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The 31-Tone Tuning System of Nicola Vicentino and the Toroidal *Tonnetz*: an Annotated Bibliography

Nicola Vicentino's treatise *L'antica musica ridotta alla moderna prattica* (1555), from here on *L'antica musica*, argues that contrapuntal practices based on modes derived from the diatonic tetrachord are insufficient to express the variety of emotions possible in vocal text settings, and that composers should be inspired by the other Ancient Greek genera as described by Boethius, the chromatic and enharmonic tetrachords. To employ these alternative genera, Vicentino devised an ingenious system that extended quarter-comma mean tone temperament to a thirty-one-tone system that can be neatly approximated by a division of the octave into thirty-one equal parts. (I will use abbreviations such as 12edo or 31edo for equal divisions of the octave in this bibliography.)

Analyses of the four enharmonic compositions presented by Vicentino in *L'antica musica* are scarce in the existing literature. These include a thorough analysis of the embedding of the chromatic and enharmonic tetrachords into Vicentino's melodies by Jonathan Wild and a thesis by Mikaela Miller that uses Neo-Riemannian techniques to examine Vicentino's voice-leading and chord-to-chord harmonic distance in *Tonnetz* space. The aim of this research project is to build on this analytic groundwork by recognizing the *Tonnetz* as a toroidal (donut-shaped) space and to adapt Neo-Riemannian tools developed to analyze 12edo music to be better suited to the 31edo compositions of Vicentino. A higher-dimensional analog to the toroidal *Tonnetz* may also be considered that includes the enharmonic diesis (the generating interval of Vicentino's system) along with the traditional thirds and fifths.

Secondary or Tertiary Monographs

1. Kaufmann, Henry W. *The Life and Works of Nicola Vicentino*, 1511-c.1576. n.p.: American Institute of Musicology, 1966.

Henry W. Kaufmann (b. 1913 – d. 1982) was an American musicologist who specialized in music of the Italian Renaissance. He finished his PhD dissertation on Nicola Vicentino at Harvard University in 1960, and subsequently published a handful of articles in the following years that reignited modern interest in Vicentino's works. He served as chair of the Rutgers University music department from 1962 to 1972. He edited the first modern edition of Vicentino's (known) complete works under the title *Opera Omnia*, published in 1963.

The Life and Works of Nicola Vicentino is divided into four chapters. The first, entitled "The Life" is a relatively short (thirty-plus page) biographical sketch of Vicentino, with copious footnotes that often take up half the page or more. The second, "The Music," catalogs the known works of Vicentino, many of which comprise the remainder of five volumes of madrigals; (published editions of the first and the fifth are extant, while the remaining three are lost.) This chapter also documents incomplete works, false attributions, and additional lost works, some of which are mentioned in *L'antica musica*. The third, "The Theory," is the most relevant to this research project as Kaufmann analyzes Vicentino's own theories as represented in *L'antica musica*. Especially pertinent are Kaufmann's examinations of the scores and score fragments where Vicentino presents the enharmonic diesis (roughly forty cents) as a viable musical interval and means to revive the application of the Ancient Greek enharmonic tetrachord to (then) modern music. Kaufmann also summarizes Vicentino's description of the construction and

tuning of one of his microtonal instruments, the archicembalo. The last chapter, "RESERVATA – A Problem of Musical Mannerism," concerns *musica reservata*, a style of textual expression in vocal music from the late sixteenth century with which Vicentino identified. The movement is discussed by Kaufmann in the context of "mannerism," an aesthetic movement in European Renaissance art.

Kaufmann's primary source for chapter three is a facsimile of Vicentino's original printed treatise, *L'antica musica*, written in Italian. Kaufmann translates bits and pieces of Vicentino's writing into English for the reader, with accompanying footnotes displaying the original Italian. The dependence of the reader on these translations demonstrates how essential knowledge of the Italian language was to Vicentino scholarship at the time. In this work, Kaufmann makes Vicentino's writings more accessible to readers of the English language, but a complete English translation of *L'antica musica* would not arrive until three decades later in 1996, translated by Maria Rika Maniates (see Vicentino below).

