THE EVALUATION AND DESIGN OF TWO MULTIVARIATE QUALITY CONTROL CHARTS USING THE METHOD OF PRINCIPAL COMPONENTS

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PREFACE

The objective of this research is to develop the multivariate (1) Exponential Weighted Moving Average principal component (MEWMAPC) charts and (2) Zone principal component (MZONEPC) charts for monitoring the mean vector of a multivariate process in a realistic environment. The statistically-based models for the evaluation of the out-of-control average run length (OOC ARL) of these charts are developed. The ARL performance comparison among these charts under both classical and optimal design approaches and other existing multivariate control schemes has been performed. Interactive FORTRAN programs have been constructed to help theoreticians and practitioners in evaluation and design of these charts.

I would like to express my sincere appreciation to my major adviser, Professor Kenneth E. Case, for his superb guidance, encouragement and support throughout this research. During my doctoral studies, Dr. Case acted not only as my mentor but a loyal friend as well. Without his inspiration and assistance, I am certain that the accomplishment of this dissertation would not have been possible.

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NOMENCLATURE

ARL = Average Run Length of a control chart

CUSUM = CUmulative SUM

C1 = Criterion 1 used in the design of the MEWMAPC chart (common h and common r)

C2 = Criterion 2 used in the design of the MEWMAPC chart (different h and common r)

C3 = Criterion 3 used in the design of the MEWMAPC chart (Common h and different r)

C4 = Criterion 4 used in the design of the MEWMAPC chart (different h and different r)

D = An integer set represents all the transient states of the Markov chain associated with the IZONEPC chart

h_i = Symmetrical control limit for the ith IEWMAPC or IZONEPC chart

h_i^- = The lower control limit for the ith IEWMAPC or IZONEPC chart (Note that $h_i^- = -h_i$)

h_i^+ = The upper control limit for the ith IEWMAPC or IZONEPC chart (Note that $h_i^+ = h_i$)

IEWMAPC = Individual EWMA Principal Component

IZONEPC = Individual ZONE Principal Component

l_i = The eigenvalue corresponding to the ith principal component.

$\boldsymbol{L}$ = Matrix of eigenvalues

LCL = Lower Control Limit

M = Subgroup size

m^{-}_{i,n_i} = The constant on which the lower bound of the run length probability $p_{i,n_i}(w_i)$ is based

m^{+}_{i,n_i} = The constant on which the upper bound of the run length probability $p_{i,n_i}(w_i)$ is based

MC1 = Multivariate Cusum scheme 1 developed by Pignatiello and Kasunic (1985)

MC2 = Multivariate Cusum scheme 2 developed by Pignatiello and Runger (1990)

EWMA = Exponential Weighted Moving Average

MEWMA = Multivariate Exponential Weighted Moving Average

MEWMAPC = Multivariate Exponential Weighted Moving Average Principal Component

MCUSUM = Multivariate CUmulative SUM

MZONEPC = Multivariate ZONE Principal Component

n = Number of observations (samples or subgroups)

n_i^* = The optimal run number of the i^{th} IEWMAPC or IZONEPC chart

n^- = Minimum value of n_i^*, for all $i=1,2,...,p$

n^+ = Maximum value of n_i^*, for all $i=1,2,...,p$

N' = Random variable represent the run length of the MEWMAPC or MZONEPC chart

N_i = Random variable represents the run length for the i^{th} IEWMAPC or IZONEPC chart

OOC = Out-Of-Control

p = The total number of characteristics monitored in a multivariate process

$p_{i,n_i}(w_i)$ = The cumulative probability that the run length of the i^{th} IEWMAPC or IZONEPC chart, starting from the initial state w_i, is greater than a given number n_i

w_i = The initial value of the i^{th} IEWMAPC or IZONEPC chart

r = The weighing factor or smoothing constant of an EWMA control chart

r_i = The weighing factor for the i^{th} IEWMAPC chart or for the EWMA chart monitoring the i^{th} characteristics.

$\boldsymbol{S}$ = Variance and covariance matrix for sample or subgroup average

$\boldsymbol{Y}$ = EWMA of the i^{th} principal component for the t^{th} subgroup average or mean

$\boldsymbol{u}_i$ = The i^{th} eigenvector

$\boldsymbol{U}$ = Matrix of eigenvectors

UCL = Upper Control Limit

$\bar{\boldsymbol{x}}_i$ = A random vector represents the i^{th} subgroup mean

$\bar{\boldsymbol{X}}$ = A random vector represents the subgroup mean

$\boldsymbol{Y}$ = A random vector represents the set of principal components.

α = The probability of type I error

θ = The direction of the shift

λ = The noncentrality parameter

μ_0 = The in-control or on-target mean of an univariate process

μ_1 = The OOC mean of an univariate process

$\boldsymbol{\mu}_0$ = The in-control or target mean vector of a multivariate process

$\boldsymbol{\mu}_1$ = The OOC mean vector of a multivariate process

Σ = The variance and covariance matrix of a multivariate process

CHAPTER I

THE RESEARCH PROBLEM

Purpose

The control chart is one of the most powerful and widely used process control tools in industry. It was introduced in 1931 by Dr. Walter Shewhart. The standard control chart is designed to detect the departure of the process level or variation from its standard.

Since the early 1930's, practitioners have been looking for new and better tools for statistical process control. In this regard, many process control techniques have been developed, such as Page's cumulative sum chart (CUSUM), Wortham and Ringer's exponential weighted moving average chart (EWMA) and recently Jaehn's Zone control chart. These univariate quality control charts have experienced tremendous success in improving product quality in various manufacturing industries.

Another important field of statistical quality control is in the simultaneous control of two or more related variables when the quality depends on the joint effect of these variables rather than on the effect of each variable separately. The widely recognized pioneer work in this area was developed by Harold Hotelling on testing of bombsights during 1947 to 1951.

He used the T^2 control chart as a tool for monitoring the overall quality of a flight sight or lot by summing over the appropriate number of bombs involved. Unfortunately, the field was hampered by the lack of adequate computational resources.

With the advance of powerful computing capabilities and the growth of interest in multivariate quality characteristics, the stage has been set for renewed interest in developing multivariate quality control techniques. During the 80's, significant effort was directed towards this area resulting in the establishment of many multivariate control charts.

One intuitively appealing method to monitor the quality of a multivariate process is to apply a Shewhart chart to each characteristic separately. The underlying process is consider to be out-of-control (OOC) if one of these charts signaled. One problem related to this approach is that the in-control ARL and OOC ARL given a specified mean vector shift of the composite set of the chart cannot be determined analytically due to the existence of the correlation among those characteristics.

Another method is to control all the characteristics jointly. Most of the existing multivariate control charts are developed under this methodology. The statistic used is either the T^2 type or χ^2 type. The major advantage of using these statistics is the proper reflection of the correlation structure of the characteristics being studied. Another advantage is the ease of calculation and simple construction of these control charts. They require only a comparison of a sample T^2 or χ^2 value with a single control limit. However, the major drawback in using these control charts is that the OOC signal provides no indication leading to the identification of the original OOC characteristic(s).

Another useful tool suggested by Jackson and Morris (1957) for multivariate quality control is the method of principal components. The basic idea of the method of principal components is to perform principal axis rotation on original intercorrelated characteristics and transform them into new uncorrelated variables.

Jackson (1959, 1980, 1985) uses either Shewhart 3-sigma control limits or 95 percent control limits to control the mean of each principal component. He shows that the principal component chart can be an effective control tool for multivariate process control. He also reiterates that the principal component chart provides information that might lead to the identification of the OOC characteristic(s).

The purpose of this research is to design and evaluate multivariate statistical control procedures employing

(1) EWMA control charts

(2) Zone control charts

on principal components for monitoring the shift in the process mean vector if the known process variance covariance matrix remains unchanged during the production process.

The Problem of the Current Design and Evaluation of Multivariate Quality Control Procedures

The use of the statistical control chart to monitor and control a production process was first introduced by Dr. Walter Shewhart in 1931. Shewhart described that the purposes of the control chart are:

(1) to understand the inherent nature of a process and identify the goal or standard of the process.

(2) to use as a tool for attaining that goal.

(3) to judge whether the goal has been changed.

The multivariate quality control chart shares the same principles and goals as described by Dr. Shewhart. Instead of monitoring a single characteristic of the output of a process, the multivariate control chart simultaneously monitors several correlated characteristics that are important and contributive to the quality of the product.

The three well-known multivariate quality control charts for controlling the mean of a multivariate Normal process that have been fully developed are: Hotelling's T^2 (or Chi-square) control chart, the Multivariate Cumulative Sum (MCUSUM) control chart, and the Multivariate Exponentially Weighted Moving Average (MEWMA) control chart. Analogous to univariate control charts, the effectiveness and performance of these multivariate control charts are measured by the average run length (ARL). The ARL is defined as the expected number of subgroups taken until a signal indicating a process change in the control chart.

Under the assumption that the observations within and between subgroups are random samples from a multivariate Normal process, the ARL of the multivariate T^2 or χ^2 chart for the subgroup mean vector can be easily determined since the underlying run length distribution is geometric. However, there is no simple analytical or numerical solution for the ARL of the MCUSUM or the MEWMA control charts. This is because the statistic plotted on the MCUSUM or the MEWMA control chart is derived not only from the most recent observation but from the previous observations. Therefore, the ARL of these control charts must be determined by simulation.

Previous research shows that the MCUSUM and the MEWMA control charting techniques possess better statistical performance than the T^2 or χ^2 control scheme. However, the fact that the statistical performance of these charts can be evaluated only by simulation makes the MCUSUM and the MEWMA control charts impractical to use in industry.

The problem described above leads to the need for further research on multivariate quality control technology. A possible solution is to use the method of principal components.

Research Objectives

The primary objective of this research is listed as follows.

Objective: Develop multivariate

(1) EWMA principal component charts

(2) Zone principal component charts

under either a classical design or a statistically optimum design approach as an alternative to various types of multivariate control charts for monitoring the mean vector of a multivariate process in a realistic environment. The statistically optimal control chart is defined as a control chart with a fixed in-control ARL which has the smallest ARL for a specified or predetermined shift in the mean vector.

To accomplish this objective, several subobjectives must be met. The subobjectives are:

(1) Develop a statistically-based model for evaluating the performance of the multivariate

a. EWMA principal component chart.

b. Zone principal component chart.

(2) Develop an algorithm to obtain the statistically optimal design of the multivariate EWMA and Zone principal component control charts under a predetermined shift in the process mean vector.

(3) Develop computer programs to evaluate the statistical performance of the multivariate EWMA and Zone principal component control charts and to assist in the classical and statistically optimal designs of these charts.

(4) Investigate and compare the classical and optimal design of the multivariate EWMA and Zone principal component charts with other existing multivariate quality control charts.

(5) To conduct sensitivity analysis to study the effects of the process parameters, which include the correlation structure and the mean vector shift, on the resulting statistical performance under the optimal design of both multivariate principal component control charts.

Contributions

This research provides benefits to both theoreticians and practitioners. This study becomes the first of its kind to provide:

(1) A design of the multivariate EWMA principal component control chart.

(2) A design of the multivariate Zone principal component control chart.

(3) Analytical models to evaluate numerically the ARL associated with both multivariate principal

component control charts.

(4) Computer programs to assist the user in the analysis of the statistical performance of the proposed principal component charts and in the designing of an optimal multivariate EWMA or Zone principal component control chart given that the user specifies the in-control ARL and a specific shift in the mean vector.

(5) Statistical performance comparisons among the χ^2, the MCUSUM, the MC1, the MEWMA, and the proposed multivariate principal component control charts under classical design approach.

(6) Statistical performance comparison among the optimal MEWMA and MEWMAPC control charts and among the optimal MEWMAPC and MZONEPC control charts.

All of these are new developments to help practitioners in the evaluation the statistical performance and design of the multivariate EWMA and Zone principal component control charts.

CHAPTER II

LITERATURE SURVEY

The concept of variables control charts was first introduced by Dr. Walter Shewhart (1931). Since then, various extensions and modifications of standard Shewhart quality control charts have been developed.

Most of the existing control charting techniques, whether univariate or multivariate in nature, are based on three important assumptions as follows:

(1) The distribution of quality characteristic(s) to be measured is assumed either univariate or multivariate Normal.

(2) The mean and variance of the measured quality characteristic are usually assumed relatively stable at the target until a shock occurs that changes the level of the process. Therefore, the state of the process can be classified as either at an in-control state or at an out-of-control state.

(3) Successive subgroups and the observations within subgroups are assumed to be independent.

The fundamental idea behind the control chart is that there are two sources of variation in the quality of a product: chance causes and assignable causes. Dr. W. Edwards Deming

(1982) refers to these as common cause and special cause variation, respectively. The process, under the influence of only common cause variations, is considered stable and predictable. The process, under the influence of special causes, is considered unstable. Thus, a search for one or more assignable causes will be conducted and the corrective action will be enforced.

Alt (1977) divides the practice of control charting techniques into two phases. Phase I of the control chart is used for analyzing past data for a lack of control and to assist in establishing control charts when no standards are given. On the other hand, phase II is used to detect any departure of the underlying process from the standard value, including the mean and the variability. The primary attention of this research is directed towards phase II control charts for the mean vector of the multivariate Normal process.

This chapter provides an overview of the existing univariate and multivariate quality control techniques that relate to the three principal component control charts under consideration. The chapter is divided into three sections:

(1) statistical design of univariate control charts.

(2) statistical design of multivariate control charts.

(3) the method of principal components.

Statistical Design of Univariate Control Charts

The EWMA Control Charts

The exponential smoothing or the exponentially weighted moving average (EWMA) techniques have found widespread application in economics, inventory control, and forecasting. Brown (1959) and Muth (1960) use this approach in short-term forecasting of sales and inventory control. Roberts (1959) develops a control chart using the EWMA (there called the geometric moving average) to control a process mean. The EWMA techniques give the most recent observation the greatest weight with all previous observation's weights decreasing in a geometric (exponential) progression from the most recent back to the first. To demonstrate the EWMA technique, suppose that subgroups of size M are taken successively and the subgroup means $\bar{x}_1, \bar{x}_2, \ldots$ are calculated. The successive values of the EWMA statistic generated by subgroup mean $\bar{x}_t$ are:

$$\text{EWMA}_t = (1-r)\,\text{EWMA}_{t-1} + r\bar{x}_t, \quad 0<r\leq 1, \quad t=1,2,\ldots$$

Here r is a smoothing constant and EWMA_t is the value of the EWMA after observation t, where the subscript t represents the observation number as well as an index of a point in time.

Roberts also presents a graphical procedure for generating the EWMA. Roberts evaluates the mean action time (MAT), also known as the ARL, of the EWMA control chart by simulation and provides several MAT curves for various smoothing constants (r). He also compares the properties of control chart tests based on the EWMA with tests based on ordinary moving averages. Roberts concludes that tests based on the EWMAs compare most favorably with multiple run tests and moving average tests with regard to simplicity and statistical properties. Freund (1962) uses the MAT to compare the ability of the CUSUM chart, the EWMA chart and the acceptance control chart to detect process mean shifts. He suggests the use of the MAT rather than the Operating Characteristic (OC) Curve to determine the power of the control charts.

Wortham (1972) declares that the EWMA control chart is a possible solution for the monitoring and controlling of continuous flow processes. Wortham and Heinrich (1972) also apply the EWMA to individual measurements. They point out that this approach may be justified when the cost of inspection is high or when expensive destructive testing is involved. Ng and Case (1989) develop methodologies to construct the EWMA control chart used for monitoring the sample means (EWMASM), sample range (EWMASR), individual observations (EWMAID) and moving range (EWMAMR). They provide extensive tables of factors for constructing the control limits of these charts.

Hunter (1986) reviews the characteristics of the EWMA control chart. He claims that the EWMA chart is easy to plot, easy to interpret, and its control limits are easy to obtain. Perhaps more important, the EWMA can be used as a method for establishing real-time dynamic control in industrial processes.

He also points out that the EWMA can be viewed as a compromise between Shewhart and CUSUM charting techniques.

Lucas and Saccucci (1990) propose several enhancements to EWMA control schemes. They are:

(1) The fast initial response (FIR) feature that makes the scheme more sensitive at start-up.

(2) A combined Shewhart-EWMA scheme that provides protection against both large and small shifts in the process mean.

(3) A robust EWMA scheme that gives extra protection against outliers.

They show that large values of the smoothing constant r are optimal for detecting small shifts.

Domangue and Patch (1991) develop the omnibus EWMA control schemes that are capable of detecting changes in both mean and standard deviation of the process. The omnibus EWMA statistic is based on the exponentiation of the absolute value of the standardized subgroup mean. Montgomery and Mastrangelo (1991) present methods for applying statistical control charts to autocorrelated data. They show the EWMA statistic can be used as an approximating procedure for monitoring autocorrelated data.

The use of EWMA to control process variance is first introduced by Wortham and Ringer (1971). They calculate the EWMA statistic on sample variance and use the fact that the limiting distribution of the statistic is Chi-square to construct the control limits. Sweet (1986) suggests two models for the construction of the coupled EWMA control charts to monitor the mean and the standard deviation or variance of a process, simultaneously. They are the mean absolute deviation model and the square deviation model. These are modifications of the model proposed by Wortham and Ringer (1971). Ng and Case (1989) suggest the use of the EWMA on the sample range. They construct an EWMASR chart to monitor the process variance. Crowder and Hamilton (1992) propose using an EWMA based on the log transformation of the sample variance. They discuss the properties of the log-variance EWMA chart and provide an optimal design strategy. They show that the chart is superior to the usual range chart or s^2 chart in terms of its ability to detect quickly the small increases in the standard deviation of a Normal process.

Two methods that are often used to evaluate the run length distribution of EWMA schemes are the Markov chain and integral equation approaches. Lucas and Saccucci (1990) evaluate the run length distribution using Markov chains. Robinson and Ho (1978) present a numerical procedure using recursive techniques and an Edgeworth expansion for the approximation of the ARL of an EWMA chart. Both one-sided and two-sided ARLs are tabulated for various settings of the control limits, smoothing constant and shift in the level of the process mean. They further show that the results agree with those obtained by Roberts (1959). Waldmann (1986) proposes a general extrapolation method using integral equations to derive the upper bound and lower bound of the run length distribution of either the one-sided or two-sided EWMA schemes. Crowder (1987a, 1987b) replaces the integral equation with a system of linear algebraic equations and solves them numerically using Gaussian quadrature. Then, the ARLs for in-control and different mean shifts of the process can be determined. Hamilton and Crowder (1992) present computer programs for calculating the ARL of the log-variance EWMA chart using the same method as Crowder (1987a, 1987b) did. Gan (1991a) provides a computer program that computes the probability function of the run length N of an EWMA chart. Then, the percentage points of the run length distribution can be obtained from the probability function.

The Zone Control Charts

The Shewhart control chart is known to be insensitive in detecting small to moderate shifts in the process mean. This deficiency can be alleviated by using the supplementary runs rules. The Western Electronic Company (1958), now AT&T, presents four runs rules to improve the sensitivity of the Shewhart control chart. Since then, various runs rules have been proposed and have been used by many companies. Nelson (1984, 1985) collects a set of runs rules for the purposes of convenience and uniformity of application. He points out that the combination of these runs rules in usage will depend on the circumstances. He also claims that the user needs to be alert to any patterns of points that might indicate the presence of special causes.

The application of the runs rules depends heavily on the visual identification of special patterns of points plotted on the control chart. A different technique in identification of the runs from a control chart has been proposed by Imaizumi (1955). He develops a Zone control chart (there, called the band-score control chart) using the sum of scores method to control the temperature of a Coke Furnace at NIPPON KOKAN. He divides the Shewhart control chart spread into 6 equal zones. The zone scores of -3, -2, -1, 1, 2, and 3 are assigned successively from the lowest zone to the highest zone. The critical values of -6 and 6 are equal to the scores of the upper and lower out-of-control zones, respectively. An out-of-control signal is triggered when the cumulative zone score is outside of the range of both critical values. Toad (1958) derives the formula to find the type I error of the Zone control chart using the Markov chain approach and Feller's theorem.

Jaehn (1987a, 1987b, 1987c, 1989) proposes the Zone control chart. He claims that the Zone control chart has several advantages over the Shewhart charts, including (1) ease of construction, (2) elimination of exact data plotting, (3) operator involvement is simplified, (4) control chart tests for process shifts are automatically incorporated, and (5) target and control limit changes are made quickly.

Jaehn's Zone control chart works the same as the band-score control chart with the exception of the values of zone scores assigned. Jaehn assigns zone scores of 1, 2, 4, and 8 to regions that are both above and below the central line. Hendrix (1989) uses simulation to obtain the ARLs of the Zone control charts with different sets of zone scores. He compares the result with the Shewhart X-bar control charts.

The problem of the Zone control chart proposed by Imaizumi, Toad and Jaehn are that the chart gives a high false alarm rate when the process is in a state of statistical control (SOSC). This violates the principle of the control chart in that the false alarm rate should be low (nearly zero) when the process is stable. Therefore, an improvement of the zone control chart is needed.

Fang and Case (1990) mathematically formulate the Zone control chart using a Markov chain. They develop an analytical model to evaluate the ARLs of the Zone control chart and provide suggestions on the improvement of the Zone control charts.

Independently, David, Homer, and Woodall (1990) also employ Markov chains to evaluate the performance of the Zone control charts. They conclude that the assigned zone scores will greatly affect the performance of the Zone control charts. When the zone scores are properly assigned, the Zone control charts outperform, based on the ARLs, the competing Shewhart X-bar control charts with supplementary runs rules.

Statistical Design of Multivariate Control Charts

Introduction

Many quality control operations in industry consist of making more than one type of measurement on a particular inspected product because there is more than one characteristic that needs to be controlled to achieve the quality goal. One common practice to control several characteristics is to use multiple Shewhart control charts. The implicit assumption in this practice is that the characteristics are independent, which is often incorrect.

Jackson (1956) shows that individual or separate control of related variables will result in error of "over" or "under" control. These errors become more pronounced if the correlation between variables is higher. Another problem associated with the use of multiple Shewhart charts to control multivariate correlated variables is that there is no scientific way to evaluate the statistical performance of the joint effect of these control charts.

Hotelling's T^2 Control Charts

Hotelling (1931) generalizes the univariate t statistic as a T^2 statistic for multivariate applications. Since then, the T^2 statistic has been used extensively in the field of multivariate analysis. In 1947, Hotelling developed a control chart using the T^2 statistic for the analysis of bombsight data. The T^2 statistic for a single observation from a p-characteristic process takes the form:

$$T^2 = (\boldsymbol{x} - \bar{\boldsymbol{x}})' \boldsymbol{S}^{-1} (\boldsymbol{x} - \bar{\boldsymbol{x}})$$

where $\boldsymbol{x}$ is a p dimensional column vector representing the observed values from p characteristics, $\bar{\boldsymbol{x}}$ is a p dimensional sample or target mean vector, and $\boldsymbol{S}$ is the p × p sample variance and covariance matrix. The distribution of the T^2 statistic is a function of the number of variables p and the number of observations n used in estimating the variance and covariance matrix $\boldsymbol{S}$. The T^2 distribution is related to the well-known F distribution by the relationship:

$$T^2 = \frac{(n-1)p}{(n-p)} F_{p,n-p,\alpha}$$

where α is the probability of type I error. The quantity on the right-hand side of the equal sign is the upper limit of the T^2 control chart. In working with squared quantities, only an upper limit is required.

Elliptical Control Chart

Jackson (1956) introduces the elliptical control chart. The equation for the ellipse in the bivariate case is,

$$\frac{S_x^2 S_y^2}{S_x^2 S_y^2 - S_{xy}^2}\left[\frac{(x-\overline{x})^2}{S_x^2}+\frac{(y-\overline{y})^2}{S_y^2}-\frac{2S_{xy}(x-\overline{x})(y-\overline{y})}{S_x^2 S_y^2}\right]=T^2$$

$$T^2 = \frac{2(n-1)F_{2,n-2,\alpha}}{n-2}$$

where $F_{2,n-2,\alpha}$ is the upper (100α) percentile of the F distribution with two and n-2 degrees of freedom, both $\overline{x}$ and $\overline{y}$ are the observation means of a two-variate process, S_x and S_y are the standard deviation of variate x and y, respectively, and S_{xy} represents the covariance of x and y.

The values being plotted are the Hotelling T^2 statistics. The statistic for the j^{th} observation is,

$$T_j^2 = (\boldsymbol{x_j}-\overline{\boldsymbol{x}})' \boldsymbol{S}^{-1}(\boldsymbol{x_j}-\overline{\boldsymbol{x}}) .$$

where $\overline{\boldsymbol{x}} = (\overline{x}, \overline{y})'$ is the observation mean vector and $\boldsymbol{x_j} = (x_j, y_j)'$ is the j^{th} observation. A point falling outside the control ellipse is consider an out-of-control condition. Therefore, proper investigation is needed. Jackson also shows that both T^2 and the elliptical charts are equivalent. Ghare and Torgersen (1968), Radharamanan (1986), and Alloway and Raghavachari (1991) show that the elliptical control chart has produced satisfactory results in practical applications involving two variables. There are two drawbacks associated with the operations of the elliptical control chart: (1) the subgroup number is not preserved, and (2) the visual display of the elliptical chart becomes impossible when the number of characteristics under consideration increases to three or more.

The Multivariate Shewhart χ^2

Control Chart

Alt (1973) first introduces the multivariate equivalent of the Shewhart control charts. The univariate Shewhart control chart or x-bar chart can be considered as repeated tests of significance of the hypotheses $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$. The areas above the UCL and below the LCL correspond to the rejection regions for the likelihood ratio test. Alt extends this fact to the multivariate case. Assume that p-variate random variables are jointly distributed as a p-variate Normal and that a subgroup of size M is selected randomly. If the covariance matrix and mean of the p-variate Normal are known, the likelihood ratio test (Anderson 1984) of $H_0: \boldsymbol{\mu} = \boldsymbol{\mu_0}$ versus $H_1: \boldsymbol{\mu} \neq \boldsymbol{\mu_0}$ will reject the null hypothesis if,

$$M(\bar{\boldsymbol{x}} - \boldsymbol{\mu_0})' \Sigma^{-1} (\bar{\boldsymbol{x}} - \boldsymbol{\mu_0}) > \chi^2_{p,\alpha}$$

where $\bar{\boldsymbol{x}}$ is the vector of subgroup means, $\boldsymbol{\mu_0}$ and Σ are the population mean and covariance matrix and $\chi^2_{p,\alpha}$ is the upper (100α) percentile of the χ^2 distribution with p degrees of freedom. The control limits are therefore defined as,

$$UCL = \chi^2_{p,\alpha}$$

$$LCL = 0.$$

where the statistics plotted are

$$M(\bar{\boldsymbol{x}}-\boldsymbol{\mu}_0)'\Sigma^{-1}(\bar{\boldsymbol{x}}-\boldsymbol{\mu}_0).$$

If this statistic exceeds the upper control limit, the process mean is considered out-of-control and the cause(s) of variation is(are) sought. Since the control limit of this chart is determined by the χ^2 distribution, this chart is often referred as the Chi-square control chart.

When the process parameters $\boldsymbol{\mu}_0$ and Σ are not known, the unbiased estimates of the parameters must be used. Assume that n rational subgroups of size M are taken from the process. Let $\boldsymbol{x}_j$ denote the (p × M) data matrix for subgroup j and $\boldsymbol{x}_{ij}$ denote the i^{th} p-variate vector in subgroup j. Then, the sample mean vector $\bar{\boldsymbol{x}}_j$ and the sample covariance matrices $\boldsymbol{S}_j$ can be calculated. The unbiased estimates for $\boldsymbol{\mu}$ and Σ are given by,

$$\hat{\boldsymbol{\mu}} = \bar{\bar{\boldsymbol{x}}} = \left(\frac{1}{n}\right)\sum_{j=1}^{n}\bar{\boldsymbol{x}}_j, \quad \text{where } \bar{\boldsymbol{x}}_j = \left(\frac{1}{M}\right)\sum_{i=1}^{M}\boldsymbol{x}_{ij}$$

$$\hat{\Sigma} = \boldsymbol{S} = \left(\frac{1}{n}\right)\sum_{j=1}^{n}\boldsymbol{S}_j, \quad \text{where } \boldsymbol{S}_j = \left(\frac{1}{M-1}\right)\sum_{i=1}^{M}(\boldsymbol{x}_{ij}-\bar{\boldsymbol{x}}_j)'(\boldsymbol{x}_{ij}-\bar{\boldsymbol{x}}_j)$$

Alt (1982) shows that the statistic

$$M(\bar{\boldsymbol{x}}_j-\bar{\bar{\boldsymbol{x}}})'\boldsymbol{S}^{-1}(\bar{\boldsymbol{x}}_j-\bar{\bar{\boldsymbol{x}}})$$

is distributed as,

$$\frac{(nMp-np-Mp+p)}{(nM-n-p+1)}F_{p,\,nM-n-p+1}.$$

where $F_{\nu 1,\nu 2}$ is Snedecor's F with $\nu 1$ and $\nu 2$ degrees of freedom. The estimates $\bar{\bar{\boldsymbol{x}}}$ and $\boldsymbol{S}$ should be updated frequently in the early stage but not so often once the process has stabilized.

Alt (1973) also discusses the power or the probability of type II error of the multivariate Shewhart chart (χ^2 chart). He shows that the power depends on $\boldsymbol{\mu}_1$ and Σ only through the value of the noncentrality parameter λ, where

$$\lambda = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)' \Sigma^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) .$$

Alt, Walker, and Goode (1980) investigate the effect, if one of the population standard deviations changes, on the power of the test under the bivariate case. They show that if the two characteristics are positively correlated, the power of the χ^2 test is not a monotonically decreasing function of σ, as is the univariate case. This phenomenon can be explained by Fisherian information theory.

Blank (1988) develops a multivariate x-bar and R chart using the vector sum technique. He claims that the charts are simple to construct and easy to use. Blank calculates the vector sum, which is the Euclidean norm, of each subgroup mean vector. The central line of the control chart for the vector mean is the average of the vector sums and the control limits are determined by either the correlation among different variables or by the standard deviation of the vector sums. However, Blank does not discuss the statistical performance of the chart.

Alloway and Raghavachari (1990) present an approach to construct a trimmed mean multivariate control chart. They develop the trimmed T^2 statistic for testing of the multivariate mean vector. They also study the proposed control chart under the bivariate Normal and bivariate contaminated Normal population using simulation. The results indicate that the Hotelling T^2 method is robust for distributions having slightly heavier tails than Normal. However, for very heavy tails, the proposed trimmed mean method comes closest to the population centers and has a smaller standard error.

The Multiple Univariate CUSUM Control Scheme

Woodall and Ncube (1985) introduce the use of a one-sided or two-sided univariate CUSUM chart to monitor a p-dimensional multivariate Normal process. They assume that the independent p-characteristic random variables $\boldsymbol{x}_j$, $j = 1, 2, \ldots$ are successive samples from a p-dimensional multivariate Normal process with mean $\boldsymbol{\mu}_0$ and variance-covariance matrix Σ. The run length of a one-sided procedure for detecting the positive mean shift of the i^{th} characteristic is:

$$N(i) = \text{MIN}\ \{\ J : T^{+}_{i,J} \geq H^{+}_{i}\ \}.$$

$$\text{where } T^{+}_{i,J} = \text{MAX}\{0,\ T^{+}_{i,J-1} + x_{i,J} - k_i\},\quad J = 1, 2, \ldots \qquad (2.1)$$

In equation (2.1), $x_{i,J}$ refers to the i^{th} characteristic of the J^{th} observation, $T^{+}_{i,J}$ is the upper CUSUM of the ith characteristic after J observations, $H_i^+ > 0$ is the upper decision interval or control limit for characteristic i, and k_i is the reference value for characteristic i. The choice of the reference value k_i depends on the shift in the mean that is considered to be important and needs to be detected quickly for the ith characteristic.

To detect shifts in either direction, the run length of the two-sided CUSUM procedure is defined as:

$$N(i) = \text{MIN} \{ J : T_{i,J}^+ \geq H_i^+ \text{ or } T_{i,J}^- \leq H_i^- = H_i^+ \},$$

$$\text{where } T_{i,J}^- = \text{MIN} \{ 0 , T_{i,J-1}^- + x_{i,J} + k_i \}.$$

and $T_{i,J}^-$ is the lower CUSUM of the ith characteristic after Jth observations. The run length of the multiple CUSUM procedure is defined as,

$$N = \text{MIN} \{ N(1), N(2), \ldots, N(p) \}.$$

Therefore, the process is considered out of control as soon as any one of the multiple CUSUM control charts indicates an out of control signal. This method has two obvious advantages. It is very easy to understand and very easy to implement. However, it has a major disadvantage in that the correlation between the various quality characteristics is not taken into account. Therefore, it is impossible to tell exactly what is the significance level of the test.

The MCUSUM Control Schemes

Pignatiello and Kasunic (1985) propose a method, denoted by MC1, to control the mean of a multivariate Normal distribution. They call it the "Truly Multivariate CUSUM Chart". The CUSUM for the mean vector of the observations in t^{th} subgroup is defined as:

$$\mathbf{C_t} = \sum_{i=t-g_t+1}^{j} (\bar{\mathbf{x}}_i - \boldsymbol{\mu}_0)$$

where $\bar{\mathbf{x}}_i$ is the mean vector of the i^{th} subgroup. Note that g_t is the number of subgroups since the most recent renewal (i.e., zero value) of the CUSUM. Therefore, the average of the difference between the accumulated subgroup average and the target value of the process mean is

$$\left(\frac{1}{g_t}\right)\mathbf{C_t} = \left[\left(\frac{1}{g_t}\right)\sum_{i=t-g_t+1}^{t} \bar{\mathbf{x}}_i\right] - \boldsymbol{\mu}_0 .$$

Consequently, at subgroup t, the multivariate process mean vector can be estimated to be $(1/g_t)\,\mathbf{C_t} + \boldsymbol{\mu}_0$. It then follows that a norm of $\mathbf{C_t}$,

$$\| \mathbf{C_t} \| = (\mathbf{C_t}'\Sigma^{-1}\mathbf{C_t})^{\frac{1}{2}}$$

is a measure of the of the distance of the estimated mean of the process from the target mean of the process. The multivariate CUSUM scheme is constructed by defining $MC1_t$ as

$$MC1_t = \mathrm{MAX}\{\| \mathbf{C_t} \| - kg_t, 0\}$$

and

$$g_t = \begin{cases} g_{t-1}, & \text{if } MC1_{t-1} > 0 \\ 1, & \text{otherwise} \end{cases}$$

where the choice of the reference value $k > 0$ is discussed in detail by Pignatiello and Runger (1990). The scheme operates by plotting $MC1_t$ on a control chart with an upper control limit h. Because the MC1 scheme can not be modeled as a simple stationary Markov chain, Pignatiello and Runger (1990) use a Monte Carlo simulation to evaluate the ARL performance of the scheme.

Pignatiello and Runger (1990) propose a method, denoted by MC2, based on the square of the distance of the each subgroup average from the target mean $\boldsymbol{\mu_0}$ and then accumulate those squared distances. They define the square distance of the i[th] subgroup mean from the target value $\boldsymbol{\mu}_0$ as

$$D_i^2 = (\bar{\boldsymbol{x}}_i - \boldsymbol{\mu}_0)' \Sigma^{-1} (\bar{\boldsymbol{x}}_i - \boldsymbol{\mu}_0) \qquad (2.2)$$

For each i, D_i^2 has a Chi-squared distribution with p degrees of freedom. A one-sided CUSUM can now be defined as:

$$MC2_i = MAX\{0, MC2_{i-1} + D_i^2 - k\}.$$

with $MC2_0 = 0$. The primary difference in the two CUSUM charts is that MC1 accumulates the subgroup mean vector prior to the production of the quadratic forms, while MC2 calculates the quadratic form for each subgroup mean and then accumulates the values of those forms.

Moreover, Pignatiello and Runger (1990) compare MC1 and

MC2 to the multiple univariate CUSUM charts given by Woodall and Ncube (1985) and to the multivariate Shewhart χ^2 charts. The results show that the ARL of the MC1 chart outperforms the other three charts in almost all cases.

Crosier (1988) also presents two multivariate cumulative sum (MCUSUM) quality control schemes. The first CUSUM scheme reduces each observation vector or subgroup mean vector to a T-statistic (the square root of the right hand side of equation (2.2)) and then forms a CUSUM of the T-statistics. Crosier states that a problem with this method is that when a shift of the mean is indicated, the procedure gives no indication of where the shift occurs.

The second method derived by Crosier is a two-sided vector-value CUSUM scheme. He shrinks the updated CUSUM toward zero after each observation. The shrinkage is performed by multiplication rather than by addition or subtraction. Crosier defines the statistic G_i, the CUSUM after the i^{th} subgroup mean vector, as

$$G_i = [(\mathbf{v}_{i-1} \quad \bar{\mathbf{x}}_i - \boldsymbol{\mu}_0)'\Sigma^{-1}(\mathbf{v}_{i-1} \quad \bar{\mathbf{x}}_i - \boldsymbol{\mu}_0)]^{\frac{1}{2}}$$

$$\text{where } \begin{cases} \mathbf{v}_i = \mathbf{0}, & \text{if } G_i \le k \\ \mathbf{v}_i = (\mathbf{v}_{i-1} + \bar{\mathbf{x}}_i - \boldsymbol{\mu}_0)[1 - (k/G_i)], & \text{otherwise} \end{cases}$$

Note that $\boldsymbol{\mu}_0$ is the target value, $k > 0$ is the reference value of the scheme or the allowable slack in the process, and G_i is the generalized length of the CUSUM vector before shrinking.

Consequently, letting

$$V_i = (\mathbf{v}_i'\Sigma^{-1}\mathbf{v}_i)^{\frac{1}{2}},$$

the scheme signals when $V_i > h$, where h is the decision interval. The statistical performance of these schemes was evaluated by simulation. Crosier states that both of the methods reduce to the multivariate Shewhart chart when h equals 0 and k equals the multivariate Shewhart control limit. Note that the latter scheme will be used to make comparison with the principal component chart proposed by this research.

Alwan (1986) presents a CUSUM control scheme based on the sequential probability ratio test. He defines the statistic E_i as

$$E_i = M(\bar{\boldsymbol{x}}_i - \boldsymbol{\mu}_0)'\Sigma^{-1}(\bar{\boldsymbol{x}}_i - \boldsymbol{\mu}_0)$$

and indicates that E_i is distributed as a noncentral χ^2 distribution. Then, he proposes a sequential test in reverse order on the noncentrality parameter of the distribution of statistic E_i. Alwan shows that the decision equation of the test is linear and therefore a standard V-mask can be constructed.

Montgomery and Wadsworth (1972) suggest a multivariate control chart for process dispersion. They use the random variable $\log|\boldsymbol{S}|$, the logarithm of the determinant of the sample variance and covariance matrix. The term $|\boldsymbol{S}|$ is also called the sample generalized variance in the area of multivariate analysis. Gnanadesikan and Gupta (1970) show that the distribution of $\log|\boldsymbol{S}|$ can be approximated by a Normal distribution. Later, Alt (1985) proposes several control charts based on the sample generalized variance. One method uses the fact that in the bivariate case, the statistic

$$\frac{2(M-1)|\boldsymbol{S}|^{\frac{1}{2}}}{|\Sigma_0|^{\frac{1}{2}}}$$

is distributed as χ^2 with 2M-4 degrees of freedom. The other method is constructed using the first two moments of the $|\boldsymbol{S}|$ and the property that most of the probability distribution of $|\boldsymbol{S}|$ is contained in the interval

$$E(|\boldsymbol{S}|) \pm 3[V(|\boldsymbol{S}|)]^{\frac{1}{2}}.$$

Healy (1987) also proposes a CUSUM scheme based on the sequential probability ratio test. He uses the concept of discriminant analysis to develop a procedure to distinguish between a multivariate Normal with a "good" mean ($\boldsymbol{\mu}_g$) and one with a "bad" mean ($\boldsymbol{\mu}_b$). Healy points out that the method does not depend on the number of variables. Therefore, the ARL does not increase for large value of p. However, he indicates that his procedure will not work if the direction of the shift is unknown in advance.

Smith (1987) develops another multivariate CUSUM procedure based on the likelihood ratio test. She also extends the procedure to study shifts in the covariance matrix of a multivariate Normal process and to study shifts in the probability of a multinominal process. Smith compares the statistical performance of the procedure with Alt, Crosier and Pignatiello's methods under the bivariate case using simulation.

Hawkins (1991) suggests a CUSUM control scheme based on the vector **z** of scaled residuals from the regression of each variable on all others. He shows that this approach can be used to detect the mean shifts in several directions. Hawkins also declares that this method is more effective than that of Woodall and Ncube (1985).

The Multivariate EWMA Charts

Lowry (1989) develops a "MEWMA" scheme that is a natural extension of the univariate EWMA procedure. She defines the MEWMA for the i^{th} subgroup mean vector as

$$\beta_i = \boldsymbol{R}\bar{\boldsymbol{x}}_i + (\boldsymbol{I}-\boldsymbol{R})\beta_{i-1}$$

where $\beta_o = \mathbf{0}$ is the initial MEWMA vector, $\bar{\boldsymbol{x}}_i$ is the mean vector of the i^{th} subgroup of size M, and

$$\boldsymbol{R} = \mathrm{diag}\{r_1, r_2, \ldots, r_p\}, \quad 0 \le r_i \le 1, \quad i=1,2,\ldots,p$$

is the weighing matrix. The MEWMA chart gives an out of control signal as soon as

$$T_i^2 = \beta_i' \Sigma_{\beta_i}^{-1} \beta_i > h$$

where $h>0$ is chosen to achieve a specified in-control ARL and Σ_{β_i} is the covariance matrix of β_i. If there is no prior reason to weight past observations differently for the p quality characteristics, Lowry suggests the use of a common r value $r_1 = r_2 = \ldots = r_p = r$. Moreover, she shows that given a common weighing factor r value, the asymptotic covariance matrix Σ_{β_i} as $i \to \infty$ of the MEWMA statistics is defined as

$$\Sigma_{\beta_i} = \left[\frac{r}{(2-r)}\right]\left(\frac{\Sigma}{M}\right).$$

Note that the MEWMA charts employed in the ARL performance comparisons are designed using the asymptotic covariance matrix.

The Method of Principal Components

The purpose of the method of principal components is to reduce the dimensionality of a data set which consists of a large number of interrelated variables, while retaining as much as possible of the variation present in the data set. This goal is achieved by transforming the original variables to a new set of uncorrelated variables, the principal components.

This transformation is a principal axis rotation of the variance and covariance matrix of the data set, and the elements of the characteristic vectors or the eigenvectors of the covariance matrix are the direction cosines of the new axes related to the old. The transformed new uncorrelated variables or the principal components are normally numbered in descending order according to the amount of the variation. If most of the variation of the original data can be attributed to the first two components, then these components can replace the original variables without much loss of information.

The use of the method of principal components in the field of multivariate quality control was first introduced by Jackson and Morris in 1957. They investigate the quality problem in a photographic process at Eastman Kodak Company. Jackson and Morris identify a large number (p) of correlated variables that account for the quality of the process. They notice that the use of Hotelling's T^2 may involve computational problems since the determinant of the variance and covariance matrix is near zero. The solution is to transform the original p variables to lesser k principal components. They use the Shewhart 3σ control limits to monitor those k new variables and find that the principal component charts can be handled by production personnel quite easily.

Jackson (1959) suggests that the method of principal components can be used both as a method of characterizing a multivariate process and as a control tool associated with control procedures. He shows that the T^2 values calculated from the principal components is the same as those calculated from the original variables.

Hawkins (1974) examines the use of principal components in the maintenance of reliability in a large data base. He considers a base consisting of data vectors from a multivariate Normal distribution. A total of 5 screening procedures is proposed. Hawkins declares that the principal component analysis has superior performance.

Jackson and Mudholkar (1979) propose procedures to test the residual associated with principal component analysis. These residuals are the difference between the original observations and the predictions of them using less than a full set of principal components. Procedures for testing the residuals associated with a single observation and for an overall test for a group of observations, given that the underlying covariance matrix is known, are developed. They declare that the proposed procedures may be quite useful in detecting outliers in the data.

Jackson (1980) thoroughly discusses the concept of principal components. He introduces and discusses two alternative ways to scale characteristic vectors. Later, Jackson (1981) extends the ability of principal component charts from controlling a single observation vector to controlling a subgroup of observation vectors. He also discusses the sampling properties of vector coefficients and characteristic roots.

Summary

A literature survey of the problems, contributions and needs related to the objectives of the research is presented. It is obvious that most of the multivariate control schemes are very complex, difficult for others to accept, and beyond the capability of most operators. Furthermore, the evaluation of the statistical performance of the multivariate control charts, except T^2 or χ^2 charts, depends heavily on simulation. This fact severely undermines the utilization of the technology in reality. Therefore, the successful applications of multivariate control schemes in industry are scarce.

This survey substantiates that the most applicable technique in the area of multivariate quality control might rely on the development of principal component charts. The currently available principal component charts use Shewhart control limits for monitoring the process mean vector. No work has been done to incorporate other quality control techniques, such as the EWMA, and the Zone control charts, with the principal components. Furthermore, the concept of the optimal control chart has been fully developed under the univariate case. It is deemed necessary to extend this concept to the multivariate quality control area. Therefore, the tasks of the formulation of the statistical models, using the EWMA and Zone statistics to monitor the principal components of a multivariate process and of the determination of the optimal design parameters for those charts are yet to be accomplished.

This survey indicates that a need exists to:

(1) Derive the statistical performance evaluation models for the proposed multivariate

    a. EWMA principal component chart, and

    b. Zone principal component control chart.

(2) Develop procedures to optimize the statistical

performance of the proposed charts and to obtain their parameters.

(3) Develop computer programs to evaluate the statistical properties of the principal component charts and to help in searching the optimal design parameters.

CHAPTER III

DEVELOPMENT OF STATISTICAL PERFORMANCE EVALUATION MODELS OF TWO MULTIVARIATE PRINCIPAL COMPONENT CONTROL SCHEMES

Introduction

The use of the method of principal components, or principal component control charts, in the field of multivariate quality control can be traced back to the late 50's. The methodology consists of (1) the transformation of a set of multivariate correlated variables to a new set of uncorrelated variables (the principal components), and (2) the supervision of the principal components instead of the original variables to maintain process integrity. Previous literature shows that only the Shewhart control scheme is used to monitor individual principal components. However, research shows that the Shewhart control scheme is not sensitive in detecting small to moderate shifts in the process mean. Therefore, it is desirable to introduce another control scheme that provides better protection against small mean shifts in each principal component.

Two multivariate principal component control schemes are under study. These are the EWMA principal component control chart and the zone principal component control chart. A p-variate multivariate principal component control chart is composed of p individual principal component charts. Therefore, the performance of the multivariate principal component chart depends on the overall performance of those individual charts.

For the multivariate EWMA principal component chart, the ARL of an individual EWMA principal component chart is a function of the control limits, the weight (r) used on the current observations, and the process shift. The integral equation approach is employed to derive the run length distribution of individual EWMA principal component control charts.

For the multivariate Zone principal component control chart, the area within the control limits of each individual zone principal component chart is partitioned into six equal regions and the zone scores from the bottom region to the top region are set to be -2, -1, 0, 0, 1, and 2, respectively with the critical scores of ±4. Therefore, the ARL is a function of the symmetrical control limits and the process shift. The Markov chain approach is used to derive the run length distribution of individual Zone principal component charts.

The ARL for the composite set of EWMA and Zone individual principal component control charts or the multivariate EWMA and Zone principal component control chart can be determined by using the fact that individual principal component control charts are mutually uncorrelated or independent given the assumption that the process under consideration is multivariate Normally distributed. In this research, the statistical models for the evaluation of the ARL of these multivariate principal component control charts are developed. Furthermore, models and algorithms for the determination of statistical optimal parameters of these multivariate principal component control charts are established.

Assumptions

The assumptions underlying this research are described as follows.

(1) The multivariate process of interest has p measurable quality characteristics and the characteristics are multivariate Normally distributed with known mean vector and covariance matrix.

(2) The process can be classified as either at an in-control state where the mean vector and the covariance matrix are stable at the target or at an out-of-control state where an assignable cause shifts the mean vector to a known value.

(3) The covariance matrix of the process is assumed unchanged even when the process reaches an out-of-

control state.

(4) The process is neither self-correcting nor does it degrade progressively. Therefore, once its mean vector has shifted, it stays at the OOC condition until being detected.

(5) Successive subgroups and the observations within subgroups are assumed to be independent.

(6) The calculation of the OOC ARL is made under the assumption that the shift of the process mean vector has occurred prior to the application of the chart.

The Multivariate Process And The Principal Component Analysis

It is a common practice in industry to make multiple measurements on a manufactured item to evaluate its quality during or after production. Such a production process is often called a multivariate process.

Assume that a p characteristic multivariate process has a multivariate Normal distribution with mean vector $\boldsymbol{\mu}_0$ and covariance matrix Σ. To monitor the process, subgroups of size M each are subsequently collected. In this research, interest is centered on the stability of the process mean vector. Therefore, when changes in the process cause the mean vector $\boldsymbol{\mu}_0$ to shift from its nominal value, it is necessary to detect the change as soon as possible to ensure a uniform product quality.

Consider the p×1 random vectors $\bar{\boldsymbol{x}}_1$, $\bar{\boldsymbol{x}}_2$, $\bar{\boldsymbol{x}}_3$, . . . , each representing the subgroup averages observed over time. Thus, the subgroup average denoted by random variable $\bar{\boldsymbol{X}}$ is multivariate Normally distributed with mean $\boldsymbol{\mu}_0$ and known subgroup covariance matrix $\boldsymbol{S}$, where

$$\boldsymbol{S} = \frac{\Sigma}{M}.$$

The assumptions of the known process mean vector $\boldsymbol{\mu}_0$ and process covariance matrix Σ are made for simplicity. In reality, these parameters will be estimated by collecting data over a substantial amount of time from the process under supervision and the Normality and the independent assumptions need to be validated.

Since $\boldsymbol{S}$ is a p×p symmetric and nonsingular matrix, it may be reduced to a diagonal matrix by premultiplying and postmultiplying with an orthonormal matrix $\boldsymbol{U}$, such that

$$\boldsymbol{U}'\boldsymbol{S}\boldsymbol{U} = \boldsymbol{L}.$$

The diagonal elements of $\boldsymbol{L}$, $l_1, l_2, \ldots, l_p$, are the eigenvalues of $\boldsymbol{S}$ and the columns of matrix $\boldsymbol{U}$, $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p$, are the eigenvectors of $\boldsymbol{S}$.

Define a p×1 random vector $\boldsymbol{Y}$ and let

$$\boldsymbol{Y} = \boldsymbol{U}'\bar{\boldsymbol{X}}.$$

Then,

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_p \end{bmatrix} \sim N_p\left(\boldsymbol{U}'\boldsymbol{\mu}_0, \boldsymbol{L}\right)$$

The transformation from the random vector $\bar{\boldsymbol{X}}$ to the random vector $\boldsymbol{Y}$ is the principal axis transformation. The set of p original correlated variables, $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_p$ are now transformed into a set of p new uncorrelated variables, $Y_1, Y_2, \ldots, Y_p$. The transformed variables are the principal components of $\bar{\boldsymbol{X}}$. Therefore, the ith principal component is defined as

$$Y_i = \mathbf{u}_i'(\bar{\boldsymbol{X}} - \boldsymbol{\mu}_0),$$

and it will have mean zero and variance l_i.

Each principal component Y_i may be scaled to have unit variance. Let Z_i represents the ith standardized principal component. Then,

$$Z_i = \frac{Y_i}{\sqrt{l_i}} \quad \text{and} \quad Z_i \sim N(0, 1^2)$$

Suppose that the mean vector of the multivariate process shifts from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$. Then, the mean vector of the subgroup average $\bar{\boldsymbol{X}}$ will shift to $\boldsymbol{\mu}_1$. Furthermore, the mean of the principal components Y_i and Z_i will change accordingly. The amount of shift corresponding to each principal component can be shown as follows.

$$\text{Amount of shift for } Y_i : \quad \mathbf{u}_i' (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$$

$$\text{Amount of shift for } Z_i : \quad \frac{\mathbf{u}_i' (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)}{\sqrt{l_i}}$$

Therefore, the principal components also provide information that can be used to monitor the multivariate process.

Formulation Of The Statistical Evaluation Model

The Cumulative Run Length Probability Of An Individual EWMA Principal Component Chart

The integral equation and the Markov chain are two approaches available for the evaluation of the run length distribution of the EWMA control scheme. Champ and Rigdon (1991) show that both approaches are equivalent. However, they suggest the use of the integral equation approach whenever an integral equation can be established.

Let the probability density function of the i^{th} principal component of the mean vector of the subgroup average $\bar{\mathbf{x}}$ denoted by random variable $Z_i = \{ z_{it} \}$, i = 1, 2, ..., p and t = 1, 2, ..., be $f_i(z_i)$. It has been shown that f_i is a standardized

Normal density function with mean of zero and variance of one if the original process is in-control. Note that the EWMA of the t[th] subgroup average, $S_{i,t}$, obtained from the i[th] individual EWMA principal component control chart or the IEWMAPC control chart is calculated by

$$S_{i,t} = (1 - r_i) S_{i,t-1} + r_i z_{it}, \quad 0 < r_i \leq 1 .$$

where $S_{i,0} = w_i$ for some specified initial value w_i and $-\infty < h_i^- < w_i < h_i^+ < \infty$. Here, h_i^- and h_i^+ are the lower and the upper control limits, respectively and r_i is the smoothing constant of the i[th] IEWMAPC control chart. Note that if the weighing factor $r_i = 1$, then the EWMA chart reduces to the classical Shewhart chart.

Let the random variable N_i represent the run length of the i[th] IEWMAPC control chart. Then, define that

$$P(N_i > n_i) = p_{i,n_i}(w_i), \quad h_i^- < w_i < h_i^+ ,$$

represents the cumulative probability that the run length of the i[th] IEWMAPC chart, starting from the initial state w_i, is greater than a specified number n_i. It is clear that $P(N_i > n_i)$ depends on the initial state $S_{i,0} = w_i$.

Let the i[th] IEWMAPC chart initially start at w_i. In order for a sequence of EWMA to reach state S_{i,n_i}, $n_i \geq 1$ without stopping or signaling OOC, the first EWMA value of the chart denoted by s_i' must stay within the symmetrical control limits.

That is, the possible values of the first observation z_{i1} must satisfy the following equation.

$$s_i' = (1-r_i)w_i + r_i z_{i1}, \quad h_i^- < s_i' < h_i^+ .$$

Therefore, the cumulative run length probability $p_{i,n_i}(w_i)$ can be obtained following Crowder's (1987a) development.

$$p_{i,n_i}(w_i) = \frac{1}{r_i}\int_{h_i^-}^{h_i^+} p_{i,n_i-1}(s_i')\, f_i\left(\frac{s_i' - (1-r_i)\,w_i}{r_i}\right) ds_i' \qquad (3.1)$$

The function $p_{i,n_i}(w_i)$, $h_i^- < w_i < h_i^+$ can be computed recursively starting with $p_{i,0}(w_i) = 1$, $h_i^- < w_i < h_i^+$. Note that f_i is the standard Normal density if the process is in-control. However, if the process mean vector changes from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$, f_i becomes a Normal density function with mean $\dfrac{\mathbf{u}_i(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)}{\sqrt{l_i}}$ and variance one.

<u>The Cumulative Run Length Probability Of An Individual Zone Principal Component Chart</u>

The Zone control scheme is designed to be simpler for quality control personnel to apply. Figure 3.1 depicts the structure of a Zone principal component control chart. The region between LCL and UCL are divided into 6 equal zones. The

	ZONE Score	Probability
	4	p7
UCL		
	2	p5
	1	p3
	0	p1
CL		
	0	p2
	-1	p4
	-2	p6
UCL		
	-4	p8

Figure 3.1 Structure Of A ZONE Principal Component Chart

scheme works as follows:

(1) Determine the initial score $d_{i,0}$ for the i[th] Zone principal component control chart. It is usually set to be 0.

(2) The zone score of the current observation will be added to the previous score if the observation falls on the same side of the central line as the previous one. Otherwise, the accumulation process ends and the chart restarts based on the zone score of the current observation.

(3) The OOC condition is signaled when the cumulated zone score is the same as or beyond the outermost zone score.

The Markov chain approach is employed to derive the run length distribution of the individual Zone principal component chart. The Markov chain representation of the i^{th} Zone principal component control chart or the IZONEPC control chart has seven transient states that correspond to the value of the cumulative score which does not result in a OOC signal. Also, there is one absorbing state that corresponds to the OOC signal. Table 3.1 shows the transient state representation of the chain and Table 3.2 contains the transition matrix.

Let $D = \{1, 2, \ldots, 7\}$ denote the integer set that represents all the transition states of the Markov chain and let $Q_i(j, j')$ represent the transition probability from state

Table 3.1

STATE REPRESENTATION FOR the I^{th}
ZONE PRINCIPAL COMPONENT CHART

State No. (D)	1	2	3	4	5	6	7
Cumulative Score	-3	-2	-1	0	1	2	3

Table 3.2

The MARKOV CHAIN ONE STEP
TRANSITION MATRIX
$Q_I(J, J')$

	State J'								
State J		1 (-3)	2 (-2)	3 (-1)	4 (0)	5 (1)	6 (2)	7 (3)	Absorbing
	1 (-3)	p2	0	0	p1	p3	p5	0	p4+p6+p7+p8
	2 (-2)	p4	p2	0	p1	p3	p5	0	p6+p7+p8
	3 (-1)	p6	p4	p2	p1	p3	p5	0	p7+p8
	4 (0)	0	p6	p4	p1+p2	p3	p5	0	p7+p8
	5 (1)	0	p6	p4	p2	p1	p3	p5	p7+p8
	6 (2)	0	p6	p4	p2	0	p1	p3	p5+p7+p8
	7 (3)	0	p6	p4	p2	0	0	p1	p3+p5+p7+p8
	Absorbing	0	0	0	0	0	0	0	1

Note: Number in parenthesis represents the corresponding cumulative score.

j to j' associated with the i^th^ IZONEPC control chart. Let $P(N_i > n_i) = p_{i,n_i}(w_i)$, $w_i \in D$, represent the cumulative probability that the run length of the ith IZONEPC chart, starting from the initial state w_i, is greater than n_i. Then, $p_{i,n_i}(w_i)$ can be derived as follows.

$$p_{i,n_i}(w_i) = \sum_{\text{for all } d' \in D} p_{i,n_i-1}(d')\, Q(w_i, d') \qquad (3.2)$$

The function $p_{i,n_i}(w_i)$, $w_i \in D$ can be determined recursively, starting with $p_{i,0}(w_i) = 1$, $w_i \in D$.

