Present-biased Government, Creative Accounting and a Pitfall in Balanced Budget Rules

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Present-biased Government, Creative Accounting and a Pitfall in Balanced Budget Rules*

Marcela De Castro-Valderrama†

Abstract

I propose a general equilibrium model with a quasi-hyperbolic discounting government that optimally decides whether or not to use creative accounting in order to evaluate a balanced budget rule and a debt rule. In that context, I find that a binding balanced budget rule could fail to properly constrain public overindebtedness when government uses creative accounting while a debt rule is effective, since targets are set on total public liabilities. Results suggest that a balanced budget fiscal rule can also deteriorate welfare due to the higher interest rates derived from doing operations under the line, implying future expenditure cuts that are harmful for households, who value public goods and services. A debt rule is also preferred for its capacity to reverse some welfare losses generated by the present-biased government.

JEL Codes: E61 E62 G28 H61 H63 E21

Keywords: Quasi-Hyperbolic Discounting, Creative Accounting, Balanced Budget Rule, Fiscal Policy, Public Overindebtness, Welfare Analysis.

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Falencia de las reglas fiscales sobre el balance: gobierno con sesgo al presente y contabilidad creativa*

Marcela De Castro-Valderrama†

Resumen

En un modelo de equilibrio general con un gobierno que descuenta cuasi-hiperbólicamente se evalúan dos reglas fiscales. Los resultados sugieren que una regla fiscal sobre el balance activa y vinculante no garantiza acotar el crecimiento del endeudamiento público cuando el gobierno puede hacer trucos contables y puede generar mayores pérdidas de bienestar en la economía. Por el otro lado, una regla sobre la deuda es efectiva y logra recuperar parte de la pérdida de bienestar generada por el gobierno que descuenta cuasi-hiperbólicamente.

Códigos JEL: E61 E62 G28 H61 H63 E21

Palabras Clave: Descuento cuasi-hiperbólico, Contabilidad Creativa, Regla Fiscal sobre el balance, Política Fiscal, Sobre endeudamiento público, Análisis de bienestar.

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1 Introduction

Public debt growth has been fast in the past 50 years for emerging economies, reaching a historic peak of about 170% of GDP by 2018 (Kose et al., 2021), and advanced economies, whose public debts after the financial crisis surged to levels not recorded since the end of World War II (Reinhart and Rogoff, 2011). Furthermore, 85 countries had implemented new fiscal rules since the nineties,[1] being the balanced budget rule the most commonly used rule: from the 92 countries having at least one active fiscal rule in 2015, 31 countries had a balanced budget rule at a national level and 23 at a supranational level. Provided that the fiscal deficit should, in principle, be equal to government's debt growth, a balanced budget rule imposing a deficit cap, indirectly endeavors to set a limit on public debt. However, since the 1980's, large discrepancies between deficits and the annual change in public debt have been observed (Weber, 2012), raising the question of whether implementing a balanced budget rule sufficiently contains public overindebtedness and reverses the welfare losses generated by a deficit bias.

Taking into account that reported fiscal aggregates can differ from real ones and that some fiscal rules leave space for government to do creative accounting, I propose a general equilibrium model to explain why a binding balanced budget rule not only could fail to constrain public debt growth but can also deteriorate welfare. First of all, the model includes a government that discounts quasi-hyperbolically, this assumption resembles the existence of political dispersion leading to a deficit bias, justifying the imposition of a fiscal rule. Second, in the model the government optimally decides whether or not to use creative accounting as a way to cheat the debt and spending limits implicitly imposed by a balanced budget rule. The quasi-hyperbolic discount drives government to be present-biased and act time-inconsistently, generating harmful distortions for households due to the overborrowing made by the government to finance a higher spending in comparison to the case where government has not such bias. Nevertheless, when a quasi-hyperbolically discounting government is constrained by a balanced budget fiscal rule attempting to undo the distortion and control the excessive spending, the government uses creative accounting, and public debt will

[1] See IMF fiscal rules data base https://www.imf.org/external/datamapper/fiscalrules/map/map.htm.

still be higher than in the case where the government is not present-biased. In fact, this ineffectiveness of the balanced budget rule might lead to a welfare deterioration, since creative accounting can be costly and enlarges the welfare losses arising from the present-bias distortion.

Studying this outcome is critical to properly assess fiscal rules' effectiveness and sheds light on why and how they might fail. In theory, fiscal rules aim to counter the deficit bias derived from government's shortsightedness due to common pool problems or time preference heterogeneity among economic agents, which accounts for excessive overborrowing, even in good times (Alesina and Passalacqua, 2016; Eslava, 2011; Cangiano et al., 2013; Eyraud et al., 2018). Precisely, an incumbent seeking to gain political power in response to the many interest groups of agents in an economy, overborrows with respect to the case where government does not have any political incentives to overspend and therefore, is not present-biased. Since fiscal rules are the main mechanism to restrain the government from reaching excessive deficits and accomplish fiscal sustainability, it is imperative to fully understand the potential pitfalls of fiscal rules in dissipating the deficit bias. To do so, the model I propose allows a general equilibrium analysis of the macroeconomic implications of implementing fiscal rules and evaluate their capacity to retrieve the welfare losses on behalf of the present-biased government decisions.

Moreover, introducing creative accounting in a general equilibrium environment is a salient feature of this model and is essential for evaluating balanced budget rules due to the fact that discrepancies between deficit and changes in public debt may occur as a result of the so-called stock-flow adjustments (SFA), a form of creative accounting (Milesi-Ferretti and Moriyama, 2006; Von Hagen and Wolff, 2006). Specifically, since SFA is the residual among deficits and debt growth, it is a commonly used tool whereby governments hide *real* deficits. For example, Weber (2012) using panel data for 163 countries finds that SFA were often positive and persistent, showing significant source of debt increases that broaden the discrepancy among debt growth and budget deficit.[2] What is more, Maltritz and Wüste (2015) show that fiscal rules have induced the government to use SFA adjustments in the

[2] There are several other papers supporting these results, see for example Abbas et al. (2011); Buti et al. (2007); Reischmann (2016); Afonso and Jalles (2020); Campos et al. (2006).

European Union, supported by findings of Von Hagen and Wolff (2006) who also find that during crisis, where fiscal rules targets are specially relevant and binding, there was a systematic significant use of SFA influencing disparity among fiscal deficits and debt growth.[3]

As stated above, both a present-bias government and its ability to use creative accounting are modeled in a general equilibrium environment. The model is a closed economy model with a firm producing the private and public consumption good, a household that consumes, supplies labor and lends resources to the government, which receives revenues from the labor income tax or by issuing debt while spends in public goods and services or debt repayment. In addition, the government can choose to use SFA, that create a wedge among the reported deficit and total debt growth. Nonetheless, using accounting gimmicks increases debt repayment costs, so government will only use them if the marginal benefit of overborrowing exceeds the marginal cost of a higher interest rate. In that case, the government decides to overspend and still can meet the fiscal rule deficit targets that is introduced to control government present-bias due to its quasi-hyperbolic discounting.

The main results emanating from the model are explained as follows. Fiscal rules not constraining total government liabilities can generate additional welfare losses because of the excessive debt acquired, due to rule malfunctioning, as well as the higher cost of debt repayment caused by the use of creative accounting. As a result of high debt levels and its high repayment costs derived from a quasi-hyperbolic discounting government doing creative accounting, a bigger government expenditure cut will be needed to repay public obligations, which is harmful for consumers that value public goods and services. Also, given that households own government's total debt and receive a higher interest rate, they reduce labor supply and therefore, government revenues fall, narrowing future public spending even more.

The results also suggest that if a debt rule limits total public liabilities, this rule is preferred for its effectiveness and its capacity to reduce the welfare losses (albeit not entirely), that are caused by a present-biased government. The latter is due to the fact that a present-

[3]See also Beetsma et al. (2009).

biased politician demands more credit than a government not facing heterogeneity among agents and has no incentives to overspend or gain political power. Therefore, when constraining a present-biased government with a debt fiscal rule, debt sustainability is guaranteed at the expense of a higher interest rate compensating for the elevated credit demand, which increases debt repayment cost and hinders the complete restoration of welfare losses.

The main contribution of this paper is a theoretical general equilibrium model that includes a present-biased government which can use creative accounting, two elements that, to the best of my knowledge, have not been put together in a model. Provided that debt-to-GDP ratios have increased since the seventies, even during peace times (Alesina and Passalacqua, 2016), and that there is evidence of systematic use of SFA explaining the disconnection between fiscal budgets and public debts, including the two aforementioned characteristics is fundamental to do a proper assessment of fiscal rules. Moreover, in this setup, agents respond endogenously to fiscal policy actions when fiscal rules are implemented. This is particularly important given that it enables a tractable analysis of the channels through which costs and benefits operate. In order to elucidate and fully analyze these channels, I perform a three period simulation of the model with a government constrained by a balanced budget fiscal rule and a debt rule. These simulations are useful to evaluate the aggregate macroeconomic dynamics and the welfare implications of the equilibrium.

In the first place, my paper contributes to the strand of the literature that evaluates fiscal rules under a quasi-hyperbolic discounting government, by allowing it to optimally decide to use creative accounting as a way to trick the debt limits of a balanced budget rule. For example, without including a government doing creative accounting, Huber and Runkel (2008) find that welfare effects of a balanced budget rule are ambiguous when government discounts quasi-hyperbolically. Alternativelly, Amador et al. (2006) and Halac and Yared (2014) find that under a quasi-hyperbolic discounting government and asymmetric information, the optimal rule is a debt cap rule. Nevertheless, they do not include the endogenous responses of households upon the actions of a constrained and present-biased government nor the existence of SFA and its costs.

Additionally, Bisin et al. (2015) present a model where government accumulates debt to respond to consumers' desire to undo their commitments of using liquid assets as a way to solve their self-control problems. Even though their results offer a rationale for balanced budget rules to restrain the present-biased governments, the authors are not taking into account a government able to use creative accounting to meet the rule and still overspend. Similarly, Azzimonti et al. (2016), based on Battaglini and Coate (2008), microfound the political economy problem leading to the government shortsightedness and find that a balanced budget rule leads to a gradual reduction of debt. However, without including creative accounting, welfare can be diminished by the greater volatility and distortions caused by the tax reforms needed to meet the rule in that context. Furthermore, Alfaro and Kanczuk (2017) evaluate deficit and debt rules in a default model with a quasi-hyperbolic discounting government. Despite that their findings suggest that a debt rule can yield welfare gains while a deficit rule does not perform well, but the welfare costs of a balanced budget rule are not derived from the government doing creative accounting, since the model does not include this possibility.

On the other hand, my paper also contributes to the existing literature by developing a model that microfounds the present-biased government's optimal decision of doing creative accounting. Models that consider government's decision of whether to do or not creative accounting as a way to deviate from fiscal rules are limited, and despite that there are a few ones, the reasons leading governments to engage in such financial gimmicks are different than the ones I propose in this paper. For instance, in Milesi-Ferretti (2004) government's decision of violating the fiscal rule, by doing creative accounting, depends on the costs and probabilities of being detected and its subsequent reputation costs. Additionally, he evaluates the welfare costs of deviating from the targets imposed by a fiscal rule in a given welfare loss function without taking into account the endogenous responses to fiscal policy by private agents. Also, Buti et al. (2007), model a government where the optimal decision of deviating from the fiscal rule depends on the transactional costs and benefits of doing financial gimmicks. However, the purpose of the model is not to evaluate fiscal rule effectiveness or welfare, instead, the intuition of the model is used to bolster the positive relation among deficits and use of SFA observed in the data.

