Borradores de ECONOMÍA

Characterizing and Communicating the Balance of Risks of Macroeconomic Forecasts: A Predictive Density Approach for Colombia

By: Juan C. Méndez-Vizcaíno
Alexander Guarin
César Anzola-Bravo
Anderson Grajales-Olarte

No. 1178
2021

BANCO DE LA REPÚBLICA
COLOMBIA

Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá

Characterizing and Communicating the Balance of Risks of Macroeconomic Forecasts: A Predictive Density Approach for Colombia

Juan C. Méndez-Vizcaíno
jmendevi@banrep.gov.co

César Anzola-Bravo
canzolbr@banrep.gov.co

Alexander Guarin
aguarilo@banrep.gov.co

Anderson Grajales-Olarte*
agrajaol@banrep.gov.co

The opinions, statements, findings and interpretations expressed in this paper are responsibility of the authors and do not represent the position of Banco de la República or its Board of Directors. As usual, all errors and omissions in this work are our responsibility.

Abstract

Since July 2021, Banco de la República strengthened its forecasting process and communication instruments, by involving predictive densities in the projections of its models, PATACON and 4GM. This paper presents the main theoretical and empirical elements of the predictive density approach for macroeconomic forecasting. This model-based methodology allows to characterize the balance of risks of the economy, and to quantify their effects through a joint probability distribution of forecasts. We estimate this distribution based on the simulation of DSGE models, preserving the general equilibrium relationships and their macroeconomic consistency. We also illustrate the technical criteria used to represent prospective factors of risk through the probability distributions of shocks.

Keywords: Macroeconomic forecasts, balance of risks, uncertainty, Bayesian forecasting, monetary policy models.

JEL codes: C11, C53, E17, E52.

*We would like to thank Hernando Vargas, Juan Ospina, Franz Hamann, Carlos Huertas, Julián Perez, Adolfo Cobo, Aarón Garavito, Vicente Romero and the other members of the Technical Staff for their valuable comments and feedback throughout the development of this project. The authors are specialized professional, specialized professional, section head and junior researcher of the Model Development Section in the Macroeconomic Modeling Department, respectively.

Caracterización y Comunicación del Balance de Riesgos de los Pronósticos Macroeconómicos: Un Enfoque de Densidad Predictiva para Colombia

Juan C. Méndez-Vizcaíno
jmendevi@banrep.gov.co

César Anzola-Bravo
canzolbr@banrep.gov.co

Alexander Guarin
aguarilo@banrep.gov.co

Anderson Grajales-Olarte
agrajaol@banrep.gov.co

Las opiniones, planteamientos, hallazgos, interpretaciones y posibles errores y omisiones contenidos en este documento son responsabilidad exclusiva de los autores y no representan la posición del Banco de la República ni de su Junta Directiva.

Resumen:

Desde julio de 2021, el Banco de la República fortaleció su proceso de pronóstico y sus instrumentos de comunicación al incorporar densidades predictivas en las proyecciones de sus modelos, PATACON y 4GM. Este artículo presenta los principales elementos teóricos y empíricos del enfoque de densidad predictiva para los pronósticos macroeconómicos. Esta metodología basada en modelos permite caracterizar el balance de riesgos de la economía y cuantificar sus efectos mediante una distribución de probabilidad conjunta de los pronósticos. Esta distribución se estima mediante la simulación de los modelos DSGE, preservando las relaciones de equilibrio general y la coherencia macroeconómica. También se ilustran los criterios técnicos utilizados para representar los factores de riesgo prospectivos a través de las distribuciones de probabilidad de los choques.

Palabras clave: Pronósticos macroeconómicos, balance de riesgos, incertidumbre, pronósticos bayesianos, modelos de política monetaria.

Clasificación JEL: C11, C53, E17, E52.

1 Introduction

Generating macroeconomic forecasts and assessing their consistency are crucial tasks in the monetary policy agenda of central banks, especially in economies pursuing an inflation targeting strategy (Svensson, 2010). Therefore, Banco de la República (BanRep) continuously forecasts the main macroeconomic variables for an eight-quarter policy horizon, in an exercise that integrates the technical staff, theoretically grounded modeling tools, and the critical evaluation of empirical aspects of the economy (González et al., 2019, 2020). The results of these forecasting exercises contribute to the monetary policy decisions adopted by BanRep's Board of Directors and are published quarterly in the Monetary Policy Report.

These forecasts are conditional on the assessment of the current and future state of the economy, including projections of external variables and the endogenous response of monetary policy that brings inflation to its target level and stabilizes output and employment. However, since monetary policy operates in an environment of uncertainty (Friedman, 1972; Batini & Nelson, 2001; Goodhart, 2001), the assessment of risk factors on macroeconomic forecasts becomes an essential element in the policy decision-making process. This assessment is formalized through a balance of risks.[1]

The balance of risks implies a prospective assessment of the shocks that the economy could face over the forecast horizon and that affect the expected dynamics of macroeconomic variables. Assessing these risks is challenging because it requires characterizing the origin of the shocks, determining whether they are permanent or transitory (their degree of persistence), as well as communicating these elements to the public.

The literature highlights mainly three methods used by central banks to characterize and communicate their prospective balance of risks: qualitative assessment, symmetric and asymmetric fan-charts and predictive densities (Knüppel & Schultefrankenfeld, 2012; Bundesbank, 2010).[2] The latter two strategies follow a quantitative approach. The qualitative assessment presents a detailed description of what the state of the economy might be in the future and its likely risks using a narrative approach, without providing an explicit quantification of the different sources of uncertainty or their magnitude.

Fan-charts characterize the balance of risks in macroeconomic forecasts using a probability distribution that is generated off-model and superimposed on the central forecast. The estimation of fan-charts follows the classical approach of confidence intervals based on the historical volatility of forecast errors and an assumption about the density function. The symmetric fan-chart (Blix & Sellin, 1999) assumes a normal distribution to represent balanced risks, while the asymmetric fan-chart (Britton et al., 1998) considers a two-piece normal distribution to characterize the skewed balance of risks. Symmetric fan-charts follow a statistical approach, i.e., they do not rely on the economic structure of a macroeconomic model or the general equilibrium relationships on which the central forecast is built. Asymmetric fan-charts are based on marginal probability distributions,

[1] In this paper we treat as indistinct the terms uncertainty and risk as is common in the DSGE literature. However, we are aware of the difference between these two concepts proposed by Knight (1921).

[2] For example, the central bank of France, the Sveriges Riskbank and the European Central Bank have adopted symmetric fan-charts for their communication. Bank of England and the central banks of Hungary, Brazil and Peru have preferred asymmetric fan-charts. Qualitative assessment is explicitly used by the central bank of Japan and the Board of Governors of the U.S. Federal Reserve, and complements the analysis of central banks using quantitative tools. Risk characterization and its communication with predictive densities have been considered by the central banks of Norway and Canada, Bank of Israel (2019), and the technical staff of the New York Federal Reserve (Del Negro et al., 2013) and the Reserve Bank of New Zealand (Benes et al., 2009).

which do not guarantee macroeconomic consistency between the densities of different variables.

