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Abstract

We construct the Labor Market Indicator (LMI) focusing on the cyclical similarities of eighteen time series from household, industrial, and opinion surveys between 2001 and 2019. The LMI summarizes the growth cycle of the labor market as defined by Mintz (1972) and is connected to the evolution of the traditional business cycle indicators as well as to that of the GDP and the Unemployment rate GAP. The evolution of the indicator provide useful information to policy makers, as it complements the characterization of expansions and turning points. Thus, improving the analysis of the current momentum of the labor market.

JEL classification: E24, E66, J6, J20
Keywords: LMI, Colombian labor market, dynamic factor model, unemployment rate.

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Indicador del mercado laboral para Colombia (LMI)

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Resumen

En este documento construimos el Indicador del Mercado Laboral (LMI) con base en las similitudes entre los componentes cíclicos de 18 series de tiempo que incluyen Encuestas de hogares, industrial y de opinión entre 2001 y 2019. El LMI resume el ciclo de crecimiento del mercado laboral tal como lo define Mintz (1972) y su evolución esta relacionada con la indicadores tradicionales del ciclo de los negocios así como a la de las brechas del PIB y de la tasa de desempleo. La evolución del indicador brinda información útil a los hacedores de política dado que complementa la caracterización de las expansiones y los puntos de quiebre. De esta manera, se obtiene un análisis más completo del momentum del mercado laboral.

Clasificación JEL : E24, E66, J6, J20
Palabras clave: LMI, Mercado Laboral, Modelo de factores dinámicos, Tasa de desempleo

1 Introduction

Economic growth is not a stable and smooth process, given that internal and external disturbances, institutions, and economic policy may affect the short-run growth rate causing instabilities, i.e. business cycles. As sources of the cycles might be different, their duration, intensity, and diffusion also vary from one cycle to the next. Research on business cycles have defined two phases for the economic activity: periods with positive economic growth in which conditions are favourable in almost all economic sectors (*expansions*), and periods with negative economic growth in which the economic indicators deteriorate for most of the economic activities (*recessions*). The labor market is also affected by business cycles as workers and firms adjust their supply and demand decisions to the prevailing conditions.[1] The response of the labor market will sum the benefits and losses from all economic sectors and agents. This response may change depending on macroeconomic conditions like the origin of the cycle, how expansions and recessions spread among economic activities, as well as to industry conditions such as labor intensity or final product orientation. For instance, as investment is more sensible to shocks than consumer goods, we expect that recessions start with a deterioration of employment, vacancies, and expectations in the production of investment goods.

Traditionally, the analysis of the business cycle's effects over the labor market has been conducted using the unemployment rate given the classical view that cyclical fluctuations are the main force behind the unemployment. However, the existence of rigidities such as mobility costs and information asymmetries also play an important role in explaining unemployment (e.g. Lilien (1982), Loungani (1986), and Davis (1987). These rigidities impede labor to freely flow among sectors during recessions. Thus, it is convenient to expand our attention to variables that reflect other aspects of the labor market not affected by these rigidities. Among these variables we find vacancies, job creation, and productivity; thus, the creation of a composite indicator provide a better portrait of how the labor market absorbs economic fluctuations over the business cycle.

Moreover, research on business cycle has analysed the behaviour of economic series during upturns and downturns, finding that: *i)* fluctuations have common patterns on economic activity, *ii)* fluctuations vary across countries despite their similar characteristics, and *iii)* contractions tend to be shorter and deeper than expansions (e.g. Mitchell (1927) and Keynes (1936)). Specialized institutions have used the first conclusion to construct indicators that describe the deepness, length, and diffusion of the business cycle providing a diagnostic of current conditions to formulate economic policy. In practice, three alternative indicators have been introduced, each definition focusing on different properties of the series. The Conference Board and the Euro committee construct *business cycle* indicators using logarithms or levels of a wide set of economic series. Mintz (1972) proposes two additional sets of indicators for the economic activity, *growth cycle*, which focus on focus on each series deviation with respect to its long run trend, and *acceleration cycle* indicators, which analyze changes in the growth rate of the variables. Phases and turning points from each definition describe different

[1]During recessions, labor supply of secondary members of the household may increase to compensate income reductions as consequence of fires or hours cut of other household members. Additionally, labor demand is adjusted as firms optimize their overtime hours, vacancies, and full-time employment to face the changing economic environment.

features from the business Cycle and therefore combining them will help to forecast recessions and to make better policy design. Although, usually statistical agencies construct traditional business cycle indicators currently more focus have been made to the growth cycle definition, as the Organisation for Economic Co-operation and Development (OECD) used this definition to compute their cyclical indicator system.

The agencies that construct this indices have found that summarizing signals from various indicators to diagnose the labor market is not straightforward. For instance, between 2005 and 2008 economic growth in Colombia exceeded 5%, improvement that lead to a reduction of the unemployment and the underemployment rates of 0.76 pp and 2.00 pp, reflecting a better momentum of the labor market. However, a measure of the whole improvement is not directly derived. The problem of diagnosing the current state of the labor market is harder with multiple sources of information, horizons, or units of measurement. Then, to understand the current state of the labor market is necessary to develop an indicator that summarizes the behaviour of a broad set of series.

A first way to deal with this problem is the use of traditional statistical techniques. For instance, the *Conference Board* constructs composite indices to summarize the evolution of a wide set of series, as the average growth of the series, correcting by its volatility and re-scaling them to equate their trends to the trend of the coincident index. Also diffusion indices are arithmetically computed summarizing the proportion of time series improving with respect to the previous year. Although both indices portraits the state of the economy depending on how much the set of series is growing or that most of the series are growing, they have two weaknesses: first, variable weights are imposed ad-hoc, and second that the aggregate indicator may be contaminated by idiosyncratic movements. An alternative solution is to portrait the state of the economy as a latent factor that drives the evolution of a set of series. Thus, in the literature statistical techniques like Principal Component Analysis (PCA) and Dynamic Factor Models (DFM) have been introduced, also overcoming the difficulties described in the previous paragraph given that first weights depend on the co-movement of each series with the underlying factor and second idiosyncratic movements are easily included in the specification. These techniques have been used in economics to construct coincident, lagged, and leading indexes of the state of the economic activity, see for instance Stock and Watson (1988, 1989, 1993), Forni et al. (2005); Kamil et al. (2010), and Cristiano et al. (2012) and Nieto and Melo (2001) for Colombia.

Four studies have constructed synthetic indicators to summarize the cyclical behavior of the labor market. Barnes et al. (2007) constructed the labor market indicator (LMI) using the first principal component of twelve series that describe the dynamics of the labor market. The indicator accounts for 68% of the variation of the underlying series. Their results show that the cyclical components of both the Unemployment Rate (UR) and the LMI are similar and they also have similar predictive power to forecast wage inflation. Zmitrowicz and Khan (2014) compute the LMI as the first principal component of a set of labor market series for the United States and Canada. The indicator is re scaled to coincide with the UR units and it is interpreted as the UR consistent with the actual labor market conditions. A lower increment in the LMI than in the UR signals a positive dynamics in other indicators, and thus by only focusing on the UR we would underestimate the improvement of the labor market. Hakkio and Willis (2013) consolidate 23 labor market series using PCA, they select the two principal components that account for 82% of the series co-movements. The first factor is related to the level of activity, while the second is related to the speed of improvement. By construction the factors are standardized, with zero-mean and one standard deviation. The authors use both factors to characterize the current and future state and how fast economic conditions would improve.

The most recent paper is Chung et al. (2014) who combine a DFM and PCA to construct the labor market conditions index (LMCI) in two steps. In the first step the three most representative dynamic latent factors are extracted from a wide set of labor market series. In the second step, the LMCI is constructed as the first principal component of the common variation of the series.[2] Sixty six percent of the total co-movement is captured by the final indicator which is useful to forecast each series one-step ahead. As far as we are concerned, there is no evidence of indicators to assess the momentum of the labor market in developing economies, which have different economic and social institutions than industrialized countries. Thus, in addition to standard labor market indicators in developing economies attention must be placed on non-wage occupation, and inactivity.

The objective of this paper is to construct an aggregate growth cycle indicator for the Colombian labor market that provides policy makers with additional information than the traditional business cycle indicators and the unemployment rate to evaluate the current state of the labor market. We follow a dynamic factor model approach to summarize information from a wide set of labor market series (households, commerce, and industrial surveys, as well as administrative records). This approach considers optimal weights to maximize the common variation, captures the dynamic behavior of both the common factors and the idiosyncratic components, and allows to efficiently handle missing observations: *i)* due to different spans of information, as more series are recently available, and *ii)* due to delays in publication or randomly missed.

Our results suggest that the LMI complements the analysis of the labor market momentum as it includes more information than the Unemployment Rate, for instance, industrial and commercial employment, bottle necks, and vacancies have an important contribution in its evolution. Additionally, its interpretation complements the diagnostic from traditional business cycle indicator in which economic conditions can be classified as expansions or recessions. On the other hand, growth cycle indicators describe economic conditions as high-growth if conditions are above the trend or low-growth otherwise. Thus, it complements the characterization of the prevailing conditions.

This document is divided in seven sections being the first this introduction. The second section describes the methodology used to construct the index, in the first step we extract the maximum common variation with a

[2]The common variation of the entire set of series is constructed as the prediction of the entire set of series using only the latent factors computed in the first step.