Bounds For The Cumulative Run Length Probability Of Individual Principal Component Control Chart

Theoretically, equations (3.1) and (3.2) define the cumulative run length probability of the IEWMAPC and the IZONEPC control charts, respectively. However, unless the ARL is small, it is not practical to evaluate (3.1) or (3.2) to obtain all of the proper values of the cumulative run length probability. Woodall (1983) shows that the limiting form of the upper tail of the run length distribution can be approximated using the geometric distribution. Thus, for a relatively small value of n_i, the cumulative probability that the ith principal component chart stops at the $(n_i + j)^{th}$ subgroup or later is

$$p_{i,n_i+j}(w_i) \cong \theta_i^j p_{i,n_i}(w_i), \; 0<\theta_i<1, \; i=1,2,\ldots,p,$$

where w_i is the initial state and θ_i is the parameter of the geometric distribution for i[th] principal component chart. Based on this fact, Waldmann (1986) derives an efficient method for the determination of the value of n_i by the construction of the upper and lower bounds for the cumulative run length probability $P(N_i > n_i)$. The constants $m^{\pm}_{i,n_i}$ on which the bounds are based, are defined by Waldmann as

$$m^{+}_{i,n_i} = \sup_{h_i^- < w_i < h_i^+} \left\{ \frac{p_{i,n_i}(w_i)}{p_{i,n_i-1}(w_i)} \right\}$$

$$m^{-}_{i,n_i} = \inf_{h_i^- < w_i < h_i^+} \left\{ \frac{p_{i,n_i}(w_i)}{p_{i,n_i-1}(w_i)} \right\} \qquad (3.3)$$

where 0/0 is defined to be 0. Utilizing the fact that $p_{i,n_i}(w_i) \le p_{i,n_i-1}(w_i)$, it follows that $m^{\pm}_{i,n_i}$ are well-defined and that $0 \le m^{-}_{i,n_i} \le m^{+}_{i,n_i} \le 1$.

Waldmann shows that for any integer n_i and j, the followings are true.

(1) $(m^{-}_{i,n_i})^j p_{i,n_i}(w_i) \le p_{i,n_i+j}(w_i) \le (m^{+}_{i,n_i})^j p_{i,n_i}(w_i)$,

(2) $(m^{-}_{i,n_i})^{j+1} p_{i,n_i}(w_i) \le (m^{-}_{i,n_i+1})^j p_{i,n_i+1}(w_i)$,

(3) $(m^{+}_{i,n_i})^{j+1} p_{i,n_i}(w_i) \ge (m^{+}_{i,n_i+1})^j p_{i,n_i+1}(w_i)$. (3.4)

Part (1) of (3.4) contains the desired bounds for the cumulative run length probability with suitable constants $m^{\pm}_{i,n_i}$, $0 \le m^{-}_{i,n_i} \le m^{+}_{i,n_i} \le 1$. Parts (2) and (3) guarantee improved bounds at each step of iteration.

Furthermore, he also shows that given some mild and natural assumptions there exists a constant $m_{i,\infty} > 0$, such that

$$\lim_{n_i \to \infty} m^{-}_{i,n_i} = \lim_{n_i \to \infty} m^{+}_{i,n_i} = m_{i,\infty}.$$

The stabilization of the weight $m^{\pm}_{i,n_i}$ usually occurs for relatively small values of n_i from a numerical point of view.

Similarly, the constant $m^{\pm}_{i,n_i}$ of the i^{th} IZONEPC control chart are defined as

$$m^{-}_{i,n_i} = \inf_{w_i \in D} \left\{ \frac{p_{i,n_i}(w_i)}{p_{i,n_i-1}(w_i)} \right\}$$

$$m^{+}_{i,n_i} = \sup_{w_i \in D} \left\{ \frac{p_{i,n_i}(w_i)}{p_{i,n_i-1}(w_i)} \right\} \qquad (3.5)$$

Thus, the smallest value of n_i for which

$$m^{+}_{i,n_i} - m^{-}_{i,n_i} < \varepsilon \qquad (3.6)$$

holds is an optimal one for stopping the iteration process. Note that the value of ε is set to be 10^{-10} in the computer program.

Determination Of The ARL Of The Multivariate EWMA And Zone Principal Component Control Charts

All the mathematical developments discussed previously can be used to obtain the run length of the univariate control chart. However, the method for the evaluation of the run length of the multivariate control chart has not been developed. This section extends the mathematical developments of the cumulative run length probability of the univariate control charts to that of the multivariate control charts.

Previous discussion (p. 42) shows that p random variables Z_i, $i = 1, 2, \ldots, p$ representing the principal components are mutually independent. Therefore, the run length of each principal component N_i, $i = 1, 2, \ldots, p$, which is a function of Z_i, are mutually independent, too. Let N' be the random variable representing the run length of the composite set of p individual principal component charts or the run length of the multivariate principal component chart. Then,

$$Pr(N' > n) = \prod_{i=1}^{p} P(N_i > n),\ n = 0, 1, 2, \ldots \qquad (3.7)$$

Let n_i^*, $i = 1, 2, \ldots, p$ denote the smallest value of n_i that satisfies (3.6) in the calculation of the cumulative run length probability of the i^{th} principal component chart. Then the upper and lower bounds of the cumulative run length probability of the i[th] principal component chart, $P^{+}(N_i > n)$ and $P^{-}(N_i > n)$, respectively, are defined as

$$P^{\pm}(N_i > n) = \begin{cases} p_{i,n}(w_i), & \text{if} \quad n \le n_i^{*}, \\ (m^{\pm}_{i,n_i^*})^{n-n_i^*} p_{i,n_i^*}(w_i), & \text{if} \quad n > n_i^{*}. \end{cases}$$

where w_i is the initial EWMA value or Zone score.

Also define n^{+} and n^{-} to be the maximum and minimum values of n_i^{*}. Thus,

$$n^{+} = \max\{n_1^{*}, n_2^{*}, \ldots, n_p^{*}\}$$

$$n^{-} = \min\{n_1^{*}, n_2^{*}, \ldots, n_p^{*}\}$$

The upper and lower bounds for the cumulative run length probability of the multivariate principal component chart, $Pr^{+}(N' > n)$ and $Pr^{-}(N' > n)$, respectively, are

$$Pr^{\pm}(N' > n) = \begin{cases} \prod_{i=1}^{p} p_{i,n}(w_i), & n \le n^{-} \\ \prod_{i=1}^{p} (m^{\pm}_{i,n_i^*})^{\max\{0,\, n-n_i^*\}} \left(p_{i,n_i^*}(w_i)\right)^{\min\{1,\, \max\{0,\, n-n_i^*\}\}} & \\ \quad \times \left(p_{i,n}(w_i)\right)^{\min\{1,\, \max\{0,\, n_i^*-n\}\}}, & n^{-} < n < n^{+} \\ \prod_{i=1}^{p} (m^{\pm}_{i,n_i^*})^{n-n_i^*} p_{i,n_i^*}(w_i), & n \ge n^{+} \end{cases}$$

Let ARL^{+} and ARL^{-} represent the upper and lower bounds of the average run length of the composite set of the individual principal component charts or the multivariate principal component control chart, respectively. Then,

$$
\begin{aligned}
ARL^{\pm} &= \sum_{j=0}^{\infty} Pr^{\pm}(N' > j) \\
&= \sum_{j=0}^{n^-} \prod_{i=1}^{p} p_{i,j}(w_i) \\
&\quad + \sum_{j=n^-+1}^{n^+-1} \left[\prod_{i=1}^{p} \left((m^{\pm}_{i,n_i^*})^{\max\{0,j-n_i^*\}} \left(p_{i,n_i^*}(w_i)\right)^{\min\{1,\max\{0,j-n_i^*\}\}} \right.\right. \\
&\quad \left.\left. \times \left(p_{i,j}(w_i)\right)^{\min\{1,\max\{0,n_i^*-j\}\}} \right) \right] \\
&\quad + \frac{\prod_{i=1}^{p} Pr(N_i > n^+)}{\left(1 - \prod_{i=1}^{p} (m^{\pm}_{i,n_i^*})\right)}
\end{aligned}
$$

In the equation listed above, the upper tail area or the cumulative probabilities of the run length N_i of the i^{th} principal component chart greater than n_i^* or more are approximated by the appropriate geometric distribution. It is clear that

$$ARL^- \le ARL \le ARL^+,$$

Then, the ARL is approximated to be

$$ARL \simeq \frac{(ARL^- + ARL^+)}{2}.$$

Development Of Search Technique For The Statistically Optimal Design Parameters

The statistically optimal control chart is defined as a control chart with a fixed in-control ARL which has the smallest ARL for a specified or predetermined shift in the mean vector. Therefore, the objective function to be minimized in the optimal design of the two principal component control charts is the out-of-control ARL. The equality constraint is the desired in-control ARL. Note that both the objective function and the constraint are multi-dimensional nonlinear equations and the closed form expression of the first derivative of the objective function with respect to the design parameters is not available. Thus, a direct search along the constraint surface must be employed to determine the optimal design parameters.

Chandler (1967) claims that a widely employed search method in multi-dimensional minimization is to vary the parameters cyclically. The search starts by varying the first parameter while all of the others are held fixed. When a local minimum along this line has been reached, this parameter is fixed and the second is varied. Cycling in this way will eventually reach a local minimum of a smooth function, if one exists. One shortfall is that the cyclic variation method usually works in a zig-zag mode when the resultant vectors of any two successive complete cycles are nearly identical and the length of those vectors is a very small fraction of the distance to the minimum. Thus, the convergence can be very slow.

Another well-known direct search method, the SIMPLEX method, was developed by Nelder-Mead in 1965. The simplex procedure solves a multivariable minimization problem by forming a simplex and moving along the response surface. It approaches the minimum by deleting the point with highest objective function value or highest resultant rather than by trying to move cyclically. Chandler (1975) claims that the SIMPLEX method is excellent if the number of variables does not exceed six or so. However, It is somewhat slow if there are more variables.

In this research, there are two procedures, STEPIT and UNICY, used in the optimization process. The algorithm used in STEPIT and UNICY is developed by Chandler (1967). STEPIT is basically a cyclic variation method with an accelerating scheme. It expedites the search sequence by adopting a criterion of collinearity of successive cycle resultants. Whenever the resultants of the preceding two cycles satisfy the collinearity criterion, the next attempt is made following the direction of the last resultant. If this is successful, the step size in the same direction will be increased. Otherwise, a parabolic interpolation is used to locate the minimum along the line and the procedure returns to the cyclic variation mode. Chandler claims that this method has been quite successful on a wide variety of minimization problems. Furthermore, it is either competitive with or outperforms other published methods with more than five or ten parameters. The UNICY procedure is a one dimensional version of STEPIT. It is used to find the root of the equality constraint.

A limited study by the author (see Appendix J) shows that STEPIT and SIMPLEX provide similar results in optimizing the multivariate EWMA principal component chart. However, the STEPIT procedure is slightly faster than the SIMPLEX procedure. Therefore, STEPIT is used throughout this research.

In this research, the optimal multivariate EWMA and Zone principal component control charts are studied. For the multivariate EWMA principal component chart, the design parameters under optimization are the symmetrical control limits $h_i^{\pm}$ and the weighing factors r_i. For the multivariate Zone principal component chart, the design parameters under optimization are the symmetrical control limits $h_i^{\pm}$. Thus, for a multivariate principal component chart with m design parameters, the search procedures employed are described as follows.

(1) Identify a set of m-1 starting points. Note that a good set of starting points will expedite the

optimization process. Crowder's (1989) paper can be used as a reference to get a good starting point for r_i. Also, experience shows that the initial values of $h_i^{\pm}$ should be set wider for a process with a larger number of characteristics.

(2) Incorporating the set of m-1 points, the UNICY procedure can be used to find the m^{th} point that satisfies the constraint. Then, the objective function is evaluated using those m points.

(3) Use STEPIT to identify the next set of m-1 points along the m-1 dimensional space.

(4) For two successive sets of points, if the advancing distance within each dimension is less than a predetermined value, then the search is over. Otherwise, go to step 3.

Summary

The multivariate process and the principal component analysis have been introduced in this chapter. Statistical models for the evaluation of the individual EWMA and Zone principal component control charts are discussed. The integral equation and the Markov chain approaches are used to derive the run length distribution of the IEWMAPC and IZONEPC control charts. Moreover, the mathematical development and derivation to evaluate the ARL of the multivariate EWMA and Zone principal component control charts are fully developed. Waldmann's bound method is adopted to facilitate the numerical evaluation of the ARL performance of the MEWMAPC and MZONEPC control charts.

This chapter also introduces the concept of the statistical optimal control chart. Based on this concept, the procedures and algorithms for searching for the design parameters of the optimal multivariate EWMA and Zone principal component control chart is developed. The optimization routines employed are STEPIT and UNICY developed by Chandler (1975). Since the direct search method does not guarantee a global minimum, it is common to use multiple starting points to provide confidence that the optimal or near optimal solution has been reached. Based on the experience gained in this research, different starting points do create different sets of design parameters in certain optimal MEWMA principal component charts. However, the value of the objective function or the OOC ARL are very similar. The computer programs developed in this research include multiple sets of initial search points that facilitate the optimization process and provide reliable results.

CHAPTER IV

VERIFICATION OF THE COMPUTER MODELS

Introduction

The statistical models for the evaluation of the performance or the ARL of either the individual or the multivariate EWMA and Zone principal component control charts have been developed in chapter III. It is obvious that the closed form solution for the ARL cannot be obtained. Therefore, numerical calculation through the use of the computer is necessary. Several computer models or programs using the FORTRAN language for the calculation of the ARL of these principal component charts have been developed in this research.

The ARL of the individual EWMA and individual Zone principal component control charts obtained from the developed models can be verified with the results from Crowder (1987a) and Davis, Homer, and Woodall (1990), respectively. Furthermore, the fact that the Shewhart chart is a special case of both the EWMA and Zone principal component control charts can be employed to verify the computer models for the evaluation of the ARL of the multivariate EWMA and Zone principal component control charts.

ARL For Individual EWMA And Zone
Principal Component Charts

Crowder (1987a) presents a computer program to calculate the ARL of the EWMA chart. The ARL values obtained from Crowder's program and from the proposed computer models are tabulated in Table 4.1 for various r values, control limits h, and mean shifts. The symmetrical control limits used in generating the ARL values are $\pm 3.5\ \sigma_{EWMA}$, and

$$\sigma^2_{EWMA} = \left(\frac{r}{(2 - r)} \right) \sigma^2$$

where σ^2_{EWMA} represents the asymptotic variance of the EWMA statistics and σ^2 is the variance of the random variable which generates the EWMA. Note that the principal component is distributed Normally with mean of zero and standard deviation of one. Therefore, the size of the mean shifts and the symmetrical control limits discussed in this research are measured in terms of the number of the standard deviation of the principal component.

It is clear that the results from Crowder's program and from the proposed computer model are identical with three decimal places except the first and second rows of Table 4.1. However, the largest percentage of difference is less than 0.005%, which is small enough to be neglected. This shows that the computer model or program used in this research for

TABLE 4.1

COMPARISON OF THE ARL OF EWMA CHARTS OBTAINED FROM CROWDER'S COMPUTER PROGRAM AND FROM THE PROPOSED COMPUTER MODEL

SHIFT	r = 0.1 h = .80295507		r = 0.25 h = 1.32287566	
0.00	4106.422*	4106.242	2640.163*	2640.145
0.25	385.291*	385.290	625.784*	625.784
0.50	64.718*	64.718	123.431*	123.431
0.75	25.331*	25.331	38.678*	38.678
1.00	14.790*	14.790	17.712*	17.712
1.25	10.372*	10.372	10.475*	10.475
1.50	8.005*	8.005	7.249*	7.249
1.75	6.540*	6.540	5.522*	5.522
2.00	5.548*	5.548	4.471*	4.471
2.25	4.832*	4.832	3.772*	3.772
2.50	4.292*	4.292	3.277*	3.277
2.75	3.871*	3.871	2.909*	2.909
3.00	3.535*	3.535	2.627*	2.627
3.25	3.262*	3.262	2.407*	2.407
3.50	3.035*	3.035	2.235*	2.235
3.75	2.839*	2.839	2.099*	2.099
4.00	2.662*	2.662	1.989*	1.989

Note: "*" represent results from Crowder's program

TABLE 4.1 (Continued)

SHIFT	r = 0.5 h = 2.02072594		r = 0.75 h = 2.71108834	
0.00	2227.344*	2227.329	2157.987*	2157.984
0.25	951.178*	951.179	1245.899*	1245.899
0.50	267.360*	267.360	468.680*	468.680
0.75	88.697*	88.697	182.123*	182.123
1.00	35.973*	35.973	78.052*	78.052
1.25	17.642*	17.642	37.150*	37.150
1.50	10.192*	10.192	19.626*	19.626
1.75	6.704*	6.704	11.456*	11.456
2.00	4.861*	4.861	7.327*	7.327
2.25	3.782*	3.782	5.076*	5.076
2.50	3.096*	3.096	3.760*	3.760
2.75	2.632*	2.632	2.940*	2.940
3.00	2.299*	2.299	2.400*	2.400
3.25	2.049*	2.049	2.026*	2.026
3.50	1.852*	1.852	1.756*	1.756
3.75	1.690*	1.690	1.556*	1.556
4.00	1.553*	1.553	1.403*	1.403

Note: "*" represent results from Crowder's program calculating the ARL of the individual EWMA principal component chart (IEWMAPC) is adequate.

Davis, Homer and Woodall (1990) provide a profile of the ARL of the Zone chart with zone scores of 0, 1, 2 and 4, which is identical to the Zone control scheme employed in this research, and the symmetrical control limits of ± 3. Table 4.2 shows that the ARL calculations from the paper of Davis et al. and from the proposed computer model are identical to two decimal places. This verifies that the program employed to calculate the ARL for the individual Zone principal component chart (IZONEPC) is adequate.

ARL For Multivariate EWMA And Zone Principal Component Charts

It is noted that the EWMA control chart with the weighing factor r=1 and the critical values of ±3 is equivalent to the Shewhart control chart. Therefore, the proposed computer model for the calculation of the ARL of the multivariate EWMA principal component chart (MEWMAPC) is verified if the results obtained from the proposed computer model for the MEWMAPC chart using r=1 are comparable to the results from multiple independent Shewhart charts given that both types of charts have the same dimension.

Table 4.3 and 4.4 depict the profile of the ARL of the multiple Shewhart charts and the MEWMAPC chart under various mean shifts with two and three variates, respectively. It is

TABLE 4.2

COMPARISON OF THE ARL OF THE ZONE CHART OBTAINED FROM DAVIS, HOMER AND WOODALL'S PAPER AND FROM THE PROPOSED COMPUTER MODEL

SHIFT	DAVIS, HOMER AND WOODALL'S PAPER	PROPOSED MODEL
0.00	95.05	95.05
0.20	67.63	67.63
0.40	35.54	35.54
0.60	19.52	19.52
0.80	12.01	12.01
1.00	8.19	8.19
1.20	6.06	6.06
1.40	4.76	4.76
1.60	3.91	3.91
1.80	3.31	3.31
2.00	2.86	2.86
2.20	2.51	2.51
2.40	2.23	2.23
2.60	2.00	2.00
2.80	1.81	1.81
3.00	1.65	1.65

TABLE 4.3

COMPARISON OF THE ARL OBTAINED FROM TWO INDEPENDENT SHEWHART CHARTS AND FROM THE MULTIVARIATE EWMA PRINCIPAL COMPONENT CHART WITH R = 1

MEAN SHIFT 1ST	MEAN SHIFT 2ND	TWO INDEPENT SHEWHART CHARTS	MULTIVARIATE EWMA CHART WITH R=1
0.0	0.0	185.4495	185.4495
0.1	0.2	164.8397	164.8397
0.3	0.4	111.9975	111.9975
0.5	0.6	67.8191	67.8191
0.7	0.8	40.5576	40.5576
0.9	1.0	24.8224	24.8224
1.1	1.2	15.7114	15.7114
1.3	1.4	10.3161	10.3161
1.5	1.6	7.0336	7.0336
1.7	1.8	4.9817	4.9817
1.9	2.0	3.6657	3.6657
2.1	2.2	2.8017	2.8017
2.3	2.4	2.2229	2.2229
2.5	2.6	1.8288	1.8288
2.7	2.8	1.5575	1.5575
2.9	3.0	1.3697	1.3697

Note : The symmetrical control limits for each Shewhart chart and for each individual EWMA principal component chart are identical at ± 3.0

TABLE 4.4

COMPARISON OF THE ARL OBTAINED FROM THREE INDEPENDENT SHEWHART CHARTS AND FROM THE MULTIVARIATE EWMA PRINCIPAL COMPONENT CHART WITH R = 1

MEAN SHIFT 1ST	2ND	3RD	THREE INDEPENT SHEWHART CHARTS	MULTIVARIATE EWMA CHART WITH R=1
0.0	0.0	0.0	123.8000	123.8000
0.1	0.2	0.3	100.0707	100.0707
0.4	0.5	0.6	50.8401	50.8401
0.7	0.8	0.9	23.7376	23.7376
1.0	1.1	1.2	11.7675	11.7675
1.3	1.4	1.5	6.3585	6.3585
1.6	1.7	1.8	3.7696	3.7696
1.9	2.0	2.1	2.4591	2.4591
2.2	2.3	2.4	1.7655	1.7655
2.5	2.6	2.7	1.3890	1.3890
2.8	2.9	3.0	1.1853	1.1853

Note : The symmetrical control limits for each Shewhart chart and for each individual EWMA principal component chart are identical at ± 3.0

observed from both tables that the ARL for both types of charts is equivalent. This shows that the computer model for the calculation of the ARL of the MEWMAPC chart employed in this research is adequate.

Analogously, the Zone chart with the upper, middle and lower zone scores of 1, 0 and -1, respectively, the critical values of ±1 and the symmetrical control limits of ± 3.0 is the same as the Shewhart control chart. This is to say that the specified Zone chart can be represented by two states which are the in-control state and the out-of-control state. Therefore, the calculation of the ARL for the chart using the Markov chain is identical to that of the Shewhart chart. Therefore, the computer model for the calculation of the ARL of the MZONEPC chart is adequate as long as that of the individual Zone principal component chart is adequate.

Summary

The computer models or programs used in the research for the calculation of the ARL of the IEWMAPC, the IZONEPC, the MEWMAPC and the MZONEPC charts are verified in this chapter. The comparisons of either the results from the published literature or from the multiple Shewhart charts show that the models are adequate and proper to use.

CHAPTER V

THE DESIGN AND COMPARISON OF THE CLASSICAL MULTIVARIATE EWMA AND ZONE PRINCIPAL COMPONENT CONTROL CHARTS

Introduction

The classical design is one of the two most popular design approaches in the area of the statistical quality control chart. This approach emphasizes the general performance of the chart under various process shifts instead of a particular one. Therefore, profiles of the ARL performance of charts for different values of parameters under various size shifts are presented for the practitioner's choice. Another approach in the design of the quality control charts is the optimal design approach. The optimal design approach is discussed in the next chapter.

The classical design approach of two multivariate principal component charts, MEWMAPC and MZONEPC, is discussed in this chapter. Moreover, the statistical performance comparisons of these charts, under the classical design approach, with respect to four existing multivariate control charts are also addressed. The four existing multivariate control charts under comparison are Hotelling's χ^2 chart, Crosier's (1988) MCUSUM chart, Pignatiello and Runger's (1990)

MC1 chart, and Lowry's (1989) MEWMA chart.

The χ^2, MCUSUM, MC1 and MEWMA charts are known to be directionally invariant. That is, the ARL performance of these charts is determined only by the statistical distance of the OOC mean vector $\boldsymbol{\mu}_1$ from the in-control mean vector $\boldsymbol{\mu}_0$, not by the particular location of that mean vector. The statistical distance is defined as the noncentrality parameter λ, where

$$\lambda = \left[(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)' \Sigma^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) \right]^{\frac{1}{2}} .$$

The value of λ will be referred to as the size or the magnitude of the mean vector shift in this research.

Note that the MEWMAPC and MZONEPC charts are non-directionally invariant. That is, the ARL performance of these principal component charts depends not only on the statistical distance between the mean vector $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_0$ but on the direction of the difference between these two means as well. In order to make meaningful comparisons among the principal component charts, which are non-directionally invariant, with four types of directionally invariant charts, it is necessary to consider all the directions of shifts that generate a size of shift equal to a given value of the noncentrality parameter λ. Therefore, the mean shifts of the MEWMAPC and MZONEPC charts are calibrated to be equivalent to certain values of the noncentrality parameter λ.

Throughout this chapter, the covariance matrix is assumed to be the identity matrix and the in-control mean vector is assumed to be **0**. That is, the mean shifts under discussion are the mean shift of each centralized principal component. Since the mean shifts of the original variables from the multivariate process can be transformed into the mean shifts of the principal components and the principal components of the process can be scaled such that their covariance matrix is an identity matrix and their mean vector is **0**, the use of an identity covariance matrix and a mean vector of **0** is not a limitation.

For the classical design of the MEWMAPC chart, there is not a priori reason to either weight past observations differently or to set the control limits differently for the p principal components being monitored. Therefore, common r and common h, denoted by C1, will be used for each classical IEWMAPC chart. Similar arguments can be followed for the classical design of the MZONEPC chart regarding the control limits h. Here, common h will be employed for each classically designed IZONEPC control chart.

The Classical Design Of The MEWMAPC And The MZONEPC Charts

The Classical MEWMAPC Charts

For the bivariate case, the shift in the mean vector of either the process characteristic or the principal component can be described by the following form:

$$\boldsymbol{\mu}_1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \delta\cos\theta \\ \delta\sin\theta \end{bmatrix}, \text{ where } \theta = \tan^{-1}\frac{b}{a} \text{ and } \delta = \sqrt{a^2+b^2} \quad (5.1)$$

where a and b are the mean shifts of the first and second process characteristics or the first and second principal components, respectively, and θ is the direction of the shift. Note that the value of δ is equal to λ in equation 5.1, if the mean vector shift under consideration is measured with respect to the centralized principal component instead of the original characteristic.

Figure 5.1 shows the location of the mean vector shift of

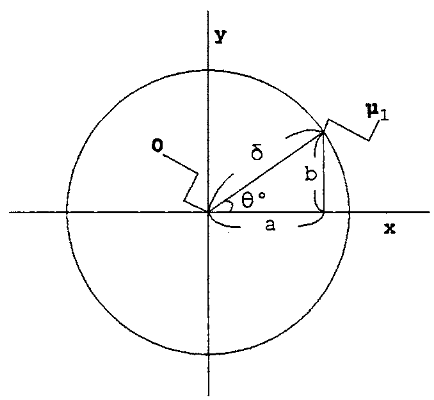

Figure 5.1 The Graphical Representation Of The Location of A Mean Vector Shift Of The Centralized Principal Component With λ = δ.

a two dimensional centralized principal component from in-control mean **0** to $\boldsymbol{\mu}_1$. The axes of the first and second principal components are x and y, respectively. Note that all possible locations of shifts of the mean vector of the centralized principal component with a specific $\lambda = \delta$ form a circle. Therefore, the circle represents the locations of the shift of the mean vector of the principal component with a size of λ.

Tables 5.1 to 5.3 display the ARL profiles of several bivariate MEWMAPC charts for varying values of r. The in-control or nominal ARL for the charts used in Tables 5.1 to 5.3 are calibrated to be 100, 200 and 370, respectively. Since the MEWMAPC charts are non-directionally invariant, the ARL performance of these charts at the size of the shift that was represented by a particular value of the noncentrality parameter, is not a constant. Thus, Tables 5.1 to 5.3 show the range of all possible ARL performance of these charts at given values of the noncentrality parameter. For example, given a shift of size $\lambda = 0.5$, the ARL of a bivariate MEWMAPC chart with parameters $r = 0.1$ and $h = 0.5597$ will be within the range of 21.200 to 22.542, depending on the direction θ of the mean vector shift.

As illustrated in these tables, small values of r are more efficient in detecting a small process mean shift. This is the same as for the univariate case. Note that the mean shift is calibrated in terms of the value of λ. Figures 5.2 to 5.7 display the ARL performance of a bivariate MEWMAPC chart with parameters $r = 0.1$ and $h = 0.5597$ (nominal ARL = 100) given that the various directions θ of the mean shifts of two principal components are calibrated to have noncentrality

Table 5.1

ARL VALUES FOR MEWMAPC CHARTS (p = 2)
NOMINAL ARL = 100

λ	r = 0.1 h = .5597	r = 0.2 h = .8760	r = 0.3 h = 1.1375
0.0	100.000	100.000	100.000
0.5	21.200-22.542	24.396-26.040	28.622-30.501
1.0	8.410-9.483	8.228-9.350	8.765-10.091
1.5	5.167-5.994	4.649-5.399	4.557-5.346
2.0	3.771-4.439	3.265-3.827	3.067-3.610
2.5	3.004-3.565	2.559-3.005	2.352-2.763
3.0	2.521-3.007	2.147-2.508	1.938-2.274

Table 5.1 (Continued)

λ	r = 0.4 h = 1.3751	r = 0.5 h = 1.6020	r = 0.6 h = 1.8260
0.0	100.000	100.000	100.000
0.5	33.234-35.227	38.034-40.031	42.933-44.847
1.0	9.735-11.339	11.084-13.004	12.826-15.069
1.5	4.675-5.571	4.959-6.020	5.415-6.697
2.0	2.996-3.567	3.008-3.641	3.098-3.832
2.5	2.229-2.641	2.160-2.593	2.137-2.610
3.0	1.795-2.127	1.701-2.032	1.644-1.983

Table 5.1 (Continued)

λ	$r = 0.7$ $h = 2.0531$	$r = 0.8$ $h = 2.2886$	$r = 0.9$ $h = 2.5376$
0.0	100.000	100.000	100.000
0.5	47.880-49.647	52.835-54.409	57.753-59.109
1.0	15.005-17.551	17.680-20.482	20.919-23.901
1.5	6.075-7.633	6.995-8.879	8.257-10.507
2.0	3.276-4.155	3.565-4.645	4.009-5.355
2.5	2.160-2.696	2.234-2.870	2.375-3.160
3.0	1.617-1.976	1.619-2.019	1.652-2.122

Table 5.2

ARL VALUES FOR MEWMAPC CHARTS (p = 2)
NOMINAL ARL = 200

λ	r = 0.1 h = .6248	r = 0.2 h = .9608	r = 0.3 h = 1.2384
0.0	200.000	200.000	200.000
0.5	27.715-30.633	34.830-38.810	43.821-48.557
1.0	9.812-11.427	9.868-11.730	10.969-13.368
1.5	5.852-6.957	5.279-6.332	5.261-6.440
2.0	4.215-5.063	3.624-4.356	3.421-4.156
2.5	3.334-4.024	2.801-3.363	2.577-3.102
3.0	2.781-3.370	2.324-2.772	2.108-2.514

Table 5.2 (Continued)

λ	r = 0.4 h = 1.4909	r = 0.5 h = 1.7325	r = 0.6 h = 1.9715
0.0	200.000	200.000	200.000
0.5	53.652-58.754	63.960-69.105	74.558-79.499
1.0	12.781-15.875	15.284-19.158	18.545-23.219
1.5	5.536-6.963	6.067-7.859	6.882-9.161
2.0	3.384-4.189	3.461-4.402	3.651-4.802
2.5	2.461-3.001	2.411-3.000	2.417-3.092
3.0	1.964-2.372	1.869-2.294	1.816-2.271

Table 5.2 (Continued)

λ	r = 0.7 h = 2.2143	r = 0.8 h = 2.4666	r = 0.9 h = 2.7339
0.0	200.000	200.000	200.000
0.5	85.315-89.872	96.116-100.170	106.843-110.327
1.0	22.674-28.105	27.806-33.886	34.085-40.638
1.5	8.056-10.940	9.700-13.302	11.977-16.383
2.0	3.979-5.430	4.492-6.355	5.271-7.680
2.5	2.486-3.294	2.633-3.641	2.888-4.193
3.0	1.799-2.308	1.822-2.419	1.893-2.632

Table 5.3

ARL VALUES FOR MEWMAPC CHARTS (p = 2)
NOMINAL ARL = 370

λ	r = 0.1 h = .6770	r = 0.2 h = 1.2097	r = 0.3 h = 1.3212
0.0	370.000	370.000	370.000
0.5	34.705-40.060	47.789-55.854	64.279-74.245
1.0	11.071-13.272	11.501-14.306	13.366-17.281
1.5	6.437-7.808	5.850-7.226	5.939-7.578
2.0	4.487-5.597	3.937-4.834	3.743-4.678
2.5	3.606-4.410	3.010-3.678	2.774-3.414
3.0	3.000-3.671	2.473-3.003	2.249-2.726

Table 5.3 (Continued)

λ	r = 0.4 h = 1.5866	r = 0.5 h = 1.8407	r = 0.6 h = 2.0926
0.0	370.000	370.000	370.000
0.5	82.576-93.517	102.034-113.188	122.268-133.053
1.0	16.340-21.680	20.482-27.433	25.966-34.589
1.5	6.417-8.524	7.267-10.059	8.563-12.269
2.0	3.750-4.823	3.908-5.224	4.226-5.914
2.5	2.669-3.343	2.643-3.406	2.690-3.602
3.0	2.109-2.593	2.021-2.540	1.978-2.558

Table 5.3 (Continued)

λ	r = 0.7 h = 2.3489	r = 0.8 h = 2.6155	r = 0.9 h = 2.8984
0.0	370.000	370.000	370.000
0.5	142.978-152.973	163.892-172.814	184.737-192.428
1.0	33.022-43.255	41.916-53.561	52.922-65.646
1.5	10.440-15.299	13.105-19.339	16.839-24.628
2.0	4.750-6.974	5.562-8.527	6.797-10.750
2.5	2.820-3.965	3.063-4.557	3.472-5.482
3.0	1.979-2.657	2.033-2.867	2.155-3.238

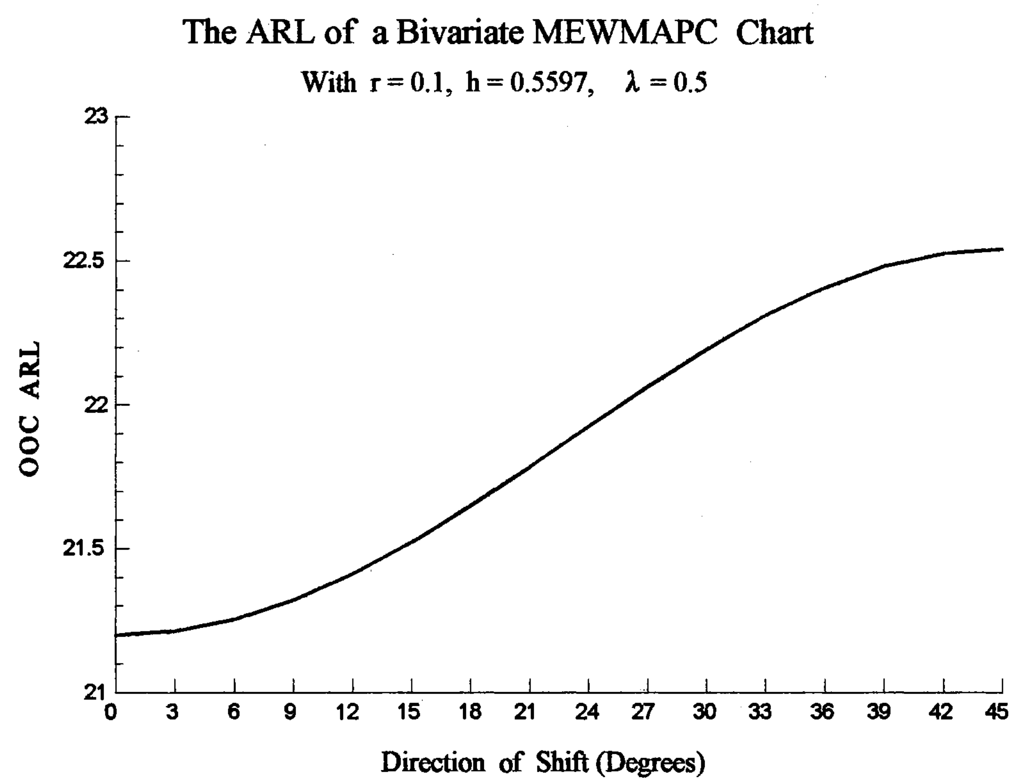

Figure 5.2 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 0.5$

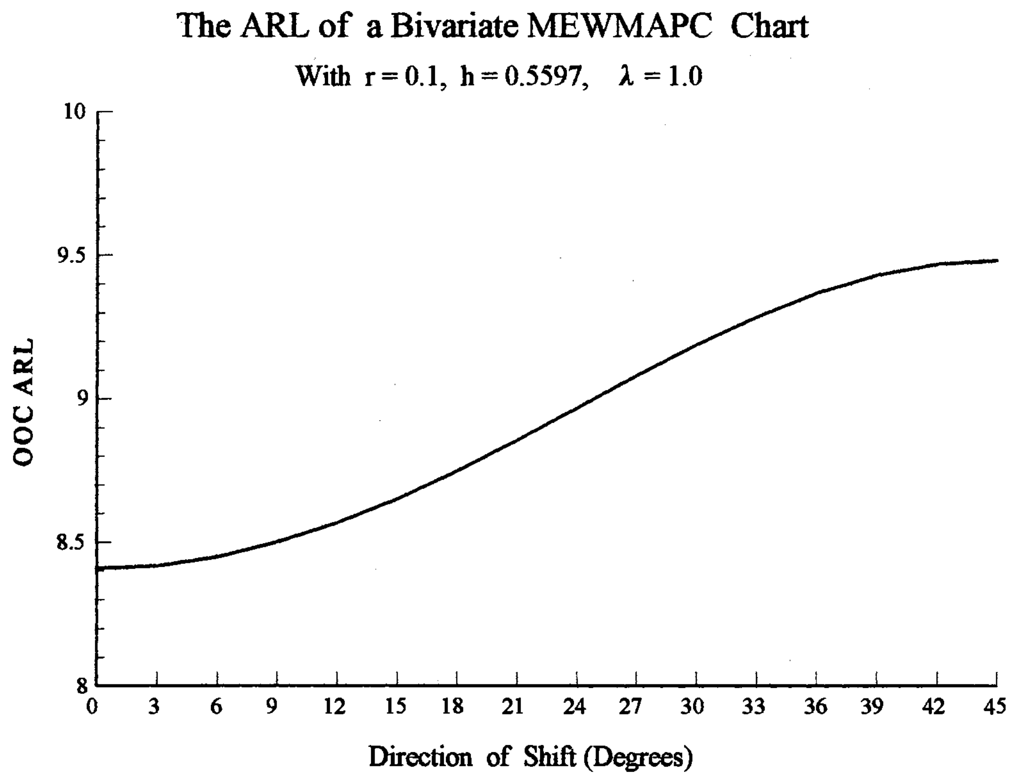

Figure 5.3 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 1.0$

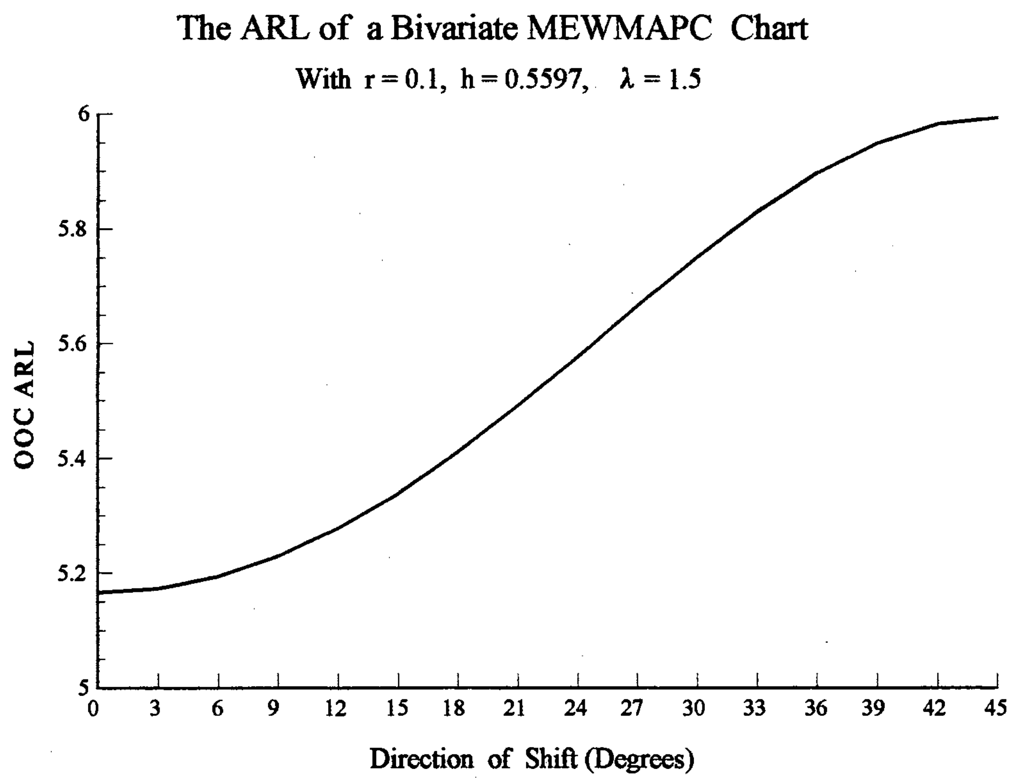

Figure 5.4 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 1.5$

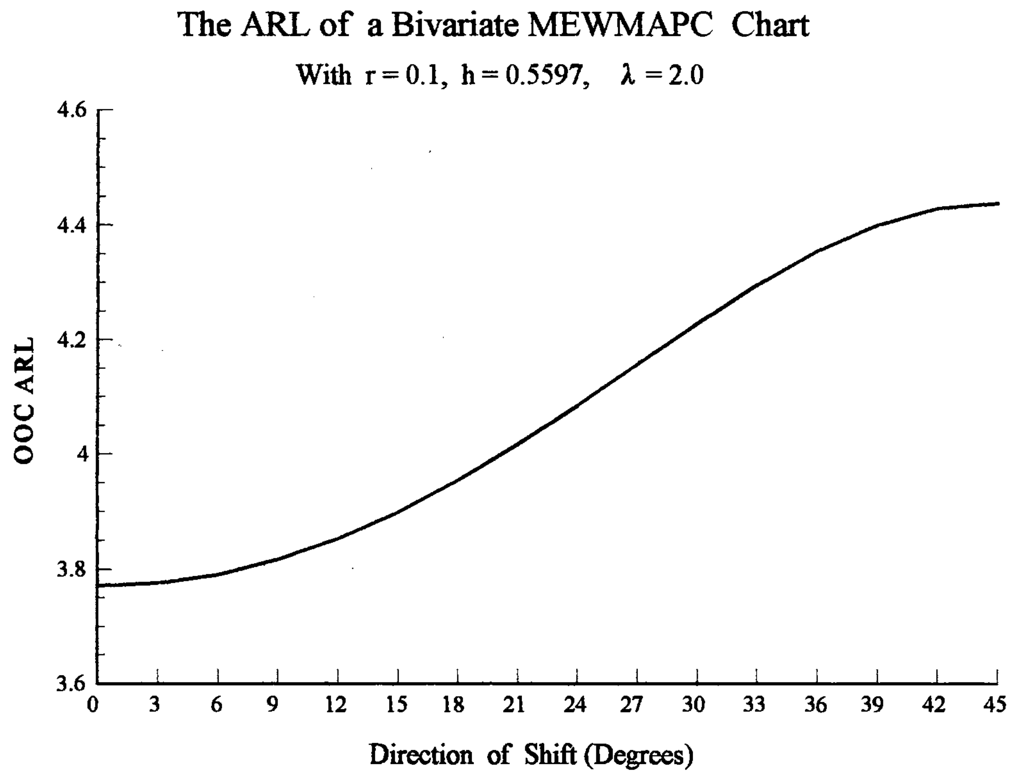

Figure 5.5 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 2.0$

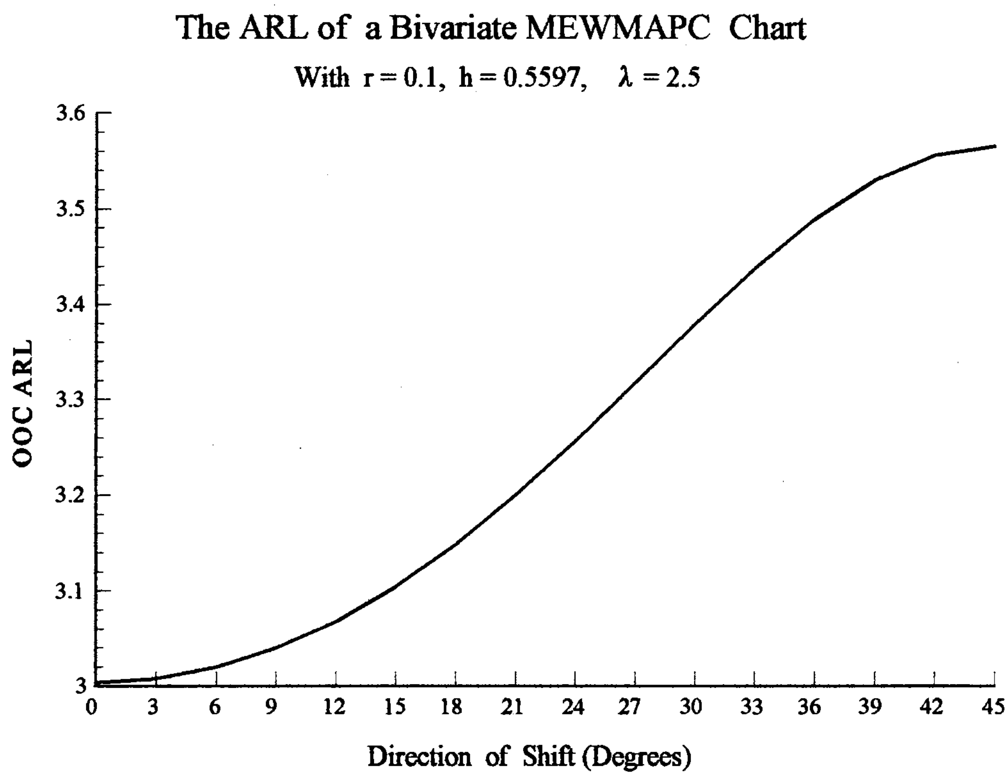

Figure 5.6 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That λ = 2.5

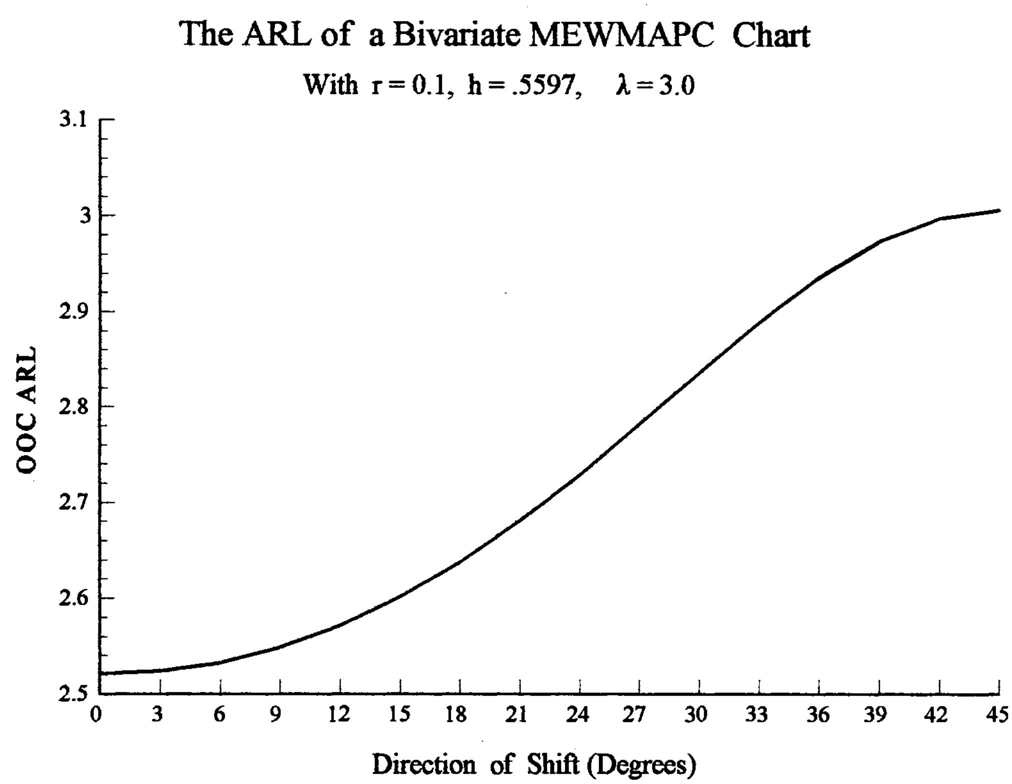

Figure 5.7 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MEWMAPC Chart With Nominal ARL = 100 Given That λ = 3.0

parameters λ with values 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0, respectively. It is clear that the bivariate MEWMAPC chart is very effective in detecting the mean shift which is along either one of the axes of the principal components. However, it is less effective in detecting a given shift which is far away from both of these axes. This phenomenon can best be explained by the follows.

(1) For a bivariate process under a given size of mean vector shift λ, the size of the mean shift of a particular principal component increases as the location of the shift moves toward it's axis and decreases as the location of the shift moves away from it's axis. Eventually, the size of the mean shift of one of the principal components will reach to it's largest, which is λ, and the other will reach 0. It is well-known that the control chart is more effective in detecting a large size mean shift than a small size mean shift. Thus, the ARL of a classical principal component chart will be the smallest if the size of the corresponding mean shift equals λ. Note that if the difference of the mean shifts in both principal components is large, the ARL performance of the multivariate principal component chart is dominated by the ARL of the individual principal component chart which encounters a large mean shift. Therefore, the OOC

ARL of the MEWMAPC chart for a given value of λ will be the smallest if the location of the shift is on one of the axes of the principal components.

(2) From equation 5.1, it is easily verified that the mean shifts of both principal components are equivalent at $\theta = 45°$. The ARL of both IEWMAPC charts is equivalent in this case. Because that the IEWMAPC charts are mutually independent, the ARL of the MEWMAPC chart will be maximized at this direction given a fixed size of shift λ. Thus, the ARL of the MEWMAPC chart for a given size of mean vector shift λ will be the largest if the direction of the mean vector shift generates an equivalent size of mean shift in each principal component.

Tables 5.4 to 5.6 show the ARL performance of various trivariate MEWMAPC charts for various values of r and λ. Analogous to the bivariate case, a small value of r provides better protection against small mean shifts and a large value of r is good at detecting a large mean shift in terms of λ.

Figures 5.8 to 5.13 display the ARL performance of a trivariate MEWMAPC chart with parameters $r = 0.1$ and $h = 0.5964$ (nominal ARL = 100) at different directions for the values of $\lambda = 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0 respectively. In these figures, the directions or forms of the shift of the mean vector under investigation are listed as follows:

Table 5.4

ARL VALUES FOR MEWMAPC CHARTS (p = 3)
NOMINAL ARL = 100

λ	r = 0.1 h = 0.5964	r = 0.2 h = 0.9250	r = 0.3 h = 1.1963
0.0	100.000	100.000	100.000
0.5	23.606-26.307	27.785-31.077	33.066-36.679
1.0	9.134-11.192	9.035-11.284	9.800-12.542
1.5	5.543-7.092	4.992-6.438	4.931-6.510
2.0	4.018-5.258	3.466-4.528	3.265-4.318
2.5	3.188-4.223	2.696-3.535	2.480-3.262
3.0	2.665-3.559	2.249-2.932	2.037-2.657

Table 5.4 (Continued)

λ	r = 0.4 h = 1.4429	r = 0.5 h = 1.6785	r = 0.6 h = 1.9116
0.0	100.000	100.000	100.000
0.5	38.621-42.265	44.190-47.658	49.658-52.817
1.0	11.105-14.459	12.896-16.886	15.188-19.767
1.5	5.122-6.976	5.519-7.770	6.138-8.894
2.0	3.209-4.349	3.253-4.561	3.392-4.956
2.5	2.361-3.158	2.300-3.163	2.292-3.268
3.0	1.892-2.510	1.797-2.435	1.740-2.421

Table 5.4 (Continued)

λ	r = 0.7 h = 2.1481	r = 0.8 h = 2.3935	r = 0.9 h = 2.6535
0.0	100.000	100.000	100.000
0.5	54.964-57.739	60.065-62.423	64.625-66.866
1.0	18.022-23.086	21.447-26.841	25.508-31.032
1.5	7.026-10.377	8.258-12.268	9.940-14.630
2.0	3.642-5.559	4.038-6.421	4.640-7.608
2.5	2.337-3.489	2.446-3.856	2.644-4.419
3.0	1.718-2.474	1.730-2.610	1.782-2.859

Table 5.5

ARL VALUES FOR MEWMAPC CHARTS (p = 3)
NOMINAL ARL = 200

λ	r = 0.1 h = 0.6585	r = 0.2 h = 1.0060	r = 0.3 h = 1.2930
0.0	200.000	200.000	200.000
0.5	31.089-37.033	40.448-48.485	51.871-61.007
1.0	10.581-13.685	10.828-14.637	12.329-17.415
1.5	6.224-8.258	5.639-7.688	5.681-8.090
2.0	4.454-6.016	3.826-5.209	3.627-5.068
2.5	3.509-4.773	2.936-3.987	2.705-3.713
3.0	2.922-3.989	2.421-3.264	2.200-2.966

Table 5.5 (Continued)

λ	r = 0.4 h = 1.5541	r = 0.5 h = 1.8041	r = 0.6 h = 2.0517
0.0	200.000	200.000	200.000
0.5	63.958-73.300	76.182-85.118	82.268-96.417
1.0	14.726-21.348	18.046-26.241	22.345-32.027
1.5	6.071-9.109	6.783-10.699	7.866-12.895
2.0	3.615-5.262	3.740-5.744	4.006-6.538
2.5	2.595-3.659	2.560-3.763	2.590-4.026
3.0	2.059-2.834	1.968-2.799	1.920-2.851

Table 5.5 (Continued)

λ	r = 0.7 h = 2.3035	r = 0.8 h = 2.5653	r = 0.9 h = 2.8431
0.0	200.000	200.000	200.000
0.5	100.039-107.190	111.366-117.431	122.144-127.124
1.0	27.752-38.689	34.392-46.230	42.364-54.651
1.5	9.424-15.769	11.611-19.421	14.634-23.971
2.0	4.448-7.710	5.133-9.354	6.172-11.599
2.5	2.696-4.484	2.899-5.202	3.245-6.271
3.0	1.914-3.007	1.955-3.301	2.056-3.791

Table 5.6

ARL VALUES FOR MEWMAPC CHARTS (p = 3)
NOMINAL ARL = 370

λ	r = 0.1 h = 0.7084	r = 0.2 h = 1.0721	r = 0.3 h = 1.3726
0.0	370.000	370.000	370.000
0.5	39.168-50.263	56.396-72.867	77.593-96.969
1.0	11.877-16.149	12.620-18.500	15.100-23.614
1.5	6.805-9.364	6.227-8.942	6.406-9.840
2.0	4.818-6.676	4.140-5.845	3.957-5.821
2.5	3.773-5.242	3.144-4.392	2.902-4.140
3.0	3.133-4.352	2.569-3.557	2.337-3.251

Table 5.6 (Continued)

λ	r = 0.4 h = 1.6462	r = 0.5 h = 1.9086	r = 0.6 h = 2.1688
0.0	370.000	370.000	370.000
0.5	100.460-120.616	123.958-143.436	147.466-165.344
1.0	19.016-30.674	24.483-39.443	31.728-49.851
1.5	7.049-11.670	8.163-14.441	9.861-18.251
2.0	4.001-6.249	4.225-7.113	4.649-8.484
2.5	2.806-4.161	2.802-4.401	2.882-4.884
3.0	2.202-3.142	2.121-3.161	2.088-3.305

Table 5.6 (Continued)

λ	r = 0.7 h = 2.4337	r = 0.8 h = 2.7095	r = 0.9 h = 3.0024
0.0	370.000	370.000	370.000
0.5	170.546-186.299	192.859-206.260	214.143-225.182
1.0	41.018-61.890	52.617-75.572	66.741-90.897
1.5	12.335-23.250	15.862-29.622	20.813-37.582
2.0	5.337-10.488	6.404-13.297	8.035-17.13
2.5	3.064-5.681	3.389-6.905	3.931-8.716
3.0	2.107-3.605	2.188-4.125	2.356-4.965