2. Lewin, David. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press, 1987.

Lewin's monograph is a cornerstone of Neo-Riemannian theory. Lewin's work can be considered a secondary source, as he uses real musical examples to illustrate his theories. However, this work could also be considered a primary monograph as it contains no bibliography; Lewin's work is supported by original research, and he gives primarily mathematical or deductive/procedural arguments to support his claims. Lewin examines how a musical interval constitutes a magnitude and a direction: the equivalent of a mathematical vector. He attempts to lend this understanding of mathematical vectors to the service of elucidating musical ones that exist both in frequency/pitch space as well as time. Though this work is foundational it is also quite complex and dependent on a catalog of mathematical/procedural operations that are developed and augmented throughout the course of the work. Mathematical group theory is the foundation for this volume (Lewin redefines all the relevant operations in music theoretic terms in Chapter 1) and therefore this book can be thought of as the tonal and chromatic music analog to the post-tonal work of Robert D. Morris and others. This particular application of mathematical group theory is now known as "transformational theory" in the music theory world. As these concepts are foundational to Neo-Riemannian theory, any valid critique of them could affect structures such as Riemann's Tonnetz that have been retrofitted to accommodate this foundation. Therefore, reexaminations of Lewin's work such as those by Dmitri Tymoczko are critical in attempting to continue to build on such transformational theories (see Tymoczko 2009).

David Lewin (b. 1933 – d. 2003) is often cited as one of the most important music theorists of his generation. He taught at UC Berkeley, SUNY Stony Brook, Yale University, and Harvard University.

3. Vicentino, Nicola. *Ancient Music Adapted to Modern Practice*. Edited by Claude V. Palisca. Translated by Maria Rika Maniates. New Haven: Yale University Press, 1996.

This English translation of Vicentino's published treatise on music theory and composition, *L'antica musica*, increased accessibility to the composer's writings to a much wider range of scholars. The translator, Maria Rika Maniates (b. 1937 – d. 2011) was a Canadian

musicologist and professor of music at the University of Toronto. She also held leadership roles in various organizations such as the International Musicological Society and the American Musicological Society. The volume is part of a series edited by Claude V. Palisca (b. 1921 – d. 2001), one of the leading musicologists of his generation, and former professor at Yale University. Palisca expresses his confidence in Maniates's translation in the editor's preface, lending authority in attesting to the quality of the work.

In addition to providing a translation of Vicentino's treatise, this volume includes a fiftyplus page introduction by Maniates, a list of errata in the printed examples, appendices, and an index. The introduction includes a biographical sketch of Vicentino and a summary of the treatise (a book on music theory plus the five books on music practice.) Found in the treatise summary and of critical use to this research project are diagrammed explanations of Vicentino's interpretations of the Greek genera (through the conduit of Boethius), as well as short explanations of how each of Vicentino's compositions presented in Book III attempt to apply the genera. Maniates concludes the introduction with an explanation of her translational approach. Of the five books in Vicentino's treatise that follow, Books III and V are the most pertinent. Book III describes Vicentino's approach to applying the Greek genera to composition, with the enharmonic tetrachord at the heart of the inspiration for Vicentino's microtonal tuning system (see Wild). Book V describes the construction and tuning of the archicembalo, a custom-built microtonal keyboard instrument. The appendices provide supplementary material, especially regarding the archicembalo tuning, and supply diagrams of the keyboard layout as well as tables of string lengths and tunings in cents.

In a brief passage from the introduction (p. *l*) Maniates notes that though Vicentino does not use enharmonic equivalents in his spellings, they must be implied for the tuning system to become closed. This closing of the system is precisely the reason my research project describes Vicentino's tuning system (or more accurately, the 31edo representation of it) as a toroidal space (see Cohn, Miller, Tymoczko 2012).

Essays in Collections

 Maniates, Maria Rika. "Bottrigari versus Sigonio: on Vicentino and his Ancient Music Adapted to Modern Practice." In *Musical Humanism and Its Legacy: Essays in Honor of Claude V. Palisca*, edited by Nancy Kovaleff Baker and Barbara Russano Hanning, 79-107. Stuyvesant, NY: Pendragon Press, 1992.

