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Solar Cells, Lambert W and the LogWright Functions

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***Abstract*—Algorithms that calculate the current-voltage (I-V) characteristics of a solar cell play an important role in processes that aim to improve the efficiency of a solar cell. I-V characteristics can be obtained from different models used to represent the solar cell, and the single diode model is a simple yet accurate model for common field implementations. However, the I-V characteristics are obtained by solving implicit equations, which involve repeated iterations and inherent errors associated with numerical methods used. Some methods use the Lambert W function to get an exact explicit formula, but often causes numerical overflow problems. The present work discusses an algorithm to calculate I-V characteristics using the LogWright function, a transformation of the Lambert W function, which addresses the problem of arithmetic overflow that occurs in the Lambert W implementation. An implementation of this algorithm is presented and compared against other algorithms in the literature. It is observed that in addition to addressing the numerical overflow problem, the algorithm based on the LogWright function offers speed benefits while retaining high precision.**

***Index Terms*—Solar Cell, Photovoltaic Cell, Single Diode Model, Lambert W function, LogWright function, I-V characteristics**

I. Introduction

SOLAR cells have been extensively used for decades to convert light energy into electricity through the photovoltaic effect. They are one of the major sustainable options for generating electricity, owing to its benefits like long equipment lifespan and absence of harmful emissions. Solar cells find their applications in electric cars, water-heating, lighting systems, telecommunication towers and so on.

A solar panel comprises many solar cells arranged in series and parallel configurations. The output of the individual cells may vary due to shading in the panel, or due to failure or poor performance of some cells due to aging. A defect in a string (series) of cells can cause loss of current through the whole string. This can be due to a shadow, not just a semi-permanent characteristic of the cell. For instance, a building chimney can shade a part of a roof panel, with the shadow moving slowly across the panel during the day. Shading can change even faster in case of a car with a panel on its roof being driven along a tree-lined avenue. Dust in the air, swirling sand deposited on a roof panel, can cause even more rapid changes in shading. Some parts of the panel receive full sunlight and keep pumping out energy which must go somewhere. When not provided with a path to flow, that energy gets dissipated as heat, possibly damaging the cells. This brings the need to perform real-time load balancing in order to minimize losses. To perform such calculations, it is essential to develop mathematical models and simulate the behavior of solar cells using the same.

Solar cells are modeled in different ways based on the diode configuration. The single diode model is extensively used to represent solar cells, owing to its simplicity and accuracy in many cases. The single diode model is described by an implicit equation which relates the current I and the voltage V in terms of the cell parameters as described in literature [1]. Active load balancing aims at establishing the optimal operation (at the maximum power point, when the product of current and voltage is maximized). Load balancing requires calculations, and in some cases, there is a need to solve an implicit equation by repeated iteration. Note that mathematical models used to characterize the voltage-current relationship of solar cells are based on approximations. Solar cells are not really single diode circuits with a series and a bypass resistor. Therefore, a practical implementation should also involve a history of past behaviour of the panel as experienced in the field. Ideally, each field installation would have its own computer which runs the model, keeps history, and communicates with the load equipment. These field installations consist of special-purpose hardware with small compute power, and using explicit formulas to calculate the I-V characteristics of the Solar Cell is much more efficient that using implicit formulas. Moreover, large errors in the output due to approximations while solving an implicit equation [2] are also avoided.

Jain and Kapoor [3] gave an exact explicit formula for V as a function of I using the Lambert W function. Since then, many publications have proposed the use of the Lambert W function for tasks such as extraction of I-V characteristics,

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parameter estimation and maximum power point tracking. This brought about the need to develop fast and efficient algorithms to evaluate the Lambert W function. Batzelis et al. [4] compare a variety of explicit non-iterative methods that use approximations to quickly evaluate the Lambert W function. Toledo et al. [5] propose using an approximation of the Lambert W as a seed for the iterative process, so that one can choose to use the approximation as is and trade-off speed with precision if needed. Nguyen et al. [6] propose a convex programming approach to accurately evaluate the Lambert W function.

