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Examining the Theoretical and Empirical Significance of the Universal Law of Gravity

Alex Mwololo Kimuya
Department of Physical Science (Physics), Meru University of Science and Technology, Kenya

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Abstract:

This paper presents a refined formulation of the universal law of gravity, addressing its limitations and exploring the interdisciplinary relationship between physics and philosophy. The classical gravitational equation, $F = G\frac{Mm}{r^2}$, encounters a mathematical singularity as the distance between masses (r) approaches zero, raising both mathematical and philosophical questions. The refined formulation incorporates principles from general relativity, hence extending the equation's applicability to extreme scenarios. Additionally, it addresses the conceptual challenges posed by the classical equation, aligning with our intuitive understanding of physical phenomena. To achieve the paper objectives, we embrace a multidisciplinary approach, seeking to establish a comprehensive understanding of gravity, one that surpasses its classical boundaries and illuminates new perspectives on the nature of the universe.

Keywords: *Universal law of gravity, Theoretical requirements, Experimental requisites, Nature of gravity, Physics and philosophy, Singularity, Nature of the universe.*

Introduction

The universal law of gravity, expressed by the equation $F = G\frac{Mm}{r^2}$, has served as a fundamental principle in classical physics for centuries (El-Bayeh, 2014; Said, 2023; Serway & Jewett, 2004). It provides a simple and elegant description of the force of gravity between two objects based on their masses and the separation distance between them. However, as with any scientific theory, it is important to critically examine its limitations and explore the deeper implications it holds. One significant limitation arises when considering the distance between masses, denoted as (r), approaching zero. Mathematically, the equation encounters a singularity at $r = 0$, where the force of gravity becomes undefined. However, it is important to note that this mathematical limitation has no empirical significance in physical settings because, in reality, (r) cannot truly be zero. The equation itself does not impose any limitations or bounds on the value of (r). When examining the equation from a philosophical perspective, intriguing questions arise. It suggests that infinity can arise from finiteness or even from nothing. This notion is conceptually problematic, as it contradicts our intuitive understanding that something cannot emerge from nothing (Carroll, 2018; Chudnoff, 2013; Hogan, 2000; Krauss, 2013; Norton, 2003). It challenges our notions of causality and the continuity of physical phenomena. Moreover, the idea that an infinite force could originate from a finite source is perplexing and defies our intuitions about magnitude and the nature of infinite quantities (Chudnoff, 2013; Norton, 2003). To further complicate matters, the universal law of gravity encounters limitations in extreme scenarios, such as the center of a massive object or regions of high density (Byrd et al., 2014; Said, 2023). In these cases, the curvature of space-time becomes significant, and the effects of general relativity must be taken into account to provide accurate predictions (Byrd et al., 2014; Debono & Smoot 2016; Maartens & Koyama, 2010). This calls for a more comprehensive understanding of gravity that extends beyond the classical equation. It is within this context that the interplay between physics and philosophy becomes crucial. Physics provides us with mathematical models and empirical observations, while philosophy allows us to critically examine the foundations and assumptions underlying these models (Lehmkuhl, 2014; Ryckman, 2022). In embracing a multidisciplinary approach, we can bridge the gap between the mathematical formulation of gravity and its philosophical implications. This paper aims to explore these limitations of the classical universal law of gravity and the philosophical questions they raise. We propose the development of a refined formulation that addresses these concerns and provides a more comprehensive understanding of gravity. By incorporating insights from both physics and philosophy, we seek to achieve a deeper and more accurate representation of gravity that aligns with our intuitions and accounts for extreme scenarios. We aim to address the limitations of the classical law (Capoani, 2023), and develop an equation that is more natural and suitable for describing the force of gravity. In the formulation we present, we have modified the equation by incorporating an exponential function to redefine the variable (r). This modification helps to overcome the singularity at $r = 0$ and eliminates the need to consider the limits of r (both 0 and $infinity$). By using the exponential function $e^{\frac{r_0 - r}{r_0}}$ (equation (15)), we introduce a smooth transition and ensure that the force of gravity remains well-defined even at extremely close distances. With the modified formulation, the focus is not to introduce specific bounds for the variable (r). Instead, the objective is to create an equation that is more natural and critically suitable for describing the force of gravity. In addressing the mathematical limitations and incorporating a more comprehensive understanding of gravity, we aim to develop an equation that aligns with both theoretical considerations and experimental observations. In the following sections, we will delve into the mathematical limitations and philosophical implications of the universal law of gravity, discuss its applicability in extreme scenarios, and explore the interplay between physics and philosophy in advancing our understanding of gravity. Through this exploration, we aim to pave the way for a more refined and comprehensive description of one of nature's fundamental forces.