BanRep has previously adopted the methods described above to characterize and communicate uncertainty about its risks. Until 2018, in its Inflation Report, BanRep made explicit the risk factors on the GDP growth rate and headline inflation using asymmetric fan-charts. In 2019, with its new forecasting process and its new Monetary Policy Report, BanRep adopted symmetric fan-charts that reflected risks on projections through the volatility of historical forecast errors (González et al., 2019). In 2020, as a consequence of the COVID-19 pandemic and the difficulty in reflecting the uncertainty caused by this shock, BanRep suspended the publication of fan-charts and adopted qualitative risk assessment.

Since July 2021, BanRep's Technical Staff (TS) introduced the *Predictive Density* (PD) approach to characterize and communicate its assessment of risk factors that could affect its macroeconomic forecasts. This is a model-based approach that aims to characterize, quantify and communicate the prospective balance of risks using a joint probability distribution over the forecast of all variables, preserving general equilibrium relationships, and therefore, macroeconomic consistency (Del Negro & Schorfheide, 2013; Del Negro et al., 2016). The PD approach considers two elements to simulate this distribution. First, the transmission mechanisms implicit in the economic structure of the model. Second, the exogenous distributions of the shocks that reflect the qualitative analysis of the risks considered by the TS.

BanRep adopted the PD methodology on the forecasts generated by PATACON (González et al., 2011) and 4GM (González et al., 2020), its two general equilibrium models for macroeconomic forecasting, and combined the results to obtain a unified PD. This approach illustrates the sensitivity of macroeconomic forecasts to risk factors, quantifies their importance in probabilistic terms, and helps to define a more robust monetary policy recommendation in uncertain environments.

In this paper we outline the main theoretical aspects of the PD approach, describe its application using the macroeconomic models, PATACON and 4GM, and illustrate how this method is implemented within the TS forecasting process. We also illustrate the technical criteria adopted to characterize the mode, variance and skewness of the probability distributions of the shocks, and hence, to reflect the prospective balance of risks.

This paper is divided into five sections. The first is this introduction. In Section 2 we present the theoretical aspects of the PD methodology. In Section 3 we characterize the probability distribution of shocks. In Section 4 we illustrate how to combine the distributions of both models to produce a unified PD. Finally, we offer some concluding remarks.

2 Predictive Densities on Macroeconomic Forecasts

Consider the reduced-form solution of a rational expectations model given by the state-space representation

$$Y_t = Z(\theta)S_t + H(\theta)\nu_t, \tag{1}$$

$$S_t = T(\theta)S_{t-1} + R(\theta)\epsilon_t, \tag{2}$$

where equations (1) and (2) are the measurement and transition equations, respectively.[3] The first equation links the vector of observed variables Y_t to the set of state variables S_t through the matrix $Z(\theta)$, where θ is the vector of structural parameters. This equation also accounts for the vector of measurement errors, ν_t, which is mapped to Y_t through the matrix $H(\theta)$.[4]

Equation (2) defines the transition dynamics for S_t from the state variables in the previous period S_{t-1} and the structural shocks or innovations ϵ_t via the matrices $T(\theta)$ and $R(\theta)$, respectively. The measurement and transition equations are linear, while the errors ν_t and ϵ_t are assumed to follow a normal distribution.

The system of equations (1) - (2) generates forecasts whose dynamics are explained by the economic structure of the model, its transmission channels and the structural shocks faced by the economy (Smets & Wouters, 2003, 2007; Christiano et al., 2003). In the framework of Bayesian statistics, the analysis is based on the probability distribution of forecasts, also called PD. This distribution reflects the probability assigned to the possible future realizations of a variable conditional on the observed data (Geweke & Whiteman, 2006).

Following Del Negro & Schorfheide (2013), for the model described by equations (1) and (2), the one-step ahead predictive density $P(Y_{T+1}|Y_{1:T})$ is given by

$$P(Y_{T+1}|Y_{1:T}) = \int_{\theta} \left[\int P(Y_{T+1}|S_{T+1}, \theta) P(S_{T+1}, S_T|\theta, Y_{1:T}) d(S_{T+1}, S_T) \right] P(\theta|Y_{1:T}) d\theta, \tag{3}$$

where $P(S_{T+1}, S_T|\theta, Y_{1:T}) = P(S_{T+1}|S_T, \theta, Y_{1:T})P(S_T|\theta, Y_{1:T})$ is the joint conditional distribution of future and current states, $P(\theta|Y_{1:T})$ is the posterior distribution of the parameters, and $P(Y_{T+1}|S_{T+1}, \theta)$ is the predictive likelihood.

Equation (3) captures three sources of uncertainty (Del Negro & Schorfheide, 2013).[5] First, the risk implicit in the estimates of the unobserved state variables within the model (e.g., the output gap and inflation expectations), which is represented by the conditional density $P(S_T|\theta, Y_{1:T})$. Second, uncertainty about future realizations of state variables, denoted by the density $P(S_{T+1}|S_T, \theta, Y_{1:T})$. The risk comes from the structural shocks ϵ_t, whose probability distribution reflects the qualitative assessment of the exogenous factors that could affect the economy in the future. Third, the uncertainty associated with the estimated parameters and their impact on the dynamics of endogenous variables and the law of motion of exogenous processes. This risk is expressed by the posterior distribution $P(\theta|Y_{1:T})$ (Geweke & Whiteman, 2006).

As usual in practice, we adopt the *practitioners shortcut* illustrated by Del Negro & Schorfheide (2013) to remove parameter uncertainty in the PD, and focus the analysis on the risks arising from latent variables and future shocks. This shortcut replaces the posterior parameter distribution $P(\theta|Y_{1:T})$ with a point estimator, such as the posterior mode θ^*. Although this strategy underestimates the uncertainty, the PDs computed with this shortcut and those provided by the full Bayesian approach should be similar, since the posterior distribution is concentrated around its mode.

Under the *practitioners shortcut*, the one-step ahead predictive density $P(Y_{T+1}|Y_{1:T})$ can be

[3]This particular reduced state-space form is obtained by using the generalized Schur decomposition to integrate future expectations (Klein, 2000).

[4]These errors are introduced to recognize that variables may be measured with noise, and to capture the variance of the observed variables, improving the explanatory power of the model.

[5]Note that measurement errors, ν_t, are zero in forecast.

written as

$$P(Y_{T+1}|Y_{1:T}) = \int P(Y_{T+1}|S_{T+1}, \theta^*)P(S_{T+1}, S_T|\theta^*, Y_{1:T})d(S_{T+1}, S_T), \tag{4}$$

where $P(S_{T+1}, S_T|\theta^*, Y_{1:T}) = P(S_{T+1}|S_T, \theta^*, Y_{1:T})P(S_T|\theta^*, Y_{1:T})$.

