
2019	1.4	1.3	0.4	6.4
2020	1.5	1.5	0.4	7.7
2021	1.7	1.5	0.4	7.3
2014-21	1.4	1.3	0.3	8.3
Earnings/price (%)				
Year				
2014	5.5	7.0	23.9	-9.5
2015	3.1	9.6	20.1	-33.5
2016	8.4	9.6	30.0	-13.8
2017	6.8	10.4	28.0	-25.1
2018	8.7	6.9	26.2	-3.8
2019	12.1	8.6	32.5	0.7
2020	6.4	9.8	34.0	-16.3
2021	12.5	11.1	53.6	-4.5
2014-21	7.9	9.6	53.6	-33.5
Spread (2014-2021) (%)				
Mean	3.7			
Std. dvn.	1.7			
Min	1.7			
Max	7.1			

Excess returns is the anual real return in year t inclusive of prices and dividends with respect to the risk free rate CDT's.
Book to market is the ratio of the book to market per share at the end of year t.
Earnings/price is the earnings in year t with respect to prices at the end of year t.
Spread is the diference between the policy interest rates in Colombia and United States in year t.

Table 2

Prior distribution for the parameters of the models

Parameter	*Distribution*	*Mean*	*Standard deviation*
β	Gamma	2.4	0.85
γ	Gamma	2.4	0.85
θ	Normal	0	10
σ_0^2	InvGamma	50	∞
σ_{id}^2	InvGamma	75	∞

Table 3

Posterior distribution for the parameters

Parameter	*Mean*	*Standard deviation*	*95% confidence interval*
β	1.458	0.399	(0.746 2.304)
γ	1.537	0.495	(0.708 2.634)
θ	-1.946	0.715	(-3.362, -0.560)
σ_0^2	695.164	71.297	(568.964, 847.991)
σ_{id}^2	29.824	16.018	(10.789, 70.959)
α	-2.434	2.017	(-6.376, 1.546)

Bayesian normal regression Metropolis-Hastings and Gibbs sampling	
MCMC iterations	375,000
Burn-in	125,000
MCMC sample size	250,000
Number of obs	200
Acceptance rate	0.483
Efficiency: min =	0.0362
avg =	0 .1173
max =	0 .7421

Table 4

Posterior predicted p-values for the mean of the simulated residuals

T	Mean	Std.Dev	E(T_obs)	P(T>=T_obs
Mean	-0.012	1.816	-1.670	0.752

Note. (P>=T_obs) close to 0 or 1 indicates lack of fit

Table 5

Posterior predicted p-values for the variance of the simulated residuals

T	Mean	Std.Dev	E(T_obs)	P(T>=T_obs
Variance	581.14	90.334	585.12	0.455

Note. (P>=T_obs) close to 0 or 1 indicates lack of fit

Table 6

Posterior predicted p-values for skewness and kurtosis of the simulated residuals

T	Mean	Std.Dev	E(T_obs)	P(T>=T_obs
Skewness	-0.002	0.268	0.238	0.238
Kurtosis	-0.029	0.339	0.449	0.100

Note. (P>=T_obs) close to 0 or 1 indicates lack of fit

Table 7

Posterior distribution for the parameters using a 2.3 standard deviation for priors of the parameters β and γ

Parameter	*Mean*	*Standard deviation*	*95% confidence interval*
β	1.056	0.563	(0.111, 2.243)
γ	0.795	0.636	(0.033, 2.361)
θ	-1.362	0.768	(-2.874, 0.130)
σ_0^2	690.463	70.419	(565.762, 841.931)
σ_{id}^2	27.440	14.115	(10.319, 63.528)
α	-1.988	2.067	(-6.037, 2.074)

Bayesian normal regression

Metropolis-Hastings and Gibbs sampling

MCMC iterations	375,000
Burn-in	125,000
MCMC sample size	250,000
Number of obs	200
Acceptance rate	0.5043
Efficiency: min =	0.028
avg =	0.107
max =	0.811

Table 8

Posterior distribution for the parameters for the firms with lower book-to-market ratio

Parameter	*Mean*	*Standard deviation*	*95% confidence interval*
β	1.331	0.404	(0.632, 2.213)
γ	1.644	0.555	(0.728, 2.883)
θ	-1.598	0.955	(-3.483, 0.255)
σ_0^2	685.211	102.919	(512.372, 915.432)
σ_{id}^2	38.949	27.408	(11.802, 109.159)
α	-1.712	2.128	(-5.879, 2.454)

Bayesian normal regression

Metropolis-Hastings and Gibbs sampling

MCMC iterations	375,000
Burn-in	125,000
MCMC sample size	250,000
Number of obs	96
Acceptance rate	0.5342
Efficiency: min =	0.064
avg =	0.138
max =	0.641

Table 9

Posterior distribution for the parameters for the firms with higher book-to-market ratio

Parameter	*Mean*	*Standard deviation*	*95% confidence interval*
β	2.136	0.622	(1.049, 3.465)
γ	1.941	0.631	(0.884, 3.327)
θ	-3.153	0.955	(-5.041, -1.292)
σ_0^2	698.531	99.261	(529.534, 917.645)
σ_{id}^2	38.745	25.499	(11.893, 105.482)
α	-1.297	2.102	(-5.408, 2.838)

Bayesian normal regression

Metropolis-Hastings and Gibbs sampling

MCMC iterations	375,000
Burn-in	125,000
MCMC sample size	250,000
Number of obs	104
Acceptance rate	0.5362
Efficiency: min =	0.066
avg =	0.156
max =	0.780

Table 10

Posterior distribution for the parameters for book-to-market and earnings-to-price assuming a normal prior distribution with mean 2 and standard deviation of 2.3[8]

Parameter	*Mean*	*Standard deviation*	*95% confidence interval*
β	1.407	0.599	(0.194, 2.569)
γ	1.412	0.958	(-0.506, 3.277)

Table 11

Probability that the parameters for book-to-market and earnings-to-price are between 0 and 2.5 and between 0 and 3.3, respectively, when a normal prior distribution with mean 2 and standard deviation of 2.3 is assumed

		Mean	Std. dev.	MCSE
β	Prob	0.956	0.205	0.004
γ	Prob	0.903	0.295	0.004

[8] The estimations are made separately for earnings-to-price ratio and book-to-market ratio including in each the spread.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