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Imprecise WareHouse Space in Aggregate Production Planning Using Fuzzy Goal Programming

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Abstract:

Considering the fluctuating market demands with variable storage capacity and available production capacity, this study examines a number of workable techniques for modeling multiproduct aggregate production planning problems with fuzzy numbers. The suggested method makes use of factors including; inventory levels, labor levels, overtime, backordering levels, workforce capacity, machine capacity, and fuzzy warehouse capacity in an effort to reduce operating costs, reduce production waste, and increase capacity utilization rate. With the aid of this formulation and interpretation, a fuzzy multiproduct aggregate production planning model is developed. Finally, the study's conclusions were arrived at using information provided by Rich Pharmaceuticals Ltd. using Lingo version 18 software (RPL).and it uses parametric programming, best balancing, and interactive techniques to give solutions that can be adjusted to fit a variety of decision-making circumstances.

Keywords: *Aggregate production planning, fuzzy space, warehouse, Decision maker, linear membership function.*

Introduction

Effective production planning has become a key approach for firms to ensure efficient operations, satisfy consumer needs, and maximize resource usage in today's dynamic and competitive business environment. Aggregate Production Planning (APP), one of the many planning methods, stands out as a crucial strategy that enables companies to strike a careful balance between production levels and inventories while staying in line with market expectations. Aggregate production planning helps businesses deal with the difficulties of varying demand, unpredictability in the supply chain, and cost considerations by concentrating on the overall picture of production over a specific time horizon.

For aggregate production planning, warehouse and storage facilities are crucial because they enable the company to adjust to changes in demand by building up seasonal stockpiles or scheduling backorders (Guillermo, 2001). They also have an impact on the price of transportation, labor, inventory, and production (Sunderesh, 2022). By taking into account the restrictions at the warehouse and other supply chain stages, a linear programming model can be used to optimize the aggregate production planning problem, (Madanhire & Mbohwa, 2015; Sunderesh, 2022).

In supply chain management and production planning, the following are the main roles of warehouse and storage facilities: Storage of products: Businesses can keep their supplies, inventory, equipment, and other materials here in a safe and secure setting. Facilitation of movement: The warehouse serves as the major center for receiving and sending out commodities. It enables businesses to stay on time and continuously satisfy client demand. Risk reduction: Warehouses can shield commodities from theft, damage, spoilage, and other environmental conditions. Price stabilization: By balancing supply and demand, warehouses can stop market price volatility. They can also offer insurance protection for the items being stored. Value-added services: In order to increase the value of the products, warehouses might provide extra services like packaging, labeling, grading, sorting, and quality control. (Leon, 2023; Bradley, 2023).

Storage facilities are crucial for companies to manage demand patterns effectively through inventory buffering. They provide a space to store excess inventory during low demand and replenish it during high demand, aligning production with market needs, Narasimhan and Talluri (2009) and Pishvaee et al. (2010). They also went further to state that this practice ensures consistent product supply and protects against supply chain disruptions. Inventory management balances production, distribution, and demand uncertainties, enhancing operational efficiency and coping with market uncertainties. Silver et al. (1998) and Nahmias (2009) emphasize the importance of storage facilities in shaping production scheduling decisions and cost efficiency. They argue that storage allows for strategic production timing, smoothing out production peaks and troughs, and optimizing resource utilization. Nahmias (2009) emphasizes the dynamic interaction between storage capacity, inventory levels, and production scheduling decisions, highlighting the strategic advantage of storage facilities in managing production activities. Ballou (2004) and Mangan et al. (2016) emphasize the importance of warehouse location in transportation and distribution strategies. They argue that strategic positioning of warehouses can optimize the flow of goods, reduce costs, and improve logistical efficiency. Mangan et al. (2016) argue that warehouse location decisions balance cost and service levels, enhancing a company's competitive edge by facilitating quick response to customer demands and efficient order fulfillment.

Aggregate Production Planning (APP) oversees the best way to meet forecast demand in the intermediate future, often from 6 to 24 months ahead, by adjusting regular and overtime production rates, inventory levels, labor levels, subcontracting and backordering rates, and other controllable variables (Wang R. et al., 2005). The primary inputs of APP are market demands and the manufacturing plan to meet those expectations. (Leung et al., 2003). Production planning does this in response to changes in demand. Changing a company's production schedule on a moment's notice can be expensive and lead to insecurity. Planning for changes in demand months in advance guarantees that the change of production schedules can occur with little effort (Hossain et al., 2016). APP is a general style to altering a company's production schedule to respond to changes in demand.

By employing integrated parametric programming, best balance, and interactive approaches, Fung et al. (2003) introduced a fuzzy multi-product aggregate production planning (FMAPP) model to cater to various situations under varied decision-making preferences. This model can also effectively improve the capability of an aggregate plan to deliver feasible disaggregate plans under varying circumstances with fuzzy demands and fuzzy capacities. In order to tackle multi-product APP choice problems in a fuzzy environment, Wang and Liang (2004) more recently created a fuzzy multi-objective linear programming model using the piecewise linear membership function. The model can yield an effective compromise solution and the decision maker's overall levels of satisfaction. Additional research on fuzzy APP problem solving may be found in Wang and Fang (1997), Tang et al. (2000), Wang and Fang (2001), and Tang et al (2003). To optimize profit, minimize repair costs, and maximize machinery usage, Leung and Chan (2009) created a preemptive goal programming approach for the APP problem. Sakall et al. (2010) discussed a probabilistic APP model for the blending issue in a brass production. They came up with the best procedures for buying raw materials.

Many aggregate planning issues do not properly take into account productivity losses brought on by non-existent or unstable warehouse or storage facilities. The productivity losses linked to other capacity changes, such backorders, multiple shifts, and overtime, are also largely unmentioned in parts of the research. When productivity losses are taken into account, traditional methodologies impute corresponding costs but do not take lost due to cost and productivity into account.

A gap in earlier works has been identified, according to the literature referenced above. In this study, an APP problem with multiple objectives, multiple periods, and multiple products is suggested. The suggested solution to the problem is a FGP. Minimizing total manufacturing costs, maximizing sales revenue, and maximizing customer satisfaction are all crucial factors for the case concern in this instance. It is therefore more reasonable to describe them as three distinct objectives so that the APP model may identify a Pareto optimum that strikes a balance between these three goals. So, for the example study, the following three-objective, multi-period, multi-product FGP-APP model is developed.