DFM and in the second step we compute the LMI as the first principal component of the projected series. In the third section we discuss the set of variables and how the cyclical component was extracted. The fourth section presents a deeper interpretation of the growth cycle indices and their relation with traditional business cycle. In the fifth section we present the LMI between 2001 and 2019 its phases and presents a robust analysis changing the sample to see whether the introduction of new information change the turning points. In the sixth section we compare the LMI with the GDP and UR gaps for Colombia and additionally we compare its evolution with a traditional business cycle indicator to capture the state of the business cycle. Finally, we conclude in the last section.

2 Methodology

This section presents twe he two-step methodology to extract t in the last sectione common information across the cyclical behavior of the series and how to combine them into an indicator that tracks the current state of the labor market. In the first step, we use a DFM to decompose the cyclical components of a set of labor market series into the common variation due to the current status of the labor market and the variation due to idiosyncratic movements. To extract the highest common variation we include several dynamic factors. In the second step, we compute the LMI as the first principal component of the projection of the series onto the common factors of the previous step.

2.1 Dynamic Factor Model

In the first step, we decompose a vector of n_v observed variables X_t into two components: a group of n_f latent factors F_t that is common across all series and a set of idiosyncratic series U_t that capture the own-evolution of each series.

$$X_t = \Lambda F_t + U_t, \tag{1}$$

where Λ is a (n_v, n_f) matrix that corresponds to the loading coefficients of the factors in each observed variable, that is, how much each factor contributes to each series dynamics at period t. U_t captures each series non-observed idiosyncratic component, this component describes the dynamics due to shocks and the proper movements of each series, in our application it is allowed to follow an autoregressive structure of lag p_u.[3]

$$U_t = C_0 + C_1 U_{t-1} + \cdots + C_{p_u} U_{t-p_u} + \eta_{u,t}, \tag{2}$$

[3]The inclusion of an autoregressive factor for each idiosyncratic component follows Stock and Watson (2011) and is the principal difference between the proposed methodology and Chung et al. (2014). We expect to isolate idiosyncratic variation that may contaminate the factors and LMI dynamic.

with C_i (n_v, n_v) diagonal matrices for $i = \{1, 2, \ldots, p_u\}$, and C_0 is a n_v vector that contains the intercepts of the AR processes. $\eta_{u,t}$ is a n_v vector assumed Gaussian and uncorrelated across time nor with the other observed variables. Thus, the dynamic of the series due to the common component is given by ΛF_t and that due to the idiosyncratic component is U_t, which does not provide information about the current state of the labor market or any other idiosyncratic component, this is a common assumption used for instance by Stock and Watson (1989).[4] As the factors describe underlying economic forces they are allowed to follow a VAR (p_f):

$$F_t = A_1 F_{t-1} + \cdots + A_{p_f} F_{t-p_f} + \eta_{f,t}, \tag{3}$$

with $A_1, \ldots, A_p$ (n_f, n_f) matrices that capture the dynamic relationship of the factors. $\eta_{f,t}$ is a n_f vector of *i.i.d* Gaussian errors, with $\{\eta_{u,t}\}$ and $\left\{\eta_{f,t}\right\}$ *i.i.d.* disturbances. Equations (1) to (3) represent the DFM that can be converted into a linear state space representation using two equations. The first is the measurement equation that relates the observed variables to the latent factors and the second is the transition equation that describes the evolution of the factors and the idiosyncratic componen The linear state space model is estimated using the Kalman Filter to evaluate the likelihood function Harvey (1990), the appropriate state-space representation can be found in appendix A.

2.2 Principal Component Analysis

Once the latent factors have been estimated we eliminate the variation of the series due to the idiosyncratic components by projecting each series using only the factors ($\hat{X}_t = \hat{\Lambda}\hat{F}_t$). Subsequently, the LMI is constructed as the first principal component of the projected series ($\hat{X}_t$).

$$LMI_t = \Theta\hat{\mathrm{X}}_{\mathrm{t}} = \Theta\hat{\Lambda}\hat{\mathrm{F}}_{\mathrm{t}}. \tag{4}$$

with Θ as the eigenvector of the matrix $\hat{\Lambda} var\left(\hat{\mathrm{F}}_{\mathrm{t}}\right)\hat{\Lambda}'$ that corresponds to the highest eigenvalue.

3 Data and statistical treatment

3.1 Description of the database

The main source of information regarding the labor market in Colombia is the household survey monthly published by DANE, but in recent years other aspects of the labor market are gathered in opinion surveys, administrative records, and industrial and commerce surveys. For instance, business expectations and vacancies are computed and published by Banco de la República (BR) and with more recent information of household

[4]It is also possible to model an idiosyncratic component that affects a small group of series.

surveys flow measures can be constructed Morales et al. (2018).[5] Although it would be desirable to include the majority of indicators, extracting a signal in large specifications is more difficult and may lead to worse performance (e.g Boivin and Ng (2006); Ruiz and Poncela (2012), and Bańbura and Modugno (2012)). Thus, to select the series we characterize all dimensions of labor market keeping a reasonable number of series. We start by including seven key indicators of the labor market that Arango et al. (2015) discuss in their analysis of the labor market and the Colombian business cycle between 1984 and 2014. Additionally, to account for more dimensions of the labor market and to take advantage of the new information available since 2002, we include eleven variables from industrial, commerce, and expectations surveys. The selected series satisfy the traditional requirements for series used in business cycle indicators, as they are quickly available, their methodology is reliable, they are affected by the cycle, and finally they are no subject to erratic behaviour.[6] Then, our series represent different dimensions such as unemployment, hours, wages, vacancies, hiring, firing, quits, and opinion surveys. With this in mind the variables are presented in Table 1.

Unemployment and Underemployment

The *unemployment rate* has been considered the most important indicator in the analysis of the labor market performance, its behavior is counter-cyclical as better conditions imply a higher level of employment and less people will look for a job. This variable was extracted from the Official Household Survey (GEIH by its acronym in Spanish). Another traditional indicator is the *labor force participation rate* (LFPR). This is computed as the percentage of the working-age population, which is either working or looking for a job, and reflects the decision to offer their labor services at the extensive margin. However, its relationship with the business cycle is not clear due to the existence of two opposite effects: the *added* and the *discouraged worker*. The first one, is countercyclical given that secondary members of the household increase their participation during recessions to compensate income reductions in the household due to layoffs or longer spells into unemployment. The second one is procyclical given that during recessions the expected wage reduces as the probability of finding a job and the actual wage reduces. The *objective underemployment rate by hours* (UER) is included as a measure of underutilization, it captures the proportion of not fully employed workers that may generate a pressure on the labor market. It is defined as the percentage of employees that *i)* are currently working less than 32 hours per week, *ii)* want to work more hours, and *iii)* are available to work.

Employment

We include three measures of labor demand as it traditionally moves along with economic conditions and is watched for gauging the health of the economy. First, we consider the industrial employment with *total industry employment* (TIE) from the Monthly Manufacturing Survey (EMM, by its acronym in Spanish), as industry represents about 10% of both the GDP and employment of the economy. This index captures the evolution of employment in firms with more than 10 employees in the manufacturing sector. The second measure cor-

[5]An alternative method to construct labor markets flows form the firms perspective is based on the *Planilla Integrada de Liquidación de Aportes* (PILA) which is an administrative record of the Ministry of Health and Social Protection, Morales and Medina (2016),

[6]The selection of the series in the OECD indicators is made based on the following criteria: the co-movements with the economic activity, the time consistency of this relationship, the reliability of the statistical methodology, the availability of the information, the relevance of the business cycle as a factor that explains the series, and the no-erraticism nor high-volatility of the series.

Table 1: Variables included in the LMI

Variable	**Acronym**	**Source**
Unemployment and Underemployment		
Unemployment rate	UR	GEIH
Labor force participation rate	LFPR	GEIH
Objective underemploymet by hours	UER	GEIH
Employment		
Occupation rate	ER	GEIH
Total employment - Industry	TIE	EMM
Private employment	PE	GEIH
Total employment index - Commerce	TCE	EMCM
Informal employment		
Proportion of non-wage workers	NWP	GEIH
Workweeks		
Average weekly hours for wage workers	HPW	GEIH
Wages		
Average wage	AW	GEIH
Average labor income	ALI	GEIH
Labor income index - Commerce	ALIC	EMCM
Vacancies, perceptions, and expectations		
Job vacancies	JV	BR
Business expectations	BP4	BR
Bottle Necks	BN	BR
Fluidity Measures		
Job creation	JC	BR
Churning	CH	BR
Other variables		
Discouraged workers rate	DC	GEIH

Table 1 presents the labor market series included in the LMI, the first column describes the series while the second column presents the acronym used in this document. The source is presented in the last column.

responds to *employees in particular firms* (PE), which corresponds to the interviewees that report to work for a non-state firm during the reference period and is computed using the GEIH. Finally, we include the total employment in commerce (TCE) as reported by DANE and its monthly commerce survey.

Non-wage employment

In a developing country after a negative shock the response of the economic agents might be different from just going to the unemployment, for instance agents may adjust the extensive margin, labor underutilization or engage in informal activities. Thus, as Mondragón-Vélez et al. (2010) found informality is countercyclical. However, informality is only included in a monthly basis since 2007, thus we use as proxy the percentage of employed that are non-wage workers *NW*.

