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Simplified Mathematical Modeling on Person-to-Person Disease Transmission: The Coronavirus Case

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Abstract:

In this paper, a simplified mathematical model is developed through a system of ordinary differential equations for the transmission of diseases from person to person, conditions for disease control are provided and cases are studied in which it is not possible to apply the Hurwitz criterion, the corresponding qualitative study is carried out to draw conclusions on the future evolution of the disease. Additionally, the ways in which the different diseases are transmitted are analyzed and the possibilities of epidemic development and the conditions that must be created to avoid them are studied A background of the Mathematical Modeling research group is also indicated.

Keywords: *Disease, mathematical model, transmission.*

Introduction

The processes of contagion by a disease in general are produced from person to person, by some type of contact, either through speech, or any other physical contact (Hethcote, 1976); there may also be transmission through sexual contact, (Soto, et al., 2019); in many of these diseases, the number of individuals infected increase at such a rapid rate that they can become an epidemic or, in the case of worldwide spread, even a pandemic. (Greenhalgh, 1992; Hamer, 1906; Hethcote, 1994).

Indirect transmission is very common, that is, through some vector, especially the mosquito that causes the transmission of dengue, zika, yellow fever and chikungunya, etc, (Esteva, & Vargas, 1998; Yang, & Nie, 2017). Due to the mosquito ability to reproduce rapidly, controlling this form of transmission is challenging and expensive. This is one of the reasons why diseases like dengue fever is becoming endemic.

A disease that has become a pandemic is COVID-19, which continues to claim lives despite the efforts made by the World Health Organization and the health ministries of various countries. Although a percentage of the world's population has been vaccinated, several reinfections have still occurred, resulting in the emergence multiple waves. (Olivero, Domínguez, & Chaveco, 2022).

Although in this work mainly analyzes the epidemic transmission caused by the SARS-Cov-2, is applicable to epidemics, of different types, such as sexually transmitted diseases, where the fundamental difference lies in the fact that in the latter, the concentrations between susceptible and exposed individuals is very high. (Ivorra, et al., 2020; Domínguez, et al., 2020).

As a consequence of the high transmissibility and lethality of COVID-19, the WHO declared it as a public health emergency spread across all continents, affecting a large percentage of people, this is among the worst situations that human health has ever faced (Domínguez, et al., 2020; Zhao, & Chen, 2020).

Números studs have been done on COVID-19, both from the perspective of its biochemical characteristics and treatment as well as from the perspective of mathematical modeling to make predictions about its future behavior. Among them are Ramirez-Torres, et al., (2021); Ivorra, et al., (2020); Domínguez, et al., (2020); Zhao, & Chen, (2020) where the models are presented by simple differential equations and provide conclusions about the future behavior of the infectious process of the considered population.

History of the mathematical modeling group

Our Mathematical Modeling group is made up of professionals from Universidad de Oriente and Universidad de Guantánamo, both in Cuba. In addition to the University of the State of Amazonas in Brazil, professionals from other institutions, both in Cuba and Brazil, have participated sporadically.

This group made itself known in 1983 with an article referring to the transmission of infectious diseases, which was linked to a doctoral thesis; again in 2004 other articles were published on the subject of hemoglobin polymerization, also related to another doctoral thesis.

The first non-autonomous article was published in 2016, with periodic coefficients with respect to time.

The group has already published more than 50 articles, which include both autonomous and non-autonomous models, linked to Medico-Biological processes; these works are related to the following themes:

- Elimination of pollution by means of oxidation ponds.
- Use of glucose as an energy source.
- Coexistence of different species in a fir ecological space, including the prey-predator model for two and three species.
- Elimination of a drug in the human body.
- Dynamics between the oxonium layer and the infrared rays.
- Blood oxygenation process.
- Elimination of toxin by the liver, circulatory system and kidneys.
- Development of epidemics with and without vectors.
- Models of COVID 19.
- The dynamics between the immune system and viruses.
- The transmission of leprosy and its dermatological effect.