Method and Procedure

Assumptions and Problem Definition

Following the findings of a real-world case study, the following presumptions are made for the mathematical model of the suggested APP problem.

- Production planning is done in a time horizon of T time periods ($\forall\, t = 1,2, \dots, T$).
- There is a Batch production system capable of producing all kinds of N types of products.
- Market demand can be fulfilled or backordered, however no backorder in the last t is allowed.
- There are two working shifts; Regular time production and Over time production
- A warehouse is allowed for holding final products.
- In advance, the holding cost of inventories are determined and well known.
- The workforce accommodates various skill levels ($k - levels$).
- Workers salary is independent of unit production cost.
- At each period T, Production quantity is considered more of the safety stock for finished products.
- Hiring and firing of Manpower based on product demand is eligible and there is an allowable limit.
- In each period T, the shortage of production is recovered by overtime production in each shift.
- In each period T, the nominal and actual capacity of production machines is not the same due to unforeseen failures. So, the actual capacity of production is usually reduced by a fixed failure percentage.
- If an unforeseen failure occurs during a shift the repair process is completed in the next. This may stop, reduce, or decrease the production rate during maintenance actions
- The impreciseness and uncertainty of real-world problem and confliction of different objectives are modeled using fuzzy goals.
- Linear membership functions are defined for fuzzy goals.
- FGP used to solve the problem.

Parameters, Indices, Decision Variables and Notations

They are as stated in Tables 1 to 3.

Table 1. Set of indices

t	Number of periods in the planning horizon; $t = 1,2, \dots, T$
i	Number of product types; $i = 1,2, \dots, I$

m	Raw material type; $m = 1,2, \dots, M$
q	Types of shifts; $q \in 1,2$
w	Types of warehouse; $w = 1,2, \dots, W$
k	Skill levels of workers; $k = 1,2, \dots, K$
j	Number of objective Functions; $j = 1,2,3$

Table 2. Decision variable Notation

Decision variable	Definition
X_{iqt}	Number of product **i** produced in shift q of period **t**
$X\beta_{iqt}$	Number batches of product *i* produced in shift *q* of period *t*
B_{it}	Backorder level of product *i* in period *t*
XW_{kt}	Number of available workers of level k in period t
XH_{kt}	Number of hired workers of level k in period t
XF_{kt}	Number of fired workers of level k in period t
XR_{mtw}	Inventory level of raw material type m at the end of period t in warehouse w
XP_{itw}	Inventory level of finished-product i in period t in warehouse w

Table 1. Notation for parameters

Parameter	Definition
CoP_{iq}	Cost of Production; for product i in shift q
DoP_{it}	Demand of product i in period t
CoB_{it}	Cost of Backordering; for product i in period t
SRe_i	Sales Revenue for product i (₦/unit)
PrT_t	Process time of product i in period t
BUL_t	The Budget upper limit in period t
AsP_{it}	Allowable shortage of product i in period t
$\overline{AMW}_t$	Available Maximum workforce in period t
$\underline{AMW}_t$	Available Minimum workforce in period t
WaO	workforce that are available for overtime (in percentage)
CoW_{kt}	Cost of workforce of level k in period t
CoH_{kt}	Cost of Hiring workforce of level k in period t
CoF_{kt}	Cost of firing workforce of level k in period t
CoR_{mtw}	Holding cost for raw material type m in period t in warehouse w
$CohP_{itw}$	Holding cost of unit of product i in period t
E_t	cumulative investment in tools and equipment in period *t* (currency unit)
FoW_t	fraction of the workforce variation in period t
MH_{it}	Machine hours needed to produce unit of product i in period t
MCi_t	Machine capacity that is lost due to **interruption** in period t (in percentage)
MCr_t	Machine capacity that is lost due to **repairs** in period t (in percentage)
MmC_{qt}	The maximum of machine capacity that is available in shift q in period t
MCo	The machine capacity that is available for overtime (in percentage)
ArT_{it}	Available Regular time in both shifts in period t
uMR_{im}	The units of type m raw material required to produce unit of product i
SSP_i	product i safety stock
SSR_m	Raw material type m safety stock
$\overline{M}SW_m$	The maximum available space of warehouse w
$WhCR_{wmt}$	The capacity of warehouse w for storage of raw-material type m in period t
$WhCP_{wit}$	The capacity of warehouse w for storage of finished-product i in period t
$\mathcal{D}d_i$	The Due date of product i
$\mathcal{B}_i$	Batch size of product i
DrF_i	Finished product i Defect rate

Model Formulation

Minimize Total Cost

$$Min\ Z_1 = \sum_{i=1}^{I} \sum_{q \in \{1,2\}} \sum_{t=1}^{T} CoP_{iq} X_{iqt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoW_{kt} XW_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoH_{kt} XH_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoF_{kt} XF_{kt} + \sum_{i=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} CohP_{iwt} XP_{iwt} + \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoR_{mwt} XR_{mwt} + \sum_{i=1}^{I} \sum_{t=1}^{T} CoB_{lt} B_{lt} \quad (1)$$

The above Minimization of Total Cost function (TFC) involves the following seven terms; the per unit Production Cost, Cost of salary of the workforce, Cost of hiring, Cost of firing, Cost of holding of products, Cost of holding of raw materials, and Cost of Backordering.

Maximize Customer Satisfaction Level

$$Min\ Z_2 = \sum_{i=1}^{I} \left| \sum_{q\in\{1,2\}} \sum_{t=1}^{T} PtP_{it} X_{iqt} - Dd_i \right| \tag{2}$$

Function (2) is for achieving the customer satisfaction, this is by minimizing the difference between the delivery date (PtP_{it}) of all products and the due date (Dd_i) of all products, this in turn maximizes the customer satisfaction level. Worthy of note that delivering the product earlier to Dd_i is not to the benefit of the producer and delivering later to Dd_i is also not to the benefit of the customer, thus (2) minimizes the imbalance concurrently.