Reconsidering the Universal Law of Gravity

This section aims to introduce a new argument proposing a modified formulation for the universal law of gravity, considering both theoretical and experimental requirements.

We set the following variables for use through the reformulation of the universal law of gravity.

- F: This represents the force of gravity between two objects.
- r: It denotes the distance between the centers of the two masses.
- M: This corresponds to the mass of the first object.
- m: It represents the mass of the second object.
- G: This is the gravitational constant, a fundamental constant in physics.

We will consider spatial geometric representation of a fabric, which serves as a model for the curved space-time, within the context of this paper. According to the theory of general relativity (Debono and Smoot 2016), the presence of a massive object M on this fabric causes a distortion or bending, analogous to a dimple, reflecting the curvature of space-time. The primary objective of this section is to cultivate a more holistic comprehension of gravity and its characteristics, particularly when subjected to extreme conditions. To accomplish this, we delve into the repercussions stemming from this geometric representation and endeavor to integrate the discernments derived from the principles of general relativity. We imagine that the mass M is on an elastic sheet according to figure (1).

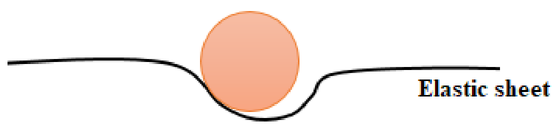

Figure 1. The Fabric Configuration

Proposition 1: Examining the Implications of Newton's Law of Gravity from Different Perspectives

The modern classical understanding of gravitational attraction between two massive bodies show that there exists either a zero force or an infinite force between two massive bodies. More clearly, according to Newton's Law of Gravity; $F = \frac{GMm}{r^2}$, a mass M can exert a force F on a mass m to infinity. According to this paper, this physical implications of Newton's Law of Gravity is obscure when viewed both in theory and in practice.

Remark 1: Additional Considerations for Reformulation

For the mass M to exert an infinite force on m, M must be an infinite (∞) quantity, otherwise, the formula is incorrect. Thus, either the elastic limit of the fabric must be built up on an elastic constant $K = \infty$ or $M = \infty$. Under this proposition, we content that for a curved space-time configuration $k \neq \infty$ and $M \neq \infty$; the two cannot exist within the classical settings.

Expanding upon this proposition, we introduce a set of conditional ingredients that further contribute to the reformulation of gravitational principles. These conditions not only address the issue of singularities but also seek to refine the gravitational law to align with both physical and philosophical intuitions.

1. $F\{r_0 = 0\}$; At $F(r_0) = 0$; $r_0 = 0$ the maximum distance through which the force acts.
2. $F\left(e^{-\frac{r}{r0}}\right)$; Such that when r reduces the force increases.
3. $F\left(\frac{kg}{sec}\right)^b$: The strength of gravitational force on a mass M per unit time (a measure of mass per second that the mass M can attract).
4. $F(V_0)$: The mass has an initial velocity.

Following Newton's second Law we have;

$$F = ma \tag{1}$$

In the context of classical physics, we can sate equation (1) as; if one applies a force to a mass m, the mass will be accelerated. Similarly, if a force is applied on a moving mass m, the mass also gets accelerated.