Related to these processes, the group has already published four books, one in Germany and the other three here in Brazil, at APPRIS and CRV publishing houses.

Mathematical model

To formulate the simplified disease transmission model, only two states will be considered, the population of those who are at risk and the population of those are infected where $\tilde{x}_1$ and $\tilde{x}_2$ are the total concentrations of susceptible persons at time t and the overall concentrations of infected persons at time t respectively. Furthermore, $\overline{x}_1$ and $\overline{x}_2$ are the values of admissible concentrations of population that are susceptible and population that are infected respectively, the variables will be introduced in addition $x_1 = \tilde{x}_1 - \overline{x}_1$ and $x_2 = \tilde{x}_2 - \overline{x}_2$. Such that, if $(x_1, x_2) \to (0,0)$ then $\tilde{x}_1 \to \overline{x}_1$ and $\tilde{x}_2 \to \overline{x}_2$, this is rearch objective, because that way the disease would be controlled and there would be no possibility of developing an epidemic.

The variation of population of the susceptible does not increase or if it increases these values would only reach a maximum value, but they are added in dependence of the concentration of the infected population; on the other hand, the variation in population that are nfected decreases according to the increase of the susceptible, due to the process of recuperation and they are added proportionally to their concentration due to the process of expansion of the disease. Therefore, the model can be represented by the subsequent set of differential equations.

$$\begin{cases} x_1' = -a_1 x_1 + a_2 x_2 + X_1(x_1, x_2) \\ x_2' = -b_1 x_1 + b_2 x_2 + X_2(x_1, x_2) \end{cases} \quad (1)$$

Where the functions $X_i(x_1, x_2)$ $(i = 1,2)$ represents the action of complementary activities or control exercised by the competent authorities and other physiological activities of the body that may influence the process, as well as any other disorder that may alter the functioning of our body favoring or not favoring the infection process. The power series expansion of these functions can be expanded as follows:

$$X_i(x_1, x_2) = \sum_{|p| \geq 2} X_i^{(p)} x_1^{p_1} x_2^{p_2} \; (i = 1,2), |p| = p_1 + p_2$$

The meaning of the coefficients specified in the system is as follows:

a_1 represents the decrease in the amount of susceptible as a function of their own concentration.

a_2 represents the growth in the amount of susceptible as a function of the concentration of infected people due to recovery.

b_1 represents the decrease in the infected people concentration due to the increase in susceptible.

b_2 represents the growth of the infected people concentration due to their own concentration.

Remark 1: It is considered that the initial process is favorable to the development of the disease, otherwise there would be no way for the infection to spead. In this case, the signs of the coefficients in the previous analysis align with the attributes problem being discussed.

The characteristic equation of the linear part of the system has the form,

$$\lambda^2 + (a_1 - b_2)\lambda + (a_2 b_1 - a_1 b_2) = 0.$$

From here, partial conclusions can be drawn regarding the behavior of the process as a function of the coefficients of variation of the concentrations.

Theorem 1: The equilibrium position $(0,0)$ is stable asymptotically only if the conditions $a_1 > b_2$ and $a_2 b_1 > a_1 b_2$ are satisfied.

The above conditions are consequences of Hurwitz's theorem, once the first approximation theorem is applied.

Remark 2: When the conditions $a_1 < b_2$ and $a_2 b_1 > a_1 b_2$ are met then, the total susceptible well and infected people concentration will become adjacent to values of ideal concentrations, so the possibilities for the development of epidemics become nil, otherwise measures must be taken to prevent further complications.