Maximize Sales Revenue

This last objective function is to realize the highest possible return from the quantities produced by regular production and overtime production including inventories and back orders.

$$Max\ Z_3 = \sum_{i=1}^{I} \sum_{q\in\{1,2\}} \sum_{t=1}^{T} SRe \times \left(XP_{iwt-1} - B_{lt-1} + X_{iqt} - XP_{iwt} + B_{lt}\right) \tag{3}$$

Constraints

The **Labor-force Constraints** are considered as follows:

$$\sum_{k=1}^{K} XW_{kt} \leq \overline{AMW}_t\,, \qquad \forall t \tag{4}$$

$$\sum_{k=1}^{K} XW_{kt} \geq \underline{AMW_t}\,, \qquad \forall t \tag{5}$$

$$XW_{kt} = XW_{k(t-1)} + XH_{kt} - XF_{kt}\,, \quad \forall k, \forall t, t > 1 \tag{6}$$

$$XW_{kt} - XW_{k(t-1)} \leq FoW_t * XW_{kt}\,, \quad \forall k, \forall t, t > 1 \tag{7}$$

Constraints (4) attests that the total labor utilized during period t does not exceed the total workforce that is available. In a similar vein, (5) guarantees that in period t, the employed workforce exceeds the available minimum workforce. Set of Constraints (6) is a workforce level balance equation that assures that the workforce with skill level k available during a given period is equal to the workforce with the same skill level k during the previous period plus the change in workforce level during the current period. The change in workforce level in each planning period cannot be greater than a benchmark number of workers in the present period, according to constraint number seven.

Time Constraints

$$\sum_{i=1}^{I} PrT_{it} * X_{iqt} \leq \sum_{k=1}^{K} ArT_{qt} * XW_{Kt} \,, \qquad \forall t, \;\; q = 1 \qquad (8)$$

$$\sum_{i=1}^{I} PrT_{it} * X_{iqt} \leq \sum_{k=1}^{K} ArT_{qt} * WaO * XW_{Kt} \,, \qquad \forall t, \;\; q = 2 \qquad (9)$$

The relationships mentioned above make sure that each working shift's necessary production time is less than or equal to the available regular production time and overtime.

Inventory Constraints

$$XP_{iwt} = XP_{iw(t-1)} + \sum_{q\in\{1,2\}} X_{iqt} - B_{it} - DoP_{it} \,, \quad \forall i, \forall w, \;\; t > 1 \qquad (10)$$

$$XR_{mwt} = XR_{mw(t-1)} + \sum_{q\in\{1,2\}} X_{iq(t-1)} - uRM_{im}, \quad \forall i, \forall w, \;\; t > 1 \qquad (11)$$

$$SSR_m \leq \sum_{w\in W} XR_{mwt}, \quad \forall m, \forall t, \qquad (12)$$

Constraints (10) ensures that the amount of finished product type I in period t in warehouse w is equal to the amount of finished product type I in period $t-1$ in warehouse w plus the quantity of produced finished goods type I in period t in both working shifts, less the amount of product type I in period t that is on backorder and the quantity of produced finished goods type I in period t in both working shifts. A set of limitations (11) assures that there is a balance between raw materials, and (12) guarantees that the safety stock of raw materials in warehouses is satisfied.

Production Constraint

$$SSP_i \leq \sum_{q\in\{1,2\}} X_{iqt}, \quad \forall i, \forall t, \qquad (13)$$

$$DoP_{it} \leq \left(1 - \frac{DrF_i}{\beta_i}\right) * \sum_{q\in\{1,2\}} X_{iqt} + XP_{i(t-1)}, \quad \forall i, \forall t, \;\; t > 1 \qquad (14)$$

Set of constraints (13), which is written for all product types and all periods of planning, guarantee the satisfaction of safety stock of finished-products in working shifts. Set of constraints (14) represents the total production of non-defected final products plus the inventory of finished-product in previous period should be greater than or equal to demand of the finished-product in current period.

Machine capacity Constraints

$$\sum_{i=1}^{I} MH_{it} * X_{iqt} \leq \overline{M}mC_{qt} - MCi_t * \overline{M}mC_{qt}, \qquad \forall t, \ \ q = 1 \tag{15}$$

$$\sum_{i=1}^{I} MH_{it} * X_{iqt} \leq MCo * \overline{M}mC_{qt} - MCr_t * MCo * \overline{M}mC_{qt}, \qquad \forall t, \ \ q = 2 \tag{16}$$

Constraints (15) and (16) pledge that in regular time and overtime, the machine capacity is assured.

Warehouse Capacity Constraint

$$\sum_{w=1}^{W} XP_{iwt} \leq \sum_{w=1}^{W} WhcP_{wit}, \ \ \forall i, \forall t, \tag{17}$$

$$\sum_{m=1.}^{M} \sum_{w=1}^{W} XR_{mwt} \leq \sum_{w=1}^{W} \sum_{m=1}^{M} WhcR_{mwt}, \ \ \ \forall t, \tag{18}$$

$$\sum_{w=1}^{W} WhcP_{wit} + \sum_{w=1}^{W} WhcR_{mwt} \leq \overline{M}SWh_m, \ \ \forall i, \forall t, \tag{19}$$

The first two constraints (17) and (18) gives the restrictions of actual inventories of finished products and raw materials. While (19) guarantees that each warehouse at each period will not be able to allow storage capacity of products and raw materials beyond its maximum warehouse available space.

Backorder, Budget limit and Non-negativity Constraints

There is backorder obeying the following;

$$\sum_{w=1}^{W} B_{it} \leq \sum_{w=1}^{W} AsP_{it} * DoP_{it} \ \ \ \forall i, \ \ \ t \neq T \tag{20}$$

$$B_{iT} = 0, \qquad\qquad \forall i \tag{21}$$

$$ToCo \ \leq \sum_{t=1}^{T} BUL_t \tag{22}$$

$$X_{iqt}, X\beta_{iqt}, B_{it}, XR_{mtw}, XP_{iwt} \geq 0, \qquad \forall i, \forall q, \forall t, \forall m, \forall w \tag{23}$$

$$XL_{kt}, XH_{kt}, XF_{kt} \ \ \geq 0, \qquad \forall t, \forall k, \forall l \tag{24}$$

Constraints (20) represent the backorder level at the end of period t cannot exceed the certain percent-age of the demand which determines the upper limit of shortage. While (21) assure that there is no possibility for backordering at the end of time horizon or last period.

A restriction on the available budget for each planning period is shown using (22), which ensures that the Total Cost (i.e., Eq. (1)) cannot go beyond the predetermined budget for the time horizon.

(23) and (24) both present non-negativity requirements on decision variables.

Fuzzy Set Theory - Definitions and Notations

For the sake of completeness, the following basic definitions and concepts related to fuzzy sets theory are provided in this section:

Definition 1 (Bellman and Zadeh 1970)

A fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$\tilde{A} = \{x, \mu_{\tilde{A}}(x)/x \in X\}$

$\mu_{\tilde{A}}(x)$ is called the membership function of x in $\tilde{A}$ which maps X into $[0,1]$. If $sup_x\, \mu_{\tilde{A}}(x) = 1$, the fuzzy set $\tilde{A}$ is called normal.