Workweeks

Labor utilization at the intensive margin is measured by the number of hours worked per week. Research considers that it is procyclical because during economic upturns working hours increase and during economic downturns they decrease (e.g. Kydland and Prescott (1991); Cho and Cooley (1994) among others), also employers tend to adjust working hours before modifying their workforce. Thus, we included *average weekly hours (HPW)* for wage-earners, taken from GEIH since 2001.

Wages

Salaries and earnings for employees are key to understand their decision to enter the labor market. The evidence suggests a strong and positive correlation between the economic cycle, the employment rate, and the wage rate. During a favorable economic environment firms raise their offered wages to attract workers, in a search framework we think that a tight labor market attract workers by an intertemporal substitution effect increasing their current labor supply. In contrast, during economic downturns the wage rate offered declines, and the add and discouraged worker effects take place. The final change in the wage rate will depend on the employment composition and the existence of rigidities. In this paper, we included two measures of labor income. For the employees in private or government firms the *average wage* (AW) represents their hourly wage, and given that in Colombia the non-wage workers represent half of the working population, we include for all non-wage earners employees the *average labor income* (ALI).

Vacancies, perceptions, and expectations

Job vacancies (JV) reflect openings and perspectives of firms. In an economy where the expectations of entrepreneurs are positive, one would expect an increase in this indicator, but if expectations are negative, then openings at firms will be closed and vacancies will reduce. In this paper, we used the monthly information of classified advertisements provided by job sites to obtain the number of job vacancies over time, as described in Arango (2013). *Employment expectations* (BP4T) reflect changes in attitudes concerning future employment and is the only expectations-based variable of the panel considered. In order to capture future employment we include this variable computed as the difference between the percentage of firms that will hire more employees and the percentage of those which will reduce their plant in the next twelve months. We used information provided by the Monthly Survey of Economic Expectations. Another variable of this survey that portraits the current difficulties faced by firms to hire is the existence of bottle necks *BN*.

Fluidity Measures

Labor flows provide more information about the functioning of the labor market than the simple changes in the stocks. *Job creation* (JC) is computed as the number of employees created from one period to the next period. In order to capture the labor overflow in the economy we also included the measure of *churning* (CH) which reflects the ratio between workers and job reallocations. We use the series proposed by Morales et al. (2018), who provide a measure of labor flows for wage and non-wage workers. This is a more comprehensive approach of understanding flows than just focusing on wage workers from firm level data Morales and Medina (2016) and Flórez et al. (2017).

3.2 Treatment of the series

In this subsection we discuss the steps to extract a cyclical component of the series for the DFM described in section 2. The empirical application requires to extract the cyclical component of each series, to avoid miss-specifications, false signals, and to get robust estimates of fitting the model in equations 1 - 3. Thus, in the construction of business cycle indicators it is usual to remove the seasonal and permanent components. First, we test for seasonal autocorrelation in the original series using the QS test proposed by Gómez and Maravall (1996). Only three series did not require seasonal adjustment: average weekly hours (HPW), average wage (AW), and average labor income (NLI). Thus, we seasonally adjust the remaining fifteen series using the TRAMO-SEATS software. Maximo et al. (2015) strongly advise to use seasonally adjusted variables in a DFM since its performance is comparable or even better than the performance of a DFM that includes a common seasonal component. Their results suggest that including seasonal structure in a DFM will have the curse of dimensionality and moreover it is miss-specified if the seasonal component is idiosyncratic, in our application there are three series that do not require adjustment.

We remove the permanent component of the data, *detrend*. Several filtering methodologies have been developed to decompose a time series into their permanent and cyclical components being the most popular used Hodrick and Prescott (HP), Baxter and King (BK), and LOWESS (Locally Weighted Scatter plot Smoothing). We choose the BK filter focusing on the cyclical component for frequencies between 6 months and 6 years, this eliminates high-frequency as well as undesired long-term components. Literature has shown that this filter has better performance at the business cycle frequencies and for instance Kaiser and Maravall (1999) show that the Hodrick-Prescott filter produces spurious effects and it has a poorer approximation at the end points.[7] As filters are characterized by end-point problems leading to false signals, we forecast the four-year *non-observed* period before the beginning and after the end of the sample. For the seven series included in Arango et al. (2015) we conduct conditional ARIMA models, assuming that the monthly series should reflect the same end of quarter change as the quarterly series. For the remaining variables, the forecasts are derived from an ARIMA(p,d,q) model that minimizes the mean square forecast error of a rolling out of sample forecasting evaluation up to one year ahead, with a forecast sample that starts in January 2013.

[7] The OECD indicators are constructed considering Phase Average Trend (PAT), Hodrick and Prescott filter, and Christiano and Fitzgerald filter.

4 Traditional vs Growth cycle indicators

Worldwide the analysis of the current state of the economy is crucial, while in the United States the *Business Cycle Dating Committee* and *the Conference Board* are the responsible institutions of this task, *the Euro Area Business Cycle Dating Committee* is the responsible for the euro zone. These institutions track the state of the main economic variables and construct indicators that summarize that behavior and determine whether current conditions signal a change in the economic phase. As it was mentioned in the introduction, there are three alternative measurements to construct the indicators: *business cycle, growth cycle* and *acceleration cycles*. Thus, in this section we discuss the *business cycle* and *growth cycle* indicators, and how to connect them for a clear interpretation of the LMI. Later, we introduce *duration*, *deepness*, and *diffusion* as concepts that must be combined with the turning points to correctly diagnose the current phase of the business cycle.

Traditionally business cycle analysis focuses on determining whether the economy is in an expansion or recession phase, being those two phases separated by *peaks* and *troughs*, denominated turning points. An expansion describes a period of economic growth and begins after economic activity falls to its minimum (*trough*) and lasts up to it reaches the maximum (*peak*). On the other hand, a recession denotes a period of contraction in the economic activity and begins right after the economy reaches the *peak* of the previous expansion and ends when the economic activity falls to its *trough.* In this literature, indicators are constructed extracting the current state of the economy from the fluctuations on the level of a variety of economic activity series. As we focus on the cyclical component of the series, the LMI belongs to the *growth cycles* indicators, that is a strand of the business cycle literature initiated by Mintz (1972). Series are analyzed around its long-run trend, instead of its level and the indicators are constructed in a similar manner than in the traditional analysis. Similarly, these cycles also have two phases: *high-growth* and *low-growth* and the turning points separating those phases are called *downturns* and *upturns*. Growth cycles tend to be more frequent and symmetric than traditional business cycle, where expansions are distinctively longer than recessions. Thus, acceleration and deceleration in growth might occur without a decline in the level of economic activity. A definition of growth cycles is provided by Mintz (1972) page 40:

> *"... Growth cycles are fluctuations in aggregate economy activity. A growth cycle consists of a period of relatively high growth rates occurring at about the same time in many economic activities, followed by a period of similarly widespread low growth rates which merges into the high-growth phase of the next cycle... "*

Despite the fact that there is not a one-to-one connection between the traditional and the growth business cycle definitions, phases, and turning points, some links between the two methodologies can be established. First, in periods of a high-growth phase (*growth definition*), the growth exceeds the long-run trend, then those favorable conditions are related to an expansion (*traditional definition*). However, such connection cannot be established with low-growth periods given that they may refer to periods of expansion with a moderate economic growth or to recessions. For these reasons, high-rate phases tend to be shorter than expansions. During recessions (*traditional definition*) the growth is negative and therefore below the long-run trend, thus the economy must also be in a low growth phase (*growth definition*). However, no association can be done with expansion periods, given that positive growth can be associated with both high and low growth phases. With respect to the turning points, Mintz (1972) highlights that while downturns tend to lead peaks, upturns tend to lag troughs.

A comparison of the phases and the turning points from these definitions is presented in Figure 1. The upper panel presents a series that can be decomposed into a linear trend (black line) and a cyclical component (red line) presented in the lower panel. The chronology of the turning points is the following: first we observe a *downturn* in economic activity, as marked by point A, this implies that the economy falls into a *low-growth phase*, that does not become a *recession* until economic activity reach its *peak* at point B. At this point there is a reduction in the level of the series, as the long-run trend growth does not compensate the reduction in the cyclical component. The recession will continue as the series continue reducing and reach its *through* in point C. Finally, the cyclical component of the series also reacts and there is an *upturn* at point D. Thus, in the example the light grey shaded region represents a *recession*, and the whole shaded region (dark and light grey) will capture the *low-growth phase*. In this example we can see that both the level and the cyclical component of the series have turning points. However, this could not be the case, if the reduction of the cyclical component is not big enough to offset the increments in the long-run trend, we will have turning points in the growth cycle indicators but not in the level indicators.

After creating the indicator the next step is to compute the chronology of the business cycle, that is the turning points that determine the phases. This is usually done with an objective methodology, being the most popular approach Bry and Boschan (1971), examples of this methodology for the Colombian economic activity are presented inArango et al. (2007); Alfonso et al. (2013). As mentioned, it is not enough to focus on the turning points to determine the current phase of the economy, given that small, brief, or sector specific changes in economic activity can lead to miss interpretations. Thus, we complement the turning points with the three D's criteria: *duration*, *depth*, and *diffusion* proposed by *Conference Board* to diagnose the current state of the economy in the traditional business cycle definition. *Duration* is defined as the number of periods that the current phase has prevailed, while recessions tend to be short, expansions are longer. For instance, it is usual to set that expansions last at least eighteen months long while recessions must be at least six in the Bry and Boschan algorithm. The *depth* refers to the magnitude of the change in the index analyzed; a small change in the indicator may not indicate a definite change in the economic activity. Finally, *diffusion* captures the fact that as more indicators are affected by economic conditions they tend to move accordingly during the phase of the cycle.

