The influence of the entropy order in stochastic image alignment based on the maximum mutual information criterion

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***Abstract*— This paper investigates the effect of entropy order on the convergence rate of a stochastic image alignment algorithm based on Renyi mutual information. A comparison is made with the case of using the Shannon mutual information.**

***Keywords*— image alignment, stochastic procedure, geometric deformation, theoretical-information measure, mutual information, Shannon, Renyi**

I. Introduction

Image alignment is used in a wide variety of fields, particularly in medicine for the alignment of positron emission, magnetic resonance and computed tomography images.

The task of image alignment comes down to the fact that points corresponding to identical elements of the scene structure are aligned by a spatial geometric transformation of two or more images formed by sensor. Image alignment techniques have been intensively developed and widely used over the last few decades. Many effective approaches oriented to different application tasks have been proposed. Conventionally, these approaches can be divided into two broad categories: image alignment based on key features and based on intensities [1].

The extraction of image features is often difficult [2, 3] for aligning images that have different nature and structure. Intensity-based alignment does not require any feature search, but involves only the estimation of the numerical value of a similarity measure. At the same time, the theoretical-informational measures have been increasingly used, as they do not require prior processing and estimation of image parameters. Such measures include Shannon and Renyi mutual information (MI) [4, 5].

II. Problem Statement

The search for optimal geometric deformations parameters $\bar{\alpha}^*$, in particular of two images $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$ is reduced, as a rule, to the search for the extremum of a multidimensional objective function $\mathbf{J}(\bar{\alpha}) = \mathbf{J}\left(\bar{\alpha}, \mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}\right)$ describing a certain similarity measure between the conditionally reference $\mathbf{Z}^{(1)}$ and deformed $\mathbf{Z}^{(2)}$ images.

An efficient approach to finding an extremum in the image alignment problem is an adaptive non-identification stochastic procedure [6], which in general reduces to:

$$\hat{\bar{\alpha}}_t = \hat{\bar{\alpha}}_{t-1} \pm \mathbf{\Lambda}_t \bar{\beta}\left(\mathbf{J}\left(\bar{\alpha}, Z_t\right)\right) \tag{1}$$

where $\hat{\bar{\alpha}}$ – vector of parameter estimates; $\bar{\beta}(\cdot)$ – stochastic gradient of the objective function $\mathbf{J}(\cdot)$ of parameter estimation quality; $\mathbf{\Lambda}$ – the gain matrix that determines the quality and rate of learning estimates; $t = \overline{1, T}$ – iteration number; Z_t – local sampling from images $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ at the t-th iteration, by which the stochastic gradient of the objective function.

Shannon MI is defined as [7]:

$$\mathbf{J}_S = \hat{H}\left(\mathbf{Z}^{(1)}\right) + H\left(\mathbf{Z}^{(2)}\right) - \hat{H}\left(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}\right), \tag{2}$$

where $\hat{H}(\mathbf{Z}) = -\sum_i p_z(z_i) \ln\left(p_z(z_i)\right)$ and $\hat{H}\left(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}\right) = -\sum_i \sum_k p_{z1,z2}(z_i, z_k) \cdot \ln(p_{z1,z2}(z_i, z_k))$ – is the estimates of single and joint entropy of images $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$; p_z and $p_{z1,z2}(z_i, z_k)$ – estimates of marginal and joint probability density image brightness over the local sampling Z_t; z_i, z_k – the pixels included from the local sampling.

Renyi MI is found from the Renyi entropy of order α, which is determined as [8]:

$$\mathbf{J}_R = \frac{\hat{H}_\alpha\left(Z^{(1)}\right) + H_\alpha\left(Z^{(2)}\right)}{\hat{H}_\alpha\left(Z^{(1)}, Z^{(2)}\right)}, \tag{3}$$

where $\hat{H}_\alpha\left(\mathbf{Z}^{(1)}\right) = (1-\alpha)^{-1} \ln \sum_i p_i^\alpha(z_i)$ and $\hat{H}_\alpha\left(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}\right) = (1-\alpha)^{-1} \ln \sum_i \sum_k p_{i,k}^\alpha(z_i, z_k)$ are single [9] and joint Renyi entropies; α is a constant, called the entropy order.

The object of the paper is to investigate the effect of the Renyi entropy order on the convergence rate of a stochastic procedure.

III. Experimental Results

The research was carried out on real and simulated images. Examples of the following results are obtained on synthesized images with Gaussian brightness distribution. In particular, Fig. 1 shows normalized Shannon (line 1) and Renyi MI at three values of the entropy of order alpha: 0; 1.1

and 6 (lines 2, 3, and 4 respectively). The mean and maximum slopes were also estimated for these similarity measures, which determine the potential convergence rate of the parameters of the stochastic procedure (1).

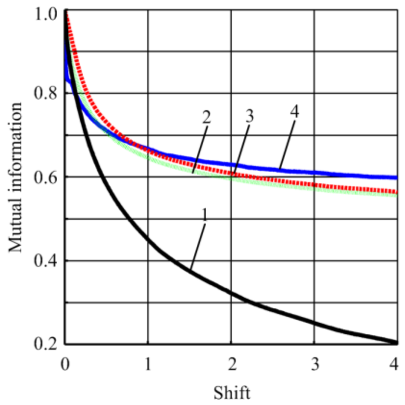

Fig. 1. Shannon and Renyi MI

The maximum slope for the Shannon MI was 3.36 and the average was 0,11. For the Renyi MI, these parameters depend on α . The corresponding dependences are shown in Fig. 2a and 2b, respectively. Note that at $\alpha = 1$ the Renyi MI is reduced to the Shannon MI. This value is missed in the Fig. 2. It can be seen that the greatest average slope is provided by selecting α from about 0.9 to 1.7. To verify this conclusion, a series of experiments were done to estimate the parameters of image deformation using procedure (1).