Suppose if the case is $a_1 = b_2$ and $a_2 b_1 > a_1 b_2$, the first approximation method is not applicable, as we have a critical case corresponding to a pair of pure imaginary eigenvalues. In correspondence with the procedures applied in previous works, Rodríguez, et al., (2019); Soto, et al., (2019) through a non-degenerate transformation $X = SY$, the system (1) is transformed into the system (2),

$$\begin{cases} y_1' = \sigma i y_1 + Y_1(y_1, y_2) \\ y_2' = -\sigma i y_2 + Y_2(y_1, y_2) \end{cases} \quad (2)$$

Similar to the works are previously published in the mathematical modeling group Rodríguez, et al., (2019) and Soto, et al., (2019) different processes are modeled and reduced to normal form for their study qualitatively. For a better understanding of the work, this theorem will be demonstrated in full.

Theorem 2: The change of variables,

$$\begin{cases} y_1 = z_1 + h_1(z_1, z_2) \\ y_2 = z_2 + h_2(z_1, z_2) \end{cases} \quad (3)$$

transforms the system (2) into normal form,

$$\begin{cases} z_1' = \sigma i z_1 + z_1 P(z_1 z_2) \\ z_2' = -\sigma i z_2 + z_2 \overline{P}_2(z_1 z_2) \end{cases} \quad (4)$$

Where$h_i(z_1, z_2)$, $i = 1{,}2$ are series similar to series $X_i(x_1, x_2)$ $(i = 1{,}2)$, $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$ are power series.

Proof: Differentiating the transformation (3) along the paths of systems (2) and (4) gives the system of equations,

$$\begin{cases} (p_1 - p_2 - 1)\sigma i h_1(z_1, z_2) + z_1 P(z_1 z_2) = \\ Y_1(z_1 + h_1, z_2 + h_2) - \frac{\partial h_1}{\partial z_1} z_1 P(z_1 z_2) - \frac{\partial h_1}{\partial z_2} z_2 \overline{P}(z_1 z_2) \\ (p_1 - p_2 + 1)\sigma i h_2(z_1, z_2) + z_2 \overline{P}(z_1 z_2) = \\ Y_2(z_1 + h_1, z_2 + h_2) - \frac{\partial h_2}{\partial z_1} z_1 P(z_1 z_2) - \frac{\partial h_2}{\partial z_2} z_2 \overline{P}(z_1 z_2) \end{cases} \quad (5)$$

The system (5) enable the coefficients of the series to be determined , $h_1(z_1, z_2)$, $h_2(z_1, z_2)$, $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$; the resonance equations are, $p_1 - p_2 - 1 = 0$ and $p_1 - p_2 + 1 = 0$, allow to determine the series $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$ which are different from zero in the resonant case, being $h_1(z_1, z_2)$ and $h_2(z_1, z_2)$ are equal to zero, in this case as they are arbitrary and to ensure uniqueness they become equal to zero, from the resonance equations it is deduced the forms of the powers of $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$. In the non-resonant case, the series $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$ are equal to zero, and in this case, $h_1(z_1, z_2)$ and $h_2(z_1, z_2)$ are uniquely determined.

The series $P(z_1 z_2)$ and $\overline{P}(z_1 z_2)$ admit the following development in power series,

$$P(z_1 z_2) = \sum_{n=k}^{\infty} a_n (z_1 z_2)^n + i \sum_{n=l}^{\infty} b_n (z_1 z_2)^n$$

Here it is proceeded in correspondence with the bibliographies indicated above.

Theorem 3: If $a_1 = b_2, a_2 b_1 > a_1 b_2$ and $a_k < 0$, then the system trajectories (5) will be asymptotically stable, else unstable.

Proof: For the demonstration, the "Lyapunov function" defined as the following positive $V(z_1, z_2) = z_1 z_2$ will be used, since the process is biological its solution belongs to the first quadrant.

The derivative of function V along system (4) can be expressed as,

$$V'(z_1, z_2) = 2a_k(z_1z_2)^{k+1} + R_1(z_1, z_2) < 0$$

It is observed that the derivative $V'(z_1, z_2)$ is negative, because the function $R_1(z_1, z_2)$ contains terms with a degree greater than $(k+1)$ only. So, $V'(z_1, z_2)$ is negative definite if the null solution of the system (4) is guaranteed with the asymptotic stability, thus proving Theorem3.