Maniates can be viewed as an English-language scholar who in her translation of Vicentino's *L'antica musica* brought Kaufmann's groundwork to a much larger audience. This volume is dedicated to Palisca, a mentor of Maniates who edited the music theory series of which Maniates's translation entitled *Ancient Music Adapted to Modern Practice* was an integral part. This essay was published while Maniates was working on this translation project. Maniates's essay is an in-depth account of the conflict between Vicentino and Gandolfo Sigonio as to the correct interpretation of Boethius when it came to integrating Ancient Greek genera (tetrachords) into (then modern) sixteenth-century contrapuntal choral writing. Vicentino claimed that the minor and major thirds were endemic to the chromatic and enharmonic tetrachords, respectively, whereas Sigonio asserted that the minor and major thirds could be derived as non-adjacent intervals in the diatonic tetrachord, and that therefore modes constructed from the diatonic genus were the only ones relevant to modern composition. A formal debate was adjudicated by officials from the church, and Sigonio won. Ercole Bottrigari was a figure who published a rebuttal against Sigonio's arguments, defending Vicentino, and Maniates encapsulates his arguments in this essay. Vicentino's bitterness over this loss may have very well have motivated the composition of his later treatise, and Maniates makes numerous specific connections between Vicentino's documented arguments, the treatise, and the novel 31-tone tuning of the archicembalo (see Kaufmann, Vicentino).

Dissertations and Theses

 Miller, Mikaela. "Nicola Vicentino and the Enharmonic Diesis: An Analytical and Empirical Study." Master's thesis, McGill University, 2011. ProQuest Dissertations & Theses Global.

Miller's thesis contains the most thorough harmonic (chord-to-chord) analysis of the four enharmonic compositions from *L'antica musica* in the scholarly literature. As the enharmonic tetrachord is the motivation for these pieces, Miller enumerates every possible chord progression (under transpositional equivalence) in 31edo that would include the enharmonic diesis as a melodic interval, and out of those, catalogs which possibilities Vicentino actually uses in these compositions. She finds that he favors certain types of melodic and harmonic motion over others, even ignoring possibilities his theories would seem to favor. She employs Neo-Riemannian theories in analyzing these chord progressions via metrics of melodic and harmonic distance. Miller claims that "no system of calculating voice-leading similarity or distance exists for a tonal system with 31 tones per octave" (58). Therefore, she shifts to a 12edo perspective in order to use the "taxi cab" metric used by theorists such as Richard Cohn. Miller attempts to analyze harmonic distance by interpreting the *Tonnetz* in both 12edo and 31edo.

Editorial aside: It is true that Neo-Riemannian theory exhibits a severe bias toward 12edo (as its tools were devised to analyze 12edo music). Though work that highly generalizes some of these spaces certainly exists, scholarly articles that specifically track the implications of switching these metrics and analytical tools to alternate divisions of the octave are scarce, and are particularly elusive when it comes to 31edo. However, there do not appear to be any factors prohibiting the extension of these metrics and tools to a space such as 31edo, and in fact this may constitute a space for new research. I intend to convert the taxi cab metric to 31edo and analyze the implications for Vicentino's progressions when compared to Miller's 12edo measurements. Furthermore, I have noticed an inconsistency in Miller's 31edo interpretation of the Tonnetz. Miller correctly notes that one can differentiate between 12edo and 31edo interpretations of the Tonnetz by noting where enharmonic equivalence takes place. Specifically, in 12edo, F-sharp and G-flat are the same pitch class, whereas in 31edo F-sharp and G-flat are separated by one enharmonic diesis. The mapping of F-sharp onto G-flat in 12edo essentially maps the Tonnetz from a cylindrical space to a toroidal one, and Miller's measures of harmonic distance should therefore be accurate. However, though F-sharp and G-flat are not equivalent in 31edo, that is not to say that Pythagorean equivalents do not exist in such a system. Miller's 31edo Tonnetz only contains 28 pitch classes (61). Perhaps these are the only pitch classes necessary in enumerating the chords Miller analyzes. However, if the Tonnetz were to continue into the triplesharps, G-triple-sharp would appear in the next row above. In 31edo, G-triple-sharp is an enharmonic equivalent of B-double-flat, which appears at the bottom of Miller's Tonnetz. These two nodes of the graph should be recognized as equivalent for the Tonnetz to map onto a toroidal

space. It is possible that Miller indeed recognized this equivalence in her calculations without mentioning it in her thesis. But if not, there may be inaccuracies in her measurements of harmonic distance in 31edo.