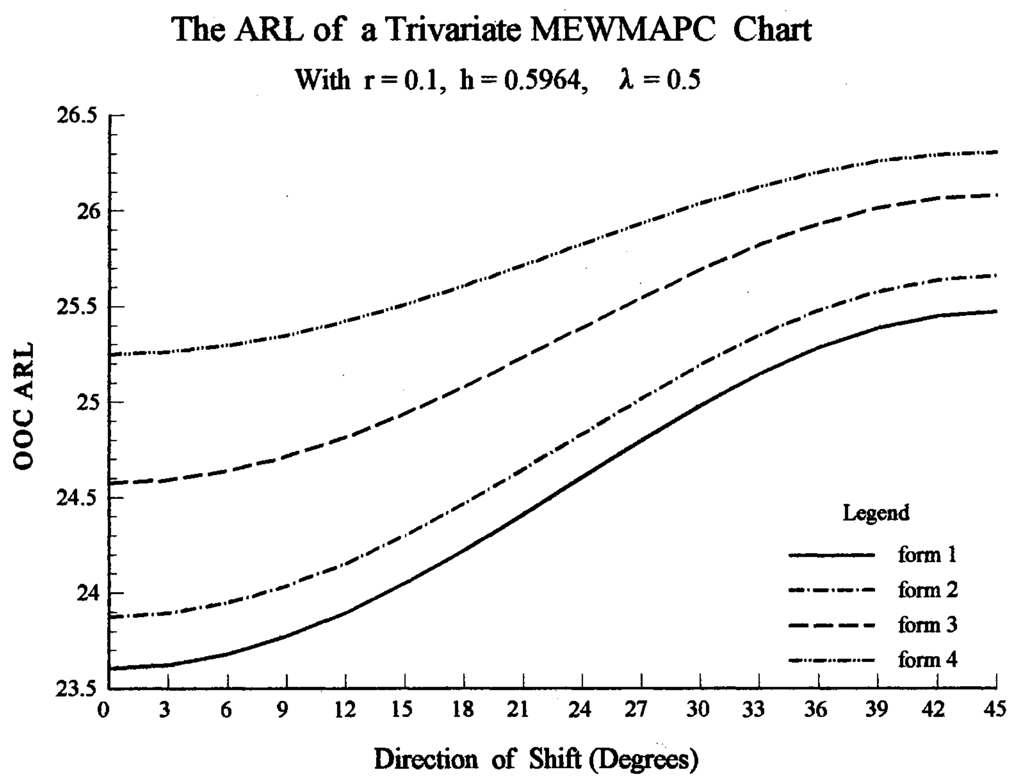

Figure 5.8 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 0.5$

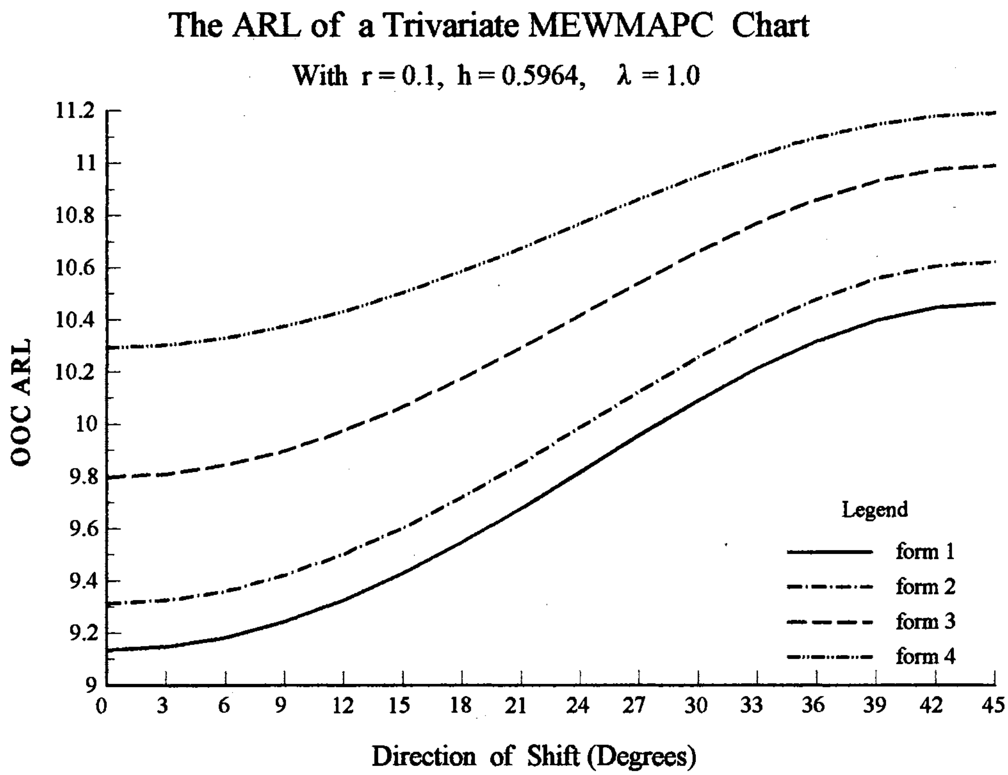

Figure 5.9 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 1.0$

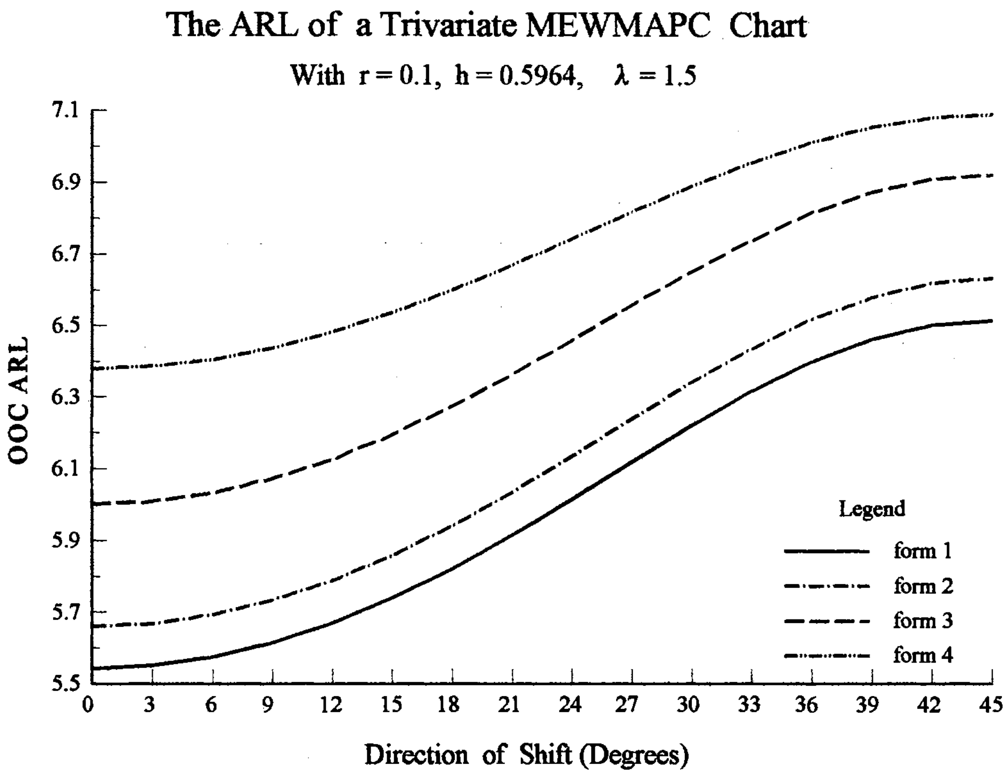

Figure 5.10 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC chart With Nominal ARL = 100 Given That $\lambda = 1.5$

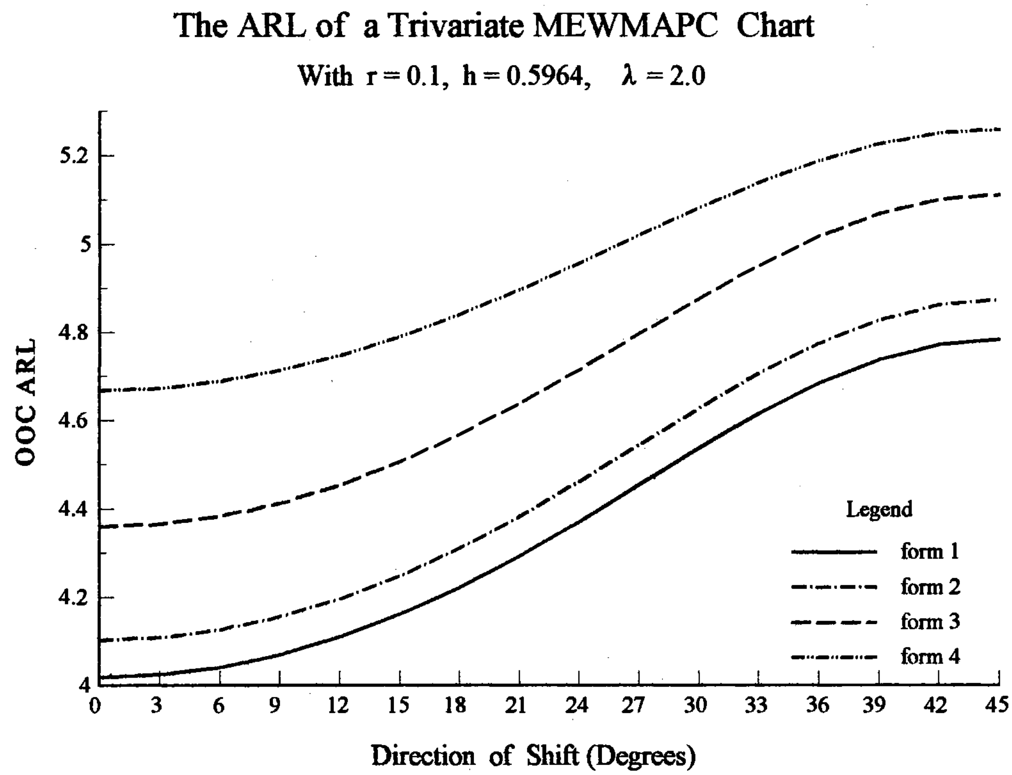

Figure 5.11 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 2.0$

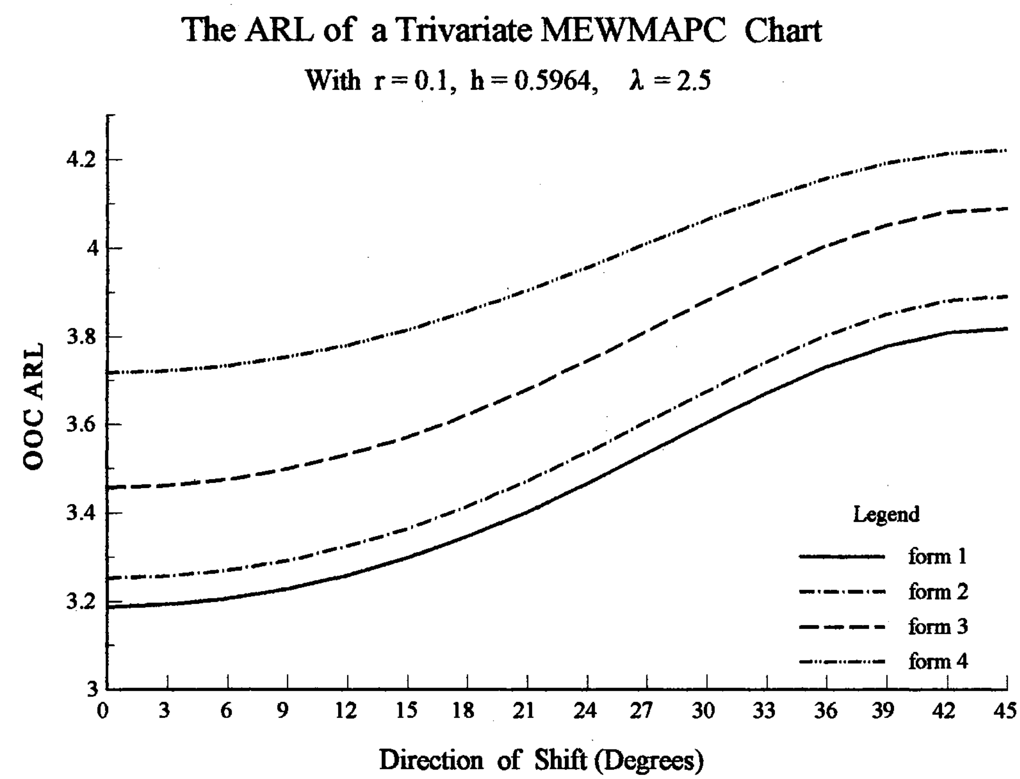

Figure 5.12 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 2.5$

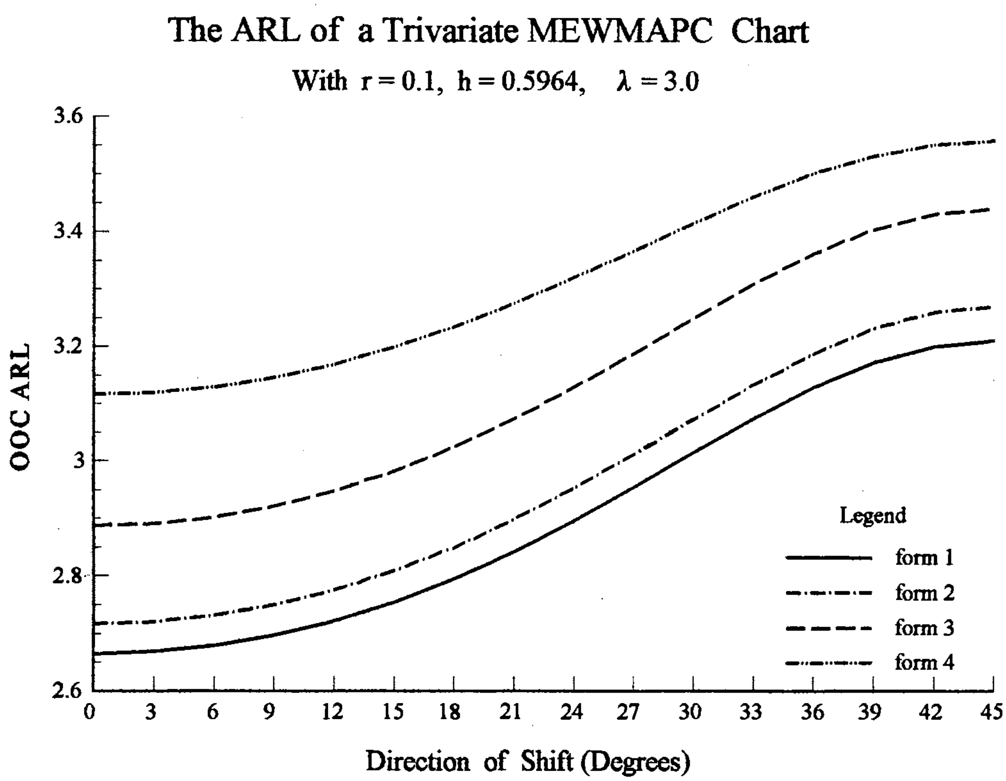

Figure 5.13 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MEWMAPC Chart With Nominal ARL = 100 Given That $\lambda = 3.0$

$$\begin{bmatrix}\delta\cos\theta\\ \delta\sin\theta\\ 0\end{bmatrix}, \begin{bmatrix}\sqrt{\frac{24}{5}}\delta\cos\theta\\ \sqrt{\frac{24}{5}}\delta\sin\theta\\ \frac{1}{5}\delta\end{bmatrix}, \begin{bmatrix}\sqrt{\frac{21}{5}}\delta\cos\theta\\ \sqrt{\frac{21}{5}}\delta\sin\theta\\ \frac{2}{5}\delta\end{bmatrix}, \begin{bmatrix}\sqrt{\frac{2}{3}}\delta\cos\theta\\ \sqrt{\frac{2}{3}}\delta\sin\theta\\ \sqrt{\frac{1}{3}}\delta\end{bmatrix} \quad (5.2)$$

Forms 1 to 4 in equation 5.2 are selected such that the shift of one of the principal components is at a fixed value while varying the directions of the other two, respectively. Also, δ and θ are defined the same as previously. The last form of shift is selected for the purpose of having equal shifts among all three principal components at $\theta = 45°$.

Figures 5.8 to 5.13 clearly show that within each form of the mean vector shift, the ARL is the smallest at $\theta = 0°$ and the ARL is the largest at $\theta = 45°$. Thus, it is obvious that for a given size shift λ, the trivariate MEWMAPC chart is very effective in detecting the mean shift which is along or nearby one of the axes of the principal components. However, it is less effective in detecting a given shift which is away from all the axes. For example, given a fixed value of λ, a trivariate MEWMAPC chart performs best in detecting the mean vector shift at a direction that result in the mean shift of only one principal component. On the other hand, it performs the worst if the direction of the mean vector shift generates equal amount of mean shift in all three principal components.

The Classical MZONEPC Charts

Tables 5.7 and 5.8 show the ARL performance of the classical design of MZONEPC charts for both bivariate and trivariate cases, respectively. Each chart is calibrated to have nominal ARL values of 100, 200 and 370.

Figures 5.14 to 5.19 display the ARL performance of a bivariate MZONEPC chart with nominal ARL = 100 at different directions for the values of λ = 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0, respectively. The directions of the shift of the mean vector presented in these figures are selected according to those used under the bivariate MEWMAPC charts.

Furthermore, Figures 5.20 to 5.25 show the ARL performance of a trivariate MZONEPC chart with in-control ARL of 100 under the directions of the shifts which are equivalent to those four forms described previously for λ = 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0, respectively. It is observed from these figures that the MZONEPC chart is also effective in detecting the mean vector shifts which are located either along or nearby one of the axes of the principal components and it is less effective in detecting a given shift which is away from all the axes.

The Comparisons

Tables 5.9 and 5.10 show the ARL comparisons for p = 2 and 3, respectively. Each chart defined in these tables is designed so that the in-control ARL is approximately 200. The

Table 5.7

ARL Values For MZONEPC Charts (p = 2)
Nominal ARL = 100, 200, 370

λ	h = 3.3292	h = 3.6318	h = 3.8898
0.0	100.000	200.000	370.000
0.5	33.373-35.599	50.784-56.274	73.311-84.685
1.0	10.002-11.687	12.420-15.365	14.930-19.566
1.5	5.020-5.975	5.800-7.180	6.526-8.384
2.0	3.297-3.953	3.728-4.583	4.109-5.163
2.5	2.429-2.957	2.740-3.392	3.011-3.773
3.0	1.882-2.339	2.126-2.686	2.344-2.986

Table 5.8

ARL Values For MZONEPC Charts (p = 3)
Nominal ARL = 100, 200, 370

λ	h = 3.5029	h = 3.8002	h = 4.0536
0.0	100.000	200.000	370.000
0.5	37.806-41.845	58.817-68.960	86.476-107.872
1.0	11.108-14.499	13.824-19.910	16.651-26.496
1.5	5.434-7.318	6.250-9.034	7.011-10.852
2.0	3.538-4.780	3.973-5.614	4.356-6.415
2.5	2.605-3.575	2.916-4.113	3.184-4.600
3.0	2.019-2.858	2.267-3.271	2.485-3.629

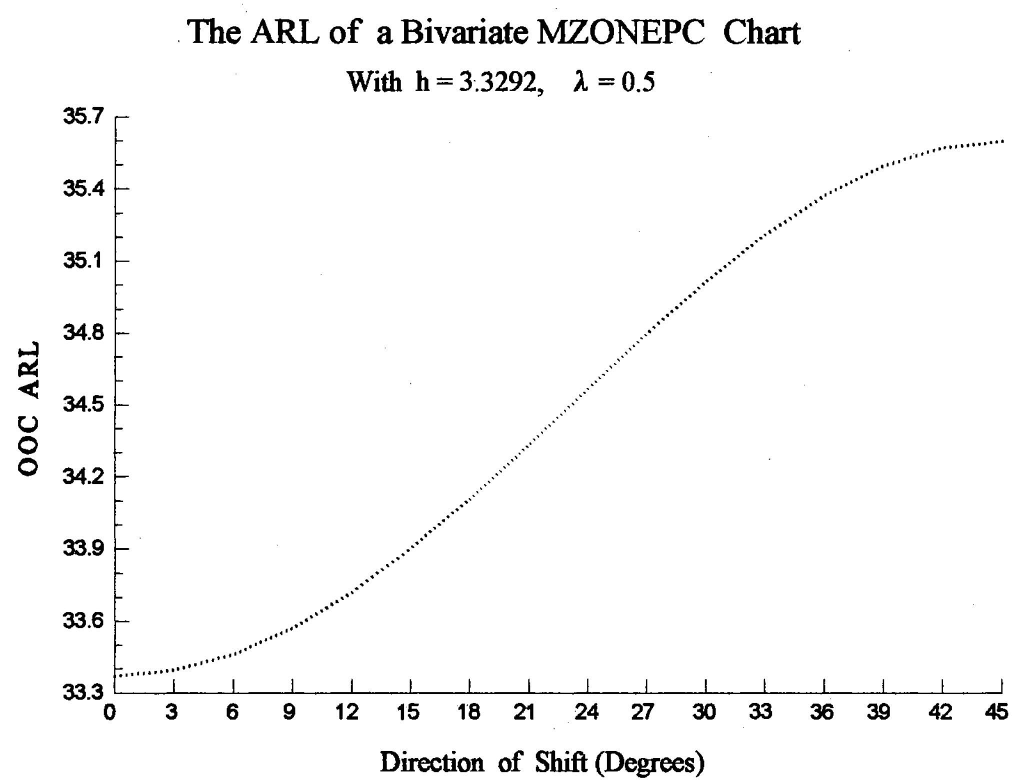

Figure 5.14 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That λ = 0.5

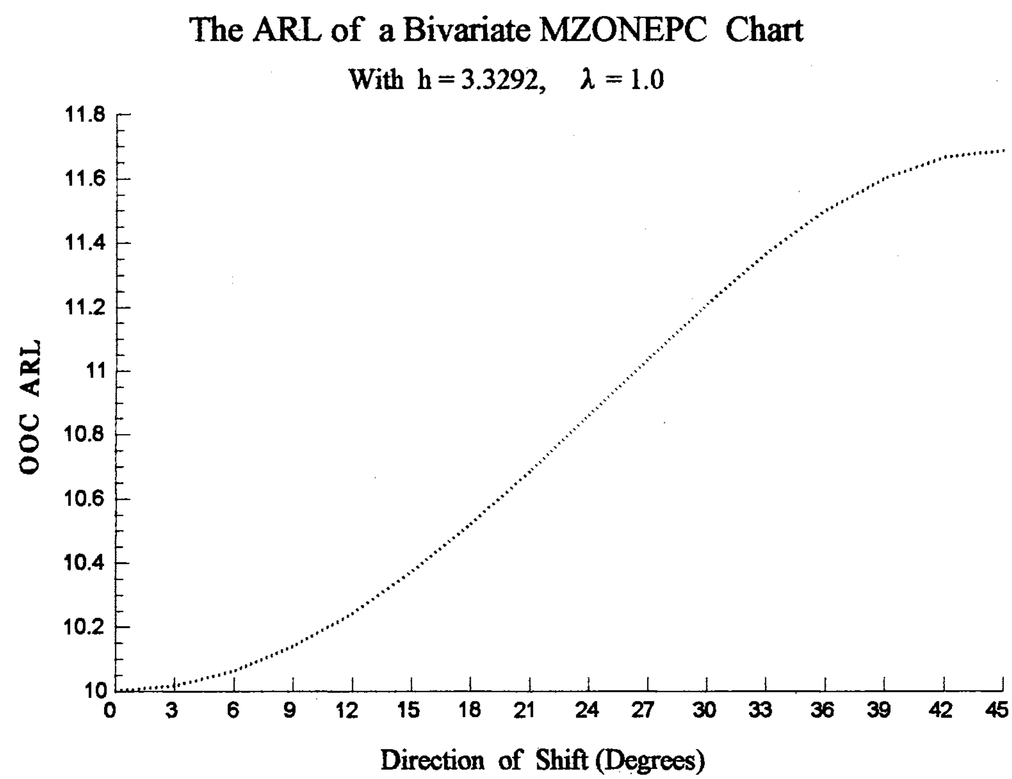

Figure 5.15 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 1.0$

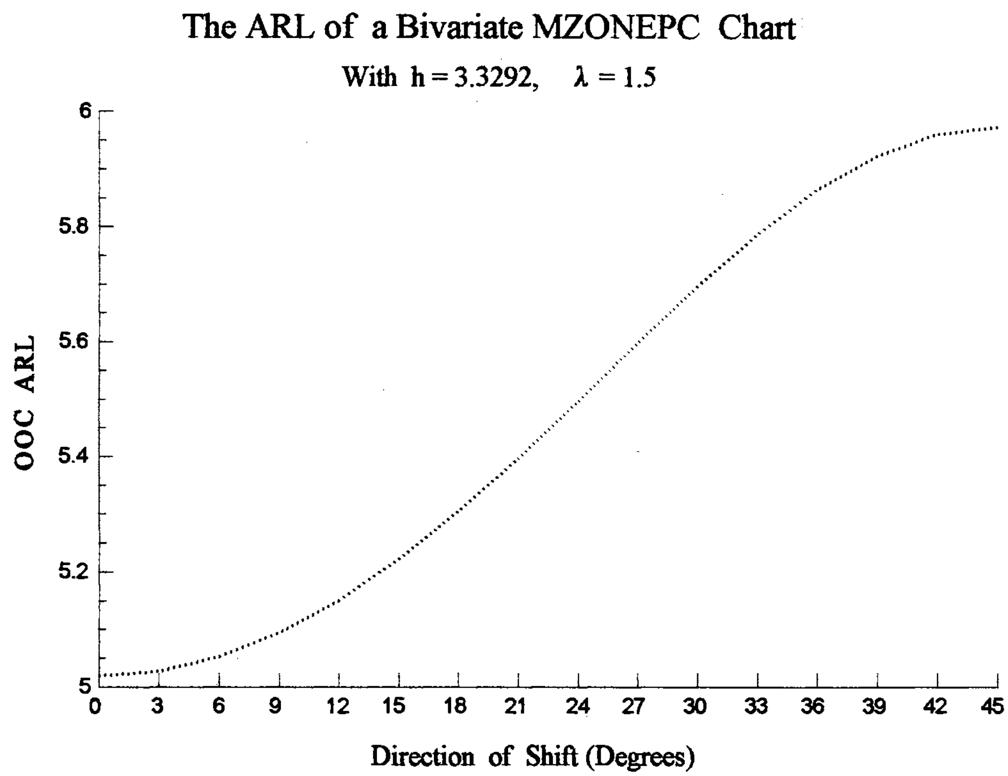

Figure 5.16 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 1.5$

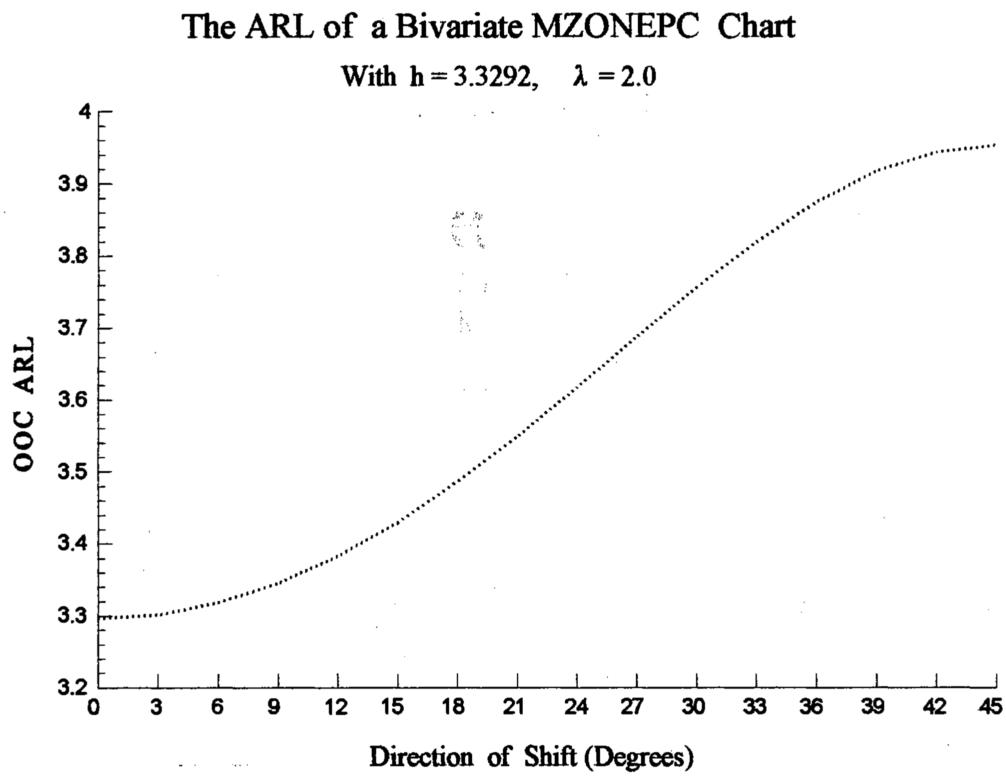

Figure 5.17 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 2.0$

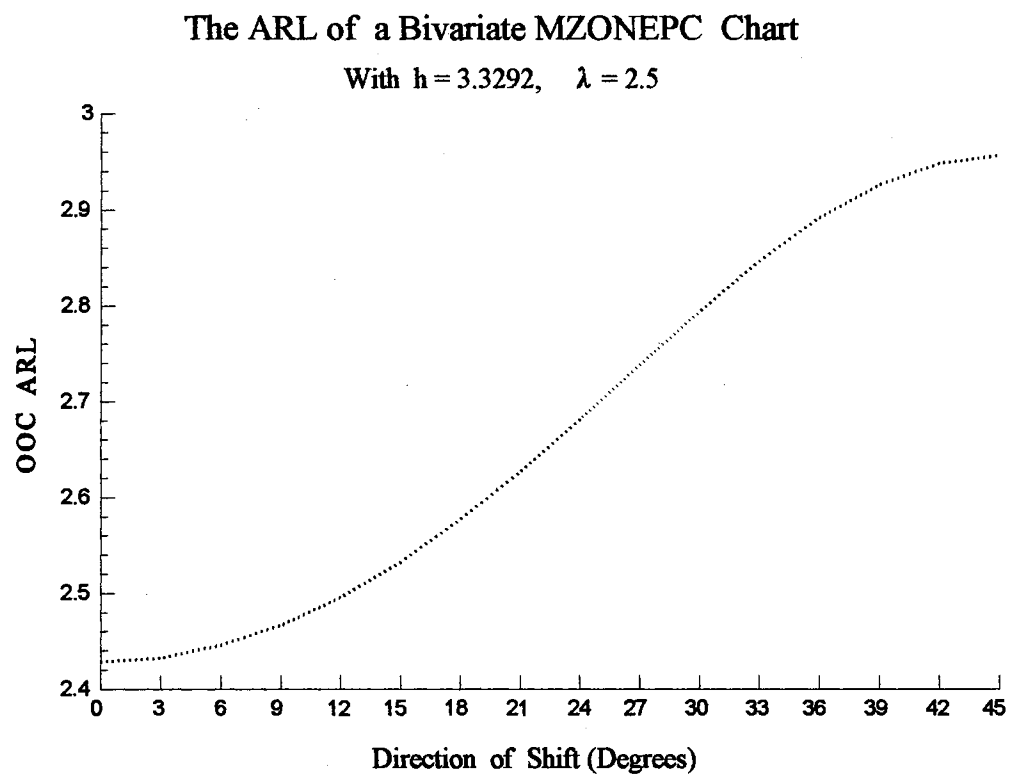

Figure 5.18 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 2.5$

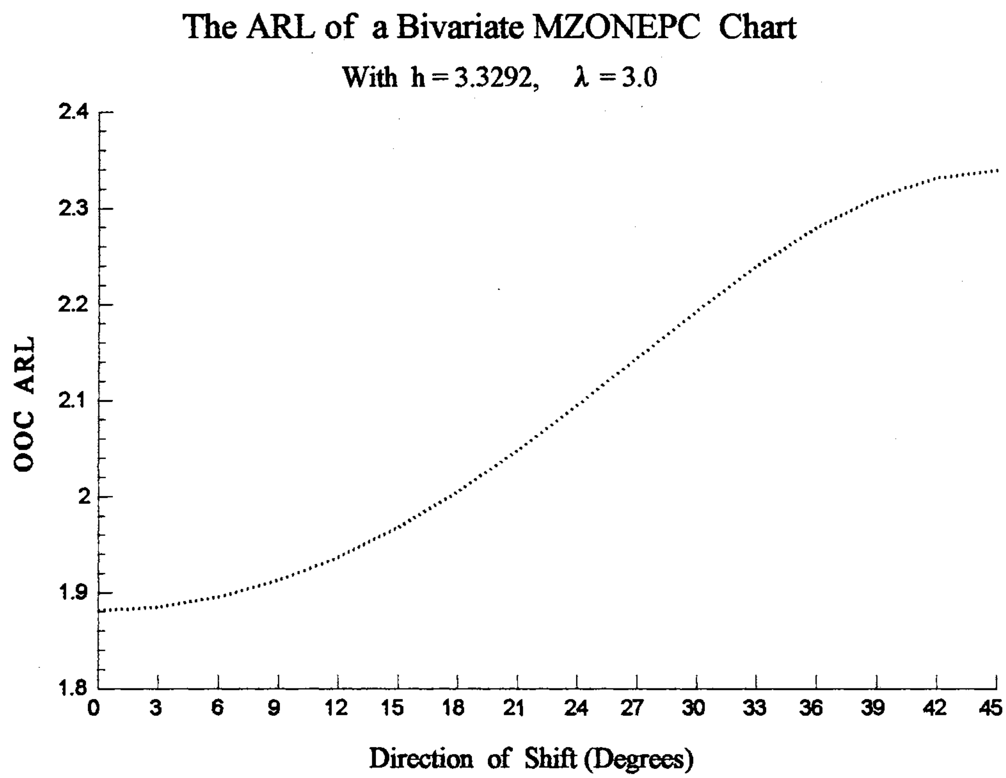

Figure 5.19 The OOC ARL At Various Directions (θ) Of Shift For A Bivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 3.0$

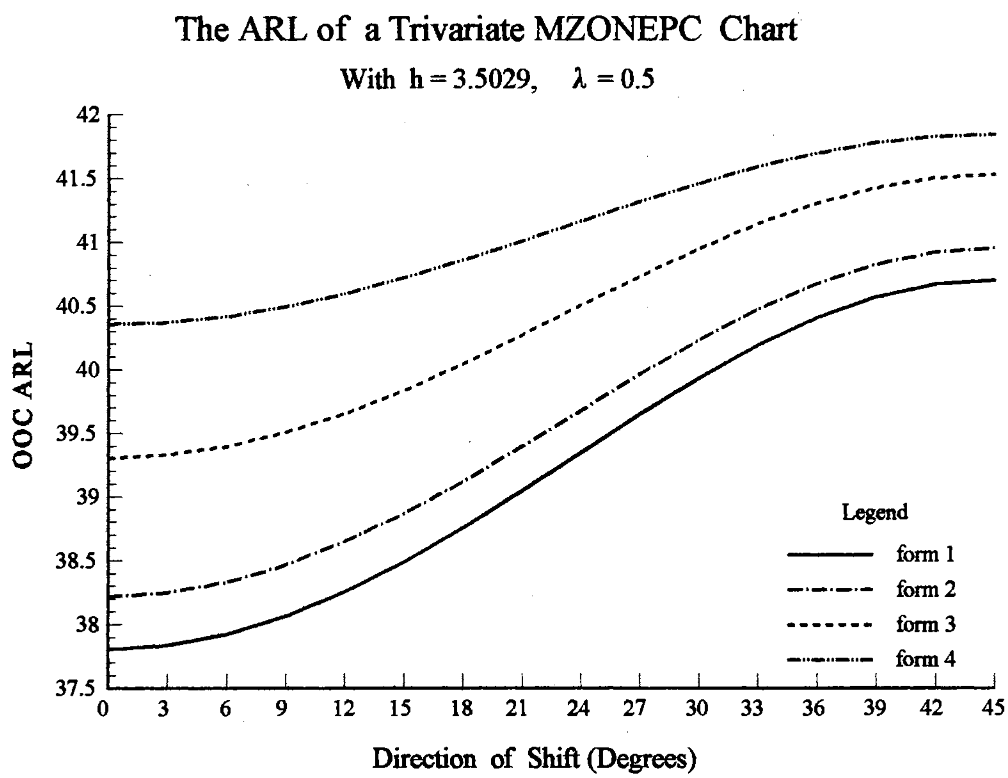

Figure 5.20 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 0.5$

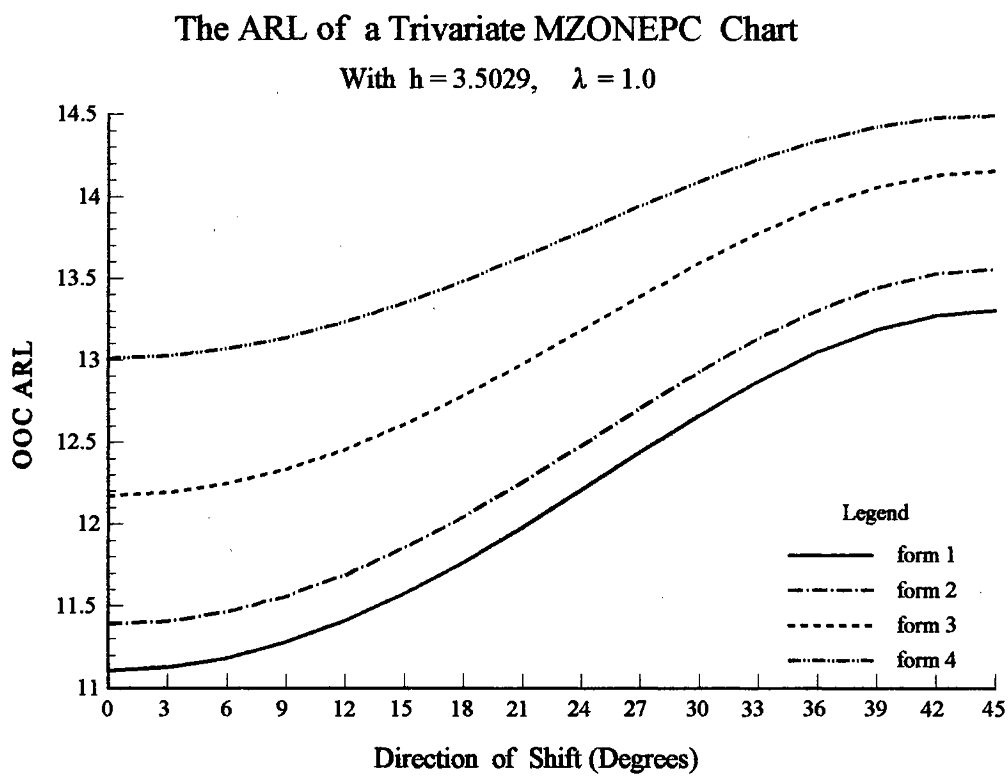

Figure 5.21 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 1.0$

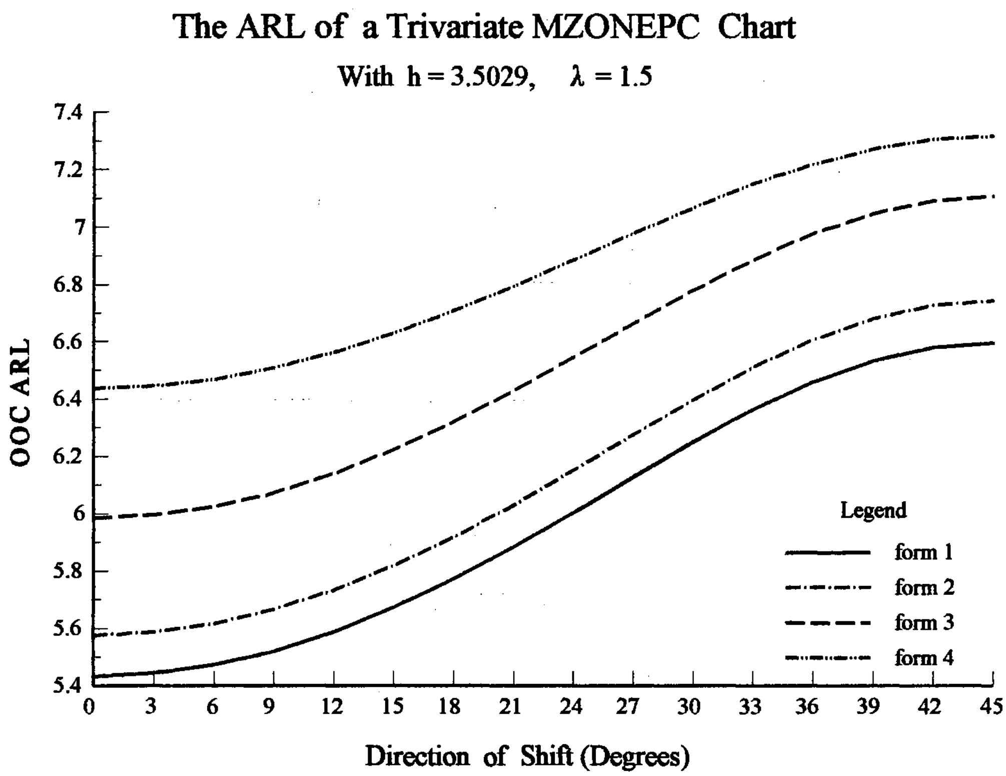

Figure 5.22 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 1.5$

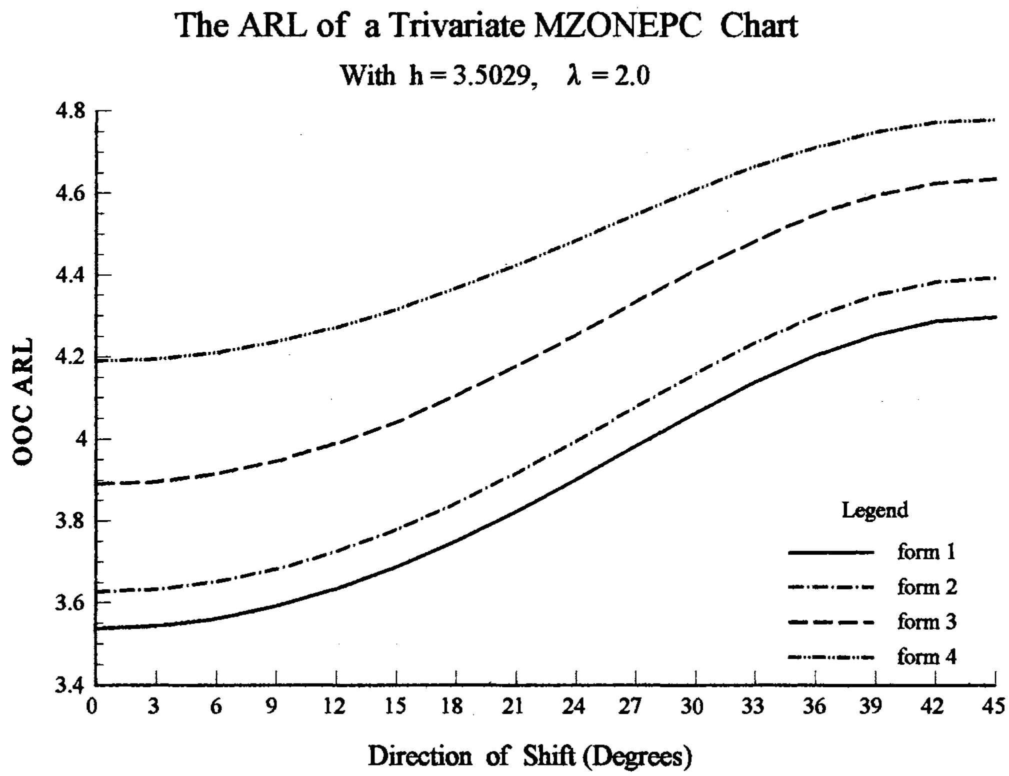

Figure 5.23 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart With Nominal ARL = 100 Given That λ = 2.0

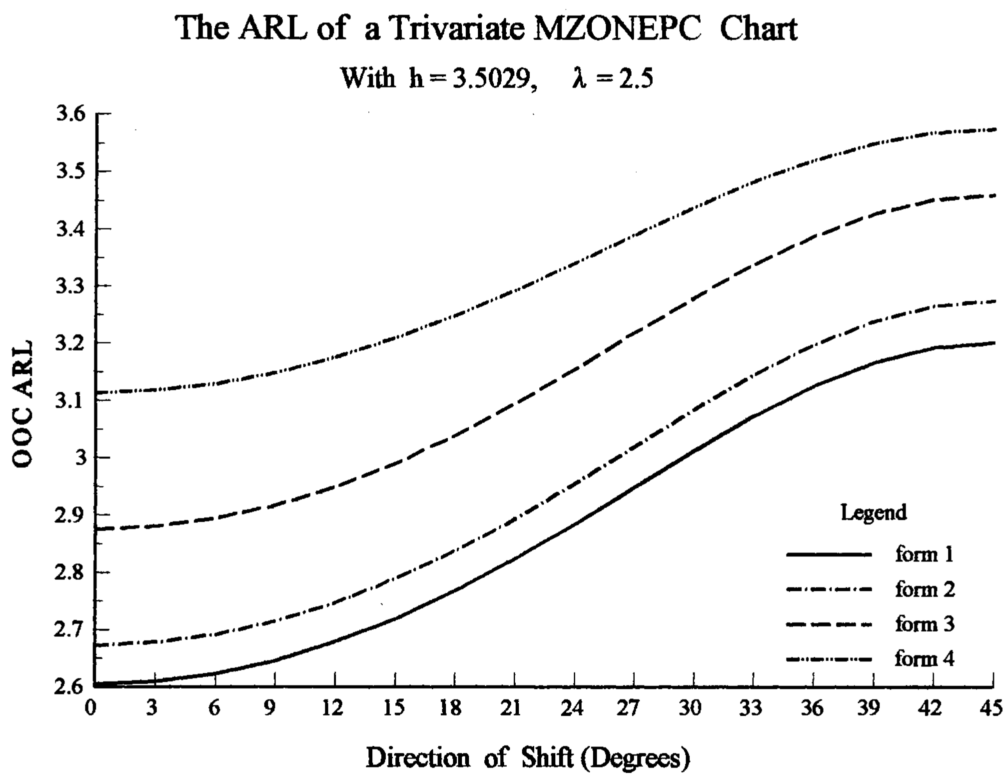

Figure 5.24 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart With Nominal ARL = 100 Given That $\lambda = 2.5$

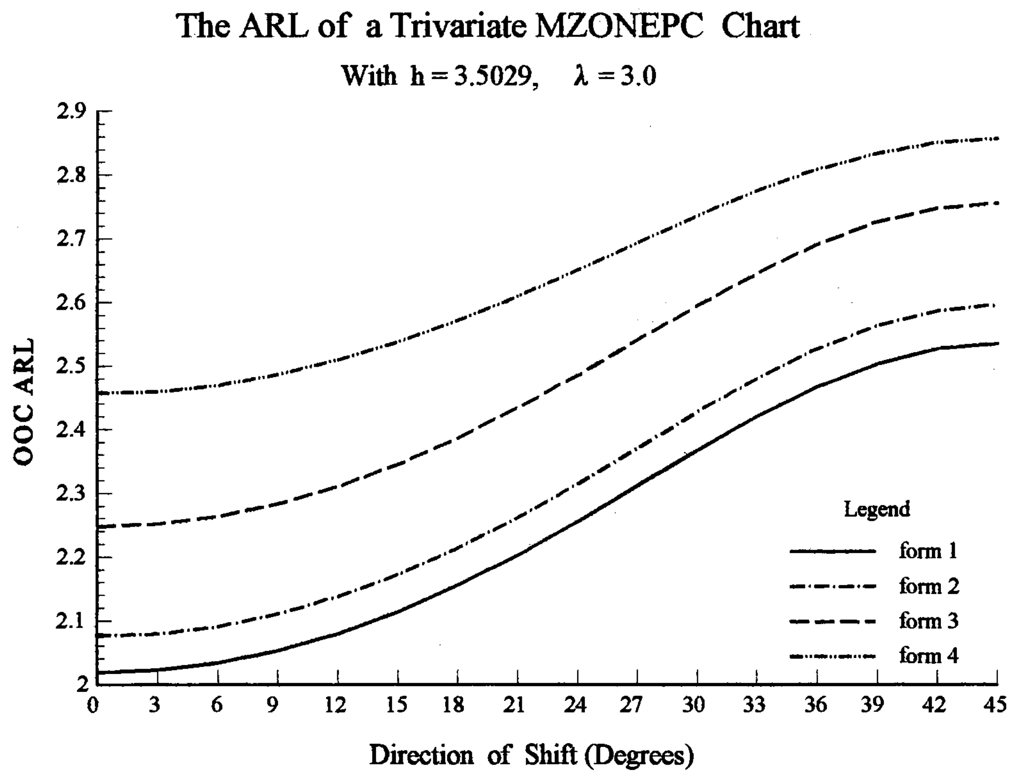

Figure 5.25 The OOC ARL At Various Directions (θ) Of Shift For A Trivariate MZONEPC Chart with Nominal ARL = 100 Given That $\lambda = 3.0$

Table 5.9

ARL COMPARISON OF SIX MULTIVARIATE CONTROL CHARTS (p = 2)

λ	χ^2 h=10.6	MCUSUM k=.50 h=5.50	MC1 k=.50 h=4.75	MEWMA r=.10 h=8.66	MEWMAPC r=.10 h=.6248	MZONEPC h=3.632
0.0	200.	200.	203.	200.	200.00	200.00
0.5	116.	28.8	31.3	28.1	27.7-30.6	50.8-56.3
1.0	42.0	9.35	9.28	10.2	9.8-11.43	12.4-15.4
1.5	15.8	5.94	5.23	6.12	5.85-6.96	5.80-7.18
2.0	6.9	4.20	3.69	4.41	4.22-5.06	3.73-4.58
2.5	3.5	3.26	2.91	3.51	3.33-4.02	2.74-3.38
3.0	2.2	2.78	2.40	2.92	2.78-3.37	2.13-2.69

Table 5.10

ARL COMPARISON OF SIX MULTIVARIATE CONTROL CHARTS (p = 3)

λ	χ^2 h=12.85	MCUSUM k=.50 h=6.88	MC1 k=.50 h=5.48	MEWMA r=.10 h=10.79	MEWMAPC r=.10 h=.6585	MZONEPC h=3.800
0.0	201.	200.	200.	202.	200.00	200.00
0.5	130.	32.7	33.5	31.8	31.1-37.0	58.8-69.0
1.0	52.6	11.2	10.1	11.30	10.6-13.7	13.8-19.9
1.5	20.5	6.69	5.66	6.69	6.22-8.29	6.25-9.03
2.0	8.8	4.70	4.00	4.86	4.45-6.02	3.97-5.61
2.5	4.4	3.83	3.17	3.83	3.51-4.77	2.92-4.11
3.0	2.6	3.17	2.63	3.2	2.92-3.99	2.27-3.27

ARL values for the MCUSUM schemes are given by Crosier (1988); for the MC1 schemes are given by Pignatiello and Runger (1990); and for the MEWMA schemes are given by Lowry et al. (1992). The in-control or nominal ARLs for schemes, MCUSUM, MC1 and MEWMA, are achieved by finding the upper control limits which produced, via simulation of in-control multivariate processes, 95% confidence intervals for the ARL which covered 200. The control limits for the Hotelling χ^2 scheme are obtained by the use of χ^2 tables.

It is clear from this comparison that the MCUSUM, MEWMA and MEWMAPC charts are more effective in detecting small shifts in the mean vector, the MC1 chart is more effective in detecting moderate shifts and the χ^2 and the MZONEPC chart are more effective in detecting larger shifts.

Figure 5.26 clearly displays that if the shifts of the mean vector are along or nearby the axes of the principal components, the bivariate MEWMAPC chart performs the best among all six charts for small values of λ.

Figure 5.27 to 5.31 show that the performance of the MEWMA and the MEWMAPC charts deteriorates compared with other charts when the value of λ grows. This result is as expected because both charts employed a small value of r. On the contrary, the performance of the MZONEPC chart is improved compared to the others as the value of λ increases.

Figures 5.27 to 5.29 show that the MC1 chart outperforms all other charts for moderate shift where λ = 1.0, 1.5 and 2.0.

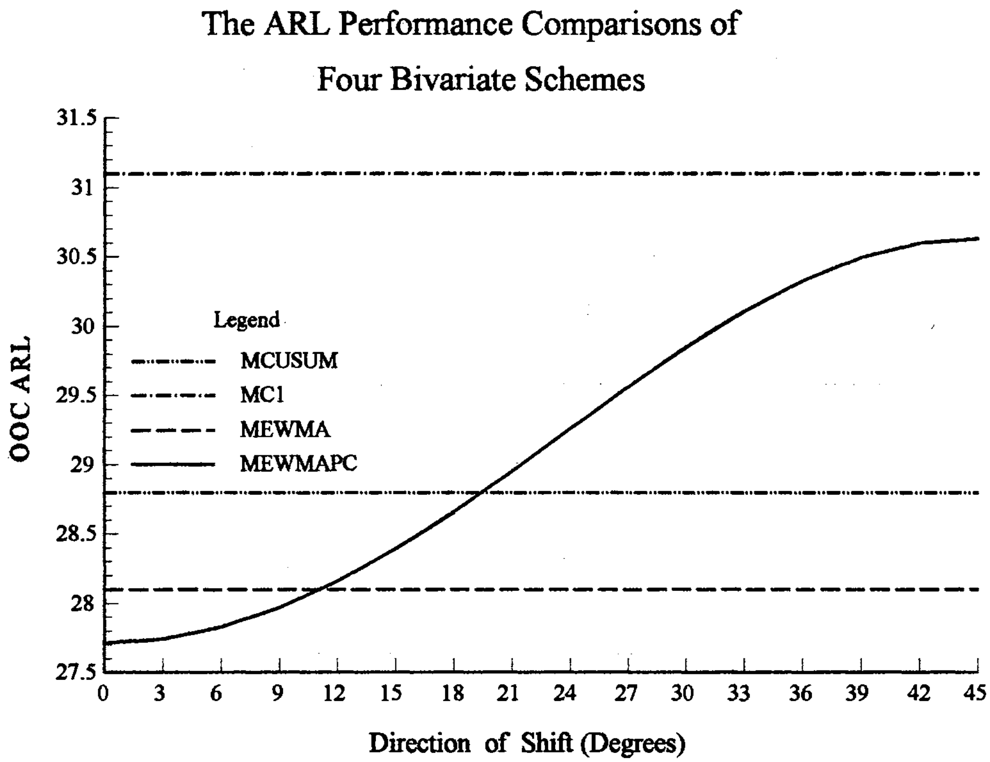

Figure 5.26 The ARL Performance Comparisons Among The Bivariate MCUSUM, MC1, MEWMA And MEWMAPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that $\lambda = 0.5$

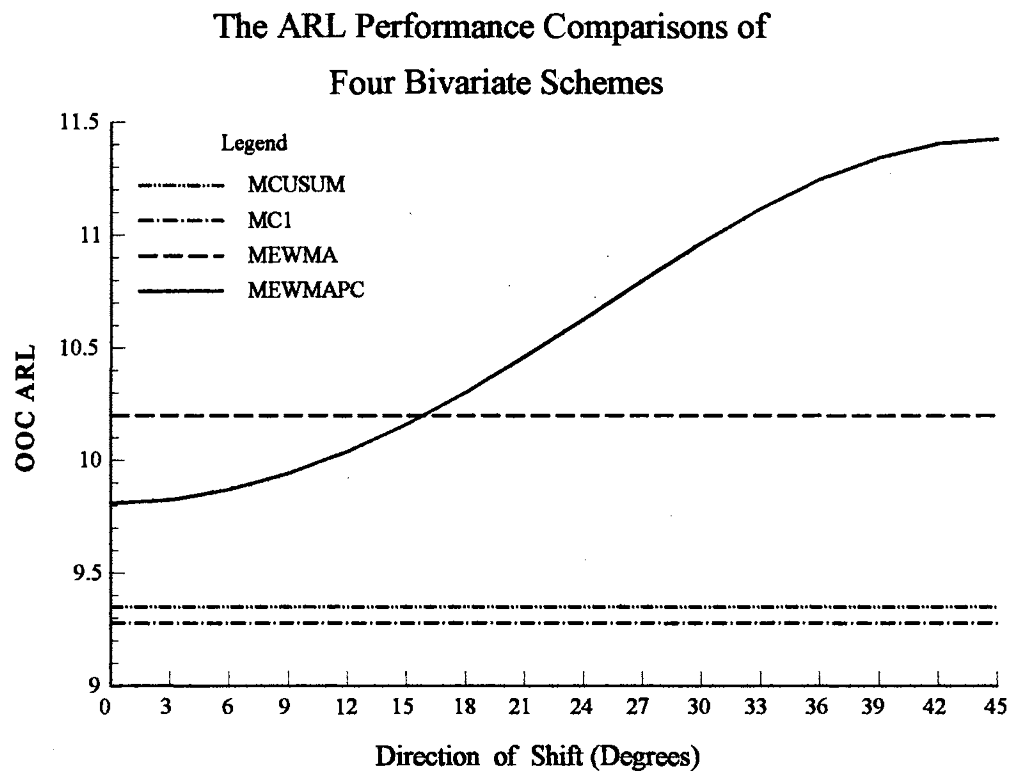

Figure 5.27 The ARL Performance Comparisons Among The Bivariate MCUSUM, MC1, MEWMA And MEWMAPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that $\lambda = 1.0$

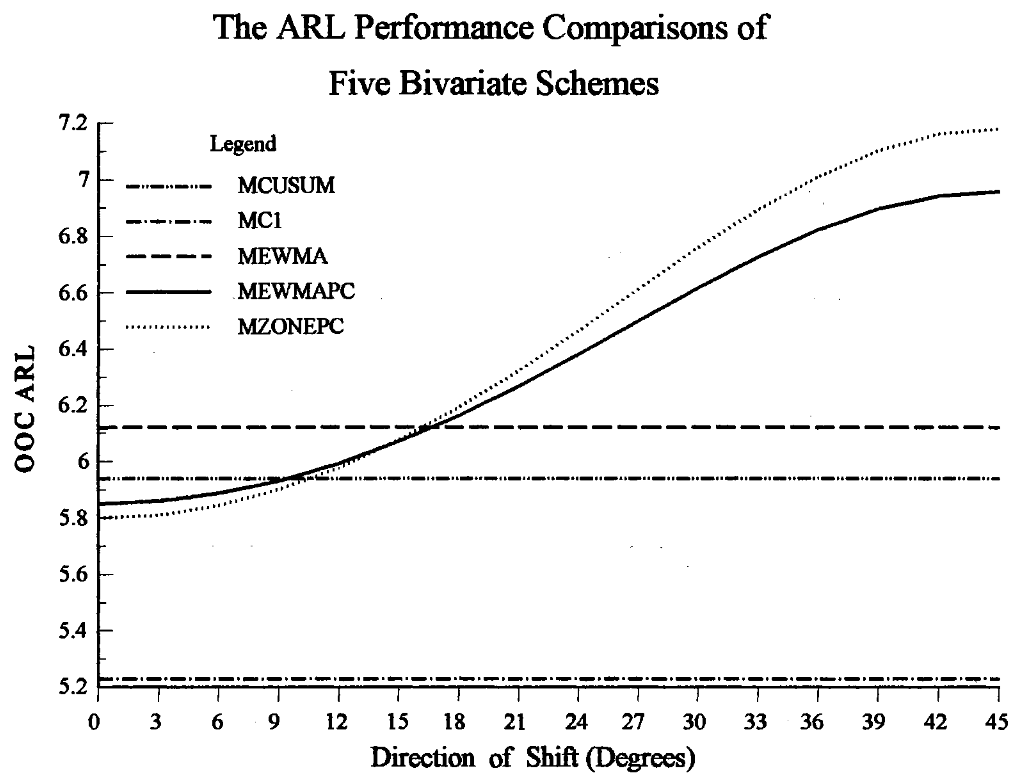

Figure 5.28 The ARL Performance Comparisons Among The Bivariate MCUSUM, MC1, MEWMA, MEWMAPC And MZONEPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that $\lambda = 1.5$

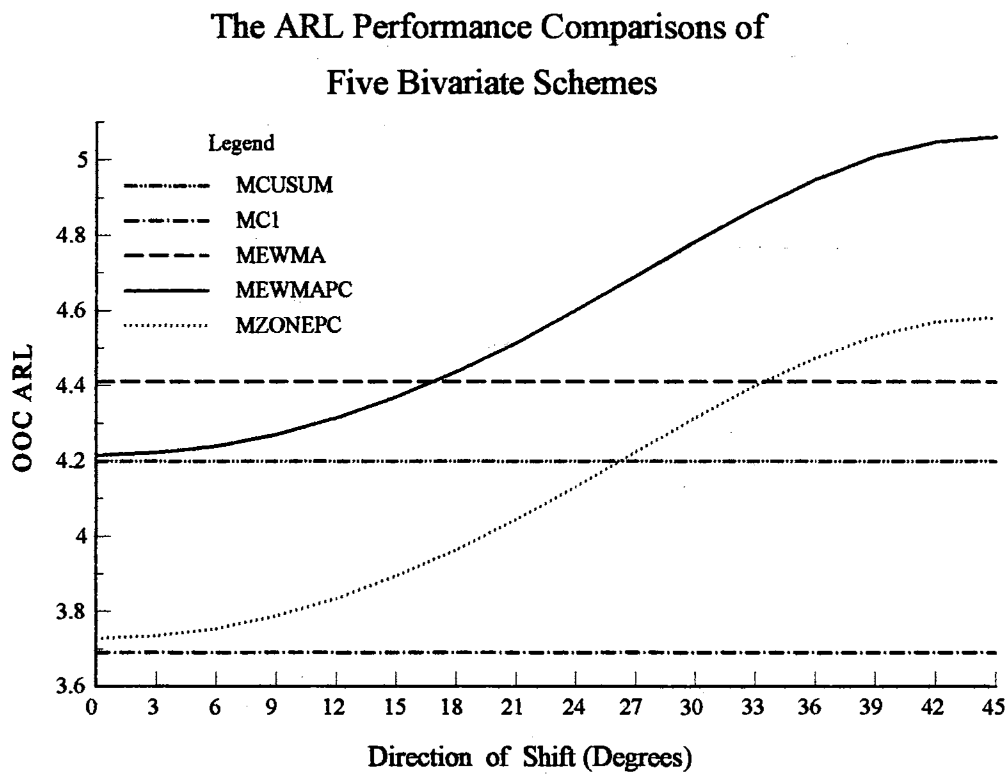

Figure 5.29 The ARL Performance Comparisons Among The Bivariate MCUSUM, MC1, MEWMA, MEWMAPC And MZONEPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that $\lambda = 2.0$

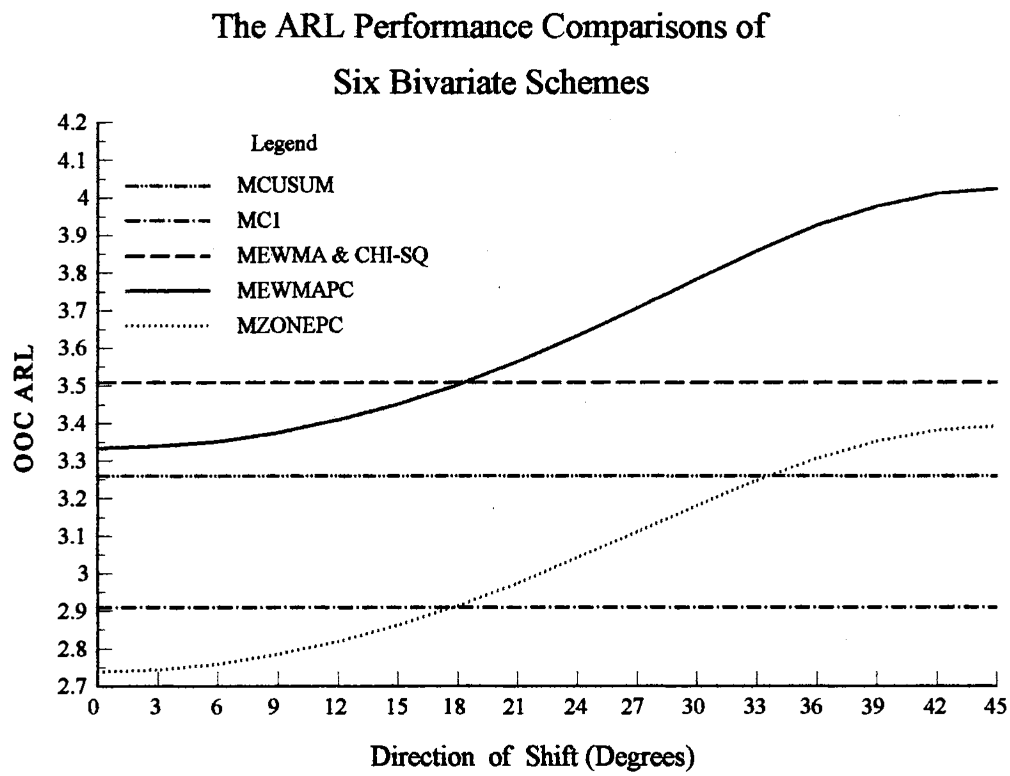

Figure 5.30 The ARL Performance Comparisons Among The Bivariate χ^2, MCUSUM, MC1, MEWMA, MEWMAPC And MZONEPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that λ = 2.5

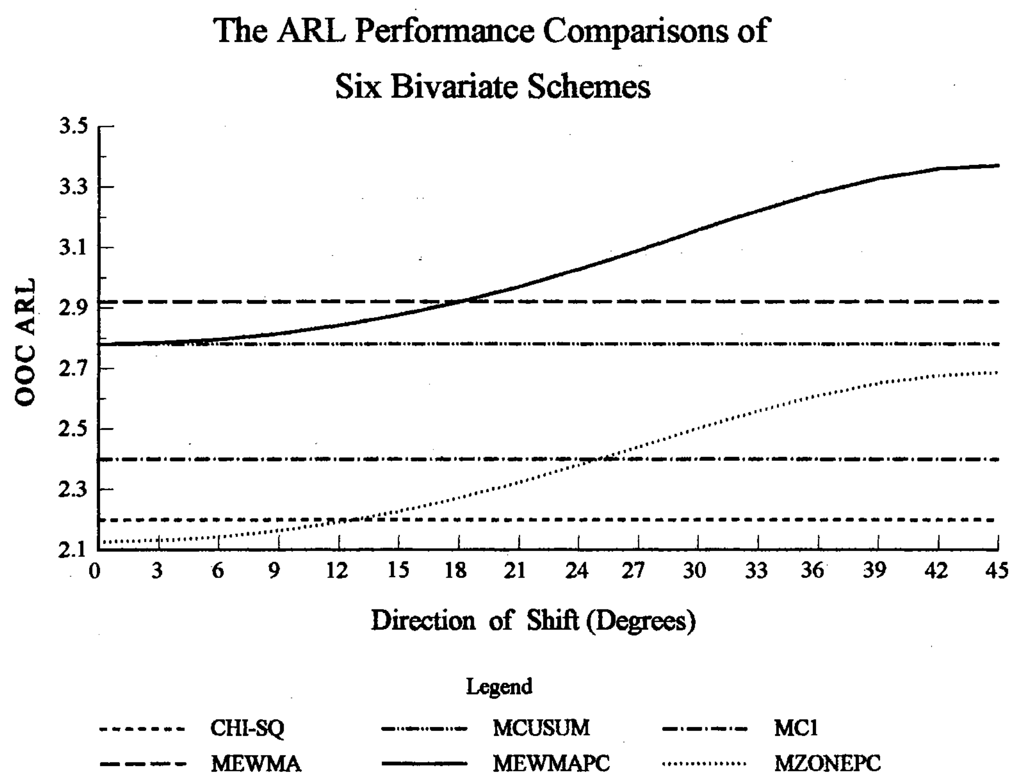

Figure 5.31 The ARL Performance Comparisons Among The Bivariate χ^2, MCUSUM, MC1, MEWMA, MEWMAPC, And MZONEPC Schemes With Nominal ARL = 200 Under Various Directions (θ) Of Shift Such that λ = 3.0

For larger values of mean shift, such as $\lambda = 2.5$ and 3.0, the MZONEPC chart performs the best among all six charts if the shifts of the mean vector are along or nearby the axes of the principal components. Note that the MZONEPC chart is not included in Figures 5.25 and 5.26 and the χ^2 chart is not included in Figures 5.25 to 5.28. This is because the performance of these charts is far worse than that of the others. In general, the ARL performance of the univariate x-bar and Zone charts is not as good as that of the CUSUM and EWMA charts when the shift is small. This phenomenon is also true for the multivariate case. Furthermore, the ARL performance comparisons for the trivariate cases lead to the same conclusions. Therefore, the graphical representations of the comparisons of the trivariate cases are not present here.