We can simulate the predicted density in equation (4) using Monte Carlo methods. Algorithm 1 describes the steps necessary to obtain this simulation, while Figure 1 shows them schematically. We first apply the Kalman filter (KF) to the system of equations (1)-(2) conditioning on the mode of the posterior distribution of the parameters, θ^*, and the data $Y_{1:T}$. Kalman filtering takes time series of noisy observations $Y_{1:T}$ and produces minimum mean-square error estimates of both the structural shocks $\epsilon_{1:T}$ and the latent state variables $S_{1:T}$, whose conditional density at time T follows $P(S_T|\theta^*, Y_{1:T})$. Next, we randomly draw M sequences of structural shock combinations, $\epsilon_{T+1:T+H}$, from specific probability distributions for periods $T+1 : T+H$, with H being the forecast horizon. In Section 3 we will describe in detail the characterization of these distributions, which are fundamental to represent the risks.

Taking the data, Y_T, and the mean of the estimates of the latent state variables in S_T at time T as a starting point, for each draw $m = 1, \ldots, M$ and forecast horizon $T+1 : T+H$, we predict the equations (1)-(2) conditional on the simulated sequence of shocks $\epsilon^n_{T+1:T+H}$. This process generates a set of M forecast trajectories of $S_{T+1:T+H}$ and $Y_{T+1:T+H}$. Finally, from these M trajectories we use a Kernel estimator (KE) to estimate the density functions for each period over the forecast horizon $h = 1, \ldots, H$, $P(S_{T+h}|S_{T+h-1}, \theta^*, Y_{1:T+h-1})$ and $P(Y_{T+h}|Y_{1:T+h-1})$, respectively. The latter corresponds to the PD defined in equation (4).

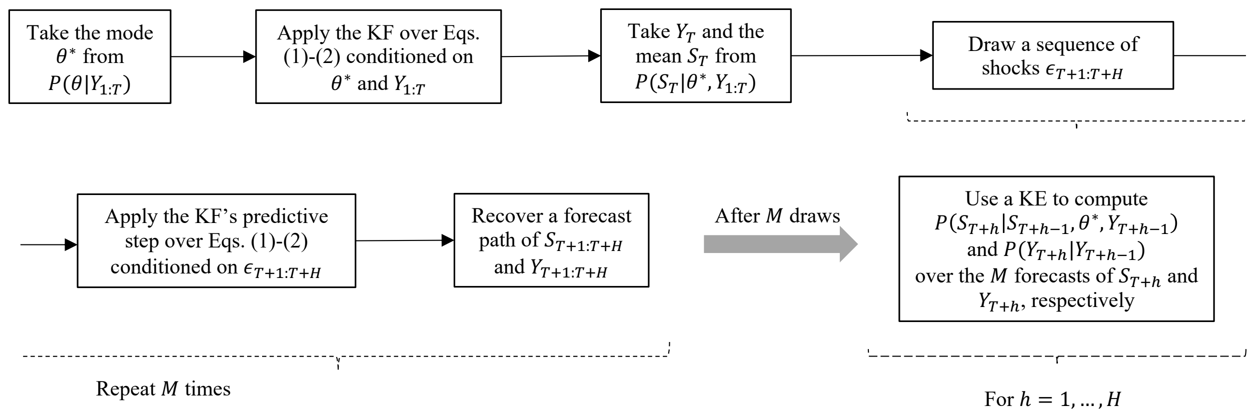
Algorithm 1 Predictive Density Estimation

1. Take the mode of the posterior distribution of the parameters, θ^*.
2. Apply the KF over equations (1)-(2) conditioning on θ^* and data $Y_{1:T}$.
3. Recover the mean estimates of $S_{1:T}$ from the density $P(S_t|\theta^*, Y_{1:t})$ for $t = 1, ..., T$.
4. Take Y_T and S_T as a starting point for forecasting.
5. For each draw $m = 1, \ldots, M$, predict equations (1)-(2) conditional on $\epsilon^n_{T+1:T+H}$.
6. Recover the M forecast paths of $S_{T+1:T+H}$ and $Y_{T+1:T+H}$.
7. For $h = 1, \ldots, H$, compute $P(S_{T+h}|S_{T+h-1}, \theta^*, Y_{1:T+h-1})$ over M forecasts of S_{T+h} using a KE.
8. For $h = 1, \ldots, H$, use a KE to compute $P(Y_{T+h}|Y_{1:T+h-1})$ over M forecasts of Y_{T+h}.

3 Risk characterization

The balance of risks presents the assessment of the different sources of uncertainty that could affect macroeconomic forecasts and the monetary policy recommendation. Its formulation involves the prospective identification of the factors that could affect the economy in the future and, therefore, the expected dynamics of macroeconomic variables.

Figure 1: Predictive Density Estimation



We quantitatively represent the risk factors through the probability distributions of the vector if structural shocks, which we initially assume to follow a normal distribution with mean vector $\mu = \mathbf{0}$ and variance matrix Σ, $\epsilon_{T+1:T+H} \sim N(\mu, \Sigma)$. As illustrated above, for the filtering process up to time T we apply a standard linear Kalman filter to the system of equations (1)-(2).

For forecasting, we follow a conditioning strategy that is different from the Kalman filtering. Under this strategy, we solve equations (1)-(2) to find the values of specific shocks to condition the measurement variables to certain values over the forecasting horizon. This approach allows considering shocks that follow distributions other than normal. Further, it allows quantifying the marginal effect of each source of uncertainty on the PD estimate.

This section discuss the technical criteria adopted by the TS to characterize the balance of risks through changes in the mode, variance and skewness of the distributions of shocks used in the PATACON and 4GM models.

3.1 Mode

Under the assumption of shocks $\epsilon_{T+1:T+H} \sim N(\mathbf{0}, \Sigma)$, the system of equations (1)-(2) generates unconditional forecasts of all variables, which return to their steady-state values according to the dilution of the filtered shocks up to time T since there are no additional innovations. However, for policy decision making it is crucial to have forecasts conditioned on exogenous information, such as GDP nowcast, short-term forecast of inflation, and assumed paths for foreign variables (e.g., oil price and FED interest rate).[6]

Over the forecast horizon, we include exogenous conditioning in macroeconomic models through structural shocks, which are drawn from distributions whose mode $\eta_{T+1:T+H}$ is set to a non-zero value to reproduce the central forecast path of these variables. We adopt the *News* strategy (Del Negro & Schorfheide, 2013) to include external information one-period-ahead, Z_{T+1}, which provides

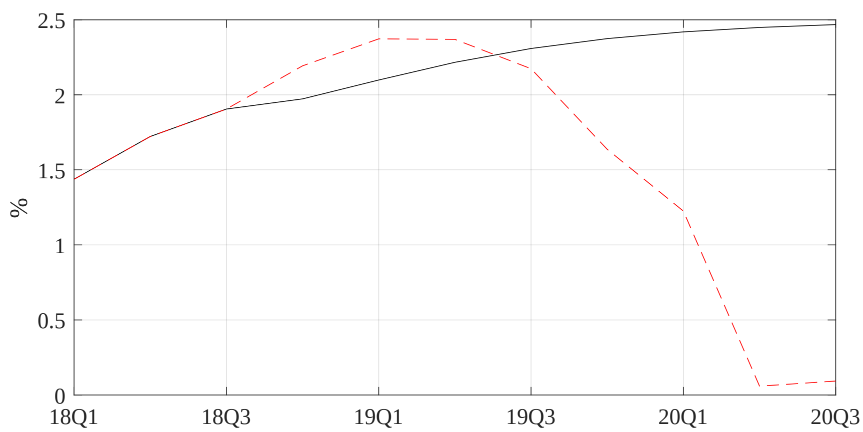
[6]Nowcasting and time series forecasting models often outperform short-term projections generated by structural DSGE models. However, the latter usually produce better medium- and long-term forecasts than the former (Smets & Wouters, 2007; Adolfson et al., 2007).