The above inconsistencies aside, Miller's work is of excellent quality and lays the groundwork for any future harmonic analysis of Vicentino's enharmonic compositions. The second half of the thesis is devoted to a study demonstrating that the differences between 12edo and 31edo interpretations of music were very audible to listeners, and therefore that concepts such as harmonic distance have a perceived reality and are not just abstract measurements. To me, this serves as further evidence for why 31edo-specific tools should be devised to describe harmonic and voice-leading distance. Miller composed this thesis at McGill University, and was supervised by Jonathan Wild, another theorist who made important contributions to the melodic analysis of these works (see Wild.)

Journal articles

6. Cohn, Richard. "Neo-Riemannian Operations, Parsimonious Trichords, and Their 'Tonnetz' Representations." Journal of Music Theory 41, no. 1 (Spring 1997): 1-66. https://www.jstor.org/stable/843761.

Richard Cohn's mathematically rigorous and highly technical article explores in detail the parsimonious nature of the PLR-type Neo-Riemannian transformations (relationships of common tone between triads, and how they enable smooth voice leading) when applied to the *Tonnetz*. He constructs a general *Tonnetz*, abstracted from any particular pitch relationships or tuning systems, which he dubs the "Parsimonious *Tonnetz*" that is embedded on an infinite plane. Critically, Cohn observes how it is the application of a modulus that establishes equivalence classes between nodes of the graph, and that these equivalence classes map the infinite twodimensional space to a closed toroidal space. Cohn notes how errors of distance may be made in using such a closed system if the toroidal nature of the *Tonnetz* is not fully realized (17-18). Cohn does not focus solely on a modulus of 12, which is equivalent to 12edo, but also examines the nature of PLR-type operations in toroidal *Tonnetze* formed by moduli of 18 and 24, equivalent to 18edo and 24edo, respectively. This level of generalization could serve as groundwork for the study of toroidal *Tonnetze* that model other equal divisions of the octave, such as 31 (see Miller).

Cohn is a music theorist, Professor of Music at Yale University, and the current editor of the peer-reviewed *Journal of Music Theory* in which this article appeared.

 Douthett, Jack and Peter Steinbach. "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition." *Journal of Music Theory* 42, no. 2 (Autumn 1998): 241-263. https://www.jstor.org/stable/843877.

Douthett and Steinbach's work examines cyclical relationships formed by parsimonious movement in the *Tonnetz* space. Interpreting the *Tonnetz* through a space of 12edo pitch class equivalence, each triangular face of the *Tonnetz* can be rendered as a node of a new graph, and each parsimonious movement or edge-flipping on the *Tonnetz* is represented by an edge of this new graph. The new graph is the hexagonal "Chicken-Wire Torus" and is the geometric dual of the *Tonnetz*. Geometric duality is the mathematical relationship between two graphs or polytopes

where an isomorphism is demonstrated between them, such that faces that share edges of one map one-to-one and onto the vertices that share edges of the other. Douthett and Steinbach demonstrate the existence of cyclic or networked chord relations in this space, naming them "HexaCycles," "Octatowers," "Octacycles" and "Enneacycles" (245-246). These are understood to define modes of limited transposition, and two more graphs called the "Cube Dance and Power Towers" are shown to interrelate these structures. These are among the structures that Tymoczko was able to unify under the generalized *Tonnetz* in later work (see Tymoczko 2012). The evidence Douthett and Steinbach present for their argument is rooted in the work of previous Neo-Riemannian scholars, Lewin and Cohn among them. A mathematical argument is presented for the most basic relationships among chords, and further argument can be understood to be self-evident, where the theoretical structures are simply shown and it is left to the reader to verify that such structures are coherent. (Editorial interpolation: Mathematical proofs can be tricky to produce in a music theoretic problem of sufficient complexity. However, such presentations of self-evident structures can sometimes be seemingly verified by the reader while certain rhetorical slippages or mathematical conflations occur unnoticed. Without claiming that this is necessarily happening here, I conjecture that this is a persistent problem in verifying music theoretical arguments.)