However, existing literature reports arithmetic overflow exceptions when using the Lambert W function to evaluate I-V characteristics. For instance, at higher shunt resistance (used for enhancing the efficiency of solar cells), for a current $I = 0$ (open-circuit), the argument of the W-function can be 4.59×10^{1141}. However, the maximum representable magnitude in IEEE-754-2008 compliant double precision is only about 10^{323} [7]! This makes it difficult for this formula to be used in hardware implementations of load balancing algorithms. To get around this issue, Batzelis et al. [4] propose a way to evaluate the Lambert W function by expressing the function argument as $x = ae^b$ and using the parameters a and b instead of using x to evaluate the function. While such an implementation solves the overflow issue, it would not help when the user wishes to calculate the derivative of the I-V characteristic.

Hence, it is desirable to replace the equation of Jain and Kapoor with an equation that: (i) is exact, analytic, explicit; (ii) is robust when used in calculations; (iii) does not pose a risk of arithmetic overflow; (iv) is desirable for lab studies, industrial applications, and field installations (load balancing); (v) is suitable for programming in Fortran or C or for microcontrollers; and (vi) reduces possible cancellations which cause loss of significant digits. A method proposed by Roberts and Valluri [8] that uses the LogWright function, also called the *g-function*, helps address the overflow problem caused by the Lambert W function and satisfies all the requirements posed above. The LogWright function has previously been used for prediction of conversion efficiency and real-time maximum power in photovoltaic modules [9].

In this paper, an approach to calculate the I-V characteristics of a solar cell using the LogWright function is discussed and compared with other methods proposed in the literature that use the Lambert W function. In section II, the single diode model is described and both the implicit and explicit equations that relate the output current and voltage are presented. In section III, the LogWright function is introduced and the SDM equations in terms of the LogWright function are obtained. An algorithm to evaluate the LogWright function, which was proposed by Roberts [10], is discussed in section III-A. Two approaches to compute the I-V characteristics using LogWright function are discussed in section IV, and the performance of their implementations is compared with other algorithms proposed in the literature. Lastly, section V concludes the comparative study and lists the scope for future work.

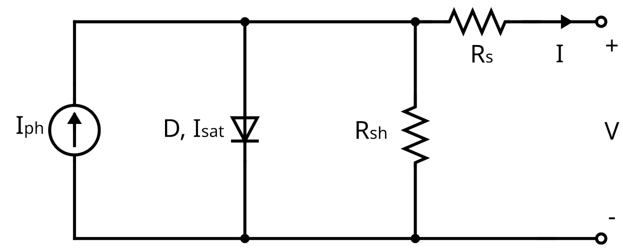

Fig. 1. Circuit representing the Single Diode Model of a Solar Cell

II. Single Diode Model of Solar Cell

The Single Diode Model (SDM) represents a solar cell using a constant current source, a diode, a series and a bypass resistor, and it approximates an inorganic solar cell quite well. The schematic of the SDM is shown in fig. 1. Modeling a solar panel using the SDM involves adjusting the model parameters, which include resistance and current values, such that the I-V characteristics of the model match that of the solar panel.

A solar panel with a string of N_s cells is modelled using the SDM, and the case of a single cell is handled as $N_s = 1$. In either situation, the output current of the solar cell I is given by eq. (1) [1].

$$I = I_{ph} - I_{sat}\left(\exp\left(\frac{V + IR_s}{a}\right) - 1\right) - \frac{V + IR_s}{R_{sh}} \quad (1)$$

where V is the output voltage of the solar panel, I_{ph} is the photocurrent of the panel, I_{sat} is the saturation current of the diode in the SDM and a is a constant defined as $a = \eta N_s V_{Th}$. Here, η is the ideality factor of the diode, and $V_{Th} = \frac{kT}{q}$ is the thermal voltage (k is the Boltzmann constant, T is the temperature and q is the charge of an electron). One can obtain an explicit expression for I using the Lambert W function [3], which can be used to calculate I if all other parameters are known.