In the real line ℝ, a fuzzy number is a fuzzy set with the membership function illustrated as:

$$u = \mu_{\tilde{a}}(x) = \begin{cases} 0 & \forall x \in (-\infty, a_1] \\ f_a(x) & \text{increasing } \forall x \in [a_1, a_2] \\ 1 & \forall x \in [a_2, a_3] \\ g_a(x) & \text{decreasing } \forall x \in [a_3, a_4] \\ 0 & \text{Otherwise} \end{cases} \quad (25)$$

in which $\tilde{a} = (a_1, a_2, a_3, a_4)$.

Definition 2 (Bellman and Zadeh 1970)

The support of a fuzzy set $\tilde{A}$ in X is the crisp set of all $x \in X$, such that $\mu_{\tilde{A}}(x) > 0$.

Definition 3 (Bellman and Zadeh 1970)

The set of elements that belong to the fuzzy set $\tilde{A}$ on X at least to the degree α is called the α-cut set:

$$\tilde{A} = \{x \in \mathbb{R}, \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\}.$$

An α-cut is a slice through the fuzzy number $\tilde{a}$ which produces a nonfuzzy set. Based on this definition, it can be written as $a_\alpha = [f_a^{-1}(u), g_a^{-1}(u)]$. In such cases when f_a and g_a are linear functions, the membership function (25) is the membership function of a trapezoidal fuzzy number denoted by $\tilde{a} = (a_1, a_2, a_3, a_4)$. If $a_2 = a_3$, then a triangular fuzzy number(TFN) is obtained.

Definition 4 (Bellman and Zadeh 1970)

A fuzzy set $\tilde{A}$ in X is called convex if:

$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\};\ x, y \in \mathbb{R}\ and\ \lambda \in [0,1].$

You should be aware that a fuzzy set is convex if all of its α-cuts are convex.

Definition 5 (Bellman and Zadeh 1970)

On the real line ℝ, a fuzzy number $\tilde{a}$ is a convex normalised fuzzy set such that:

1. There is at least one $x_0 \in \mathbb{R}$ with $\mu_{\tilde{a}}(x_0) = 1$.

2. $\mu_{\tilde{a}}(x)$ is piecewise continuous.

Following Heilpern (1992) and considering (25) the Expected Interval of a fuzzy number $\tilde{a}$, denoted by $EI(\tilde{a})$ is defined as $EI(\tilde{a}) = [E_1^a E_2^a] = \left[\int_0^1 f_a^{-1}(u)du,\ \int_0^1 g_a^{-1}(u)du\right]$.

Similarly given a fuzzy number $\tilde{a}$, the expected value denoted by $EV(\tilde{a})$, is the half point of the expected interval, which is given as:

$$EV(\tilde{a}) = \frac{E_1^a + E_2^a}{2}$$

Thus, if a fuzzy number $\tilde{a}$ is trapezoidal or triangular, its expected interval and expected value can be easily calculated as follows:

$$EI(\tilde{a}) = \left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_2 + a_3)\right]; \qquad EV(\tilde{a}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$$

$$EI(\beta\tilde{a} + \gamma\tilde{b}) = \beta EI(\tilde{a}) + \gamma EI(\tilde{b})$$

$$EV(\beta\tilde{a} + \gamma\tilde{b}) = \beta EV(\tilde{a}) + \gamma EV(\tilde{b})$$

Definition 6 (Jime´nez, M. 1996)

The extent to which $\tilde{a}$ is larger than $\tilde{b}$ for any pair of fuzzy numbers, $\tilde{a}$ and $\tilde{b}$, may be expressed as follows:

$$\mu_M(\tilde{a},\tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & if\ 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \quad (26)$$

Where $[E_1^a, E_2^a]$ and $[E_1^b, E_2^b]$ are the expected interval of $\tilde{a}$ and $\tilde{b}$.

If $\mu_M(\tilde{a},\tilde{b}) = 0.5$, its said that $\tilde{a}$ and $\tilde{b}$ are different and if $\mu_M(\tilde{a},\tilde{b}) \geq \alpha$ it is said that $\tilde{a}$ is bigger than or equal to $\tilde{b}$ at least in degree α and its indicated by $\tilde{a} \geq_\alpha \tilde{b}$

Formation of Fuzzy Warehouse Space

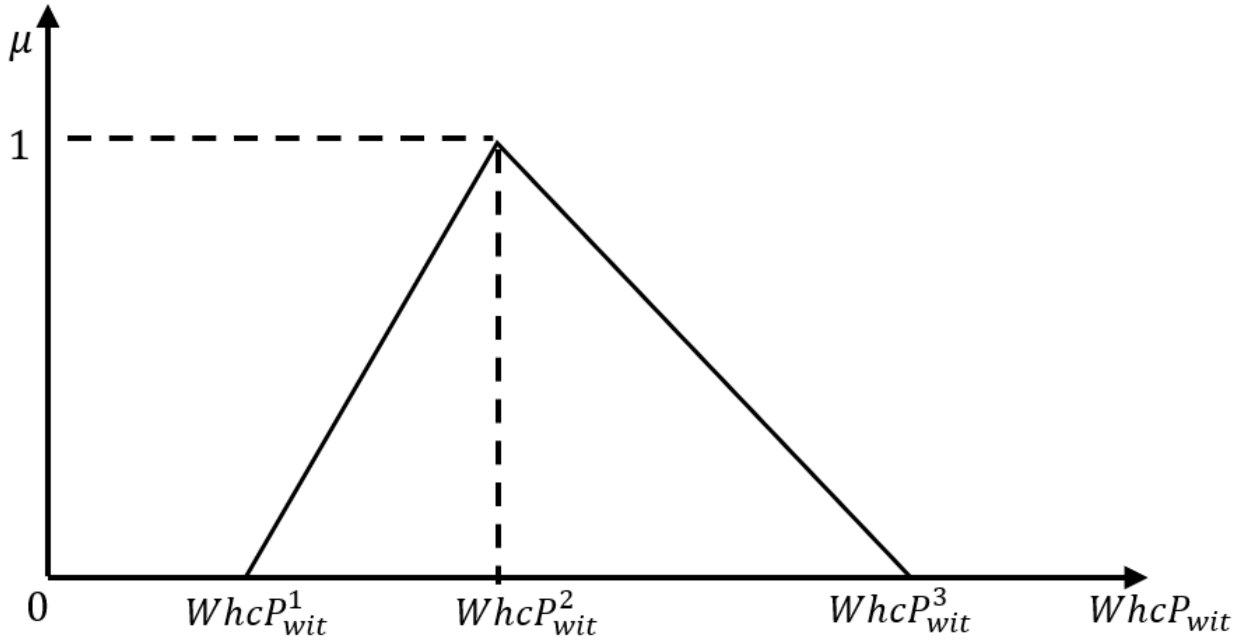


Figure 1. A Triangular Distribution of the Fuzzy Available Space for Finished Products