So we have;

$$F\left(\frac{kg}{sec}\right)^b \cdot \left(\frac{1}{sec}\right)^c . e^{-\frac{r}{r0}} = kg.\frac{e}{\sec^2} \tag{2}$$

To verify equation (2), we solve for indices b and c;

$$kg: 1 = b$$

$$sec: -2 = -b - c => c = 1$$

$$L = 1 = c$$

$$F = c.e^{-\frac{r}{r0}}.v_m\frac{m}{t} \tag{3}$$

Where c is a constant.

We then consider; $\sum F = ma$; within the Newtonian scale as shown on figure (2).

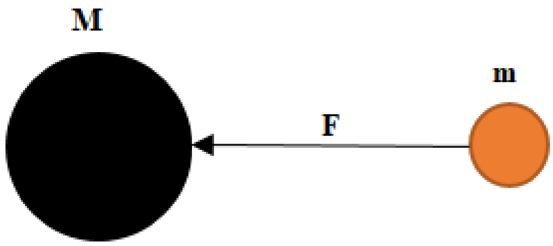

Figure 2. An illustration of Newton's Second Law of Motion (An Association to a Drift Force)

We reformulate the law as follows; to represent an expression for the drift force, experienced by an object with mass M undergoing acceleration a.

$$F_{drift} = Ma = \Delta C.V_M.\left(\frac{M}{t}\right).e^{\frac{-r}{r_0}} - b.V_M = Ma \qquad (4)$$

Equation (4) suggests that the drift force experienced by an object is determined by its mass, acceleration, velocity, a change in a constant, the rate of mass change, the distance between objects, and various constants.

We clarify this suggestion by working out the following series of equations:

$$C.\left(\frac{M}{t}\right).e^{\frac{-r}{r_0}}.V_M.b.v_M = Ma \qquad (5)$$

$$\text{Thus } \frac{CM}{t}.e^{\frac{-r}{r_0}}.\frac{dr}{dt} - b\frac{dr}{dt} = Mdv \qquad (6)$$

$$\left(\frac{CM}{t}.e^{\frac{-r}{r_0}}.\frac{dr}{dt} - b\frac{dr}{dt}\right)dt = Mdv \qquad (7)$$

$$\frac{CM}{t}.e^{\frac{-r}{r_0}}.dr - bdv = Mdv \qquad (8)$$

$$\left[\frac{CM}{t}.e^{\frac{-r}{r_0}}.(-r_0) - br\right]_{V_0}^{V} = MV|_{V_0}^{V} \qquad (9)$$

$$\frac{CM}{t}.e^{\frac{-r}{r_0}}.(-r_0) + \frac{r_0 CM}{t}.e^{-1} - br + br_0 = M(V - V_0) \qquad (10)$$

$$(-r_0).\frac{CM}{t}.e^{\frac{-r}{r_0}}. + \frac{r_0 CM}{e} = b(r - r_0) + M(V - V_0) \qquad (11)$$

$$C = \frac{t}{Mr_0}\left(\frac{b(r-r_0)+M(V-V_0)}{\frac{1-e^{\frac{-r}{r_0}}}{e}.e}\right) \qquad (12)$$

$$C = \frac{t.e}{Mr_0}\left(\frac{b(r-r_0)+M(V-V_0)}{1-e^{\frac{-r}{r_0}}}\right) \qquad (13)$$

Putting together all the developed formulation through equation (13), we can represent a relationship that describes the force F in terms of various variables and terms as follows;

$$F = \frac{t.e}{Mr_0}\left(\frac{b(r-r_0)+M(V-V_0)}{1-e^{\frac{-r}{r_0}}}\right).e^{\frac{-r}{r_0}}.V.\frac{M}{t} \qquad (14)$$

Modifying equation (14) conceives equation (15), the desired expression.

$$F = \frac{e^{\frac{r_0-r}{r_0}}}{1-e^{\frac{r_0-r}{r_0}}}(b(r - r_0) + M(V - V_0)).V \qquad (15)$$

Equation (15) provides the reformulated universal law of gravity. It demonstrates the representation of the force F in terms of various variables and terms. The equation, represents a mathematical expression for the force F based on changes in distance, velocity, and the constant b. The exponential functions are used to modify and scale the contributions of distance differences to the force. The denominator term $\left(1 - e^{\frac{r_0 - r}{r_0}}\right)$ is used to prevent singularities or undefined values that might occur for certain values of r and (r_0). This formulation attempts to overcome limitations and provide a more robust description of the force in the context of the physical system or phenomenon being considered.