$$\begin{aligned} I &= \frac{1}{R_{sh} + R_s}(R_{sh}(I_{ph} + I_{sat}) - V) \\ &- \frac{a}{R_s} W_0\left(\frac{I_{sat} R_{sh} R_s}{a(R_{sh} + R_s)} \exp\left(\frac{R_{sh}(R_s(I_{ph} + I_{sat}) + V)}{a(R_{sh} + R_s)}\right)\right) \end{aligned} \quad (2)$$

Similarly, an explicit expression can be obtained for the output current V as well.

$$\begin{aligned} V &= R_{sh}(I_{ph} + I_{sat}) - (R_{sh} + R_s)I \\ &- aW_0\left(\frac{I_{sat} R_{sh}}{a} \exp\left(\frac{R_{sh}}{a}(I_{ph} + I_{sat} - I)\right)\right) \end{aligned} \quad (3)$$

Either of eq. (2) or eq. (3) can be used to obtain the I-V characteristics of the solar panel, by manually varying V or I respectively. However, such computations are often impractical as numerical overflows tend to occur during calculations. For example, the argument of the Lambert W function in eq. (3) can reach the order of 10^{1100}, which cannot be represented by

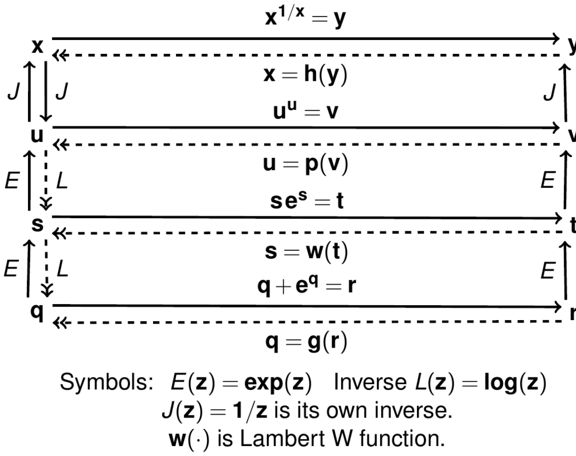

Fig. 2. The Lambert W Function Ladder [8] [13]

a IEEE-754-2008 standard hardware floating point in double precision (which has a limit around 10^{323}). Such overflows occur due to the presence of exponentials in the argument of the Lambert W function. Such an overflow problem can be addressed using the LogWright function.

III. The LogWright function

The LogWright function, also called the g-function [10], [11], is the logarithm of the Wright ω function [12]. It addresses the overflow problem by mathematically modifying the Lambert W function such that intermediate arguments with large numerical magnitudes are avoided. The following shows the relation between the LogWright function $g(x)$ and the Lambert W function.

$$g(x) = \log\left(W(e^x)\right) = x - W(e^x). \tag{4}$$

Note that $\log$ denotes the natural logarithm (to base e). The LogWright function can be obtained from the Lambert W function by taking the logarithm of the function parameters. However, a more efficient way to compute the LogWright function is discussed in section III-A. This logarithmic transformation prevents extremely large numbers from occurring during intermediate calculations. The Lambert W ladder [8] [13] shown in fig. 2 gives a better understanding of how the LogWright function is obtained from the Lambert W function. In this diagram, the dashed lines with double arrowheads represent multi-valued functions.