additional knowledge of the state variables, S_{T+1}, and the forecasts of the observed variables, Y_{T+1}.[7]

By conditioning on the exogenous data, Z_{T+1}, we can replace the conditional distribution of future states in equation (4) by $P(S_{T+1}|S_T, \theta^*, Y_{1:T}, Z_{T+1})$. To simulate this density function for $h = 1, \ldots, H$ steps ahead, we generate a sequence of innovations $\epsilon_{T+1:T+H} \sim N(\mu_{T+1:T+H}, \Sigma))$, $\mu \in \mathbb{R}$, such that the mean vector $\mu_{T+1:T+H} = \eta_{T+1:T+H}$ guarantees that $Y_{T+1:T+H} = Z_{T+1:T+H}$.[8]

As an illustration, Figure 2 shows a hypothetical conditioning for the future path of the U.S. FED interest rate.[9] This figure shows the conditioned path (red line) for this rate, and the path that would have been projected (black line) with the 4GM model. The assumption about the central path for the FED interest rate is the result of the analysis performed by TS on data from U.S. financial analysts' projections, traders' surveys, futures market operations, FED press releases, among others.

Figure 2: Conditioning of the FED Interest Rate



Note: In this example, the conditioned path (red dashed-line) corresponds to the observed value of the variable.

3.2 Variance

So far we have simulated innovations $\epsilon_{T+1:T+H}$ assuming a diagonal, time-invariant variance-covariance matrix Σ. However, to capture different levels of uncertainty over the policy horizon, we relax this assumption by allowing the variance of the distribution of shocks to be time-varying, $\Sigma_{T+1:T+H}$.

For $h = 1, \ldots, H$ steps ahead, we simulate structural shocks $\epsilon_{T+1:T+H} \sim N(\mu_{T+1:T+H}, \Sigma_{T+1:T+H})$ that follow a normal distribution with mean vector μ and variance matrix $\Sigma_{T+1:T+H}$. As in Section 3.1, $\mu_{T+1:T+H} = \eta_{T+1:T+H}$ guarantees that $Y_{T+1:T+H} = Z_{T+1:T+H}$.

In what follows we introduce three methodologies for defining the volatility of shocks: external information sources, shocks estimated within models, and the Markov switching approach.

External Information Sources

[7]This information set is orthogonal to the measurement error ν_t.

[8]The mean and mode are equal in symmetric distributions.

[9]The FED interest rate is used in the macroeconomic models, PATACON and 4GM, as a proxy for the relevant foreign interest rate for Colombian.

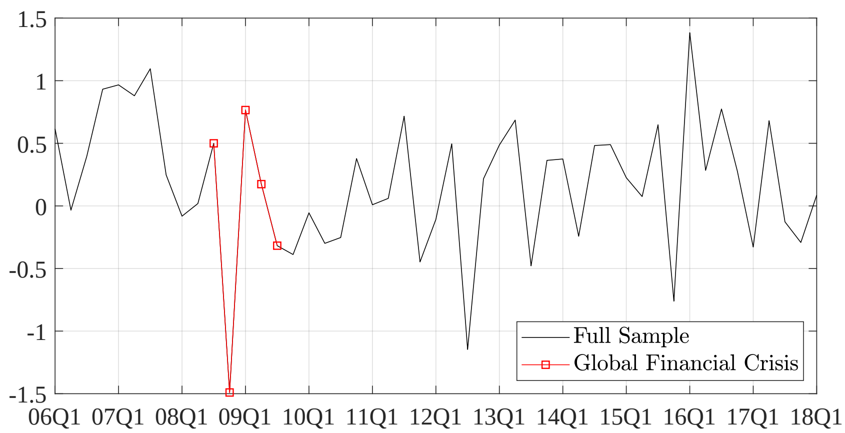
In the first methodology, we estimate a time-varying variance using information from external sources (e.g., financial market projections or Bloomberg surveys) to illustrate the different levels of uncertainty in the macroeconomic forecasts over the policy horizon. For instance, oil prices are continuously monitored by a variety of market agents, whose projections are captured through surveys. This information, together with the TS analysis, allows us to construct a distribution that reflects diverse views about the future dynamics of this variable. The estimation of the time-varying variance from these data is used to simulate the distributions of the structural shocks.

From a set of projections of an exogenous variable, for each period $t = \{T+1, \ldots, T+H\}$ we can fit a density function using a KE that captures a specific probability mass. In particular, we estimate a density with 90% probability, compute the 5% and 95% percentiles, and extract the implied volatility for each time t.

Estimated Shocks within Models

In the second approach, we caculate the time-varying variance from time series of shocks estimated with the PATACON and 4GM models, using Kalman filtering. These estimates are based on the structure of each model and the observed variables (Durbin & Koopman, 2012). The variance of these shocks could be conditioned on specific spans of time, in order to reflect the uncertainty of past periods as a proxy for the risk of future events with similar characteristics.

Figure 3: Demand Shock



For example, Figure 3 presents the estimated demand shocks between 2006 and 2018 with the 4GM. These shocks capture exogenous changes in aggregate demand that are not explained by the dynamics of the output gap, its determinants and other exogenous factors within the model. This figure illustrates a strong negative shock (red line) in late 2008, which characterizes the downward demand pressures associated with the global financial crisis. The variance of these innovations in this particular time period could be used to simulate shocks, whose probability distribution represents a high level of risk associated with future episodes of economic crises or turbulent financial markets.

Markov Switching Model

We consider a third methodology that uses a Markov Switching (MS) model to capture different regimes for the variance of innovations. These regimes are implicit in historical data and reveal different levels of risk over time as a result of particular events.

Specifically, we adopt a two-regime MS model (Hamilton, 1994, 2005; Perlin, 2015), given by

$$y_t = \mu + \epsilon_{S_t}, \quad \epsilon_{S_t} \sim N(0, \sigma^2_{S_t}) \quad for \quad S_t = [Low, High] \tag{5}$$

where y_t is the explained variable, μ is the long-run mean, ϵ_{S_t} is a shock following a normal distribution with zero mean and variance $\sigma^2_{S_t}$. Over time, volatility moves between regimes S_t with transition probability matrix $P(S_{t+1}|S_t) = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, where p_{ii} (p_{ij}) is the probability that σ^2 stays in *(moves to)* regime i (j) at time $t+1$ given that the current regime is i at time t.

For each forecast period, h, we apply the MS model on historical forecast error data to estimate two regimes, low and high volatility, as well as their expected duration. In addition, these estimates are used to derive a conditional regime, which is an average of the low and high volatility states weighted by the elapsed time in each. The estimated volatility regimes are used to simulate a probability distribution of shocks that illustrates the qualitative assessment of risks over the forecast horizon. Algorithm 2 describes the steps to implement this empirical strategy.