Jack Douthett (b. 1942 – d. 2021) was a musician, mathematician and music theorist who was known for his contributions to Neo-Riemannian theory. Peter Steinbach seems to be primarily known as a collaborator to Douthett. This article appeared in the peer-reviewed *Journal of Music Theory*.

8. Gollin, Edward. "Some Aspects of Three-Dimensional '*Tonnetze*."" *Journal of Music Theory* 42, no. 2 (Autumn 1998): 195-206. http://www.jstor.com/stable/843873.

Gollin illustrates how the addition of a fourth pitch class to a chord extends the *Tonnetz* into an additional dimension. The [0258] tetrachord is selected as a particular example, which extends a major triad to a dominant-seventh chord or a minor-triad to a half-diminished (or Tristan) chord. Each instance of the tetrachord corresponds to a tetrahedron in the threedimensional Tonnetz, with dominant-seventh and half-diminished tetrachords in opposite orientations. Various degrees of parsimony can be established by whether two tetrahedrons share a face (three common tones,) an edge (two common tones,) or a single vertex (one-common tone.) Each of these relationships engenders a family of possible chord transformations. As a parallel to Cohn's non-pitch-specific "Parsimonious Tonnetz," Gollin presents the reader with a "generalized 3-D Tonnetz" in which each independent vector represents some different interval, and therefore each tetrahedron represents some (non-specific) tetrachord. Gollin demonstrates that though some of the particulars of chord-to-chord transformations depend on specific mappings of the axes to intervals, there is an underlying group structure that is invariant for this generalized version of the 3-D Tonnetz. Gollin's claims are supported by a mix of citations that appeal to the authority of earlier sources as well as the kind of self-evidence posited by music theoretic arguments mentioned above.

Gollin is professor and chair of the music department at Williams College. He received his PhD from Harvard University in 2000.

9. Maniates, Maria Rika. "Nicola Vicentino's Reconstruction of the Ancient Greek Genera."

Revista de Musciología 16, no. 3, Del XV Congreso de la Sociedad Internacional de Musicología: Culturas Musicales Del Mediterráneo y sus Ramificaciones (1993): 1294-1314. http://www.jstor.com/stable/20795987.

While translating Vicentino's *L'Musica Antica*, Maniates reinvigorated interest in modern scholarship of this composer, building on the groundwork laid by Kaufmann. This essay published during the execution of her translation project outlines some of her key findings, many of which were revisited in the introduction to *Ancient Music Adapted to Modern Practice*. Elaborating on Kaufmann's work, Maniates enumerates some of the inconsistencies of Vicentino's work, such as the sloppy substitution of 39/38 for 128/125 as a representation of the enharmonic diesis in the tuning of the archicembalo. Such a preference for superparticular (n / n-1) ratios was later noted by Jonathan Wild as emblematic of Vicentino's conflation of ideas from Ptolemaic (5-limit just intonation) harmonic practice as presented by Boethius with Pythagorean and mean tone systems, none of which are mutually compatible without severe alterations (see Wild). Though much of these ideas do find their way into the introduction to her translation of Vicentino's treatise, Maniates draws a connection between Vicentino and the larger Renaissance movement of humanism (which drew upon Ancient Greek and Roman sources for inspiration) in her separately published essays, which in turn add nuance to her body of arguments regarding Vicentino.

10. Tymoczko, Dmitri. "The Generalized Tonnetz." *Journal of Music Theory* 56, no. 1 (Spring 2012): 1-52. https://www.jstor.org/stable/41508604.