Figure 1: Comparison of the business cycle indicators

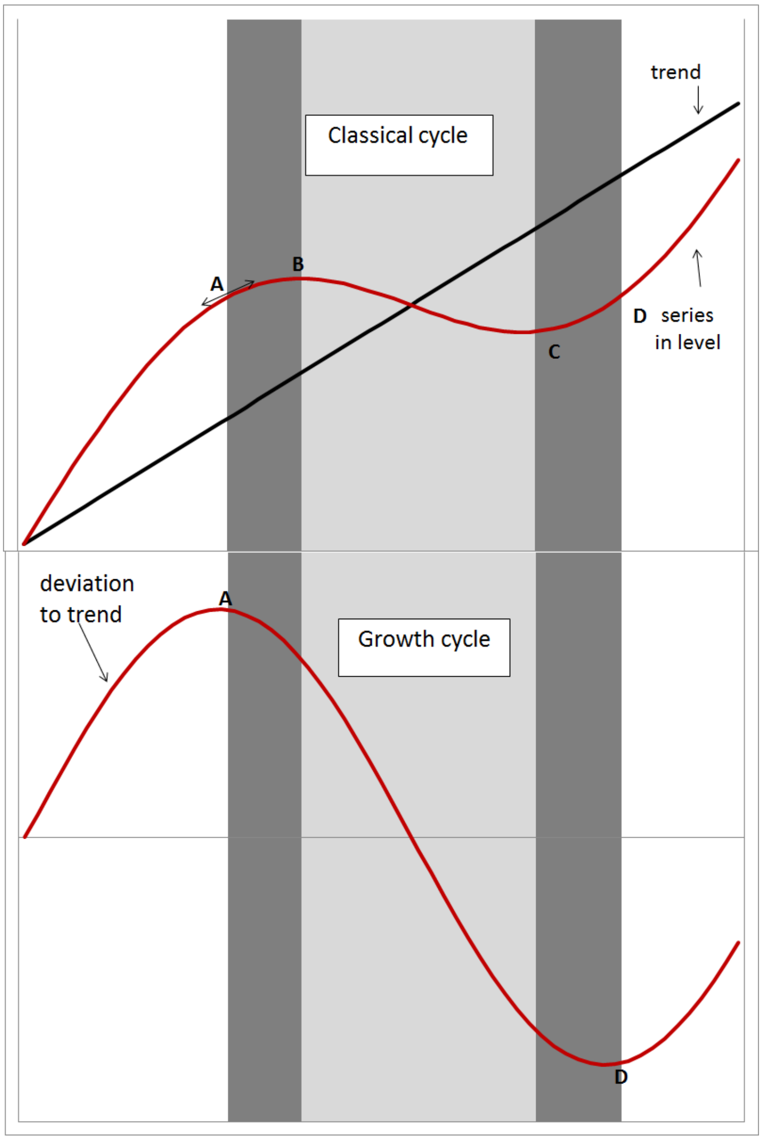


Figure 1 Compares the classical and Growth cycle phases and turning points. A and D correspond to the downturn and upturn turning points, and the shaded area corresponds to a low-growth phase. Points B and C are the peak and through of the classical definition and the light gray shaded corresponds to a recession.

Now that we presented the growth cycle interpretation and how the phases are defined, in the next section we present the estimated LMI and its chronology using the Bry and Boschan (1971) methodology, which identifies nine phases between 2001 and 2017. After that we define whether the turning points detected are consistent with the three D's criteria, and explain why it is important to consider both elements.

5 LMI history

The LMI is computed between March 2001 and December 2019 using the variables in Table 1. The statistical fit of the model was the best with three latent factors, each of which can be approximated as an AR(2) process, whilst the idiosyncratic processes were better approximated by a series of AR(1) processes.[8] The LMI is presented as a momentum of the labor market, cyclical series, instead of a trend restored series.[9] Although the trend restoration procedure eases the comparison with any reference series, it has two main disadvantages: the cyclical patterns can be obscured or even missed as the long-term trend dominates the cyclical movements and it is an extra source of revision of the LMI as the trend is re-estimated in each publication.

Figure 2: The Labor Market Indicator

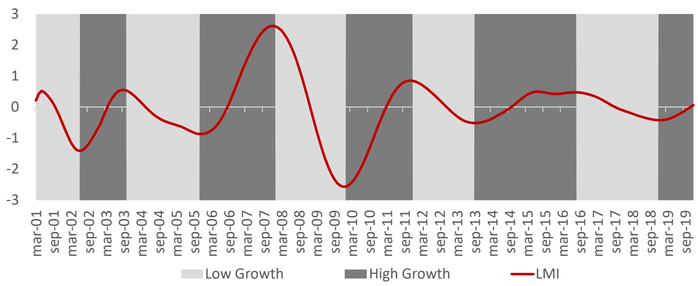


Figure 2 presents the LMI between march 2001 and December 2019.

As discussed in the previous subsection, it is crucial for a system of business cycle indicators to determine the turning point of the indicators, thus in table 2 we use the Bry and Boschan (1971) algorithm to determine the *upturns* and *downturns* of the LMI and its phases. During the period of analysis we found nine turning points

[8]The selection of this model is based on the usual AIC and BIC criteria.
[9]Graphs of the cyclical components of the series and the LMI are presented in appendix B.

that define nine phases. The second column shows whether the period is a high-growth or low-growth phase, the third and fourth columns describe the starting and ending dates while the duration in months is reported in column 5. Finally, amplitude is shown in column 6, OECD (2004), pp. 27 defines amplitude "...the difference between values at peak and trough is referred to as an "amplitude"... In general, the larger the amplitude is, the more volatile the business cycle will be.

Table 2: Phases of the LMI

Period	Growth-Phase	Start	End	Duration	Depth
1	Low	NA	July 2002	NA	NA
2	High	July 2002	October 2003	15	1.91
3	Low	October 2003	December 2005	22	-1.40
4	High	December 2005	January 2008	25	3.47
5	Low	January 2008	February 2010	25	-5.12
6	High	February 2010	December 2011	23	3.37
7	Low	December 2011	October 2013	22	-1.36
8	High	October 2013	August 2016	34	0.98
9	Low	August 2016	January 2019	29	-0.90

Table 2 presents the low and high growth phases of the LMI computed with the Bry and Boschan (1971) algorithm. We impose restrictions on the minimum duration of both phases and the whole cycle. We use the standard values of six and eighteen months.

Before 2001 the Colombian economic activity experienced the biggest recession in its history, and the initial years of the decade show the recovery of the economic activity. Table 2 shows that between 2001 and 2019 there were nine growth cycle phases, five of which are characterized as low-growth, although we have complete information only for the last four phases. Additionally, there are four high-growth phases completely observed. While the average length or duration of the low-growth phases is twenty one months it is twenty eight months for the high growth phase, this finding is consistent with the similar phases duration in the growth cycle. The highest downturn point of the indicator was reached in January 2008, while the lowest was reached in November 2009. Each phase duration is at least 15 months and cycles depth (amplitude is higher than 1,7), thus, the phases in the table satisfy this two criteria.

To describe *diffusion* we focus on each series contribution into the LMI, doing so we determine whether the variables evolution coincide with the aggregate signal. However, as the estimates come from a Kalman filter the computation of the contributions is not a trivial task. We follow Koopman and Harvey (2003), who express the factors as linear combination of the observed variables.[10] In the next paragraphs we present the evolution of the LMI by phases defined by the Bry and Boschan algorithm.

[10]The idea behind this procedure, is to use the Kalman filter recursions to compute weights for the observed variables. The filtered estimator of the state vector based on information available at time $t-1$ is given by $a_{t|t-1} = \sum_{j=1}^{t-1} w_j\left(a_{t|t-1}\right) Y_j$. For the case in which the transition equation corresponds to VAR(1), the weight vectors can be computed by the backward recursion $w_j\left(a_{t|t-1}\right) = B_{t,j}K_j, B_{t,j-1} = B_{t,j}A_j - w_j\left(a_{t|t-1}\right)\Lambda_j, = t-1, t-2, \ldots, 1$, with $B_{t,t-1} = I$. The calculation of weights for smoothing follows a similar procedure to the described in the previous equations.

Figure 3: Decomposition of the LMI

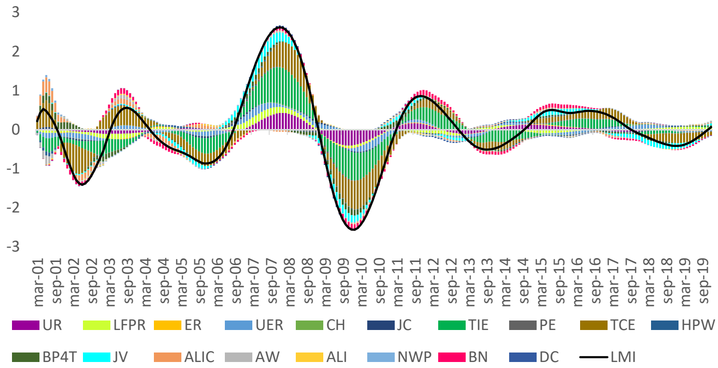


Figure 3 presents each series contribution to the LMI. We follow Koopman and Harvey (2003) to express the factors as linear combination of the observed variables.