Algorithm 2 Simulation of a Distribution of Shocks based on the MS Model

For each step $h = \{1, ..., H\}$:

1. Build a time series y_t^h, with the h-periods-ahead absolute forecasting errors for variable y_t.
2. Estimate the MS model stated by Eq.(5) on data y_t^h, obtain μ, $\sigma^2_{S_t}$, and $P(S_{t+1}|S_t)$ for $S = [Low, High]$, and compute the variance for a conditional regime $\sigma^2_{\bar{S}_t}$. The latter is a weighted average of $\sigma^2_{S_t}$ for $S = [Low, High]$ according to the time spent on each state.
3. Simulate the probability distribution of shocks linked to y_t^h with the variances $\sigma^2_{\bar{S}_t}$ for high, low and conditional regimes, reflecting the future risk factors assessed by the TS.

For example, we use the MS model to characterize the variance of the food-basket Phillips curve shocks. The historical volatility of these innovations is high, and has changed over time as a consequence of, for example, the dynamics of international food prices and climatic factors such as *El Niño* phenomenon, among other risks. In this sense, we make considerations on these factors and their uncertainty over the policy horizon as these elements are important to simulate the probability distribution of shocks on the Phillips curve of this basket.

We apply the Algorithm 2 to the forecast errors of the food-basket inflation rate. Figure 4 illustrates the standard deviation estimated with the MS model for each forecasting step, h, and the low- (black line), conditional- (green line) and high-volatility (red line) regimes. The risk levels in the conditional- and low-state are similar, suggesting that periods of high volatility, associated with the *El Niño*, are less frequent and of relatively short duration.

As an example, Figure 5 shows, for the forecast period between the second quarter of 2018 and the first quarter of 2020, the PD for the food-basket inflation rate calculated using the probability distribution of the simulated shocks with the estimated variances for the low- (Panel A), conditional- (Panel B) and high- (Panel C) regimes.

Figure 4: Food-basket Inflation Shocks: Estimated Volatility

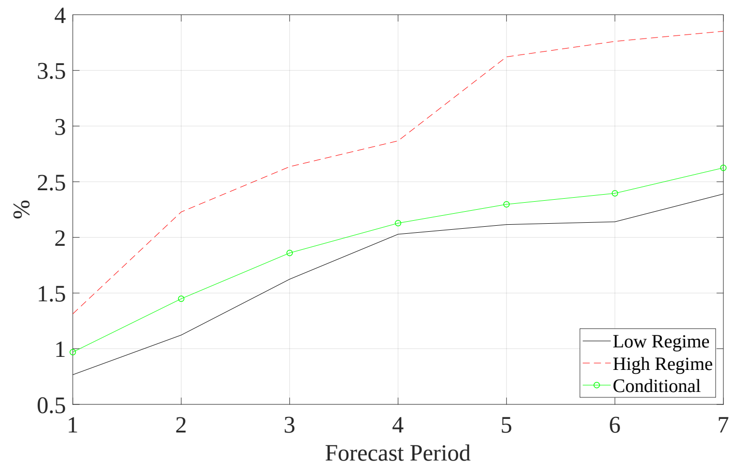
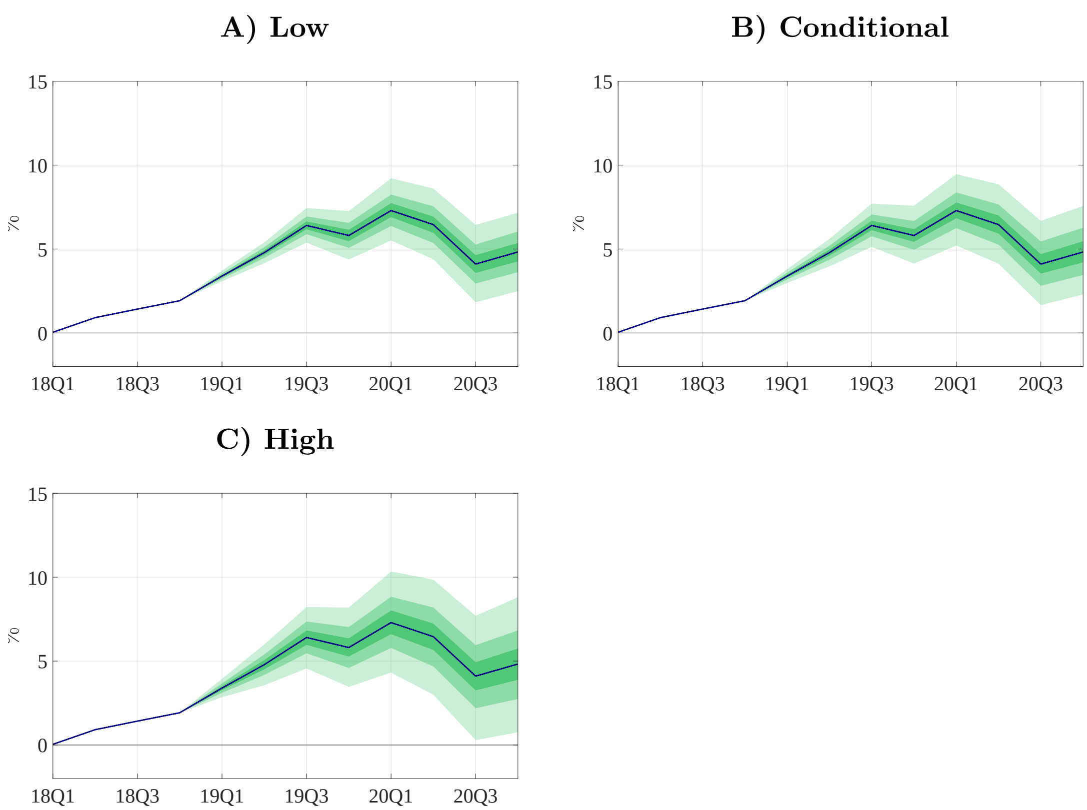


Figure 5: PD on the Food-basket Inflation Rate for Different Volatility Regimes



Note: In this example, the mode corresponds to the observed value of the variable. The shaded areas show the 30%, 60%, and 90% of probability around the mode of the distribution (i.e., the central forecast path).

3.3 Skewness

So far we have relied on the assumption that the shocks $\epsilon_{T+1:T+H}$ follow a normal distribution that is symmetric around the mode $\eta_{T+1:T+H}$. By relaxing this assumption, we consider asymmetric distributions of innovations to reflect a skewed balance of risks (Britton et al., 1998). This approach draws the innovations $\epsilon_{T+1:T+H}$ from an asymmetric distribution keeping the mode of the distribution of $\eta_{T+1:T+H}$ conditioned on specific values, for instance, the central forecast performed by the TS.