The present paper is structured as follows. Section 2 describes the model and the main intuition of the optimal response of government upon the balanced budget and debt fiscal rules. Section 3 presents the three period model simulation results; in this section, the macroeconomic dynamics are presented first, followed by a welfare analysis and discussion. Finally, Section 4 concludes.

2 The Model

The model in this paper is a closed economy model with three agents. A firm hires labor to produce the final good that is bought by households and the government. A household consumes, saves, works and receives utility from public goods and services. Finally, government receives revenues from the labor tax and also issues non-contingent one-period debt, which is held entirely by households, while its expenditures are public spending and debt service and repayment. In the model, government maximizes public spending from household's utility function subject to the government's budget constraint but is present-biased due to a quasi-hyperbolic discount as modeled in Halac and Yared (2014); Amador et al. (2006); Alfaro and Kanczuk (2017). To undo the distortions created by the quasi-hyperbolic discounting that lead to a deficit-bias, in some of the presented cases government is also subject to a fiscal rule.

2.1 Private Sector

2.1.1 Firms

The representative firm produces the final good using a linear production function with labor as its only input. Firms maximize their benefits in perfect competition subject to the following production function: $Y_t = A_t N_t$, where A_t is a deterministic time-varying productivity and N_t is employment. The problem of the firm is given by:

$$\Pi_t = \max_{N_t} Y_t - W_t N_t$$

Since firms operate in perfect competition, their benefits are zero ($\Pi_t = 0$), consequently, in the optimum firms demand labor inelastically and real wage W_t must equal marginal productivity:

$$W_t = A_t \tag{1}$$

2.1.2 Households

The representative household consumes the final good, decides how much to work to receive a labor income which is taxed, and saves by lending money to the government, that is, owns government's total debt. Finally, households dislike working and value public goods and services (i.e. government spending is included in the utility function). Thus, the household problem is given by:

$$\max_{\{C_t, N_t, D_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t U(C_t, N_t, G_t)$$

Subject to:

$$C_t + D_t = N_t W_t (1-\tau) + D_{t-1}(1 + r_t^D)$$

where $\beta \in (0,1)$ is the discount factor, C_t is private consumption, N_t are hours worked, G_t government spending, D_t is total public debt, τ is the labor tax rate and r_t^D the endogenous real interest rate. Utility function is given by:

$$U(C_t, N_t, G_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \chi \frac{G_t^{1-\theta}}{1-\theta}$$

Household's first order conditions are:

$$\frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} = \beta(1 + r_{t+1}^D) \tag{2}$$

$$\psi N_t^{\gamma} = C_t^{-\sigma} W_t (1-\tau) \tag{3}$$

$$C_t + D_t = N_t W_t (1-\tau) + D_{t-1}(1 + r_t^D) \tag{4}$$

Equation (2) is the Euler equation in terms of marginal utility of consumption and shows that the inter-temporal private consumption decision. This equation shows that household's savings, that are lent to government, depend on the interest rate and consumers' impatience.

Additionally, equation (3) is the optimal labor decision of households and shows that the marginal disutility of labor equals the the marginal benefit of working, which is given by the marginal utility of consumption derived from receiving a labor income net of taxes.

2.2 Government

This agent provides public goods and services with revenues coming from a distorting labor income tax and by issuing non-contingent one-period debt. Government acts as a benevolent agent, considering it maximises household's utility function, though, it is not a central planner since it is not willing to choose the tax rate that minimizes the distortions created by the labor tax nor its objective function is a welfare loss function. On the contrary, government takes labor tax rate as given and is present-biased due to a quasi-hyperbolic discount that accounts for a aggregation of heterogeneous, time consistent citizens' preferences as demonstrated by (Jackson and Yariv, 2014).[4] Finally, government is deciding whether to use or not creative accounting as a way to finance part of its expenditures by choosing the amount of SFA, which creates a wedge among the reported debt growth and budget deficit. The precise definitions of government accounting are presented first, followed by the problem of the government.

2.2.1 Accounting identities

Government's "reported" fiscal balance differs from the total "real" balance[5] since government can decide to do creative accounting to make its reported deficit look appealing while financing its total deficit by issuing additional debt. This is explained by the fact that government's deficit is a net concept in the sense that excludes financial transactions, while debt is measured in gross terms including all fiscal liabilities. The following definitions formally present the relationship among deficits, debts and SFA with the notation used for

[4]There are other reasons leading to agents' heterogeneity or political frictions accounting for deficit-bias. The quasi-hyperbolic discounts resembles any political dispersion as microfunded by Alesina and Tabellini (1990); Azzimonti et al. (2016); Bisin et al. (2015); Battaglini and Coate (2008); Jackson and Yariv (2014, 2015), among others. See Alesina and Tabellini (1990); Eslava (2011) for a survey.

[5]This also the case in the model proposed by Milesi-Ferretti (2004), who distinguishes among measured and economic "meaningful" fiscal balances.

this paper.

Definition 1. Government's *real total balance* is the government's budget balance of the total fiscal aggregates:

$$G_t + r_t^D D_{t-1} - \tau_t W_t N_t = D_t - D_{t-1} \equiv DEF_t \tag{5}$$

Where D_t and G_t are real total debt and real public expenditure respectively, and r_t^D is the interest rate of total debt.

Equation (5) shows that the total deficit is equivalent to the total debt growth. However, if government decides to under-report its expenditures, that is, if government reports g_t instead of G_t and $g_t < G_t$, the debt growth consistent with the reported deficit differs from the one consistent with the real total balance.

Definition 2. Government's *reported total balance* $\overline{DEF_t}$ is the government's reported budgetary expenditure and revenues:

$$r_t^b b_{t-1} + g_t - \tau_t W_t N_t = b_t - b_{t-1} \equiv \overline{DEF_t} \tag{6}$$

Where b_t is the variable that accounts for the issued debt that consistently finances the reported deficits and r_t^b the corresponding interest rate.

However, in practice, (6) might not be equal to the total debt growth since there are other debt-creating flows affecting total debt besides from the reported deficit.[6] That discrepancy between total government's liabilities and report-consistent debt is a consequence of a government that finances part of its deficit under the line using creative accounting.

Definition 3. *Stock-Flow Adjustments* SFA_t are the residuals among total debt growth and reported deficit.

$$\underbrace{D_t - D_{t-1}}_{DEF_t} - \underbrace{(b_t - b_{t-1})}_{\overline{DEF_t}} \equiv SFA_t \tag{7}$$

[6]This is also the definition presented in Von Hagen and Wolff (2006).

Using creative accounting implies that total debt is equal to the debt consistent with reported deficits plus the stock of debt that results from doing accounting gimmicks (F_t):

$$D_t = b_t + F_t \tag{8}$$

where $F_t > 0$, based on the definition of Milesi-Ferretti (2004) whereby a financing operation is considered creative accounting if it does not imply an improvement in the intertemporal budgetary position of the government sector (i.e the government's net worth decreases when using it).[7] Note that SFA_t account for an increase of government's debt between period t and $t-1$ by more than what is implied by the reported deficit, therefore $F_t = F_{t-1} + SFA_t$.[8]

Additionally, the repayment cost of the total debt deviates from the interest rate r_t^b corresponding to b_t, since total debt does take into account the complete government's liabilities and therefore, a higher debt must imply a higher interest rate. Also, markets penalize a government that under-reports fiscal aggregates, and require a higher return when government does creative accounting as shown by Bernoth and Wolff (2008), who find that creative accounting significantly rises country's risk premium.

Assumption 1. Using SFA increases debt's interest rate. The interest rate of the issued debt that is acquired by doing creative accounting (r_t^F) has an ad-hoc risk premium component that increases exponentially as more accounting gimmicks are done by government.

$$1 + r_{t+1}^F = (1 + r_{t+1}^b) \underbrace{e^{\kappa F_t}}_{\text{risk premium}} \tag{9}$$

where $\kappa > 0$ represents the exponential factor growth of risk premium in response to the use of accounting gimmicks. This assumption captures the fact that that there is a cost of doing creative accounting.

Recall that households buy total government's debt (D_t) and receive r_t^D as the total interest debt for their savings. The interest rate of the total debt must be a composition of

[7] Buti et al. (2007) similarly models a differentiated stock of debt that represents other ways of financing or accounting gimmicks and does not enter in the measured/target budget balance.

[8] This can be seen by subtracting (8) in $t-1$ to (8) and rearranging: $D_t - D_{t-1} - (b_t - b_{t-1}) = (F_t - F_{t-1})$ and replacing the right side of the latter in the left side of equation (7) $\Rightarrow SFA_t = F_t - F_{t-1}$.

the interest rates of the two parts that constitute the total government debt.

Definition 4. *Total debt interest rate* r_t^D is a weighted average of each of the interest rates of government's liabilities:

$$1 + r_t^D = (1 + r_t^b)\frac{b_{t-1}}{D_{t-1}} + (1 + r_t^F)\frac{F_{t-1}}{D_{t-1}} \tag{10}$$

Using (10) and (8), government's real total balance (5) can be written as:

$$G_t + (1 + r_t^b)b_{t-1} + (1 + r_t^F)F_{t-1} = N_t W_t \tau + b_t + F_t \tag{11}$$

2.2.2 Government's problem

Political economy suggests various reasons for a present-biased government expenditure that explains *excessive* levels of public debt (Alesina and Passalacqua, 2016). Moreover, these theories encompass the potential political distortions accounting for a government that acts time inconsistently and overaccumulates debt. Among these reasons, Alesina and Tabellini (1990) suggest that debt is used as a strategic variable to assure that future governments, which might have different political preferences, follow policies closer to the actual incumbent party's preferences. Similarly, Amador et al. (2008); Battaglini and Coate (2008) propose models of possible cases that lead to a common pool problem where agents do not fully internalize the implied tax burden of excessive spending. Alternatively, Jackson and Yariv (2014, 2015) show that when there is any heterogeneity in time inconsistent citizens, an utilitarian government is present-biased.[9]

Assumption 2. Government discounts quasi-hyperbolically.

Even though I do not explicitly model the political frictions, the government has a quasi-hyperbolic discount as a way to capture that spending and indebtedness decisions respond to a present-biased government.

[9] Other microfundations of agets heterogeneity and conflict of interest which lead to fiscal deficits are Aguiar and Amador (2011); Caballero and Yared (2010); Persson and Svensson (1989). See Alesina and Passalacqua (2016); Eslava (2011); Alesina and Perotti (1994) for a survey.