Summary

The classical design of the MEWMAPC and MZONEPC charts is introduced. Profiles of the ARL performance of several bivariate and trivariate MEWMAPC and MZONEPC charts with in-control ARLs of 100, 200, and 370 under a classical design approach are presented. An analysis of the graphics reveals that the MEWMAPC and MZONEPC charts under classical design approach perform best if the locations of the shift of the process mean vector are along or nearby one of the axes of the principal components. The charts perform worst if the location of the mean vector shift is away from all the axes of the principal components. A comparison of the ARL performance among the (1) χ^2 charts, (2) MCUSUM charts, (3) MC1 charts, (4) MEWMA charts with r = 0.1, (5) MEWMAPC charts with r = 0.1, and (6) MZONEPC charts under a classical design approach is performed. The result show that the MCUSUM, MEWMAPC and MEWMA charts are efficient in detecting small process mean shifts in terms of the noncentrality parameter λ (due to the small value of r employed). The MC1 chart is best in detecting moderate sized mean shifts and the χ^2 and MZONEPC charts are good at detecting larger mean shifts.

CHAPTER VI

THE DESIGN, COMPARISON AND ANALYSIS OF THE OPTIMAL MULTIVARIATE EWMA AND ZONE PRINCIPAL COMPONENT CONTROL CHARTS

Introduction

The optimal design of a control chart is employed when a particular shift of the process mean or variance is to be detected as quickly as possible while allowing the process to run a desired period without interruption if the process is in a SOSC. Thus, the optimal designed control chart will provide the best performance under a given circumstance. The optimal design approach in the area of the quality control chart was first introduced by Bowker and Lieberman in 1972. They develop a table for selecting the parameters of an optimal CUSUM chart with a V-mask. Gan (1991b) reviews the optimal design of CUSUM control charts. He presents plots which enable the parameters of an optimal chart to be determined easily. Gan also recommends a four-step procedure to design an optimal CUSUM control chart.

Crowder (1989) reviews the design procedures of the EWMA control schemes. He presents various plots that can be used to identify quickly the optimal EWMA chart parameters.

The selected parameters are optimal in the sense that for a fixed in-control ARL, they produce the smallest possible out-of-control ARL for a specified shift in the process mean. Lucas and Saccucci (1990) provide a table of chart parameters for a similarly defined optimal EWMA chart.

One important aspect that needs to be determined prior to the design of the optimal multivariate principal component chart is the relationship among the parameters of each individual principal component chart. In the optimal design of the MEWMAPC chart, for example, the criterion regarding the relationship among each optimal IEWMAPC chart can be any one of the following: (1) common r and common h, (2) common r and different h, (3) common h and different r and (4) different r and different h. These criteria are coded as C1 to C4 throughout this research, respectively. It is natural to use C4 among all IEWMAPC charts in order to obtain a MEWMAPC chart with the best performance. Similar arguments can be followed for the design of the MZONEPC chart. Thus, for the optimal design of the MZONEPC chart, different h is employed in each IZONEPC chart.

Without loss of generality, the covariance matrix used in this chapter is the identity matrix and the in-control mean vector used is **0** unless otherwise addressed.

This chapter starts by showing that the ARL performance of the optimal MEWMAPC chart derived using C4 and C2 are very similar. Therefore, for the purposes of facilitating the optimization process and easier application, C2 is adapted in this research. Furthermore, C3 is not investigated in this research due to the increase in complexity on the computation of the EWMA statistics by using a different r for each principal component. However, it is intuitively appealing that the performance of the MEWMAPC chart using C3 should be similar to that using C2.

Only Lowry's (1992) MEWMA scheme provides optimal design and evaluation. Therefore, all optimal design of MEWMAPC and MZONEPC charts are made in comparison with the MEWMA chart only. Moreover, the ARL performance of the optimal bivariate MEWMAPC charts given that the covariance matrix has diagonal elements of 1 and off-diagonal elements of $\rho = 0.2$, 0.5 and 0.8 is also discussed. In this case, the mean shifts are measured with respect to the original variables instead of the principal components and the shifts are calibrated to have a statistical distance λ of 0.5.

Finally, sensitivity analyses are performed. A bivariate process with in-control mean vector of $\mathbf{0}$ and the covariance matrix with 1 in the diagonal and 0.5 in the off diagonal is selected as the basis or real environment for study and illustration.

All optimal MEWMAPC and MZONEPC charts discussed in this chapter are derived by computer search and optimization procedures. The bounds in the computer search procedure with respect to parameters of each control chart are described as follows.

(1) The bound of the symmetrical control limit h for each optimal standardized IEWMAPC chart is set at 5.5 σ_{EWMA}, where

$$\sigma_{EWMA} = \sqrt{\frac{r_i}{(2-r_i)}}\,\sigma .$$

σ_{EWMA} represents the asymptotic standard deviation of the EWMA statistics of the i^{th} IEWMAPC chart, $\sigma = 1$ is the standard deviation of the standardized principal component, and r_i is the weighing factor for the i^{th} IEWMAPC chart.

(2) The upper and lower bound for the parameter r_i of each optimal IEWMAPC chart is set at 1 and 0.03, respectively.

(3) The bound for the symmetrical control limit h of each optimal standardized IZONEPC chart is set at 6.0σ and $\sigma = 1$ is the standard deviation of the standardized principal component.

The Optimal Design Of The MEWMAPC And The MZONEPC Charts

Comparisons Of The Optimal MEWMAPC Charts Using C2 and C4

The criteria employed in the design of the MEWMAPC charts have been discussed previously. Tables 6.1 and 6.2 illustrate

Table 6.1

COMPARISON OF THE ARL PERFORMANCE FOR THE BIVARIATE OPTIMAL MEWMAPC CHARTS UNDER C2 AND C4

λ	$\theta = 0°$		$\theta = 15°$	
	C2	C4	C2	C4
0.0	200.000	200.000	200.000	200.000
0.5	21.9676	21.9676	23.0064	23.0063
1.0	8.3180	8.3180	8.7468	8.7468
1.5	4.5925	4.5925	4.8328	4.8328
2.0	3.0063	3.0063	3.1646	3.1646
2.5	2.1455	2.1455	2.2631	2.2631
3.0	1.6186	1.6186	1.7066	1.7066

Table 6.1 (Continued)

λ	$\theta = 30°$		$\theta = 45°$	
	C2	C4	C2	C4
0.0	200.000	200.000	200.000	200.000
0.5	26.3217	26.3128	28.7473	28.7473
1.0	10.1598	10.1594	11.3033	11.3033
1.5	5.6355	5.6355	6.3121	6.3121
2.0	3.6931	3.6930	4.1455	4.1455
2.5	2.6544	2.6540	2.9889	2.9889
3.0	2.0081	2.0079	2.2708	2.2708

Table 6.2

COMPARISON OF THE ARL PERFORMANCE FOR THE TRIVARIATE OPTIMAL MEWMAPC CHARTS USING FORM 4 UNDER C2 And C4

λ	$\theta = 0°$		$\theta = 15°$	
	C2	C4	C2	C4
0.0	200.000	200.000	200.000	200.000
0.5	27.6180	27.6070	28.6383	28.6275
1.0	10.7561	10.7553	11.2067	11.2058
1.5	5.9848	5.9847	6.2449	6.2448
2.0	3.9256	3.9255	4.0975	4.0974
2.5	2.8262	2.8258	2.9518	2.9515
3.0	2.1427	2.1425	2.2421	2.2419

Table 6.2 (Continued)

λ	$\theta = 30°$		$\theta = 45°$	
	C2	C4	C2	C4
0.0	200.000	200.000	200.000	200.000
0.5	31.6624	31.6519	33.8276	33.8276
1.0	12.5784	12.5775	13.6403	13.6403
1.5	7.0460	7.0459	7.6873	7.6873
2.0	4.6300	4.6300	5.0645	5.0645
2.5	3.3391	3.3390	3.6565	3.6565
3.0	2.5482	2.5478	2.7982	2.7982

the similarity in the optimal design of MEWMAPC charts using criterion 2 (C2) and criterion 4 (C4) for both bivariate and trivariate cases. For a given size of mean vector shift λ, the directions of the shifts of the principal components under investigation are 0°, 15°, 30° and 45°. Equation 5.1 is used to represent the form of the mean vector shift for the bivariate principal component. Furthermore, the form of the mean vector shift for the trivariate principal component is selected to be Form 4 of equation 5.2. The parameters of the charts used to establish Tables 6.1 and 6.2 are listed in Appendix A.

It is obvious from these tables that the ARL performance of the optimal MEWMAPC charts using C2 and C4 are very similar. Note that the OOC ARL performance of the IEWMAPC chart under a fixed nominal ARL depends on the pair of parameters h and r. It is always true that the ARL performance of an IEWMAPC is changed if r is changed while h is kept constant or vice versa. However, in the optimization procedure, the change of r must be incorporated with the change of h in order to obtain the desired nominal ARL. This is to say that the parameters h and r of an optimal IEWMAPC chart are somewhat highly correlated. Since h and r of an optimal IEWMAPC chart are correlated, it is sufficient to use either different h or different r in the design of the MEWMAPC chart without losing much from the optimized OOC ARL. Therefore, the use of C2 or a common value of r, but different values of h, in the optimal design of the MEWMAPC chart is equally effective as using C4.

Another disadvantage of using criterion C4 in the design of the optimal MEWMAPC chart is that the optimal parameters of the chart often depend on the initial search point. This is to say that different initial search points create different sets of optimal control chart parameters and the parameters of the true optimal chart are hard to obtain. Therefore, the research suggests that C2, a common r and different h, should be used in the design of the optimal multivariate EWMA principal component control chart. In this chapter, the optimal design of the MEWMAPC chart using C4 will not be discussed further.

The Optimal MEWMAPC Charts

Another design criterion that may be of interest is criterion C1. This section evaluates the advantages and disadvantages of the optimal design of the MEWMAPC chart using both C1 and C2. Also, a comparison of the ARL performance of the optimal MEWMAPC chart with respect to that of the MEWMA chart is also addressed.

Since the optimal MEWMA charts provided by Lowry et al. (1992) are calibrated to have a nominal ARL of 200 and $\lambda = 0.5$, 1.0, 1.5 and 2.0, the comparisons and discussion in this section are limited to these cases. The parameters and the ARL performances of the optimal MEWMA charts are listed in Appendix B. Furthermore, the directions of shifts employed in the optimal design of the MEWMAPC charts are selected following equations 5.1 and 5.2 for the bivariate and trivariate cases, respectively.

Optimal Design Using C1. Figures 6.1 to 6.4 display the ARL performance comparisons among various bivariate optimal MEWMAPC charts and a MEWMA chart. The comparisons of the ARL of the trivariate cases are displayed in Figures 6.5 to 6.8.

These figures show that the optimal MEWMA chart performs better than the optimal MEWMAPC chart most of the time. However, for shift in the direction which is along or near one of the axes of the principal components, the ARL performance of the optimal MEWMAPC chart shows a slight edge over that of the optimal MEWMA chart. The parameters and the ARL values of the optimal MEWMAPC charts used to generate Figures 6.1 to 6.8 are tabulated in Appendix C.

The advantages of employing C1, which is to apply common h and common r to all IEWMAPC charts, in the optimal design of the MEWMAPC charts are as follows.

(1) The optimal design parameters of the chart can be obtained efficiently. Note that only two parameters, h and r, are involved in the computer optimization procedure for the optimal design of a MEWMAPC chart using C1.

(2) Since the parameters used in each IEWMAPC chart are equivalent, the MEWMAPC chart is easier to implement under a real environment.

(3) The charts thus designed are insensitive to moderate

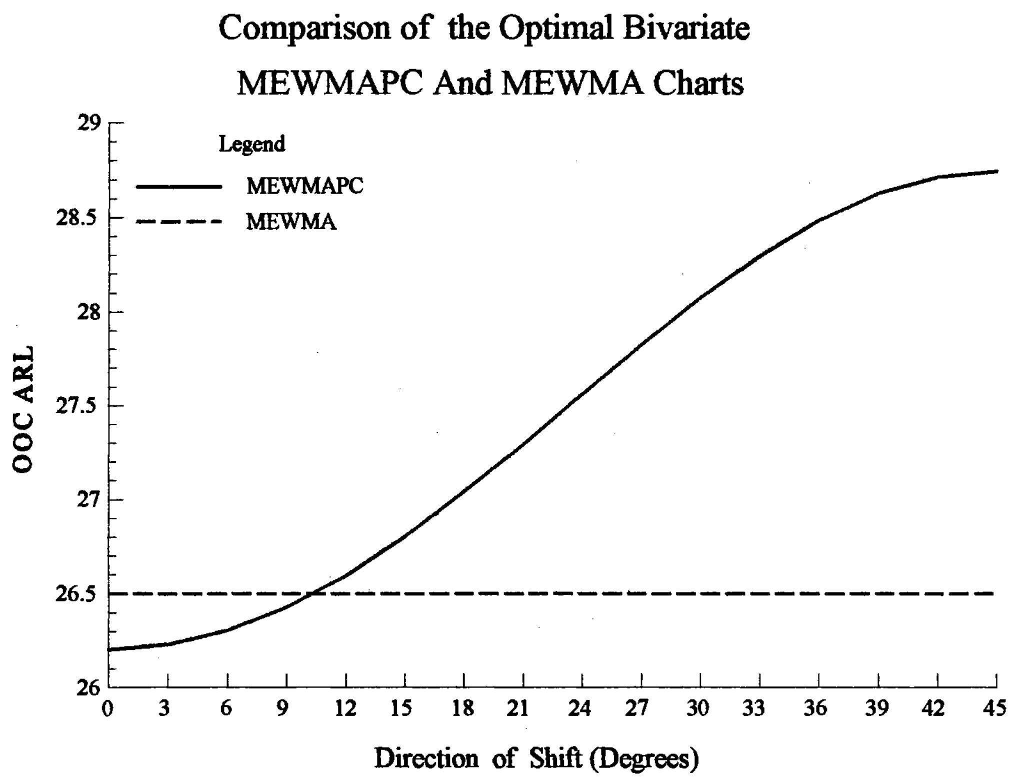

Figure 6.1 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 0.5

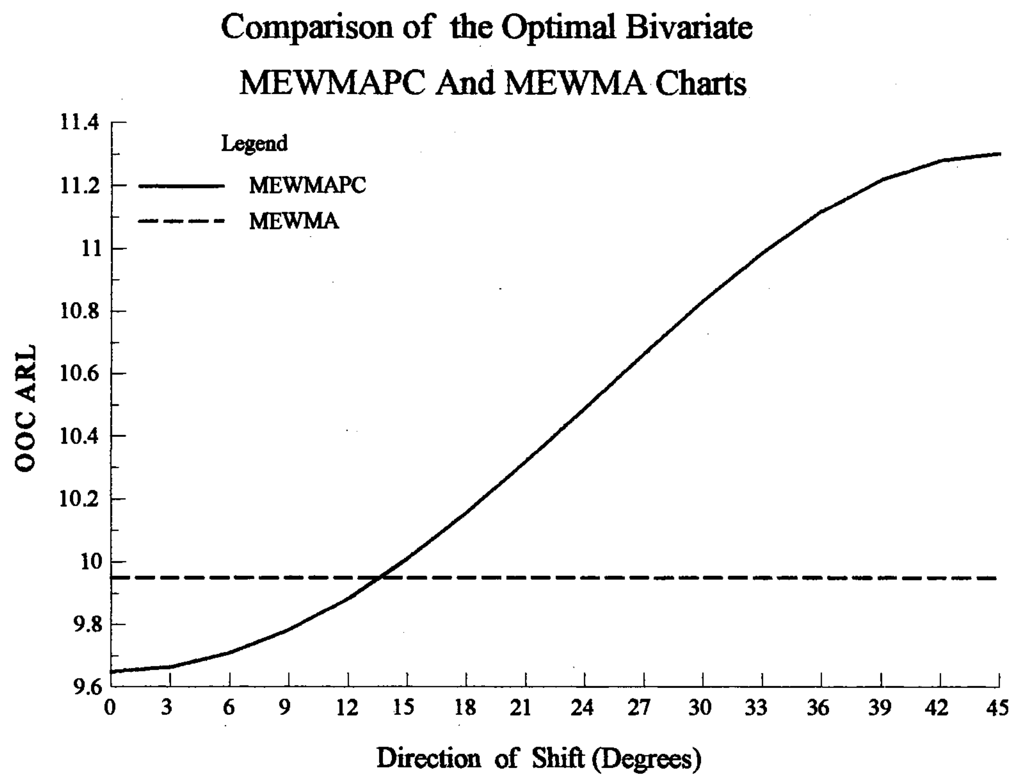

Figure 6.2 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.0

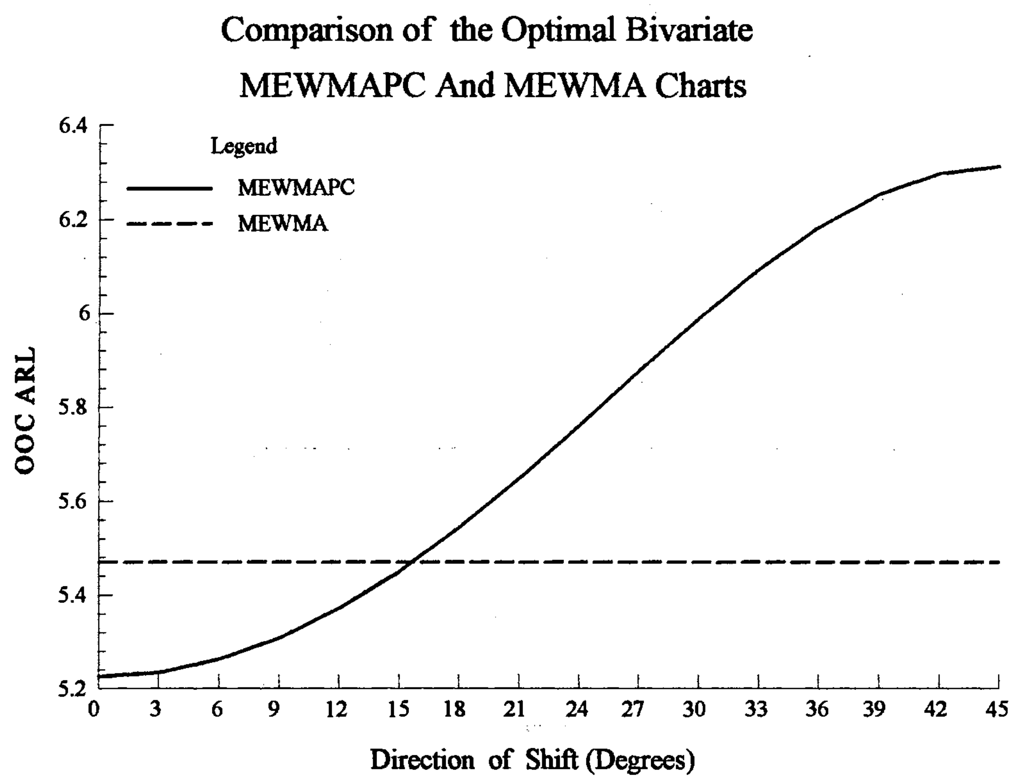

Figure 6.3 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.5

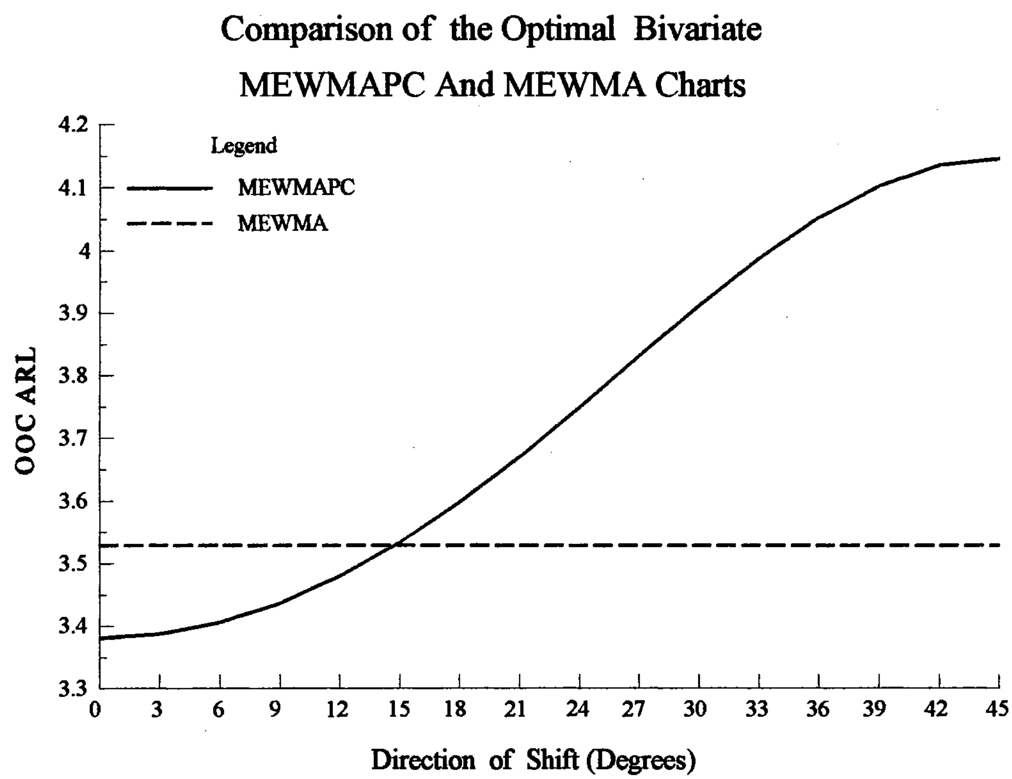

Figure 6.4 ARL Performance Comparisons Among The Optimal Bivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 2.0

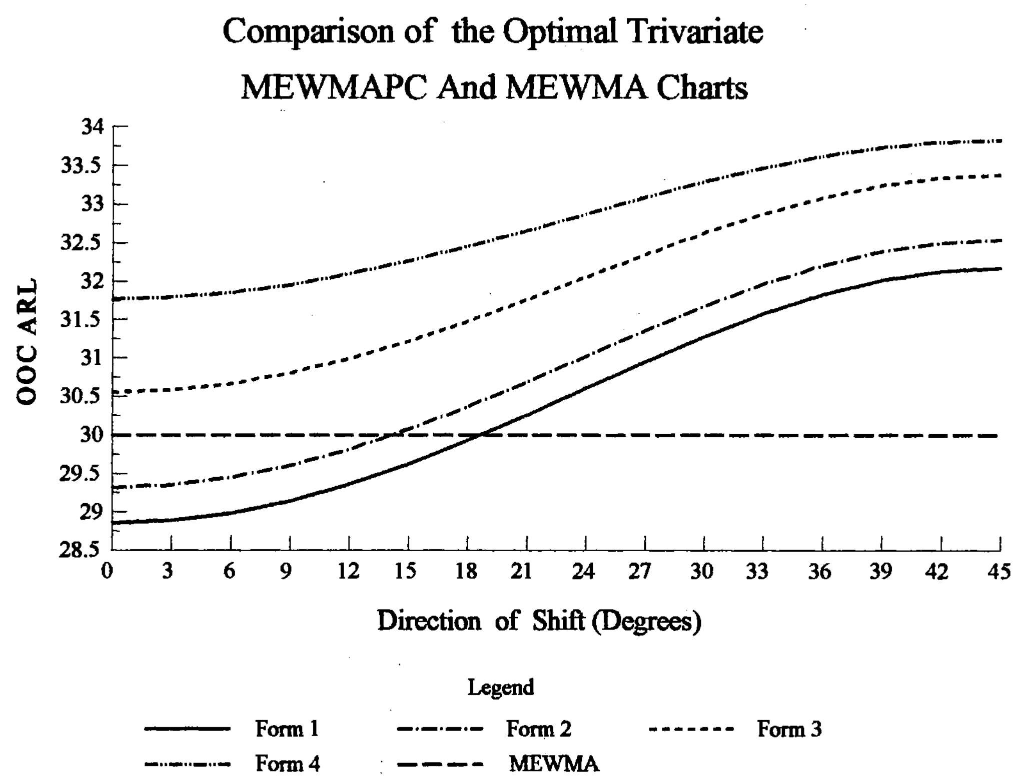

Figure 6.5 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 0.5

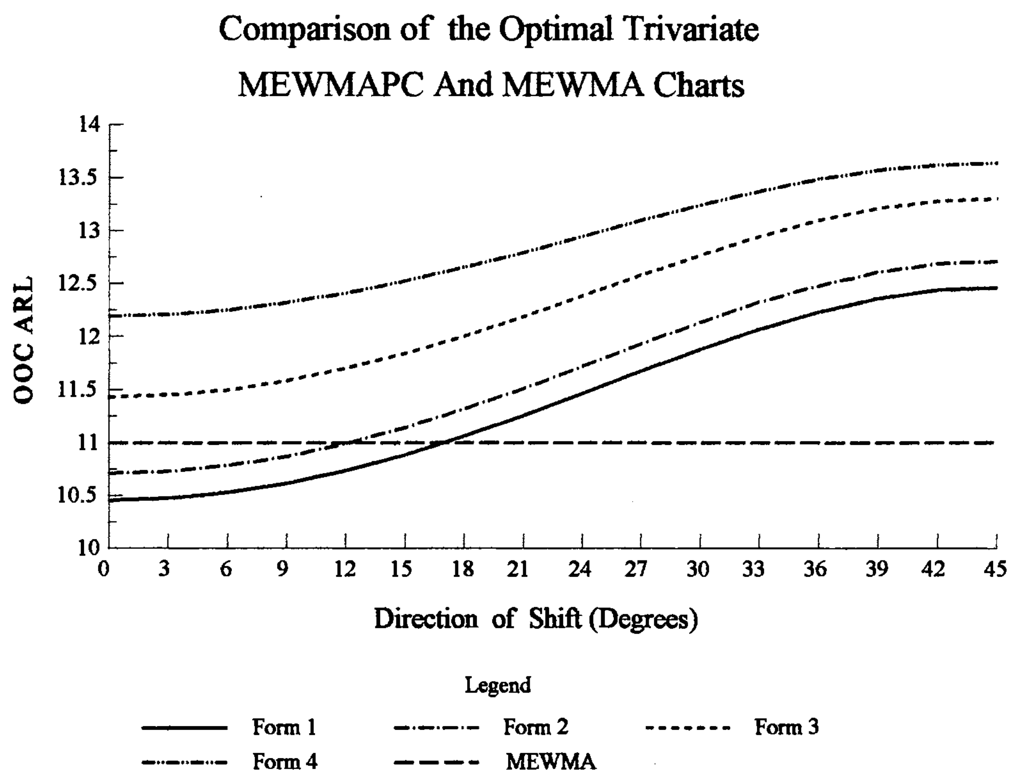

Figure 6.6 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.0

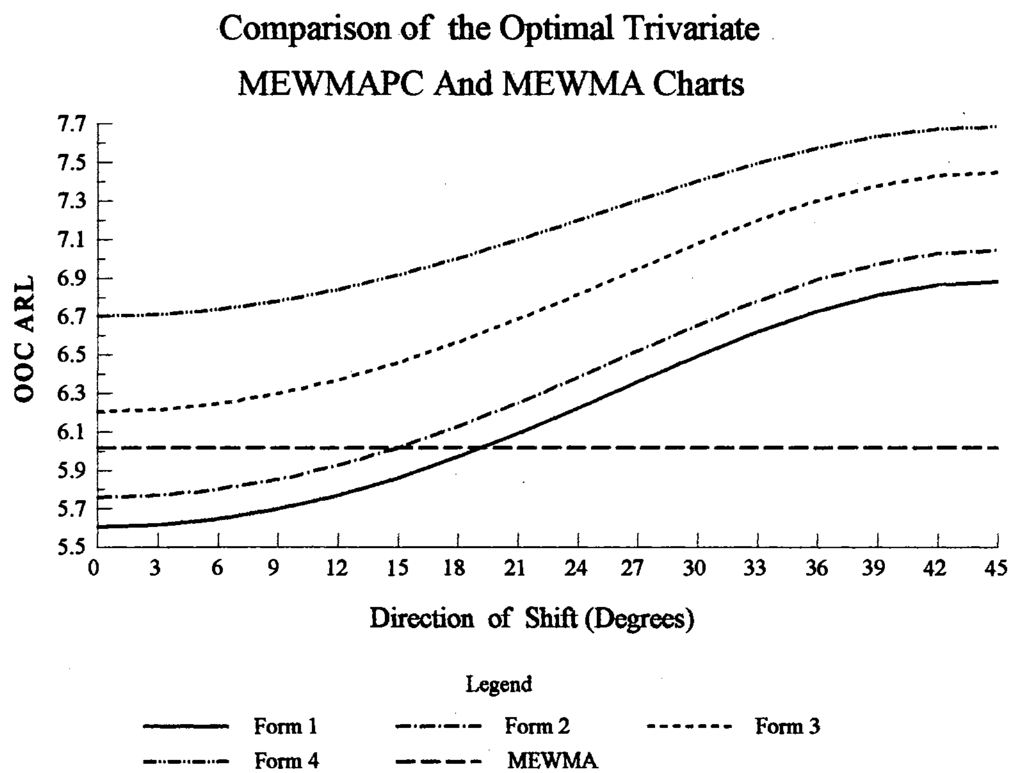

Figure 6.7 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.5

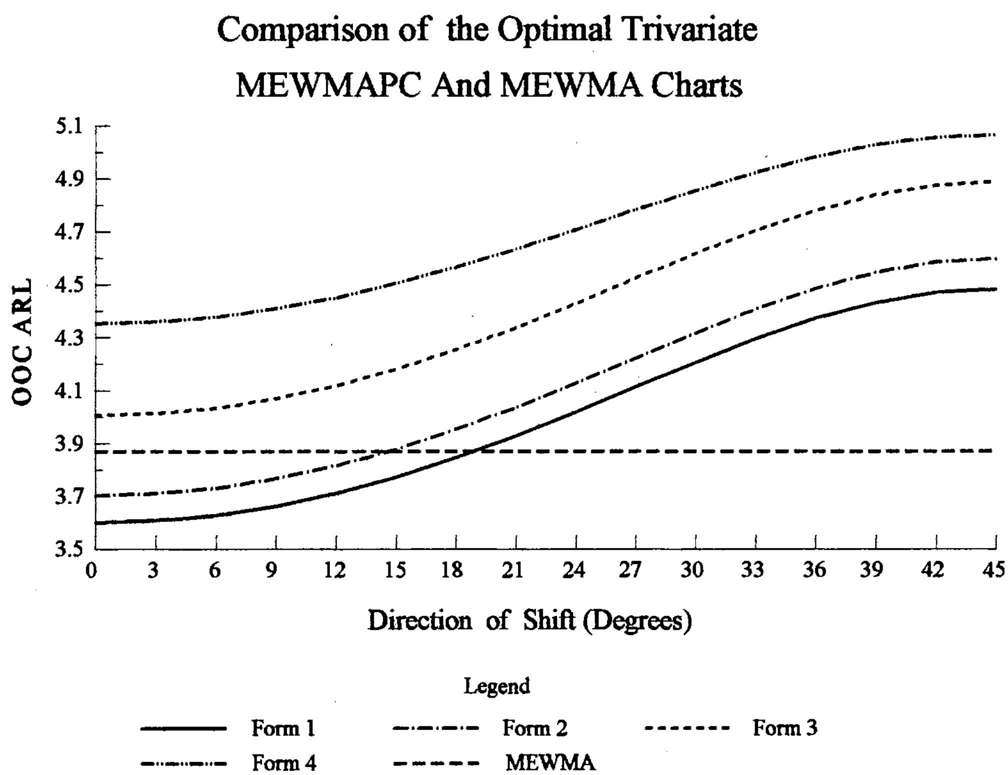

Figure 6.8 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C1 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 2.0

errors in the estimation of the covariance of the original variables and the size and the direction of the process sudden mean shift. This can be shown in the sensitivity analysis section later.

Optimal Design Using C2. Figures 6.9 to 6.12 display the ARL performance comparisons among various bivariate optimal MEWMAPC charts using C2 and the optimal MEWMA charts at various λ values. These figures show that the optimal MEWMAPC charts perform better than the optimal MEWMA chart two thirds of the time.

The performance comparisons of the optimal trivariate MEWMAPC charts with the MEWMA charts at various λ are shown in Figures 6.13 to 6.16. These figures show that at certain directions of shift the optimal MEWMAPC charts perform better than the optimal MEWMA charts. However, for other directions, the optimal MEWMA charts perform better. The parameters and the ARL performance of the optimal MEWMAPC charts employed in the construction of Figures 6.9 to 6.16 can be obtained from Appendix D.

It is intuitively appealing that the ARL performance of the MEWMAPC charts using C2 is better than that using C1. This can be confirmed by examining the corresponding ARL in Appendices C and D. Note that the performance of the MEWMAPC charts under both C1 and C2 will be the same if the mean vector shift of the original variables is transformed to have equal size of mean shift in each principal component.

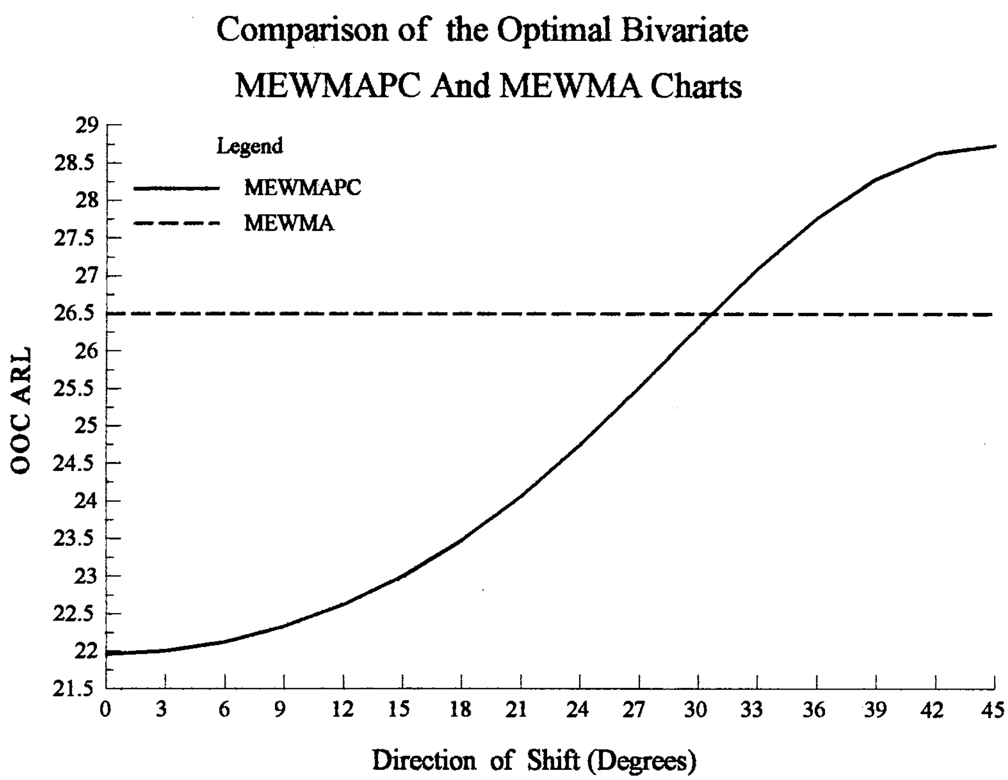

Figure 6.9 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 0.5

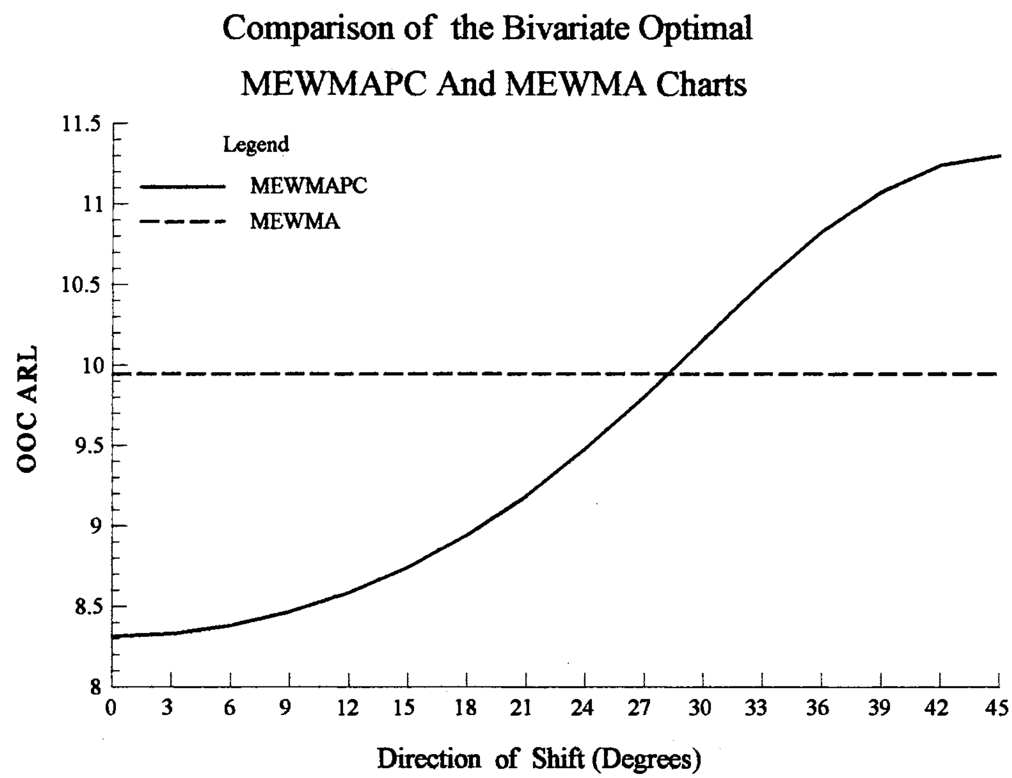

Figure 6.10 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.0

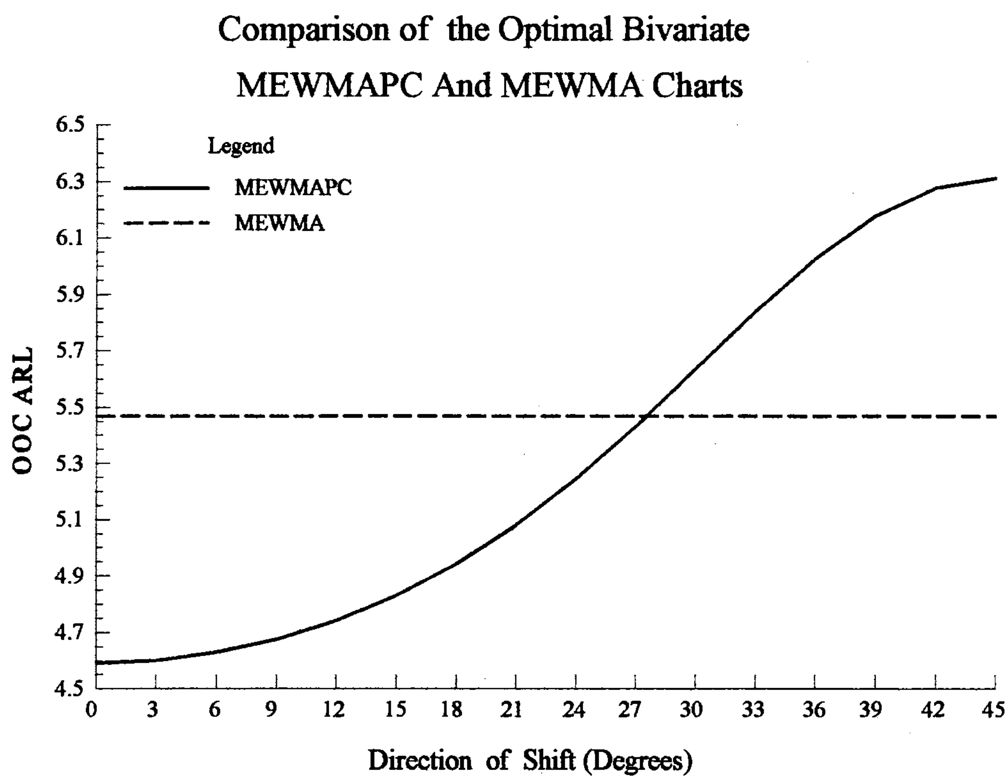

Figure 6.11 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.5

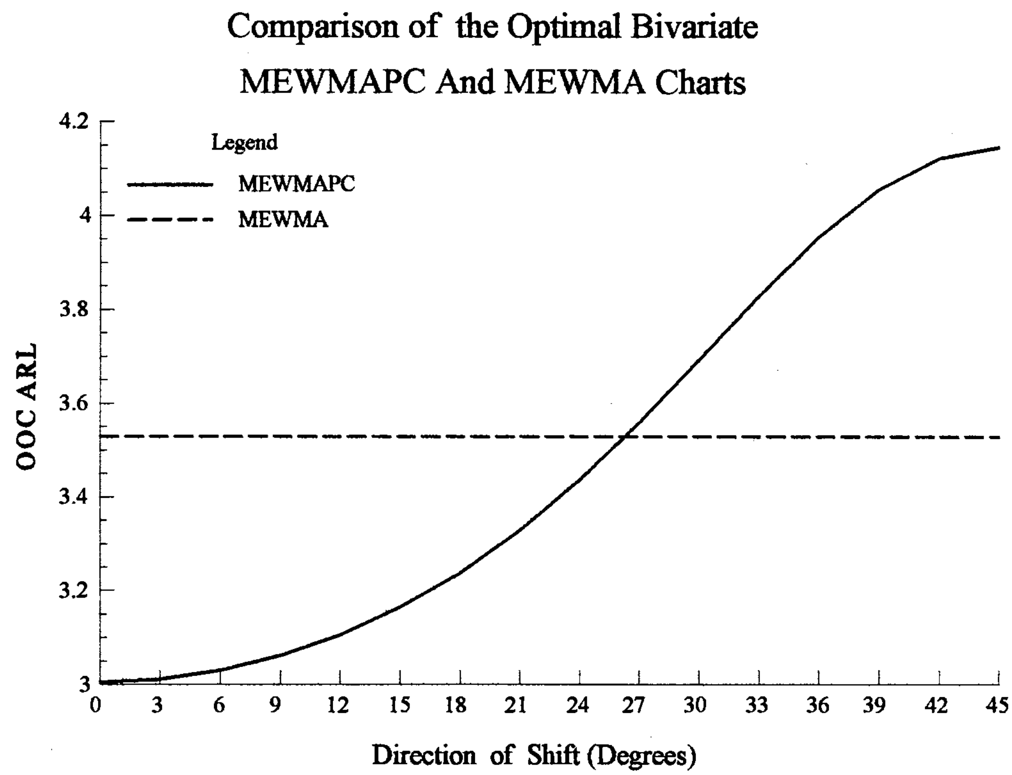

Figure 6.12 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Bivariate MEWMA Chart With Nominal ARL = 200 And λ = 2.0

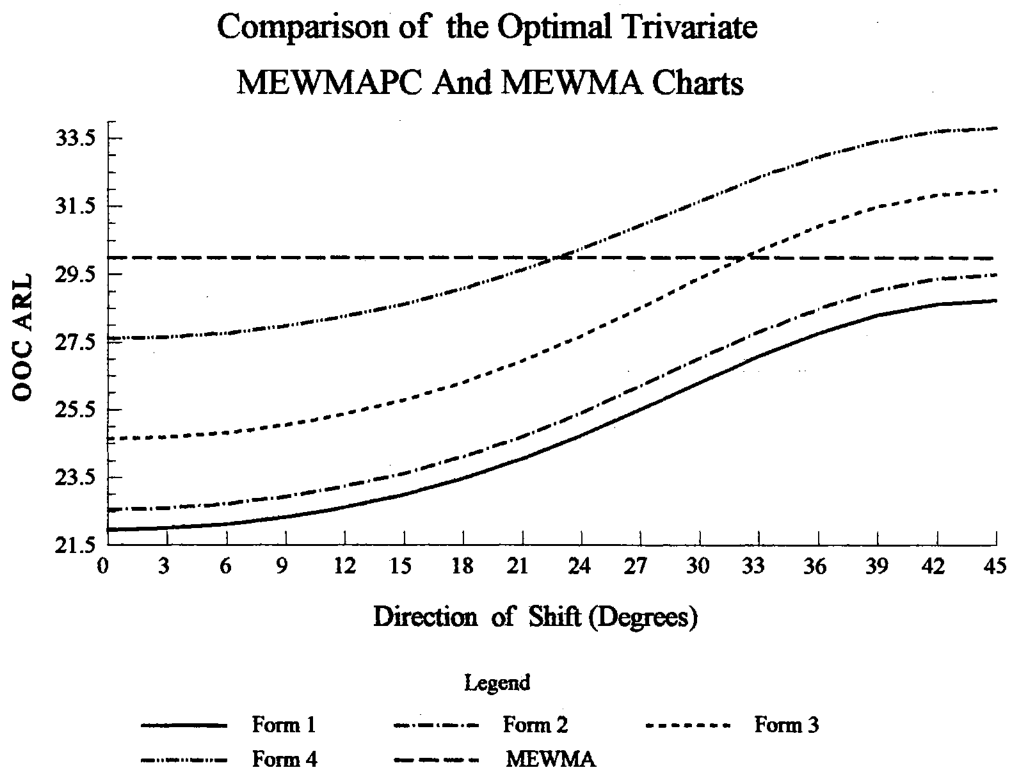

Figure 6.13 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C2 Under Various Directions (Θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And $\lambda = 0.5$

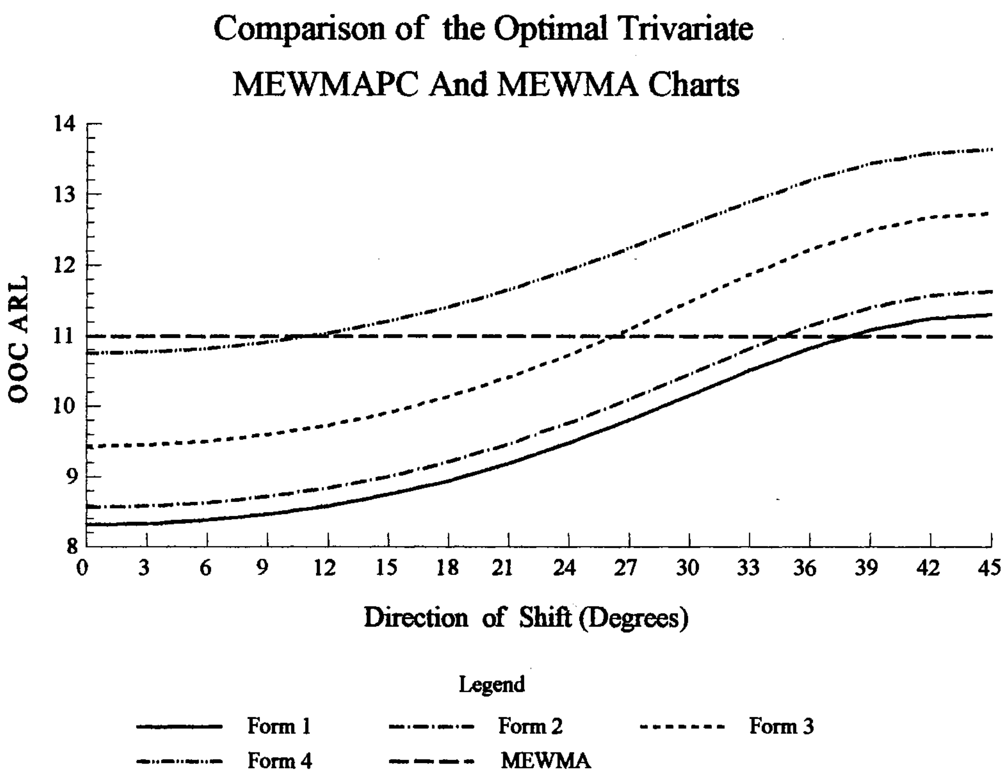

Figure 6.14 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.0

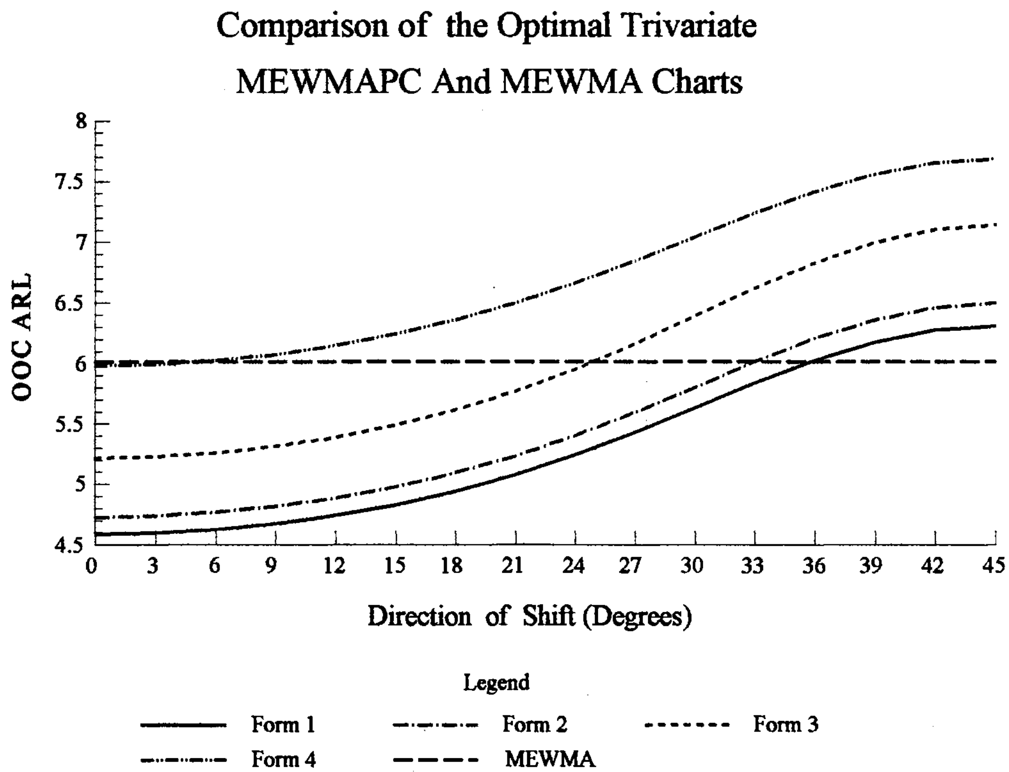

Figure 6.15 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 1.5

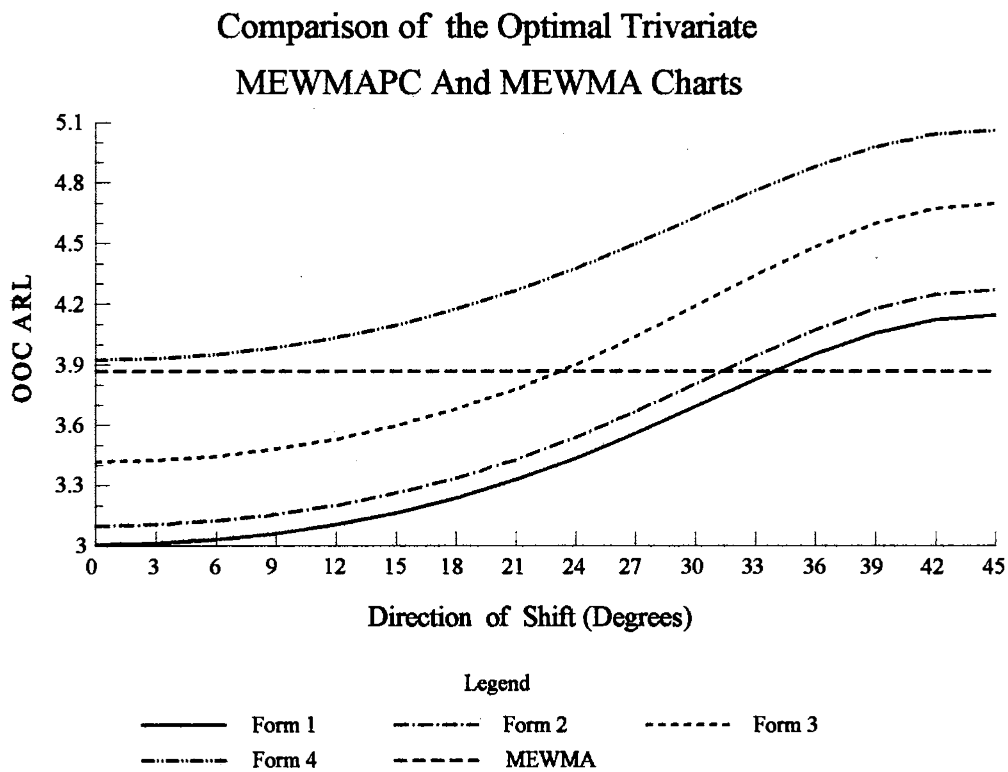

Figure 6.16 ARL Performance Comparison Among The Optimal Trivariate MEWMAPC Charts Using C2 Under Various Directions (θ) Of Shift And An Optimal Trivariate MEWMA Chart With Nominal ARL = 200 And λ = 2.0

Previous discussion on the comparison of the performance of the optimal MEWMAPC and the optimal MEWMA charts concentrates solely on the transformed principal components. It is necessary to investigate the performance of these two types of charts with respect to the original variables where the covariance matrix is not an identity matrix.