To draw the innovations $\epsilon^{\Delta}_{T+1:T+H}$ from a skewed probability distribution, we assume that shocks follow a triangular distribution $\Delta(\eta^{\Delta}_{T+1:T+H}, \epsilon^{\Delta,L}_{T+1:T+H}, \epsilon^{\Delta,U}_{T+1:T+H})$ with mode $\eta^{\Delta}_{T+1:T+H}$, lower bound $\epsilon^{\Delta,L}_{T+1:T+H}$, and upper bound $\epsilon^{\Delta,U}_{T+1:T+H}$.[10] This distribution allows the researcher to provide information about the most likely, pessimistic and optimistic outcomes based on previous experience or knowledge of the underlying process. Algorithm 3 introduces the steps for simulating shocks that follow a skewed probability distribution.

Figure 6 compares a normal distribution $N(\mu, \sigma^2)$ with a triangular distribution $\Delta(\eta^{\Delta}, \epsilon^{\Delta,L}, \epsilon^{\Delta,U})$ with positive skewness. The latter is simulated using Algorithm 3. The mode and lower bound are equal in both distributions ($\eta = \eta^{\Delta}$) and ($\epsilon^{L} = \epsilon^{\Delta,L}$). The upper bound will be higher in the triangular distribution ($\epsilon^{\Delta,U} > \epsilon^{U}$), so the probability of observing an innovation above the mode is higher.[11]

Algorithm 3 Simulation of a Skewed Probability Distribution of Shocks

For each shock ϵ_t considered over the forecasting horizon (e.g. demand shocks, oil price shocks):

1. Generate a sequence of M innovations $\epsilon_{T+1:T+H} \sim N(\mu_{T+1:T+H}, \Sigma_{e_{T+1:T+H}})$, $\mu \in \mathbb{R}$, such that $\mu_{T+1:T+H} = \eta_{T+1:T+H}$ guarantees that $Y_{T+1:T+H} = Z_{T+1:T+H}$
2. For each step $h = \{1, \ldots, H\}$, approximate a density function on the set of M simulated draws $\epsilon_{T+1:T+H}$, and compute percentiles at 1% and 99% as lower and upper bounds, respectively.
3. Consider a neutral, negative or positive skewness on $\epsilon_{T+1:T+H}$ from the qualitative analysis of risks, and calibrate a skewness multiplier ($\psi_{T+1:T+H} > 1$), using historical variance ratios between a high- and low-volatile periods (see Section 3.2).
4. Set the lower and upper bounds $[\epsilon^{\Delta,L}_{T+1:T+H}, \epsilon^{\Delta,U}_{T+1:T+H}] = [\beta^{L}_{T+1:T+H} * \epsilon^{L}_{T+1:T+H}, \beta^{U}_{T+1:T+H} * \epsilon^{U}_{T+1:T+H}]$ on the triangular distribution so that $\beta^{L}_{T+1:T+H} = \psi_{T+1:T+H}$ ($\beta^{U}_{T+1:T+H} = \psi_{T+1:T+H}$) if skewness is negative (*positive*) and $\beta^{L}_{T+1:T+H} = 1$ ($\beta^{U}_{T+1:T+H} = 1$) otherwise.
5. Generate a sequence of innovations from a triangular distribution $\Delta(\eta^{\Delta}_{T+1:T+H}, \epsilon^{\Delta,L}_{T+1:T+H}, \epsilon^{\Delta,U}_{T+1:T+H})$ with mode $\eta^{\Delta}_{T+1:T+H}$, lower bound $\epsilon^{\Delta,L}_{T+1:T+H}$, and upper bound $\epsilon^{\Delta,U}_{T+1:T+H}$.

Prospective analysis of skewed risks, in most cases, comes from the assessment of current and past episodes. For example, lockdowns and other social isolation measures to dela with the Covid-19 pandemic had adverse effects on economic activity in 2020, and implied foreseeing negatively

[10] The triangular distribution has been used previously in the literature on risk analysis and uncertainty problems (Johnson, 1997).

[11] ϵ^{L} and ϵ^{U} correspond to percentiles at 1% and 99% of the normal distribution, respectively.

skewed demand shocks over the policy horizon to reflect a higher probability of GDP growth and output gap projections below the central forecast path.

Figure 6: Simulation of the Distribution of Shocks

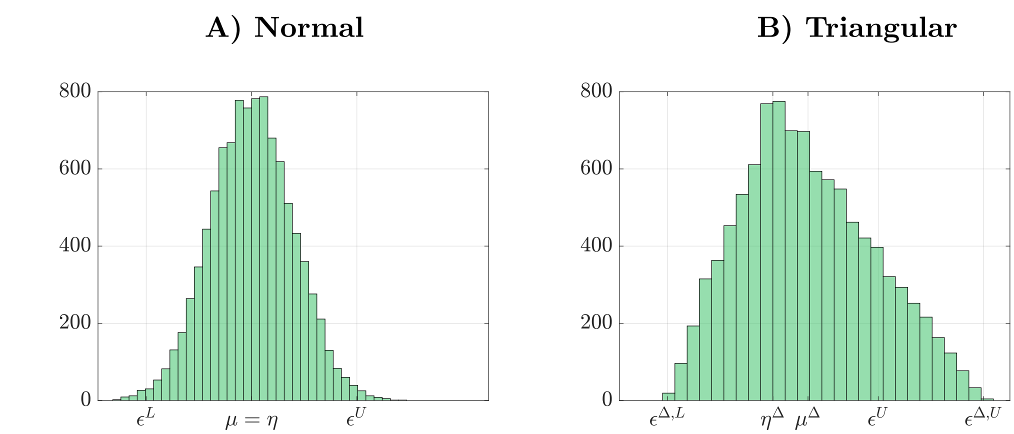
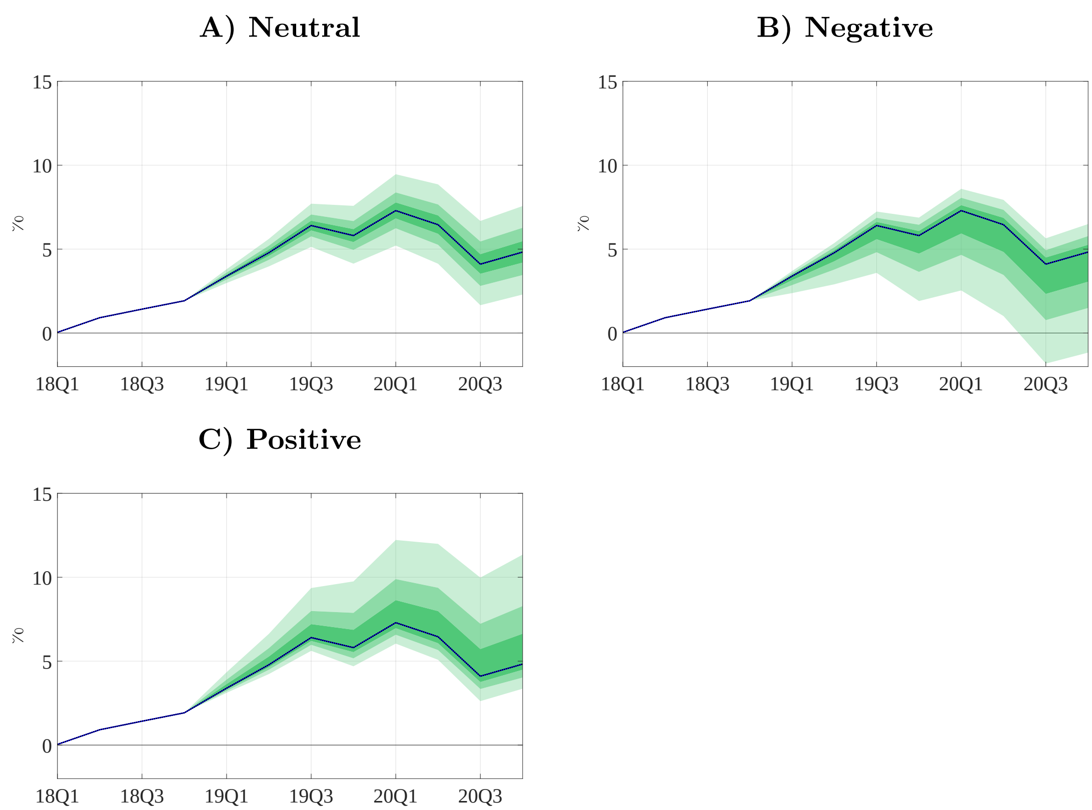