Government's problem is:

$$\max_{\{G_t,b_t,F_t\}_{t=1}^{T}} \frac{C_t^{1-\sigma}}{1-\sigma} - \psi\frac{N_t^{1+\gamma}}{1+\gamma} + \chi\frac{G_t^{1-\theta}}{1-\theta} + \delta\sum_{s=1}^{T}\beta^s\left\{\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \psi\frac{N_{t+s}^{1+\gamma}}{1+\gamma} + \chi\frac{G_{t+s}^{1-\theta}}{1-\theta}\right\}$$

Subject to:

$$G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} = N_tW_t\tau + b_t + F_t$$

Where $\delta \in (0,1]$ is the quasi-hyperbolic discounting factor, and when $\delta = 1$ is the standard case where agent discounts exponentially. First order conditions for this case are given by:

$$\frac{G_{t+1}^{\theta}}{G_t^{\theta}} = \delta\beta(1+r_{t+1}^b) \tag{12}$$

$$\frac{G_{t+1}^{\theta}}{G_t^{\theta}} = \delta\beta(1+r_{t+1}^F) \tag{13}$$

$$\frac{G_{t+s+1}^{\theta}}{G_{t+s}^{\theta}} = \beta(1+r_{t+s+1}^b) \tag{14}$$

$$\frac{G_{t+s+1}^{\theta}}{G_{t+s}^{\theta}} = \beta(1+r_{t+s+1}^F) \tag{15}$$

First order conditions are the Euler equations determining the public's credit demand which depend on the government's impatience rate (β and δ for the near future) and the corresponding interest rate of issuing debt using or not SFA. Note that government discounts the near future (the immediate following period) differently than what it discounts the periods further on. The quasi-hyperbolic discount factor δ makes government more impatient in the present, which decreases the marginal cost of government spending in the near future,[10] implying that present public expenditure, relative to the following period, optimally increases with this kind of time preferences. However, after period $t+1$ the marginal cost of government spending in the following periods is discounted only at rate β, which can be seen when comparing equations (12) to (14) and (13) to (15).[11] As a consequence of a government that

[10] Equations (12) and (13) can be respectively re-written as

$$G_t^{-\theta} = \delta\beta(1+r_{t+1}^b)G_{t+1}^{-\theta}, \qquad G_t^{-\theta} = \delta\beta(1+r_{t+1}^F)G_{t+1}^{-\theta}$$

Where the left hand side of both equations is the marginal benefit of consuming today while the right hand side is the marginal cost.

[11] See Laibson (1997) for details on quasi-hyperbolic discount.

is more impatient in the near future, public expenditure will be present-biased, and therefore government goods and services provided in t will be higher than in case where $\delta = 1$.

Moreover, quasi-hyperbolic discounting makes government decisions time-inconsistent. To gain intuition about this, think of the government as different "selves" who represent the government of time t.[12] Viewed from $t = 1$, for "self 1" utility value of one unit of government spending at time 3, relative to the value at time 2 is $\frac{\beta\delta}{\beta^2\delta} = \beta$, while from the perspective of self 2 the utility value of that unit of government expenditure at time 3 relative to time 2 is $\frac{\beta\delta}{1} = \beta\delta$. In other words, government today values the three periods from now public spending more than what "self 2 government" will value it when period two arrives. Consequently, when period two arrives, government will change its optimal decision and consume more in $t = 2$ relative to period three, than what "self 1" decided in $t = 1$.

Even though government wants to spend more in the present than what it does in the case where $\delta = 1$, it will optimally choose not to use creative accounting to finance its deficit.

Lemma 1. *In absence of a fiscal rule, government optimally chooses not to use creative accounting as a way to finance its deficit. Thereupon, total debt is equal to measurable debt, and interest rates for b_t, D_t and F_t are equal.*

Proof See appendix. Intuitively, since doing creative accounting is costly, government will not use it if does not face any limit constrain.

Finally, a quasi-hyperbolic discounting will initially rise government's credit demand with respect to the case where $\delta = 1$. Using Lemma 1, government's budget constraint can be written as $G_t + (1 + r_t^D)D_{t-1} =_t W_t + D_t$ and equation (12) as $G_t = [\delta\beta(1 + r_{t+1}^b)]^{-\frac{1}{\theta}} G_{t+1}$. Replacing the latter in the former and rearranging:

$$\frac{G_{t+1}}{[\delta\beta(1 + r_{t+1}^b)]^{\frac{1}{\theta}}} + (1 + r_t^D)D_{t-1} -_t W_t = D_t$$

When $\delta \in (0, 1)$, debt demand will initially be higher than in the case of exponential discount (which corresponds to $\delta = 1$). However, an increase in debt demand will affect household

[12] Is standard to think of the quasi-hyperbolic consumer as different selves like in Laibson (1997); Peleg and Yaari (1973); Goldman (1980).

decisions which in turn will affect government revenue and hence, in a second round its net debt demand.

The following definition presents the equilibrium of this economy.

Definition 5. The *equilibrium* of the economy where government is unconstrained is given by prices $\{W_t, r_t^b, r_t^F, r_t^D\}_{t=1}^T$ and allocations $\{C_t, G_t, N_t, b_t, F_t, D_t\}_{t=1}^T$ such that

1. Optimal decisions determining the behavior of agents are satisfied:

   i. Firms optimal labor demand condition equation (1) holds.
   ii. Households optimal decisions equations (2) -(4) hold.
   iii. Government optimal decisions (12) - (15) hold.

2. All markets clear:

   N_t clears the labor market, $D_t = b_t + F_t$ in the debt market and $Y_t = C_t + G_t$ in the goods market.

Public overspending, measured as the deviation from the non-distorted case given by $\delta = 1$ is the reason to constrain government aggregates (Wyplosz, 2005). However, when government is unable to default on its debt or monetize the debt through inflation, as is the case in this model, the constant higher public debt levels would eventually imply a fiscal adjustment that enables government to repay its debt. Under the assumption that government is not allowed to do a fiscal reform to increase its revenue (i.e. tax rates remain constant), a public expenditure reduction is eventually needed to repay its debt obligations.

Whereas the future expenditure cut would be harmful for consumers in this economy, given that household does not discount quasi-hyperbolically and would prefer balanced budgets across time, it is not obvious that household's life-time utility worsens when government discounts quasi-hyperbolically. At the outset, since household values public goods and services, an increase in government spending rises consumers utility. Nevertheless, a higher public debt means that the household is lending more resources to government, henceforth, its consumption in the initial period potentially decreases or pushes her to work more hours, both of which could decrease their utility. Additionally, a higher level of public debt raises the interest rate, meaning that households receive a higher income in the following periods.

On the other hand, public spending needs to decrease eventually in order to repay government's obligations, and the reduction in public goods and services will reduce household's utility.

2.2.3 Fiscal Rules

A present-biased government is an economic reason to adopt institutions that can constrain a government overspending and achieve debt sustainability. To avoid the fiscal decisions that lead to debt overaccumulation, countries have adopted fiscal rules as their main mechanism to constraint public deficit bias (Alesina and Passalacqua, 2016; Wyplosz, 2005). To constrain the present-biased incumbent, a balanced budget rule and a debt rule are introduced to the government described in section 2.2.2. These rules are the two most common used rules in 2015, according to the IMF Fiscal Rules Data base.[13] First, the balanced budget rule is presented in definition 4, followed by the government's problem including this rule. Afterwards, the debt rule and its corresponding problem is presented.

Definition 6. A *Balanced Budget Rule (BBR)* constrains fiscal spending by setting a threshold on total public balance as GDP percentage:

$$\frac{g_t + r_t b_{t-1} - \tau_t W_t N_t}{Y_t} \leq \overline{BBY_t} \tag{16}$$

Where $\overline{BBY_t}$ is the imposed limit on fiscal balance as percentage of GDP.

Note that balanced budget rule's targets are set on fiscal deficit as it directly derives from (6).

$$\Rightarrow \qquad \frac{r^b_{t\,t-1} + g_t - \tau_t W_t N_t}{Y_t} = \frac{b_t - b_{t-1}}{Y_t} \leq \overline{BBY_t}$$

Therefore, a balance budget rule is equivalent to a rule that sets a limit on debt growth, meaning that balanced budget rules aim to guarantee debt sustainability indirectly. Even so, it is easy to see that a balanced budget rule leaves space for government to use SFA to create a discrepancy between government's spending established by the balanced budget rule and

[13]Info from IMF fiscal rules data base: https://www.imf.org/external/datamapper/fiscalrules/map/map.htm for more details.

the real spending laced to the growth of real total debt. The corresponding government's problem when subject to a balanced budget fiscal rule is presented hereafter.

Government's problem with BBR

$$\max_{\{G_t,b_t,F_t\}_{t=1}^{T}} \frac{C_t^{1-\sigma}}{1-\sigma} - \psi\frac{N_t^{1+\gamma}}{1+\gamma} + \chi\frac{G_t^{1-\theta}}{1-\theta} + \delta\sum_{s=1}^{T}\beta^s\left\{\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \psi\frac{N_{t+s}^{1+\gamma}}{1+\gamma} + \chi\frac{G_{t+s}^{1-\theta}}{1-\theta}\right\}$$

Subject to:

$$G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} = N_t W_t \tau + b_t + F_t$$
$$\frac{b_t - b_{t-1}}{Y_t} \leq \overline{BBY_t}$$

Note that the balanced budget fiscal rule is an additional constraint to the problem of section 2.2.2. Optimal conditions are given by:[14]

$$G_t^{-\theta} + \frac{\mu_t^b}{Y_t} = \delta\beta\left(G_{t+1}^{-\theta}(1+r_{t+1}^b) + \frac{\mu_{t+1}^b}{Y_{t+1}}\right) \tag{17}$$
$$\frac{G_{t+1}^{\theta}}{G_t^{\theta}} = \delta\beta(1+r_{t+1}^F) \tag{18}$$
$$G_{t+s}^{-\theta} + \frac{\mu_{t+s}^b}{Y_{t+s}} = \beta\left(G_{t+s+1}^{-\theta}(1+r_{t+s+1}^b) + \frac{\mu_{t+s+1}^b}{Y_{t+s+1}}\right) \tag{19}$$
$$\frac{G_{t+s+1}^{\theta}}{G_{t+s}^{\theta}} = \beta(1+r_{t+s+1}^F) \tag{20}$$
$$\mu_t^b\left(\frac{b_t - b_{t-1}}{Y_t} - \overline{BBY_t}\right) = 0 \tag{21}$$
$$\mu_{t+s}^b\left(\frac{b_{t+s} - b_{t+s-1}}{Y_{t+s}} - \overline{BBY_{t+s}}\right) = 0 \tag{22}$$

Where μ_t^b is the Lagrange multiplier of the balanced budget fiscal rule.

Equations (17)-(22) represent the optimal decisions of a quasi-hyperbolic discounting government facing a balanced budget fiscal rule. First of all, equations (21) and (22) are the Kuhn-Tucker conditions showing that when $\mu_t^b \neq 0$, the BBR constraint is binding, and if so, equations (17) and (19) show that government spending decisions are constrained. However, the optimal decisions with respect to F_t, (18) and (20), are unconstrained. As a result,

[14] The complete derivation of the optimal conditions is available in the Appendix

when BBR constraint binds, government may decide to optimally reallocate use other sorts of financing, that is, doing creative accounting, to spend more than what is stipulated by the rule. Results are formalized in the following Lemma.

Lemma 2. *If the balanced budget constraint is binding, government may optimally decide to use creative accounting to finance a higher spending. In that case,* $D_t \neq b_t$ *and interest rates for all debts* (r_t^D, r_t^b, r_t^F) *are different.*

Proof See Appendix. Intuitively, if the marginal benefits of doing creative accounting are greater than the marginal costs, government will use creative accounting.