The first is a low-growth phase that finishes in July 2002, however we cannot determine the amplitude of the fall given that this is the beginning of the sample. Seven of the available series have a negative contribution with non wage workers, the unemployment rate and vacancies leading the reduction of the index. The high-growth phase afterwards lasts fifteen months and the downturn point of the LMI improves 1.9 with respect to the previous upturn. Despite the fact that this is a high growth phase the LMI is negative in almost objective underemployment, average wage, and industrial employment the entire period. Ten series show an improvement with objective underemployment, average wage, and industrial employment showing the greatest contribution to the change. On the other hand, the unemployment, labor force participation, and occupation rates show a reduction. Labor market conditions were not good, during these two initial phases, this behaviour is consistent with general economic conditions. During 2001 and 2002 domestic demand was low and internal demand showed weak signs of recuperation. External demand was also weak as economic growth in Latin America was lower than the observed in Colombia. Additionally, the recovery was sluggish during 2003, mainly due to a better world demand. This whole period coincides with a labor market recession using the traditional definition as described in Alfonso et al. (2013).

The following low-growth phase lasts twenty two months and finishes in December 2005. The index evolution reflects the low economic growth experienced by the Colombian economy and a world demand that does not recover in 2004. Employment in the commerce and industrial sectors as well as objective underemployment lead the 1.4 reduction of the index, eight series have a negative behaviour during this period. While the labor force participation, occupation, and unemployment rates contribute positively. The fourth phase is the longest high-growth phase and the LMI reach its maximum in January 2008. In this period, all the variables but labor income for non-wage workers, show an improvement, being the highest contributions to this positive dynamic industrial and commercial employment, and the unemployment rate. This downturn point is close to the economic activity peak found by Alfonso et al. (2013). Labor market conditions improved as a result of an outstanding economic growth during 2005 and 2007, with industry and construction as the leading sectors.

The next phase is the deepest low-growth phase that also leads to the minimum of the LMI in November 2009. This period is also categorized as a recession according to Alfonso et al. (2013), annual economic growth declines to 1.7% after the 3.5% observed in 2008. During this period, industrial and commerce employment, the unemployment rate, expectations, and vacancies have the highest contributions to this decline. The upturn point lags the through found by Alfonso et al. (2013), consistent with the timing of the turning points described in Mintz (1972). The sixth phase shows a recovery of 3.37 and lasts twenty three months. Almost all series contribute to this improvement, which is lead by industrial and commerce employment and the unemployment rate, while only LFPR show a small deterioration. This period is characterized by the recovery of economic activity in 2009 and 2010, that leads to a GDP and domestic demand growths in 2011 of 5.9% and 6.5%. Commerce, industry and construction were the economic sectors with a better performance. However, improvements in job formality remains as a challenge.

After that, the LMI enters into a low-growth phase the following months and reach the upturn point in October 2013. Thirteen series show a reduction being expectations, industrial and commerce employment the series that contribute the most to the deterioration. During this period, LFPR and discouraged workers rate had a positive contribution. These years coincide with an economic deceleration, but the unemployment rate show a positive behaviour and the minimum wage increment was close to the inflation rate. The eight is a high-growth phase that finishes in August 2016 after thirty four months. The 0.98 amplitude leads to the LMI to a similar downturn point as the previous one, industrial and commerce employment and bottle necks lead the increase of the LMI. During this period, economic conditions improved as consequence of the oil price shock and an increment in government expenditure. The highest contribution to the index was given by the industrial and particular employment, job creation, and vacancies. The highest improvement was in 2014 as economic growth was 4.6% compared to 3.1% in 2015. After August 2016, the index starts a low growth phase, that coincides with a reduction in economic activity as the GDP growth falls in 2016 and 2017 to 2.0% and 1.4%. Vacancies, expectations, and industrial employment lead the reduction. However, the growth of objective underemployment, the participation rate, and the churning rate show positive signals and alleviate the reduction of the index.

The analysis of the previous paragraphs shows that the relationship of the LMI with the traditional business cycle indicators is consistent with the description provided by Mintz (1972). First, the number of phases is higher in the growth cycle approach as we found nine phases during the same period while Alfonso et al. (2013) found four phases. Second, the downturn points of the LMI lags the two through dates of the business cycle. Another important evidence is provided by the fact that besides the unemployment rate, other variables are crucial to explain the short-term dynamics during the low and high growth phases. It is noteworthy that expectations, job creation, bottle necks, and vacancies are highly important to describe the current state of the labor market.

In table 3 we assess each series' importance in the final LMI using two criteria: the Kalman gain and the correlation with the indicator. In column 2, we present the Kalman gain that measures how new information on each variable affects the estimate of the LMI indicator, while in column four we present the correlation coefficient. The key difference among them according to Chung et al. (2014) is that while the Kalman gain reflects the influence of a series on the LMI given the whole set of variables included in the estimation, the correlation reflects not only each series direct contribution but also the contribution due to the correlation with other indicators. Despite this difference, it is important to note that variables with high gain coefficients tend also to have a higher correlation with the LMI. The first five series according to the Kalman gain have a similar influence in the LMI.

Table 3: Relations between Individual Variables and the LMI

	Kalman Gain		Correlation with LMI	
Variable	K_t	Rank	ρ	Rank
BP4	-0.084	1	-0.794	2
JV	-0.083	2	-0.806	1
TIE	-0.066	3	-0.754	4
UR	0.065	4	0.77	3
BN	-0.055	5	-0.545	6
TCE	-0.052	6	-0.658	5
ER	-0.047	7	-0.273	11
ALIC	-0.046	8	-0.404	8
AW	-0.042	9	-0.365	9
PE	-0.028	10	-0.407	7
ALI	-0.019	11	-0.128	14
DC	0.017	12	0.129	13
NWP	-0.016	13	-0.021	17
CH	-0.007	14	-0.047	16
UER	0.005	15	0.279	10
JC	-0.004	16	-0.002	18
LFPR	0.003	17	0.2	12
HPW	0.001	18	-0.061	15

Table 3 presents alternative measures of the association between the LMI and the series, the second and third column presents the kalman gain and its ranking, while the third and fourth columns present the correlation of each series with the composite indicator and its ranking.

5.1 Robustness analysis

The most important issue in the construction of business cycle indicators is to determine the *correct* chronology of the turning points, which allows to determine the low and high growth phases. Thus, it is desirable that new information has little impact on the phases previously defined. Thus, in this subsection we inspect how new information affect the LMI. There are two sources that may lead to a revision of the LMI. First, there is a one month publication lag of the micro data from household survey, then when new data is available it will replace the estimates of those missing values based on the latent factors and the idiosyncratic structure.[11] The second source of revision is the estimation of the model, as it is a statistical technique that provides the best fit conditional on the information available, thus, as new information is included those estimates may change.

In order to check whether the introduction of new information affects the chronology of the LMI we conduct a rolling estimation with four samples, with lower information. The first sample includes information between March 2001 and December 2015, the second sample includes twelve additional months, that is up to December 2016, and each subsequent sample includes twelve additional months. Table 4 compares each sample turning points with those in Table 2. Columns one to four present the different samples while rows report the date of the original turning point. The numbers reported indicate for how many months each sample turning point lags the original turning point. Thus, a negative number indicate that the turning point identified with that sample leads the turning point.

First, all estimates have the same number of turning points, that is no additional upturns or downturns are reported with the sub samples analyzed. The first is the upturn in July 2002, that is similar in all samples as it is identified between June and August. The next turning point is a downturn in December 2003 that depending on the sample is identified between three months later or two months earlier. The timing is similar for the rest of upturns and downturns, with the greatest difference being a lag or lead of 2 months. In conclusion, There are no big differences in the phases defined and the LMI is robust to the introduction of new information. Thus, the introduction of new information does not affect the diagnostic made. Given that the majority of the series are published and not revised, and the estimates of the parameters of the statistical model does not change between estimations.

6 The LMI and the Colombian business cycle

In this section we provide insight of how the LMI fits when describing the effect of the business cycle on the labor market. In the first subsection we compare the LMI with the most traditional growth cycle indicators the GDP GAP and the UR GAP, which are computed as the cyclical component of the series. In the second subsection, we describe the state of the labor market combining the traditional and growth cycle definitions with a diffusion index and the LMI.

[11] The variables that are affected by this delay in publication are: job creation, churning, weekly hours, and the earnings measurements: average wage and average labor income.

Table 4: Number of months that the turning points of the LMI change with alternative samples

	December 2015	December 2016	December 2017	December 2018
July 2002 - Upturn	1	-1	-1	-1
October 2003 - Downturn	2	-1	-2	-1
December 2005 - Upturn	0	1	0	-1
January 2008 - Downturn	2	0	0	-2
February 2010 - Upturn	2	1	0	1
December 2011 - Downturn	1	-1	-1	-2
October 2013 - Upturn	2	-1	0	-1
August 2016 - Downturn	NA	0	0	1
January 2019 - Upturn	NA	NA	NA	2

Table 4 presents the difference between the dates of the turning points computed with different spans of information and those computed with information up to December 2018 that are presented in table 2.