Figure 7: PD on the Food-basket Inflation Rate for Different Skews



Note: In this example, the mode corresponds to the observed value of the variable. The shaded areas show to the 30%, 60%, and 90% probability around the mode of the distribution (i.e. the central forecast path).

Another example is the assessment of climatic factors. For instance, previous episodes of *El Niño* had led to a rapid increase in inflation through the food basket. To reflect this risk factor, we would have to consider shocks to the food-basket Phillips curve drawn from a positively skewed probability distribution.

Figure 7 illustrates, as an example, the PD of the food-basket inflation rate we generate with cost shocks on this basket for the horizon period between the second quarter of 2018 and the first quarter of 2020. We assume a triangular probability distribution with neutral (Panel A), negative (Panel B) and positive (Panel C) skewness, respectively.

4 Unified PD

Once the central macroeconomic projection is defined, we initiate a new iterative process to generate PDs, resulting in a distribution that assigns probabilities to the forecasts of each variable. This procedure was illustrated in both Algorithm 1 and Figure 1.

The process begins with the presentation of the factors that could affect the future state of the economy, and their analysis within the TS. These prospective factors of risk are represented through the probability distributions of the structural shocks and considerations about their mode, variance and skewness, as described in Section 3. From these distributions, we randomly draw M sequences of innovations $\epsilon_{T+1:T+H}$ that feed the system of equations (1) - (2) to simulate M forecast paths for all latent state variables $S_{T+1:T+H}$, and measurement variables $Y_{T+1:T+H}$.

From the above results, we use a KE to approximate a probability density function over the set of M projections of every variable for both PATACON and 4GM. We then generate a unified PD that combines the probability distributions of the forecasts from both models. We build this density from the weighted average of each model's forecasts using weights $w_i = 0.5$.[12]

Under this framework, for each variable and horizon $h = \{1, \ldots, H\}$, we calculate the average across sequences ordered in ascending order, of the forecasts from both models. Finally, we estimate the PD by applying the KE to this set of weighted forecasts. With this methodology, the mode of the unified density corresponds to the average of the modes of the PATACON and 4GM forecast densities, which facilitates the communication of the results.

We give equal weight to each model avoiding arbitrary bias towards either model. The literature has found that a forecast combination that assigns equal weights to its components often outperforms the predictive ability of more sophisticated combinations.[13]

Figure 8 shows an example of the PD over the 8-quarter ahead projections of food-basket inflation rate for the 4GM (Panel A) and PATACON (Panel B) models, as well as the unified density (Panel C). In this example, we use the unconditional forecast path of each model as the mode of the PDs.

There are other methodologies for combining PDs. For example, the *linear pooling* approach, which directly calculates a weighted average of the densities of each model as $P_c\left(Y_{T+1}|Y_{1:T}\right) = \sum_{i=1}^{m} w_i P_i\left(Y_{T+1}|Y_{1:T}\right)$, and $\sum_{i=1}^{m} w_i = 1$, where $P_i\left(Y_{T+1}|Y_{1:T}\right)$ represents the forecast distributions for PATACON and 4GM.[14] Another example is the *dynamic linear pooling* technique, which allows

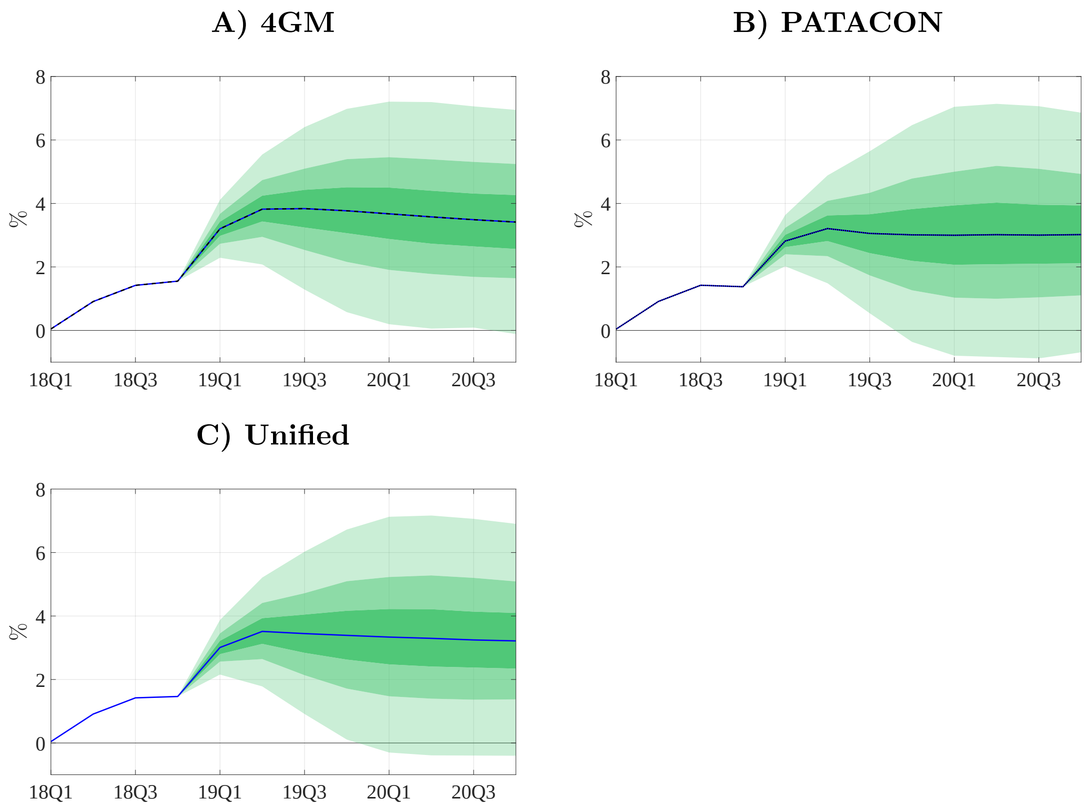
[12] The averaging of the data is briefly discussed in Hill & Miller (2011)

[13] This empirical fact is known in the literature as the forecast combination puzzle. Graefe et al. (2014) summarize the history of this puzzle, while Claeskens et al. (2016) propose a theoretical explanation to its occurrence.