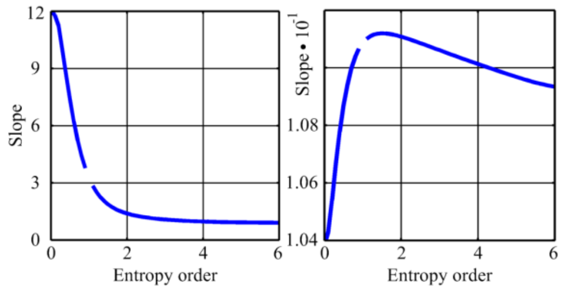

Fig. 2. Dependences of the maximum and average slope of Renyi MI on entropy order

The Euclidean mismatch distance [10], which fully describes the behavior of the deformation parameter estimates vector, was used as the accuracy of alignment of the reference and deformed images. The convergence rate was determined by the number of iterations until the Euclidean mismatch distance minimum was reached. An example of the corresponding dependence for the deformation parameters: shifts along the basic axes -4.5 and 4.5, rotation angle -5.5º, scale factor 1.1, is shown in Fig. 3. In Fig. 3, the median convergence rate corresponds to line 1 and the mean to the line 2. The local sample size Z_t was 100. The results were averaged over 60 realizations. The minimum number of convergence iterations is achieved at $\alpha = 1{,}1$. In this case the average convergence rate as well as the median is 112. When using the Shannon MI, the average number of convergence iterations is 116 and the median is 114.

IV. Conclusion

Thus, in stochastic image alignment procedures based on the Renyi MI, the choice of the order of the Renyi entropy has an effect on the convergence rate of the procedure. The experiments (Fig. 3) also confirmed the assumption that the order of the Renyi entropy can be chosen a priori before the design of the alignment procedure only by the appearance of the similarity measure graph (Fig. 2b). In particular, the range from 0.9 to 1.7 turned out to be preferable in the conducted studies.

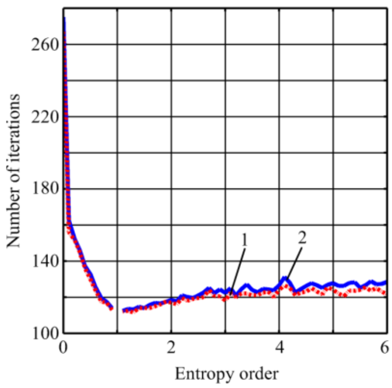

Fig. 3. Dependencies of the convergence rate of the stochastic procedure on entropy order

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References

[1] Maintz, J.B.A. A survey of medical image registration / J.B.A. Maintz, M.A. Viergever // Medical image analysis. – 1998. – Vol. 2(1). – P. 1-36.

[2] Park, H. Adaptive registration using local information measures / H. Park, P.H. Bland, K.K. Brock // Medical Image Analysis. – 2004. – Vol. 8(4). – P. 465-473.

[3] Can, A.A feature-based, robust, hierarchical algorithm for registering pairs of images of the curved human retina / A. Can, C.V. Stewart, B. Roysam // IEEE transactions on pattern analysis and machine intelligence. – 2002. – Vol. 24(3). – P. 347-364.

[4] Tashlinskii, A.G. Usage of mutual information as similarity measures for stochastic binding images / A.G. Tashlinskii, R.O. Kovalenko, G.L. Safina, R.M. Ibragimov // 2021 International Conference on Information Technology and Nanotechnology. – 2021. – P. 1-6.

[5] Kovalenko, R.O. Optimization of the histogram intervals number which approximate brightness probability distributions in stochastic image alignment based on mutual information / R.O. Kovalenko, A.G. Tashlinskii // IEEE Xplore (VIII International Conference on Information Technology and Nanotechnology (ITNT)). - 2022, Vol. 21860541. – P. 1-5.

[6] Tashlinskii, A.G. Pseudogradient estimation of digital images interframe geometrical deformations / A.G. Tashlinskii // Vision Systems: Segmentation and Pattern Recognition. – 2007. – P. 465-494.

[7] Duncan, T.E. On the calculation of mutual information / T.E. Duncan // SIAM Journal on Applied Mathematics. – 1970. – Vol. 19(1). – P. 215-220.

[8] Wachowiak, M.P. Similarity metrics based on nonadditive entropies for 2D-3D multimodal biomedical image registration / M.P. Wachowiak, R. Smolikova, G.D. Tourassi, A.S. Elmaghraby // Medical Imaging 2003: Image Processing. – 2003. – Vol. 5032. – P. 1090-1100.

[9] Renyi, A. On measures of entropy and information / A. Renyi // Proc. Fourth Berkeley Symposium on Mathematical Statistics Probability. – 1961. – Vol. 1. – P. 547–561.

[10] Tashlinskii, A.G. Pseudogradient optimization in the estimation of geometric interframe image deformations / A.G. Tashlinskii, G.L. Minkina // Pattern recognition and image analysis. – 2008. – Vol. 18(4). – P. 706-711.