Figure 6.17 illustrates the ARL performance comparisons of the optimal MEWMAPC charts and an optimal MEWMA chart for three bivariate processes with common in-control mean vector **0** and correlation at $\rho = 0.2$, 0.5 and 0.8, respectively. The in-control ARL is selected to be 200 and the shifts of the process mean vectors are calibrated to have $\lambda = 0.5$. It is observed that:

(1) Most of the optimal bivariate MEWMAPC charts perform better than the MEWMA chart.

(2) For a bivariate process with low correlation between both variables ($\rho < 0.5$), the optimal MEWMAPC chart performs worse than the optimal MEWMA chart when the magnitude of the shift is concentrated on one of the original variables.

(3) Regardless of the size of the correlation ρ, the optimal MEWMAPC chart using C2 performs better than the optimal MEWMA chart when the magnitude of the mean shift in the original variables is equal or nearly equal.

The parameters and ARL performance of the optimal MEWMAPC

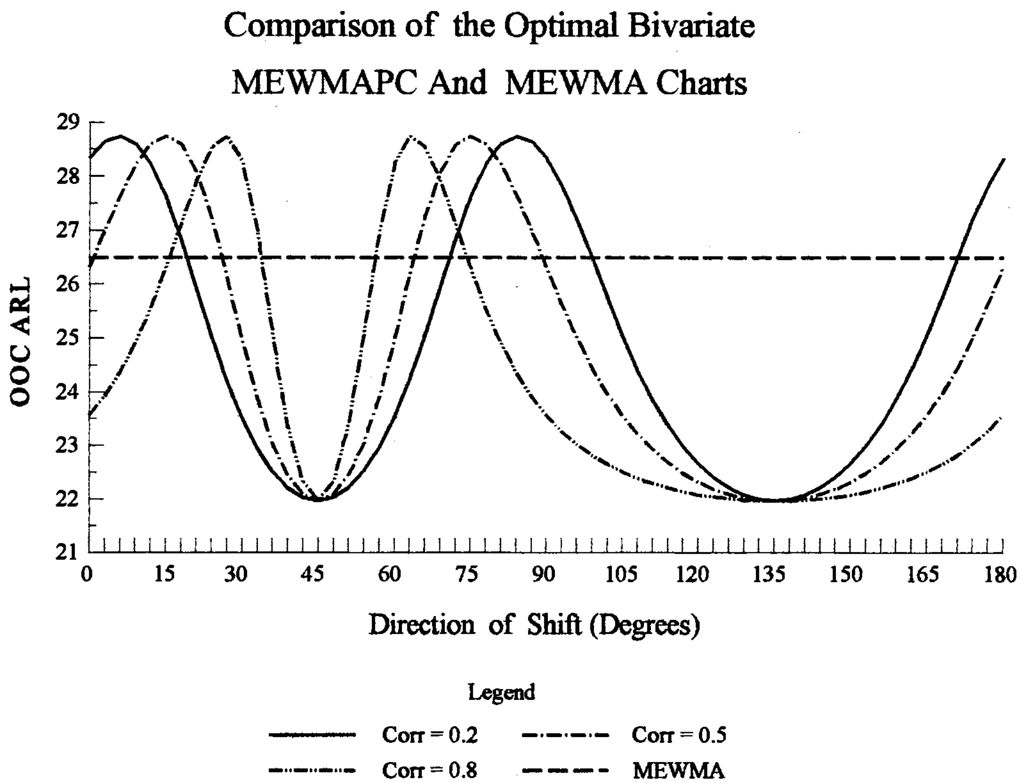

Figure 6.17 ARL Performance Comparison Of The Optimal Bivariate MEWMAPC Charts Using C2 With The Covariance ρ = 0.2, 0.5 And 0.8 And An Optimal MEWMA Chart Under Various Directions (θ) Of Shift Given That λ = 0.5 (Nominal ARL = 200)

charts employed in the construction of Figure 6.17 are tabulated in Appendix E.

The Optimal MZONEPC Charts

Figures 6.18 and 6.19 display the ARL performance comparisons of the bivariate optimal MZONEPC charts using different h and the optimal MEWMAPC charts using design criterion C2 at $\lambda = 2.5$ and 3.0, respectively. These figures clearly show that the ARL performance of the optimal MZONEPC chart is not as good as that of the optimal MEWMAPC and MEWMA charts in general.

Although the optimal MZONEPC chart is less effective than the optimal MEWMAPC chart, it has several advantages not available in the MEWMAPC chart. The advantages include:

(1) Easier to construct

(2) Simpler to apply

(3) Eliminates exact data plotting

(4) Easily understood process performance.

Sensitivity Analysis

Bennett, Case, and Schmidt (1974) suggest conducting sensitivity analyses based on three performance measures. They show that the use of these measures will aid in assessing the effects of sampling error on the total expected cost of an existing optimal sampling plan. They define three performance measures as:

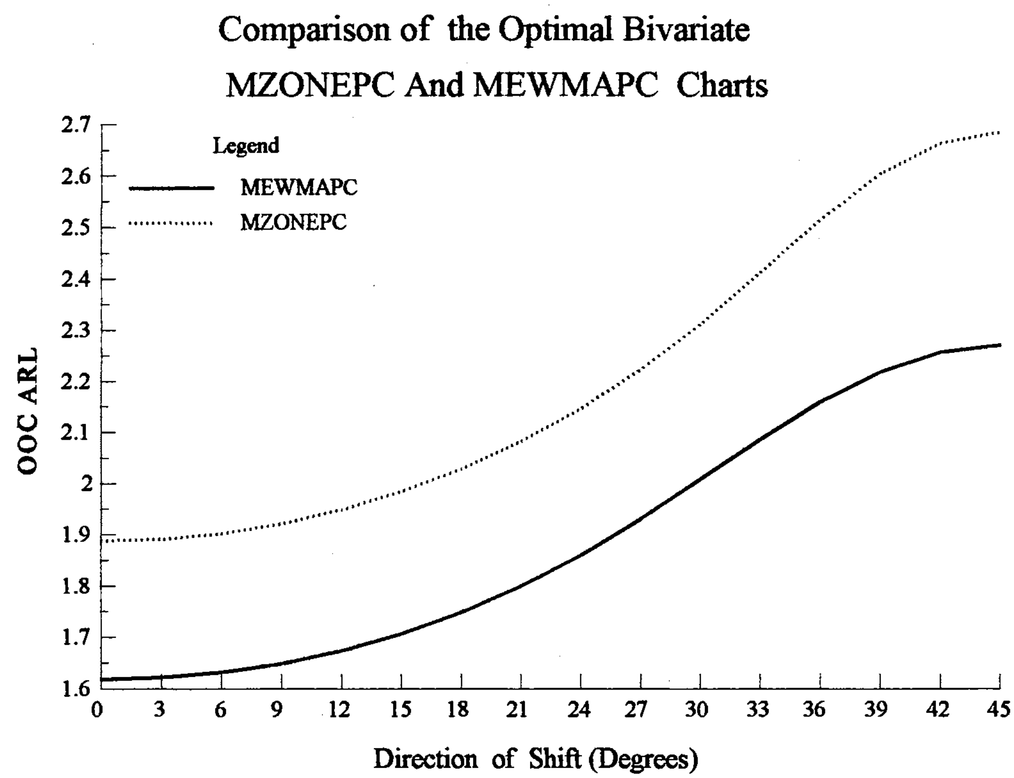

Figure 6.18 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 And The Optimal Bivariate MZONEPC Charts Under Various Directions (θ) Of Shift With Nominal ARL = 200 And λ = 2.5

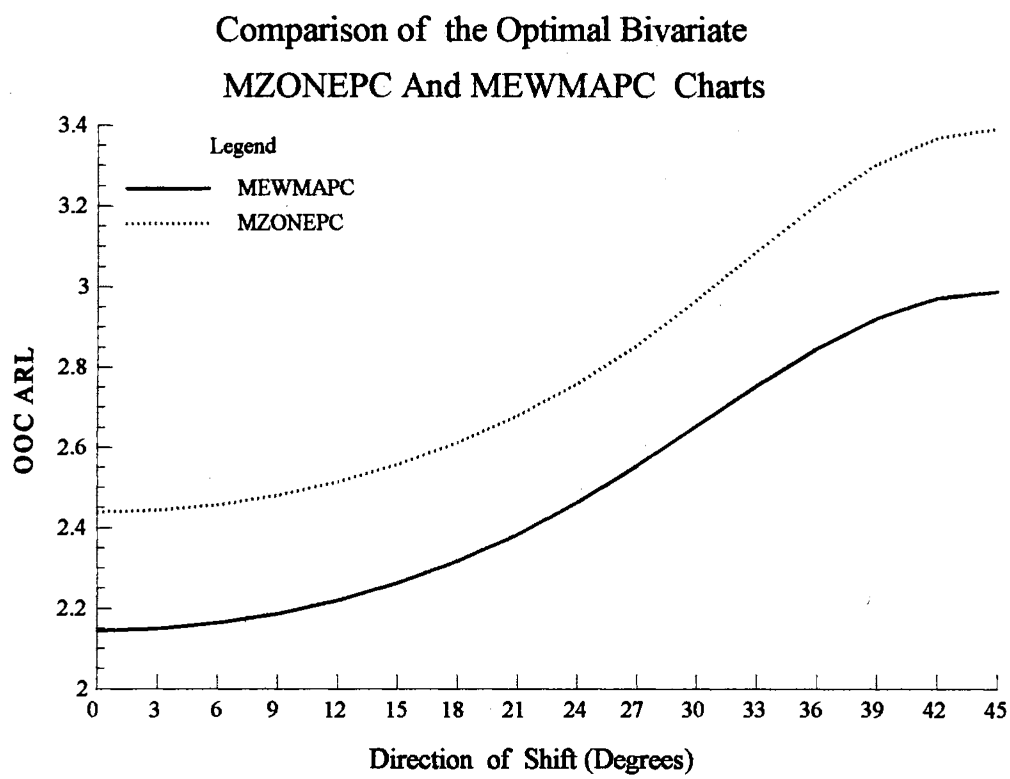

Figure 6.19 ARL Performance Comparison Among The Optimal Bivariate MEWMAPC Charts Using C2 And The Optimal Bivariate MZONEPC Charts Under Various Directions (θ) Of Shift With Nominal ARL = 200 And λ = 3.0

(1) Δ_1 measures the increased cost of using an error-free optimal sampling plan in an error-prone environment as compared to that cost expected in the error-free environment.

(2) Δ_2 measures the least increase in cost achievable when operating in an error-prone inspection environment as opposed to one which is error-free.

(3) Δ_3 measures the increased cost of neglecting error in the design of a sampling plan for use in an error-prone environment.

Here, Δ_2 measure is employed in the sensitivity analysis. The purpose of the sensitivity study is to investigate the effect of the presence of the error in the estimation of the process parameters on the statistical performance of the control chart.

Procedures For Sensitivity Analysis

The procedures for conducting the sensitivity analysis in this research are outlined as follows.

(1) Select a real process environment and shift. Then, determine the real optimal OOC ARL.

(2) Determine the factors or process parameters and the levels for each factor to examine sensitivity. A combination of the levels for each factor represents a conceived process environment and shift due to the presence of estimation error.

(3) Find the optimal design parameters of the principal

component control charts based on the conceived process environment and shift. The control chart thus obtained is optimum for the conceived environment and shift.

(4) Apply the conceived optimal principal component control charts found in (3) to the real process environment and shift, and determine the OOC ARL performance.

(5) Conduct the sensitivity analysis based on the data obtained from (1) and (4).

The Process, Factors and Levels Under Analysis

A real bivariate process environment with in-control mean vector **0** and covariance matrix with 1 in the diagonal and 0.5 in the off diagonal is selected for investigation. The in-control ARL for the optimal MEWMAPC and MZONEPC charts is calibrated to be 200. The real sudden shift of the mean vector is selected at the direction and size of $\theta = 30°$ and $\lambda = 1.0$, respectively.

The factors or process parameters under study are (1) the direction of the shift in terms of θ, (2) the magnitude of the shift in terms of the noncentrality parameter λ, and (3) the covariance ρ between two variables.

Table 6.3 shows the combinations of the levels of the factors employed in the analysis. Note that the sizes of the

Table 6.3

COMBINATIONS OF LEVELS FOR FACTORS
USED IN THE SENSITIVITY ANALYSIS

No	ρ	θ	λ
1	0.4	24°	0.8
2	0.4	24°	0.9
3	0.4	24°	1.1
4	0.4	24°	1.2
5	0.4	27°	0.8
6	0.4	27°	0.9
7	0.4	27°	1.1
8	0.4	27°	1.2
9	0.4	33°	0.8
10	0.4	33°	0.9
11	0.4	33°	1.1
12	0.4	33°	1.2
13	0.4	36°	0.8
14	0.4	36°	0.9
15	0.4	36°	1.1
16	0.4	36°	1.2
17	0.45	24°	0.8
18	0.45	24°	0.9
19	0.45	24°	1.1
20	0.45	24°	1.2
21	0.45	27°	0.8
22	0.45	27°	0.9
23	0.45	27°	1.1
24	0.45	27°	1.2
25	0.45	33°	0.8
26	0.45	33°	0.9
27	0.45	33°	1.1
28	0.45	33°	1.2
29	0.45	36°	0.8
30	0.45	36°	0.9
31	0.45	36°	1.1
32	0.45	36°	1.2

Table 6.3 (Continued)

No	ρ	θ	λ
33	0.55	24°	0.8
34	0.55	24°	0.9
35	0.55	24°	1.1
36	0.55	24°	1.2
37	0.55	27°	0.8
38	0.55	27°	0.9
39	0.55	27°	1.1
40	0.55	27°	1.2
41	0.55	33°	0.8
42	0.55	33°	0.9
43	0.55	33°	1.1
44	0.55	33°	1.2
45	0.55	36°	0.8
46	0.55	36°	0.9
47	0.55	36°	1.1
48	0.55	36°	1.2
49	0.6	24°	0.8
50	0.6	24°	0.9
51	0.6	24°	1.1
52	0.6	24°	1.2
53	0.6	27°	0.8
54	0.6	27°	0.9
55	0.6	27°	1.1
56	0.6	27°	1.2
57	0.6	33°	0.8
58	0.6	33°	0.9
59	0.6	33°	1.1
60	0.6	33°	1.2
61	0.6	36°	0.8
62	0.6	36°	0.9
63	0.6	36°	1.1
64	0.6	36°	1.2

estimation error or the levels of each factor are selected to be ±10% and ±20% away from the value of the real process environment. Error of this magnitude is considered as moderate. The parameters of the optimal charts derived from the table are the conceived optimal design under the conceived process environment and shift as opposed to the real optimal design under the real process environment and shift.

Analysis Of The Optimal Bivariate
MEWMAPC And MZONEPC Charts

Table 6.4 depicts the design parameters of the optimal MEWMAPC charts using C1 under the conceived process environment and shift and the resulting increases of the OOC ARL with respect to the real optimal OOC ARL obtained using C1 and C4 at the real process environment and shift. It is observed that the % increment of the OOC ARL is between 0.194% and 1.602% compared with the real optimal ARL value using c1. However, the % increment of the OOC ARL is dramatically increased to double digits and the range of the increment is within 10.341% to 11.908% compared with the real optimal ARL using C4.

Table 6.5 shows the design parameters and the % increment of the OOC ARL of the optimal MEWMAPC charts using C2 compared with the real optimal OOC ARL using C2 and C4. The ranges of the % increment of the OOC ARL is within 0.106% and 5.199% for both cases.

It can be observed from Tables 6.4 and 6.5 as follows.

Table 6.4

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING ARL INCREASES OF THE MEWMAPC CHARTS USING C1 AT THE CONCEIVED PROCESS VS. THE REAL OPTIMAL ARL OBTAINED USING C1 AND C4 AT THE REAL PROCESS

				% Increased	
No	h	r	OOC ARL	vs. C1	vs. C4
1	0.61578	0.09771	10.70148	1.50856	11.80575
2	0.68191	0.11506	10.58401	0.39425	10.57840
3	0.81049	0.15198	10.56781	0.24060	10.40916
4	0.87302	0.17136	10.65172	1.03651	11.28581
5	0.61880	0.09848	10.69433	1.44074	11.73103
6	0.68564	0.11607	10.57973	0.35367	10.53370
7	0.81602	0.15366	10.57290	0.28892	10.46238
8	0.87954	0.17343	10.66373	1.15049	11.41135
9	0.62573	0.10024	10.67863	1.29175	11.56694
10	0.69462	0.11852	10.57038	0.26498	10.43601
11	0.82895	0.15761	10.58660	0.41885	10.60549
12	0.89461	0.17826	10.69386	1.43629	11.72614
13	0.62904	0.10110	10.67143	1.22349	11.49176
14	0.69886	0.11969	10.56646	0.22777	10.39503
15	0.83515	0.15952	10.59404	0.48945	10.68325
16	0.90182	0.18058	10.70942	1.58380	11.88862
17	0.61472	0.09744	10.70403	1.53272	11.83235
18	0.68058	0.11470	10.58560	0.40932	10.59500
19	0.80854	0.15139	10.56612	0.22461	10.39155
20	0.87077	0.17065	10.64770	0.99841	11.24385
21	0.61769	0.09819	10.69695	1.46556	11.75839
22	0.68434	0.11572	10.58119	0.36753	10.54897
23	0.81395	0.15303	10.57095	0.27036	10.44194
24	0.87713	0.17267	10.65921	1.10763	11.36414
25	0.62477	0.10000	10.68074	1.31182	11.58905
26	0.69353	0.11822	10.57144	0.27505	10.44710
27	0.82736	0.15712	10.58477	0.40152	10.58641
28	0.89275	0.17766	10.68997	1.39938	11.68549
29	0.62846	0.10095	10.67267	1.23529	11.50476
30	0.69811	0.11948	10.56712	0.23409	10.40199
31	0.83406	0.15918	10.59269	0.47665	10.66916
32	0.90056	0.18017	10.70664	1.55744	11.85958

Table 6.4 (Continued)

No	h	r	OOC ARL	% Increased	
				vs. C1	vs. C4
33	0.61267	0.09692	10.70901	1.57997	11.88440
34	0.67801	0.11400	10.58876	0.43930	10.62802
35	0.80466	0.15022	10.56293	0.19435	10.35823
36	0.86635	0.16925	10.64005	0.92581	11.16388
37	0.61529	0.09759	10.70266	1.51971	11.81803
38	0.68130	0.11489	10.58474	0.40120	10.58605
39	0.80944	0.15166	10.56689	0.23189	10.39957
40	0.87197	0.17103	10.64984	1.01868	11.26618
41	0.62283	0.09950	10.68508	1.35299	11.63440
42	0.69090	0.11750	10.57409	0.30016	10.47476
43	0.82350	0.15594	10.58052	0.36117	10.54196
44	0.88827	0.17622	10.68078	1.31219	11.58945
45	0.62699	0.10057	10.67586	1.26554	11.53808
46	0.69624	0.11897	10.56884	0.25039	10.41995
47	0.83128	0.15833	10.58933	0.44470	10.63396
48	0.89737	0.17915	10.69973	1.49196	11.78746
49	0.61173	0.09669	10.71133	1.60198	11.90864
50	0.67675	0.11366	10.59035	0.45439	10.64464
51	0.80259	0.14960	10.56132	0.17910	10.34142
52	0.86434	0.16862	10.63666	0.89370	11.12851
53	0.61405	0.09727	10.70566	1.54815	11.84935
54	0.67971	0.11446	10.58665	0.41936	10.60606
55	0.80719	0.15098	10.56499	0.21382	10.37967
56	0.86930	0.17018	10.64512	0.97394	11.21689
57	0.62157	0.09918	10.68796	1.38027	11.66444
58	0.68929	0.11706	10.57576	0.31601	10.49222
59	0.82115	0.15522	10.57804	0.33765	10.51605
60	0.88553	0.17535	10.67533	1.26048	11.53250
61	0.62609	0.10034	10.67783	1.28419	11.55861
62	0.69508	0.11865	10.56994	0.26080	10.43141
63	0.82962	0.15782	10.58737	0.42612	10.61351
64	0.89538	0.17850	10.69548	1.45162	11.74303
C1	0.75335	0.13507	10.54244	0.0	10.14415
C4	h_1 = 0.69649	r_1 = 0.13882	h_2 = 1.13334	r_2 = 0.14639	OOC ARL = 9.57150

Table 6.5

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING ARL INCREASES OF THE MEWMAPC CHARTS USING C2 AT THE CONCEIVED PROCESS VS. THE REAL OPTIMAL ARL OBTAINED USING C2 AND C4 AT THE REAL PROCESS

					% Increased	
No	h_1	h_2	r	OOC ARL	vs. C2	vs. C4
1	0.55755	0.77001	0.09695	9.77552	2.13121	2.13157
2	0.62160	0.85125	0.11477	9.66305	0.95615	0.95651
3	0.74614	1.00860	0.15274	9.61957	0.50187	0.50224
4	0.80684	1.08509	0.17274	9.67343	1.06457	1.06494
5	0.56127	0.86453	0.09912	9.70832	1.42912	1.42949
6	0.62711	0.95576	0.11767	9.60899	0.39134	0.39171
7	0.75506	1.13257	0.15723	9.59387	0.23344	0.23380
8	0.81737	1.21856	0.17805	9.66288	0.95441	0.95478
9	0.58786	1.26621	0.10689	9.65549	0.87722	0.87758
10	0.65717	1.39767	0.12703	9.58698	0.16138	0.16175
11	0.79172	1.65323	0.16993	9.63895	0.70442	0.70478
12	0.85717	1.77788	0.19246	9.74436	1.80564	1.80601
13	0.60011	1.32676	0.11035	9.63831	0.69768	0.69804
14	0.67066	1.45412	0.13112	9.58163	0.10554	0.10591
15	0.80760	1.70412	0.17530	9.65994	0.92365	0.92401
16	0.87419	1.82490	0.19848	9.77991	2.17710	2.17747
17	0.55871	0.74241	0.09653	9.81762	2.57108	2.57145
18	0.62221	0.82062	0.11411	9.70124	1.35511	1.35548
19	0.74570	0.97211	0.15157	9.64985	0.81822	0.81858
20	0.80589	1.04575	0.17129	9.69968	1.33887	1.33924
21	0.55885	0.82642	0.09816	9.72648	1.61883	1.61920
22	0.62402	0.91368	0.11643	9.62201	0.52739	0.52775
23	0.75068	1.08276	0.15540	9.59569	0.25237	0.25273
24	0.81239	1.16497	0.17591	9.65867	0.91036	0.91073
25	0.58444	1.19131	0.10593	9.66044	0.92888	0.92924
26	0.65340	1.31527	0.12590	9.58861	0.17848	0.17884
27	0.78728	1.55618	0.16844	9.63329	0.64522	0.64558
28	0.85241	1.67370	0.19078	9.73467	1.70442	1.70479
29	0.59807	1.32538	0.10977	9.64102	0.72605	0.72641
30	0.66842	1.45230	0.13043	9.58236	0.11318	0.11355
31	0.80496	1.69933	0.17440	9.65625	0.88509	0.88545
32	0.87136	1.82043	0.19747	9.77378	2.11299	2.11336

Table 6.5 (Continued)

No	h_1	h_2	r	OOC ARL	% Increased vs. C2	% Increased vs. C4
33	0.56615	0.69259	0.09615	9.95705	4.02778	4.02816
34	0.62890	0.76531	0.11334	9.83414	2.74370	2.74407
35	0.75090	0.90615	0.14990	9.77211	2.09553	2.09590
36	0.81036	0.97461	0.16913	9.81750	2.56978	2.57015
37	0.55791	0.75692	0.09673	9.79340	2.31799	2.31836
38	0.62169	0.87673	0.11444	9.67906	1.12342	1.12379
39	0.74572	0.99131	0.15217	9.63174	0.62901	0.62937
40	0.80618	1.06645	0.17204	9.68361	1.17099	1.17136
41	0.57588	1.04918	0.10352	9.67284	1.05849	1.05885
42	0.64392	1.15900	0.12305	9.59240	0.21806	0.21843
43	0.77063	1.37221	0.16466	9.61827	0.48829	0.48866
44	0.84031	1.47606	0.18653	9.70937	1.44010	1.44047
45	0.59273	1.31292	0.10826	9.64842	0.80334	0.80371
46	0.66254	1.43910	0.12865	9.58459	0.13643	0.13680
47	0.79804	1.68681	0.17206	9.64697	0.78821	0.78858
48	0.86395	1.80566	0.19485	9.75814	1.94958	1.94995
49	0.57297	0.66990	0.09614	10.0691	5.19871	5.19909
50	0.63565	0.74017	0.11318	9.94394	3.89077	3.89115
51	0.75745	0.87629	0.14939	9.88049	3.22794	3.22832
52	0.81679	0.94247	0.16842	9.92646	3.70816	3.70853
53	0.56033	0.72541	0.09634	9.85360	2.94700	2.94738
54	0.62352	0.80174	0.11348	9.73487	1.70653	1.70690
55	0.74640	0.94958	0.15093	9.67913	1.12412	1.12448
56	0.80630	1.02145	0.17047	9.72688	1.62301	1.62337
57	0.57073	0.98141	0.10205	9.68119	1.14567	1.14603
58	0.63813	1.08447	0.12129	9.59503	0.24546	0.24582
59	0.76905	1.28442	0.16228	9.60851	0.38633	0.38670
60	0.83276	1.38176	0.18384	9.69291	1.26818	1.26854
61	0.58918	1.29869	0.10726	9.65358	0.85721	0.85758
62	0.65863	1.43243	0.12747	9.58634	0.15469	0.15505
63	0.79344	1.67847	0.17051	9.64112	0.72701	0.72737
64	0.85901	1.79802	0.19311	9.74807	1.84446	1.84482
C2	0.69676	1.10150	0.13890	9.57153	0.0	0.00034
C4	h_1 = 0.69649	r_1 = 0.13882	h_2 = 1.13334	r_2 = 0.14639	OOC ARL = 9.57150	***

(1) The small variation within each % increased column of Table 6.4 indicates that the optimal MEWMAPC chart using C1 is relatively insensitive to moderate error in the estimation of the process correlation ρ and the direction and size of the mean vector shift. Note that C1 provides the same protection against the shift on each principal component by using common h and common r in each IEWMAPC chart. Therefore, the estimated error in the direction and the size of the mean vector shift will not have significant influence on the performance of the charts thus designed.

(2) The large variation within each % increased column of Table 6.5 indicates that the optimal MEWMAPC chart using C2 is sensitive to moderate error in the estimation of the process correlation ρ and the direction and size of the mean vector shift. Note that the optimal MEWMAPC chart using C2 tends to provide a tighter control limit on the principal component that incurs a higher size of mean shift and to provide a looser control limit on the principal component that incurs a smaller size of mean shift. The chart thus designed guarantees the optimal performance against a shift of a specified size and direction. However, the optimal MEWMAPC chart using C2 does not provide equal protection on

the mean shift in each principal component. Therefore, it is more susceptible to estimation error in the process correlation parameter as well as the direction and size of the mean shift.

(3) The difference or the bias between the OOC ARL of the conceived optimal MEWMAPC charts using C1 and the real optimal MEWMAPC charts using C4 can be more than 10% as shown in the last % increased column of Table 6.4. Note that the higher value in that column is due to the use of C1 instead of the estimation error in the process parameters.

Table 6.6 shows the optimal design parameters and the ARL increases of the MZONEPC charts using different h. The range of the % increment of the OOC ARL is between 0.104% and 4.1%. Thus, the variation within the % increased column of Table 6.6 is smaller than the variation within the % increased column of Table 6.5. However, the bias will be large if the OOC ARL of the conceived optimal MZONEPC charts is compared with the real optimal OOC ARL of the MEWMAPC charts using C4.

Summary

The optimal design of the MEWMAPC and MZONEPC charts are introduced. Four design criteria for the optimal MEWMAPC charts are discussed. This chapter shows that the ARL performance of the optimal MEWMAPC charts under design criteria C2 and C4 are almost identical. Also, the comparisons of the

Table 6.6

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING OOC ARL INCREASES OF THE MZONEPC CHARTS USING DIFFERENT H AT THE CONCEIVED PROCESS VS. THE REAL OPTIMAL ARL AT THE REAL PROCESS

No	h_1	h_2	OOC ARL	% Increased
1	3.37415	4.34959	12.49840	0.56494
2	3.37534	4.33824	12.50260	0.59877
3	3.37550	4.33667	12.50320	0.60354
4	3.37447	4.34654	12.49951	0.57387
5	3.34898	4.74945	12.43168	0.02809
6	3.34954	4.73384	12.43242	0.03404
7	3.34937	4.73844	12.43219	0.03221
8	3.34866	4.75868	12.43128	0.02490
9	3.33647	5.99998	12.44903	0.16766
10	3.33647	5.99998	12.44903	0.16766
11	3.33647	5.99998	12.44903	0.16766
12	3.33647	5.99998	12.44903	0.16766
13	3.33647	5.99998	12.44903	0.16766
14	3.33647	5.99998	12.44903	0.16766
15	3.33647	5.99998	12.44903	0.16766
16	3.33647	5.99998	12.44903	0.16766
17	3.38859	4.23034	12.55363	1.00937
18	3.38999	4.22058	12.55940	1.05573
19	3.39035	4.21814	12.56087	1.06761
20	3.38928	4.22554	12.55644	1.03197
21	3.35604	4.58891	12.44444	0.13075
22	3.35682	4.57482	12.44622	0.14511
23	3.35673	4.57640	12.44602	0.14344
24	3.35587	4.59211	12.44405	0.12763
25	3.33647	5.99998	12.44903	0.16766
26	3.33647	5.99998	12.44903	0.16766
27	3.33647	5.99998	12.44903	0.16766
28	3.33647	5.99998	12.44903	0.16766
29	3.33647	5.99998	12.44903	0.16766
30	3.33647	5.99998	12.44903	0.16766
31	3.33647	5.99998	12.44903	0.16766
32	3.33647	5.99998	12.44903	0.16766
basis	3.34397	4.92832	12.42819	0.0

Table 6.6 (Continued)

No	h_1	h_2	OOC ARL	% Increased
33	3.43204	4.00933	12.75407	2.62211
34	3.43375	4.00295	12.76270	2.69157
35	3.43451	4.00018	12.76651	2.72222
36	3.43350	4.00387	12.76144	2.68142
37	3.38039	4.29327	12.52127	0.74898
38	3.38168	4.28265	12.52619	0.78855
39	3.38193	4.28062	12.52716	0.79634
40	3.38087	4.28928	12.52310	0.76368
41	3.33744	5.55078	12.44097	0.10283
42	3.33753	5.53011	12.44048	0.09893
43	3.33742	5.55729	12.44112	0.10404
44	3.33725	5.60459	12.44219	0.11263
45	3.33647	5.99998	12.44903	0.16766
46	3.33647	5.99998	12.44903	0.16766
47	3.33647	5.99998	12.44903	0.16766
48	3.33647	5.99998	12.44903	0.16766
49	3.46424	3.90611	12.92315	3.98258
50	3.46594	3.90147	12.93246	4.05745
51	3.46681	3.89912	12.93722	4.09576
52	3.46593	3.90149	12.93241	4.05712
53	3.40033	4.15580	12.60361	1.41150
54	3.40187	4.14712	12.61041	1.46616
55	3.40236	4.14438	12.61260	1.48381
56	3.40128	4.15040	12.60782	1.44533
57	3.33929	5.24953	12.43343	0.04220
58	3.33947	5.23040	12.43296	0.03839
59	3.33930	5.24805	12.43340	0.04191
60	3.33900	5.28461	12.43432	0.04931
61	3.33647	5.99998	12.44903	0.16766
62	3.33647	5.99998	12.44903	0.16766
63	3.33647	5.99998	12.44903	0.16766
64	3.33647	5.99998	12.44903	0.16766
basis	3.34397	4.92832	12.42819	0.0

ARL performance of the bivariate and trivariate optimal (1) MEWMA charts, (2) MEWMAPC charts and (3) MZONEPC charts are presented. An analysis of these results shows that (1) the optimal MEWMAPC chart performs best among these three charts if the directions of the shift of the process mean vector are along or nearby the axes of the principal components, (2) the ARL performance of the optimal MEWMAPC chart performs better than the optimal MZONEPC chart and (3) the MEWMA chart outperforms both the MEWMAPC and MZONEPC charts if the direction of the mean vector shift is away from all the axes of the principal components.

The sensitivity analysis with respect to a selected real process environment and shift is performed. The approach suggested by Collins, Case and Bennett (1974) is adopted in the analysis. The OOC ARL of the real optimal bivariate MEWMAPC charts using C1, C2 and C4, and MZONEPC charts using different h is selected as the base for the analysis. The analysis shows that the optimal MEWMAPC charts using C1 are relatively insensitive to moderate error in the estimation of the process environment and shift. However, the difference or bias between the OOC ARL of the conceived optimal MEWMAPC charts using C1 and that of the real optimal MEWMAPC charts using C4 is large. Finally, the optimal MEWMAPC charts using C2 create small bias but are susceptible to estimation error.

CHAPTER VII

USING THE INTERACTIVE COMPUTER PROGRAMS

Introduction

This chapter details the use of the interactive computer programs which permit easy utilization of the evaluation and design methodologies presented in previous chapters, which are (1) the MEWMAPC quality control chart and (2) the MZONEPC quality control chart. The actual programs using FORTRAN 77 language are well-documented and appear in Appendix G. The programs have been implemented on an IBM personal computer and RS6000 work station successfully.

The user is prompted for all necessary inputs by the computer. All these values, together with some preprogrammed parameter values, are presented to the user for verification or change. Only when a set of inputs has been verified does the program continue.

When several values are to be entered, they only need be separated by a space. Integer numbers must be entered without a decimal point. The input mechanism is virtually self-explanatory, as long as the user understands the terms being input as well as their mathematically feasible range.

In the remainder of this chapter, actual interactive output is interspersed with comments and explanations. All computer output to follow are automatically generated. Note that the Times New Roman Regular font is used to represent the computer input and output.

Statistical Performance Evaluation

And Design Of The MEWMAPC

Control Chart

Evaluation Of The OOC ARL OF

An Existing MEWMAPC Chart

The program begins by prompting the option menu. The Selection of "1" indicates that the OOC ARL of a desired MEWMAPC control chart at a desired location of the mean vector shift is to be evaluated.

```
**************************
***    MAIN    MENU    ***
**************************

(1) EVALUATION OF THE ARL OF A MEWMAPC CHART
(2) CLASSICAL DESIGN OF THE MEWMAPC CHART
(3) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C1
(4) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C2
(5) EXIT THE PROGRAM

===> PLEASE ENTER YOUR OPTION (1, 2, 3, 4, OR 5) <===
1
```

Then, the user is asked to enter the number of variables being monitored in the multivariate process.

```
==> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <==
==> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <==
3
```

```
  *** THE TOTAL NUMBER OF VARIABLES IS 3

  ===> IS THE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1
```

After verification, the user is prompted for the input values of the standardized covariance matrix. The following interactive exchange now takes place:

```
  *** THE STANDARDIZED COVARIANCE MATRIX
  *** IS INITIALLY AN IDENTITY MATRIX ***

  ===> IS THE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
2

  ===> PLEASE ENTER THE DESIRED STANDARDIZED <===
  ===> COVARIANCE MATRIX ROW BY ROW <===
1   0 .6  0.4
0.6 1     0.7
0.4  0 .7 1

  *** THE STANDARDIZED COVARIANCE MATRIX IS ***

   1.000   0.600   0.400
   0.600   1.000   0.700
   0.400   0.700   1.000

  ===> ARE THESE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1
```

Then, the user is sequentially prompted for the input values of the parameters of three IEWMAPC charts.

```
  *** PLEASE ENTER THE PARAMETERS OF THE EXISTING CHARTS ***

  *** PLEASE ENTER THE SYMMETRICAL CONTROL LIMITS H
  *** AND FACTOR R FOR NO. 1 IEWMAPC CHART ***
0.68825  0.1
```

```
  *** PLEASE ENTER THE SYMMETRICAL CONTROL LIMITS H
  *** AND FACTOR R FOR NO. 2 IEWMAPC CHART ***
1.73205 0.5

  *** PLEASE ENTER THE SYMMETRICAL CONTROL LIMITS H
  *** AND FACTOR R FOR NO. 3 IEWMAPC CHART ***
1.0394 0.25

  *** THE PARAMETERS FOR THE CHARTS ARE : ***

  H1 = 0.688250    R1 = 0.100000
  H2 = 1.732050    R2 = 0.500000
  H3 = 1.039400    R3 = 0.250000

  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1
```

After verification, the user is asked to enter the desired location of the shift of the mean vector of the process. Note that the standardized mean vector shift should be entered throughout the computer programs.

```
 *** PLEASE ENTER THE DESIRED SHIFT OF THE MEAN VECTOR ***
1.0 0.2 2.0

  *** THE DESIRED MEAN VECTOR SHIFT IS ***
    1.000000    0.200000    2.000000


  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1
```

After receiving the desired shift, the program transfers the location of the shift from the original axes to the corresponding position in the axes of the principal components.

*** THE CORRESPONDING SHIFTS ON THE PRIN. COMP. ARE ***

1.227067 0.5951731 2.526707

After transformation, the program prompts the user to verify that the initial EWMA values of all IEWMAPC charts are 0.0.

*** THE INITIAL EWMA VALUES FOR ALL IEWMAPC CHARTS ***
*** ARE SET AT 0.D0 ***

===> ARE THESE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

In the previous example, the user enters "1" to verify that all initial EWMA values are 0.0. However, if "2" is entered, the program will prompt the user to enter the desired initial EWMA value for each IEWMAPC chart. After verification, the program reminds the user that the original process mean vector is centralized at **0** and prompts to notify the user that the evaluation process is undergoing.

*** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
*** ARE SET AT 0.D0 (CENTRALIZED) ***

*** EVALUATION IN PROGRESS ***

Now, the OOC ARL for the desired MEWMAPC chart at the desired location of the mean vector shift is obtained. The program also prompts the user to hit the "enter" key to return to the main menu.

*** THE OOC ARL FOR THE DESIRED MEWMAPC CHART ***
*** AT THE DESIRED SHIFT IS 2.467869

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***

The Classical Design Of

The MEWMAPC Chart

A selection of "2" from the main menu leads to the design of the MEWMAPC chart using a classical design approach. The following is a sequence of interactive exchanges between the user and the computer.

```
***************************
***    MAIN    MENU   ***
***************************

(1) EVALUATION OF THE ARL OF A MEWMAPC CHART
(2) CLASSICAL DESIGN OF THE MEWMAPC CHART
(3) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C1
(4) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C2
(5) EXIT THE PROGRAM

===> PLEASE ENTER YOUR OPTION (1, 2, 3, 4, OR 5) <===
2

===> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <===
===> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <===
2

*** THE TOTAL NUMBER OF VARIABLES IS 2

===> IS THE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** THE STANDARDIZED COVARIANCE MATRIX
*** IS INITIALLY AN IDENTITY MATRIX ***
```

```
  ===> IS THE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
2

  ===> PLEASE ENTER THE DESIRED STANDARDIZED <===
  ===> COVARIANCE MATRIX ROW BY ROW <===
1   0.8
0.8 1

  *** THE STANDARDIZED COVARIANCE MATRIX IS ***

  1.000    0 .800
  0.800    1.000

  ===> ARE THESE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1
```

After verifying the standardized covariance matrix, the user is prompted to enter the desired in-control ARL and desired weighing factor r.

```
 *** PLEASE ENTER THE DESIRED IN-CONTROL ARL ***
100.0

  *** PLEASE ENTER THE VALUE OF THE DESIRED COMMON R FACTOR ***
  *** NOTE : USE 0.03 < R < 1.0 ***
0.15

  *** THE DESIRED IN-CONTROL ARL IS   100.0000
  *** AND THE DESIRED COMMON R FACTOR IS   0.150000

  ===> ARE THESE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1
```

A feature has been added to the program that enables the user to evaluate the ARL performance at a specified direction of interest. Therefore, the program prompts the user to input the desired direction of the shift. Note that the direction is represented by the standardized values of the desired mean shift at each characteristic. Then, the program will convert the original direction into the direction with respect to the principal components.

```
  *** PLEASE ENTER THE DIRECTION OF THE EXPECTED ***
  *** SHIFT OF THE MEAN VECTOR ***
0.75  0.5

  *** THE DIRECTION OF THE EXPECTED MEAN ***
  *** VECTOR SHIFT IS ***
    0.750000      0.500000

  ===> ARE THESE DATA CORRECT ?  <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

  *** THE CORRESPONDING DIRECTION OF THE SHIFT ***
  *** IN THE PRIN. COMP. WITH UNIT LENGTH IS ***

    0.8574929      0.5144958
```

Then, the program prompts the user to verify the initial EWMA values of all IEWMAPC charts and further notifies that the original mean vector is centralized at **0**.

```
  *** THE INITIAL EWMA VALUES FOR ALL IEWMAPC CHARTS ***
  *** ARE SET AT 0.D0 ***


  ===> ARE THESE DATA CORRECT ?  <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

  *** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
  *** ARE SET AT 0.D0 (CENTRALIZED) ***
```

The optimization is then performed. The chart parameters, eigenvectors, the profile of the OOC ARL in the specified direction, and the best and worst performance in all directions with respect to the noncentrality parameter λ of 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0 are presented.

```
*** OPTIMIZATION IN PROGRESS ***

*** THE PARAMETERS OF A 2 VARIATE MEWMAPC CHART
*** WITH IN-CONTROL ARL OF 100.00  ARE :
---------------------------------------------------------------------------
*** COMMON R = 0.150    AND COMMON H =   0.7283024

*** THE MATRIX U OF THE EIGEN VECTORS ARE : ***
*** u1 :  0.52705  0.52705
*** u2 :  1.58114 -1.58114

*** THE OOC ARL PROFILE ARE SHOWN AS FOLLOWS ***

LAMBDA          OOC ARL              OOC   ARL
                AT DIRECTION          OVERALL
-----------------------------------------------------------------------------
  0.50          23.7280           22.5832 -  24.0728
  1.00           8.9948            8.1862 -   9.2549
  1.50           5.3927            4.8232 -   5.5905
  2.00           3.8886            3.4535 -   4.0523
  2.50           3.0770            2.7286 -   3.2186
  3.00           2.5744            2.2945 -   2.6980

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***
```

The Optimal Design Of The

MEWMAPC Chart Using C1

A selection of "3" from the main menu leads to the design of the MEWMAPC chart using design criterion 1 (common h and common r). The interactive procedures are shown as follows.

```
     ***************************
     ***    MAIN    MENU    ***
     ***************************

  (1) EVALUATION OF THE ARL OF A MEWMAPC CHART
  (2) CLASSICAL DESIGN OF  THE MEWMAPC CHART
  (3) OPTIMAL DESIGN OF  THE MEWMAPC CHART USING C1
  (4) OPTIMAL DESIGN OF  THE MEWMAPC CHART USING C2
  (5) EXIT THE PROGRAM

  ====> PLEASE ENTER YOUR OPTION (1, 2, 3, 4, OR 5) <====
3

  ===> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <===
  ===> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <===
3

  *** THE TOTAL NUMBER OF VARIABLES IS  3

  ===> IS THE DATA CORRECT ?  <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

  *** THE STANDARDIZED COVARIANCE MATRIX
  *** IS INITIALLY AN IDENTITY MATRIX ***

  ===> IS THE DATA CORRECT ?  <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1


  *** PLEASE ENTER THE DESIRED IN-CONTROL ARL ***
150.0

  *** THE DESIRED IN-CONTROL ARL IS   150.0000

  ===> IS THE DATA CORRECT ?  <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

  *** PLEASE ENTER THE EXPECTED SHIFT OF ***
  *** THE ORIGINAL MEAN VECTOR ***
1.7  0.9  0.4

  *** THE EXPECTED ORIGINAL MEAN VECTOR SHIFT IS ***
    1.700000      0.900000      0.400000
```

===> ARE THESE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** THE CORRESPONDING SHIFT OF THE ***
*** MEAN OF THE PRIN. COMP. IS ***

1.700000 0.900000 0.400000

*** THE INITIAL EWMA VALUES FOR ALL IEWMAPC CHARTS ***
*** ARE SET AT 0.D0 ***

===> ARE THESE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** THE IN-CONTROL PROCESS MEAN FOR ALL VARIABLES ***
*** ARE SET AT 0.D0 (CENTRALIZED) ***

*** OPTIMIZATION IN PROGRESS ***

.... TOTAL OF 5 FUNCTION EVALUATIONS DONE ....

.... TOTAL OF 10 FUNCTION EVALUATIONS DONE ....

.... TOTAL OF 15 FUNCTION EVALUATIONS DONE ....

*** THE OPTIMAL 3 VARIATE MEWMAPC CHART ***
*** WITH IN-CONTROL ARL OF 150.00
*** AT THE SHIFT OF THE ORIGINAL MEAN VECTOR OF : ***
1.700000 0.9000000 0.4000000
*** IS LISTED AS FOLLOWS ***

THE COMMON SYMMETRIC CONTROL LIMIT = 1.322603
THE COMMON R = 0.3262823

*** THE OPTIMAL OOC ARL = 4.141323

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***

The Optimal Design Of The
MEWMAPC Chart Using C2

A selection of "4" from the main menu leads to the design of the MEWMAPC chart using design criterion 2 (different h and common r). The interactive procedures are shown as follows.

```
**************************
***    MAIN    MENU    ***
**************************

 (1) EVALUATION OF THE ARL OF A MEWMAPC CHART
 (2) CLASSICAL DESIGN OF  THE MEWMAPC CHART
 (3) OPTIMAL DESIGN OF  THE MEWMAPC CHART USING C1
 (4) OPTIMAL DESIGN OF  THE MEWMAPC CHART USING C2
 (5) EXIT THE PROGRAM

 ====> PLEASE ENTER YOUR OPTION (1, 2, 3, 4, OR 5) <====
4


 ===> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <===
 ===> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <===
2

 ***THE TOTAL NUMBER OF VARIABLES IS  2

 ===> IS THE DATA CORRECT ?  <===
 ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

 *** THE STANDARDIZED COVARIANCE MATRIX
 *** IS INITIALLY AN IDENTITY MATRIX ***

 ===> IS THE DATA CORRECT ?  <===
 ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

 *** PLEASE ENTER THE DESIRED IN-CONTROL ARL ***
250.0

 *** THE DESIRED IN-CONTROL ARL IS   250.0000
```

```
  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

  *** PLEASE ENTER THE EXPECTED SHIFT OF ***
  *** THE ORIGINAL MEAN VECTOR ***
0.9 1.2

  *** THE EXPECTED ORIGINAL MEAN VECTOR SHIFT IS ***
    0.900000      1.200000

  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

  *** THE CORRESPONDING SHIFT OF THE ***
  *** MEAN OF THE PRIN. COMP. IS ***

    0.900000      1.200000

  *** THE INITIAL EWMA VALUES FOR ALL IEWMAPC CHARTS ***
  *** ARE SET AT 0.D0 ***

  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

  *** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
  *** ARE SET AT 0.D0 (CENTRALIZED) ***


  *** OPTIMIZATION IN PROGRESS ***


 ....... TOTAL OF   5 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  10 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  15 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  20 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  25 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  30 FUNCTION EVALUATIONS DONE .......

 ....... TOTAL OF  35 FUNCTION EVALUATIONS DONE .......
```

.... TOTAL OF 40 FUNCTION EVALUATIONS DONE ....

.... TOTAL OF 45 FUNCTION EVALUATIONS DONE ....

.... TOTAL OF 50 FUNCTION EVALUATIONS DONE ....

```
*** THE OPTIMAL 2 VARIATE MEWMAPC CHART ***
*** WITH IN-CONTROL ARL OF  250.00
*** AT THE SHIFT OF THE ORIGINAL MEAN VECTOR OF : ***
  0.900000     1.200000
*** IS LISTED AS FOLLOWS ***
------------------------------------------------------------

*** H1 =   1.140306    R =   .2186067
*** H2 =   .9869904    R =   .2186067

*** THE OPTIMAL OOC ARL =   6.384885

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***
```

The program ended when the user select "5" from the main menu.

Statistical Performance Evaluation And

Design Of The MZONEPC Control Chart

The statistical performance evaluation and design of the MZONEPC chart follows very similar to those of the evaluation and design of the MEWMAPC chart. The interactive dialog and procedures are shown as follows.

Evaluation Of The OOC ARL Of

An Existing MZONEPC Chart

```
    ***************************
    ***    MAIN    MENU    ***
    ***************************

  (1) EVALUATION OF THE ARL OF A MZONEPC CHART
  (2) CLASSICAL DESIGN OF THE MZONEPC CHART
  (3) OPTIMAL DESIGN OF THE MZONEPC CHART
  (4) EXIT THE PROGRAM

  ====> PLEASE ENTER YOUR OPTION (1, 2, 3, OR 4) <====
1

  ===> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <===
  ===> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <===
3

  ***THE TOTAL NUMBER OF VARIABLES IS 3

  ===> IS THE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

  *** THE STANDARDIZED COVARIANCE MATRIX
  *** IS INITIALLY AN IDENTITY MATRIX ***

  ===> IS THE DATA CORRECT ? <===
  ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
2

  ===> PLEASE ENTER THE DESIRED STANDARDIZED <===
  ===> COVARIANCE MATRIX ROW BY ROW <===
1 0.6 0.4
0.6 1 0.7
0.4 0.7 1

  *** THE STANDARDIZED COVARIANCE MATRIX IS ***

  1.000   0.600   0.400
  0.600   1.000   0.700
  0.400   0.700   1.000
```

===> ARE THESE DATA CORRECT ? <===

===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** PLEASE ENTER THE PARAMETERS OF THE EXISTING CHARTS ***

*** THE SYMMETRICAL CONTROL LIMIT H ***
*** FOR NO. 1 IZONEPC CHART IS : ***
3.0

*** THE SYMMETRICAL CONTROL LIMIT H ***
*** FOR NO. 2 IZONEPC CHART IS : ***
3.0

*** THE SYMMETRICAL CONTROL LIMIT H ***
*** FOR NO. 3 IZONEPC CHART IS : ***
2.5

*** THE PARAMETERS FOR THE CHARTS ARE : ***

H1 = 3.000000
H2 = 3.000000
H3 = 2.500000

===> ARE THESE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** PLEASE ENTER THE EXPECTED SHIFT OF THE MEAN VECTOR ***
1. 0.2 2.0

*** THE EXPECT MEAN VECTOR SHIFT IS ***
1.000000 .2000000 2.000000

===> ARE THESE DATA CORRECT ? <===
===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

*** THE CORRESPONDING SHIFTS ON THE PRIN. COMP. ARE ***

1.227067 0.5951731 2.526707

*** THE INITIAL ZONE SCORES FOR ALL IZONEPC CHARTS ***
*** ARE SET AT 0 ***

==> ARE THESE DATA CORRECT ? <==
==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

*** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
*** ARE SET AT 0.D0 (CENTRALIZED)***

*** THE OOC ARL FOR THE DESIRED MZONEPC CHART ***
*** AT THE DESIRED SHIFT IS 1.616788

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***

The Classical Design Of

The MZONEPC Chart

****
*** MAIN MENU ***
****

(1) EVALUATION OF THE ARL OF A MZONEPC CHART
(2) CLASSICAL DESIGN OF THE MZONEPC CHART
(3) OPTIMAL DESIGN OF THE MZONEPC CHART
(4) EXIT THE PROGRAM

==> PLEASE ENTER YOUR OPTION (1, 2, 3, OR 4) <==
2

==> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <==
==> THE NUMBER SHOULD BE BETWEEN 2 TO 5 <==
2

*** TOTAL NUMBER OF VARIABLES ARE 2

==> IS THE DATA CORRECT ? <==
==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

*** THE STANDARDIZED COVARIANCE MATRIX
*** IS INITIALLY AN IDENTITY MATRIX ***

==> IS THE DATA CORRECT ? <==
==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
2

```
  ==> PLEASE ENTER THE DESIRED STANDARDIZED <==
  ==> COVARIANCE MATRIX ROW BY ROW <==
1 0.8
0.8 1

  *** THE STANDARDIZED COVARIANCE MATRIX IS ***

  1.000    0.800
  0.800    1.000

  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

  *** PLEASE ENTER THE DESIRED IN-CONTROL ARL ***
100.0

  *** THE DESIRED IN-CONTROL ARL IS   100.0000

  ==> IS THE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1

  *** PLEASE ENTER THE DIRECTION OF THE EXPECTED ***
  *** SHIFT OF THE MEAN VECTOR ***
0.75 0.5

  *** THE DIRECTION OF THE EXPECTED MEAN ***
  *** VECTOR SHIFT IS ***
    0.750000     0.500000

  ==> ARE THESE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1


  *** THE CORRESPONDING DIRECTION OF THE SHIFT ***
  *** IN THE PRIN. COMP. WITH UNIT LENGTH IS ***

    0.8574929     0.5144958

  *** THE INITIAL ZONE SCORES FOR ALL IZONEPC CHARTS ***
  *** ARE SET AT 0 ***

  ==> IS THE DATA CORRECT ? <==
  ==> PLEASE ENTER (1) FOR YES, (2) FOR NO <==
1
```

*** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
*** ARE SET AT 0.D0 (CENTRALIZED)***

THE COMMON CONTROL LIMIT OF A 2 VARIATE MZONEPC CHART
WITH IN CONTROL ARL OF 100.00 IS :
3.329191

*** THE MATRIX U OF THE EIGEN VECTORS ARE : ***
*** u1 : 0.52705 0.52705
*** u2 : 1.58114 -1.58114

*** THE OOC ARL PROFILE ARE SHOWN AS FOLLOWS ***

LAMBDA	OOC ARL AT DIRECTION	OOC ARL OVERALL
0.50	35.0798	33.3729 - 35.5993
1.00	11.2619	10.0021 - 11.6872
1.50	5.7266	5.0199 - 5.9747
2.00	3.7769	3.2968 - 3.9525
2.50	2.8112	2.4291 - 2.9571
3.00	2.2086	1.8818 - 2.3391

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***

<u>The Optimal Design Of</u>

<u>The MZONEPC Chart</u>

****
*** MAIN MENU ***
****

(1) EVALUATION OF THE ARL OF A MZONEPC CHART
(2) CLASSICAL DESIGN OF THE MZONEPC CHART
(3) OPTIMAL DESIGN OF THE MZONEPC CHART
(4) EXIT THE PROGRAM

===> PLEASE ENTER YOUR OPTION (1, 2, 3, OR 4) <===
3

===> PLEASE ENTER THE NUMBER OF VARIABLES MONITORED <===
===> THE NUMBER SHOULD BE BETWEEN 2 AND 5 <===
2

***THE TOTAL NUMBER OF VARIABLES IS 2

```
   ===> IS THE DATA CORRECT ? <===
   ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

   *** THE STANDARDIZED COVARIANCE MATRIX
   *** IS INITIALLY AN IDENTITY MATRIX ***

   ===> IS THE DATA CORRECT ? <===
   ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

   *** PLEASE ENTER THE DESIRED IN-CONTROL ARL ***
250.0

   *** THE DESIRED IN-CONTROL ARL IS   250.0000

   ===> IS THE DATA CORRECT ? <===
   ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

   *** PLEASE ENTER THE EXPECTED SHIFT OF ***
   *** THE ORIGINAL MEAN VECTOR ***
0.9  1.2


   *** THE EXPECTED ORIGINAL MEAN VECTOR SHIFT IS ***
     0.9000000      1.200000

   ===> ARE THESE DATA CORRECT ? <===
   ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

   *** THE CORRESPONDING SHIFT OF THE  ***
   *** MEAN OF THE PRIN. COMP. IS ***

     0.9000000      1.200000

   *** THE INITIAL ZONE SCORES FOR ALL IZONEPC CHARTS ***
   *** ARE SET AT 0 ***

   ===> IS THE DATA CORRECT ? <===
   ===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===
1

   *** THE IN-CONTROL PROCESS MEANS FOR ALL VARIABLES ***
   *** ARE SET AT 0.D0 (CENTRALIZED)***
```

.... TOTAL OF 15 FUNCTION EVALUATIONS DONE ....

.... TOTAL OF 30 FUNCTION EVALUATIONS DONE ....

*** THE OPTIMAL 2 VARIATE MZONEPC CHART ***
*** WITH IN-CONTROL ARL OF 250.00
*** AT THE SHIFT OF THE ORIGINAL MEAN VECTOR OF : ***
0.9000000 1.200000
*** IS LISTED AS FOLLOWS ***

*** H1 = 4.047463
*** H2 = 3.543947

*** THE OPTIMAL OOC ARL = 7.285275

*** PAUSE! PLEASE HIT ENTER TO RETURN TO MAIN MENU ***

Summary

Nearly every feature of the interactive computer programs of this research has been illustrated in this chapter. Examples and step by step procedures are given to describe the convenience, flexibility, and comprehension of the interactive features of the computer program for the evaluation and design of the MEWMAPC and MZONEPC control charts. The implementation of this program provides an efficient and powerful tool for practitioners to use in the field of multivariate quality control.

CHAPTER VIII

SUMMARY, CONCLUSIONS AND FUTURE WORK

This chapter details all the steps leading to the fulfillment of the objective and subobjectives of this research. First a summary of the research is presented. Secondly, the contributions of the research are pointed out. Then, the conclusions are provided. Finally, possible future work and extension of this research are cited.

Summary

The objective of this research is to develop multivariate EWMA and Zone principal component control charts under either a classical design or a statistically optimal design approach as an alternative to various types of multivariate control charts to monitor the mean vector of a multivariate process in a realistic environment. To fulfill this objective, five subobjectives are identified for this research. These subobjectives and the associated accomplishments are discussed as follows.

Statistical Model Development

The first subobjective is the development of the statistically-based model for evaluating the ARL performance of the MEWMAPC and MZONEPC charts. The accomplishments and contributions are:

(1) Combining Crowder's integral equation and Waldmann's bound method to establish an efficient analytical model to evaluate the ARL performance of the IEWMAPC chart.

(2) Employ Waldmann's bound method to develop an efficient analytical model to evaluate the ARL performance of the IZONEPC chart.

(3) Develop the analytical model to evaluate the ARL performance of the multivariate EWMA and Zone principal component control charts.

Optimal Design Algorithm Development

The second subobjective is to develop an algorithm to obtain the parameters of the statistically optimal MEWMAPC and MZONEPC charts. The following contributions are achieved.

(1) A four-step optimization algorithm is developed.

(2) The UNICY and STEPIT programs developed by Chandler (1967) are selected to perform the optimization process.

Computer Programs Development

The third subobjective is to develop computer programs to evaluate the ARL performance of the MEWMAPC and MZONEPC charts and to assist in the classical and statistically optimal designs of these charts. The contributions are:

(1) An interactive program is developed and implemented to (i) evaluate the ARL of a MEWMAPC chart, (ii) design a classical MEWMAPC chart, (iii) design an optimal MEWMAPC chart using C1, (iv) design an optimal MEWMAPC chart using C2.

(2) An interactive program is developed and implemented to (i) evaluate the ARL of a MZONEPC chart, (ii) design a classical MZONEPC chart, (iii) design an optimal MZONEPC chart.

Comparisons Of Newly Developed And Existing Multivariate Control Charts

The fourth subobjective is to investigate and compare the developed multivariate principal component charts with other existing control charts. The contributions are listed as follows.

(1) A comparison of the ARL performance among the classical designs of the bivariate MEWMAPC, MZONEPC, MCUSUM, MC1, MEWMA and χ^2 charts is performed.