Remark 3: If the conditions $a_1 = b_2$, $a_2b_1 > a_1b_2$ and $a_k < 0$ the convergence of susceptible and infected populations to admissible values is guaranteed, this indicates that there will be no possibility of an epidemic developing.

Description of the results obtained: From observations 2 and 3 it can be deduced that it is sufficient to identify the matrix coefficients of the linear part to determine whether the disease is under control or there are risks of an epidemic.

Example 1: Let the system,

$$\begin{cases} x_1' = -x_1 - x_2 - x_1^3x_2^2 \\ x_2' = 5x_1 + x_2 - x_1^2x_2^3 \end{cases}$$

The eigenvalues of the system matrix are $\lambda_1 = 2i$ and $\lambda_2 = -2i$ this case corresponds with the theoretical view, to reach conclusions it is necessary to apply the second method of Lyapunov, but we will make the graph so that we can see the phase trajectories and draw conclusions.

The transformation $X = SY$, where

$$S = \begin{pmatrix} \frac{-1+2i}{5} & \frac{-1-2i}{5} \end{pmatrix}$$

Reduces the system in the equivalent system

$$\begin{cases} y_1' = 2iy_1 + 25y_1\left((2+i)y_1^2 + 2iy_1y_2 - (2-i)y_2^2\right)^2 \\ y_2' = -2iy_2 - 25y_2\left((1-2i)y_1^2 + 2y_1y_2 + (1+2i)y_2^2\right)^2 \end{cases}$$

The change of variables (3) transform the system in the normal form

$$\begin{cases} z_1' = 2iz_1 - 350z_1^3z_2^2 + (12500 - 25000i)z_1^5z_2^4 + \ldots \\ z_2' = -2iz_2 - 350z_1^2z_2^3 + (12500 + 25000i)z_1^4z_2^5 + \ldots \end{cases}$$

where

$$z_1P(z_1z_2) = -350z_1^3z_2^2 + (12500 - 25000i)z_1^5z_2^4 + \ldots$$

$$z_2\bar{P}(z_1z_2) = -350z_1^2z_2^3 + (12500 + 25000i)z_1^4z_2^5 + \ldots$$

$$h_1(z_1, z_2) = (75 + 100i)z_1^5 - 350z_1^2z_2^3 - (100 + 200i)z_1^2z_2^3 + (75 - 100i)z_1z_2^4 + \ldots$$

$$h_2(z_1, z_2) = (75 + 100i)z_1^4z_2 - (100 - 200i)z_1^3z_2^2 - (100 + 200i)z_1z_2 + \ldots$$

Let the Lyapunov function $V(z_1, z_2) = z_1z_2$, the derivative of the system trajectories is expressed as $V'(z_1, z_2) = -700(z_1z_2)^3 + R(z_1, z_2) < 0$; therefore the system is asymptotically stable, the geometrical behavior of its solutions are illustrated by the following graphs.