It follows from Lemma 2 that in the case where government decides to use accounting gimmicks to finance a higher spending, government will still demand more debt than when there is no present-bias, but market will supply it at a higher cost, since it request an additional risk premium for the risk of a bigger quantity of debt than what is stipulated by the rule.

Additionally, equations (18) to (22) bring to light one of the limitations of fiscal rules imposed on flow fiscal aggregates by showing that as long as there is a disconnection to the total fiscal stocks, the rule leaves space for debt overaccumulation with respect to what the rule aims to constrain (Cangiano et al., 2013; Von Hagen and Wolff, 2006). In other words, even when a balanced budget rule is implemented and the limits imposed are relevant and binding, public spending might be higher in comparison to the case where government is not present-biased and therefore, the rule can fail to undo the distortions of a quasi-hyperbolic discounting government.

In order to directly constrain total government debt, another commonly used rule is the debt rule as defined hereafter.

Definition 7. A *Debt rule (DR)* constrains public debt as GDP percentage:

$$\frac{D_t}{Y_t} \leq \overline{DY_t} \tag{23}$$

Where $\overline{DY_t}$ is the imposed limit on total fiscal debt as a percentage of GDP in each period.

Note that imposing a debt rule does constrain the government from using accounting gimmicks and indirectly constrains the total real balance plus debt repayment. Using (11):

$$\begin{aligned} G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} - \tau_t W_t N_t =& D_t \\ \Rightarrow \qquad \frac{G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} - \tau_t W_t N_t}{Y_t} =& \frac{b_t + F_t}{Y_t} \leq \overline{DY_t} \end{aligned} \tag{24}$$

Government's problem with DR

$$\max_{\{G_t,b_t,F_t\}_{t=1}^T} \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \chi \frac{G_t^{1-\theta}}{1-\theta} + \delta \sum_{s=1}^{T} \beta^s \left\{ \frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \psi \frac{N_{t+s}^{1+\gamma}}{1+\gamma} + \chi \frac{G_{t+s}^{1-\theta}}{1-\theta} \right\}$$

Subject to:

$$G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} = N_t W_t \tau + b_t + F_t$$

$$\frac{b_t + F_t}{Y_t} \leq \overline{DY_t}$$

As in the case of the balanced budget rule, the debt rule is an additional constraint to the problem of the quasi-hyperbolic discounting government and is introduced using (24). Optimal conditions are given by:[15]

$$G_t^{-\theta} + \frac{\mu_t^D}{Y_t} = \delta\beta G_{t+1}^{-\theta}(1+r_{t+1}^b) \tag{25}$$

$$G_t^{-\theta} + \frac{\mu_t^D}{Y_t} = \delta\beta G_{t+1}^{-\theta}(1+r_{t+1}^F) \tag{26}$$

$$G_{t+s}^{-\theta} + \frac{\mu_{t+s}^D}{Y_{t+s}} = \beta G_{t+s+1}^{-\theta}(1+r_{t+s+1}^b) \tag{27}$$

$$G_{t+s}^{-\theta} + \frac{\mu_{t+s}^D}{Y_{t+s}} = \beta G_{t+s+1}^{-\theta}(1+r_{t+s+1}^F) \tag{28}$$

$$\mu_t^D \left(\frac{b_t + F_t}{Y_t} - \overline{DY_t} \right) = 0 \tag{29}$$

$$\mu_{t+s}^D \left(\frac{b_{t+s} + F_{t+s}}{Y_{t+s}} - \overline{DY_{t+s}} \right) = 0 \tag{30}$$

Where μ_t^D is the Lagrange multiplier of the debt fiscal rule.

[15]The complete derivation of the optimal conditions is available in the Appendix

Equations (25)- (30) represent the optimal decisions of a quasi-hyperbolic discounting government that has a debt rule. In this case, in comparison with the balanced budget rule, equations (29) and (30) show that when the fiscal rule is binding, this rule should effectively constrain government from doing creative accounting. This is why the optimal conditions with respect to the debt acquired by doing accounting gimmicks liabilities (equations (26) and (28)), are also different from the ones obtained when the government is not facing a fiscal rule. In this case, when the fiscal rule is binding, the whole set of debt demand is restricted and therefore the Lagrange multiplier of the debt rule creates a wedge in the Euler equations, reducing government's expenditure in t relative to the following period.

The following definition presents the equilibrium of the economy when government discounts quasi-hyperbolically and is subject to a debt rule.

Definition 8. The *equilibrium* of the economy where government discounts quasi-hyperbolically and is subject to a debt rule is given by prices $\{W_t, r_t^b, r_t^F, r_t^D\}_{t=1}^T$ and allocations $\{C_t, G_t, N_t, b_t, F_t, D_t\}_{t=1}^T$ such that

1. Optimal decisions determining the behavior of agents are satisfied:
   i. Firms optimal labor demand condition equation (1) holds.
   ii. Households optimal decisions equations (2) -(4) hold.
   iii. Government optimal decisions hold:
   Equations (17)-(22) if government is subject to a balanced budget rule, or equations (25)-(30) if it is subject to a debt rule.
2. All markets clear:
   N_t clears the labor market, $D_t = b_t + F_t$ in the debt market and $Y_t = C_t + G_t$ in the goods market.

Albeit fiscal rules aim to constrain overspending and higher levels of public debt of a quasi-hyperbolic discounting government, the effects of imposing such constraints on household's welfare are not obvious since households decisions are responding to changes in borrowing market. At the outset, when the fiscal rule is binding, a higher interest rate compensates for a high credit demand, increasing future costs for government and thereby rising household's income in following periods. Also, the rise in total interest rate increases present consumption marginal cost, hence the intertemporal consumption decision of households changes, which in turn, modifies the labor supply. Finally, given that labor demand is perfectly elastic, it directly affects equilibrium hours,[16] meaning that movements in labor supply also affect public revenues and consequently, total government spending. In summary, welfare will depend on the general equilibrium effects derived from the adoption of a fiscal rule.

3 Results

As outlined by Alesina and Passalacqua (2016) and Wyplosz (2005) there is a desirability to constrain present-biased governments by imposing fiscal rules, aiming to restrain overindebtedness and attain fiscal sustainability. Furthermore, fiscal rules intend to undo the distortions created by the political incentives that lead government to be present-biased as in Azzimonti et al. (2016); Chatterjee and Eyigungor (2019). Nevertheless, the effects in consumer's welfare depend on the responses of private sector to the government decisions when facing a fiscal rule.

With the purpose of analyzing the effectiveness of fiscal rules and its effects on household's welfare, I analyze four different scenarios using a 3 period version of the model with an economic boom in $t = 2$, which is captured using a deterministic increase of productivity. The purpose of the simulation is to show all channels present in the model and therefore three periods is the minimum amount of periods needed to evince government's time-inconsistent decisions: its plans for the third period, taken from the government of the first period, are different when it plans its third period from $t = 2$.

To depict the effects of imposing fiscal rules on a present-biased government, I use a predetermined list of parameters that guarantee a solution (i.e. Satisfy equilibrium for all scenarios).[17] To evaluate welfare, several scenarios are presented. First the government presented in section 2.2.2 with $\delta = 1$ works as the benchmark scenario because given that government is not present biased, there is no need to implement a fiscal rule. The second case corresponds to the quasi-hyperbolic discounting government. The purpose of this scenario is to evaluate the general equilibrium effects of the present-biased government and its welfare

[16] Whether the supplied labor increases or decreases depend on the income and substitution effects given by the utility function parameters.

[17] Table 1 in the appendix summarizes the parametrization used in the numerical simulation.

implications when comparing it to the benchmark case. The two remaining scenarios are cases where the quasi-hyperbolic discounting government is constrained with fiscal rules that are calibrated using benchmark scenario results as targets. The following table summarizes the cases.

Table 1: Simulation Cases and Description

Case	Discounting	Fiscal Rule	Description
Benchmark	$\delta = 1$	No	Reference case. No present-bias.
Q.Hyperbolic	$\delta < 1$	No	Distorted (present-bias) unconstrained scenario.
BBR	$\delta < 1$	Yes	Q-Hyperbolic with a balanced budget rule.
DR	$\delta < 1$	Yes	Q-Hyperbolic with a debt rule.

Finally, it is important to clarify that a numerical solution of the model is computed without doing any approximations. Welfare is evaluated from the point of view of the household and is defined as the present value of all time household's utility:

$$W^k = \sum_{t=1}^{3} \beta^t \left\{ \frac{C_t^{k\,1-\sigma}}{1-\sigma} - \psi \frac{N_t^{k\,1+\gamma}}{1+\gamma} + \chi \frac{G_t^{k\,1-\theta}}{1-\theta} \right\}$$

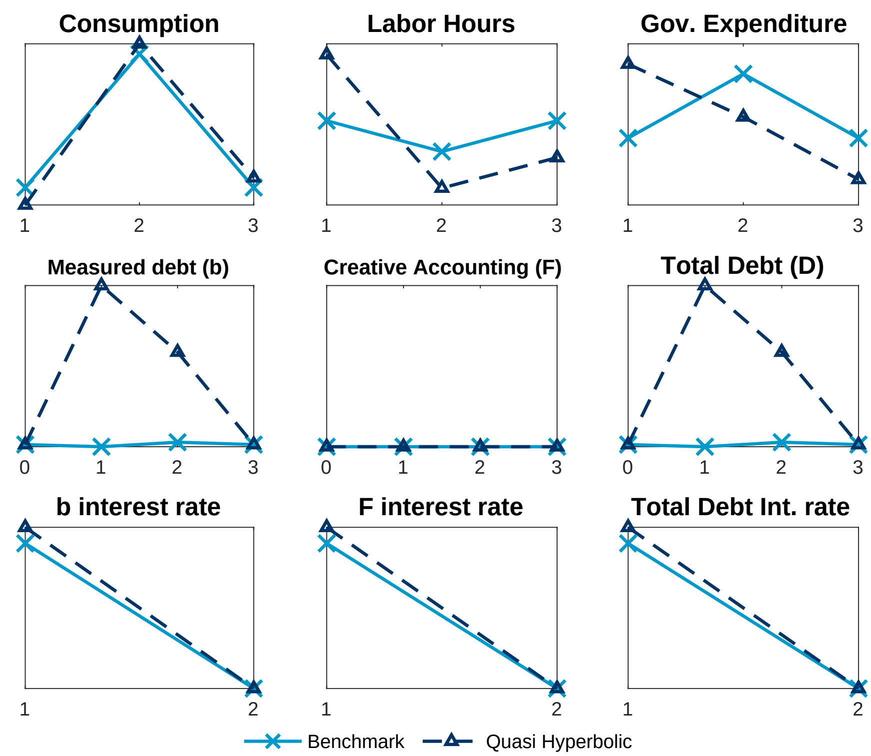
where $k = \{Benchmark, Q.Hyperbolic, BBR, DR\}$ denotes the scenario as presented in table 3. Then, welfare is compared across scenarios to get an ordinal measure of the implications of having a quasi-hyperbolic discounting government and fiscal rules implementation.

3.1 Macroeconomic Dynamics

Figure 1 shows the results for the benchmark and quasi-hyperbolic scenarios. In the benchmark scenario, due to an expected increase in total productivity in $t = 2$, households consume more and work less in period 2 than what they do in period 1. Also, a higher productivity raises government revenue and hence, public expenditure increases too in the second period. Additionally, total debt in this economy stays almost the same, since income is increasing equally proportional for both agents. In the third period, macro aggregates behave as in the first period in response to a productivity that goes back to the initial level.