(2) The comparison of the optimal bivariate and trivariate MEWMAPC chart under design criteria C2 and C4 shows that the ARL performance of the optimal MEWMAPC chart under both design criteria is very similar.

(3) A comparison of the ARL performance among the optimal MEWMAPC charts using C1 and the optimal MEWMA charts under both bivariate and trivariate cases is discussed.

(4) A comparison of the ARL performance among the optimal MEWMAPC charts using C2 and the optimal MEWMA charts under both bivariate and trivariate cases is discussed.

(5) Comparisons of the ARL performance of the optimal MEWMAPC charts and an optimal MEWMA chart for three bivariate processes with correlation at $\rho = 0.2$, 0.5 and 0.8, respectively, are performed.

(6) A comparison of the ARL performance among the bivariate optimal MEWMAPC charts using C2 and the optimal MZONEPC charts has been addressed.

Sensitivity Analysis

The fifth subobjective is to conduct sensitivity analysis to study the effect of the erroneous estimation of the process parameters on the ARL performance of the optimal MEWMAPC and MZONEPC charts. The following contribution has been obtained.

(1) Sensitivity analysis of the optimal MEWMAPC charts using design criteria C1 and C2 are carried out. The study shows that the optimal MEWMAPC chart using C1 is insensitive to moderate error in the estimation of the true process parameters.

(2) Sensitivity analysis of the optimal MZONEPC charts is also performed in a similar manner.

Conclusions

Based on the results obtained in this research, the conclusions are listed as follows.

(1) For a given size of mean vector shift, the MEWMAPC and MZONEPC control charts under either classical or optimal design approache perform best if the locations of the shift of the process mean vector are along or nearby one of the axes of the principal components. Note that the size of the mean vector shift is measured according to the statistical distance between the in-control and OOC mean vector, or to the value of the noncentrality parameter λ.

(2) For a given size of mean vector shift, the ARL of either the MEWMAPC or MZONEPC control charts will be the largest if the direction of the mean vector shift generates an equivalent size of mean shift in each principal component.

(3) In the design of MEWMAPC control charts, small values of r are more efficient in detecting small process mean shifts and large values of r are more efficient in detecting large process mean shifts. This is the same as in the design of the univariate EWMA control chart.

(4) The ARL performance of the optimal MEWMAPC control chart is superior to the MZONEPC control chart. The superiority is especially significant when the size of the mean vector shift is small.

(5) The ARL performance of the optimal MEWMAPC control chart using C2 is better than that of using design criterion C1 most of the time. Note that if the mean vector shift of the original variables is transformed to have equal size of mean shift in each principal component, the ARL of the optimal MEWMAPC chart using C1 and C2 is equivalent.

(6) For a bivariate process, regardless of the size of the correlation ρ, the optimal MEWMAPC chart using C2 performs better than the optimal MEWMA chart when the magnitude of the mean shift in the original variables is equal or nearly equal.

(7) The optimal MEWMAPC control charts using C1 are relatively insensitive to moderate error in the estimation of the process ccvariance, the sizes, and the directions of the shift of the process mean vector. However, the bias or the difference of the OOC ARL of the optimal MEWMAPC charts obtained from the use of C1 versus C4 can be large. On the contrary, the optimal MEWMAPC control charts using C2 are sensitive to estimation error in the process parameters. Finally, the bias can be large if the

OOC ARL of the conceived optimal MZONEPC charts is compared with the real optimal OOC ARL of the MEWMAPC charts using C4.

Future Work

The following are topics for possible future work related to the extension of this research.

(1) Develop the multivariate CUSUM principal component control chart. The research and experience gained in this research will help in the design of this chart.

(2) Develop a chart that can be incorporated or combined with the principal component chart thus developed to monitor the change of the process covariance matrix. In this research, the possible change of the process covariance matrix is not considered.

(3) Extend Duncan's economic model for the univariate control chart to these multivariate principal component control charts. Therefore, the economically optimal multivariate principal component chart can be developed.

(4) Investigate the fast initial response and worst case scenarios of the MEWMAPC and MZONEPC charts. This can be done by using the proper initial value in each individual principal component chart.

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A P P E N D I C E S

APPENDIX A

PARAMETERS OF THE OPTIMAL BIVARIATE AND TRIVARIATE MEWMAPC CHARTS EMPLOYED IN THE COMPARISON OF CRITERIA C2 AND C4

Parameters Of The Optimal Bivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 Under Various Values Of λ For Directions (θ) Of Shift At 0°, 15°, 30° And 45°

θ	λ	h_1	h_2	r
0°	0.5	0.3888	0.9426	0.05718
	1.0	0.7595	1.6176	0.15922
	1.5	1.0962	2.2277	0.28254
	2.0	1.4444	2.8760	0.42954
	2.5	1.8230	3.5908	0.59809
	3.0	2.1511	4.2226	0.74176
15°	0.5	0.3752	0.9160	0.05427
	1.0	0.7355	1.5732	0.15144
	1.5	1.0625	2.1690	0.26918
	2.0	1.3939	2.7819	0.40742
	2.5	1.7602	3.4716	0.57011
	3.0	2.0900	4.1050	0.71553
30°	0.5	0.3561	0.5167	0.04934
	1.0	0.6847	0.9399	0.13366
	1.5	0.9844	1.3202	0.23646
	2.0	1.2740	1.6863	0.35253
	2.5	1.5924	2.0914	0.49097
	3.0	1.9210	2.4948	0.63642
45°	0.5	0.4057	0.4057	0.05077
	1.0	0.7380	0.7380	0.13067
	1.5	1.0382	1.0382	0.22654
	2.0	1.3197	1.3197	0.33146
	2.5	1.6148	1.6148	0.45096
	3.0	1.9489	1.9489	0.59059

Parameters Of The Optimal Bivariate MEWMAPC Charts Using C4 With Nominal ARL = 200 Under Various Values Of λ For Directions (θ) Of Shift At 0°, 15°, 30° And 45°

θ	λ	h_1	r_1	h_2	r_2
0°	0.5	0.3888	0.05719	0.6787	0.03000
	1.0	0.7595	0.15922	0.6787	0.03000
	1.5	1.0962	0.28254	0.6787	0.03000
	2.0	1.4444	0.42954	0.6787	0.03000
	2.5	1.8230	0.59810	0.6787	0.03000
	3.0	2.1511	0.74176	0.6787	0.03000
15°	0.5	0.3752	0.05427	0.6787	0.03000
	1.0	0.7356	0.15144	0.6787	0.03000
	1.5	1.0625	0.26918	0.6787	0.03000
	2.0	1.3939	0.40742	0.6787	0.03000
	2.5	1.7603	0.57011	0.6787	0.03000
	3.0	2.0900	0.71553	0.6787	0.03000
30°	0.5	0.3505	0.04816	0.5792	0.06038
	1.0	0.6825	0.13297	0.9694	0.14128
	1.5	0.9840	0.23629	1.3273	0.23869
	2.0	1.2760	0.35341	1.6511	0.34036
	2.5	1.6006	0.49477	1.9360	0.43516
	3.0	1.9290	0.64019	2.3038	0.56685
45°	0.5	0.4057	0.05077	0.4057	0.05077
	1.0	0.7380	0.13067	0.7380	0.13067
	1.5	1.0382	0.22654	1.0382	0.22654
	2.0	1.3197	0.33146	1.3197	0.33146
	2.5	1.6148	0.45096	1.6148	0.45096
	3.0	1.9489	0.59059	1.9489	0.59059

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 Under Various Values Of λ For Directions (θ) Of Shift At 0°, 15°, 30° And 45° Of Form 4

θ	λ	h_1	h_2	h_3	r
0°	0.5	0.3685	0.8804	0.4634	0.04996
	1.0	0.6899	1.4579	0.8394	0.13133
	1.5	0.9830	1.9822	1.1776	0.22991
	2.0	1.2619	2.4865	1.4984	0.33940
	2.5	1.5612	3.0365	1.8440	0.46721
	3.0	1.8888	3.6444	2.2127	0.61021
15°	0.5	0.3638	0.8673	0.4471	0.04855
	1.0	0.6788	1.4324	0.8102	0.12704
	1.5	0.9661	1.9430	1.1369	0.22219
	2.0	1.2378	2.4330	1.4448	0.32750
	2.5	1.5242	2.9576	1.7707	0.44861
	3.0	1.8433	3.5473	2.1269	0.58754
30°	0.5	0.3603	0.5255	0.4119	0.04583
	1.0	0.6592	0.9158	0.7401	0.11708
	1.5	0.9316	1.2669	1.0362	0.20330
	2.0	1.1871	1.5952	1.3131	0.29831
	2.5	1.4429	1.9239	1.5909	0.40282
	3.0	1.7300	2.2950	1.9013	0.52563
45°	0.5	0.4076	0.4076	0.4076	0.04645
	1.0	0.7168	0.7168	0.7168	0.11484
	1.5	0.9956	0.9956	0.9956	0.19664
	2.0	1.2556	1.2556	1.2556	0.28627
	2.5	1.5070	1.5070	1.5070	0.38151
	3.0	1.7813	1.7813	1.7813	0.49079

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C4 With Nominal ARL = 200 Under Various
Values Of λ For Directions (θ) Of Shift
At 0°, 15°, 30° And 45° Of Form 4

θ	λ	h_1	r_1	h_2	r_2	h_3	r_3
0°	0.5	0.3584	0.04791	0.7343	0.03224	0.5001	0.05670
	1.0	0.6843	0.12961	1.1110	0.07182	0.8637	0.13794
	1.5	0.9805	0.22893	2.0317	0.16409	1.1896	0.23392
	2.0	1.2635	0.34009	2.7023	0.21618	1.4901	0.33636
	2.5	1.5740	0.47307	3.6440	0.37460	1.7790	0.44157
	3.0	1.9017	0.61620	2.8515	0.34384	2.1391	0.58101
15°	0.5	0.3533	0.04647	0.77089	0.03247	0.4796	0.05446
	1.0	0.6727	0.12519	1.1802	0.07393	0.8332	0.13326
	1.5	0.9628	0.22098	2.2243	0.10914	1.1492	0.22630
	2.0	1.2385	0.32777	2.0881	0.21606	1.4422	0.32652
	2.5	1.5357	0.45381	1.8217	0.16881	1.7232	0.42974
	3.0	1.8578	0.59424	2.6867	0.33405	2.0593	0.56028
30°	0.5	0.3486	0.04360	0.5758	0.05401	0.4291	0.04889
	1.0	0.6517	0.11495	0.9409	0.12303	0.7554	0.12111
	1.5	0.9278	0.20191	1.2760	0.20586	1.0425	0.20541
	2.0	1.1860	0.29784	1.5838	0.29469	1.3168	0.29973
	2.5	1.4504	0.40612	1.8652	0.38302	1.5781	0.39767
	3.0	1.7486	0.53406	2.1136	0.46336	1.8698	0.51246
45°	0.5	0.4076	0.04645	0.4076	0.04645	0.4076	0.04645
	1.0	0.7168	0.11484	0.7168	0.11484	0.7168	0.11484
	1.5	0.9956	0.19664	0.9956	0.19664	0.9956	0.19664
	2.0	1.2556	0.28627	1.2556	0.28627	1.2556	0.28627
	2.5	1.5070	0.38151	1.5070	0.38151	1.5070	0.38151
	3.0	1.7813	0.49079	1.7813	0.49079	1.7813	0.49079

APPENDIX B

PARAMETERS AND THE ARL OF THE OPTIMAL BIVARIATE AND TRIVARIATE MEWMA CHARTS EMPLOYED IN THE PERFORMANCE COMPARISON WITH THE MEWMAPC CHARTS

Parameters And The ARL Performance Of The

Optimal Bivariate MEWMA Charts With

Nominal ARL ≅ 200

λ	h	r	ARL
0.5	7.70	0.06	26.50
1.0	9.35	0.16	9.95
1.5	9.90	0.24	5.47
2.0	10.17	0.34	3.53

Parameters And The ARL Performance Of The

Optimal Trivariate MEWMA Charts With

Nominal ARL ≅ 200

λ	h	r	ARL
0.5	9.8	0.06	30.0
1.0	11.52	0.16	11.0
1.5	11.96	0.22	6.02
2.0	12.31	0.30	3.87

APPENDIX C

PARAMETERS AND THE OOC ARL OF THE OPTIMAL BIVARIATE AND TRIVARIATE MEWMAPC CHARTS USING C1 EMPLOYED IN THE PERFORMANCE COMPARISON WITH THE MEWMA CHARTS

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 0.5$

θ	h	r	ARL
0°	0.4124	0.05206	26.204
3°	0.4123	0.05205	26.230
6°	0.4121	0.05201	26.306
9°	0.4117	0.05194	26.430
12°	0.4113	0.05186	26.598
15°	0.4108	0.05175	26.803
18°	0.4102	0.05163	27.039
21°	0.4095	0.05150	27.294
24°	0.4088	0.05137	27.560
27°	0.4081	0.05123	27.824
30°	0.4075	0.05111	28.074
33°	0.4069	0.05100	28.299
36°	0.4064	0.05091	28.487
39°	0.4060	0.05084	28.629
42°	0.4058	0.05079	28.717
45°	0.4057	0.05078	28.747

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 1.0$

θ	h	r	ARL
0°	0.7756	0.14155	9.650
3°	0.7752	0.14143	9.665
6°	0.7739	0.14107	9.710
9°	0.7719	0.14047	9.784
12°	0.7692	0.13968	9.884
15°	0.7659	0.13873	10.010
18°	0.7623	0.13767	10.156
21°	0.7584	0.13654	10.318
24°	0.7545	0.13540	10.490
27°	0.7507	0.13430	10.665
30°	0.7472	0.13328	10.833
33°	0.7441	0.13239	10.988
36°	0.7415	0.13166	11.119
39°	0.7396	0.13112	11.219
42°	0.7384	0.13078	11.282
45°	0.7380	0.13067	11.303

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 1.5$

θ	h	r	ARL
0°	1.1122	0.25296	5.226
3°	1.1114	0.25265	5.235
6°	1.1089	0.25175	5.263
9°	1.1050	0.25033	5.309
12°	1.0997	0.24842	5.372
15°	1.0933	0.24612	5.450
18°	1.0861	0.24355	5.543
21°	1.0785	0.24081	5.648
24°	1.0707	0.23804	5.760
27°	1.0632	0.23536	5.876
30°	1.0562	0.23288	5.989
33°	1.0501	0.23072	6.094
36°	1.0450	0.22892	6.184
39°	1.0413	0.22762	6.253
42°	1.0390	0.22682	6.297
45°	1.0382	0.22654	6.312

Parameters Of The Optimal Bivariate MEWMAPC Charts Using C1 With Nominal ARL = 200 For Various Directions (θ) Of Shift At $\lambda = 2.0$

θ	h	r	ARL
0°	1.4419	0.38012	3.382
3°	1.4406	0.37956	3.388
6°	1.4365	0.37790	3.406
9°	1.4298	0.37522	3.437
12°	1.4209	0.37166	3.480
15°	1.4103	0.36737	3.534
18°	1.3983	0.36258	3.598
21°	1.3856	0.35750	3.670
24°	1.3728	0.35241	3.749
27°	1.3604	0.34747	3.831
30°	1.3488	0.34291	3.911
33°	1.3389	0.33898	3.987
36°	1.3308	0.33579	4.052
39°	1.3247	0.33340	4.103
42°	1.3210	0.33195	4.135
45°	1.3197	0.33146	4.146

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 1
At $\lambda = 0.5$

θ	h	r	ARL
0°	0.4211	0.04887	28.857
3°	0.4210	0.04885	28.890
6°	0.4206	0.04879	28.987
9°	0.4203	0.04873	29.146
12°	0.4196	0.04861	29.361
15°	0.4186	0.04841	29.625
18°	0.4178	0.04828	29.929
21°	0.4168	0.04809	30.260
24°	0.4157	0.04789	30.606
27°	0.4146	0.04770	30.952
30°	0.4136	0.04751	31.281
33°	0.4127	0.04735	31.577
36°	0.4119	0.04721	31.826
39°	0.4113	0.04711	32.015
42°	0.4110	0.04705	32.132
45°	0.4109	0.04703	32.172

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 1
At $\lambda = 1.0$

θ	h	r	ARL
0°	0.7813	0.13222	10.458
3°	0.7808	0.13208	10.476
6°	0.7793	0.13166	10.529
9°	0.7769	0.13100	10.616
12°	0.7736	0.13010	10.735
15°	0.7698	0.12904	10.884
18°	0.7654	0.12784	11.059
21°	0.7608	0.12658	11.254
24°	0.7561	0.12530	11.462
27°	0.7515	0.12407	11.675
30°	0.7473	0.12293	11.882
33°	0.7436	0.12193	12.072
36°	0.7405	0.12111	12.235
39°	0.7383	0.12051	12.360
42°	0.7369	0.12014	12.438
45°	0.7364	0.12001	12.465

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C1 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 1 At $\lambda = 1.5$

θ	h	r	ARL
0°	1.1188	0.23777	5.606
3°	1.1179	0.23745	5.617
6°	1.1151	0.23650	5.648
9°	1.1107	0.23498	5.700
12°	1.1048	0.23294	5.772
15°	1.0976	0.23050	5.863
18°	1.0895	0.22773	5.971
21°	1.0809	0.22481	6.092
24°	1.0720	0.22182	6.224
27°	1.0634	0.21894	6.360
30°	1.0554	0.21626	6.495
33°	1.0484	0.21395	6.620
36°	1.0427	0.21205	6.728
39°	1.0385	0.21064	6.812
42°	1.0358	0.20977	6.865
45°	1.0349	0.20948	6.883

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 1
At λ = 2.0

θ	h	r	ARL
0°	1.4431	0.35672	3.602
3°	1.4417	0.35618	3.609
6°	1.4376	0.35459	3.630
9°	1.4307	0.35194	3.664
12°	1.4217	0.34848	3.712
15°	1.4106	0.34423	3.773
18°	1.3982	0.33951	3.846
21°	1.3850	0.33448	3.928
24°	1.3715	0.32939	4.019
27°	1.3585	0.32446	4.113
30°	1.3464	0.31991	4.207
33°	1.3358	0.31594	4.296
36°	1.3267	0.31253	4.373
39°	1.3207	0.31030	4.433
42°	1.3168	0.30883	4.471
45°	1.3155	0.30834	4.484

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At $\lambda = 0.5$

θ	h	r	ARL
0°	0.4198	0.04863	29.324
3°	0.4197	0.04861	29.356
6°	0.4194	0.04856	29.451
9°	0.4187	0.04843	29.606
12°	0.4183	0.04835	29.815
15°	0.4175	0.04821	30.072
18°	0.4163	0.04800	30.367
21°	0.4156	0.04788	30.689
24°	0.4146	0.04770	31.025
27°	0.4136	0.04751	31.360
30°	0.4127	0.04734	31.679
33°	0.4118	0.04719	31.966
36°	0.4111	0.04707	32.207
39°	0.4106	0.04698	32.389
42°	0.4103	0.04693	32.503
45°	0.4102	0.04691	32.541

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At λ = 1.0

θ	h	r	ARL
0°	0.7742	0.13026	10.715
3°	0.7737	0.13011	10.733
6°	0.7723	0.12973	10.786
9°	0.7700	0.12911	10.873
12°	0.7670	0.12827	10.992
15°	0.7633	0.12727	11.141
18°	0.7592	0.12615	11.316
21°	0.7548	0.12496	11.510
24°	0.7504	0.12376	11.718
27°	0.7461	0.12260	11.929
30°	0.7421	0.12153	12.135
33°	0.7386	0.12059	12.324
36°	0.7357	0.11982	12.485
39°	0.7335	0.11925	12.609
42°	0.7322	0.11890	12.687
45°	0.7318	0.11878	12.713

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At λ = 1.5

θ	h	r	ARL
0°	1.1058	0.23329	5.760
3°	1.1049	0.23298	5.771
6°	1.1023	0.23209	5.802
9°	1.0980	0.23065	5.855
12°	1.0923	0.22870	5.928
15°	1.0855	0.22637	6.020
18°	1.0778	0.22377	6.128
21°	1.0694	0.22097	6.251
24°	1.0610	0.21814	6.384
27°	1.0528	0.21541	6.521
30°	1.0452	0.21288	6.656
33°	1.0386	0.21068	6.782
36°	1.0330	0.20885	6.891
39°	1.0290	0.20752	6.975
42°	1.0265	0.20670	7.028
45°	1.0257	0.20643	7.046

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At $\lambda = 2.0$

θ	h	r	ARL
0°	1.4231	0.34904	3.704
3°	1.4219	0.34855	3.711
6°	1.4178	0.34701	3.732
9°	1.4113	0.34451	3.767
12°	1.4027	0.34122	3.816
15°	1.3923	0.33726	3.878
18°	1.3804	0.33276	3.952
21°	1.3679	0.32801	4.036
24°	1.3551	0.32317	4.127
27°	1.3426	0.31849	4.223
30°	1.3311	0.31416	4.318
33°	1.3209	0.31038	4.408
36°	1.3127	0.30730	4.486
39°	1.3065	0.30503	4.547
42°	1.3027	0.30362	4.585
45°	1.3015	0.30315	4.598

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C1 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 3 At $\lambda = 0.5$

θ	h	r	ARL
0°	0.4159	0.04792	30.557
3°	0.4158	0.04791	30.585
6°	0.4156	0.04786	30.670
9°	0.4152	0.04780	30.807
12°	0.4147	0.04771	30.993
15°	0.4141	0.04760	31.220
18°	0.4134	0.04748	31.481
21°	0.4126	0.04734	31.764
24°	0.4119	0.04720	32.059
27°	0.4111	0.04707	32.351
30°	0.4105	0.04695	32.629
33°	0.4098	0.04684	32.879
36°	0.4093	0.04675	33.088
39°	0.4089	0.04668	33.246
42°	0.4087	0.04664	33.344
45°	0.4086	0.04663	33.377

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 3
At $\lambda = 1.0$

θ	h	r	ARL
0°	0.7567	0.12548	11.433
3°	0.7563	0.12537	11.450
6°	0.7552	0.12506	11.500
9°	0.7533	0.1246	11.584
12°	0.7508	0.12388	11.698
15°	0.7478	0.12307	11.840
18°	0.7444	0.12216	12.005
21°	0.7409	0.12120	12.188
24°	0.7372	0.12023	12.383
27°	0.7337	0.11928	12.581
30°	0.7304	0.11841	12.772
33°	0.7275	0.11765	12.947
36°	0.7251	0.11702	13.096
39°	0.7233	0.11655	13.210
42°	0.7222	0.11626	13.281
45°	0.7219	0.11616	13.305

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 3
At $\lambda = 1.5$

θ	h	r	ARL
0°	1.0732	0.22224	6.205
3°	1.0725	0.22198	6.215
6°	1.0702	0.22123	6.247
9°	1.0666	0.22003	6.299
12°	1.0619	0.21843	6.371
15°	1.0561	0.21649	6.461
18°	1.0495	0.21431	6.568
21°	1.0426	0.21200	6.688
24°	1.0355	0.20966	6.816
27°	1.0286	0.20739	6.949
30°	1.0222	0.20529	7.079
33°	1.0166	0.20346	7.199
36°	1.0119	0.20194	7.303
39°	1.0085	0.20082	7.382
42°	1.0064	0.20014	7.433
45°	1.0057	0.19991	7.450

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 3
At λ = 2.0

θ	h	r	ARL
0°	1.3734	0.33010	4.006
3°	1.3723	0.32967	4.013
6°	1.3690	0.32842	4.034
9°	1.3635	0.32637	4.070
12°	1.3563	0.32364	4.119
15°	1.3476	0.32034	4.181
18°	1.3377	0.31664	4.255
21°	1.3267	0.31254	4.338
24°	1.3164	0.30867	4.429
27°	1.3059	0.30479	4.524
30°	1.2961	0.30118	4.617
33°	1.2876	0.29801	4.704
36°	1.2805	0.29542	4.780
39°	1.2753	0.29351	4.839
42°	1.2721	0.29233	4.876
45°	1.2710	0.29193	4.889

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 0.5$

θ	h	r	ARL
0°	0.4121	0.04724	31.770
3°	0.4120	0.04723	31.792
6°	0.4119	0.04720	31.855
9°	0.4117	0.04717	31.957
12°	0.4114	0.04711	32.095
15°	0.4110	0.04705	32.263
18°	0.4106	0.04698	32.455
21°	0.4102	0.04690	32.662
24°	0.4097	0.04682	32.877
27°	0.4093	0.04674	33.090
30°	0.4088	0.04666	33.291
33°	0.4085	0.04660	33.471
36°	0.4082	0.04654	33.621
39°	0.4079	0.04649	33.734
42°	0.40774	0.046465	33.804
45°	0.40769	0.046456	33.828

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 1.0$

θ	h	r	ARL
0°	0.7412	0.12130	12.198
3°	0.7409	0.12122	12.212
6°	0.7401	0.12101	12.253
9°	0.7387	0.12063	12.320
12°	0.7371	0.12020	12.411
15°	0.7350	0.11964	12.523
18°	0.7326	0.11901	12.653
21°	0.7301	0.11835	12.796
24°	0.7276	0.11768	12.946
27°	0.7251	0.11702	13.097
30°	0.7228	0.11641	13.242
33°	0.7208	0.11588	13.374
36°	0.7191	0.11544	13.485
39°	0.7179	0.11511	13.570
42°	0.7171	0.11491	13.622
45°	0.7168	0.11484	13.640

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 1.5$

θ	h	r	ARL
0°	1.0440	0.21247	6.704
3°	1.0435	0.21230	6.713
6°	1.0419	0.21177	6.739
9°	1.0393	0.21092	6.782
12°	1.0358	0.20978	6.842
15°	1.0317	0.20841	6.916
18°	1.0270	0.20688	7.002
21°	1.0221	0.20525	7.098
24°	1.0170	0.20360	7.200
27°	1.0121	0.20199	7.304
30°	1.0075	0.20049	7.405
33°	1.0034	0.19918	7.498
36°	1.0001	0.19810	7.576
39°	0.9976	0.19730	7.637
42°	0.9961	0.19681	7.674
45°	0.9956	0.19664	7.687

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C1 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 2.0$

θ	h	r	ARL
0°	1.3291	0.31341	4.355
3°	1.3282	0.31311	4.362
6°	1.3259	0.31222	4.380
9°	1.3220	0.31078	4.410
12°	1.3168	0.30886	4.452
15°	1.3106	0.30655	4.504
18°	1.3036	0.30393	4.565
21°	1.2960	0.30112	4.633
24°	1.2882	0.29826	4.707
27°	1.2808	0.29551	4.782
30°	1.2738	0.29293	4.855
33°	1.2676	0.29067	4.923
36°	1.2625	0.28880	4.982
39°	1.2587	0.28742	5.027
42°	1.2563	0.28656	5.055
45°	1.2556	0.28628	5.064

APPENDIX D

PARAMETERS AND THE OOC ARL OF THE OPTIMAL BIVARIATE AND TRIVARIATE MEWMAPC CHARTS USING C2 EMPLOYED IN THE PERFORMANCE COMPARISON WITH THE MEWMA CHARTS

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At λ = 0.5

θ	h_1	h_2	r	ARL
0°	0.3888	0.9426	0.05718	21.968
3°	0.3882	0.9422	0.05707	22.008
6°	0.3866	0.9377	0.05671	22.129
9°	0.3839	0.9345	0.05613	22.334
12°	0.3801	0.9257	0.05531	22.625
15°	0.3752	0.9160	0.05427	23.006
18°	0.3693	0.8499	0.05301	23.485
21°	0.3626	0.7190	0.05160	24.067
24°	0.3567	0.6278	0.05030	24.753
27°	0.3541	0.5629	0.04950	25.520
30°	0.3561	0.5167	0.04934	26.322
33°	0.3621	0.4831	0.04964	27.094
36°	0.3708	0.4577	0.05007	27.772
39°	0.3811	0.4374	0.05045	28.299
42°	0.3927	0.4204	0.05069	28.633
45°	0.4057	0.4057	0.05077	28.747

Parameters Of The Optimal Bivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift At λ = 1.0

θ	h_1	h_2	r	ARL
0°	0.7595	1.6176	0.15922	8.318
3°	0.7585	1.6158	0.15890	8.335
6°	0.7556	1.6106	0.15796	8.385
9°	0.7508	1.6008	0.15640	8.469
12°	0.7441	1.5889	0.15422	8.589
15°	0.7355	1.5732	0.15144	8.747
18°	0.7250	1.5254	0.14807	8.945
21°	0.7129	1.2986	0.14419	9.188
24°	0.7002	1.1397	0.14008	9.477
27°	0.6899	1.0246	0.13639	9.806
30°	0.6847	0.9399	0.13366	10.160
33°	0.6855	0.8767	0.13201	10.511
36°	0.6920	0.8288	0.13117	10.829
39°	0.7032	0.7916	0.13082	11.082
42°	0.7186	0.7620	0.13070	11.247
45°	0.7380	0.7380	0.13067	11.303

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 1.5$

θ	h_1	h_2	r	ARL
0°	1.0962	2.2277	0.28254	4.592
3°	1.0948	2.2255	0.28200	4.602
6°	1.0908	2.2190	0.28038	4.630
9°	1.0840	2.2084	0.27769	4.677
12°	1.0746	2.1895	0.27395	4.744
15°	1.0625	2.0690	0.26918	4.833
18°	1.0478	2.1405	0.26341	4.944
21°	1.0306	1.8236	0.25672	5.081
24°	1.0122	1.6013	0.24943	5.244
27°	0.9957	1.4399	0.24237	5.432
30°	0.9844	1.3202	0.23647	5.635
33°	0.9807	1.2306	0.23215	5.840
36°	0.9847	1.1627	0.22934	6.028
39°	0.9959	1.1105	0.22767	6.180
42°	1.0138	1.0699	0.22681	6.278
45°	1.0382	1.0382	0.22654	6.312

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At λ = 2.0

θ	h_1	h_2	r	ARL
0°	1.4444	2.8760	0.42954	3.006
3°	1.4424	2.8708	0.42864	3.012
6°	1.4362	2.8580	0.42594	3.031
9°	1.4260	2.8417	0.42148	3.062
12°	1.4119	2.8109	0.41528	3.106
15°	1.3939	2.7819	0.40742	3.165
18°	1.3721	2.7393	0.39797	3.238
21°	1.3470	2.3456	0.38707	3.328
24°	1.3197	2.0553	0.37513	3.435
27°	1.2939	1.8437	0.36316	3.558
30°	1.2740	1.6863	0.35253	3.693
33°	1.2632	1.5681	0.34413	3.829
36°	1.2627	1.4787	0.33815	3.955
39°	1.2721	1.4107	0.33428	4.056
42°	1.2911	1.3589	0.33215	4.122
45°	1.3197	1.3197	0.33146	4.146

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 1 At $\lambda = 0.5$

θ	h_1	h_2	h_3	r	ARL
0°	0.3888	0.9436	0.9436	0.05718	21.968
3°	0.3882	0.9426	0.9426	0.05707	22.008
6°	0.3866	0.9396	0.9396	0.05671	22.129
9°	0.3839	0.9346	0.9346	0.05613	22.334
12°	0.3801	0.9276	0.9276	0.05531	22.625
15°	0.3752	0.9185	0.9185	0.05427	23.006
18°	0.3693	0.8499	0.9076	0.05301	23.485
21°	0.3626	0.7190	0.8946	0.05160	24.067
24°	0.3567	0.6278	0.8812	0.05030	24.753
27°	0.3541	0.5629	0.8762	0.04950	25.520
30°	0.3561	0.5167	0.8729	0.04934	26.322
33°	0.3621	0.4831	0.8774	0.04963	27.094
36°	0.3708	0.4577	0.8810	0.05007	27.772
39°	0.3811	0.4374	0.8847	0.05045	28.299
42°	0.3927	0.4204	0.8869	0.05069	28.633
45°	0.4057	0.4057	0.8877	0.05078	28.747

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 1
At $\lambda = 1.0$

θ	h_1	h_2	h_3	r	ARL
0°	0.7595	1.6176	1.6176	0.15922	8.318
3°	0.7585	1.6158	1.6158	0.15890	8.335
6°	0.7556	1.6106	1.6106	0.15796	8.385
9°	0.7508	1.6019	1.6019	0.15640	8.469
12°	0.7441	1.5898	1.5898	0.15422	8.589
15°	0.7355	1.5742	1.5742	0.15144	8.747
18°	0.7250	1.5254	1.5552	0.14807	8.945
21°	0.7128	1.2986	1.5323	0.14419	9.188
24°	0.7002	1.1397	1.5089	0.14008	9.477
27°	0.6899	1.0246	1.4879	0.13639	9.806
30°	0.6847	0.9399	1.4749	0.13366	10.160
33°	0.6855	0.8767	1.4621	0.13201	10.511
36°	0.6920	0.8288	1.4572	0.13118	10.829
39°	0.7032	0.7916	1.4551	0.13082	11.083
42°	0.7186	0.7620	1.4542	0.13070	11.247
45°	0.7380	0.7380	1.4541	0.13067	11.303

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 1 At $\lambda = 1.5$

θ	h_1	h_2	h_3	r	ARL
0°	1.0962	2.2308	2.2308	0.28254	4.592
3°	1.0948	2.2283	2.2283	0.28200	4.602
6°	1.0908	2.2208	2.2208	0.28038	4.630
9°	1.0840	2.2084	2.2084	0.27769	4.677
12°	1.0746	2.1911	2.1911	0.27395	4.744
15°	1.0624	2.1689	2.1689	0.26917	4.833
18°	1.0478	2.1406	2.1414	0.26341	4.944
21°	1.0306	1.8236	2.1106	0.25672	5.081
24°	1.0122	1.6013	2.0761	0.24943	5.244
27°	0.9957	1.4399	2.0424	0.24238	5.432
30°	0.9844	1.3202	2.0131	0.23646	5.636
33°	0.9807	1.2306	1.9931	0.23215	5.840
36°	0.9847	1.1626	1.9794	0.22934	6.028
39°	0.9959	1.1105	1.9713	0.22767	6.180
42°	1.0138	1.0699	1.9670	0.22681	6.278
45°	1.0382	1.0382	1.9657	0.22654	6.312

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 1
At λ = 2.0

θ	h_1	h_2	h_3	r	ARL
0°	1.4444	2.8764	2.8764	0.42954	3.006
3°	1.4424	2.8726	2.8726	0.42864	3.012
6°	1.4362	2.8611	2.8611	0.42594	3.031
9°	1.4260	2.8420	2.8420	0.42148	3.062
12°	1.4119	2.8155	2.8155	0.41528	3.106
15°	1.3939	2.7819	2.7819	0.40742	3.165
18°	1.3721	2.7413	2.7413	0.39797	3.238
21°	1.3470	2.3456	2.6943	0.38707	3.328
24°	1.3197	2.0553	2.6423	0.37512	3.435
27°	1.2939	1.8437	2.5907	0.36316	3.558
30°	1.2740	1.6863	2.5442	0.35252	3.693
33°	1.2632	1.5680	2.5073	0.34413	3.829
36°	1.2627	1.4787	2.4810	0.33815	3.955
39°	1.2721	1.4107	2.4639	0.33428	4.056
42°	1.2911	1.3589	2.4544	0.33214	4.123
45°	1.3197	1.3197	2.4514	0.33146	4.146

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 2 At λ = 0.5

θ	h_1	h_2	h_3	r	ARL
0°	0.3807	0.9283	0.9283	0.05545	22.574
3°	0.3802	0.9278	0.9278	0.05534	22.615
6°	0.3786	0.9224	0.9224	0.05499	22.740
9°	0.3759	0.9199	0.9199	0.05442	22.949
12°	0.3722	0.9107	0.9107	0.05363	23.247
15°	0.3674	0.9041	0.9041	0.05262	23.638
18°	0.3616	0.8351	0.8933	0.05140	24.128
21°	0.3550	0.7063	0.8788	0.05004	24.724
24°	0.3493	0.6165	0.8691	0.04878	25.425
27°	0.3469	0.5528	0.8623	0.04803	26.210
30°	0.3491	0.5074	0.8600	0.04792	27.028
33°	0.3552	0.4745	0.8644	0.04824	27.816
36°	0.3639	0.4496	0.8687	0.04868	28.507
39°	0.3742	0.4296	0.8722	0.04907	29.044
42°	0.3857	0.4129	0.8744	0.04931	29.383
45°	0.3985	0.3985	0.8752	0.04939	29.499

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At λ = 1.0

θ	h_1	h_2	h_3	r	ARL
0°	0.7453	1.5919	1.5919	0.15459	8.568
3°	0.7444	1.5902	1.5902	0.15429	8.585
6°	0.7415	1.5850	1.5850	0.15337	8.636
9°	0.7368	1.5765	1.5765	0.15185	8.723
12°	0.7302	1.5646	1.5646	0.14973	8.847
15°	0.7217	1.5492	1.5492	0.14072	9.009
18°	0.7114	1.4996	1.5306	0.14375	9.213
21°	0.6994	1.2765	1.5080	0.13997	9.463
24°	0.6871	1.1202	1.4846	0.13599	9.760
27°	0.6770	1.0071	1.4646	0.13243	10.098
30°	0.6720	0.9238	1.4488	0.12983	10.461
33°	0.6730	0.8617	1.4396	0.12827	10.822
36°	0.6796	0.8147	1.4349	0.12751	11.148
39°	0.6909	0.7781	1.4332	0.12720	11.408
42°	0.7061	0.7490	1.4327	0.12709	11.576
45°	0.7253	0.7253	1.4326	0.12707	11.633

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At λ = 1.5

θ	h_1	h_2	h_3	r	ARL
0°	1.0762	2.1941	12.1941	0.27459	4.732
3°	1.0749	2.1917	2.1917	0.27407	4.742
6°	1.0709	2.1844	2.1844	0.27249	4.771
9°	1.0643	2.1723	2.1723	0.26988	4.819
12°	1.0550	2.1523	2.1523	0.26625	4.889
15°	1.0432	2.1337	2.1337	0.26162	4.980
18°	1.0288	2.1039	2.1039	0.25601	5.095
21°	1.0120	1.7927	2.0749	0.24951	5.236
24°	0.9939	1.5742	2.0427	0.24243	5.404
27°	0.9777	1.4156	2.0097	0.23559	5.597
30°	0.9669	1.2980	1.9815	0.22988	5.807
33°	0.9634	1.2099	1.9604	0.22574	6.018
36°	0.9676	1.1431	1.9473	0.22307	6.211
39°	0.9788	1.0918	1.9409	0.22149	6.366
42°	0.9965	1.0519	1.9368	0.22068	6.467
45°	1.0206	1.0206	1.9357	0.22043	6.502

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 2
At λ = 2.0

θ	h_1	h_2	h_3	r	ARL
0°	1.4143	2.8201	2.8201	0.41635	3.099
3°	1.4123	2.8164	2.8164	0.41548	3.105
6°	1.4064	2.8052	2.8052	0.41288	3.124
9°	1.3965	2.7868	2.7868	0.40858	3.156
12°	1.3828	2.7604	2.7604	0.40261	3.201
15°	1.3654	2.7287	2.7287	0.39505	3.261
18°	1.3443	2.6895	2.6895	0.38596	3.337
21°	1.3200	2.3011	2.6437	0.37547	3.429
24°	1.2937	2.0169	2.5934	0.36401	3.540
27°	1.2689	1.8098	2.5443	0.35256	3.667
30°	1.2500	1.6559	2.4998	0.34244	3.806
33°	1.2401	1.5403	2.4649	0.33451	3.946
36°	1.2401	1.4530	2.4400	0.32889	4.075
39°	1.2499	1.3865	2.4237	0.32527	4.179
42°	1.2689	1.3357	2.4150	0.32328	4.248
45°	1.2972	1.2972	2.4121	0.32264	4.271

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 3 At $\lambda = 0.5$

θ	h_1	h_2	h_3	r	ARL
0°	0.3574	0.8849	0.6388	0.05046	24.650
3°	0.3569	0.8817	0.6374	0.05036	24.695
6°	0.3555	0.8812	0.6331	0.05007	24.828
9°	0.3532	0.8769	0.6261	0.04958	25.053
12°	0.3499	0.8690	0.6164	0.04891	25.373
15°	0.3458	0.8630	0.6041	0.04805	25.791
18°	0.3408	0.7960	0.5894	0.04704	26.315
21°	0.3354	0.6737	0.5730	0.04592	26.951
24°	0.3310	0.5888	0.5571	0.04494	27.697
27°	0.3300	0.5289	0.5450	0.04445	28.527
30°	0.3333	0.4864	0.5382	0.04453	29.390
33°	0.3401	0.4556	0.5354	0.04496	30.217
36°	0.3491	0.4320	0.5348	0.04545	30.938
39°	0.3593	0.4130	0.5348	0.04585	31.497
42°	0.3706	0.3970	0.5351	0.04610	31.850
45°	0.3831	0.3831	0.5352	0.04618	31.970

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 3
At λ = 1.0

θ	h_1	h_2	h_3	r	ARL
0°	0.7019	1.5119	1.1590	0.14065	9.434
3°	0.7011	1.5111	1.1565	0.14038	9.452
6°	0.6985	1.5056	1.1492	0.13956	9.508
9°	0.6941	1.4985	1.1369	0.13820	9.603
12°	0.6881	1.4870	1.1201	0.13631	9.738
15°	0.6804	1.4734	1.0987	0.13392	9.914
18°	0.6711	1.4248	1.0730	0.13103	10.137
21°	0.6604	1.2134	1.0438	0.12772	10.408
24°	0.6495	1.0656	1.0131	0.12429	10.729
27°	0.6412	0.9589	0.9850	0.12129	11.095
30°	0.6377	0.8805	0.9629	0.11919	11.486
33°	0.6370	0.8221	0.9475	0.18037	11.873
36°	0.6473	0.7778	0.9377	0.11755	12.223
39°	0.6589	0.7432	0.9319	0.11741	12.500
42°	0.6740	0.7154	0.9288	0.11740	12.679
45°	0.6927	0.6927	0.9279	0.11741	12.741

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 3
At $\lambda = 1.5$

θ	h_1	h_2	h_3	r	ARL
0°	1.0148	2.0800	1.6283	0.25047	5.220
3°	1.0136	2.0781	1.6249	0.25000	5.230
6°	1.0099	2.0714	1.6145	0.24858	5.262
9°	1.0038	2.0601	1.5975	0.24623	5.315
12°	0.9954	2.0445	1.5739	0.24297	5.391
15°	0.9845	2.0253	1.5441	0.23881	5.492
18°	0.9714	1.9967	1.5084	0.23380	5.618
21°	0.9561	1.7020	1.4674	0.22799	5.772
24°	0.9400	1.4956	1.4235	0.22173	5.956
27°	0.9258	1.3460	1.3816	0.21576	6.166
30°	0.9168	1.2353	1.3461	0.21088	6.394
33°	0.9149	1.1524	1.3194	0.20744	6.623
36°	0.9202	1.0895	1.3009	0.2053	6.832
39°	0.9319	1.0410	1.2892	0.20405	7.000
42°	0.9495	1.0030	1.2827	0.20343	7.109
45°	0.9729	0.9729	1.2807	0.20325	7.146

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 3 At λ = 2.0

θ	h_1	h_2	h_3	r	ARL
0°	1.3236	2.6501	2.0906	0.37684	3.419
3°	1.3218	2.6468	2.0859	0.37608	3.426
6°	1.3166	2.6360	2.0719	0.37381	3.446
9°	1.3078	2.6197	2.0487	0.37005	3.482
12°	1.2957	2.5971	2.0168	0.36485	3.532
15°	1.2804	2.5684	1.9765	0.35827	3.597
18°	1.2619	2.5339	1.9286	0.35039	3.680
21°	1.2406	2.1723	1.8740	0.34132	3.781
24°	1.2178	1.9064	1.8152	0.33145	3.902
27°	1.1968	1.7133	1.7580	0.32174	4.041
30°	1.1815	1.5702	1.7083	0.31331	4.192
33°	1.1747	1.4628	1.6694	0.30685	4.345
36°	1.1770	1.3815	1.6416	0.30237	4.485
39°	1.1880	1.3194	1.6234	0.29955	4.599
42°	1.2073	1.2716	1.6132	0.29802	4.673
45°	1.2348	1.2348	1.6099	0.29753	4.699

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 0.5$

θ	h_1	h_2	h_3	r	ARL
0°	0.3685	0.8804	0.4634	0.04996	27.618
3°	0.3683	0.8798	0.4627	0.04991	27.658
6°	0.3677	0.8783	0.4607	0.04973	27.780
9°	0.3667	0.8757	0.4574	0.04945	27.983
12°	0.3654	0.8717	0.4529	0.04905	28.269
15°	0.3638	0.8673	0.4471	0.04855	28.638
18°	0.3618	0.8569	0.4402	0.04795	29.093
21°	0.3597	0.7275	0.4323	0.04726	29.634
24°	0.3579	0.6376	0.4240	0.04657	30.258
27°	0.3578	0.5729	0.4169	0.04605	30.947
30°	0.3603	0.5255	0.4119	0.04583	31.662
33°	0.3657	0.4900	0.4091	0.04589	32.351
36°	0.3735	0.4636	0.4080	0.04606	32.956
39°	0.3832	0.4408	0.4076	0.04626	33.427
42°	0.3945	0.4228	0.4076	0.04640	33.725
45°	0.4076	0.4076	0.4076	0.04645	33.828

Parameters Of The Optimal Trivariate MEWMAPC Charts Using C2 With Nominal ARL = 200 For Various Directions (θ) Of Shift Of Form 4 At $\lambda = 1.0$

θ	h_1	h_2	h_3	r	ARL
0°	0.6899	1.4579	0.8394	0.13133	10.756
3°	0.6894	1.4569	0.8382	0.13115	10.774
6°	0.6881	1.4539	0.8346	0.13063	10.827
9°	0.6858	1.4486	0.8287	0.12976	10.917
12°	0.6827	1.4414	0.8206	0.12856	11.043
15°	0.6788	1.4324	0.8102	0.12704	11.207
18°	0.6742	1.4215	0.7978	0.12523	11.409
21°	0.6690	1.2564	0.7836	0.12314	11.650
24°	0.6638	1.1057	0.7681	0.12089	11.931
27°	0.6600	0.9967	0.7530	0.11877	12.245
30°	0.6592	0.9158	0.7401	0.11708	12.578
33°	0.6626	0.8546	0.7304	0.11595	12.907
36°	0.6702	0.8075	0.7238	0.11531	13.202
39°	0.6818	0.7705	0.7197	0.11500	13.437
42°	0.6973	0.7408	0.7175	0.11487	13.588
45°	0.7168	0.7168	0.7168	0.11484	13.640

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At $\lambda = 1.5$

θ	h_1	h_2	h_3	r	ARL
0°	0.9830	1.9822	1.1776	0.22991	5.985
3°	0.9823	1.9806	1.1760	0.22959	5.995
6°	0.9802	1.9758	1.1710	0.22865	6.026
9°	0.9768	1.9670	1.1627	0.22709	6.077
12°	0.9720	1.9578	1.1513	0.22493	6.150
15°	0.9661	1.9430	1.1369	0.22219	6.245
18°	0.9590	1.9282	1.1196	0.21891	6.362
21°	0.9510	1.7362	1.0997	0.21514	6.502
24°	0.9427	1.5288	1.0779	0.21101	6.665
27°	0.9355	1.3786	1.0560	0.20690	6.849
30°	0.9316	1.2669	1.0362	0.20330	7.046
33°	0.9328	1.1823	1.0204	0.20053	7.242
36°	0.9397	1.1174	1.0088	0.19863	7.420
39°	0.9525	1.0668	1.0012	0.19746	7.563
42°	0.9710	1.0270	0.9969	0.19683	7.655
45°	0.9956	0.9956	0.9956	0.19664	7.687

Parameters Of The Optimal Trivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift Of Form 4
At λ = 2.0

θ	h_1	h_2	h_3	r	ARL
0°	1.2619	2.4865	1.4983	0.33940	3.926
3°	1.2609	2.4843	1.4962	0.33891	3.932
6°	1.2579	2.4761	1.4896	0.33745	3.953
9°	1.2530	2.4671	1.4787	0.33503	3.987
12°	1.2463	2.4524	1.4637	0.33170	4.035
15°	1.2378	2.4330	1.4448	0.3275	4.097
18°	1.2279	2.4098	1.4223	0.32249	4.175
21°	1.2167	2.1890	1.3965	0.31676	4.268
24°	1.2049	1.9267	1.3682	0.31047	4.376
27°	1.1942	1.7365	1.3394	0.30411	4.498
30°	1.1871	1.5952	1.3131	0.29831	4.630
33°	1.1860	1.4882	1.2913	0.29362	4.762
36°	1.1920	1.4063	1.2749	0.29019	4.882
39°	1.2054	1.3430	1.2639	0.28793	4.979
42°	1.2265	1.2938	1.2576	0.28667	5.043
45°	1.2556	1.2556	1.2556	0.28627	5.064

APPENDIX E

PARAMETERS AND THE ARL OF THE OPTIMAL MEWMAPC CHARTS USING C2 OBATAINED FROM THREE BIVARIATE PROCESSES WITH ρ = 0.2, 0.5 AND 0.8 EMPLOYED IN THE PERFORMANCE COMPARISON WITH THE MEWMA CHARTS

Parameters Of The Optimal MEWMAPC Charts Using C2 With Nominal ARL = 200 Designed Under A Bivariate Process With $\rho = 0.2$ For Various Directions (θ) Of Shift At $\lambda = 0.5$

θ	h_1	h_2	r	ARL
0°	0.4359	0.3820	0.05047	28.332
3°	0.4193	0.3936	0.05070	28.648
6°	0.4046	0.4068	0.05078	28.746
9°	0.3914	0.4222	0.05067	28.606
12°	0.3793	0.4405	0.05039	28.223
15°	0.3685	0.4634	0.04996	27.618
18°	0.3597	0.4936	0.04950	26.865
21°	0.3546	0.5363	0.04934	25.948
24°	0.3552	0.5996	0.04991	25.047
27°	0.3611	0.6948	0.05128	24.218
30°	0.3689	0.8411	0.05294	23.515
33°	0.3759	0.9174	0.05441	22.954
36°	0.3815	0.9302	0.05561	22.516
39°	0.3855	0.9364	0.05648	22.210
42°	0.3879	0.9420	0.05701	22.028
45°	0.3888	0.9426	0.05718	21.968
48°	0.3879	0.9420	0.05701	22.028
51°	0.3855	0.9364	0.05648	22.210
54°	0.3815	0.9302	0.05561	22.516
57°	0.3759	0.9174	0.05441	22.954
60°	0.3689	0.8411	0.05294	23.515
63°	0.3611	0.6948	0.05128	24.218
66°	0.3552	0.5996	0.04991	25.047
69°	0.3546	0.5363	0.04934	25.948
72°	0.3597	0.4936	0.04950	26.865
75°	0.3685	0.4634	0.04996	27.618
78°	0.3793	0.4405	0.05039	28.223
81°	0.3914	0.4222	0.05067	28.606
84°	0.4046	0.4068	0.05078	28.746
87°	0.4193	0.3936	0.05070	28.648
90°	0.4359	0.3820	0.05047	28.332

θ	h_1	h_2	r	ARL
93°	0.4553	0.3718	0.05012	27.837
96°	0.4787	0.3634	0.04970	27.209
99°	0.5082	0.3572	0.04938	26.501
102°	0.5468	0.3542	0.04939	25.769
105°	0.5983	0.3551	0.04989	25.062
108°	0.6668	0.3592	0.05067	24.418
111°	0.7582	0.3649	0.05209	23.857
114°	0.8826	0.3705	0.05328	23.381
117°	0.9165	0.3755	0.05432	22.986
120°	0.9261	0.3796	0.05520	22.663
123°	0.9328	0.3829	0.05592	22.407
126°	0.9362	0.3855	0.05647	22.212
129°	0.9399	0.3873	0.05687	22.076
132°	0.9423	0.3884	0.05711	21.994
135°	0.9426	0.3888	0.05718	21.968
138°	0.9423	0.3884	0.05711	21.994
141°	0.9399	0.3873	0.05687	22.076
144°	0.9362	0.3855	0.05647	22.212
147°	0.9328	0.3829	0.05592	22.407
150°	0.9261	0.3796	0.05520	22.663
153°	0.9165	0.3755	0.05432	22.986
156°	0.8826	0.3705	0.05328	23.381
159°	0.7582	0.3649	0.05209	23.857
162°	0.6668	0.3592	0.05067	24.418
165°	0.5983	0.3551	0.04989	25.062
168°	0.5468	0.3542	0.04939	25.769
171°	0.5082	0.3572	0.04938	26.501
174°	0.4787	0.3634	0.04970	27.209
177°	0.4553	0.3718	0.05012	27.837
180°	0.4359	0.3820	0.05047	28.332

Parameters Of The Optimal MEWMAPC Charts Using C2 With Nominal ARL = 200 Designed Under A Bi-variate Process With $\rho = 0.5$ For Various Directions (θ) Of Shift At $\lambda = 0.5$

θ	h_1	h_2	r	ARL
0°	0.5167	0.3561	0.04934	26.322
3°	0.4864	0.3613	0.04959	27.012
6°	0.4616	0.3692	0.05000	27.666
9°	0.4407	0.3792	0.05039	28.220
12°	0.4223	0.3913	0.05067	28.604
15°	0.4057	0.4057	0.05077	28.747
18°	0.3904	0.4234	0.05066	28.586
21°	0.3763	0.4461	0.05029	28.062
24°	0.3638	0.4773	0.04972	27.244
27°	0.3553	0.5252	0.04933	26.153
30°	0.3555	0.6060	0.04999	24.975
33°	0.3644	0.7488	0.05198	23.904
36°	0.3746	0.9148	0.05414	23.056
39°	0.3823	0.9317	0.05580	22.451
42°	0.3871	0.9398	0.05683	22.088
45°	0.3888	0.9426	0.05718	21.968
48°	0.3871	0.9398	0.05683	22.088
51°	0.3823	0.9317	0.05580	22.451
54°	0.3746	0.9148	0.05414	23.056
57°	0.3644	0.7488	0.05198	23.904
60°	0.3555	0.6060	0.04999	24.975
63°	0.3553	0.5252	0.04933	26.153
66°	0.3638	0.4773	0.04972	27.244
69°	0.3763	0.4461	0.05029	28.062
72°	0.3904	0.4234	0.05066	28.586
75°	0.4057	0.4057	0.05077	28.747
78°	0.4223	0.3913	0.05067	28.604
81°	0.4407	0.3792	0.05039	28.220
84°	0.4616	0.3692	0.05000	27.666
87°	0.4864	0.3613	0.04959	27.012
90°	0.5167	0.3561	0.04934	26.322

θ	h_1	h_2	r	ARL
93°	0.5167	0.3561	0.04934	26.322
96°	0.6024	0.3553	0.04995	25.016
99°	0.6618	0.3589	0.05080	24.457
102°	0.7356	0.3636	0.05182	23.974
105°	0.8277	0.3683	0.05282	23.563
108°	0.9121	0.3726	0.05371	23.216
111°	0.9205	0.3762	0.05449	22.925
114°	0.9253	0.3793	0.05515	22.681
117°	0.9293	0.3819	0.05571	22.480
120°	0.9338	0.3841	0.05617	22.317
123°	0.9372	0.3858	0.05654	22.188
126°	0.9394	0.3871	0.05683	22.090
129°	0.9420	0.3880	0.05703	22.022
132°	0.9424	0.3886	0.05714	21.981
135°	0.9426	0.3888	0.05718	21.968
138°	0.9424	0.3886	0.05714	21.981
141°	0.9420	0.3880	0.05703	22.022
144°	0.9394	0.3871	0.05683	22.090
147°	0.9372	0.3858	0.05654	22.188
150°	0.9338	0.3841	0.05617	22.317
153°	0.9293	0.3819	0.05571	22.480
156°	0.9253	0.3793	0.05515	22.681
159°	0.9205	0.3762	0.05449	22.925
162°	0.9121	0.3726	0.05371	23.216
165°	0.8277	0.3683	0.05282	23.563
168°	0.7356	0.3636	0.05182	23.974
171°	0.6618	0.3589	0.05080	24.457
174°	0.6024	0.3553	0.04995	25.016
177°	0.5547	0.3541	0.04943	25.644
180°	0.5167	0.3561	0.04934	26.322

Parameters Of The Optimal MEWMAPC Charts Using C2 With Nominal ARL = 200 Designed Under A Bivariate Process With ρ = 0.8 For Various Directions (θ) Of Shift At λ = 0.5

θ	h_1	h_2	r	ARL
0°	0.8277	0.3683	0.05282	23.563
3°	0.7448	0.3641	0.05193	23.925
6°	0.6733	0.3597	0.05097	24.369
9°	0.6119	0.3558	0.05008	24.912
12°	0.5599	0.3541	0.04947	25.565
15°	0.5167	0.3561	0.04934	26.322
18°	0.4813	0.3626	0.04966	27.141
21°	0.4517	0.3735	0.05018	27.934
24°	0.4259	0.3886	0.05062	28.542
27°	0.4024	0.4091	0.05077	28.740
30°	0.3804	0.4386	0.05043	28.269
33°	0.3610	0.4878	0.04957	26.976
36°	0.3549	0.5944	0.04984	25.107
39°	0.3703	0.8771	0.05324	23.398
42°	0.3839	0.9347	0.05614	22.329
45°	0.3888	0.9426	0.05718	21.968
48°	0.3839	0.9347	0.05614	22.329
51°	0.3703	0.8771	0.05324	23.398
54°	0.3549	0.5944	0.04984	25.107
57°	0.3610	0.4878	0.04957	26.976
60°	0.3804	0.4386	0.05043	28.269
63°	0.4024	0.4091	0.05077	28.740
66°	0.4259	0.3886	0.05062	28.542
69°	0.4517	0.3735	0.05018	27.934
72°	0.4813	0.3626	0.04966	27.141
75°	0.5167	0.3561	0.04934	26.322
78°	0.5599	0.3541	0.04947	25.565
81°	0.6119	0.3558	0.05008	24.912
84°	0.6733	0.3597	0.05097	24.369
87°	0.7448	0.3641	0.05193	23.925
90°	0.8277	0.3683	0.05282	23.563

θ	h_1	h_2	r	ARL
93°	0.9099	0.3720	0.05358	23.266
96°	0.9157	0.3750	0.05423	23.022
99°	0.9219	0.3776	0.05478	22.818
102°	0.9243	0.3798	0.05524	22.649
105°	0.9300	0.3816	0.05564	22.507
108°	0.9332	0.3832	0.05597	22.389
111°	0.9340	0.3845	0.05625	22.289
114°	0.9367	0.3855	0.05649	22.207
117°	0.9383	0.3864	0.05668	22.139
120°	0.9406	0.3872	0.05684	22.084
123°	0.9405	0.3878	0.05697	22.041
126°	0.9420	0.3882	0.05706	22.008
129°	0.9414	0.3885	0.05713	21.986
132°	0.9420	0.3887	0.05717	21.972
135°	0.9426	0.3888	0.05718	21.968
138°	0.9420	0.3887	0.05717	21.972
141°	0.9414	0.3885	0.05713	21.986
144°	0.9420	0.3882	0.05706	22.008
147°	0.9405	0.3878	0.05697	22.041
150°	0.9406	0.3872	0.05684	22.084
153°	0.9383	0.3864	0.05668	22.139
156°	0.9367	0.3855	0.05649	22.207
159°	0.9340	0.3845	0.05625	22.289
162°	0.9332	0.3832	0.05597	22.389
165°	0.9300	0.3816	0.05564	22.507
168°	0.9243	0.3798	0.05524	22.649
171°	0.9219	0.3776	0.05478	22.818
174°	0.9157	0.3750	0.05423	23.022
177°	0.9099	0.3720	0.05358	23.266
180°	0.8277	0.3683	0.05282	23.563