Fuzzy numbers like triangular and trapezoidal fuzzy numbers, can be used to represent the available space of warehouse for finished products ($WhcP_{wit}$) in order to reflect this informational ambiguity. TFNs are used in this study to represent warehouse space-related fuzzy data. Assuming the TFN of $WhcP_{wit}$ is $\widetilde{WhcP_{wit}} = (WhcP_{wit}^1, WhcP_{wit}^2, WhcP_{wit}^3)$, in which $WhcP_{wit}^2$ is the most possible available space that certainly belongs to the set of available values (with a membership value of 1 after it is normalized). The lower bound value $WhcP_{wit}^1$ is the most pessimistic available space that has a small likelihood to belong to the set of available values (with a membership value of zero if normalized) and the upper bound value $WhcP_{wit}^3$ is the most optimistic available space with a small likelihood to belong to the set of available values (with a membership value of zero if normalized). Let $\mu(\widetilde{WhcP_{wit}})$ represent the arbitrary measurement of fuzzy available space in view of the Decision-maker, i.e. membership function, that defines the degree of x in the fuzzy space $\widetilde{WhcP_{wit}}$ and figure 1 depicts the relationships of this function.

As seen in Figure 1 the membership function of fuzzy demand may be expressed as follows:

$$\mu(\overline{M}SWh_m) = \begin{cases} 0 & WhcP_{wit} \le WhcP^1_{wit} \\ (WhcP_{wit} - WhcP^1_{wit})/(WhcP^2_{wit} - WhcP^1_{wit}) & WhcP^1_{wit} \le WhcP_{wit} \le WhcP^2_{wit} \\ (WhcP^2_{wit} - WhcP_{wit})/(WhcP^3_{wit} - WhcP^2_{wit}) & WhcP^2_{wit} \le WhcP_{wit} \le WhcP^3_{wit} \\ 1 & Otherwise \end{cases} \quad (27)$$

Supposing the decision-maker desires that APP meets the available warehouse space for product i in period t with a possibility level. Using the fuzzy available warehouse space information, the constraint equation (17)-(19) will be replaced with the following equations (28)-(30):

$$\sum_{w=1}^{W} XP_{iwt} \le \sum_{w=1}^{W} \widetilde{WhcP}_{wit}, \quad \forall i, \forall t, \quad (28)$$

$$\sum_{m=1.}^{M} \sum_{w=1}^{W} XR_{mwt} \le \sum_{w=1}^{W} \sum_{m=1}^{M} \widetilde{WhcR}_{mwt}, \quad \forall t, \quad (29)$$

$$\sum_{w=1}^{W} WhcP_{wit} + \sum_{w=1}^{W} WhcR_{mwt} \le \widetilde{\overline{M}SWh}_m, \quad \forall i, \forall t, \quad (30)$$

Based on the ranking method developed by Jim'enez(1996), all fuzzy (imprecise) available warehouse space constraints in the model are translated to their corresponding crisp constraints as follows:

$$\sum_{w=1}^{W} XP_{iwt} \le \sum_{w=1}^{W} \left(\alpha \frac{WhcP^1_{wit} + WhcP^2_{wit}}{2} + (1-\alpha) \frac{WhcP^2_{wit} + WhcP^3_{wit}}{2} \right), \quad \forall i, \forall t, \quad (31)$$

$$\sum_{m=1.}^{M} \sum_{w=1}^{W} XR_{mwt} \le \sum_{w=1}^{W} \sum_{m=1}^{M} \left(\alpha \frac{WhcR^1_{mwt} + WhcR^2_{mwt}}{2} + (1-\alpha) \frac{WhcR^2_{mwt} + WhcR^3_{mwt}}{2} \right), \quad \forall t, \quad (32)$$

$$\sum_{w=1}^{W} WhcP_{wit} + \sum_{w=1}^{W} WhcR_{mwt} \le \left(\alpha \frac{\overline{M}SWh^1_m + \overline{M}SWh^2_m}{2} + (1-\alpha) \frac{\overline{M}SWh^2_m + \overline{M}SWh^3_m}{2} \right), \forall i, w \quad (33)$$

Fuzzy Multi-objective Goal Programing Development

In classic models of GP, the decision maker has to specify a precise aspiration level (goal) for each of the objectives. In general, especially in large-scale problems, this is a very difficult task, and the use of the Fuzzy Set theory in GP models can overcome such problem, allowing decision makers to work with imprecise aspiration levels (Yaghoobi and Tamiz, 2007). In multiobjective programming, In fuzzifying the inequality signs; " = " " ≤ " and " ≥ ", Zimmermann (1978) used the symbol "∼", they are to be understood as "essentially greater than or equal to" and "essentially less than or equal to". if an imprecise aspiration level is introduced to each of the objective functions then these fuzzy objectives are termed as fuzzy goals. Let g_k be the aspiration level assigned to the kth objective $Z_k(x)$. Then the fuzzy goals are:

$$Z_k(x) \gtrsim g_k \ \text{[for maximizing } Z_k(x)\text{] and}$$

$$Z_k(x) \lesssim g_k \ \text{[for minimizing } Z_k(x)\text{]}$$

In solving the problem, a general form of FGP model is considered:

$$
\begin{aligned}
&\text{find} \quad x \\
&\text{to satisfy;} \\
&\qquad Z_k(x) \gtrapprox g_k \qquad k = 1 \dots n \\
&\qquad Z_k(x) \lessapprox g_k \qquad k = n+1 \dots J \qquad (34) \\
&\text{subjet to} \\
&\qquad AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \\
&\qquad X \geq 0
\end{aligned}
$$

FGP is employed in solving the APP system (1) – (24). Being able to use FGP approach with fuzzy goals, the aspiration levels should be calculated. Payoff table is used when the decision maker has no enough view point to determine the aspiration levels. Zimmermann (1978) used a Payoff table to develop an upper and lower limit that was used to formulate the membership functions of the fuzzy goals.