6.1 LMI and the growth cycle in Colombia

In this subsection we compare the LMI with the cyclical component of the quarterly Gross Domestic Product and the unemployment rate GAP, averaging the monthly values of the LMI in each quarter. Panel A of the figure 4 shows our indicator and the the GDP GAP, in order to determine which economic activity oscillations are also presented in the labor market.[12] The synthetic index closely follows the GDP GAP, with synchronization of their turning points. However, there are some differences in the dynamics. First, until 2004 the GDP GAP was below its potential and steadily increases, while the LMI is negative but close to its trend. This might be caused by the lack of information before 2001 of the series included in our index. As the economy was getting out of the 1998 - 1999 recession, series back-casting may not able to recover the crisis and the posterior adjustment. The second difference is presented between 2013 and 2014 where the GDP GAP quickly increases and the LMI does not respond at the same speed. This may be caused by the fact that the improvement in the economic activity was mainly originated by the oil shock and may have not fully spread to labor intensive economic activities. Finally, between 2016 and 2018 the GDP GAP shows a deterioration that is not reflected in the aggregate labor market indicator.

Panel B compares the LMI and the unemployment rate GAP, computed as the deviation of the observed unemployment rate with respect to an average of five measures of the NAIRU.[13] In this panel then we compare whether the Unemployment rate and the LMI provide the same signal about the current status of the labor market, as it is expected there is a negative correlation as a higher unemployment rate GAP signals a deterioration of the labor market. Some differences emerge from this comparison. The first difference is that the UR GAP has a higher volatility than the LMI, this is consistent with different facts previously discussed, *i)* the LMI captures more dimensions of the labor market and therefore has more variables to adjust, *ii)* The LMI does not include the variation from idiosyncratic factors that may remain in the UR. The second difference is that the

[12]The GDP GAP corresponds to its cyclical component of and is computed using the Hodrick and Prescott filter.

[13]The measures of NAIRU included are computed following: Shimer (2012); Ball and Mankiw (2002); King and Morley (2007); Perry (1970) and Julio (2001) as presented in Arango and Flórez (2016).

improvement of the UR gap between 2001 and 2007 is not reflected in the LMI. As in the previous case the difference in the trend might be caused by the issue of the beginning of sample but some other factors that might contribute to these effect is the higher flexibility that the Colombian labor market undergoes at the beginning of the century given a series of reforms that reduce costs associated to hiring, separations, and working hours. Despite this changes are mostly structural, the cyclical component may also be affected until economy reach its new equilibrium. After 2007 both indicators exhibit a similar evolution, with very close turning points. Finally, in the most recent years while the unemployment rate is higher than the average of the NAIRU measures, signaling a deterioration of the labor market the LMI shows that its momentum is close to the trend as some other variables counter-off the reduction of the unemployment rate.

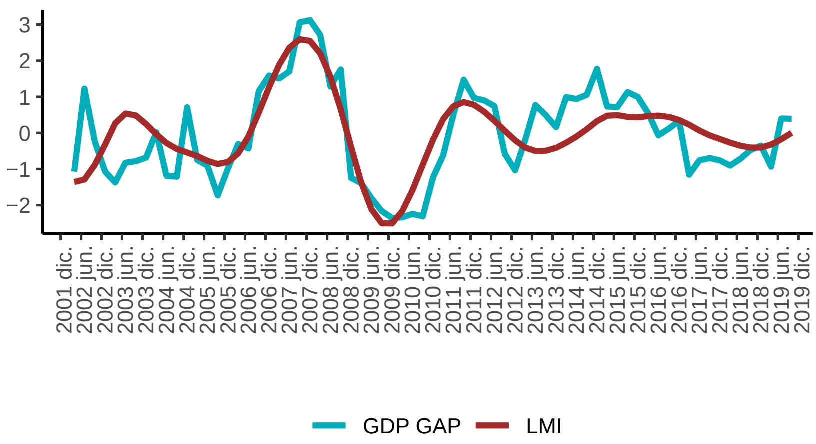
Then when diagnosing the status of the growth cycle for economic activity and the labor market in recent years, we can see that while the economic growth not experienced a positive behaviour as the GDP GAP remains opened between 2016 and 2018, the labor market signals a better momentum, specially if we consider the aggregate indicator including the information of expectations, industrial employment, and vacancies which have not deteriorate as much as the unemployment rate.

6.2 The LMI and the diagnostic of the labor market momentum

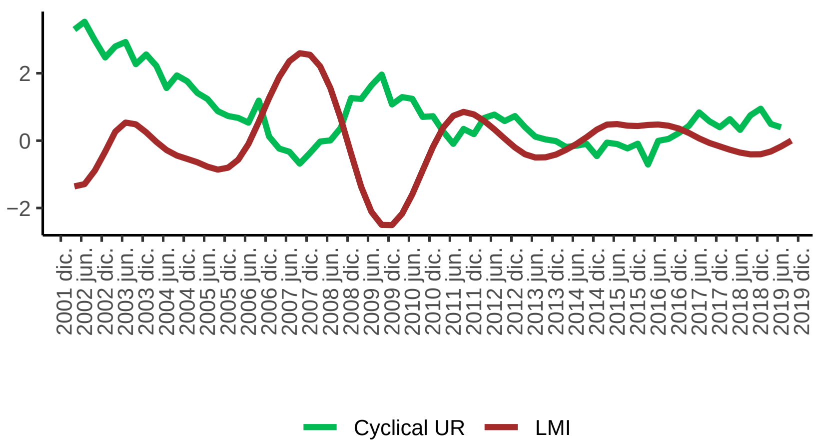
As discussed in section 4, while the LMI diagnoses the state of the cyclical component of the Labor Market, the traditional business cycle indicators diagnoses the state of the overall state of the economy, as series are considered in levels. Thus we can combine the signals that provide alternative indicators to get a better picture of the current state of the labor market. There are two features of the LMI that will contribute to policy analysis. First, the LMI is useful to give more information during expansions, as it characterizes them as low or high growth phases, second, as the downturns tend to lead peaks they might be an early signal of the beginning of a recession, which has been used by Anas and Ferrara (2004) to construct the ABCD approach to predict signs of deterioration that might lead to a recession. Understanding then these connections is very useful for economic policy design and analysis.

Thus, to provide a more complete analysis of the labor market we introduce a diffusion index and we compare the phases of the LMI with those obtained in the diffusion index (e.g. Alfonso et al. (2013)). The diffusion index is computed as the percentage of series for which the annual change improves minus the percentage of series for which conditions worsen, then if the index is above zero the majority of the series are improving but if the index is negative it may indicate a recession. Then, when the index crosses zero it marks a turning point, to determine the business cycle phase we also consider the duration of the phases in the same fashion than in the Bry and Boschan (1971) algorithm.

Figure 4: LMI and cyclical component of quarterly reference series



(a) Panel A



(b) Panel B

Figure 5: Diffusion Index of the labor market

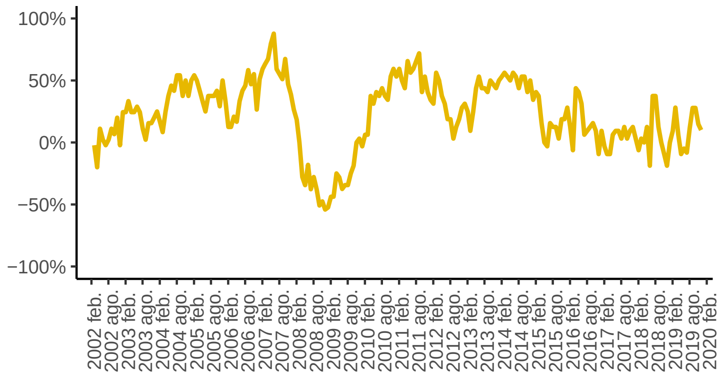


Figure 5 presents the LMI between march 2001 and December 2019.

Figure 5 shows that between 2001 and 2019 we identify one recession that starts in June 2008 and finishes in January 2009, then the period before and after that recession is considered as an expansion. The relationship between both indices is consistent with the predictions of economic theory as: *i)* there are more phases in the LMI than in the diffusion indicator, *ii)* the peak of the diffusion index occurs in June 2008, that lags by five months the downturn point (January 2008), and *iii)* the trough in the diffusion index, July 2009, leads by four months the upturn point (October). Moreover, the LMI shows its maximum and minimum at the turning points associated with the peak and through of the diffusion index, which may indicate that in those periods the permanent and transitory components of the labor market experience a similar behavior.

7 Conclusions

The unemployment rate is not the best indicator to track the current state of the labor market. The existence of rigidities impede labor to freely flow among occupations, for instance in developing countries with an important informal sector, workers who lose their job go out the active labor force or engage in informal activities, movements that are not captured by the UR. In this document we present the Labor Market Index as a tool to analyze of the current state of the labor market focusing on the cyclical similarities of a broad set of labor market series, removing the trend which might obscure the similarities in the cyclical behaviour. Its construction is based on a two step methodology similar to Chung et al. (2014), extracting the maximum common variation in the first stage and computing the first principal component of the projected series in the second stage. This indicator is based on the assumption that the momentum of the labor market is described by latent factors that drives the evolution of all variables. The proposed methodology accounts for missing observations, optimally determine the influence of each series into the final indicator, and remove idiosyncratic behaviour that will contaminate the evolution of the index. Our estimations show ten phases between 2001 and 2019, in which industrial and commercial employment, expectations, the presence of bottle necks, and the unemployment rate play an important role in the evolution of the indicator.