[14] A taxonomy of solutions to the problem of combining distributions can be found in Genest & Zidek (1986). Linear pooling was proposed by Stone (1961) and was attributed to Laplace by Bacharach (1979).

the calculation of optimal weights based on the historical predictive performance of each model, once a sufficient pool of forecasts is available (Del Negro et al., 2016).

Figure 8: PD on the Food-basket Inflation



Note: The mode corresponds to the unconditioned forecast path in each model. The shaded areas correspond to the 30%, 60%, and 90% probability around the mode of the distribution (i.e. the central forecast path).

Conclusions

Since July 2021, BanRep implemented the PD approach in its forecasting process, using the projections generated by its two general equilibrium models, PATACON and 4GM. The PDs from both models are combined to obtain a unified probability distribution of the macroeconomic forecasts, for each variable considered. The PD reflects the probability assigned to the possible future realizations of a variable conditional on the observed data.

The PD technique provides a suitable technique for characterizing and communicating the qualitative assessment of factors that could affect the economy in the future and thus the macroeconomic forecasts. This approach illustrates the sensitivity of macroeconomic forecasts to risk factors, quantifies their importance in probabilistic terms and helps to delineate a more robust monetary policy recommendation in uncertain environments.

The PD approach provides a flexible method to characterize the balance of risks through probability distributions of shocks and technical considerations on their mode, variance and skewness.

Unlike other methodologies, the PD technique considers the economic structure of macroeconomic models to approximate, by simulation, a joint probability distribution over the forecasts of all variables, ensuring that projections preserve general equilibrium relationships and thus their macroeconomic consistency.

References

Adolfson, M., Lindé, J., & Villani, M. (2007). Forecasting performance of an open economy dynamic stochastic general equilibrium model. *Econometric Reviews 26(2-4)*.

Bacharach, M. (1979). Normal Bayesian dialogues. *Journal of the American Statistical Association*, *74*(368), 837–846.

Bank of Israel. (2019). *Research Department Staff Forecast, April 2019.*

Batini, N., & Nelson, E. (2001). *The lag from monetary policy actions to inflation: Friedman revisited* (Discussion Papers No. 06). Monetary Policy Committee Unit, Bank of England.

Benes, J., Binning, A., Fukac, M., Lees, K., & Matheson, T. (2009). *K.I.T.T.: Kiwi inflation targeting technology.*

Blix, M., & Sellin, P. (1999). Inflation forecasts with uncertainty intervals. *Quarterly Review (Sveriges Riksbank)*, *2*, 12-28.

Britton, E., Fisher, P., & Whitley, J. (1998). The inflation report projections: Understanding the fan chart. *Bank of England Quarterly Bulletin, Feb. 1998*.

Bundesbank, D. (2010). *Deutsche Bundesbank Monthly Report, June 2010: Uncertainty of macroeconomic forecasts.*

Christiano, L., Eichenbaum, M., & Evans, C. (2003). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113(1)*.

Claeskens, G., Magnus, J. R., Vasnev, A. L., & Wang, W. (2016). The forecast combination puzzle: A simple theoretical explanation. *International Journal of Forecasting*, *32*(3), 754-762.

Del Negro, M., Eusepi, S., Giannoni, M., Sbordone, A., Tambalotti, A., Cocci, M., ... Linder, H. (2013). The FRBNY DSGE model. *Federal Reserve Bank of New York Staff Reports*, *647*.

Del Negro, M., Hasegawa, R. B., & Schorfheide, F. (2016). Dynamic prediction pools: An investigation of financial frictions and forecasting performance. *Journal of Econometrics*, *192*(2), 391-405.

Del Negro, M., & Schorfheide, F. (2013). DSGE model-based forecasting. *Handbook of Economic Forecasting, Volume 2A.*

Durbin, J., & Koopman, S. (2012). *Time series analysis by state space methods.*

Friedman, M. (1972). Have monetary policies failed? *American Economic Review*, *62*(1), 11-18.

Genest, C., & Zidek, J. V. (1986). Combining probability distributions: A critique and an annotated bibliography. *Statistical Science*, *1*(1), 114–135.

Geweke, J., & Whiteman, C. (2006). Bayesian forecasting. *Handbook of Economic Forecasting, Volume 1*.

González, A., Guarín, A., Rodriguez-Guzmán, D., & Vargas-Herrera, H. (2020). 4GM: A new model for the monetary policy analysis in Colombia. *Borradores de Economía, Volume 1106*.

González, A., Huertas, C., Parra, J., & Vargas, H. (2019). Proceso de toma de decisiones de política monetaria del Banco de la República y comunicación sobre política monetaria. *Documentos Técnicos o de Trabajo*.

González, A., Mahadeva, L., Prada, J. D., & Rodríguez-Guzmán, D. (2011). Policy analysis tool applied to Colombian needs: PATACON model description. *Borradores de Economía, Volumen 656*.

Goodhart, C. (2001). Monetary transmission lags and the formulation of the policy decision on interest rates. *Review, Federal Reserve Bank of St. Louis*, *83*, 165-186.

Graefe, A., Armstrong, J. S., Jones, R. J., & Cuzán, A. G. (2014). Combining forecasts: An application to elections. *International Journal of Forecasting*, *30*(1), 43-54.

Hamilton, J. D. (1994). Time series analysis. *Princeton, NJ: Princeton University Press*.

Hamilton, J. D. (2005). Regime-switching models. *Palgrave Dictionary of Economics*.

Hill, T. P., & Miller, J. (2011). How to combine independent data sets for the same quantity. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *21*(3), 033102. Retrieved from https://doi.org/10.1063/1.3593373 doi: 10.1063/1.3593373

Johnson, D. (1997). The triangular distribution as a proxy for the beta distribution in risk analysis. *Journal of the Royal Statistical Society 46(3).*

Klein, P. (2000). Using the generalized Schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, *24(10)*, 1405–1423.

Knight, F. H. (1921). *Risk, uncertainty and profit* (Vol. 31). Houghton Mifflin.

Knüppel, M., & Schultefrankenfeld, G. (2012). How informative are central bank assessments of macroeconomic risks? *International Journal of Central Banking*, *8*(3), 87-139.

Perlin, M. (2015). *MS_Regress - the MATLAB package for Markov regime switching models.* Retrieved from https://ssrn.com/abstract=1714016

Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association 3(2-3).*

Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: a Bayesian DSGE approach. *American Economic Review, 97(3).*

Stone, M. (1961). The opinion pool. *The Annals of Mathematical Statistics*, *32*(4), 1339–1342.

Svensson, L. E. (2010). Inflation targeting. *NBER Working Paper 16654*.


BANCO DE LA REPÚBLICA
COLOMBIA