Eq. 2 can be rewritten in terms of the LogWright function as

$$I = \frac{a}{R_s}\left(g(u(V)) - \log\left(\frac{I_{sat}R_s}{a(1+\frac{R_s}{R_{sh}})}\right)\right) - \frac{V}{R_s} \tag{5}$$

where $u(V)$ is

$$u(V) = \log\left(\frac{I_{sat}R_sR_{sh}}{a(R_s+R_{sh})}\right) + \frac{R_sR_{sh}(I_{PV}+I_{sat})+VR_{sh}}{a(R_{sh}+R_s)}. \tag{6}$$

Similarly eq. (3) can be rewritten as

$$V = a\left(g(v(I)) - \log\left(\frac{I_{sat}R_{sh}}{a}\right)\right) - IR_s \tag{7}$$

where $v(I)$ is

$$v(I) = \log\left(\frac{I_{sat}R_{sh}}{a}\right) + \frac{(I_{PV}+I_{sat}-I)R_{sh}}{a}. \tag{8}$$

Note that certain terms in eqs. (5) to (8) have different signs from the equations derived by Roberts and Valluri [11], owing to the reversal in the direction of output current convention (as seen in fig. 1).

A. Algorithm to evaluate the LogWright function

The LogWright function is evaluated using the following 3-step process, as proposed by Roberts and Valluri [10]:

1) Make an initial (crude) estimate of y_0 in the following manner:
   - If $x \le -e$, take $y_0 = x$.
   - If $x \ge e$, take $y_0 = \log x$.
   - If $-e \le x \le e$, take $y_0 = -e + \frac{1+e}{2e}(x+e)$ (linear interpolation between points (-e,-e) and (e,1)).
2) Refine the estimate y_0 by calculating
   $$y_1 = y_0 - \frac{2(y_0+e^{y_0}-x)(1+e^{y_0})}{2(1+e^{y_0})^2-(y_0+e^{y_0}-x)e^{y_0}}$$
   This iteration formula is referred to as Halley's method and has cubic convergence (Note that the Newton Raphson method was used in [5], which has quadratic convergence).
3) Perform additional iterations to get a better approximation
   $$y_n = y_{n-1} - \frac{2(y_{n-1}+e^{y_{n-1}}-x)(1+e^{y_{n-1}})}{2(1+e^{y_{n-1}})^2-(y_{n-1}+e^{y_{n-1}}-x)e^{y_{n-1}}}$$

In the interest of computational efficiency, it is desirable to calculate e^{y_0} in step 2 and $e^{y_{n-1}}$ in step 3 only once and reuse the calculated values. This prevents repeated calls to the exponential function and speeds up the algorithm. Also, one can avoid unnecessary iterations with careful design in order to improve efficiency. In most practical application situations, only one iteration of step 3 is needed as y_2 is accurate to about 8 significant digits. This depends upon the voltage levels present in the equipment.

IV. Experiments and Results

One can obtain the I-V characteristics of a single diode model using the LogWright function by following one of the two approaches:

1) **I-approach**: Solve for current by using eq. (5) and varying the voltage values.
2) **V-approach**: Solve for voltage by using eq. (7) and varying current values.

Both the above approaches require evaluating the LogWright function. These approaches, along with the algorithm to evaluate the LogWright function, are implemented in the Python programming language and compared with other methods used to evaluate I-V characteristics. We compare the LogWright-based approaches against the following methods.

- The SciPy [14] implementation of the Lambert W function, which is based on the algorithm proposed by [15].

TABLE I
PARAMETER VALUES OF DIFFERENT SINGLE DIODE MODELS CONSIDERED FOR THE COMPARATIVE STUDY.

Parameters (Units)	Set 1	Set 2	Set 3
I_{ph} (A)	15.88	1.032	3.654
I_{sat} (A)	7.44×10^{-10}	2.513×10^{-6}	3.999×10^{-21}
R_s (Ω)	2.04	1.239	2.69
R_{sh} (Ω)	425.2	744.714	2329
a (V)	14.67	1.3	0.516
I_{sc} (A)	15.804	1.031	3.650
V_{oc} (V)	348.1	16.775	24.893
Parameters (Units)	**Set 4**	**Set 5**	**Set 6**
I_{ph} (A)	0.578	0.761	4.802
I_{sat} (A)	1.34×10^{-10}	3.107×10^{-7}	4.016×10^{-7}
R_s (Ω)	0.0127	0.037	0.5906
R_{sh} (Ω)	612	52.89	1167
a (V)	0.0118	0.039	0.037
I_{sc} (A)	0.578	0.760	1.006
V_{oc} (V)	0.262	0.573	0.603

TABLE II
PERFORMANCE ANALYSIS OF ALGORITHMS USING THE I-APPROACH ON SOLAR CELL SDMS WITH PARAMETER SETS TAKEN FROM TABLE I.