APPENDIX F

PARAMETERS AND THE OOC ARL OF THE OPTIMAL BIVARIATE MEWMAPC CHARTS USING C2 AND THE OPTIMAL MZONEPC CHARTS EMPLOYED IN THE PERFORMANCE COMPARISON OF THESE CHARTS

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 2.5$

θ	h_1	h_2	r	ARL
0°	1.8230	3.5908	0.59809	2.145
3°	1.8205	3.5874	0.59698	2.150
6°	1.8130	3.5734	0.59365	2.164
9°	1.8005	3.5494	0.58807	2.189
12°	1.7829	3.5159	0.58024	2.220
15°	1.7602	3.4716	0.57011	2.263
18°	1.7324	3.4200	0.55767	2.318
21°	1.6995	2.9338	0.54293	2.384
24°	1.6626	2.5675	0.52613	2.464
27°	1.6250	2.2961	0.50822	2.555
30°	1.5924	2.0914	0.49097	2.654
33°	1.5697	1.9360	0.47615	2.755
36°	1.5601	1.8181	0.46475	2.848
39°	1.5642	1.7291	0.45692	2.923
42°	1.5824	1.6628	0.45243	2.972
45°	1.6148	1.6148	0.45096	2.989

Parameters Of The Optimal Bivariate MZONEPC Charts
With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 2.5$

θ	h_1	h_2	ARL
0°	3.3365	6.0000	2.439
3°	3.3365	6.0000	2.444
6°	3.3365	6.0000	2.458
9°	3.3365	6.0000	2.481
12°	3.3365	6.0000	2.514
15°	3.3365	6.0000	2.558
18°	3.3365	6.0000	2.613
21°	3.3365	6.0000	2.680
24°	3.3365	6.0000	2.760
27°	3.3367	5.8318	2.856
30°	3.3407	5.1231	2.967
33°	3.3551	4.6062	3.087
36°	3.3876	4.2372	3.204
39°	3.4426	3.9717	3.303
42°	3.5227	3.7771	3.369
45°	3.6318	3.6318	3.392

Parameters Of The Optimal Bivariate MEWMAPC Charts
Using C2 With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At λ = 3.0

θ	h_1	h_2	r	ARL
0°	2.1511	4.2226	0.74176	1.619
3°	2.1487	4.2182	0.74074	1.622
6°	2.1415	4.2033	0.73766	1.632
9°	2.1294	4.1804	0.73247	1.649
12°	2.1123	4.1479	0.72513	1.674
15°	2.0900	4.1050	0.71553	1.707
18°	2.0624	4.0360	0.70357	1.748
21°	2.0293	3.4534	0.68915	1.799
24°	1.9920	3.0357	0.67246	1.860
27°	1.9540	2.7272	0.65436	1.931
30°	1.9210	2.4948	0.63642	2.008
33°	1.8977	2.3179	0.62033	2.087
36°	1.8876	2.1829	0.60736	2.160
39°	1.8921	2.0805	0.59801	2.219
42°	1.9122	2.0040	0.59244	2.257
45°	1.9489	1.9489	0.59059	2.271

Parameters Of The Optimal Bivariate MZONEPC Charts
With Nominal ARL = 200 For Various
Directions (θ) Of Shift
At $\lambda = 3.0$

θ	h_1	h_2	ARL
0°	3.3365	6.0000	1.889
3°	3.3365	6.0000	1.892
6°	3.3365	6.0000	1.903
9°	3.3365	6.0000	1.922
12°	3.3365	6.0000	1.949
15°	3.3365	6.0000	1.984
18°	3.3365	6.0000	2.028
21°	3.3365	6.0000	2.082
24°	3.3365	6.0000	2.147
27°	3.3365	6.0000	2.223
30°	3.3371	5.6573	2.312
33°	3.3447	4.8954	2.413
36°	3.3721	4.3702	2.515
39°	3.4275	4.0267	2.605
42°	3.5128	3.7953	2.665
45°	3.6318	3.6318	2.686

APPENDIX G

FORTRAN PROGRAM FOR DESIGN AND EVALUATION OF THE MEWMAPC CHART

```
      PROGRAM MAIN
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      CHARACTER CHAR*1
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C
      LP=6
      IN=5
11    WRITE(LP,50)
50    FORMAT(//13X,26(1H*)/13X,'***  MAIN      MENU    ',
     *'***'/13X,26(1H*)//
     *5X,'(1) EVALUATION OF THE ARL OF A MEWMAPC CHART'/
     *5X,'(2) CLASSICAL DESIGN OF THE MEWMAPC CHART'/5X,
     *'(3) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C1'/
     *5X,'(4) OPTIMAL DESIGN OF THE MEWMAPC CHART USING C2'
     */5X,'(5) EXIT THE PROGRAM'//
     *5X,'====> PLEASE ENTER YOUR OPTION (1, 2, 3, 4, OR'
     *' 5) <===='')
      READ(IN,*) ISELECT
      IF(ISELECT .LT. 1 .OR. ISELECT .GT. 5) THEN
      WRITE(LP,57)
        GO TO 11
      ENDIF
      IF(ISELECT .EQ. 5) GO TO 26
1     WRITE(LP,51)
51    FORMAT(/5X,'===> PLEASE ENTER THE NUMBER OF VARIABLE'
     *,'S MONITORED <==='/5X,'===> THE NUMBER SHOULD BE'
     *   ,' BETWEEN 2 AND 5 <===')
      READ(IN,*) NP
      IF (NP .LT. 2 .OR. NP .GT. 5) THEN
         WRITE(LP,57)
         GO TO 1
      ENDIF
2     WRITE(LP,55) NP
55    FORMAT(/5X,'*** THE TOTAL NUMBER OF VARIABLES IS ',I2)
      WRITE(LP,56)
56    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <===' /5X,
     *  '===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ (IN,*) IANS
      GO TO (13,1) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 2
13    IDTMX=1
      WRITE(LP,70)
70    FORMAT(/5X,'*** THE STANDARDIZED COVARIANCE MATRIX'
     */5X,'*** IS INITIALLY AN IDENTITY MATRIX ***')
      WRITE(LP,56)
      READ(IN,*) IANS
      GO TO (20,3) IANS
```

```
      WRITE(LP,57)
      GO TO 13
3     WRITE(LP,60)
60    FORMAT(/5X,'===> PLEASE ENTER THE DESIRED '
     *'STANDARDIZED <==='/5X,'===> COVARIANCE MATRIX '
     * 'ROW BY ROW <===')
      IDTMX=0
      DO 500 I=1,NP
         K=(I-1)*NP+1
         L=K+NP-1
         READ(IN,*) (COVAR(J),J=K,L)
500   CONTINUE
4     WRITE(LP,65)
65    FORMAT(/5X,'*** THE STANDARDIZED COVARIANCE MATRIX'
     * ,' IS ***'/)
      DO 505 I=1,NP
         K=(I-1)*NP+1
         L=K+NP-1
         WRITE(LP,66) (COVAR(J),J=K,L)
66       FORMAT(2X,5G11.4)
505   CONTINUE
      WRITE(LP,75)
75    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (20,3)IANS
      WRITE(LP,57)
      GO TO 4
20    GO TO (22,23,24,25,26) ISELECT
22    CALL EVMEWMA
      WRITE(LP,100)
100   FORMAT(/5X,'*** PAUSE! PLEASE HIT ENTER TO RETURN'
     *  ' TO MAIN MENU ***')
      READ(IN,101) CHAR
101   FORMAT(A1)
      GO TO 11
23    CALL CDMEWMA
      WRITE(LP,100)
      READ(IN,101) CHAR
      GO TO 11
24    CALL OPMEWC1
      WRITE(LP,100)
      READ(IN,101) CHAR
      GO TO 11
25    CALL OPMEWC3
      WRITE(LP,100)
      READ(IN,101) CHAR
      GO TO 11
26    STOP
C
C END MAIN
C
```

```
      END
C
      SUBROUTINE EVMEWMA
C--------------------------------------------------------------
C  EVALUATION OF THE MEWMAPC CHART
C--------------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL ARLPC,RLEWMA
C
      DIMENSION OGMEANSF(5),PCMEANSF(5),XK(5),R(5),
     * S(5),D(5),V(25)
C
      DIMENSION PU(500,5),PL(500,5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C--------------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C--------------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE PARAMETERS OF THE'
     * ' EXISTING CHARTS ***')
      DO 510 I=1,NP
        WRITE(LP,80) I
80      FORMAT(/5X,'*** PLEASE ENTER THE SYMMETRICAL '
     *    'CONTROL LIMITS H'/5X,'*** AND FACTOR R FOR '
     *     'NO.',I2,' IEWMAPC CHART ***')
         READ(IN,*) XK(I),R(I)
510   CONTINUE
      WRITE(LP,86)
86    FORMAT(/5X,'*** THE PARAMETERS FOR THE CHARTS ARE'
     *  ' : ***'/)
6     DO 520 I=1,NP
        WRITE(LP,85) I,XK(I),I,R(I)
85      FORMAT(5X,'H',I1,' =',G15.7,3X,'R',I1,' =',G15.7)
520   CONTINUE
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE EXPECTED SHIFT OF'
     * ' THE MEAN VECTOR ***')
      READ(IN,*) (OGMEANSF(I),I=1,NP)
8     WRITE(LP,95) (OGMEANSF(I),I=1,NP)
```

```
95    FORMAT(/5X,'*** THE EXPECT MEAN VECTOR SHIFT IS ***'/
     * 5X,3G15.7/(5X,2G15.7))
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C-----------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C-----------------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DO 521 I=1,NP
521        PCMEANSF(I)=OGMEANSF(I)
        GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
      DO 530 I=1,NP
          IDX=(I-1)*NP
          PCMEANSF(I)=0.D0
          DO 530 J=1,NP
530          PCMEANSF(I)=PCMEANSF(I)+OGMEANSF(J)*V(IDX+J)
20    WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING SHIFTS ON THE PRIN.'
     *     ' COMP. ARE ***')
      WRITE(LP,105) (PCMEANSF(I),I=1,NP)
105   FORMAT(/5X,3G15.7/(5X,2G15.7))
      DO 531 I=1,NP
531     S(I)=0.D0
10    WRITE(LP,110)
110   FORMAT(/5X,'*** THE INITIAL EWMA VALUES FOR ALL '
     * 'IEWMAPC CHARTS ***'/5X,'*** ARE SET AT 0.D0 ***')
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (14,11) IANS
      WRITE(LP,57)
      GO TO 10
11    WRITE(LP,115)
115   FORMAT(/5X,'*** PLEASE ENTER THE DESIRED S VALUES'
     *  ' ONE BY ONE ***')
      READ(IN,*) (S(I),I=1,NP)
12    WRITE(LP,120)
120   FORMAT(/5X,'*** THE DESIRED S VALUES ARE ***')
      WRITE(LP,125) (S(I),I=1,NP)
125   FORMAT(/5X,5F10.4)
      WRITE(LP,70)
      READ(IN,*) IANS
```

```
      GO TO (14,11) IANS
      WRITE(LP,57)
      GO TO 12
14    WRITE(LP,132)
132   FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED) ***')
      WRITE(LP,135)
135   FORMAT(//5X,'*** EVALUATION IN PROGRESS ***'/)
C---------------------------------------------------------------
-
C   EVALUATION OF THE OOC ARL OF THE EXISTING MEWMAPC CHART
C---------------------------------------------------------------
-
      LGN=0
      DO 535 I=1,NP
         SIGEW=XK(I)*DSQRT((2.D0-R(I))/R(I))
         CALL RLEWMA(SIGEW,R(I),S(I),PCMEANSF(I),PL(1,I))
         NVAL=IDINT(PL(1,I))
         PU(NVAL+2,I)=PL(NVAL+3,I)
         IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
         DO 535 J=1,NVAL+1
            PU(J,I)=PL(J,I)
535   CONTINUE
      LGNP2=LGN+2
      DO 540 I=1,NP
         IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
            NP2=PL(1,I)+2
            XMNMN=PL(NP2,I)
            XMNPL=PU(NP2,I)
            DO 545 J=NP2,LGN+1
               PL(J,I)=PL(J-1,I)*XMNMN
545            PU(J,I)=PU(J-1,I)*XMNPL
            PL(LGNP2,I)=XMNMN
            PU(LGNP2,I)=XMNPL
          ENDIF
540   CONTINUE
      ARL=ARLPC(NP,LGN,PL,PU)
      WRITE(LP,130) ARL
130   FORMAT(/5X,'*** THE OOC ARL FOR THE DESIRED MEWMAPC'
     * ' CHART ***' /5X,'*** AT THE EXPECTED SHIFT IS '
     * ,G15.7)
      RETURN
C
C   END EVMEWMA
C
      END
C
      SUBROUTINE CDMEWMA
C
      IMPLICIT REAL*8(A-H,O-Z)
C
```

```
      EXTERNAL RLEWMA,ARLPC
C
      DIMENSION V(25),XSARL(6),XBARL(6),
     * DIRARL(6),D(5),DIRPCSF(5),DIROGSF(5)
C
      DIMENSION PU(500,5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C
      COMMON /CD/ PL(500,5),XMUIC,S,XK,R,
     * ARLIC,FBEST,XUBEST,NTRACU
C-----------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C-----------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED IN-CONTROL'
     * ' ARL ***')
      READ(IN,*) ARLIC
      WRITE(LP,76)
76    FORMAT(/5X,'*** PLEASE ENTER THE VALUE OF THE DESIRED'
     * ' COMMON R FACTOR ***'/5X,'*** NOTE : USE 1.0 < R'
     *  ' < 0.03 ***')
      READ(IN,*) R
6     WRITE(LP,80) ARLIC,R
80    FORMAT(/5X,'*** THE DESIRED IN-CONTROL ARL IS',G15.7
     */5X,'*** AND THE DESIRED COMMON R FACTOR IS ',G15.7)
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE DIRECTION OF THE '
     *'EXPECTED ***'/5X,'*** SHIFT OF THE MEAN VECTOR ***')
      READ(IN,*) (DIROGSF(I),I=1,NP)
8     WRITE(LP,95) (DIROGSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE DIRECTION OF THE EXPECTED MEAN'
     * ' ***'/5X,'*** VECTOR SHIFT IS ***'/5X,3G15.7
     * /(5X,2G15.7))
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C----------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
```

```
C  COMPONENT
C-----------------------------------------------------------
9      IF (IDTMX .EQ. 1) THEN
         DSUM=0.D0
         DO 521 I=1,NP
            DIRPCSF(I)=DIROGSF(I)
521         DSUM=DSUM+DIRPCSF(I)*DIRPCSF(I)
         GO TO 20
       ENDIF
       CALL JACOBI(COVAR,NP,NP,D,V,NROT)
       CALL EIGSRT(D,V,NP,NP)
       DO 525 I=1,NP
          IM1=I-1
          DO 525 J=1,NP
525          V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
       DSUM=0.D0
       DO 530 I=1,NP
           IDX=(I-1)*NP
           DIRPCSF(I)=0.D0
           DO 531 J=1,NP
531           DIRPCSF(I)=DIRPCSF(I)+DIROGSF(J)*V(IDX+J)
530        DSUM=DSUM+DIRPCSF(I)*DIRPCSF(I)
C-------------------------------------------------------------
C  CONVERT A DIRECTION OF SHIFT IN PRIN. COMP. TO
C  UNIT LENGTH
C-------------------------------------------------------------
20     DO 532 I=1,NP
532       DIRPCSF(I)=DIRPCSF(I)/DSQRT(DSUM)
       WRITE(LP,100)
100    FORMAT(/5X,'*** THE CORRESPONDING DIRECTION OF THE '
     * 'SHIFT ***'/5X,'*** IN THE PRIN. COMP. WITH UNIT '
     * 'LENGTH IS ***')
       WRITE(LP,105) (DIRPCSF(I),I=1,NP)
105    FORMAT(/5X,3G15.7/(5X,2G15.7))
       S=0.D0
10     WRITE(LP,110)
110    FORMAT(/5X,'*** THE INITIAL EWMA VALUES FOR ALL '
     * 'IEWMAPC CHARTS ***'/5X,'*** ARE SET AT 0.D0 ***')
       WRITE(LP,70)
       READ(IN,*) IANS
       GO TO (13,11) IANS
       WRITE(LP,57)
       GO TO 10
11     WRITE(LP,115)
115    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED COMMON'
     *  ' S VALUE ***')
       READ(IN,*) S
12     WRITE(LP,120)
120    FORMAT(/5X,'*** THE DESIRED COMMON S VALUE IS ***')
       WRITE(LP,125) S
125    FORMAT(5X,F12.4)
       WRITE(LP,71)
```

```
71     FORMAT(/5X,'===> IS THE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
       READ(IN,*) IANS
       GO TO (13,11) IANS
       WRITE(LP,57)
       GO TO 12
13     XMUIC=0.D0
       WRITE(LP,130)
130    FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED) ***')
       WRITE(LP,210)
210    FORMAT(//5X,'*** OPTIMIZATION IN PROGRESS ***'/)
C-----------------------------------------------------
C   DO ONE VARIABLE OPTIMIZATION
C-----------------------------------------------------
       XK=3.D0
       XUBEST=XK
C
C  NOW DO THE CONSTRAINED MINIMIZATION,
C  MOVING ALONG THE CONSTRAINT SURFACE.
C
       CALL CDFUNK
C
       XK=XUBEST
C
       CL=XK*DSQRT(R/(2.D0-R))
       WRITE(LP,135) NP,ARLIC,R,CL
135    FORMAT(//5X,'*** THE PARAMETERS OF A',I2,' VARIATE'
     *' MEWMAPC CHART'/5X,'*** WITH IN-CONTROL ARL OF',
     *G9.3,' ARE :'/5X,50(1H-)/5X,'*** COMMON R = ',G9.3,
     *' AND COMMON H = ',G15.7)
C
       IF (IDTMX .NE. 1) THEN
         WRITE(LP,201)
201      FORMAT(/5X,'*** THE MATRIX U OF THE EIGEN VECTORS'
     *   ' ARE : ***')
         DO 755 I=1,NP
           K=(I-1)*NP+1
           WRITE(LP,202) I,(V(J),J=K,K+NP-1)
202        FORMAT(5X,'*** u',I1,' :',5F10.5)
755      CONTINUE
       ENDIF
C------------------------------------------------------
C  CALCUALTE OOC ARL AT GIVEN DIRECTION
C------------------------------------------------------
       DO 550 L=1,6
          LGN=0
          DO 535 I=1,NP
             XLAMBDA=DINT(L)*0.5D0*DIRPCSF(I)
             CALL RLEWMA(XK,R,S,XLAMBDA,PL(1,I))
             NVAL=IDINT(PL(1,I))
```

```
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=NVAL
            DO 535 J=1,NVAL+1
               PU(J,I)=PL(J,I)
535      CONTINUE
         LGNP2=LGN+2
         DO 540 I=1,NP
            IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
              NP2=PL(1,I)+2
              XMNMN=PL(NP2,I)
              XMNPL=PU(NP2,I)
              DO 545 J=NP2,LGN+1
                 PL(J,I)=PL(J-1,I)*XMNMN
545              PU(J,I)=PU(J-1,I)*XMNPL
              PL(LGNP2,I)=XMNMN
              PU(LGNP2,I)=XMNPL
            ENDIF
540   CONTINUE
      DIRARL(L)=ARLPC(NP,LGN,PL,PU)
550   CONTINUE
C-------------------------------------------------------
C  CALCUALTE OVERALL OOC ARL AT MINIMUM
C-------------------------------------------------------
      DO 650 L=1,6
         LGN=0
         DO 635 I=1,NP
            FACTOR=0.D0
            IF (I .EQ. 1) FACTOR=1.D0
            XLAMBDA=DFLOAT(L)*FACTOR*0.5D0
            IF (I .LE. 2) THEN
               CALL RLEWMA(XK,R,S,XLAMBDA,PL(1,I))
            ELSE
               DO 636 K=1,IDINT(PL(1,2))+3
636               PL(K,I)=PL(K,2)
            ENDIF
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=NVAL
            DO 635 J=1,NVAL+1
               PU(J,I)=PL(J,I)
635      CONTINUE
         LGNP2=LGN+2
         DO 640 I=1,NP
            IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
              NP2=PL(1,I)+2
              XMNMN=PL(NP2,I)
              XMNPL=PU(NP2,I)
              DO 645 J=NP2,LGN+1
                 PL(J,I)=PL(J-1,I)*XMNMN
645              PU(J,I)=PU(J-1,I)*XMNPL
              PL(LGNP2,I)=XMNMN
              PU(LGNP2,I)=XMNPL
```

```
            ENDIF
640     CONTINUE
        XSARL(L)=ARLPC(NP,LGN,PL,PU)
650     CONTINUE
C-------------------------------------------------------
C  CALCUALTE OVERALL OOC ARL AT MAXIMUM
C-------------------------------------------------------
        FACTOR=1.D0/DSQRT(DFLOAT(NP))
        DO 750 L=1,6
           LGN=0
           XLAMBDA=DFLOAT(L)*FACTOR*0.5D0
           DO 735 I=1,NP
              IF (I .LE. 1) THEN
                 CALL RLEWMA(XK,R,S,XLAMBDA,PL(1,I))
              ELSE
                 DO 736 K=1,IDINT(PL(1,1))+3
736                 PL(K,I)=PL(K,1)
              ENDIF
              NVAL=IDINT(PL(1,I))
              PU(NVAL+2,I)=PL(NVAL+3,I)
              IF (NVAL .GT. LGN) LGN=NVAL
              DO 735 J=1,NVAL+1
                 PU(J,I)=PL(J,I)
735       CONTINUE
          LGNP2=LGN+2
          DO 740 I=1,NP
             IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
               NP2=PL(1,I)+2
               XMNMN=PL(NP2,I)
               XMNPL=PU(NP2,I)
               DO 745 J=NP2,LGN+1
                  PL(J,I)=PL(J-1,I)*XMNMN
745               PU(J,I)=PU(J-1,I)*XMNPL
               PL(LGNP2,I)=XMNMN
               PU(LGNP2,I)=XMNPL
             ENDIF
740     CONTINUE
        XBARL(L)=ARLPC(NP,LGN,PL,PU)
750     CONTINUE
C
        WRITE(LP,140)
140     FORMAT(/5X,'*** THE OOC ARL PROFILE ARE SHOWN AS'
       *' FOLLOWS ***')
        WRITE(LP,145)
145     FORMAT(/5X,'LAMBDA',12X,'OOC ARL',14X,'OOC   ARL'/
       *21X,'AT DIRECTION',12X,'OVERALL'/4X,56(1H-))
        DO 700 I=1,6
           XLAMBDA=DFLOAT(I)*0.5D0
           WRITE(LP,150)XLAMBDA,DIRARL(I),XSARL(I),XBARL(I)
150        FORMAT(5X,F5.2,9X,F11.4,5X,F11.4,' -',F11.4)
700     CONTINUE
        RETURN
```

```
C
C END CDMEWMA
C
      END
C
      SUBROUTINE CDFUNK
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL CDFUNKU,UNICY
C
      COMMON /NBVAR/ NP
      COMMON /CD/ PL(500,5),XMUIC,S,XK,R,
     * ARLIC,FBEST,XUBEST,NTRACU
C
      KW=6
      XUBEST=XK
C
      NTRACU=-2
      XU=XUBEST
      XMAXU=5.5D0
      XMINU=0.03D0
      DELTXU=0.009D0
C
C  START WITH A LARGE DELMNU, AND DECREASE IT LATER.
C
      DELMNU=0
      DELMNU=0.0001D0
C
      NFMAXU=200
      KWU=KW
C
      CALL UNICY(CDFUNKU,NTRACU,FOBJU,XU,XMAXU,XMINU,DELTXU,
     *   DELMNU,NFMAXU,KWU)
      XUBEST=XU
C
      RETURN
C
C  END CDFUNK
C
      END
C
C
      SUBROUTINE CDFUNKU (XU,FOBJU)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL RLEWMA
C
      COMMON /NBVAR/ NP
      COMMON /CD/ PL(500,5),XMUIC,S,XK,R,
     * ARLIC,FBEST,XUBEST,NTRACU
```

```
C
      KW=6
      XK=XU
C
      CALL RLEWMA(XK,R,S,XMUIC,PL(1,1))
      NVAL=IDINT(PL(1,1))
      NVALP1=NVAL+1
      NVALP2=NVALP1+1
      NVALP3=NVALP2+1
      SUMP=1.D0
      DO 1 I=2,NVAL
         PNGTI=1.D0
         DO 2 J=1,NP
2           PNGTI=PNGTI*PL(I,1)
1        SUMP=SUMP+PNGTI
      XMNMN=1.D0
      XMNPL=1.D0
      PLST=1.D0
      DO 3 I=1,NP
           PLST=PLST*PL(NVALP1,1)
           XMNMN=XMNMN*PL(NVALP2,1)
3          XMNPL=XMNPL*PL(NVALP3,1)
      ARLMN=SUMP+PLST/(1.D0-XMNMN)
      ARLPL=SUMP+PLST/(1.D0-XMNPL)
      ARL=0.5D0*(ARLMN+ARLPL)
C
      FOBJU=(ARL-ARLIC)**2
      IF(NTRACU.GE.2) WRITE(KW,10)XK,R,S,ARL,FOBJU
   10 FORMAT(' CDFUNKU:  XK =',1PG15.7,
     *    5X,'R =',G15.7,
     *    5X,'S =',G15.7/
     *    5X,'ARL =',G15.7,5X,'FOBJU =',G15.7)
C
      RETURN
C
C   END CDFUNKU
C
      END
C
      SUBROUTINE OPMEWC1
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL RLEWMA,ARLPC,OP1FUNK,STEPIT,STSET,UNICY
C
      DIMENSION V(25),D(5),OGMEANSF(5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *    DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *    NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
```

```
C
      COMMON /PCSF/ PCMEANSF(5),PU(500,5)
      COMMON /OPC1/ PL(500,5),XMUIC,S,XK,R,ARLIC,FBEST,
     * XUBEST,NTRACU
C-----------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C-----------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED IN-CONTROL'
     * ' ARL ***')
      READ(IN,*) ARLIC
6     WRITE(LP,80) ARLIC
80    FORMAT(/5X,'*** THE DESIRED IN-CONTROL ARL IS',G15.7)
      WRITE(LP,71)
71    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE EXPECTED SHIFT OF'
     *' ***'/5X,'*** THE ORIGINAL MEAN VECTOR ***')
      READ(IN,*) (OGMEANSF(I),I=1,NP)
8     WRITE(LP,95) (OGMEANSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE EXPECTED ORIGINAL MEAN VECTOR '
     * 'SHIFT IS ***'/5X,3G15.7/(5X,2G15.7))
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C----------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C----------------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DO 521 I=1,NP
521        PCMEANSF(I)=OGMEANSF(I)
        GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
```

```
      DO 530 I=1,NP
          IDX=(I-1)*NP
          PCMEANSF(I)=0.D0
          DO 530 J=1,NP
530          PCMEANSF(I)=PCMEANSF(I)+OGMEANSF(J)*V(IDX+J)
C-----------------------------------------------------------
C  CONVERT A DIRECTION OF SHIFT IN PRIN. COMP. TO
C  UNIT LENGTH
C-----------------------------------------------------------
20    WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING SHIFT OF THE '
     * ' ***'/5X,'*** MEAN OF THE PRIN. COMP. IS ***')
      WRITE(LP,105) (PCMEANSF(I),I=1,NP)
105   FORMAT(/5X,3G15.7/(5X,2G15.7))
      S=0.D0
10    WRITE(LP,110)
110   FORMAT(/5X,'*** THE INITIAL EWMA VALUES FOR ALL '
     * 'IEWMAPC CHARTS ***'/5X,'*** ARE SET AT 0.D0 ***')
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 10
11    WRITE(LP,115)
115   FORMAT(/5X,'*** PLEASE ENTER THE DESIRED COMMON'
     *  ' S VALUE ***')
      READ(IN,*) S
12    WRITE(LP,120)
120   FORMAT(/5X,'*** THE DESIRED COMMON S VALUE IS ***')
      WRITE(LP,125) S
125   FORMAT(5X,F12.4)
      WRITE(LP,71)
      READ(IN,*) IANS
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 12
13    XMUIC=0.D0
      WRITE(LP,130)
130   FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED) ***')
      IF (IDTMX .NE. 1) THEN
        WRITE(LP,201)
201     FORMAT(/5X,'*** THE MATRIX U OF THE EIGEN VECTORS'
     *  ' ARE : ***'/)
        DO 755 I=1,NP
          K=(I-1)*NP+1
          WRITE(LP,202) I,(V(J),J=K,K+NP-1)
202       FORMAT(5X,'*** u',I1,' :',5F10.5)
755     CONTINUE
      ENDIF
      WRITE(LP,210)
```

```
210   FORMAT(//5X,'*** OPTIMIZATION IN PROGRESS ***'/)
C-----------------------------------------------------------
C SWITCH POSITION BETWEEN THE CHART WITH THE LARGEST
C SHIFT AND THE LAST
C-----------------------------------------------------------
      ISWCH=0
      PCSFMAX=0.D0
      DO 535 I=1,NP
         IF(DABS(PCMEANSF(I)) .LT. 1.D-5) PCMEANSF(I)=0.D0
         IF (DABS(PCMEANSF(I)) .GT. PCSFMAX) THEN
            ISWCH=I
            PCSFMAX=DABS(PCMEANSF(I))
         ENDIF
535   CONTINUE
      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=PCMEANSF(NP)
         PCMEANSF(NP)=TEMP
      ENDIF
C-----------------------------------------------------------
C     SET THE INITIAL SEARCH POINTS
C-----------------------------------------------------------
      R=0.8D0
      SPREAD=3.D0
      CONST=1.D-4
      DELTX(1)=0.03
      DXX=0.05
      IF (PCMEANSF(NP) .LE. 3.D0) R=0.7D0
      IF (PCMEANSF(NP) .LE. 2.5D0) R=0.5D0
      IF (PCMEANSF(NP) .LE. 2.D0) R=0.3D0
      IF (PCMEANSF(NP) .LE. 1.5D0) R=0.2D0
      IF (PCMEANSF(NP) .LE. 1.D0) R=0.1D0
      IF (PCMEANSF(NP) .LE. .75D0) R=0.5D-1
      IF (PCMEANSF(NP) .LE. .5D0) R=0.4D-1
      IF (R .LE. .5D0) SPREAD=2.9D0
      IF (R .LE. .25D0) SPREAD=2.75D0
      IF (R .LE. .1D0) THEN
         SPREAD=2.6D0
         DELTX(1)=0.01D0
      ENDIF
C-----------------------------------------------------------
C  INITIAL STEPIT ROUTINE
C-----------------------------------------------------------
      CALL STSET
C
      NV=1
      MATRX=0
      XK=SPREAD
      XUBEST=XK
      NTRACE=-2
C
      X(1)=R
```

```
C-----------------------------------------------------------
C  NOW DO THE CONSTRAINED MINIMIZATION,
C  MOVING ALONG THE CONSTRAINT SURFACE.
C-----------------------------------------------------------
      DELMIN(1)=CONST
C
      FBEST=1.0D30
      XMAX(1)=1.D0
      XMIN(1)=0.03D0
C
      CALL STEPIT (OP1FUNK)
C
      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(NP)
         PCMEANSF(NP)=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=TEMP
      ENDIF
      CL=XK*DSQRT(R/(2.D0-R))
C
      WRITE(KW,135) NP,ARLIC
135   FORMAT(//5X,'*** THE OPTIMAL',I2,' VARIATE MEWMAPC'
     *' CHART ***'/5X,'*** WITH IN-CONTROL ARL OF',G11.3
     * /5X,'*** AT THE SHIFT OF THE ORIGINAL MEAN '
     * 'VECTOR OF : ***')
      IF (NP .LT. 4) THEN
         WRITE(LP,141) (OGMEANSF(I),I=1,NP)
141      FORMAT(5X,3G15.7)
         WRITE(LP,151)
151      FORMAT(5X,'*** IS LISTED AS FOLLOWS ***')
      ELSE
         WRITE(LP,146) (OGMEANSF(I),I=1,NP)
146      FORMAT(5X,3G15.7)
         WRITE(LP,151)
      ENDIF
      WRITE(KW,161)
161   FORMAT(5X,50(1H-))
      WRITE(KW,140) CL,R,FOBJ
140   FORMAT(/5X,'THE COMMON SYMMETRIC CONTROL LIMIT = '
     * ,G15.7/5X,'THE COMMON R = ',G15.7//5X,'*** THE '
     * ' OPTIMAL OOC ARL = ',1PG15.7)
      RETURN
C
C  END MAIN
C
      END
C
      SUBROUTINE OP1FUNK
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL OP1FUNKU,UNICY,RLEWMA,ARLPC
C
```

```
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
      COMMON /NBVAR/ NP
      COMMON /PCSF/ PCMEANSF(5),PU(500,5)
      COMMON /OPC1/ PL(500,5),XMUIC,S,XK,R,ARLIC,FBEST,
     * XUBEST,NTRACU
C
      I=MOD(NF,5)
      IF (I .EQ. 0 .AND. NF .NE. 0) WRITE(KW,15) NF
15    FORMAT(/3X,'....... TOTAL OF ',I4,' FUNCTION'
     *  ' EVALUATIONS DONE .......')
      FUMAX=1.0D-6
      BIG=1.0D30
C
      R=X(1)
C
      NTRACU=-2
      XU=XUBEST
      XMAXU=5.5D0
      XMINU=1.D0
      DELTXU=0.009D0
C
      DELMNU=0
      DELMNU=0.0001D0
C
      NFMAXU=200
      KWU=KW
C
      CALL UNICY(OP1FUNKU,NTRACU,FOBJU,XU,XMAXU,XMINU,DELTXU,
     *   DELMNU,NFMAXU,KWU)
C-------------------------------------------------------------
C  IN CASE NO ROOT WAS FOUND, USE A PENALTY.
C-------------------------------------------------------------
      IF(FOBJU.GT.FUMAX) THEN
         FOBJ=BIG
         RETURN
      ENDIF
C
      XK=XU
C
      LGN=0
      DO 2 I=1,NP
         IM1=I-1
         IF (I .EQ. 1) THEN
            CALL RLEWMA(XK,R,S,PCMEANSF(I),PL(1,I))
            GO TO 12
         ENDIF
         DO 20 K=1,IM1
            IF (PCMEANSF(I) .EQ. PCMEANSF(K)) THEN
              DO 25 L=1,PL(1,K)+3
```

```
25                PL(L,I)=PL(L,K)
            GO TO 12
            ENDIF
20        CONTINUE
          CALL RLEWMA(XK,R,S,PCMEANSF(I),PL(1,I))
12        NVAL=IDINT(PL(1,I))
          PU(NVAL+2,I)=PL(NVAL+3,I)
          IF (NVAL .GT. LGN) LGN=NVAL
          DO 3 J=1,NVAL+1
3             PU(J,I)=PL(J,I)
2     CONTINUE
      LGNP2=LGN+2
      DO 5 I=1,NP
          IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
            NP2=PL(1,I)+2
            XMNMN=PL(NP2,I)
            XMNPL=PU(NP2,I)
            DO 4 J=NP2,LGN+1
               PL(J,I)=PL(J-1,I)*XMNMN
4              PU(J,I)=PU(J-1,I)*XMNPL
            PL(LGNP2,I)=XMNMN
            PU(LGNP2,I)=XMNPL
          ENDIF
5     CONTINUE
      FOBJ=ARLPC(NP,LGN,PL,PU)
C
      IF(FOBJ.LT.FBEST) THEN
         FBEST=FOBJ
         XUBEST=XU
      ENDIF
C
      RETURN
C
C   END OP1FUNK
C
      END
C
C
      SUBROUTINE OP1FUNKU (XU,FOBJU)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL RLEWMA,ARLPC
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
      COMMON /NBVAR/ NP
      COMMON /OPC1/ PL(500,5),XMUIC,S,XK,R,ARLIC,FBEST,
     * XUBEST,NTRACU
C
```

```
      XK=XU
C
      CALL RLEWMA(XK,R,S,XMUIC,PL(1,1))
      NVAL=IDINT(PL(1,1))
      NVALP1=NVAL+1
      NVALP2=NVALP1+1
      NVALP3=NVALP2+1
      SUMP=1.D0
      DO 1 I=2,NVAL
         PNGTI=1.D0
         DO 2 J=1,NP
2           PNGTI=PNGTI*PL(I,1)
1        SUMP=SUMP+PNGTI
      XMNMN=1.D0
      XMNPL=1.D0
      PLST=1.D0
      DO 3 I=1,NP
           PLST=PLST*PL(NVALP1,1)
           XMNMN=XMNMN*PL(NVALP2,1)
3          XMNPL=XMNPL*PL(NVALP3,1)
      ARLMN=SUMP+PLST/(1.D0-XMNMN)
      ARLPL=SUMP+PLST/(1.D0-XMNPL)
      ARL=0.5D0*(ARLMN+ARLPL)
C
      FOBJU=(ARL-ARLIC)**2
      RETURN
C
C  END OP1FUNKU
C
      END
C
      SUBROUTINE OPMEWC3
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL RLEWMA,ARLPC,OP3FUNK,STEPIT,STSET,UNICY
C
      DIMENSION V(25),D(5),OGMEANSF(5),CL(5)
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
      COMMON /OPC3/ XK(5),PL(500,5),R,S,XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU
      COMMON /PCSF/ PU(500,5),PCMEANSF(5)
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
      COMMON /FLG/ IFLAG
C---------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C---------------------------------------------------------
```

```
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED IN-CONTROL'
     * ' ARL ***')
      READ(IN,*) ARLIC
6     WRITE(LP,80) ARLIC
80    FORMAT(/5X,'*** THE DESIRED IN-CONTROL ARL IS',G15.7)
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE EXPECTED SHIFT OF'
     *' ***'/5X,'*** THE ORIGINAL MEAN VECTOR ***')
      READ(IN,*) (OGMEANSF(I),I=1,NP)
8     WRITE(LP,95) (OGMEANSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE EXPECTED ORIGINAL MEAN VECTOR '
     * 'SHIFT IS ***'/5X,3G15.7/(5X,2G15.7))
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C-------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C-------------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DO 521 I=1,NP
521        PCMEANSF(I)=OGMEANSF(I)
        GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
      DO 530 I=1,NP
          IDX=(I-1)*NP
          PCMEANSF(I)=0.D0
          DO 530 J=1,NP
530          PCMEANSF(I)=PCMEANSF(I)+OGMEANSF(J)*V(IDX+J)
C---------------------------------------------------------
C  CONVERT A DIRECTION OF SHIFT IN PRIN. COMP. TO
C  UNIT LENGTH
C---------------------------------------------------------
```

```
20    WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING SHIFT OF THE '
     * ' ***'/5X,'*** MEAN OF THE PRIN. COMP. IS ***')
      WRITE(LP,105) (PCMEANSF(I),I=1,NP)
105   FORMAT(/5X,3G15.7/(5X,2G15.7))
      S=0.D0
10    WRITE(LP,110)
110   FORMAT(/5X,'*** THE INITIAL EWMA VALUES FOR ALL '
     * 'IEWMAPC CHARTS ***'/5X,'*** ARE SET AT 0.D0 ***')
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 10
11    WRITE(LP,115)
115   FORMAT(/5X,'*** PLEASE ENTER THE DESIRED COMMON'
     *  ' S VALUE ***')
      READ(IN,*) S
12    WRITE(LP,120)
120   FORMAT(/5X,'*** THE DESIRED COMMON S VALUE IS ***')
      WRITE(LP,125) S
125   FORMAT(5X,F12.4)
      WRITE(LP,71)
71    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 12
13    XMUIC=0.D0
      WRITE(LP,130)
130   FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED) ***')
      IF (IDTMX .NE. 1) THEN
        WRITE(LP,201)
201     FORMAT(/5X,'*** THE MATRIX U OF THE EIGEN VECTORS'
     *  ' ARE : ***'/)
        DO 755 I=1,NP
          K=(I-1)*NP+1
          WRITE(LP,202) I,(V(J),J=K,K+NP-1)
202       FORMAT(5X,'*** u',I1,' :',5F10.5)
755     CONTINUE
      ENDIF
      WRITE(LP,210)
210   FORMAT(//5X,'*** OPTIMIZATION IN PROGRESS ***'/)
C-----------------------------------------------------------
C SWITCH POSITION BETWEEN THE CHART WITH THE LARGEST
C SHIFT AND THE LAST
C-----------------------------------------------------------
      ISWCH=0
      PCSFMAX=0.D0
```

```
      DO 535 I=1,NP
         IF(DABS(PCMEANSF(I)) .LT. 1.D-5) PCMEANSF(I)=0.D0
         IF (DABS(PCMEANSF(I)) .GT. PCSFMAX) THEN
            ISWCH=I
            PCSFMAX=DABS(PCMEANSF(I))
         ENDIF
535   CONTINUE
      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=PCMEANSF(NP)
         PCMEANSF(NP)=TEMP
      ENDIF
C-----------------------------------------------------------
C     SET THE INITIAL SEARCH POINTS
C-----------------------------------------------------------
      R=0.8D0
      SPREAD=3.D0
      CONST=1.D-4
      DELTX(1)=0.03
      DXX=0.05
      IF (PCMEANSF(NP) .LE. 3.D0) R=0.7D0
      IF (PCMEANSF(NP) .LE. 2.5D0) R=0.5D0
      IF (PCMEANSF(NP) .LE. 2.D0) R=0.3D0
      IF (PCMEANSF(NP) .LE. 1.5D0) R=0.2D0
      IF (PCMEANSF(NP) .LE. 1.D0) R=0.1D0
      IF (PCMEANSF(NP) .LE. .75D0) R=0.5D-1
      IF (PCMEANSF(NP) .LE. .5D0) R=0.4D-1
      IF (R .LE. .5D0) SPREAD=2.9D0
      IF (R .LE. .25D0) SPREAD=2.75D0
      IF (R .LE. .1D0) THEN
         SPREAD=2.6D0
         DELTX(1)=0.01D0
      ENDIF
C-----------------------------------------------------------
C  INITIAL STEPIT ROUTINE
C-----------------------------------------------------------
      CALL STSET
C
      NV=NP
      MATRX=0
      NTRACE=-2
      X(1)=R
      XK(NP)=SPREAD
      DO 780 I=1,NV-1
         XK(I)=3.2D0
780      X(I+1)=XK(I)
C-----------------------------------------------------------
C  NOW DO THE CONSTRAINED MINIMIZATION,
C  MOVING ALONG THE CONSTRAINT SURFACE.
C-----------------------------------------------------------
      DO 825 J=2,NV
         DELTX(J)=DXX
```

```
825   CONTINUE
      DO 826 J=1,NV
         DELMIN(J)=CONST
826   CONTINUE
C
      XUBEST=XK(NP)
      FBEST=1.0D30
      XMAX(1)=1.D0
      XMIN(1)=0.03D0
      DO 820 I=2,NV
         XMIN(I)=1.0D0
820      XMAX(I)=5.5D0
C
      CALL STEPIT (OP3FUNK)
C

      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(NP)
         PCMEANSF(NP)=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=TEMP
         TEMP=XK(NP)
         XK(NP)=XK(ISWCH)
         XK(ISWCH)=TEMP
      ENDIF
      DO 821 I=1,NP
821      CL(I)=XK(I)*DSQRT(R/(2.D0-R))
C
      WRITE(KW,135) NP,ARLIC
135   FORMAT(//5X,'*** THE OPTIMAL',I2,' VARIATE MEWMAPC'
     * ' CHART ***'/5X,'*** WITH IN-CONTROL ARL OF',G11.3
     * /5X,'*** AT THE SHIFT OF THE ORIGINAL MEAN '
     * 'VECTOR OF : ***')
      IF (NP .LT. 4) THEN
         WRITE(LP,161) (OGMEANSF(I),I=1,NP)
161      FORMAT(5X,3G15.7)
         WRITE(LP,151)
151      FORMAT(5X,'*** IS LISTED AS FOLLOWS ***')
      ELSE
         WRITE(LP,146) (OGMEANSF(I),I=1,NP)
146      FORMAT(5X,3G15.7/5X,2G15.7)
         WRITE(LP,151)
      ENDIF
      WRITE(KW,144)
144   FORMAT(5X,50(1H-)/)
      DO 141 I=1,NP
         WRITE(KW,140)  I,CL(I),R
140   FORMAT(5X,'*** H',I1,' = ',G15.7,' R = ',G15.7)
141   CONTINUE
      WRITE(KW,143) FOBJ
143   FORMAT(/5X,'*** THE OPTIMAL OOC ARL = ',1PG15.7)
      RETURN
C
```

```
C  END MAIN
C
      END
C
      SUBROUTINE OP3FUNK
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL OP3FUNKU,UNICY,RLEWMA,ARLPC
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
      COMMON /OPC3/ XK(5),PL(500,5),R,S,XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU
      COMMON /PCSF/ PU(500,5),PCMEANSF(5)
      COMMON /FLG/ IFLAG
      COMMON /NBVAR/ NP
C
      I=MOD(NF,5)
      IF (I .EQ. 0 .AND. NF .NE. 0) WRITE(KW,15) NF
15    FORMAT(/3X,'....... TOTAL OF ',I4,' FUNCTION'
     *  ' EVALUATIONS DONE .......')
      FUMAX=1.0D-6
      BIG=1.0D30
      DO 100 I=1,NP-1
100      XK(I)=X(I+1)
      R=X(1)
C
      NTRACU=-2
      XU=XUBEST
      XMAXU=5.5D0
      XMINU=1.D0
      DELTXU=0.009D0
C
      DELMNU=0
      DELMNU=0.0001D0
C
      NFMAXU=200
      KWU=KW
C
      LGN=0
      DO 35 I=1,NP-1
35       CALL RLEWMA(XK(I),R,S,XMUIC,PL(1,I))
      IFLAG=0
C
      CALL UNICY(OP3FUNKU,NTRACU,FOBJU,XU,XMAXU,XMINU,DELTXU,
     *   DELMNU,NFMAXU,KWU)
C-----------------------------------------------------------
C  IN CASE NO ROOT WAS FOUND, USE A PENALTY.
C-----------------------------------------------------------
      IF(FOBJU.GT.FUMAX) THEN
```

```
         FOBJ=BIG
         RETURN
      ENDIF
C
      XK(NP)=XU
C
      LGN=0
      DO 55 I=1,NP
         IF (PCMEANSF(I) .NE. XMUIC) THEN
            CALL RLEWMA(XK(I),R,S,PCMEANSF(I),PL(1,I))
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
            DO 56 J=1,NVAL+1
56             PU(J,I)=PL(J,I)
            GO TO 55
         ENDIF
         IF (PL(1,I) .GT. LGN) LGN=IDINT(PL(1,I))
55    CONTINUE
      LGNP2=LGN+2
      DO 57 I=1,NP
         IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
            NP2=PL(1,I)+2
            XMNMN=PL(NP2,I)
            XMNPL=PU(NP2,I)
            DO 58 J=NP2,LGN+1
               PL(J,I)=PL(J-1,I)*XMNMN
58             PU(J,I)=PU(J-1,I)*XMNPL
            PL(LGNP2,I)=XMNMN
            PU(LGNP2,I)=XMNPL
          ENDIF
57    CONTINUE
      FOBJ=ARLPC(NP,LGN,PL,PU)
C
      IF(FOBJ.LT.FBEST) THEN
         FBEST=FOBJ
         XUBEST=XU
      ENDIF
      RETURN
C
C   END OP3FUNK
C
      END
C
C
      SUBROUTINE OP3FUNKU (XU,FOBJU)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL RLEWMA,ARLPC
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
```

```
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
      COMMON /OPC3/ XK(5),PL(500,5),R,S,XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU
      COMMON /PCSF/ PU(500,5),PCMEANSF(5)
      COMMON /FLG/ IFLAG
      COMMON /NBVAR/ NP
C
      XK(NP)=XU
C
      CALL RLEWMA(XK(NP),R,S,XMUIC,PL(1,NP))
      IF (IFLAG .NE. 0) THEN
         NVAL=IDINT(PL(1,NP))
         PU(NVAL+2,NP)=PL(NVAL+3,NP)
         DO 36 J=1,NVAL+1
36          PU(J,NP)=PL(J,NP)
         IF (PL(1,NP) .LT. PL(1,1)) THEN
            LGN=PL(1,1)
            LGNP2=LGN+2
            NP2=NVAL+2
            XMNMN=PL(NP2,NP)
            XMNPL=PU(NP2,NP)
            DO 55 J=NP2,LGN+1
               PL(J,NP)=PL(J-1,NP)*XMNMN
55             PU(J,NP)=PU(J-1,NP)*XMNPL
            PL(LGNP2,NP)=XMNMN
            PU(LGNP2,NP)=XMNPL
            PL(1,NP)=DFLOAT(LGN)
            PU(1,NP)=PL(1,NP)
         ELSEIF (PL(1,NP) .GT. PL(1,1)) THEN
               LGN=PL(1,NP)
               LGNP2=LGN+2
               DO 56 I=1,NP-1
                     NP2=PL(1,I)+2
                     XMNMN=PL(NP2,I)
                     XMNPL=PU(NP2,I)
                     DO 57 J=NP2,LGN+1
                        PL(J,I)=PL(J-1,I)*XMNMN
57                      PU(J,I)=PU(J-1,I)*XMNPL
                     PL(LGNP2,I)=XMNMN
                     PU(LGNP2,I)=XMNPL
                     PL(1,I)=DFLOAT(LGN)
                     PU(1,I)=PL(1,I)
56             CONTINUE
         ENDIF
         LGN=PL(1,NP)
         ARL=ARLPC(NP,LGN,PL,PU)
         GO TO 200
      ENDIF
          LGN=0
          DO 135 I=1,NP
             NVAL=IDINT(PL(1,I))
```

```
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
            DO 136 J=1,NVAL+1
136             PU(J,I)=PL(J,I)
135       CONTINUE
         LGNP2=LGN+2
         DO 140 I=1,NP
            IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
               NP2=PL(1,I)+2
               XMNMN=PL(NP2,I)
               XMNPL=PU(NP2,I)
               DO 145 J=NP2,LGN+1
                  PL(J,I)=PL(J-1,I)*XMNMN
145               PU(J,I)=PU(J-1,I)*XMNPL
               PL(LGNP2,I)=XMNMN
               PU(LGNP2,I)=XMNPL
               PL(1,I)=DFLOAT(LGN)
               PU(1,I)=PL(1,I)
             ENDIF
140       CONTINUE
      IFLAG=1
      ARL=ARLPC(NP,LGN,PL,PU)
C
200   FOBJU=(ARL-ARLIC)**2
      RETURN
C
C  END OP3FUNKU
C
      END
C
      DOUBLE PRECISION FUNCTION ARLPC(NP,LGN,PBMN,PBPL)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PBMN(500,NP),PBPL(500,NP)
C
      SUMPMN=0.D0
      SUMPPL=0.D0
      LGNP1=LGN+1
      LGNP2=LGN+2
      DO 1 I=2,LGN
         PGTJMN=1.D0
         PGTJPL=1.D0
         DO 2 J=1,NP
            PGTJMN=PGTJMN*PBMN(I,J)
2           PGTJPL=PGTJPL*PBPL(I,J)
         SUMPMN=SUMPMN+PGTJMN
         SUMPPL=SUMPPL+PGTJPL
1     CONTINUE
      XMNMN=1.D0
      XMNPL=1.D0
      PLSTMN=1.D0
      PLSTPL=1.D0
      DO 3 I=1,NP
```

```
         PLSTMN=PLSTMN*PBMN(LGNP1,I)
         PLSTPL=PLSTPL*PBPL(LGNP1,I)
         XMNMN=XMNMN*PBMN(LGNP2,I)
3        XMNPL=XMNPL*PBPL(LGNP2,I)
      ARLMN=1.D0+SUMPMN+PLSTMN/(1.D0-XMNMN)
      ARLPL=1.D0+SUMPPL+PLSTMN/(1.D0-XMNPL)
      ARLPC=0.5D0*(ARLMN+ARLPL)
      RETURN
      END
C
C
      SUBROUTINE RLEWMA(XK,R,S,XMU,UTAILP)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TEMP(100),UTAILP(500),P(24),ww(24)
      COMMON /PRVP/ PNM1(100)
      COMMON /CENDPT/ POINTS(100),W(24),HOVER2,int
      COMMON /TABLE/ TAB(97,96)
      DATA DELTA,NPOINT/1.0D-10,24/
      DATA P/-.9951872199970214D0,-.9747285559713095D0,
     2       -.9382745520027328D0,-.8864155270044010D0,
     3       -.8200019859739029D0,-.7401241915785544D0,
     4       -.6480936519369756D0,-.5454214713888395D0,
     5       -.4337935076260451D0,-.3150426796961634D0,
     6       -.1911188674736163D0,-.0640568928626056D0,
     7        0,0,0,0,0,0,0,0,0,0,0,0/
      DATA wW/.0123412297999872D0,.0285313886289337D0,
     2      .0442774388174198D0,.0592985849154368D0,
     3      .0733464814110803D0,.0861901615319533D0,
     4      .0976186521041139D0,.1074442701159656D0,
     5      .1155056680537256D0,.1216704729278034D0,
     6      .1258374563468283D0,.1279381953467522D0,
     7      0,0,0,0,0,0,0,0,0,0,0,0/
C
C
      QEXP(ARG)=DEXP(DMIN1(ARGMAX,DMAX1(-ARGMAX,ARG)))
      F(SX,SP)=3.989422804014327D-1*
     +   QEXP(-.5D0*((SP-(1.D0-R)*SX)/R-XMU)*((SP-
     +   (1.D0-R)*SX)/R-XMU))

      ARGMAX=50.D0
C
C
C     INITIALIZE PARAMETERS
C
C
      CL=XK*DSQRT(R/(2.D0-R))
      int=4
      IMAXX=500
      NMAXX=500
      NMAXM3=NMAXX-3
      ITOP=INT*NPOINT
      ITOPD2=ITOP/2
```

```
      ITOP1=ITOP+1
      PMIN=1.D0-1.D-12
      DO 10 L=1,12
         w(l)=ww(l)
         W(25-L)=W(L)
10       P(25-L) = -P(L)
      DO 1 I=1,ITOP
         PNM1(I)=0.D0
1        POINTS(I)=0.D0
      DO 5 I=1,NMAXX
5        UTAILP(I)=0.D0
C
C   TRANSFORM GAUSS POINTS AND WEIGHTS FROM THE
C   (-1, 1) INTERVAL TO THE (A, B) INTERVAL
C

      H = 2.D0*CL/DBLE(INT)
      HOVER2 = H/2.D0
      DO 20 I=0,INT-1
         XMID = -CL+DBLE(I)*H+HOVER2
         DO 50 J = 1, NPOINT
            POINTS(I*NPOINT+J) = HOVER2*P(J)+XMID
50       CONTINUE
20    CONTINUE
      DO 31 I = 1,96
         DO 31 J=1,96
31          TAB(I,J) = F(POINTS(I),POINTS(J))
      DO 32 J=1,96
32       TAB(97,J)= F(S,POINTS(J))
C   L IS THE INDEX OF THE INITIAL S VALUE
      L=97
C
      DO 3 I=1,ITOP
         PNM1(I)=1.D0
3        TEMP(I)=1.D0
C
      IF ( XMU .EQ. 0.D0) GO TO 500
C
      DO 120 N=1,NMAXM3
         NP1=N+1
         DO 12 IDX=1,ITOP
12          PNM1(IDX) = TEMP(IDX)
         UTAILP(NP1)=PROB(R,L)
         IF ( UTAILP(NP1) .GE. PMIN )  UTAILP(NP1)=1.D0
         XMAXMN=0.D0
         XMINMN=1.D0
         DO 11 J=1,ITOP
            TEMP(J) = PROB(R,J)
            XMAXMN=DMAX1(TEMP(J)/PNM1(J),XMAXMN)
11          XMINMN=DMIN1(TEMP(J)/PNM1(J),XMINMN)
         IF ((XMAXMN-XMINMN) .LE. DELTA) GO TO 30
120   CONTINUE
```

```
      GO TO 30
500   DO 220 N=1,NMAXM3
         NP1=N+1
         DO 22 IDX=1,ITOP
22          PNM1(IDX) = TEMP(IDX)
         UTAILP(NP1)=PROB(R,L)
         IF ( UTAILP(NP1) .GE. PMIN )  UTAILP(NP1)=1.D0
         XMAXMN=0.D0
         XMINMN=1.D0
         DO 51 J=1,ITOPD2
            TEMP(J) = DMIN1(PROB(R,J),1.D0)
            TEMP(ITOP1-J)=TEMP(J)
            XMAXMN=DMAX1(TEMP(J)/PNM1(J),XMAXMN)
            XMINMN=DMIN1(TEMP(J)/PNM1(J),XMINMN)
            IF (TEMP(J) .GE. 1.D0) GO TO 15
51       CONTINUE
         GO TO 16
15       JP1=J+1
         DO 17 I=JP1,ITOPD2
            TEMP(I)=1.D0
            XMAXMN=DMAX1(TEMP(I)/PNM1(I),XMAXMN)
            XMINMN=DMIN1(TEMP(I)/PNM1(I),XMINMN)
17          TEMP(ITOP1-I)=1.D0
16       IF ((XMAXMN-XMINMN) .LE. DELTA) GO TO 30
220   CONTINUE
30       UTAILP(1)=DBLE(N)
         UTAILP(N+2)=XMINMN
         UTAILP(N+3)=XMAXMN
      RETURN
      END


      DOUBLE PRECISION FUNCTION PROB(R,INDEX)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /PRVP/ PNM1(100)
      COMMON /CENDPT/ POINTS(100),W(24),HOVER2,int
      COMMON /TABLE/ TAB(97,96)
      DATA NPOINT/24/
      PROB=0.D0
      DO 2 I=0,INT-1
         DO 3 J=1,NPOINT
            IX=I*NPOINT+J
            PROB=PROB+HOVER2*W(J)*PNM1(IX)*TAB(INDEX, IX)
3        CONTINUE
2     CONTINUE
      PROB=PROB/R
      RETURN
      END
```