In the general form (34), the purpose of FGP is to find compromise solution X such that all fuzzy goals are satisfied. g_k is the aspiration level for kth goal, $AX \leq b$ are system constraints in vector notation. $Z_k(x) \lessapprox g_k$ Means that the kth fuzzy goal is approximately less than or equal to the aspiration level g_k, and $Z_k(x) \gtrapprox g_k$ gives the reverse, (Hannan, 1981).

The fuzzy decision-making concept of Bellman and Zadeh (1970) can be used to solve the planned multi-objective APP problem (1)–(24). Linear membership functions as proposed by Zimmermann (1978) are used to represent the fuzzy goals of decision makers.

Now, the membership function μ_k for the kth fuzzy goal $Z_k(x) \lessapprox g_k$ can be expressed as follows:

$$
\mu(Z_k(x)) = \begin{cases} 1 & Z_k(x) \leq g_k \\ \dfrac{u_k - Z_k(x)}{u_k - g_k} & g_k \leq Z_k(x) \leq u_k \\ 0 & Z_k(x) \geq u_k \end{cases} \qquad (35)
$$

where u_k is the upper tolerance limit for the kth fuzzy goal and $u_k - g_k$ is the tolerance p_k which is subjectively chosen and the function is as depicted in Figure 2a.

Again, the membership function μ_k for the kth fuzzy goal $Z_k(x) \gtrapprox g_k$ can be expressed as follows:

$$
\mu(Z_k(x)) = \begin{cases} 1 & Z_k(x) \geq g_k \\ \dfrac{Z_k(x) - l_k}{g_k - l_k} & l_k \leq Z_k(x) \leq g_k \\ 0 & Z_k(x) \leq l_k \end{cases} \qquad (36)
$$

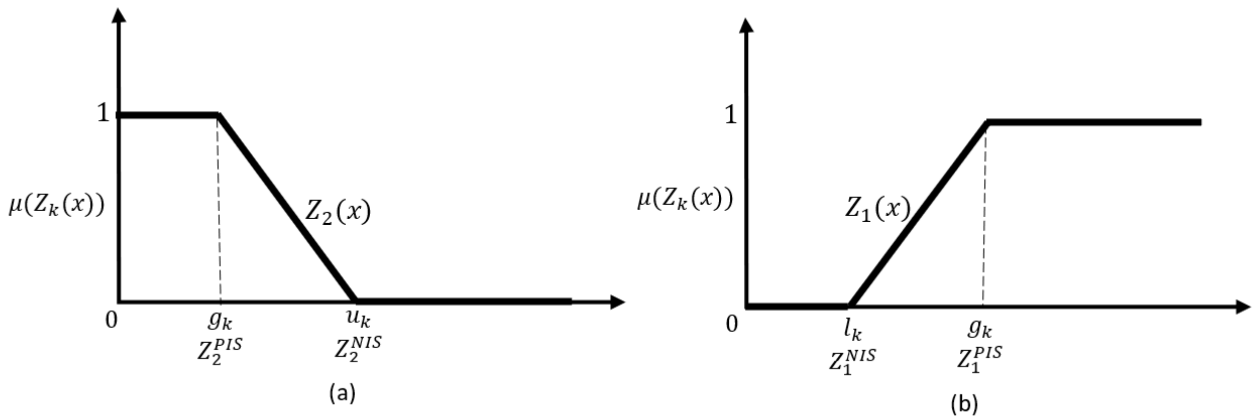


Figure 2. Linear Membership Form

where l_k is the lower tolerance limit for the kth fuzzy goal and $g_k - l_k$ is the tolerance p_k which is subjectively chosen and the function is as depicted in Figure 2b.

Hence, the associated FGP model for the multiobjective APP problem (1)-(32) is formulate as follows:

$$\begin{gathered} \text{find} \quad x \\ Maximize \quad \lambda \\ \text{to satisfy;} \\ \lambda \le \mu(Z_1(x)) = \frac{u_k - Z_k(x)}{u_k - g_k} \\ \lambda \le \mu(Z_2(x)) = \frac{u_k - Z_k(x)}{u_k - g_k} \\ \lambda \le \mu(Z_3(x)) = \frac{Z_k(x) - l_k}{g_k - l_k} \\ \mu(Z_j(x)) \in [0,1], \quad j = 1,2,3 \\ Constraints\ (4)-(16), (20)-(24), (31)-(33) \\ x_i \ge 0, i = 1 \ldots\ldots n \end{gathered}$$

This suggested approach states that goal weights are decided by DM, and goal aspiration levels are derived using a payout table. The positive ideal solutions (PIS) and negative ideal solutions (NIS) of the objective functions can be respectively specified as follows, (Hwang & Yoon,1981; Lai & Hwang, 1992);

$$Z_1^{PIS} = MinZ_1;\ Z_1^{NIS} = Max\{Z_1(v_j^*)\}$$

$$Z_2^{PIS} = MinZ_2;\ Z_2^{NIS} = Max\{Z_2(v_j^*)\}$$

$$Z_3^{PIS} = MaxZ_3;\ Z_3^{NIS} = Min\{Z_3(v_j^*)\}$$

Where v_j^* is the positive ideal solution of objective function Z_k.

Implementation

An industrial case study.

Data description

The case study of Rich Pharmaceuticals Limited(RPL) was utilized to show how useful the suggested methodology is. RPL is one of the leading producers of pharmaceuticals in Nigeria. RPL's goods are mostly sold in Southern and Middle belt of Nigeria, some parts of West and East Africa, they have recently experienced fluctuations in demand. RPL's business APP approach is to keep a stable labor force level over the planning horizon, allowing for the flexible meeting of demand through the use of inventories, overtime, and backorders.