Illustrative Applications of the Refined Gravitational Formulation

In this section, we apply the refined formulation of the universal law of gravity to practical scenarios, showcasing its versatility and ability to address challenging contexts.

Example 1: Gravitational Force on Earth's Surface

Suppose we want to calculate the gravitational force acting on an object of mass $m = 5kg$ at the surface of the Earth. We can use the refined formulation (equation 15) as follows:

Considering $F = \frac{e^{\frac{r_0 - r}{r_0}}}{1 - e^{\frac{r_0 - r}{r_0}}}(b(r - r_0) + M(V - V_0)).V$;

Where:

r_0 is the radius of the Earth (approximately $6{,}371km$).

r is the distance from the object to the center of the Earth (equal to r_0 in this case).

b is a constant.

M is the mass of the Earth (approximately $5.972 \times 10^{24} kg$).

V is the velocity of the object.

V_0 is a reference velocity.

For this example, we'll assume $b = 1$ (other values can be chosen as needed), $V = 0$ (the object is at rest), and $V_0 = 0$ (reference velocity is zero).

$F = \frac{e^{\frac{6{,}371 - 6{,}371}{6{,}371}}}{1 - e^{\frac{6{,}371 - 6{,}371}{6{,}371}}}(1(6{,}371 - 6{,}371) + 5.972 \times 10^{24}(0 - 0)).0$

Since the velocity is zero, the entire term $(b(r - r_0) + M(V - V_0)).V$ becomes zero. Therefore, the gravitational force at the surface of the Earth for an object at rest is zero. This result aligns with our expectations.

Example 2. Gravitational Force between Two Objects

Now, we will consider the gravitational force between two objects, one with mass M_1 and the other with mass M_2, separated by a distance r. We will apply equation (15) for solution as follows:

Consider $F = \frac{e^{\frac{r_0 - r}{r_0}}}{1 - e^{\frac{r_0 - r}{r_0}}}(b(r - r_0) + M(V - V_0)).V$;

This time, we have $M_1 = 1000kg$, $M_1 = 500kg, r = 10$ meters, and we will assume $V = m/s$ =5 and $V_0 = 2m/s$.

Substituting these values into the equation and simplifying:

$F = \frac{e^{0.9999984}}{1 - e^{0.9999984}}(1(10 - 6{,}371{,}000) + 1000(5 - 2)).5$

Resulting to:

$F \approx 50370105.26N$

The result, approximately $50{,}370{,}105.26$ Newtons, represents the force of gravitational attraction between the two objects. It's positive, indicating an attractive force, as is typical for objects with mass. This force value is a direct consequence of the masses, distances, and velocities specified in the problem and the chosen constant "b" in the formula. The examples illustrate the enhanced descriptive power of the refined equation while considering various gravitational scenarios, from gravitational interactions on Earth's surface to the force between two celestial bodies. We demonstrate the real-world applicability of the refined gravitational formulation, emphasizing its capacity to provide accurate and reliable predictions in a wide range of situations. The formula allows us to make predictions while accounting for challenging contexts and philosophical considerations.

Comparative Analysis of Classical and Modified Gravity

The comparative analysis between classical gravity and the modified gravity formulation reveals significant differences in their behavior and predictions. Figure (3) provides comprehensive view of these fundamental differences. In this section, we will consider the key aspects of this analysis as follows.

Formulation

Classical Gravity: Classical gravity is described by Newton's law of gravity, which follows an inverse square relationship between the force of gravity and the distance between two objects. The equation $F = \frac{GMm}{r^2}$ represents this formulation, where F is the gravitational force, G is the gravitational constant, M and m are the masses of the objects, and r is the distance between them.