In the second scenario, when government discounts quasi-hyperbolically, public spending in the first period is higher than in the benchmark scenario as a result of a present-biased government. To finance a higher spending, government's credit demand curve shifts right, raising the interest rate. Nonetheless, government does not do creative accounting, since using it would be more costly. Therefore, total real debt is equal to reported debt in both periods as stated by lemma 1. Meanwhile, a higher interest rate increases the amount of resources that households are willing to lend to government since the opportunity cost of present consumption relative to future consumption rises. Conversely, households labor supply also increases since lending more resources to government implies that their present consumption decreases; in other words, marginal utility of private consumption in period one rises. Therefore, household prefers to work more in comparison to the benchmark scenario to lend more resources to government without sacrificing much present consumption. Notwithstanding, private consumption in the first period is lower than in the benchmark scenario.

Figure 1: Macroeconomic Dynamics for benchmark and Q.Hyperbolic scenarios



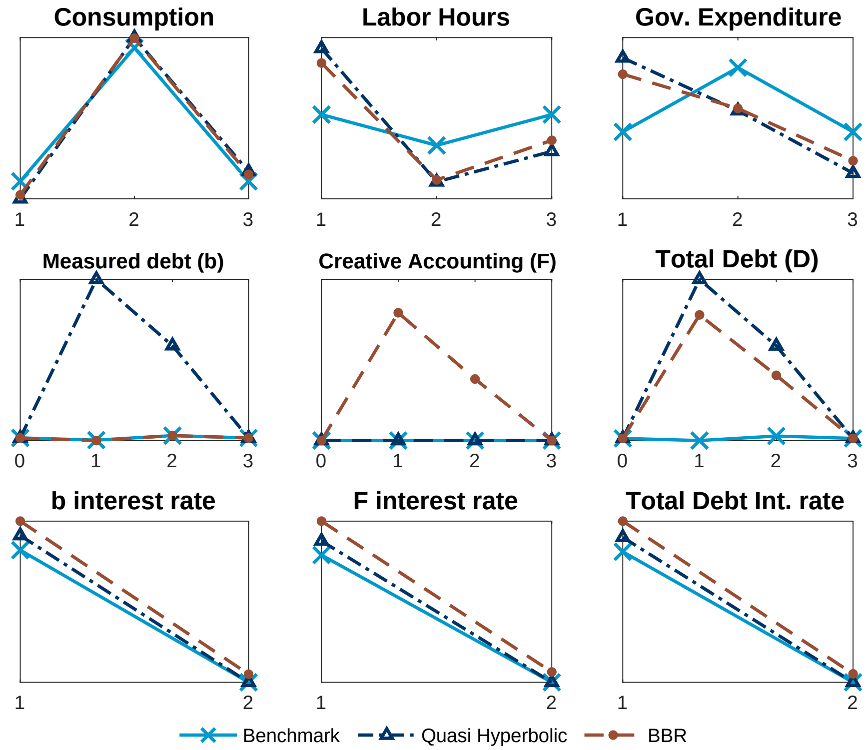
In the second period, public spending decreases with respect to period one since government needs to make an expenditure cut to start repaying its debt. On the other hand, household has a higher income as a consequence of lending resources to government and receiving an interest for those savings, allowing her to consume more and work less than in the benchmark case. Note that lower worked hours in equilibrium also decrease government's revenue, so government decides not to repay all its obligation in the second period. As a result, public spending in the third period decreases even more so that government can pay back to households. Once again, in the last period, private consumption is higher and labor hours are lower than in benchmark scenario due to household's savings and its interest returns.

The effects on household's welfare in the quasi-hyperbolic scenario depend on the movements in the labor and consumption good market because household derives utility from private consumption, leisure and government spending. In this case, welfare worsens with respect to the benchmark for three reasons. First, more work hours in the first period imply that household enjoys less leisure time, second; less private consumption in $t = 1$ as a consequence of lending more resources to government, and lastly, less government spending in $t = 2, 3$. Those results outweigh the higher utility generated by the increase of government expenditure in first period and the higher returns households are receiving due to higher interest rates.

3.1.1 Implementation of Fiscal Rules

Figure 2 illustrates the case when a quasi-hyperbolic discounting government is subject to a balanced budget rule. As outlined by Lemma 2, when government is subject to a relevant constraint deficit, the incumbent optimally decides to use accounting gimmicks, and therefore accumulates additional debt F if the marginal benefit of doing operations under the line is higher than its marginal cost. In this case, as a consequence of a government that desires to spend more than in the benchmark case due to the quasi-hyperbolic discounting, benefits of doing accounting gimmicks exceed the implied costs, and government decides to finance a higher spending. As a matter of fact, this scenario shows that when government faces a balanced budget constraint, it can increase its spending in $t = 1$ in comparison to the benchmark scenario, by using "other type of financing" and still meet the rule. However, the use of creative accounting entails a higher interest risk premium of its debt and rises interest rate of the total public debt. Note that the risk premium component of equation (9) represents a debt supply curve that increases faster from the point where debt is implicitly constrained by the rule, as depicted in Panel A of Figure 3.

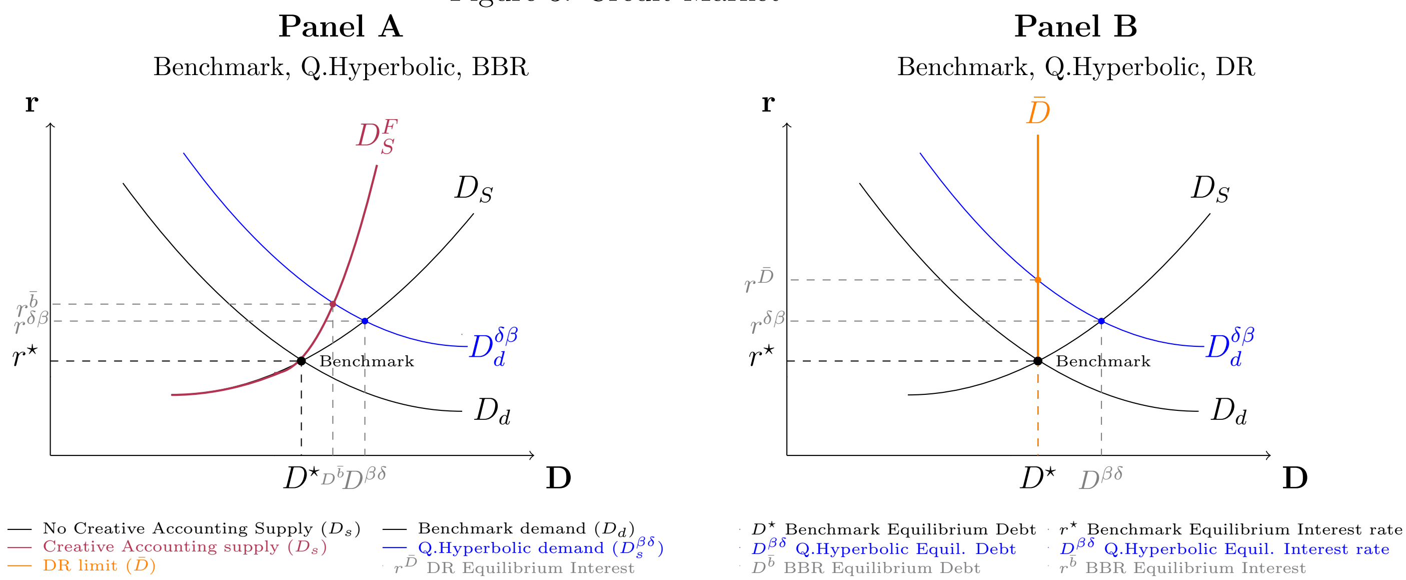
Figure 2: Macroeconomic Dynamics for benchmark, Q.Hyperbolic and BBR scenarios



Doing creative accounting allows government to finance more spending vis-a-vis a rapidly rising risk premium. In this respect, a higher risk premium increases the marginal cost of borrowing, so government can not get as much total debt as it would like to. To sum up, government expending is higher than what the rule aims to achieve, but since doing creative accounting is costly, government cannot spend as much as in the unconstrained quasi-hyperbolic scenario. Additionally, in the following periods government needs to reduce its expenditure to repay the debt obligations acquired to finance the first period expenditure.

As well as in the quasi-hyperbolic scenario, higher interest rates change household's intertemporal savings and labor optimal allocations. Although private consumption reduces in the first period in comparison to the benchmark scenario, it is not reduced as much as in the unconstrained quasi-hyperbolic case, while while total hours worked remain between

Figure 3: Credit Market

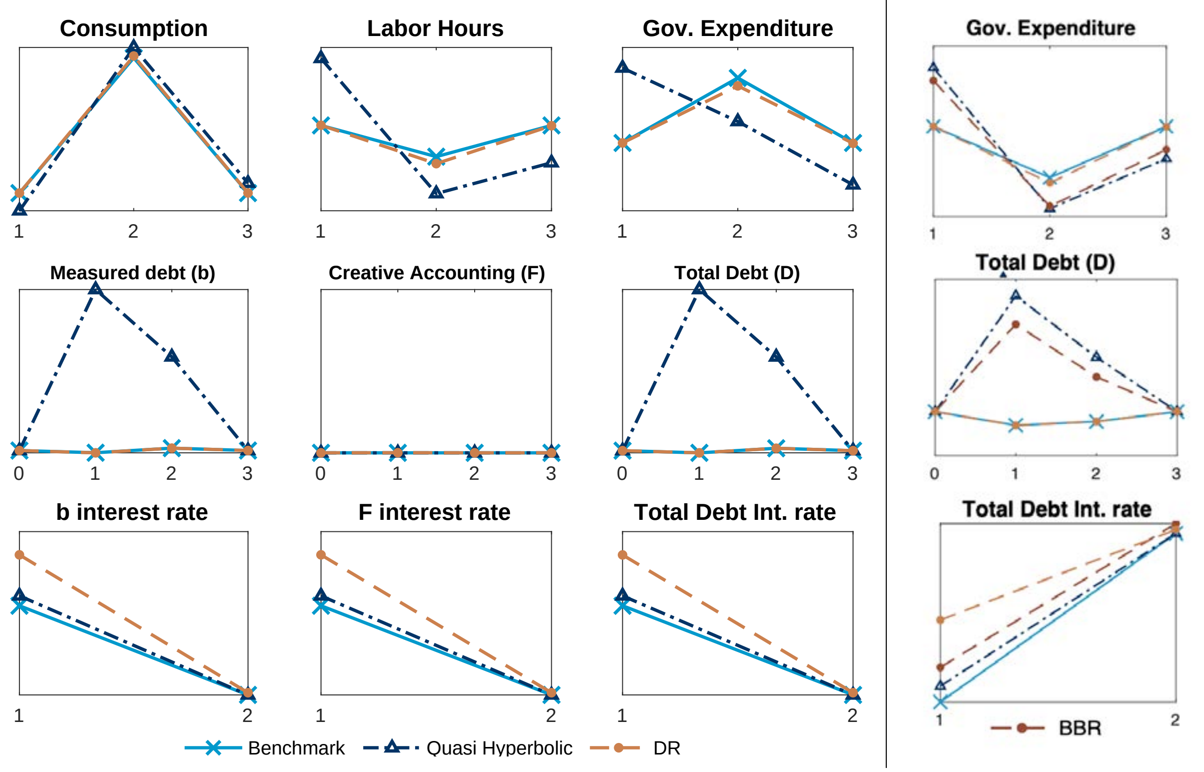


the benchmark and quasi-hyperbolic equilibria. Similarly occurs for periods 2 and 3, where consumption and labor allocations are distorted with respect to the benchmark case but less than in the unconstrained scenario. Nonetheless, consumption in period 2 is almost the same as in the unconstrained scenario since household is receiving a higher interest rate due to the higher risk premium derived from government doing creative accounting.