Alternately, the DM can use a mathematical programming technique to create an aggregate production schedule for RPL factory. Based on company reports, the planning horizon spans for six months, May to October. The model includes two types of standard products. Each period, the standard payroll is ₦64. The expenses for hiring and firing employees are ₦30 and ₦40 per employee every day, respectively. Production expenses for overtime are capped at 30% of production expenses for regular hours. Additionally, it is assumed that each product has no beginning inventory and no backorders at the last period. Table 4 gives the forecasted monthly available warehouses spaces for production

Table 4. Available Space Forecasting $\widetilde{WhcP}_{wit} = (WhcP^1_{wit}, WhcP^2_{wit}, WhcP^3_{wit})$ (m^3/month)

	Period t					
Space	1	2	3	4	5	$WhcP_{wit}$
$WhcR_{mwt}$	(3900, 2800, 4000)	(3100, 2800, 3100)	(4100, 4200, 3440)	(2350, 3260, 4280)	(3380, 3300, 2300)	(1320, 1270, 1290)
$WhcR_{mwt}$	(3000, 4100, 3200)	(3050, 3300, 3500)	(4050, 3001, 4500)	(3300, 3260, 4305)	(3410, 4300, 3320)	(1030, 1270, 1260)

In a day, there are two working shifts. 8 hours are allotted for regular production per shift, while 3 hours allotted for overtime production. To produce these products, 10 types of raw materials are required and the Selling price for finished products is ₦470. Repairs are done just in shift 2 (i.e., overtime). When demand for a certain period exceeds production capacity during regular hours and inventory levels are likewise insufficient to meet this demand, production is continued during overtime.

The APP decision problem for the industrial case that is discussed here focuses on the creation of a multiple fuzzy goal programming model for figuring out the best way to meet forecasted demand by modifying output rates, hiring and firing, inventory levels, overtime and backorders. The anticipated outcomes of this APP decision include minimizing total production cost, production waste minimization and maximization of the capacity utilization rate.

Findings and Outcomes

From the Triangular Fuzzy warehouse Space $\widetilde{WhcR}_{wit}$ the crispy number $WhcR_{wit}$ needs to be found. Taking $\widetilde{WhcR}_{111}$ which is warehouse space for product 1 in period 1 as stated in Table 4 and depicted in Fig. 3, the crispy warehouse space $WhcR_{111} = 0.8 * \left(\frac{3900+2800}{2}\right) + (1 - 0.8) * \left(\frac{2800+4000}{2}\right) = 3360.$

The membership function of the fuzzy demand may be expressed as follows:

$$\mu(WhcR_{111}) = \begin{cases} 0 & WhcR_{111} \leq 3900 \\ (WhcR_{111} - 3900)/(2800 - 3900) & 3900 \leq WhcR_{111} \leq 2800 \\ (2800 - WhcR_{111})/(4000 - 2800) & 2800 \leq WhcR_{111} \leq 4000 \\ 1 & Otherwise \end{cases}$$

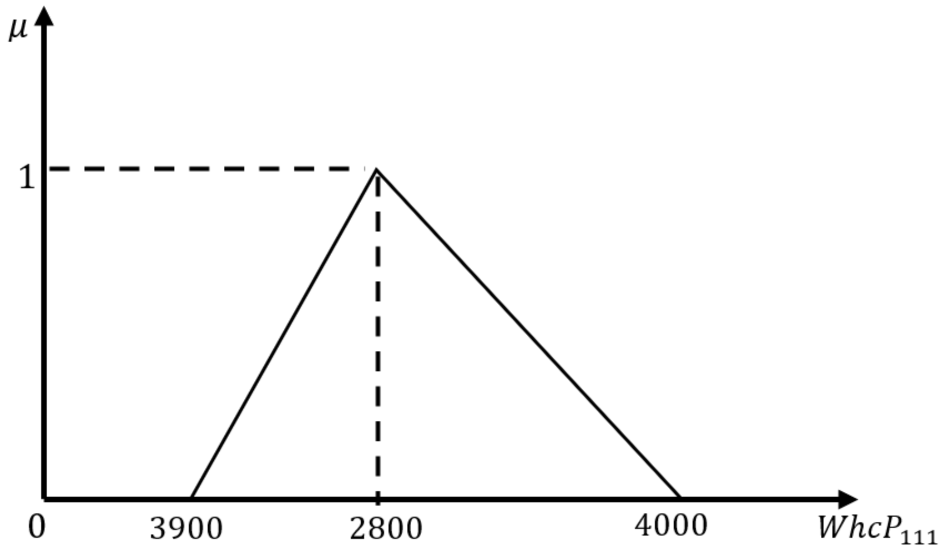


Figure 3. Triangular Distribution of the Fuzzy Warehouse Space for $\mathbf{WhcP_{111}}$

Other data on demand are also interpreted similarly.

The recommended APP is programmed and executed with LINGO 18 solution. The minimum and maximum values for objectives are determined by the payout matrix and are shown in Table 5. Thus, the goals and aspirational levels have been determined to be $g_1 = 1368835;\ g_2 = 10344.02;\ g_3 = 1760481$.

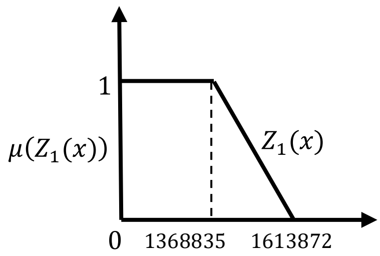
Table 5. Payoff Matrix

		$Z_k(v_j^*)$		
Objectives	PIS			NIS
Min $Z_1(x)$	1368835	1613872	1589018	1613872
Min $Z_2(x)$	10344.02	10344.06	11835.14	11835.14
Max $Z_3(x)$	1760481	1650640	1650640	1650640

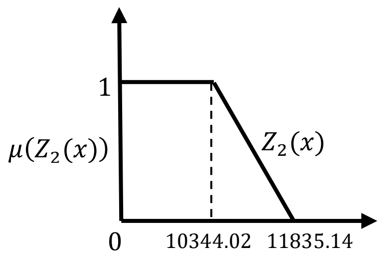
The linear membership function of each objective function is determined with its PIS and NIS as the interval of the objective values, and also to specify the equivalence of these objective values as a membership value in the interval [0, 1]. The fuzzy aspiration levels can be quantified using the linear and continuous membership function. According to Eq. 35 and 36, the relevant linear membership functions can be defined as shown below.