Method	Mean time (μs)	Median time (μs)	RMSE
Parameter Set 1			
Lambert W	1704	1696	—
Hybrid	813	805	2.69×10^{-3}
Toledo et al.	992	978	1.28×10^{-15}
LogWright	874	865	1.44×10^{-14}
Parameter Set 2			
Lambert W	1695	1678	—
Hybrid	810	805	1.11×10^{-4}
Toledo et al.	1017	998	2.14×10^{-17}
LogWright	934	923	1.23×10^{-15}
Parameter Set 3			
Lambert W	1798	1793	—
Hybrid	841	839	7.03×10^{-4}
Toledo et al.	1122	1112	3.84×10^{-16}
LogWright	955	950	1.05×10^{-15}
Parameter Set 4			
Lambert W	1768	1759	—
Hybrid	830	821	7.47×10^{-5}
Toledo et al.	986	979	7.02×10^{-18}
LogWright	870	866	1.38×10^{-15}
Parameter Set 5			
Lambert W	1698	1684	—
Hybrid	810	804	1.04×10^{-4}
Toledo et al.	993	980	1.95×10^{-17}
LogWright	905	900	1.01×10^{-15}
Parameter Set 6			
Lambert W	1837	1823	—
Hybrid	847	843	8.42×10^{-6}
Toledo et al.	1150	1145	5.22×10^{-16}
LogWright	1102	1096	6.31×10^{-16}

- Approach proposed by Toledo et al. [5]. This algorithm chooses between two different variants of the Lambert W function depending upon the value of the argument x of the Lambert W function in the interest of precision.
- The hybrid method proposed by Batzelis et al. [4]. This is a non-iterative method that aims to give a quick yet reasonable result at the cost of precision.

To test and compare performance, we generate I-V characteristics using each of the above approaches for 20000 iterations and compare the mean time and median time taken to generate I-V characteristics throughout those iterations. We also calculate the root mean square error (RMSE) of the generated I-V characteristics by considering the output of the SciPy implementation as the baseline. The above metrics are calculated for various parameter values taken from [5] to ensure that we include cases in which the default Lambert W implementation encounters numerical overflow. All the parameter values used are listed in table I. The iterative algorithms are configured such that their output precision is at least 10^{-8}. To ensure a fair comparison, all the above algorithms are implemented and benchmarked in Python (version 3.11.3) on a machine with AMD Ryzen 7 5800H (3.2 GHz) processor and 16GB DDR4 RAM running the Manjaro Linux operating system. Our implementation of these algorithms has been posted to the GitHub repository [16].

TABLE III
PERFORMANCE ANALYSIS OF ALGORITHMS USING THE V-APPROACH ON SOLAR CELL SDMS WITH PARAMETERS TAKEN FROM TABLE I. BASELINE LAMBERT W ISN'T INCLUDED FOR PARAMETER SETS 3-6 BECAUSE IT ENCOUNTERS NUMERICAL OVERFLOW DURING CALCULATION.