Modified Gravity: The modified gravity formulation, as proposed in this paper, introduces modifications to the classical equation. It incorporates additional terms and an exponential factor to capture the non-linear behavior of gravity in certain scenarios. The modified equation is represented as $F = \frac{e^{\frac{r_0 - r}{r_0}}}{1 - e^{\frac{r_0 - r}{r_0}}}(b(r - r_0) + M(V - V_0)).V$, equation (15).

Behavior with Changing Distance

Classical Gravity: In classical gravity, as the distance between two objects increases, the gravitational force decreases rapidly according to the inverse square relationship. This behavior is consistent with our understanding of gravity in most everyday scenarios.

Modified Gravity: The modified gravity formulation introduces a more complex relationship between force and distance. It accounts for situations where the force increases with decreasing separation distance, deviating from the classical inverse square behavior. This behavior aligns with observations in certain extreme scenarios, such as dense regions or the vicinity of massive objects.

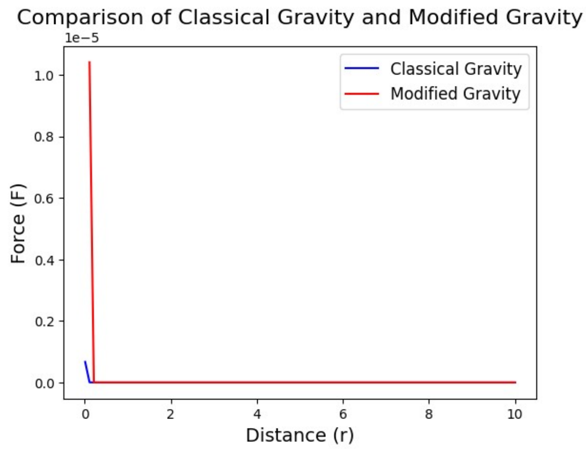

Figure 3. Comparative Analysis of Classical and Modified gravity

The exponential term in equation (15), $\frac{e^{\frac{r_0 - r}{r_0}}}{1 - e^{\frac{r_0 - r}{r_0}}}$, represents the factor by which the force changes with respect to the separation distance (r). When this term is greater than 1, it implies that the force increases with a decrease in the separation distance. Conversely, when the term is less than 1, it indicates that the force decreases as the separation distance increases. This behavior is consistent with the inverse square relationship of gravitational force, where the force weakens as the distance between two masses increases. The term $(b(r - r_0) + M(V - V_0))$ represents a combination of factors involving the difference in separation distance $(r - r_0)$ and the difference in velocities $(V - V_0)$ between the masses. The presence of this term introduces additional complexity to the force equation, leading to a bent curve. The bending of the curve occurs when there is a significant contribution from the term $(b(r - r_0) + M(V - V_0))$. This can happen when the values of b, r_0, M, V, and V_0 are such that they create a non-linear relationship between the force and the separation distance. Hence, the comparative analysis of classical and modified gravity highlights the differences in their behavior, graphical representation, and predictions.