Finally, panel A of figure 4 shows the macroeconomic dynamics of the economy when government is subject to a debt rule. In this scenario, the fiscal rule imposed over the total debt does constrain the government from doing creative accounting and the rule effectively limits public overspending, even when government would still want to do it because of its quasi-hyperbolic discounting. Indeed, the debt rule manages to approach almost all macroeconomic variables to the results of the benchmark scenario. This is particularly true in the first period, when government aggregates are properly constrained and thus, total debt and public spending are exactly the same than in the benchmark scenario. Despite that the debt rule does constrain government overspending, its credit demand is still higher than in benchmark scenario since it discounts quasi-hyperbolically. As depicted in Figure 3 Panel B, in equilibrium, a higher interest rate adjusts for the elevated credit's demand when debt quantities are constrained, even at a higher level than in the balanced budget case, as shown in panel B figure 4. As a result, household's income marginally rises in period two, allowing her to reduce total hours worked thereby, reducing government's revenue. The latter explains why government spending is slightly bellow than in benchmark.

Figure 4: Macroeconomic Dynamics

Panel A - Benchmark, Q.Hyperbolic and DR | **Panel B -** All scenarios



In summary, the macroeconomic dynamics presented above show that when a present-biased government is constrained by a fiscal rule and the rule leaves space to do creative accounting, public debt will be higher in comparison to the case where government is not present-biased. Even though creative accounting rises government's repayment cost, its shortsightedness outweighs the higher risk premium resulting of augmenting its liabilities "under the line", rendering the balanced budget rule not completely effective: debt and government spending is not at high as in the unconstrained distorted case, but is still higher than what the rule implicitly stipulates. On the contrary, a debt rule that constrains total liabilities in each period, ensures that government does not use creative accounting and therefore effectively constrains total government spending.

3.2 Welfare Analysis of fiscal rules

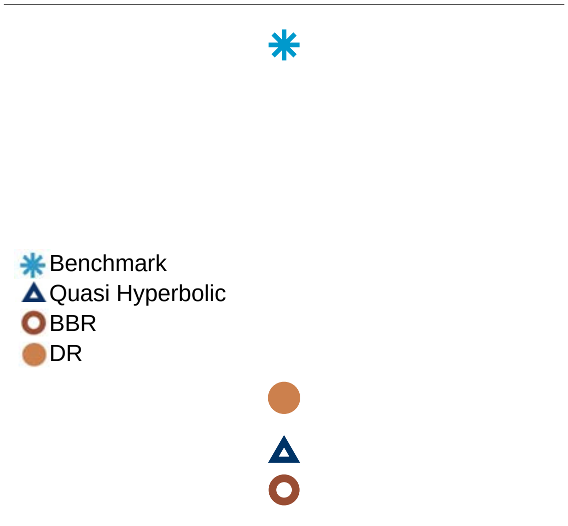
If fiscal rules were to be judged in terms of their effectiveness, that is, to constrain fiscal overindebtedness, the results of previous section suggest that a debt rule that constrains total government liabilities is preferred than a balanced budget rule where government uses creative accounting to overspend. Nevertheless, the efficiency of a balanced budget rule or a fiscal rule remains unclear. Do fiscal rules achieve to undo the *welfare losses* generated from a quasi-hyperbolically discounting government? It is better for consumers that government implement a balanced budget rule even if it does not fully constrain government from overspending? Does a debt rule always diminishes welfare losses of a present-biased government? This section is oriented to evaluate fiscal rules efficiency in terms of improving consumer welfare losses.

As mentioned, the welfare effect of implementing fiscal rules varies depending on the general equilibrium effects entailed by the endogenous reaction of household in response to government's behaviour in each scenario. Recall that there is a discrepancy between household and government's time preferences and therefore, economy's life-time value function should be measured using household's exponential discount and not the present-biased government discount.

In the scenario where government is subject to a balanced budget rule, various opposite effects impacted household's welfare when comparing it to the unconstrained quasi-hyperbolic scenario. In the first place, due to the exponentially increasing credit supply curve, the equilibrium debt level of this scenario is lower than in the unconstrained case. The latter allows household to increase her consumption and work less hours in the first period, increasing its instant utility. On the contrary, government spending is lower than in the quasi-hyperbolic scenario because government borrowed less resources. That lower government consumption reduces household's utility in $t = 1$.

Thereafter, the higher interest rate increases household's resources by easing its budget constrain, although implying that government keeps its expenditure still low, similar to the one in the unconstrained case and far from benchmark scenario, because it is facing a higher repayment cost for its debt. The opposite occurs in the third period, the lower debt obligations, in comparison to the quasi-hyperbolic case, make households poorer so they work more and consume less, but government can rise its expenditure. In conclusion, there are positive and negative effects for household's derived from the implementation of this fiscal rule. Figure 5 shows that for the given parametrization, the elevated repayment cost of leaning on other ways of financing, which in turn imply a greater contraction of government expenditure, ends up surpassing the benefits brought to the household by the implementation of a balanced budget fiscal rule.

Figure 5: Welfare - All Scenarios

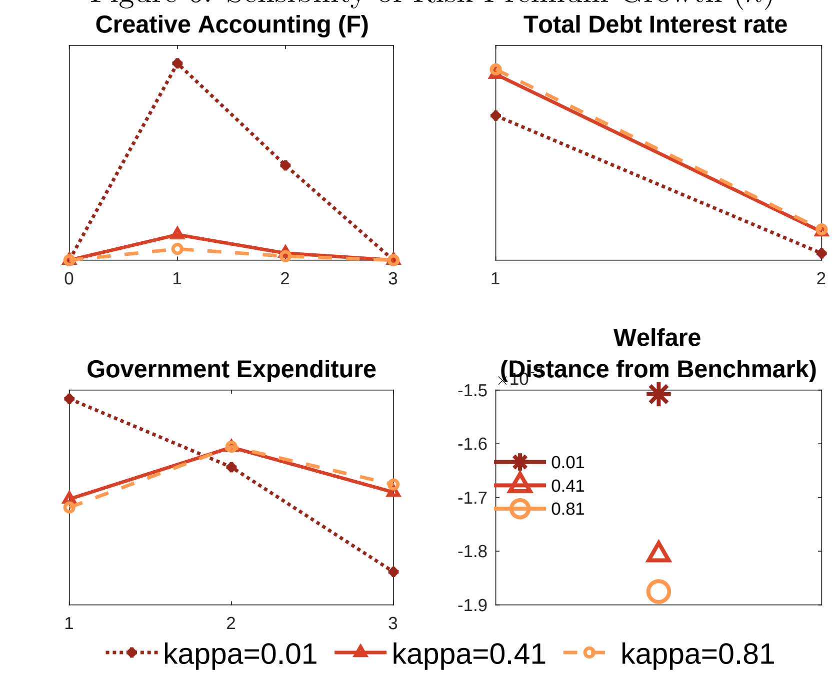


Welfare is defined as the present value of the 3 period household's utility using exponential discount since it is evaluated from the point of view of private consumers. It is computed for four cases: Benchmark, Q.Hyperbolic, Balanced Budger Rule(BBR) and Debt Rule (DR).

How harmful is that government circumvents the debt growth limit implicit in the balanced budget fiscal rule depends upon how much country's risk premium rises when government reverts to creative accounting. To evaluate the effects of a greater increase in risk perception when government uses creative accounting, two higher values of κ are evaluated in additional simulations of the BBR scenario; concretely, κ is increased in 40 and 80 basic points respectively. The main results presented in Figure 6 show that a higher response of markets to creative accounting, rises debt repayment cost much more than in the baseline parametrization. Note that a linear increase of 40bp traduces in a non-linear rise in the interest rate with respect to the benchmark scenario.

Additionally, the higher the risk premium responds to the use of creative accounting, the less the government turns to use SFA. Although a greater κ deters total use creative accounting, getting government spending closer to that of the benchmark scenario in the first period, the repayment cost of the acquired obligations is so high that welfare distances even more from benchmark's. In conclusion, if investors perceive that when politicians do accounting gimmicks sovereign default risk widely rises, then households end up worse off when government resorts to creative accounting.

Figure 6: Sensibility of Risk Premium Growth (κ)



Welfare is defined as the present value of the 3 period household's utility using exponential discount since it is evaluated from the point of view of private consumers. It is computed for the three values of risk premium growth factor ($\kappa = 0.01$, $\kappa = 0.41$, $\kappa = 0.81$) and then the life-time welfare is subtracted from the benchmark life-time welfare where $\kappa = 0.01$

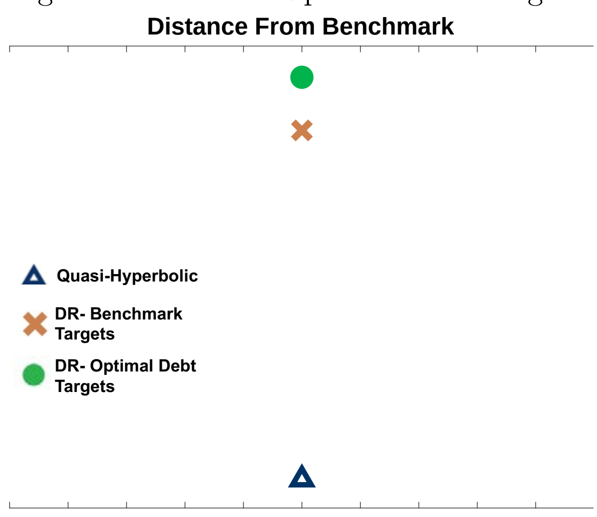
On the contrary, the implementation of debt rule achieves to approach allocations of the benchmark scenario as depicted in Figure 5. In the debt rule scenario, consumption and labor bundles are more balanced across time, as in the not-distorted reference case, making welfare get closer to the benchmark. Besides, the higher asset returns marginally increase household's income and therefore, they need to work less. The net effect on welfare is positive with respect to the quasi-hyperbolic scenario, but the highest interest rate implies a marginal reduction of public spending in the second period in comparison to the not-distorted scenario, hence, welfare is still lower than in the benchmark.