Method	Mean time (μs)	Median time (μs)	RMSE
Parameter Set 1			
Lambert W	1722	1701	—
Hybrid	776	769	1.76×10^{-2}
Toledo et al.	1072	1060	2.08×10^{-13}
LogWright	1030	1025	4.81×10^{-13}
Parameter Set 2			
Lambert W	1708	1686	—
Hybrid	778	772	1.38×10^{-3}
Toledo et al.	1075	1065	4.81×10^{-14}
LogWright	1046	1036	6.95×10^{-14}
Parameter Set 3			
Hybrid	810	807	—
Toledo et al.	921	915	—
LogWright	883	870	—
Parameter Set 4			
Hybrid	802	797	—
Toledo et al.	917	905	—
LogWright	877	873	—
Parameter Set 5			
Hybrid	775	772	—
Toledo et al.	1079	1071	—
LogWright	1056	1051	—
Parameter Set 6			
Hybrid	813	808	—
Toledo et al.	894	885	—
LogWright	868	862	—

All the performance analysis of the algorithms in terms of mean time, median time and RMSE is summarized in tables II and III. Table II contains the metrics obtained when the I-approach is used to calculate the I-V characteristics for SDMs with parameter sets 1-6 taken from table I, whereas table III contains the metrics when the V-approach is used for the same. As the Lambert W method encounters numerical overflow when calculating the I-V characteristics for models with parameters sets 3-6 using the V-approach, we neither include Lambert W in the results nor calculate the RMSE for any of the algorithms for those parameter sets. From the tables, one can observe that the Hybrid algorithm take the least amount of time to run, in both the I-approach and the V-approach, closely followed by the LogWright method. However, the LogWright method gives an exact solution to