Moreover, the evolution of the LMI is similar to that of the most traditional growth cycle indicators the GDP and the unemployment rate gaps, despite there are some differences that we attribute to the beginning of the sample and methodological differences. Then, we want to investigate what causes the differences in other periods, that is, what variables explain the discrepancies between the signals of the LMI and the cyclical component of the UR, as during the 2016 - 2018 period in which the negative conditions that lead to a negative GDP GAP, have not been present in the labor market as were in the UR GAP. These differences are explained by the positive contribution of vacancies, industrial and commerce employment. Moreover, the LMI helps to characterize better the state of the business cycle, combining it with a diffusion index, we see that before the 2008 expansion there were four growth cycle phases, the majority with a modest growth, with the higher variation in the high-growth phase that coincides with the end of the expansion. Afterwards, the recession also coincides with the low-growth phase that leads to the minimum of the LMI. The next expansion in our diffusion indicator have had five growth cycle phases, none with similar amplitude to the ones that marks the end of the expansion and lead to the crisis. Moreover, the timing between the turning points behaves as predicted by Mintz (1972) as downturn leads the peak and upturns lag the through. natbib

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A State space representation of the Dynamic Factor Model

The dynamic factor explained in the text for n_v variables is

$$X_{t(n_v,1)} = \Lambda_{(n_v,n_f)} F_{t(n_f,1)} + u_{t(n_v,1)} \tag{5}$$

$$F_{t(n_f,1)} = \sum_{i=1}^{p_f} A_{i(n_f,n_f)} F_{t-i(n_f,1)} + \eta_{f(n_f,1)} \tag{6}$$

$$U_t = \sum_{j=1}^{p_u} C_{j_{(n_v,n_v)}} U_{(t-j)} + \eta_{u(n_v,1)} \tag{7}$$

Using a standard state space representation, we redefine the above model in the following two equations:

$$Y_t = Z\alpha_t \tag{8}$$

$$\alpha_t = T\alpha_{(t-1)} + H\epsilon_t \tag{9}$$

Equation 8 is know as the measurement equation that linearly relates the observed variables and the states: $Y_t = X_t\alpha_t$, with $\alpha_t = \left\{F_t, F_{t-1}, F_{t-p_f}, U_t, U_{t-1}, ..., U_{t-p_u}\right\}'$
and $Z = \left\{\Lambda_{(n_v,n_f)}, O_{(n_v,n_f(p_f-1))}, I_{(n_v}, O_{(n_v,n_v(p_u-1))}\right\}$, equation 8 can be rewritten as:

$$Y_{t(n_v,1)} = [\Lambda_{(n_v,n_f)}, O_{(n_v,n_f(p_f-1))}, I_{(n_v,n_v)}, O_{(n_v,n_v(p_u-1))}] \begin{bmatrix} F_t \\ F_{t-1} \\ \vdots \\ F_{t-p_f+1} \\ U_t \\ U_{t-1} \\ \vdots \\ U_{t-p_u+1} \end{bmatrix}_{(n_f p_f + n_v p_u,1)}$$

Equation 9 is known as the transition equation and describes the dynamic of the state vector α_t, In this particular case this will be:

$$
\begin{bmatrix} F_t \\ F_{t-1} \\ \vdots \\ F_{t-p_t+1} \\ U_t \\ U_{t-1} \\ \vdots \\ U_{t-p_u+1} \end{bmatrix} = \begin{bmatrix} \begin{matrix} A_1 \dots A_{p_f-1} & A_{p_f} \\ I_{n_f}(p_f-1) & O_{n_f \times (p_f-1), n_f} \end{matrix} & O_{(n_f \times p_f, n_v \times p_u)} \\ O_{(n_v \times p_u, n_f \times p_f)} & \begin{matrix} C_1 \cdots C_{p-1} & C_{p_u} \\ I_{n_v \times (p_{u-1})} & O_{(n_v \times (p_u-1), n_u)} \end{matrix} \end{bmatrix} \begin{bmatrix} F_{t-1} \\ F_{t-2} \\ \vdots \\ F_{t-p_f} \\ U_{t-1} \\ U_{t-2} \\ \vdots \\ U_{t-p_u} \end{bmatrix} +
$$

$$
\begin{bmatrix} I_{n_f} & 0_{n_f, n_f \times (p_f-1)} & 0_{n_f, n_v} & 0_{n_f, n_v \times (p_u-1)} \\ 0_{n_f \times (p_f-1), n_f} & 0_{n_f \times (p_f-1), n_f \times (p_f-1)} & 0_{n_f \times (p_f-1), n_v} & 0_{n_f \times (p_f-1), n_v \times (p_u-1)} \\ 0_{n_v, n_f} & 0_{n_v, n_f \times (p_f-1)} & I_{n_v} & 0_{n_v} \\ 0_{n_v \times (p_u-1), n_f} & 0_{n_v \times (p_u-1), n_f \times (p_f-1)} & 0_{n_v \times (p_u-1), n_v} & 0_{n_v \times (p_u-1), n_v \times (p_u-1)} \end{bmatrix} \begin{bmatrix} \eta_f \\ 0_{n_f \times (p_f-1)} \\ \eta_u \\ 0_{n_v \times (p_u-1)} \end{bmatrix}
$$

Thus we define

$$
T = \begin{bmatrix} \begin{matrix} A_1 \cdots A_{p_{f-1}} & A_{p_f} \\ I_{(n_f \times p_f-1)} & 0_{(n_f \times p_f-1, n_f)} \end{matrix} & 0_{(n_f \times p_f, n_v \times p_u)} \\ 0_{(n_v \times p_u, n_f \times p_f)} & \begin{matrix} C_1 \cdots C_{p-1} & C_{p_u} \\ I_{(n_v \times p_u-1)} & 0_{(n_v \times p_u-1, n_u)} \end{matrix} \end{bmatrix},
$$

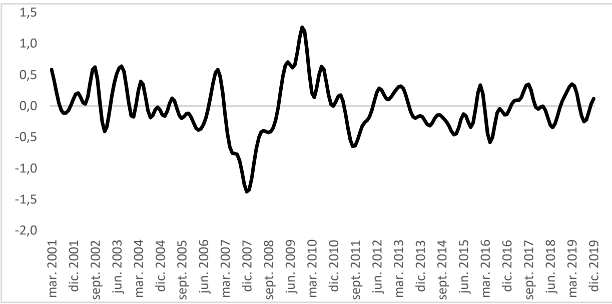
$$
H = \begin{bmatrix} I_{n_f} & 0_{n_f, n_f \times (p_f-1)} & 0_{n_f, n_v} & 0_{n_f, n_v \times (p_u-1)} \\ 0_{n_f \times (p_f-1), n_f} & 0_{n_f \times (p_f-1), n_f \times (p_f-1)} & 0_{n_f \times (p_f-1), n_v} & 0_{n_f \times (p_f-1), n_v \times (p_u-1)} \\ 0_{n_v, n_f} & 0_{n_v, n_f \times (p_f-1)} & I_{n_v} & 0_{n_v} \\ 0_{n_v \times (p_u-1), n_f} & 0_{n_v \times (p_u-1), n_f \times (p_f-1)} & 0_{n_v \times (p_u-1), n_v} & 0_{n_v \times (p_u-1), n_v \times (p_u-1)} \end{bmatrix},
$$

and

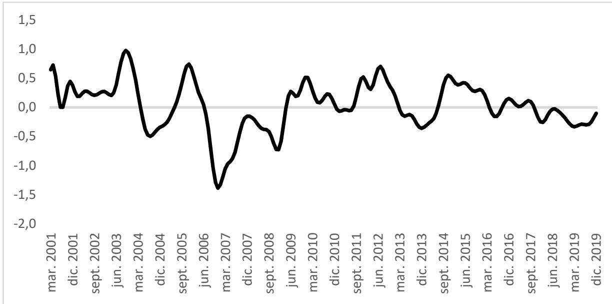
$$
\epsilon_t = \begin{bmatrix} \eta_f \\ 0_{n_f \times (p_f-1)} \\ \eta_u \\ 0_{n_v \times (p_u-1)} \end{bmatrix}
$$

Where the matrices Z, T, and H are called design matrices.

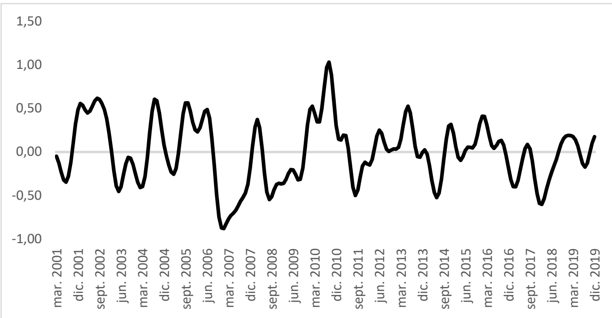
B cyclical components included in the construction of the LMI