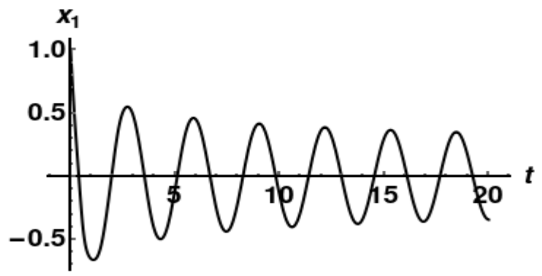

Figure 1. Fraph of $x_1(t)$

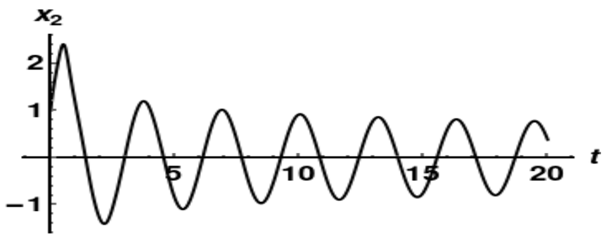

Figure 2. Graph of $x_2(t)$

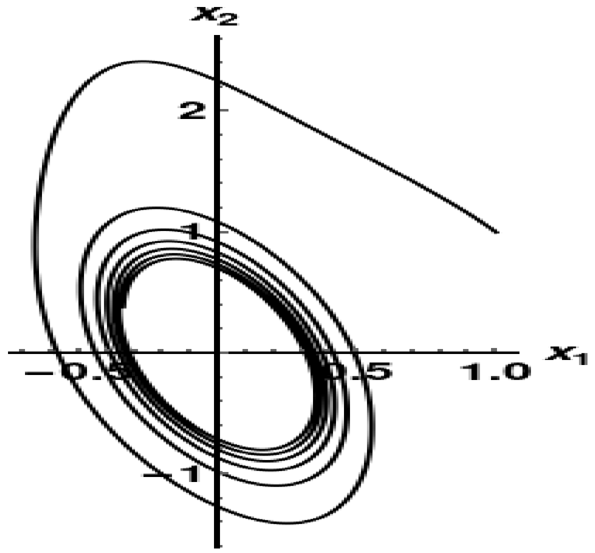

Figure 3. Graph of x_1 against x_2.

Description of the results obtained: From the simulation carried out and shown in example1, the convergence to admissible concentrations is perceived; in that case that satisfies the theoretically obtained conditions, here the System solutions converge to the origin in correspondence with theorem 3.

In an absolutely possible the case of the model where $a_1 > b_2$ and $a_2b_1 = a_1b_2$ causes linear part of the system matrix (1) to have a noll as well as an eigenvalue that is negative; using a linear transformation that is non-degenerate $X = SY$ the system (1) is transformed into the system (6).

$$\begin{cases} y_1' = Y_1(y_1, y_2) \\ y_2' = \lambda_2 y_2 + Y_2(y_1, y_2) \end{cases} \qquad (6)$$

This critical case was treated in previous works of our group Domínguez, et al., (2020), Rodríguez-Rodríguez, et al., (2020) where these systems were reduced to the quasi-form. Normal, for a better understanding, the result will be fully demonstrated. This process is accomplished through the use of the following theorem.

Theorem 4: The change of variables,

$$\begin{cases} y_1 = z_1 + h_1(z_1) + h^0(z_1, z_2) \\ y_2 = z_2 + h_2(z_1) \end{cases} \qquad (7)$$

Transforms the system (6) into quasi-normal form,

$$\begin{cases} z_1' = Z_1(z_1) \\ z_2' = \lambda_2 z_2 + Z_2(z_1, z_2) \end{cases} \qquad (8)$$

Where $h^0(z_1, z_2)$ and $Z_2(z_1, z_2)$ cancel out when $z_1 = z_2 = 0$.

Proof: This proof corresponds to what was indicated before in relation to the cited bibliography, transformation derivative (7) of system trajectory of systems (6) and (8), the partial derivative of the system equations is attained,

$$\begin{cases} p_2\lambda_2 h^0(z_1, z_2) + Z_1(z_1) = Y_1(z_1 + h_1 + h^0, z_2 + h_2) \\ \qquad -\frac{dh_1}{dz_1} Z_1(z_1) - \frac{\partial h^0}{\partial z_1} Z_1(z_1) - \frac{\partial h^0}{\partial z_2} Z_2(z_1, z_2) \\ Z_2(z_1, z_2) - \lambda_2 h_2 = Y_2(z_1 + h_1 + h^0, z_2 + h_2) - \frac{dh_2}{dz_1} Z_1(z_1) \end{cases} \qquad (9)$$