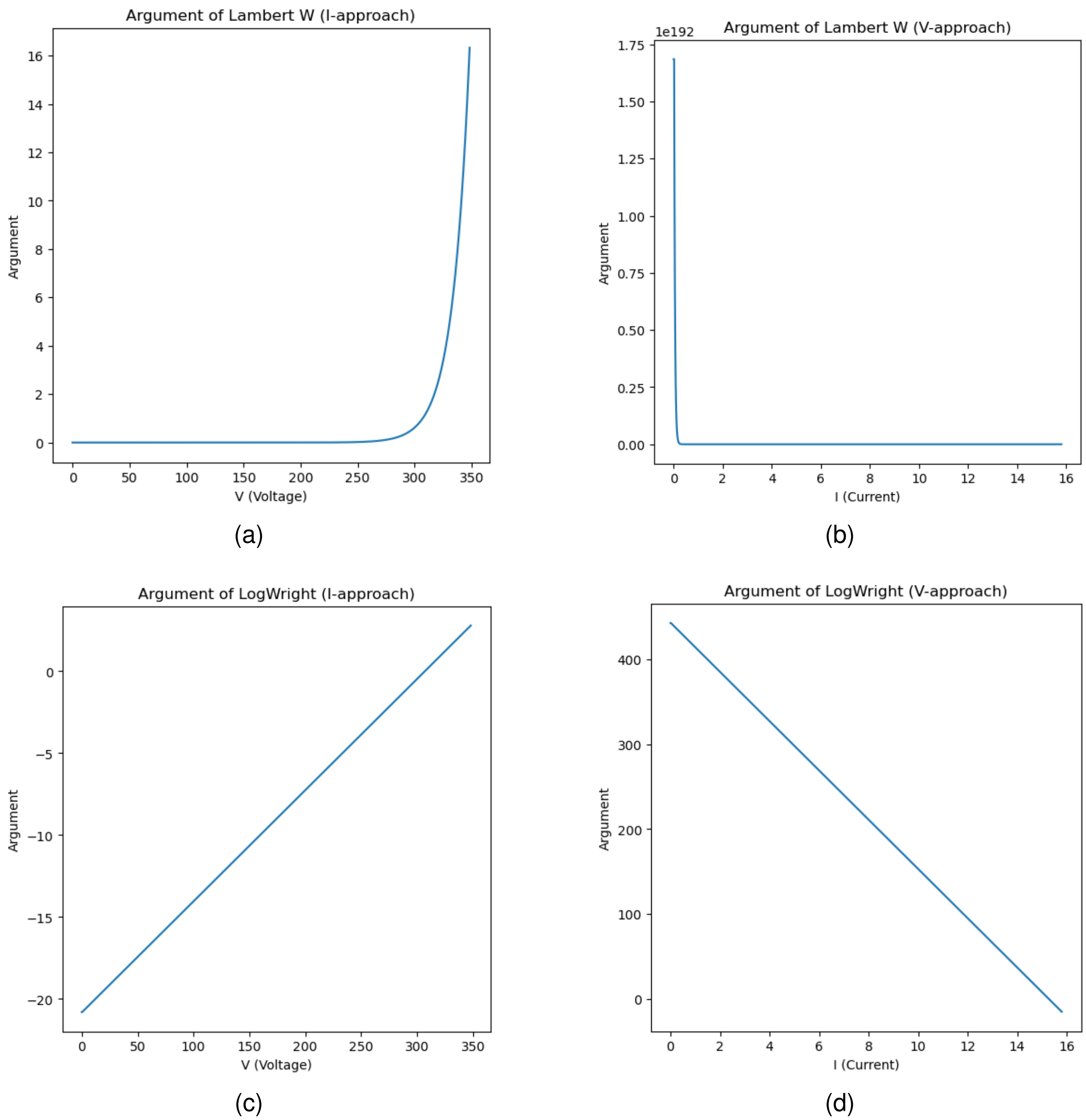

Fig. 3. Comparing arguments of the Lambert W function and the LogWright function when calculating I from V (Figures (a) and (c)) and when calculating I from V (Figures (b) and (d)). Figures (a) and (c) have been plotted from 0 to open circuit voltage, whereas Figures (b) and (d) have been plotted from 0 to the short circuit current for a solar cell, with SDM parameters as $I_{ph} = 15.88A$, $I_{sat} = 7.44 \times 10^{-10}A$, $a = 14.67$, $R_s = 2.04\Omega$ and $R_{sh} = 425.2\Omega$.

the characteristic equation whereas the Hybrid method gives an approximate solution with error of the order 10^{-3}. We also observe that I-approach is faster than the V-approach when working with parameter sets 1, 2, and 5, whereas V-approach seems to be faster than the I-approach when working with parameter sets 3, 4 and 6. No algorithm encounters numerical overflow when using the I-approach on any of the parameter sets. However, there are exceptions to the above observation, specifically in the cases of Hybrid method on parameter set 2 and Logwright method on parameter set 4.

The values of arguments of the Lambert W function and the LogWright function that are obtained when calculating the I-V characteristics of a SDM (with parameters from set 1 of table I) using the I-approach and V-approach are plotted in fig. 3. One can observe that the argument of Lambert W reaches magnitudes of the order of 10^{192} when using the V-approach on the parameter set 1 from table I, whereas its counterpart in the LogWright function takes values that are a little over 400. This plot shows us how the LogWright function maps the arguments of the Lambert W function to the logarithmic scale. Doing so not only helps prevent numerical overflow, but also makes the computation faster.

V. Conclusion

The present work implements an effective method to obtain the current-voltage (I-V) characteristics of a solar cell represented using the single diode model and the LogWright function. This implementation based on the LogWright function, a logarithmic transformation of the Lambert W, solves the numerical overflow problems that one could encounter when using the Lambert W function to solve for the I-V characteristics. It also possesses properties which makes it desirable for field applications such as active load balancing. This implementation is compared with other methods that achieve the same purpose and is shown to be the fastest among the methods that give an exact solution, making it very useful for deployment in field installations.

There is a lot of potential for future work in exploring the relationship between solar cells and the LogWright function. Using the LogWright function can prevent numerical overflow issues wherever the argument given to the Lambert W function is of the form $x = ae^b$. One can study the usage of LogWright function in two-diode models and three-diode models, and also potential applications in modeling organic solar cells by studying [11] as a starting point. Tasks such as parameter estimation of diode models and maximum power point tracking can benefit from using the LogWright function.

Note: A preliminary version of this research was presented at the Canadian Association of Physics June-2022 annual conference, under the title "Solar Cells and the Lambert W Function", presented by S. R. Valluri on behalf of all co-authors.

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