The information in Table 5 can be used to draw the conclusion that the suggested FGP is capable of locating a high-quality compromise solution even in the face of numerous competing objective functions and constraints. As is obvious, there is a high level of satisfaction for all objective functions, and this is seen as a good Compromising solution for the problem.

$$\mu(Z_1(x)) = \begin{cases} 1 & Z_1(x) \le 1368835 \\ \dfrac{1613872 - Z_1(x)}{1613872 - 1368835} & 1368835 \le Z_1(x) \le 1613872 \\ 0 & Z_1(x) \ge 1613872 \end{cases}$$



$$\mu(Z_2(x)) = \begin{cases} 1 & Z_2(x) \le 10344.02 \\ \dfrac{11835.14 - Z_2(x)}{11835.14 - 10344.02} & 10344.02 \le Z_2(x) \le 11835.1 \\ 0 & Z_2(x) \ge 11835.14 \end{cases}$$



$$\mu(Z_3(x)) = \begin{cases} 1 & Z_k(x) \ge 1650640 \\ \dfrac{Z_3(x) - 1650640}{1760481 - 1650640} & 1650640 \le Z_k(x) \le 1760481 \\ 0 & Z_k(x) \le 1760481 \end{cases}$$

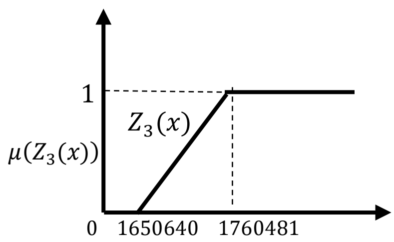


Table 6. The fuzzy goal programming

Satisfaction Level			Objective values			
μ_1	μ_2	μ_3	Z_1	Z_2	Z_3	λ
0.8735213	0.6856462	0.6856462	1399827	10812.72	1725952	0.6856462

Considering the various fuzzy goal values $(Z_1, Z_2 \text{ and } Z_3)$, the suggested model gives the overall levels of DM satisfaction (λ value). Each goal is fully satisfied if the answer is $\lambda = 1$. If $\lambda = 0$, none of the goals are satisfied. If $0 < \lambda < 1$, all of the goals are satisfied at some level. For instance, the initial calculation of the overall DM satisfaction (λ) with the goal values $(Z_1 = 1399827\ , Z_2 = 10812.72$, and $Z_3 = 1725952)$ was 0.6856462. The λ value can be adjusted to look for a set of superior compromise options if the DM did not accept the initial overall degree of this satisfaction value.

Additional Breakdown

A significant influence on production costs is held by the allocation of variable warehouse space within the framework of aggregate production planning. This dynamic relationship incorporates a number of important elements, such as the price of keeping inventory, the timing of production, the responsiveness of the market to demand, and the cost of transportation. Greater storage capacity is made possible by larger warehouse areas; however, this may come at a cost in terms of higher costs for insurance, storage fees, and probable obsolescence, see Table 7 below. The cost of ordering may increase as a result of smaller warehouses needing more regular replenishments with reference to changes in α-value, see equations (31) to (33) where α-value is 0.8. In addition, the ability to adjust to fluctuations in demand can be lost due to a lack of warehouse space, resulting in oversupply during peak demand periods and lost sales opportunities. Lastly, transportation costs can also be negatively impacted. Larger warehouses allow bulk shipments, which may result in cost savings as economies of scale are realized. On the other hand, smaller warehouses are more likely to have more frequent and smaller shipments, which can result in higher transportation costs. So, to sum up, optimizing your warehouse space allocation requires complex tradeoffs to balance these factors, allowing you to optimize your overall production planning and operate cost-effectively.

Table 7. The MultiObjective goal Values (Abridged Table)

λ	μ_1	μ_2	μ_3	Prod. Cost	Process Time	Sells	WH Space for Prod 1	WH Space for Prod 1	WH Space for Raw Mat 1	WH Space for Raw Mat 2
0	0.9899	0.9999	0	1371317	10344.06	1650640	1292	1173	3360	2950
0.1	0.9888	0.9610	0.1	1371588	10402.11	1661624	1292	1173	3360	2950
0.2	0.9859	0.9140	0.2	1372270	10472.22	1672608	1292	1173	3360	2950
0.3	0.9752	0.8669	0.3	1374918	10542.33	1683592	1292	1173	3360	2950
0.4	0.9571	0.8199	0.4	1379344	10612.45	1694576	1292	1173	3360	2950
0.5	0.9275	0.7729	0.5	1386590	10682.56	1705560	1292	1173	3360	2950
0.6	0.9005	0.7259	0.6	1393209	10752.67	1716545	1292	1173	3360	2950
0.7	0.7	0.7	0.6551	1442346	10791.31	1722599	1292	1173	3360	2950
0.8	0.8	0.8	0.4425	1417842	10642.22	1699241	1292	1173	3360	2950
0.9	0.9	0.9	0.2298	1393339	10493.12	1675882	1292	1173	3360	2950
1	1	0.8902	0.2507	1368835	10507.8	1678182	1292	1173	3360	2950

The balance between production capacity, inventory management and cost effectiveness is achieved through aggregate production planning, where variable warehouse space is a key factor in the planning process. To sum up, variable warehouse space is a key parameter in APP. The APP system choses the appropriate cost effective warehouse space (see the last four columns of Table 7) based on stated demand, raw material and the calculated output. By understanding how space variability affects inventory, production planning, cost reduction, and supply chain performance, companies make better decisions that lead to cost-efficient production, better customer support, and better space management.

Conclusion and Recommendations

Incorporating imprecise warehouse space into aggregate production planning using fuzzy goal programming presents a robust approach to addressing the ambiguity of space allocation within a dynamic manufacturing environment. This methodology recognizes the uncertainties and vagueness associated with warehouse space availability and integrates them into the decision-making process. By employing fuzzy goal programming, companies can systematically balance conflicting objectives, such as production efficiency, inventory holding costs, and demand changes, while accounting for imprecision in space limitations. The fuzzy goal programming approach provides a flexible framework that helps decision-makers quantify and manage uncertainty, allowing them to make more informed and adaptable production planning decisions.

Organizations can navigate the complexities of production planning while accommodating the uncertainties inherent in warehouse space allocation by adopting imprecise warehouse space in aggregate production planning using fuzzy goal programming and adhering to these recommendations. This will ultimately improve decision-making and increase operational efficiency by; adopting and accepting the effectiveness of the model, using the stated reliable guide for data which enhances the model's ability to generate realistic and effective production plans under fuzzy constraints, the iterative process will ensure that the model remains aligned with the evolving production environment, improving the accuracy of the decision-making process.

This work is capable in providing training and instruction to the team in charge of putting the fuzzy goal programming technique into practice. Effective implementation and interpretation of the results will need a complete grasp of the approach and its consequences. Future work will be to investigate effective incorporation of renewable and green house effects in building new APP models.

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