The coefficients of the powers of degree are separed to identity the series intervention in de system and the transformation the. It is expressed as follows $p = (p_1, p_2)$ in the following two cases:

Case I) Doing $z_2 = 0$ in (9), is to say to the vector $p = (p_1, 0)$ the system results

$$\begin{cases} Z_1(z_1) = Y_1(z_1 + h_1, h_2) - \frac{dh_1}{dz_1} Z_1 \\ -\lambda_2 h_2 = Y_2(z_1 + h_1, h_2) - \frac{dh_2}{dz_1} Z_1 \end{cases} \quad (10)$$

The system (10) enables to calculate the series coefficient Z_1,h_i, $i = 1,2$, under resonant cases where $h_1 = 0$, and the other series are uniquely determined.

Case II) On the other hand when $z_2 \neq 0$ the system (9) it follows that,

$$\begin{cases} p_2 \lambda_2 h^0(z_1, z_2) = Y_1(z_1 + h^0, z_2) - \frac{\partial h^0}{\partial z_1} Z_1(z_1) - \frac{\partial h^0}{\partial z_2} Z_2(z_1, z_2) \\ Z_2(z_1, z_2) = Y_2(z_1 + h^0, z_2) \end{cases} \quad (11)$$

Given that system (6) series are well-known statements, system (11) enables to determine the series $h^0(z_1, z_2)$ and $Z_2(z_1, z_2)$. This demonstrates the presence of a change in variables.

In system (8) the function $Z_1(z_1)$ has the development in power series

$$Z_1(z_1) = \alpha z_1^s + \ldots$$

Where α denotes as "initial non-zero coefficient" of the function and the corresponding power is denoted as s..

Theorem 5: The conditions $a_1 > b_2$, $a_2 b_1 = a_1 b_2$, $\alpha < 0$ and s is odd, are necessary and sufficient conditions for the system (8) to be asymptotically stable.

Proof: Consider the positive definite Lyapunov function,

$$V(z_1, z_2) = \frac{1}{2}(z_1^2 + z_2^2) \quad (12)$$

The derivative of the system (8) path for the function V is expressed as,

$$V'(z_1, z_2) = \alpha z_1^{s+1} + \lambda_2 z_2^2 + R_2(z_1, z_2)$$

The derivative $V'(z_1, z_2)$ along of the system (8) is negatively, because in the function $R_2(z_1, z_2)$ holds greater degree than s+1 respect to z_1 and greater than the second respect to z_2, thus the null solution of system (8) is asymptotically stable, This completes the proof of the theorem 5.

Remark 4: If the conditions $a_1 > b_2$, $a_2 b_1 = a_1 b_2$, $\alpha < 0$ and s is odd are satisfied then the total concentrations of the susceptible and infected populations converge to ideal values, there being no possibility of complications with the disease in the meantime these conditions are satisfied.

Description of the results obtained: From the observation4 it can be concluded that even in the case of a critical case, these conditions are sufficient to guarantee the control of the disease; this means that once the coefficients are identified, it can be determined what may happen in the future, with regard to the disease, whether or not it can become an epidemic.

Example 2: Let the system be

$$\begin{cases} x_1' = -2x_1 - x_2 - x_1^3 \\ x_2' = 2x_1 + x_2 - 2x_1^2 x_2 \end{cases}$$

The eigenvalues associated with this system are $\lambda_1 = 0$ and $\lambda_2 = -1$, by means of the transformation, this is transformed into the equivalent system

$$\begin{cases} y_1' = -5y_1^3 - 13y_1^2 y_2 - 11y_1 y_2^2 - 3y_2^3 \\ y_2' = -y_2 + 6y_1^3 + 16y_1^2 y_2 + 14y_1 y_2^2 + 4y_2^3 \end{cases}$$

where the matrix $S = \begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix}$, by means of the transformation (7) transforms into the following quasi-normal form

$$\begin{cases} z_1' = -5z_1^3 - 96z_1^5 + \ldots \\ z_2' = -z_2 + 16z_1^2 z_2 + 14z_1 z_2^2 + \ldots \end{cases}$$