In this article, Tymoczko synthesizes much previous Neo-Riemannian scholarship concerning the Tonnetz, including works by Cohn, Douthett, and Gollin that appear on this list, as well as his own work. In doing so he generalizes the traditional *Tonnetz* as one variant of several possible families of note-based graphs. Building on Douthett's concept of the "chickenwire torus" as the chord-based analog to the Tonnetz, Tymoczko shows that these families of note-based graphs are the geometric duals to other families of chord-based graphs. The geometric representations as well as topologies of these graphs can vary as well. The *Tonnetz* is shown to be toroidal only in the context of three-note chords; adding more notes to the chord corresponds to the increase in number of dimensions of the Tonnetz. Tymoczko also clears up misconceptions that appear in others works. For instance, he notes that "Cohn's early work, like neo-Riemannian theory more generally, often conflated voice-leading efficiency and commontone retention" (10). He also corrects Gollin's conception of a "3D Tonnetz," noting that it is not a "three-dimensional torus" but rather "the twisted product of a circle and a two-dimensional sphere" (42-43). As Tymoczko increases the number of chordal possibilities and harmonic relationships, the Tonnetz explodes into a menagerie of exotic configurations, some of which are higher-dimensional and can only be represented as two- or three-dimensional cross-sections of larger structures. Many of the constructions that other theorists had concocted as extensions or variations of the Tonnetz are shown by Tymoczko to be similar or even isomorphic to specific cases of the generalized Tonnetz. By first drawing connections between many disparate structural ideas in previous Neo-Riemannian scholarship concerning the Tonnetz and then clarifying them through mathematical reasoning, Tymoczko reifies his argument for a generalized Tonnetz as a theoretical space that all these structures inhabit and in which they interrelate.

Tymoczko is a music theorist, composer, and professor at Princeton University.

11. ——. "Generalizing Musical Intervals." *Journal of Music Theory* 53, no. 2 (Fall 2009): 227-254. https://www.jstor.org/stable/40925744.

This article by Tymoczko critiques the idea of musical intervals as introduced in the work of David Lewin, specifically from a mathematical perspective. In music theory, if these intervals are to be interpreted at all from a mathematical viewpoint, the usual perspectives include group theory and/or linear and abstract algebras. Tymoczko instead takes differential geometry as a starting point: a mathematical field that views geometric surfaces as continuous transformations of tangential vectors. From this viewpoint, Tymoczko finds limitations in previous music theoretic conceptions of interval, including works by Cohn, Douthett, Gollin, Lewin, and his own earlier works. Though the *Tonnetz* is only mentioned once, in a footnote, Tymoczko's concerns are foundational, and should be confronted if abstracting the *Tonnetz* to musical situations outside the domain of 12edo theory.

 Wild, Jonathan. "Genus, Species and Mode in Vicentino's 31-tone Compositional Theory." Music Theory Online 20, no. 2 (June 2014): 1-19. https://mtosmt.org/issues/mto.14.20.2/mto.14.20.2.wild.html.

This article by Jonathan Wild is the most thorough published discussion of the melodic pitch structures implied by Vicentino's attempts to apply the Ancient Greek genera to his compositions. After a brief introduction, Wild gives a thorough yet concise explanation of how quarter-comma mean tone yields an implied interval called the enharmonic diesis, the difference between three just major thirds and an octave, or equivalently, the difference between a wolf fifth and a tempered fifth in this temperament. Quarter-comma mean tone limited to twelve pitch classes yields an unusable wolf fifth at its outer extremes. However, by extending this chain of tempered fifths to thirty-one pitch classes, the remaining fifth is close to just and closes the system. Wild demonstrates how the difference between this tuning system and 31edo is negligible, and so the latter may be substituted seamlessly for the former in analysis.

Inspired by Boethius, Vicentino found a lack of expression in the church modes that had been composed from diatonic tetrachords and sought to expand expressive possibilities by implementing the other two Greek genera: the chromatic and enharmonic tetrachords. Wild demonstrates how the basics of Vicentino's tuning theory are sound in that the tetrachords map seamlessly into 31edo. However, Wild's analysis also shows that attempting to restrict melodic intervals to the confines of a single tetrachord results in severe restrictions in available triadic voicings when using the chromatic tetrachord, and a total breakdown in contrapuntal possibilities when using the enharmonic tetrachord. Wild concludes that "by invoking the possibility of mixing genera or species at will, Vicentino has allowed himself to write any melodic line conceivable" at least within the confines of triadic harmony (8). As Wild shows how the genera may have been the inspiration for Vicentino's tuning system, but don't function effectively as organizational units in his music, he alludes to a future article which will find a new theoretical approach. This article has never materialized, but perhaps this investigation was continued by one of his graduate students (see Miller.)

Music Theory Online is a peer-reviewed academic journal, but unlike many other paywalled journals is free for anyone to access online. Jonathan Wild is Associate Professor of Music at McGill University.