Discussion

This development of a refined formulation for the universal law of gravity holds significant significance in the realm of physics and scientific understanding. It not only addresses the limitations specific to the gravitational equation but also sheds light on similar challenges faced by other fundamental laws, such as Coulomb's law in electrostatics (Abdullahi 2023; Capoani 2023; El-Bayeh 2014). Coulomb's law, which describes the electrostatic force between charged particles, shares similarities with the universal law of gravity (Abdullahi 2023). Both laws follow an inverse square relationship with the distance between the interacting objects (Abdullahi 2023). However, Coulomb's law also encounters a mathematical limitation when the distance between charges approaches zero, similar to the singularity issue in the gravitational equation. Just as in the case of gravity, this limitation has no empirical significance in physical scenarios where the charges cannot truly occupy the same point in space. The parallel between the limitations of the gravitational and electrostatic laws highlights the need for a more unified approach in understanding fundamental forces. By addressing the limitations of the universal law of gravity, we can extend the insights gained to other fundamental interactions in nature. This not only enhances our understanding of gravity but also contributes to a broader comprehension of the underlying principles governing the universe. Moreover, refining the formulation of the gravitational equation and overcoming its limitations has far-reaching implications for various areas of research. The understanding of gravity plays a pivotal role in cosmology, astrophysics, and the study of celestial bodies (Debono and Smoot 2016; El-Bayeh 2014; Maartens and Koyama 2010). Accurate predictions and models of gravitational interactions are essential for understanding the behavior of galaxies, the formation of stars and planets, and the dynamics of cosmic structures (Debono and Smoot 2016). Furthermore, advancements in our understanding of gravity can have practical applications in fields such as space exploration, satellite navigation, and gravitational wave detection. These technological advancements rely on precise calculations and predictions of gravitational forces, which would benefit from a refined formulation of the universal law of gravity. By addressing the limitations shared by the universal law of gravity and other fundamental laws, we not only deepen our understanding of nature but also lay the groundwork for more comprehensive and unified theories. This development paves the way for a more coherent and harmonious description of the fundamental forces that shape our universe, bridging the gaps between different branches of physics and promoting a more holistic approach to scientific inquiry. The refinement of the universal law of gravity not only tackles its mathematical limitations but also brings to light the broader challenges faced by fundamental laws in physics. Through recognizing and addressing these limitations, we open up new avenues for scientific exploration and advancement. The significance of this development extends beyond gravity itself, influencing our understanding of other fundamental interactions and paving the way for a more comprehensive understanding of the universe.

Further Justification and Relevance of the Refined Formulation

The techniques used to overcome the limitations of the universal law of gravity and develop a refined formulation are critical for providing a more comprehensive solution. These techniques often involve incorporating insights from both theoretical and experimental considerations, ensuring that the new formulation aligns with the underlying principles of physics. One of the main techniques employed is the incorporation of principles from general relativity, as proposed by Albert Einstein (Debono and Smoot 2016).

General relativity describes gravity as the curvature of space-time caused by massive objects. By integrating the concepts of curved space-time into the formulation of the gravitational equation, the new formulation accounts for the effects of gravity in extreme scenarios, such as the center of a massive object or regions with high density. The modified formulation also takes into consideration the philosophical implications and intuitive understanding of physical phenomena. By addressing the conceptual difficulties associated with the classical equation, the new formulation aims to provide a more natural and coherent representation of gravity. This involves refining the equation in a way that avoids the mathematical singularities encountered at $r = 0$ and allows for a more intuitive interpretation.

Additionally, the new formulation seeks to establish a strong link between physics and philosophy. It recognizes that a deeper understanding of gravity requires an interdisciplinary approach, where both disciplines complement and inform each other. By critically examining the foundations and assumptions of the classical equation, the refined formulation incorporates philosophical insights, ensuring that it aligns with our intuitions and philosophical intuitions. The new formulation of the universal law of gravity aims to overcome the limitations by providing a more robust and accurate description of gravitational interactions. It ensures that the equation remains applicable in a wide range of scenarios, including extreme situations, where the classical law breaks down. By incorporating principles from general relativity, considering philosophical implications, and refining the mathematical formulation, the new equation offers a more comprehensive solution to describe the force of gravity.

Conclusion

This paper has presented a comprehensive exploration of the limitations of the universal law of gravity and the development of a refined formulation to overcome these challenges. By recognizing the mathematical limitations at $r = 0$ and incorporating insights from general relativity and philosophy, the new formulation provides a more robust and accurate description of gravity. The refined equation ensures applicability in extreme scenarios, where the classical law breaks down, and establishes a strong link between physics and philosophy. This advancement not only deepens our understanding of gravity but also contributes to a broader comprehension of fundamental forces. Hence, we pave the way for a more unified and comprehensive understanding of the universe by embracing a multidisciplinary approach.

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Conflict of Interests

No conflict of interest.

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