Where

$$Z_1(z_1) = -5z_1^3 - 96z_1^5 + \ldots$$

$$h_2(z_1) = -6z_1^3 + 78z_1^5 + \ldots$$

$$Z_2(z_1, z_2) = 16z_1^2 z_2 + 14z_1 z_2^2 + \ldots$$

$$h^0(z_1, z_2) = -13z_1^2 z_2 - \frac{11}{2} z_1 z_2^2 + \ldots$$

The derivative of (12) around the quasi-normal form is

$$V'(z_1, z_2) = -5z_1^4 - z_2^2 + R_2(z_1, z_2) < 0,$$

therefore the system is stable.

The trajetórias have the following graphical behaviors:

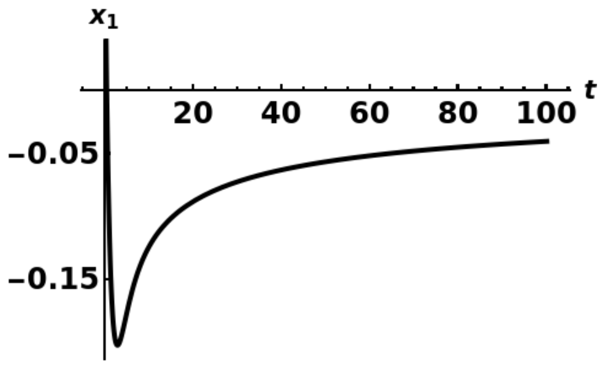

Figure 4. Graph of $x_1(t)$

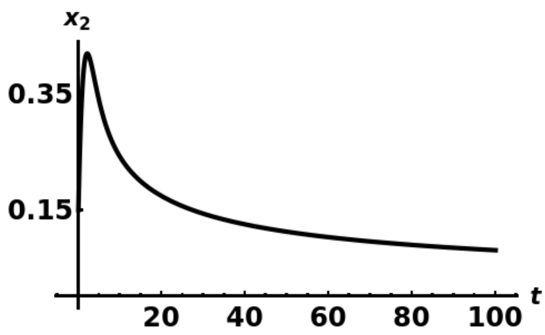

Figure 5. Graph of $x_2(t)$

Description of the analysis of the results of the graphical simulation: As shown in graphs 4 and 5 corresponding to example 2, it can be deduced that even in this critical case where the matrix of the linear part of the system has a zero eigenvalue, sufficient conditions can be provided for convergence from total concentrations to admissible concentrations, that is, ensuring disease controllability, as in this case the System trajectories converge to zero as shown in the graphs. The figure 1-5 depicts the graphs of the results observed.

Conclusion

From the Medical and Biological point of view, it is of great importance to use simplified systems that give conclusions regarding the development of some type of disease, especially those with rapid transmission.

If the conditions $a_1 > b_2$ and $a_2 b_1 > a_1 b_2$ are satisfied, the total concentrations if susceptible and of infected people will converge to ideal concentration values, and therefore there will be no possibilities for the development of epidemics, otherwise measures must be taken to prevent further complications.

If the conditions $a_1 = b_2$, $a_2 b_1 > a_1 b_2$ and $a_k < 0$ if the convergence of susceptible populations and infected populations converge to admissible values, this is still a critical case.

If the conditions $a_1 > b_2, a_2 b_1 = a_1 b_2, \alpha < 0$ e s is not even are satisfied, then the total concentrations of the susceptible and infected populations converge to ideal values, with no possibility of complications with the disease, as long as these conditions are satisfied.

The developed example1 makes it possible to verify the outcomes, treated from a theoretical perspective, Thus the investigated system's trajectories may be seen to be convergent towards their equations position.

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