Furthermore, since there are welfare costs and benefits of implementing a debt rule, it is relevant to analyze if the debt target is too strict, perhaps, it is not set at the point where benefits perfectly exceed the costs. Precisely, should the target be closer to the quasi-hyperbolic discounting scenario or should it take the exact targets of the debt in the benchmark scenario? For answering this question, I numerically find the value of the parameter that determines the optimal distance of the debt target between the debt from the benchmark case (D_t^B), and the one of the quasi-hyperbolic scenario ($D_t^{\beta\delta}$) such that welfare is maximized. Specifically, I find the debt targets that maximize welfare by solving the following problem:

$$\max_{\{\phi_t\}_{t=1}^{2}} \mathcal{W}(D_t^\star, X_t) \tag{31}$$

Where $D_t^\star = \phi_t D_t^B + (1-\phi_t)D_t^{\delta\beta}$ is the optimal debt target, $\mathcal{W}(D_t^\star, X_t)$ is the present value of household's life-time welfare and X_t is the vector of all variables in the model. In the optimum, I find $\phi_1 = 0.2766$ and $\phi_2 = 0.2669$, meaning that for the given parametrization, relaxing the debt target in the margin, can increase the benefits: allowing a higher public spending of government, while the costs of more working hours do not outweigh the additional utility of more public goods and services. Welfare results are depicted in Figure 7 and the macroeconomic dynamics of the scenario with the optimal debt targets are the appendix.

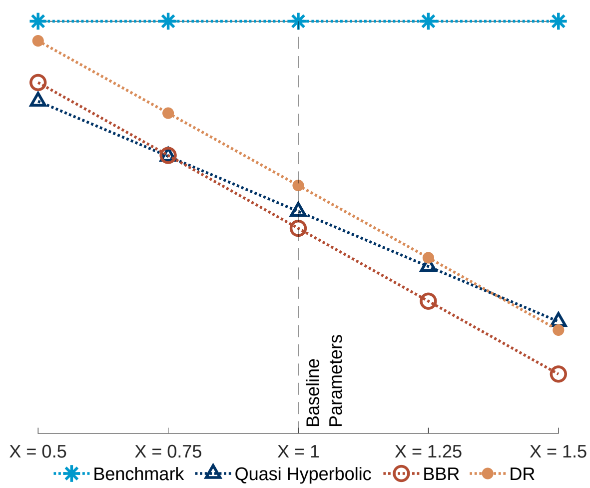
Figure 7: Welfare - Optimal Debt Target



Welfare is defined as the present value of the 3 period household's utility using exponential discount since it is evaluated from the point of view of private consumers. It is computed for the three cases: Benchmark, Q.Hyperbolic and Optimal Debt Rule (DR) targets from the benchmark life-time welfare.

Finally, its worth highlighting the role of parameter measuring the relative weight among public and private goods (χ). Even though this parameter does not interfere in the macroeconomic dynamics, since it does not appear in the first order conditions, it is an important factor for evaluating welfare. The first thing that stands out from Figure 8, is that the more the household value public goods, the more harmful are the distortions derived from a present-biased government, explained by the fact that expenditure cuts of periods 2 and 3 are relatively more painful for households. For instance, when the value of public goods is low ($\chi = 0.25$), the balanced budget rule can regain some welfare losses. Nevertheless, as public goods gain relative value in households utility, the balanced budget fiscal rule is increasingly detrimental. Similarly, if public goods and services are relatively less important

Figure 8: Welfare - Relative Value of Public Goods



Welfare is defined as the present value of the 3 period household's utility using exponential discount since it is evaluated from the point of view of private consumers. It is computed for four cases: Benchmark, Q.Hyperbolic, Balanced Budger Rule(BBR) and Debt Rule (DR), and for each case, with 5 different values of the relative values of public goods(χ). For each evaluated value of χ, welfare is subtracted from benchmark life-time welfare.

for households, a debt rule is better to diminish some welfare losses. However, that ability of a debt rule to retrieve welfare losses decline as government spending gain relative utility value, up to the point where it can be also damaging, in terms of welfare, to implement this rule. In the latter case, where $\chi = 1.5$, the costs of debt repayment that imply a lower public expenditure are bigger than the benefits of working less and increasing in the margin the private consumption, explaining why households would prefer no rule, even if a debt rule guarantees to constain fiscal overindebtedness.

4 Conclusions

This paper presents a general equilibrium closed economy model that works as a theoretical framework including two fundamental elements for the evaluation of the effectiveness and efficiency of fiscal rules: a deficit bias and a government that optimally decides to use creative accounting. Concretely, the model incorporates a quasi-hyperbolic discounting government that can revert to use Stock-Flow Adjustments as an accounting gimmick to overspend while meeting the fiscal rule requirements. The model allows analyzing the costs and benefits of the implementation of different fiscal rules in a general equilibrium framework.

Results indicate that fiscal rules with debt targets are preferred for their effectiveness and efficiency over rules that limit deficits. Albeit a balanced budget rule aims to undo the negative effects derived from a present-biased government, it can fail to properly constrain government from over spending and borrowing since it leaves space for creative accounting. Also it can be counter-productive: it decreases welfare if there exists a high cost of using accounting gimmicks. On the contrary, implementing a debt rule can effectively constrain a present-biased from overspending if it effectively limits its total public liabilities. Also, this kind of rule can diminish some of the welfare losses created by a quasi-hyperbolically discounting government.

Additionally, a relevant policy recommendation that stems from the results of this paper is the importance of having mechanisms that do not allow government to use creative accounting when fiscal rules are in use. Results reveals the importance of mechanisms, such as fiscal council or enforcement the use of international accounting reporting guidelines. Otherwise, rules may not be fully effective and could generate additional welfare losses if there is a high cost associated with the use of accounting gimmicks.

There is much to be done to extend this framework for doing quantitative investigation of optimal fiscal policy. The model containing a present-biased government that can use creative accounting is a novel framework to properly select and calibrate fiscal rules and its periodic targets. The methodology I propose can be extended for longer period simulations in country specific models including the two salient characteristics already mentioned. This framework is an approach to calibrate debt targets alternative to the IMF approach. What is most valuable of the model I propose is its ability to evaluate fiscal rules and their outcomes, in terms of the economy's welfare and in a general equilibrium framework, rather than setting targets based on fiscal reaction function estimations.

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A Appendix

A.1 Government Problem Subject to Fiscal Rules

A.1.1 Balanced Budget Rule

The Lagrangian associated with the government's problem when subject to a balanced budget rule is:

$$
\begin{aligned}
\mathcal{L}_t = & \sum_{t=0}^{T} (\delta\beta)^t \Bigg[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \chi \frac{G_t^{1-\theta}}{1-\theta} \\
& + \lambda_t \left(\tau N_t W_t + b_t + F_t - G_t - (1+r_t^b) b_{t-1} - (1+r_t^F) F_{t-1} \right) + \mu_t^b \left(\frac{b_t - b_{t-1}}{Y_t} - \overline{BBY_t} \right) \Bigg] \\
& + \delta \Bigg\{ \sum_{s=1}^{T} \beta^s \Bigg[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \psi \frac{N_{t+s}^{1+\gamma}}{1+\gamma} + \chi \frac{G_{t+s}^{1-\theta}}{1-\theta} \\
& + \lambda_{t+s} \left(\tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} - G_{t+s} - (1+r_{t+s}^b) b_{t+s-1} - (1+r_{t+s}^F) F_{t+s-1} \right) \\
& + \mu_{t+s}^b \left(\frac{b_{t+s} - b_{t+s-1}}{Y_{t+s}} - \overline{BBY}_{t+s} \right) \Bigg] \Bigg\}
\end{aligned}
$$

Note that government's borrowing decision is different in the near future in comparison to the future after $t+1$, which is capture by $s \geq 1$. For that reason, government chooses variables in t and in $t+s$. First order conditions are:

$$
\begin{aligned}
[G_t]: \quad & (\delta\beta)^t G_t^{-\sigma} - (\delta\beta)^t \lambda_t = 0 \\
\Rightarrow \quad & G_t^{-\sigma} = \lambda_t && \text{(A.1)} \\
[b_t]: \quad & (\delta\beta)^t \left(\lambda_t + \frac{\mu_t^b}{Y_t}\right) - (\delta\beta)^{t+1} \left[\lambda_{t+1}(1+r_{t+1}^b) + \frac{\mu_{t+1}^b}{Y_{t+1}}\right] = 0 \\
\Rightarrow \quad & \lambda_t + \frac{\mu_t^b}{Y_t} = \delta\beta \left[\lambda_{t+1}(1+r_{t+1}^b) + \frac{\mu_{t+1}^b}{Y_{t+1}}\right] && \text{(A.2)} \\
[F_t]: \quad & (\delta\beta)^t \lambda_t - (\delta\beta)^{t+1} \lambda_{t+1}(1+r_{t+1}^F) = 0 \\
\Rightarrow \quad & \lambda_t = \delta\beta\lambda_{t+1}(1+r_{t+1}^F) && \text{(A.3)} \\
[b_{t+s}]: \quad & \delta\beta^{t+s} \left(\lambda_{t+s} + \frac{\mu_{t+s}^b}{Y_{t+s}}\right) - \delta\beta^{t+s+1} \left[\lambda_{t+s+1}(1+r_{t+s+1}^b) + \frac{\mu_{t+s+1}^b}{Y_{t+s+1}}\right] = 0 \\
\Rightarrow \quad & \lambda_{t+s} + \frac{\mu_{t+s}^b}{Y_{t+s}} = \beta \left[\lambda_{t+s+1}(1+r_{t+s+1}^b) + \frac{\mu_{t+1}^b}{Y_{t+1}}\right] && \text{(A.4)} \\
[F_{t+s}]: \quad & \delta\beta^{t+s} \lambda_{t+s} - \delta\beta^{t+s+1} \lambda_{t+s+1}(1+r_{t+s+1}^F) = 0 \\
\Rightarrow \quad & \lambda_{t+s} = \beta\lambda_{t+s+1}(1+r_{t+s+1}^F) && \text{(A.5)} \\
[\lambda_t]: \quad & \tau N_t W_t + b_t + F_t - G_t - (1+r_t^b)b_{t-1} - (1+r_t^F)F_{t-1} = 0 \\
\Rightarrow \quad & \tau N_t W_t + b_t + F_t = G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} && \text{(A.6)} \\
[\lambda_{t+s}]: \quad & \tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} - G_{t+s} - (1+r_{t+s}^b)b_{t+s-1} - (1+r_{t+s}^F)F_{t+s-1} = 0 \\
\Rightarrow \quad & \tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} = G_{t+s} + (1+r_{t+s}^b)b_{t+s-1} + (1+r_{t+s}^F)F_{t+s-1} && \text{(A.7)} \\
& \mu_t^b \left(\frac{b_t - b_{t-1}}{Y_t} - \overline{BBY_t}\right) = 0 && \text{(A.8)} \\
& \mu_{t+s}^b \left(\frac{b_{t+s} - b_{t+s-1}}{Y_{t+s}} - \overline{BBY_{t+s}}\right) = 0 && \text{(A.9)}
\end{aligned}
$$