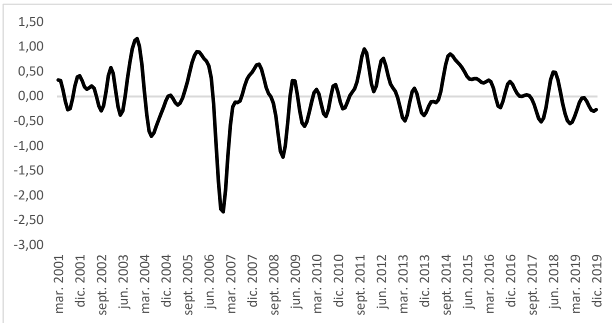
Unemployment rate



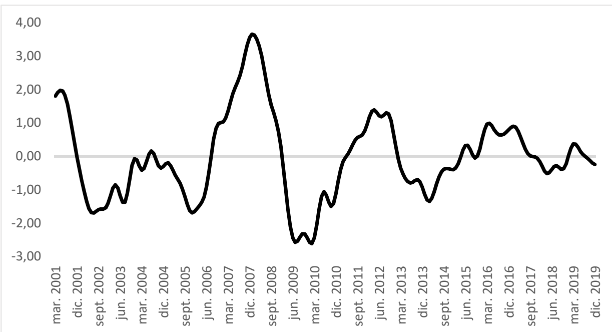
Labor force Participation rate



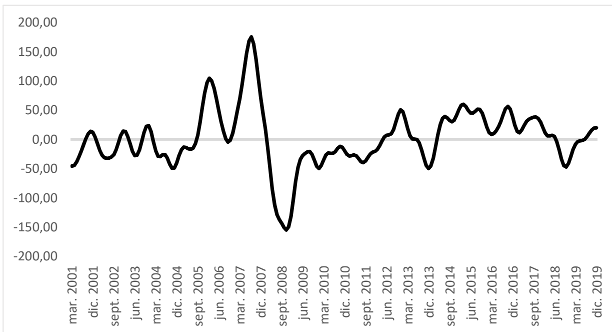
Objective Underemployment by hours



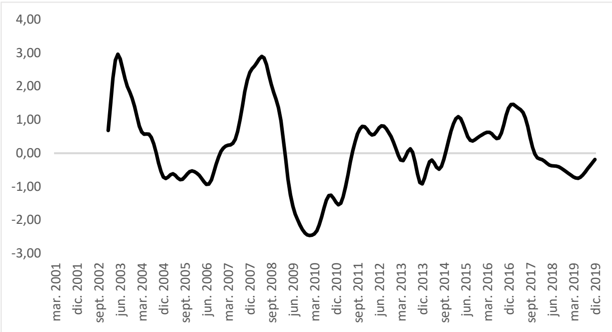
Occupation Rate



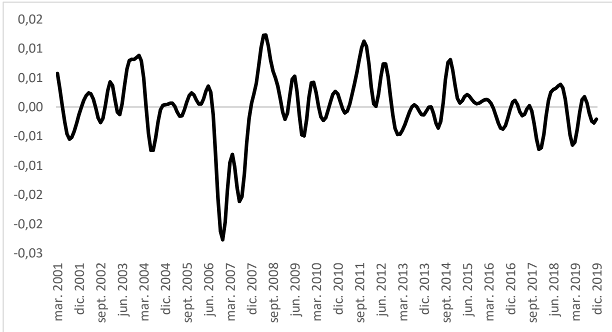
Total employment - Industry



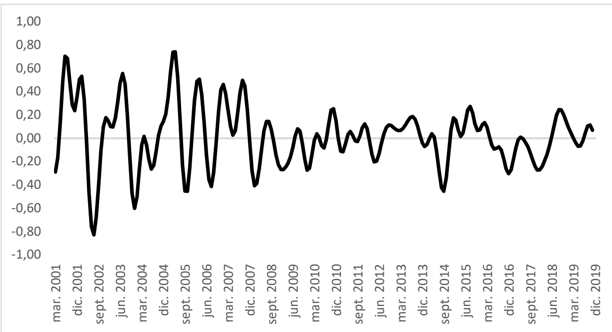
Private Employment



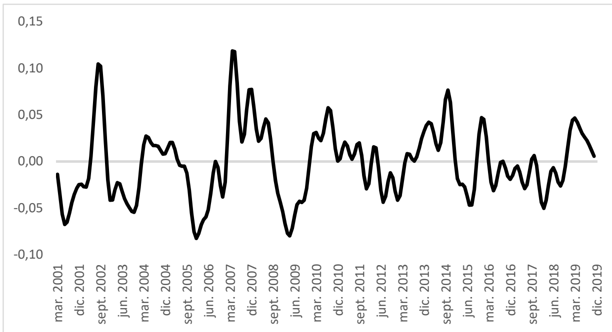
Total employment - Commerce



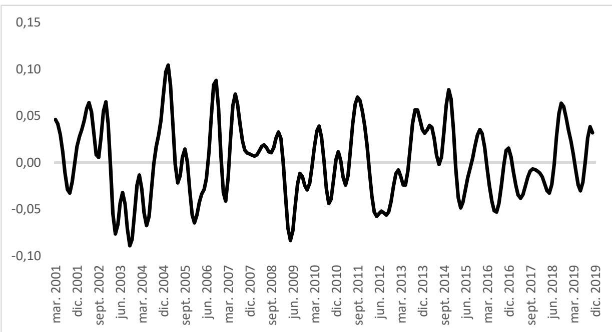
Proportion of non-wage workers



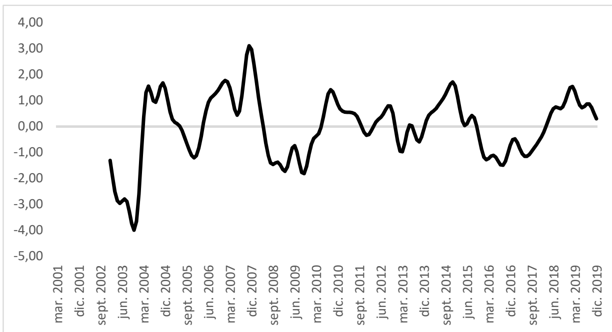
Average weekly hours - wage workers



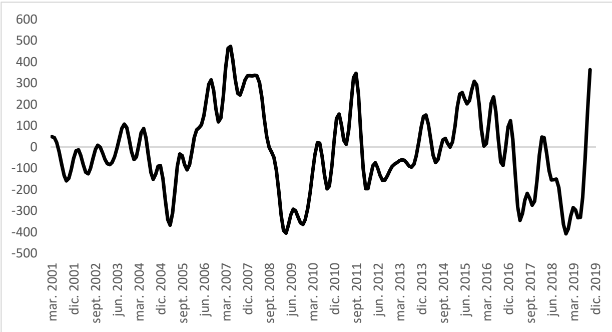
Average wage



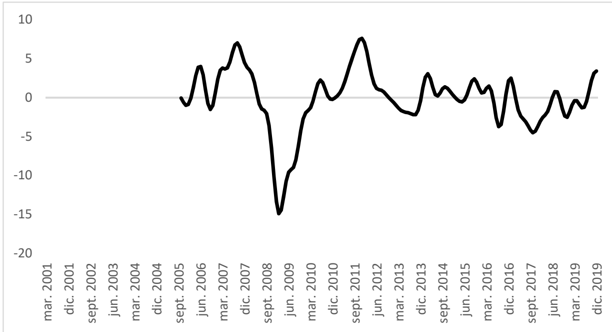
Average labor income



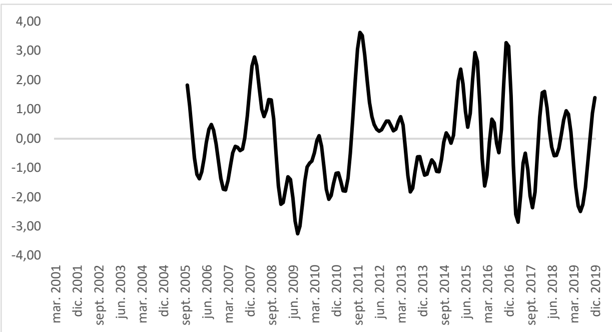
Labor income commerce



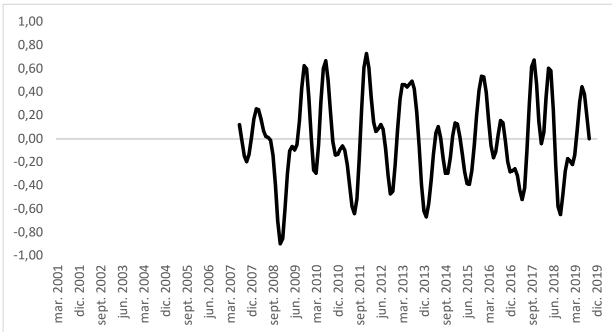
Job Vacancies



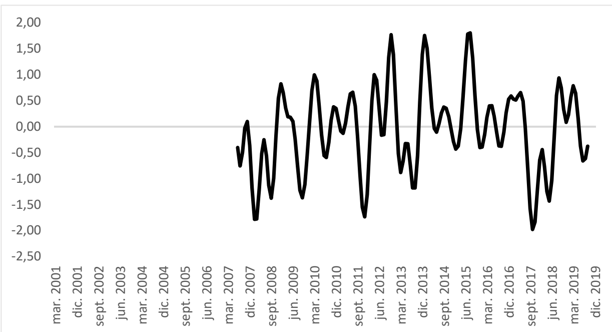
Business expectations



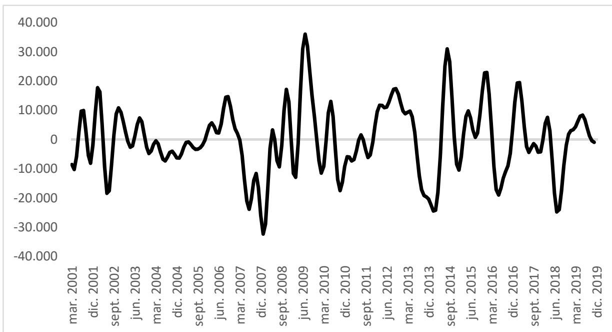
Bottle necks



Job creation



Churning



Discouraged workers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