Replacing (A.1) in (A.2), (A.4) , (A.3) and (A.5):

$$G_t^{-\sigma} + \frac{\mu_t^b}{Y_t} = \delta\beta \left[G_{t+1}^{-\sigma}(1 + r_{t+1}^b) + \frac{\mu_{t+1}^b}{Y_{t+1}} \right] \tag{A.10}$$

$$G_t^{-\sigma} = \delta\beta G_{t+1}^{-\sigma}(1 + r_{t+1}^F) \tag{A.11}$$

$$G_{t+s}^{-\sigma} + \frac{\mu_{t+s}^b}{Y_{t+s}} = \beta \left[G_{t+s+1}^{-\sigma}(1 + r_{t+s+1}^b) + \frac{\mu_{t+s+1}^b}{Y_{t+s+1}} \right] \tag{A.12}$$

$$G_{t+s}^{-\sigma} = \beta G_{t+s+1}^{-\sigma}(1 + r_{t+s+1}^F) \tag{A.13}$$

A.1.2 Debt Rule

The Lagrangian associated with the government's problem when subject to a debt rule is:

$$\begin{aligned}
\mathcal{L}_t = &\sum_{t=0}^{T} (\delta\beta)^t \Bigg[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \chi \frac{G_t^{1-\theta}}{1-\theta} \\
&+\lambda_t \left(\tau N_t W_t + b_t + F_t - G_t - (1 + r_t^b) b_{t-1} - (1 + r_t^F) F_{t-1} \right) + \mu_t^D \left(\frac{b_t - F_t}{Y_t} - \overline{DY_t} \right) \Bigg] \\
&+\delta \Bigg\{ \sum_{s=1}^{T} \beta^s \Bigg[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \psi \frac{N_{t+s}^{1+\gamma}}{1+\gamma} + \chi \frac{G_{t+s}^{1-\theta}}{1-\theta} \\
&+\lambda_{t+s} \left(\tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} - G_{t+s} - (1 + r_{t+s}^b) b_{t+s-1} - (1 + r_{t+s}^F) F_{t+s-1} \right) \\
&+\mu_{t+s}^D \left(\frac{b_{t+s} - F_{t+s}}{Y_{t+s}} - \overline{DY}_{t+s} \right) \Bigg] \Bigg\}
\end{aligned}$$

Just like the case where the government is subject to a balanced budged rule, government chooses variables in t and in $t+s$. First order conditions are:

$$[G_t]: \quad (\delta\beta)^t G_t^{-\sigma} - (\delta\beta)^t \lambda_t = 0$$

$$\Rightarrow \quad G_t^{-\sigma} = \lambda_t \tag{A.14}$$

$$[b_t]: \quad (\delta\beta)^t \left(\lambda_t + \frac{\mu_t^D}{Y_t}\right) - (\delta\beta)^{t+1} \lambda_{t+1}(1 + r_{t+1}^b) = 0$$

$$\Rightarrow \quad \lambda_t + \frac{\mu_t^D}{Y_t} = \delta\beta\lambda_{t+1}(1 + r_{t+1}^b) \tag{A.15}$$

$$[F_t]: \quad (\delta\beta)^t \left(\lambda_t + \frac{\mu_t^D}{Y_t}\right) - (\delta\beta)^{t+1} \lambda_{t+1}(1 + r_{t+1}^F) = 0$$

$$\Rightarrow \quad \lambda_t = \delta\beta\lambda_{t+1}(1 + r_{t+1}^F) \tag{A.16}$$

$$[b_{t+s}]: \quad \delta\beta^{t+s} \left(\lambda_{t+s} + \frac{\mu_{t+s}^D}{Y_{t+s}}\right) - \delta\beta^{t+s+1} \lambda_{t+s+1}(1 + r_{t+s+1}^b) = 0$$

$$\Rightarrow \quad \lambda_{t+s} + \frac{\mu_{t+s}^D}{Y_{t+s}} = \beta\lambda_{t+s+1}(1 + r_{t+s+1}^b) \tag{A.17}$$

$$[F_{t+s}]: \quad \delta\beta^{t+s} \left(\lambda_{t+s} + \frac{\mu_{t+s}^D}{Y_{t+s}}\right) - \delta\beta^{t+s+1} \lambda_{t+s+1}(1 + r_{t+s+1}^F) = 0$$

$$\Rightarrow \quad \lambda_{t+s} + \frac{\mu_{t+s}^D}{Y_{t+s}} = \beta\lambda_{t+s+1}(1 + r_{t+s+1}^F) \tag{A.18}$$

$$[\lambda_t]: \quad \tau N_t W_t + b_t + F_t - G_t - (1 + r_t^b) b_{t-1} - (1 + r_t^F) F_{t-1} = 0$$

$$\Rightarrow \quad \tau N_t W_t + b_t + F_t = G_t + (1 + r_t^b) b_{t-1} + (1 + r_t^F) F_{t-1} \tag{A.19}$$

$$[\lambda_{t+s}]: \quad \tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} - G_{t+s} - (1 + r_{t+s}^b) b_{t+s-1} - (1 + r_{t+s}^F) F_{t+s-1} = 0$$

$$\Rightarrow \quad \tau N_{t+s} W_{t+s} + b_{t+s} + F_{t+s} = G_{t+s} + (1 + r_{t+s}^b) b_{t+s-1} + (1 + r_{t+s}^F) F_{t+s-1} \tag{A.20}$$

$$\mu_t^D \left(\frac{b_t - F_t}{Y_t} - \overline{DY_t}\right) = 0 \tag{A.21}$$

$$\mu_{t+s}^D \left(\frac{b_{t+s} - F_{t+s}}{Y_{t+s}} - \overline{DY_{t+s}}\right) = 0 \tag{A.22}$$

Replacing (A.14) in (A.15), (A.17) , (A.16) and (A.18):

$$G_t^{-\sigma} + \frac{\mu_t^D}{Y_t} = \delta\beta G_{t+1}^{-\sigma}(1 + r_{t+1}^b) \tag{A.23}$$

$$G_t^{-\sigma} + \frac{\mu_t^D}{Y_t} = \delta\beta G_{t+1}^{-\sigma}(1 + r_{t+1}^F) \tag{A.24}$$

$$G_{t+s}^{-\sigma} + \frac{\mu_{t+s}^D}{Y_{t+s}} = \beta G_{t+s+1}^{-\sigma}(1 + r_{t+s+1}^b) \tag{A.25}$$

$$G_{t+s}^{-\sigma} + \frac{\mu_{t+s}^D}{Y_{+s}t} = \beta G_{t+s+1}^{-\sigma}(1 + r_{t+s+1}^F) \tag{A.26}$$

B Appendix-Lemmas Proofs

B.1 Lemma 1 Proof

Proof. From the first order conditions ((12) and (13) or (14) and (15)) it is easy to see that $1 + r_{t+1}^b = 1 + r_{t+1}^F$. By replacing (9) from assumption 2:

$$\begin{aligned} 1 + r_{t+1}^b &= 1 + r_{t+1}^F \\ &= (1 + r_{t+1}^b) * e^{\kappa F_t} \\ 1 &= e^{\kappa F_t} \quad \Rightarrow F_t = 0 \end{aligned}$$

Then, it directly derives from (8) that $D_t = b_t$, from (9) that $r_{t+1}^F = r_{t+1}^b$ and from (10) that $r_{t+1}^D = r_{t+1}^b$. □

B.2 Lemma 2 proof

Proof. From equation (21):

$$\begin{aligned} & if \quad \frac{b_t - b_{t-1}}{Y_t} - \overline{BBY_t} = 0 \\ & \Rightarrow \frac{b_t - b_{t-1}}{Y_t} = \overline{BBY_t} \qquad (*) \\ & \Rightarrow \mu_t^b \neq 0 \end{aligned}$$

Which means that the fiscal rule is binding. Also, (∗) implies that b_t is limited by the fiscal rule target and other variables that government takes as given: $b_t = \overline{BBY_t} * \frac{Y_t}{b_{t-1}}$. Replacing the latter in government balanced budget constraint (11) rearranging, and iterating forward one period:

$$G_t + (1+r_t^b)b_{t-1} + (1+r_t^F)F_{t-1} = N_t W_t \tau + \overline{BBY_t} * \frac{Y_t}{b_{t-1}} + F_t$$

$$G_t = N_t W_t \tau + \overline{BBY_t} * \frac{Y_t}{b_{t-1}} + F_t - (1+r_t^b)b_{t-1} - (1+r_t^F)F_{t-1} \tag{B.1a}$$

$$G_{t+1} = N_{t+1} W_{t+1} \tau + \overline{BBY_{t+1}} * \frac{Y_{t+1}}{b_t} + F_{t+1} - (1+r_{t+1}^b)b_t - (1+r_{t+1}^F)F_t \tag{B.1b}$$

Re-writing (18) (Euler equation corresponding to accounting gimmicks liabilities), to see the marginal costs and benefits of using creative accounting:

$$\underbrace{G_t^{-\theta}}_{\text{Marginal Benefit}} = \underbrace{\delta\beta G_{t+1}^{-\theta}(1+r_{t+1}^F)}_{\text{Marginal Cost}} \tag{B.2}$$

First, the left hand side (LHS) of (B.2) is going to be partially derived with respect to F_t to identify the marginal benefits of doing creative accounting. Replacing (B.1a) in (B.2) :

$$\text{LHS} = \left(\frac{1}{N_t W_t \tau + \overline{BBY_t} * \frac{Y_t}{b_{t-1}} + F_t - (1+r_t^b)b_{t-1} - (1+r_t^F)F_{t-1}}\right)^{\theta}$$

$$\frac{\partial \text{LHS}}{\partial F_t} = \theta \left(\frac{1}{G_t}\right)^{\theta-1}(1) \tag{B.3}$$

Secondly, the right hand side (RHS) of (B.2) is going to be partially derived with respect to F_t to identify the marginal costs of doing creative accounting. Take into account that $+r_{t+1}^F$ is a function of F_t and thus, the chain rule is used to do the following derivation. Replacing (B.1b) in (B.2) :

$$\text{RHS} = \delta\beta(1+r_{t+1}^F)\left(\frac{1}{N_{t+1} W_{t+1} \tau + \overline{BBY_{t+1}} * \frac{Y_{t+1}}{b_t} + F_{t+1} - (1+r_{t+1}^b)b_t - (1+r_{t+1}^F)F_t}\right)^{\theta}$$

$$\frac{\partial \text{RHS}}{\partial F_t} = \theta \left(\frac{1}{G_{t+1}}\right)^{\theta-1}\left[(1+r_{t+1}^F) + F_t \kappa e^{\kappa F_t}\right]\delta\beta(1+r_{t+1}^F) + \delta\beta\kappa e^{\kappa F_t} G_{t+1}^{\theta-1} \tag{B.4}$$

If (B.3) > (B.4), then the marginal benefits of doing creative accounting are greater than the marginal costs and so, government will use creative accounting. That this occurs, depends on the parameters θ, κ, β and δ after the endogenous response of household, that also influences the interest rates.

Finally, it directly derives from (8) that if $F_t > 0 \Rightarrow D_t > b_t$ and from (9) that if $F_t > 0 \Rightarrow r_t^F > r_t^D$, which implies in (10) that $r_t^D \neq r_t^b$. $\square$

C Appendix

Table C.1: Parameter values for numerical simulation

Parameter	**Value**	**Description**
δ	0.9	Quasi-hyperbolic discount
β	0.95	Time discount factor
γ	1	Frisch elasticity
ψ	2	Labor disutility
σ	1.1	Intertemporal substitution elasticity of consumption
θ	1.3	Intertemporal substitution elasticity of government spending
τ	0.25	Labor tax rate
χ	0.9	Relative value of public spending
κ	0.01	Risk premium growth factor
B_0, B_3	1	Initial and last debt level

D Appendix- Macroeconomic Dynamics, Optimal Debt Targets

Figure D.1: Macroeconomic Dynamics for benchmark, Q.Hyperbolic, Basline DR and optimal DR targets scenarios


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