APPENDIX H

FORTRAN PROGRAM FOR DESIGN AND EVALUATION OF THE MZONEPC CHART

```
      PROGRAM MAIN
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER CHAR*1
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C
      LP=6
      IN=5
11    WRITE(LP,50)
50    FORMAT(//13X,26(1H*)/13X,'***  MAIN      MENU    ',
     *'***'/13X,26(1H*)//
     *5X,'(1) EVALUATION OF THE ARL OF A MZONEPC CHART'/
     *5X,'(2) CLASSICAL DESIGN OF THE MZONEPC CHART'/
     *5X,'(3) OPTIMAL DESIGN OF THE MZONEPC CHART'/
     *5X,'(4) EXIT THE PROGRAM'//
     *5X,'====> PLEASE ENTER YOUR OPTION (1, 2, 3, OR'
     *' 4) <===='
      READ(IN,*) ISELECT
      IF(ISELECT .LT. 1 .OR. ISELECT .GT. 4) THEN
      WRITE(LP,57)
        GO TO 11
      ENDIF
      IF(ISELECT .EQ. 4) GO TO 25
1     WRITE(LP,51)
51    FORMAT(/5X,'===> PLEASE ENTER THE NUMBER OF VARIABLE'
     *,'S MONITORED <==='/5X,'===> THE NUMBER SHOULD BE'
     *   ,' BETWEEN 2 AND 5 <===')
      READ(IN,*) NP
      IF (NP .LT. 2 .OR. NP .GT. 5) THEN
         WRITE(LP,57)
         GO TO 1
      ENDIF
2     WRITE(LP,55) NP
55    FORMAT(/5X,'*** THE TOTAL NUMBER OF VARIABLES IS ',I2)
      WRITE(LP,56)
56    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <===' /5X,
     *  '===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ (IN,*) IANS
      GO TO (13,1) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 2
13    IDTMX=1
      WRITE(LP,70)
70    FORMAT(/5X,'*** THE STANDARDIZED COVARIANCE MATRIX'
     */5X,'*** IS INITIALLY AN IDENTITY MATRIX ***')
      WRITE(LP,56)
      READ(IN,*) IANS
      GO TO (20,3) IANS
      WRITE(LP,57)
```

```
      GO TO 13
3     WRITE(LP,60)
60    FORMAT(/5X,'===> PLEASE ENTER THE DESIRED '
     *'STANDARDIZED <==='/5X,'===> COVARIANCE MATRIX '
     * 'ROW BY ROW <===')
      IDTMX=0
      DO 500 I=1,NP
         K=(I-1)*NP+1
         L=K+NP-1
         READ(IN,*) (COVAR(J),J=K,L)
500   CONTINUE
4     WRITE(LP,65)
65    FORMAT(/5X,'*** THE STANDARDIZED COVARIANCE MATRIX'
     * ,' IS ***'/)
      DO 505 I=1,NP
         K=(I-1)*NP+1
         L=K+NP-1
         WRITE(LP,66) (COVAR(J),J=K,L)
66       FORMAT(2X,5G11.4)
505   CONTINUE
      WRITE(LP,75)
75    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (20,3)IANS
      WRITE(LP,57)
      GO TO 4
20    GO TO (22,23,24,25) ISELECT
22    CALL EVZONEPC
      WRITE(LP,100)
100   FORMAT(/5X,'*** PAUSE! PLEASE HIT ENTER TO RETURN'
     *  ' TO MAIN MENU ***')
      READ(IN,101) CHAR
101   FORMAT(A1)
      GO TO 11
23    CALL CDZONEPC
      WRITE(LP,100)
      READ(IN,101) CHAR
      GO TO 11
24    CALL OPZONEPC
      WRITE(LP,100)
      READ(IN,101) CHAR
      GO TO 11
25    STOP
C
C END MAIN
C
      END
C
      SUBROUTINE EVZONEPC
C-------------------------------------------------------
C  EVALUATION OF THE MEWMA CHART
```

```
C-----------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL ARLPC,RLZONE
C
      DIMENSION OGMEANSF(5),PCMEANSF(5),XK(5),
     * D(5),V(25),ISCORE(5)
C
      DIMENSION PU(500,5),PL(500,5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C-----------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C-----------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE PARAMETERS OF THE'
     * ' EXISTING CHARTS ***')
      DO 510 I=1,NP
        WRITE(LP,80) I
80      FORMAT(/5X,'*** THE SYMETRICAL '
     *  'CONTROL LIMIT H ***'/5X,'*** FOR NO.',I2,
     *    ' IZONEPC CHART IS : ***')
         READ(IN,*) XK(I)
510   CONTINUE
      WRITE(LP,86)
86    FORMAT(/5X,'*** THE PARAMETERS FOR THE CHARTS ARE'
     *  ' : ***'/)
6     DO 520 I=1,NP
        WRITE(LP,85) I,XK(I)
85      FORMAT(5X,'H',I1,' =',G15.7)
520   CONTINUE
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE EXPECTED SHIFT OF'
     * ' THE MEAN VECTOR ***')
      READ(IN,*) (OGMEANSF(I),I=1,NP)
8     WRITE(LP,95) (OGMEANSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE EXPECT MEAN VECTOR SHIFT IS ***'/
     * 5X,3G15.7/(5X,2G15.7))
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (9,7) IANS
```

```
      WRITE(LP,57)
      GO TO 8
C-------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C-------------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DO 521 I=1,NP
521        PCMEANSF(I)=OGMEANSF(I)
        GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
      DO 530 I=1,NP
          IDX=(I-1)*NP
          PCMEANSF(I)=0.D0
          DO 530 J=1,NP
530          PCMEANSF(I)=PCMEANSF(I)+OGMEANSF(J)*V(IDX+J)
20    WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING SHIFTS ON THE PRIN.'
     *     ' COMP. ARE ***')
      WRITE(LP,105) (PCMEANSF(I),I=1,NP)
105   FORMAT(/5X,3G15.7/(5X,2G15.7))
      DO 531 I=1,NP
531     ISCORE(I)=0
10    WRITE(LP,110)
110   FORMAT(/5X,'*** THE INITIAL ZONE SCORES FOR ALL '
     * 'IZONEPC CHARTS ***'/5X,'*** ARE SET AT 0 ***')
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (14,11) IANS
      WRITE(LP,57)
      GO TO 10
11    WRITE(LP,115)
115   FORMAT(/5X,'*** PLEASE ENTER THE DESIRED INITIAL '
     *'ZONE SCORES ONE BY ONE ***'/5X,'*** NOTE : THE'
     *' ZONE SCORE IS AN INTEGER VALUE BETWEEN -3 TO 3'
     * ' ***')
      READ(IN,*) (ISCORE(I),I=1,NP)
      DO 571 I=1,NP
         IF (ISCORE(I) .LT. -3 .OR. ISCORE(I) .GT. 3) THEN
           WRITE(LP,57)
           GO TO 11
         ENDIF
571   CONTINUE
12    WRITE(LP,120)
120   FORMAT(/5X,'*** THE DESIRED INITIAL ZONE'
     *  ' SCORES ARE : ***')
```

```
      WRITE(LP,125) (ISCORE(I),I=1,NP)
125   FORMAT(/5X,5(2X,I3))
      WRITE(LP,70)
      READ(IN,*) IANS
      GO TO (14,11) IANS
      WRITE(LP,57)
      GO TO 12
C-------------------------------------------------------------
C   EVALUATION OF THE OOC ARL OF THE EXISTING MEWMAPC CHART
C-------------------------------------------------------------
14    WRITE(LP,131)
131   FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED)***'/)
      LGN=0
      DO 535 I=1,NP
         CALL RLZONE(XK(I),ISCORE(I),PCMEANSF(I),PL(1,I))
         NVAL=IDINT(PL(1,I))
         PU(NVAL+2,I)=PL(NVAL+3,I)
         IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
         DO 535 J=1,NVAL+1
            PU(J,I)=PL(J,I)
535   CONTINUE
      LGNP2=LGN+2
      DO 540 I=1,NP
         IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
            NP2=PL(1,I)+2
            XMNMN=PL(NP2,I)
            XMNPL=PU(NP2,I)
            DO 545 J=NP2,LGN+1
               PL(J,I)=PL(J-1,I)*XMNMN
545            PU(J,I)=PU(J-1,I)*XMNPL
            PL(LGNP2,I)=XMNMN
            PU(LGNP2,I)=XMNPL
          ENDIF
540   CONTINUE
      ARL=ARLPC(NP,LGN,PL,PU)
      WRITE(LP,130) ARL
130   FORMAT(/5X,'*** THE OOC ARL FOR THE DESIRED MZONEPC'
     * ' CHART ***'/5X,'*** AT THE DESIRED SHIFT IS'
     *  ,G15.7)
      RETURN
C
C   END EVZONEPC
C
      END
C
      SUBROUTINE CDZONEPC
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL RLZONE,ARLPC
```

```
C
      DIMENSION V(25),XSARL(6),XBARL(6),
     * DIRARL(6),D(5),DIRPCSF(5),DIROGSF(5)
C
      DIMENSION PU(500,5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
C
      COMMON /CDZ/ PL(500,5),XMUIC,XK,
     * ARLIC,FBEST,XUBEST,NTRACU,ISCORE
C-----------------------------------------------------------
C   INPUT OPERATION PARAMETERS
C-----------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER DESIRED IN-CONTROL ARL'
     * ' ***')
      READ(IN,*) ARLIC
6     WRITE(LP,80) ARLIC
80    FORMAT(/5X,'*** THE DESIRED IN-CONTROL ARL IS',G15.7)
      WRITE(LP,71)
71    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE DIRECTION OF THE '
     *'EXPECTED ***'/5X,'*** SHIFT OF THE MEAN VECTOR ***')
      READ(IN,*) (DIROGSF(I),I=1,NP)
8     WRITE(LP,95) (DIROGSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE DIRECTION OF THE EXPECTED MEAN'
     * ' ***'/5X,'*** VECTOR SHIFT IS ***'/5X,3G15.7
     * /(5X,2G15.7))
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C----------------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C----------------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DSUM=0.D0
        DO 521 I=1,NP
```

```
            DIRPCSF(I)=DIROGSF(I)
521         DSUM=DSUM+DIRPCSF(I)*DIRPCSF(I)
         GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
      DSUM=0.D0
      DO 530 I=1,NP
          IDX=(I-1)*NP
          DIRPCSF(I)=0.D0
          DO 531 J=1,NP
531          DIRPCSF(I)=DIRPCSF(I)+DIROGSF(J)*V(IDX+J)
530       DSUM=DSUM+DIRPCSF(I)*DIRPCSF(I)
C----------------------------------------------------------
C  CONVERT A DIRECTION OF SHIFT IN PRIN. COMP. TO
C  UNIT LENGTH
C----------------------------------------------------------
20    DO 532 I=1,NP
532      DIRPCSF(I)=DIRPCSF(I)/DSQRT(DSUM)
      WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING DIRECTION OF THE '
     * 'SHIFT ***'/5X,'*** IN THE PRIN. COMP. WITH UNIT '
     * 'LENGTH IS ***')
      WRITE(LP,105) (DIRPCSF(I),I=1,NP)
105   FORMAT(/5X,3G15.7/(5X,2G15.7))
      ISCORE=0
10    WRITE(LP,110)
110   FORMAT(/5X,'*** THE INITIAL ZONE SCORES FOR ALL '
     * 'IZONEPC CHARTS ***'/5X,'*** ARE SET AT 0 ***')
      WRITE(LP,71)
      READ(IN,*) IANS
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 10
11    WRITE(LP,115)
115   FORMAT(/5X,'*** PLEASE ENTER THE DESIRED COMMON '
     *'ZONE SCORE ***'/5X,'*** NOTE : THE ZONE SCORE IS'
     *' AN INTEGER VALUE BETWEEN -3 TO 3 ***')
      READ(IN,*) ISCORE
      IF (ISCORE .LT. -3 .OR. ISCORE .GT. 3) THEN
         WRITE(LP,57)
         GO TO 11
      ENDIF
12    WRITE(LP,120) ISCORE
120   FORMAT(/5X,'*** THE DESIRED COMMON INITIAL ZONE'
     * ' SCORE IS ',I3)
      WRITE(LP,71)
      READ(IN,*) IANS
```

```
      GO TO (13,11) IANS
      WRITE(LP,57)
      GO TO 12
13    XMUIC=0.D0
      WRITE(LP,130)
130   FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED)***'/)
C-----------------------------------------------------
C   DO ONE VARIABLE OPTIMIZATION
C-----------------------------------------------------
      XK=3.D0
      XUBEST=XK
C
C  NOW DO THE CONSTRAINED MINIMIZATION,
C  MOVING ALONG THE CONSTRAINT SURFACE.
C
      CALL CDZFUNK
C
      XK=XUBEST
C
      WRITE(LP,135) NP,ARLIC,XK
135   FORMAT(//5X,'THE COMMON CONTROL LIMIT OF A'
     *,I2,' VARIATE MZONEPC CHART'/5X,' WITH IN-CONTROL'
     * ' ARL OF',G9.3,' IS :'/5X,G15.7)
C
      IF (IDTMX .NE. 1) THEN
        WRITE(LP,201)
201     FORMAT(/5X,'*** THE MATRIX U OF THE EIGEN VECTORS'
     *  ' ARE : ***')
        DO 755 I=1,NP
          K=(I-1)*NP+1
          WRITE(LP,202) I,(V(J),J=K,K+NP-1)
202       FORMAT(5X,'*** u',I1,' :',5F10.5)
755     CONTINUE
      ENDIF
C------------------------------------------------------
C  CALCUALTE OOC ARL AT GIVEN DIRECTION
C------------------------------------------------------
      DO 550 L=1,6
         LGN=0
         DO 535 I=1,NP
            XLAMBDA=DINT(L)*0.5D0*DIRPCSF(I)
            CALL RLZONE(XK,ISCORE,XLAMBDA,PL(1,I))
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=NVAL
            DO 535 J=1,NVAL+1
               PU(J,I)=PL(J,I)
535     CONTINUE
        LGNP2=LGN+2
        DO 540 I=1,NP
```

```
            IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
              NP2=PL(1,I)+2
              XMNMN=PL(NP2,I)
              XMNPL=PU(NP2,I)
              DO 545 J=NP2,LGN+1
                 PL(J,I)=PL(J-1,I)*XMNMN
545              PU(J,I)=PU(J-1,I)*XMNPL
              PL(LGNP2,I)=XMNMN
              PU(LGNP2,I)=XMNPL
            ENDIF
540   CONTINUE
      DIRARL(L)=ARLPC(NP,LGN,PL,PU)
550   CONTINUE
C-------------------------------------------------------
C  CALCUALTE OVERALL OOC ARL AT MINIMUM
C-------------------------------------------------------
      DO 650 L=1,6
         LGN=0
         DO 635 I=1,NP
            FACTOR=0.D0
            IF (I .EQ. 1) FACTOR=1.D0
            XLAMBDA=DFLOAT(L)*FACTOR*0.5D0
            IF (I .LE. 2) THEN
               CALL RLZONE(XK,ISCORE,XLAMBDA,PL(1,I))
            ELSE
               DO 636 K=1,IDINT(PL(1,2))+3
636               PL(K,I)=PL(K,2)
            ENDIF
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=NVAL
            DO 635 J=1,NVAL+1
               PU(J,I)=PL(J,I)
635     CONTINUE
        LGNP2=LGN+2
        DO 640 I=1,NP
           IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
             NP2=PL(1,I)+2
             XMNMN=PL(NP2,I)
             XMNPL=PU(NP2,I)
             DO 645 J=NP2,LGN+1
                PL(J,I)=PL(J-1,I)*XMNMN
645             PU(J,I)=PU(J-1,I)*XMNPL
             PL(LGNP2,I)=XMNMN
             PU(LGNP2,I)=XMNPL
           ENDIF
640   CONTINUE
      XSARL(L)=ARLPC(NP,LGN,PL,PU)
650   CONTINUE
C-------------------------------------------------------
C  CALCUALTE OVERALL OOC ARL AT MAXIMUM
C-------------------------------------------------------
```

```
      FACTOR=1.D0/DSQRT(DFLOAT(NP))
      DO 750 L=1,6
         LGN=0
         XLAMBDA=DFLOAT(L)*FACTOR*0.5D0
         DO 735 I=1,NP
            IF (I .LE. 1) THEN
               CALL RLZONE(XK,ISCORE,XLAMBDA,PL(1,I))
            ELSE
               DO 736 K=1,IDINT(PL(1,1))+3
736               PL(K,I)=PL(K,1)
            ENDIF
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=NVAL
            DO 735 J=1,NVAL+1
               PU(J,I)=PL(J,I)
735     CONTINUE
        LGNP2=LGN+2
        DO 740 I=1,NP
           IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
             NP2=PL(1,I)+2
             XMNMN=PL(NP2,I)
             XMNPL=PU(NP2,I)
             DO 745 J=NP2,LGN+1
                PL(J,I)=PL(J-1,I)*XMNMN
745             PU(J,I)=PU(J-1,I)*XMNPL
             PL(LGNP2,I)=XMNMN
             PU(LGNP2,I)=XMNPL
           ENDIF
740   CONTINUE
      XBARL(L)=ARLPC(NP,LGN,PL,PU)
750   CONTINUE
C
      WRITE(LP,140)
140   FORMAT(/5X,'*** THE OOC ARL PROFILE ARE SHOWN AS'
     *' FOLLOWS ***')
      WRITE(LP,145)
145   FORMAT(/5X,'LAMBDA',12X,'OOC ARL',14X,'OOC   ARL'/
     *21X,'AT DIRECTION',12X,'OVERALL'/4X,56(1H-))
      DO 700 I=1,6
         XLAMBDA=DFLOAT(I)*0.5D0
         WRITE(LP,150)XLAMBDA,DIRARL(I),XSARL(I),XBARL(I)
150      FORMAT(5X,F5.2,9X,F11.4,5X,F11.4,' -',F11.4)
700   CONTINUE
      RETURN
C
C END CDZONEPC
C
      END
C
      SUBROUTINE CDZFUNK
C
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL CDZFUNKU,UNICY
C
      COMMON /NBVAR/ NP
      COMMON /CDZ/ PL(500,5),XMUIC,XK,
     * ARLIC,FBEST,XUBEST,NTRACU,ISCORE
C
      KW=6
      XUBEST=XK
C
      NTRACU=-2
      XU=XUBEST
      XMAXU=5.5D0
      XMINU=0.03D0
      DELTXU=0.009D0
C
C   START WITH A LARGE DELMNU, AND DECREASE IT LATER.
C
      DELMNU=0
      DELMNU=0.0001D0
C
      NFMAXU=200
      KWU=KW
C
      CALL UNICY(CDZFUNKU,NTRACU,FOBJU,XU,XMAXU,XMINU,DELTXU,
     *   DELMNU,NFMAXU,KWU)
      XUBEST=XU
C
      RETURN
C
C   END CDFUNK
C
      END
C
C
      SUBROUTINE CDZFUNKU (XU,FOBJU)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL RLZONE
C
      COMMON /NBVAR/ NP
      COMMON /CDZ/ PL(500,5),XMUIC,XK,
     * ARLIC,FBEST,XUBEST,NTRACU,ISCORE
C
      KW=6
      XK=XU
C
      CALL RLZONE(XK,ISCORE,XMUIC,PL(1,1))
      NVAL=IDINT(PL(1,1))
      NVALP1=NVAL+1
```

```
      NVALP2=NVALP1+1
      NVALP3=NVALP2+1
      SUMP=1.D0
      DO 1 I=2,NVAL
         PNGTI=1.D0
         DO 2 J=1,NP
2           PNGTI=PNGTI*PL(I,1)
1        SUMP=SUMP+PNGTI
      XMNMN=1.D0
      XMNPL=1.D0
      PLST=1.D0
      DO 3 I=1,NP
           PLST=PLST*PL(NVALP1,1)
           XMNMN=XMNMN*PL(NVALP2,1)
3          XMNPL=XMNPL*PL(NVALP3,1)
      ARLMN=SUMP+PLST/(1.D0-XMNMN)
      ARLPL=SUMP+PLST/(1.D0-XMNPL)
      ARL=0.5D0*(ARLMN+ARLPL)
C
      FOBJU=(ARL-ARLIC)**2
      RETURN
C
C   END CDZFUNKU
C
      END
C
      SUBROUTINE OPZONEPC
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      EXTERNAL RLZONE,ARLPC,OPZFUNK,STEPIT,STSET,UNICY
C
      DIMENSION V(25),D(5),OGMEANSF(5)
C
      COMMON /COV/ COVAR(25),IDTMX
      COMMON /NBVAR/ NP
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
      COMMON /PCSF/ PCMEANSF(5),PU(500,5)
      COMMON /OPZ/ XK(5),PL(500,5),XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU,ISCORE
C------------------------------------------------------------
C    INPUT OPERATION PARAMETERS
C------------------------------------------------------------
      LP=6
      IN=5
5     WRITE(LP,75)
75    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED IN-CONTROL'
     * ' ARL ***')
      READ(IN,*) ARLIC
```

```
6     WRITE(LP,80) ARLIC
80    FORMAT(/5X,'*** THE DESIRED IN-CONTROL ARL IS',G15.7)
      WRITE(LP,71)
71    FORMAT(/5X,'===> IS THE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (7,5) IANS
      WRITE(LP,57)
57    FORMAT(/5X,'*** ERROR INPUT! PLEASE TRY AGAIN ***')
      GO TO 6
7     WRITE(LP,90)
90    FORMAT(/5X,'*** PLEASE ENTER THE EXPECTED SHIFT OF'
     *' ***'/5X,'*** THE ORIGINAL MEAN VECTOR ***')
      READ(IN,*) (OGMEANSF(I),I=1,NP)
8     WRITE(LP,95) (OGMEANSF(I),I=1,NP)
95    FORMAT(/5X,'*** THE EXPECTED ORIGINAL MEAN VECTOR '
     * 'SHIFT IS ***'/5X,3G15.7/(5X,2G15.7))
      WRITE(LP,70)
70    FORMAT(/5X,'===> ARE THESE DATA CORRECT ?  <==='/
     * 5X,'===> PLEASE ENTER (1) FOR YES, (2) FOR NO <===')
      READ(IN,*) IANS
      GO TO (9,7) IANS
      WRITE(LP,57)
      GO TO 8
C-----------------------------------------------------
C  CALCULATE THE CORRESPONEING SHIFTS IN EACH PRINCIPAL
C  COMPONENT
C-----------------------------------------------------
9     IF (IDTMX .EQ. 1) THEN
        DO 521 I=1,NP
521        PCMEANSF(I)=OGMEANSF(I)
        GO TO 20
      ENDIF
      CALL JACOBI(COVAR,NP,NP,D,V,NROT)
      CALL EIGSRT(D,V,NP,NP)
      DO 525 I=1,NP
         IM1=I-1
         DO 525 J=1,NP
525         V(IM1*NP+J)=V(IM1*NP+J)/DSQRT(D(I))
      DO 530 I=1,NP
          IDX=(I-1)*NP
          PCMEANSF(I)=0.D0
          DO 530 J=1,NP
530          PCMEANSF(I)=PCMEANSF(I)+OGMEANSF(J)*V(IDX+J)
C-------------------------------------------------------
C  CONVERT A DIRECTION OF SHIFT IN PRIN. COMP. TO
C  UNIT LENGTH
C-------------------------------------------------------
20    WRITE(LP,100)
100   FORMAT(/5X,'*** THE CORRESPONDING SHIFT OF THE '
     * ' ***'/5X,'*** MEAN OF THE PRIN. COMP. IS ***')
      WRITE(LP,105) (PCMEANSF(I),I=1,NP)
```

```
105    FORMAT(/5X,3G15.7/(5X,2G15.7))
10     WRITE(LP,110)
110    FORMAT(/5X,'*** THE INITIAL ZONE SCORES FOR ALL '
     * 'IZONEPC CHARTS ***'/5X,'*** ARE SET AT 0 ***')
       WRITE(LP,71)
       READ(IN,*) IANS
       GO TO (13,11) IANS
       WRITE(LP,57)
       GO TO 10
11     WRITE(LP,115)
115    FORMAT(/5X,'*** PLEASE ENTER THE DESIRED COMMON '
     *'ZONE SCORE ***'/5X,'*** NOTE : THE ZONE SCORE IS'
     *' AN INTEGER VALUE BETWEEN -3 TO 3 ***')
       READ(IN,*) ISCORE
       IF (ISCORE .LT. -3 .OR. ISCORE .GT. 3) THEN
          WRITE(LP,57)
          GO TO 11
       ENDIF
12     WRITE(LP,120) ISCORE
120    FORMAT(/5X,'*** THE DESIRED COMMON INITIAL ZONE'
     * ' SCORE IS ',I3)
       WRITE(LP,71)
       READ(IN,*) IANS
       GO TO (13,11) IANS
       WRITE(LP,57)
       GO TO 12
13     XMUIC=0.D0
       WRITE(LP,130)
130    FORMAT(/5X,'*** THE IN-CONTROL PROCESS MEANS FOR ALL'
     * ' VARIABLES ***'/5X,'*** ARE SET AT 0.D0 '
     * '(CENTRALIZED)***'/)
       IF (IDTMX .NE. 1) THEN
         WRITE(LP,201)
201      FORMAT(/5X,'*** THE MATRIX U OF THE EIGEN VECTORS'
     *   ' ARE : ***'/)
         DO 755 I=1,NP
           K=(I-1)*NP+1
           WRITE(LP,202) I,(V(J),J=K,K+NP-1)
202        FORMAT(5X,'*** u',I1,' :',5F10.5)
755      CONTINUE
       ENDIF
C-----------------------------------------------------
C SWITCH POSITION BETWEEN THE CHART WITH THE LARGEST
C SHIFT AND THE LAST
C-----------------------------------------------------
       ISWCH=0
       PCSFMAX=0.D0
       DO 535 I=1,NP
          IF(DABS(PCMEANSF(I)) .LT. 1.D-5) PCMEANSF(I)=0.D0
          IF (DABS(PCMEANSF(I)) .GT. PCSFMAX) THEN
             ISWCH=I
             PCSFMAX=DABS(PCMEANSF(I))
```

```
         ENDIF
535   CONTINUE
      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=PCMEANSF(NP)
         PCMEANSF(NP)=TEMP
      ENDIF
C
      SPREAD=4.D0
      CONST=1.D-6
      DXX=0.05
C-----------------------------------------------------------
C  INITIALIZE THE STEPIT ROUTINE
C-----------------------------------------------------------
      CALL STSET
C
      NV=NP-1
      MATRX=0
      NTRACE=-2
      DO 538 I=1,NP-1
        XK(I)=SPREAD
538     X(I)=XK(I)
      XK(NP)=SPREAD
      XUBEST=XK(NP)
C
      DO 539 I=1,NV
         DELMIN(I)=CONST
         DELTX(I)=DXX
         XMAX(I)=6.D0
539      XMIN(I)=2.D0
C
      FBEST=1.0D30
C-----------------------------------------------------------
C  NOW DO THE CONSTRAINED MINIMIZATION,
C  MOVING ALONG THE CONSTRAINT SURFACE.
C-----------------------------------------------------------
      CALL STEPIT (OPZFUNK)
C
      IF (ISWCH .LT. NP) THEN
         TEMP=PCMEANSF(NP)
         PCMEANSF(NP)=PCMEANSF(ISWCH)
         PCMEANSF(ISWCH)=TEMP
         TEMP=XK(NP)
         XK(NP)=XK(ISWCH)
         XK(ISWCH)=TEMP
      ENDIF
C
      WRITE(KW,135) NP,ARLIC
135   FORMAT(/5X,'*** THE OPTIMAL',I2,' VARIATE MZONEPC'
     * ' CHART ***'/5X,'*** WITH IN-CONTROL ARL OF',G11.3
     * /5X,'*** AT THE SHIFT OF THE ORIGINAL MEAN VECTOR'
     * ' OF : ***')
```

```
      IF (NP .LT. 4) THEN
         WRITE(LP,161) (OGMEANSF(I),I=1,NP)
161      FORMAT(5X,3G15.7)
         WRITE(LP,151)
151      FORMAT(5X,'*** IS LISTED AS FOLLOWS ***')
      ELSE
         WRITE(LP,146) (OGMEANSF(I),I=1,NP)
146      FORMAT(5X,3G15.7/5X,2G15.7)
         WRITE(LP,151)
      ENDIF
      WRITE(KW,144)
144   FORMAT(5X,50(1H-)/)
      DO 141 I=1,NP
         WRITE(KW,140)  I,XK(I)
140   FORMAT(5X,'*** H',I1,' = ',G15.7)
141   CONTINUE
      WRITE(KW,143) FOBJ
143   FORMAT(/5X,'*** THE OPTIMAL OOC ARL = ',1PG15.7)
      RETURN
C
C  END OPZONEPC
C
      END
C
      SUBROUTINE OPZFUNK
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL OPZFUNKU,UNICY,RLZONE,ARLPC
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
      COMMON /OPZ/ XK(5),PL(500,5),XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU,ISCORE
      COMMON /PCSF/ PCMEANSF(5),PU(500,5)
      COMMON /FLG/ IFLAG
      COMMON /NBVAR/ NP
C
      I=MOD(NF,15)
      IF (I .EQ. 0 .AND. NF .NE. 0) WRITE(KW,15) NF
15    FORMAT(/3X,'....... TOTAL OF ',I4,' FUNCTION'
     *  ' EVALUATIONS DONE .......')
      FUMAX=1.0D-6
      BIG=1.0D30
      DO 100 I=1,NP-1
100      XK(I)=X(I)
C
      NTRACU=-2
      XU=XUBEST
      XMAXU=6.D0
      XMINU=1.D0
```

```
      DELTXU=0.009D0
C
      DELMNU=0
      DELMNU=0.0001D0
C
      NFMAXU=200
      KWU=KW
C
      LGN=0
      DO 35 I=1,NP-1
35       CALL RLZONE(XK(I),ISCORE,XMUIC,PL(1,I))
      IFLAG=0
C
      CALL UNICY(OPZFUNKU,NTRACU,FOBJU,XU,XMAXU,XMINU,DELTXU,
     *   DELMNU,NFMAXU,KWU)
C----------------------------------------------------------
C  IN CASE NO ROOT WAS FOUND, USE A PENALTY.
C----------------------------------------------------------
      IF(FOBJU.GT.FUMAX) THEN
         FOBJ=BIG
         RETURN
      ENDIF
C
      XK(NP)=XU
C
      LGN=0
      DO 55 I=1,NP
         IF (PCMEANSF(I) .NE. XMUIC) THEN
            CALL RLZONE(XK(I),ISCORE,PCMEANSF(I),PL(1,I))
            NVAL=IDINT(PL(1,I))
            PU(NVAL+2,I)=PL(NVAL+3,I)
            IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
            DO 56 J=1,NVAL+1
56             PU(J,I)=PL(J,I)
            GO TO 55
         ENDIF
         IF (PL(1,I) .GT. LGN) LGN=IDINT(PL(1,I))
55    CONTINUE
      LGNP2=LGN+2
      DO 57 I=1,NP
         IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
            NP2=PL(1,I)+2
            XMNMN=PL(NP2,I)
            XMNPL=PU(NP2,I)
            DO 58 J=NP2,LGN+1
               PL(J,I)=PL(J-1,I)*XMNMN
58             PU(J,I)=PU(J-1,I)*XMNPL
            PL(LGNP2,I)=XMNMN
            PU(LGNP2,I)=XMNPL
          ENDIF
57    CONTINUE
      FOBJ=ARLPC(NP,LGN,PL,PU)
```

```
C
      IF(FOBJ.LT.FBEST) THEN
         FBEST=FOBJ
         XUBEST=XU
      ENDIF
      RETURN
C
C  END OPZFUNK
C
      END
C
C
      SUBROUTINE OPZFUNKU (XU,FOBJU)
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      EXTERNAL RLZONE,ARLPC
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
      COMMON /OPZ/ XK(5),PL(500,5),XMUIC,ARLIC,FBEST,
     * XUBEST,NTRACU,ISCORE
      COMMON /PCSF/ PCMEANSF(5),PU(500,5)
      COMMON /FLG/ IFLAG
      COMMON /NBVAR/ NP
C
      XK(NP)=XU
C
      CALL RLZONE(XK(NP),ISCORE,XMUIC,PL(1,NP))
      IF (IFLAG .NE. 0) THEN
         NVAL=IDINT(PL(1,NP))
         PU(NVAL+2,NP)=PL(NVAL+3,NP)
         DO 36 J=1,NVAL+1
36          PU(J,NP)=PL(J,NP)
         IF (PL(1,NP) .LT. PL(1,1)) THEN
            LGN=PL(1,1)
            LGNP2=LGN+2
            NP2=NVAL+2
            XMNMN=PL(NP2,NP)
            XMNPL=PU(NP2,NP)
            DO 55 J=NP2,LGN+1
               PL(J,NP)=PL(J-1,NP)*XMNMN
55             PU(J,NP)=PU(J-1,NP)*XMNPL
            PL(LGNP2,NP)=XMNMN
            PU(LGNP2,NP)=XMNPL
            PL(1,NP)=DFLOAT(LGN)
            PU(1,NP)=PL(1,NP)
         ELSEIF (PL(1,NP) .GT. PL(1,1)) THEN
               LGN=PL(1,NP)
               LGNP2=LGN+2
               DO 56 I=1,NP-1
```

```
                    NP2=PL(1,I)+2
                    XMNMN=PL(NP2,I)
                    XMNPL=PU(NP2,I)
                    DO 57 J=NP2,LGN+1
                       PL(J,I)=PL(J-1,I)*XMNMN
57                     PU(J,I)=PU(J-1,I)*XMNPL
                    PL(LGNP2,I)=XMNMN
                    PU(LGNP2,I)=XMNPL
                    PL(1,I)=DFLOAT(LGN)
                    PU(1,I)=PL(1,I)
56             CONTINUE
         ENDIF
         LGN=PL(1,NP)
         ARL=ARLPC(NP,LGN,PL,PU)
         GO TO 200
      ENDIF
          LGN=0
          DO 135 I=1,NP
             NVAL=IDINT(PL(1,I))
             PU(NVAL+2,I)=PL(NVAL+3,I)
             IF (NVAL .GT. LGN) LGN=IDINT(PL(1,I))
             DO 136 J=1,NVAL+1
136              PU(J,I)=PL(J,I)
135        CONTINUE
          LGNP2=LGN+2
          DO 140 I=1,NP
             IF (PL(1,I) .LT. DFLOAT(LGN)) THEN
                NP2=PL(1,I)+2
                XMNMN=PL(NP2,I)
                XMNPL=PU(NP2,I)
                DO 145 J=NP2,LGN+1
                   PL(J,I)=PL(J-1,I)*XMNMN
145                PU(J,I)=PU(J-1,I)*XMNPL
                PL(LGNP2,I)=XMNMN
                PU(LGNP2,I)=XMNPL
                PL(1,I)=DFLOAT(LGN)
                PU(1,I)=PL(1,I)
              ENDIF
140        CONTINUE
      IFLAG=1
      ARL=ARLPC(NP,LGN,PL,PU)
C
200   FOBJU=(ARL-ARLIC)**2
      RETURN
C
C  END OPZFUNKU
C
      END
C
      DOUBLE PRECISION FUNCTION ARLPC(NP,LGN,PBMN,PBPL)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PBMN(500,NP),PBPL(500,NP)
```

```
C
      SUMPMN=0.D0
      SUMPPL=0.D0
      LGNP1=LGN+1
      LGNP2=LGN+2
      DO 1 I=2,LGN
         PGTJMN=1.D0
         PGTJPL=1.D0
         DO 2 J=1,NP
            PGTJMN=PGTJMN*PBMN(I,J)
2           PGTJPL=PGTJPL*PBPL(I,J)
         SUMPMN=SUMPMN+PGTJMN
         SUMPPL=SUMPPL+PGTJPL
1     CONTINUE
      XMNMN=1.D0
      XMNPL=1.D0
      PLSTMN=1.D0
      PLSTPL=1.D0
      DO 3 I=1,NP
         PLSTMN=PLSTMN*PBMN(LGNP1,I)
         PLSTPL=PLSTPL*PBPL(LGNP1,I)
         XMNMN=XMNMN*PBMN(LGNP2,I)
3        XMNPL=XMNPL*PBPL(LGNP2,I)
      ARLMN=1.D0+SUMPMN+PLSTMN/(1.D0-XMNMN)
      ARLPL=1.D0+SUMPPL+PLSTMN/(1.D0-XMNPL)
      ARLPC=0.5D0*(ARLMN+ARLPL)
      RETURN
      END
C
      SUBROUTINE RLZONE(XK,ISCORE,XMU,UTAILP)
C
C  XK IS THE SYMETRIC CONTROL LIMIT
C  IZSCORE IS THE INITIAL SCORE OF THE ZONE CHART
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TEMP(7),UTAILP(500),TRANX(7,7)
      COMMON /PRVP/ PNM1(7)
      DATA JSTATE,DELTA/7,1.D-10/
C
C     INITIALIZE PARAMETERS
C
C
      DO 1 I=1,JSTATE
1        PNM1(I)=0.D0
      DO 5 I=1,500
5        UTAILP(I)=0.D0
C
C   CALCUALTES THE PROBABILITY OF EACH ZONE
C
      H=XK/3.D0
      P1=DUMNOR(H-XMU)-DUMNOR(0.D0-XMU)
      P3=DUMNOR(2.D0*H-XMU)-DUMNOR(H-XMU)
```

```
      P5=DUMNOR(XK-XMU)-DUMNOR(2.D0*H-XMU)
      P2=DUMNOR(0.D0-XMU)-DUMNOR(-H-XMU)
      P4=DUMNOR(-H-XMU)-DUMNOR(-2.D0*H-XMU)
      P6=DUMNOR(-2.D0*H-XMU)-DUMNOR(-XK-XMU)

C
C   SET THE TRANSITION METRIX
C
C
      DO 50 I=1,3
         TRANX(I,I)=P2
         DO 60 J=1,3
            IF (J-I) 70,60,90
90             TRANX(I,J)=0.D0
               GO TO 60
70             TRANX(I,J)=P4
60       CONTINUE
50    CONTINUE
      TRANX(3,1)=P6
      DO 150 I=5,7
         TRANX(I,I)=P1
         DO 160 J=5,7
            IF (J-I) 190,160,170
190             TRANX(I,J)=0.D0
               GO TO 160
170             TRANX(I,J)=P3
160       CONTINUE
150    CONTINUE
      TRANX(5,7)=P5
      DO 51 I=1,4
         TRANX(I,4)=P1
         TRANX(I,5)=P3
         TRANX(I,6)=P5
         TRANX(I,7)=0.D0
51    CONTINUE
      DO 52 I=4,7
         TRANX(I,1)=0.D0
         TRANX(I,2)=P6
         TRANX(I,3)=P4
         TRANX(I,4)=P2
52    CONTINUE
      TRANX(4,4)=P1+P2
C
C    CALCULATE P(N,S)
C
      ISTATE=ISCORE+4
      DO 3 I=1,JSTATE
         PNM1(I)=1.D0
3        TEMP(I)=1.D0
      DO 120 N=1,500
         NP=N+1
         UTAILP(NP)=0.D0
```

```
         DO 12 IDX=1,JSTATE
            PNM1(IDX) = TEMP(IDX)
12          UTAILP(NP)=UTAILP(NP)+PNM1(IDX)*TRANX(ISTATE,IDX)
         XMAXMN=0.D0
         XMINMN=1.D0
         DO 11 J=1,JSTATE
            TEMP(J)=0.D0
            DO 21 I=1,JSTATE
21             TEMP(J) = TEMP(J)+PNM1(I)*TRANX(J,I)
            XMAXMN=DMAX1(TEMP(J)/PNM1(J),XMAXMN)
11          XMINMN=DMIN1(TEMP(J)/PNM1(J),XMINMN)
         IF ((XMAXMN-XMINMN) .LE. DELTA) GO TO 30
120   CONTINUE
30       UTAILP(1)=DBLE(N)
         UTAILP(N+2)=XMINMN
         UTAILP(N+3)=XMAXMN
      RETURN
      END

      DOUBLE PRECISION FUNCTION DUMNOR(X)
C**********************************************************************
C
C     DOUBLE PRECISION FUNCTION DUMNOR(X)
C
C
C                               FUNCTION
C
C
C     COMPUTES THE CUMULATIVE OF THE NORMAL DISTRIBUTION,
C     I.E., THE INTEGRAL FROM -INFINITY TO X OF
C          (1/SQRT(2*PI)) EXP(-U*U/2) DU
C
C
C                               METHOD
C
C
C     THE RATIONAL FUNCTION APPROXIMATION FROM PAGES
C     90  - 92 OF KENNEDY  AND GENTLE,  STATISTICAL
C     COMPUTING,  MARCEL  DEKKER,  NY 1980.
C
C
C                               ARGUMENTS
C
C
C     X --> ARGUMENT AT WHICH CUMULATIVE NORMAL IS EVALUATED
C                    DOUBLE PRECISION X
C
C**********************************************************************
C
C
C     PIM12 IS PI**(-1/2)
C     SQRT2 IS SQRT(2)
```

```
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      LOGICAL QDIRCT
C     ..
C     .. LOCAL ARRAYS ..
      DIMENSION XDEN1(4),XDEN2(8),XDEN3(5),XNUM1(4),XNUM2(8),
     +                 XNUM3(5)
C     .. DATA STATEMENTS ..
      DATA XNUM1/2.4266795523053175D2,2.1979261618294152D1,
     +     6.9963834886191355D0,-3.5609843701815385D-2/
      DATA XDEN1/2.1505887586986120D2,9.1164905404514901D1,
     +     1.5082797630407787D1,1.0000000000000000D0/
      DATA XNUM2/3.004592610201616005D2,4.519189537118729422D2,
     +     3.393208167343436870D2,1.529892850469404039D2,
     +     4.316222722205673530D1,7.211758250883093659D0,
     +     5.641955174789739711D-1,-1.368648573827167067D-7/
      DATA XDEN2/3.004592609569832933D2,7.909509253278980272D2,
     +     9.313540948506096211D2,6.389802644656311665D2,
     +     2.775854447439876434D2,7.700015293522947295D1,
     +     1.278272731962942351D1,1.000000000000000000D0/
      DATA XNUM3/-2.99610707703542174D-3,-4.94730910623250734D-2
     +     ,-2.26956593539686930D-1,-2.78661308609647788D-1,
     +     -2.23192459734184686D-2/
      DATA XDEN3/1.06209230528467918D-2,1.91308926107829841D-1,
     +     1.05167510706793207D0,1.98733201817135256D0,
     +     1.00000000000000000D0/
      DATA PIM12/0.564189583547756286948079 5D0/
      DATA SQRT2/1.414213562373095048 8D0/
C     ..
C     .. EXECUTABLE STATEMENTS ..
      IF (.NOT. (DABS(X).LT.1.0D-30)) GO TO 10
      DUMNOR = 0.5D0
      RETURN

      GO TO 50

   10 IF (.NOT. (X.LE.-3.8D1)) GO TO 20
      DUMNOR = 0.D0
      RETURN

      GO TO 50

   20 IF (.NOT. (X.LE.-1.5D1)) GO TO 30
      DUMNOR = DEXP(DLANOR(X))
      RETURN

      GO TO 50

   30 IF (.NOT. (X.GT.6.D0)) GO TO 40
      DUMNOR = 1.D0
```

```
      RETURN

      GO TO 50

   40 CONTINUE
   50 Z = DABS(X/SQRT2)
      Z2 = Z*Z
      ZM2 = 1.0D0/Z2
      IF (Z.LT.0.5D0) THEN
          DERF = Z*DEVLPL(XNUM1,4,Z2)/DEVLPL(XDEN1,4,Z2)
          QDIRCT = .TRUE.

      ELSE IF (Z.LT.4.0D0) THEN
          DERFC = DEXP(-Z2)*DEVLPL(XNUM2,8,Z)/DEVLPL(XDEN2,8,Z)
          QDIRCT = .FALSE.

      ELSE
          DERFC = (DEXP(-Z2)/Z)* (PIM12+ZM2*DEVLPL(XNUM3,5,ZM2)/
     +                  DEVLPL(XDEN3,5,ZM2))
          QDIRCT = .FALSE.
      END IF

      IF (.NOT. (X.GE.0.D0)) GO TO 60
      IF (.NOT. (QDIRCT)) DERF = 1.0D0 - DERFC
      DUMNOR = (1.0D0+DERF)/2.0D0
      GO TO 70

   60 IF (QDIRCT) DERFC = 1.0D0 - DERF
      DUMNOR = DERFC/2.0D0
   70 RETURN

      END


      DOUBLE PRECISION FUNCTION DEVLPL(A,N,X)
C
C**********************************************************************
C
C     DOUBLE PRECISION FUNCTION DEVLPL(A,N,X)
C              DOUBLE PRECISION EVALUATE A POLYNOMIAL AT X
C
C
C                               FUNCTION
C
C
C     RETURNS
C          A(1) + A(2)*X + ... + A(N)*X**(N-1)
C
C
C                               ARGUMENTS
C
C
```

```
C     A --> ARRAY OF COEFFICIENTS OF THE POLYNOMIAL.
C                                   A IS DOUBLE PRECISION(N)
C
C     N --> LENGTH OF A, ALSO DEGREE OF POLYNOMIAL - 1.
C                                   N IS INTEGER
C
C     X --> POINT AT wHICH THE POLYNOMIAL IS TO BE EVALUATED.
C                                   X IS DOUBLE PRECISION
C
C**************************************************************
C

      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(N)
C     ..
      TERM = A(N)
      DO 10,I = N - 1,1,-1
          TERM = A(I) + TERM*X
   10 CONTINUE
      DEVLPL = TERM
      RETURN

      END


      DOUBLE PRECISION FUNCTION DLANOR(X)
C**************************************************************
C
C      DOUBLE PRECISION FUNCTION DLANOR( X )
C           DOUBLE PRECISION LOGARITH OF THE ASYMPTOTIC NORMAL
C
C
C                               FUNCTION
C
C
C      COMPUTES THE LOGARITHM OF THE CUMULATIVE NORMAL
C      DISTRIBUTION FROM ABS( X ) TO INFINITY FOR ABS( X ) >= 5.
C
C
C                               ARGUMENTS
C
C
C      X --> VALUE AT wHICH CUMULATIVE NORMAL TO BE EVALUATED
C                     DOUBLE PRECISION X
C
C
C                               METHOD
C
C
C      23 TERM EXPANSION OF FORMULA 26.2.12 OF ABRAMOwITZ AND
C      STEGUN. THE RELATIVE ERROR AT X = 5 IS ABOUT 0.5E-5.
C
```

```
C
C                                 NOTE
C
C
C      ABS(X) MUST BE >= 5 ELSE THERE IS AN ERROR STOP.
C
C**********************************************************************
C     .. PARAMETERS ..
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (DLSQPI=0.91893853320467274177D0)
C     .. LOCAL ARRAYS ..
      DIMENSION COEF(12)
C     ..
C     .. DATA STATEMENTS ..
      DATA COEF/-1.0D0,3.0D0,-15.0D0,105.0D0,-945.0D0,10395.0D0,
     +     -135135.0D0,2027025.0D0,-34459425.0D0,654729075.0D0,
     +     -13749310575D0,316234143225.0D0/
C     ..
C     .. EXECUTABLE STATEMENTS ..

      XX = ABS(X)
      IF (XX.LT.5.0D0) STOP ' ARGUMENT TOO SMALL IN DLANOR'

      APPROX = -DLSQPI - 0.5*XX*XX - DLOG(XX)

      XX2 = XX*XX
      CORREC = DEVLPL(COEF,12,1.0D0/XX2)/XX2
      CORREC = DLN1PX(CORREC)

      DLANOR = APPROX + CORREC

      RETURN

      END


      DOUBLE PRECISION FUNCTION DLN1PX(A)
C**********************************************************************
C
C     DOUBLE PRECISION FUNCTION DLN1PX(X)
C               DOUBLE PRECISION LN(1+X)
C
C
C                                 FUNCTION
C
C
C     RETURNS LN(1+X)
C     NOTE THAT THE OBVIOUS CODE OF
C               LOG(1.0+X)
C     WON'T WORK FOR SMALL X BECAUSE 1.0+X LOSES ACCURACY
C
C
```

```
C                              ARGUMENTS
C
C
C     X --> VALUE FOR wHICH LN(1-X) IS DESIRED.
C                                      X IS DOUBLE PRECISION
C
C
C                              METHOD
C
C
C     RENAMES ALNREL FROM:
C     DIDINATO, A. R. AND MORRIS,  A.   H.  ALGORITHM 708:
C     SIGNIFICANT DIGIT COMPUTATION OF THE INCOMPLETE  BETA
C     FUNCTION RATIOS.  ACM TRANS. MATH.  SOFTw. 18 (1993),
C     360-373.
C**********************************************************************
C-----------------------------------------------------------------------
C              EVALUATION OF THE FUNCTION LN(1 + A)
C-----------------------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
C     .. DATA STATEMENTS ..
      DATA P1/-.129418923021993D+01/,P2/.405303492862024D+00/,
     +     P3/-.178874546012214D-01/
      DATA Q1/-.162752256355323D+01/,Q2/.747811014037616D+00/,
     +     Q3/-.845104217945565D-01/
C     ..
C     .. EXECUTABLE STATEMENTS ..
C-------------------------
      IF (DABS(A).GT.0.375D0) GO TO 10
      T = A/ (A+2.0D0)
      T2 = T*T
      w = (((P3*T2+P2)*T2+P1)*T2+1.0D0)/ (((Q3*T2+Q2)*T2+Q1)*T2+
     * 1.0D0)
      DLN1PX = 2.0D0*T*w
      RETURN
C
   10 X = 1.D0 + DBLE(A)
      DLN1PX = DLOG(X)
      RETURN
C
      END
```

APPENDIX I

FORTRAN PROGRAMS FOR OPTIMIZATION (STEPIT AND UNICY) AND EIGENVALUE AND EIGENVECTOR EVALUATION

```
      SUBROUTINE UNICY(FUNK,NTRAC,FOBJ,X,XMAX,XMIN,DELTX,
     *   DELMN,NFMAX,KW)
C
C  UNICY 5.0        MARCH 1990
C  A.N.S.I. 1966 STANDARD FORTRAN
C
C  COPYRIGHT (C) 1990 BY
C  J. P. CHANDLER, COMPUTER SCIENCE DEPARTMENT,
C  OKLAHOMA STATE UNIVERSITY, STILLWATER, OKLAHOMA 74078
C
C  FINDS A LOCAL MINIMUM OF A FUNCTION OF ONE VARIABLE.
C
C  FOR USAGE, SEE THE STEPIT WRITEUP.
C  FUNK IS NOW A SUBROUTINE WITH TWO ARGUMENTS,
C       SUBROUTINE FUNK (X,FOBJ)    .
C
      DOUBLE PRECISION      FOBJ,X,XMAX,XMIN,DELTX,DELMN,
     *   HUGE,RELAC,ACK,STCUT,DFRAC,RZERO,RTWO,XX,SX,SN,DX,DN,
     *   UNITR,RUNIT,FBEST,DIST,XSAVE,FOB,FPREV,DENOM,TRIAL,
     *   DEL,CINDR,ZABS,ARG,      DABS
C
      INTEGER NTRAC,NFMAX,KW,
     *   NFLAT,NFX,NF,JFLAT,NFLAG
C
      ZABS(ARG)=DABS(ARG)
C
      HUGE=1.0D35
      NFLAT=1
      ACK=2.0D0
      STCUT=10.0D0
      DFRAC=0.01D0
      RZERO=0.0D0
      RUNIT=1.0D0
      UNITR=RUNIT
      RTWO=2.0D0
C
C  MOVE INPUT QUANTITIES AND SET DEFAULTS.
C
      XX=X
      SX=XMAX
      SN=XMIN
      DX=DELTX
      DN=DELMN
      NFX=NFMAX
      IF(SX.GT.SN) GO TO 10
      SX=HUGE
      SN=-HUGE
   10 IF(DX.NE.RZERO) GO TO 20
      DX=DFRAC*XX
      IF(DX.EQ.RZERO) DX=DFRAC
   20 DN=ZABS(DN)
      IF(DN.GT.RZERO) GO TO 40
```

```
C
C  COMPUTE RELAC, THE RELATIVE PRECISION OF THE MACHINE AND
C  ARITHMETIC BEING USED.
C
      RELAC=RUNIT
   30 RELAC=RELAC/RTWO
      XSAVE=RUNIT+RELAC
      IF(XSAVE.GT.UNITR) GO TO 30
      DN=DX*RELAC
C
   40 IF(XX.GT.SX) XX=SX
      IF(XX.LT.SN) XX=SN
C
      CALL FUNK (XX,FBEST)
      NF=1
      IF(NTRAC.GE.0) WRITE(KW,50)XX,SX,SN,DX,DN,FBEST
   50 FORMAT(/' UNICY.   X =',1PG16.8,5X,'XMAX =',G16.8,5X,
     *    'XMIN =',G16.8/12X,'DELTX =',G16.8,5X,'DELMN =',
     *    G16.6,15X,'FOBJ =',G16.6/' ')
C
C  THE FIRST TRIAL STEP IS SEPARATE.
C
   60 JFLAT=0
      DIST=RZERO
      XSAVE=XX
      XX=XX+DX
      IF(XX.EQ.XSAVE) GO TO 170
      NFLAG=1
      IF(XX.GE.SN .AND. XX.LE.SX) GO TO 70
      NFLAG=NFLAG+3
      FOB=FBEST
      GO TO 80
   70 CALL FUNK (XX,FOB)
      NF=NF+1
      IF(FOB.LT.FBEST) GO TO 120
      IF(FOB.EQ.FBEST) NFLAG=NFLAG+1
   80 XX=XSAVE-DX
      IF(XX.EQ.XSAVE) GO TO 170
      FPREV=FOB
      IF(XX.LT.SN .OR. XX.GT.SX) GO TO 100
      CALL FUNK (XX,FOB)
      NF=NF+1
      IF(FOB.LT.FBEST) GO TO 110
      IF(FOB.EQ.FBEST) NFLAG=NFLAG+1
      IF(NFLAG.GT.3) GO TO 100
      IF(NFLAG.EQ.3) GO TO 90
      IF(FOB.EQ.FPREV) GO TO 100
      DENOM=(FPREV-FBEST)-(FBEST-FOB)
      IF(DENOM.LE.RZERO) GO TO 100
      TRIAL=DX*(FOB-FPREV)/(DENOM+DENOM)
      XX=XSAVE+TRIAL
      CALL FUNK (XX,FOB)
```

```
      NF=NF+1
      IF(FOB.GE.FBEST) GO TO 100
      FBEST=FOB
      DIST=TRIAL/DX
      GO TO 150
   90 JFLAT=1
  100 XX=XSAVE
      GO TO 150
  110 DX=-DX
C
C  A LOWER VALUE OF FOBJ HAS BEEN FOUND.
C  HENCE X WILL CHANGE.
C
  120 DEL=DX
  130 FPREV=FBEST
      IF(NF.GT.NFX) GO TO 220
      FBEST=FOB
      DIST=DIST+DEL/DX
      DEL=ACK*DEL
      XSAVE=XX
      XX=XSAVE+DEL
      IF(XX.LT.SN .OR. XX.GT.SX) GO TO 140
      CALL FUNK (XX,FOB)
      NF=NF+1
      IF(FOB.LT.FBEST) GO TO 130
C
C  PERFORM PARABOLIC INTERPOLATION IN ORDER TO REFINE
C  THE POSITION OF THE MINIMUM.
C
      DENOM=ACK*(FPREV-FBEST)-(FBEST-FOB)
      IF(DENOM.LE.RZERO) GO TO 140
      CINDR=((FPREV-FBEST)*ACK+(FBEST-FOB)/ACK)/(DENOM+DENOM)
      XX=XSAVE+CINDR*DEL
      CALL FUNK (XX,FOB)
      NF=NF+1
      IF(FOB.GE.FBEST) GO TO 140
      FBEST=FOB
      DIST=DIST+CINDR*DEL/DX
      GO TO 150
  140 XX=XSAVE
C
C  DIST=SIGN(DIST,DX)
C
  150 IF((DX.GT.RZERO .AND. DIST.LT.RZERO) .OR.
     *   (DX.LT.RZERO .AND. DIST.GT.RZERO)) DIST=-DIST
      IF(NTRAC.GE.1) WRITE(KW,160)DIST,XX,FBEST
  160 FORMAT(' DIST =',1PG10.2,5X,'X =',G16.8,5X,
     *   'FOBJ =',G16.8)
C
      IF(ZABS(DIST).GT.RTWO) GO TO 60
C
C  CHECK FOR CONVERGENCE.
```

```
C
      IF(ZABS(DX).GT.DN) GO TO 190
  170 IF(NTRAC.GE.0) WRITE(KW,180)
  180 FORMAT(/' CONVERGED WITH STEP SIZE AS SMALL AS DELMN.' )
      GO TO 260
  190 IF(NFLAT.EQ.0 .OR. JFLAT.EQ.0) GO TO 210
      IF(NTRAC.GE.0) WRITE(KW,200)
  200 FORMAT(/' CONVERGED WITH ALL TRIAL VALUES',
     *    ' OF FOBJ EXACTLY EQUAL.'  )
      GO TO 260
C
C  CONVERGENCE HAS NOT YET BEEN ACHIEVED.
C  CUT THE STEP SIZE.
C
  210 IF(NF.LE.NFX) GO TO 240
  220 WRITE(KW,230)NFX
  230 FORMAT(/' NFMAX =',I8,' EXCEEDED IN UNICY.')
      GO TO 260
  240 DX=DX/STCUT
      IF(NTRAC.GE.1) WRITE(KW,250)DX
  250 FORMAT(' STEP SIZE DECREASED TO',1PG12.5/' ')
      GO TO 60
C
C  EITHER CONVERGENCE HAS BEEN ACHIEVED OR ELSE
C  NF EXCEEDED NFMAX.  RETURN.
C
  260 X=XX
      CALL FUNK (XX,FOB)
      NF=NF+1
      IF(NTRAC.GE.0) WRITE(KW,270)XX,FOB,NF
  270 FORMAT(/' FINAL VALUE OF X =',1PG16.8,10X,
     *    ' FINAL VALUE OF FOBJ =',G16.8,10X,
     *    I5,' FUNCTION CALLS'/' ')
      FOBJ=FOB
      RETURN
C
C  END UNICY
C
      END
C
      SUBROUTINE STEPIT (FUNK)
C
C  STEPIT 7.7           DECEMBER 1991
C
C  A.N.S.I. STANDARD FORTRAN 77
C
C  COPYRIGHT (C) 1965, 1975, 1991 J. P. CHANDLER
C     (PRESENT ADDRESS ....  COMPUTER SCIENCE DEPARTMENT,
C        OKLAHOMA STATE UNIVERSITY,
C        STILLWATER, OKLAHOMA 74078
C        (405)-744-5676 )
C
```

```
C  STEPIT FINDS LOCAL MINIMA OF A SMOOTH FUNCTION OF SEVERAL
C  PARAMETERS.
C
C  "STEPIT IS A PHLEGMATIC METHOD OF SOLVING A PROBLEM."
C       --  J. H. BURRILL, JR.,  360 STEPIT - A USER'S MANUAL
C
C  STEPIT 7.4 AND A WRITE-UP ARE AVAILABLE FROM THE
C     QUANTUM CHEMISTRY PROGRAM EXCHANGE
C     DEPT. OF CHEMISTRY, INDIANA UNIVERSITY
C     BLOOMINGTON, INDIANA 47401
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INPUT QUANTITIES.....  FUNK,NV,NTRACE,MATRX,MASK,X,XMAX,
C                            XMIN,DELTX,DELMIN,NFMAX,NFLAT,KW
C
C  OUTPUT QUANTITIES....  X,FOBJ,ERR,       KFLAG,NOREP,KERFL
C
C     FUNK        --  THE NAME OF THE SUBROUTINE THAT COMPUTES
C                        FOBJ GIVEN X(1),X(2),...,X(NV) (EACH
C                        SUCH SUBROUTINE MUST BE NAMED IN AN
C                        EXTERNAL STATEMENT IN THE CALLING
C                        PROGRAM)
C
C     NV          --  THE NUMBER OF PARAMETERS, X
C
C     NTRACE      --  = 0 FOR NORMAL OUTPUT,
C                     =+1 FOR TRACE OUTPUT,
C                     =-1 FOR NO OUTPUT
C                     =-2 FOR NO ERROR MESSAGE FOR ABNORMAL
C                          END OF PROGRAM
C
C     MATRX       --  = 0 FOR NO ERROR CALCULATION,
C                     = 100+M TO APPROXIMATE THE ERRORS IN THE
C                        X(J) USING STEPS 10**(-M) TIMES AS
C                        LARGE AS X(J), IF NONZERO
C
C     FOBJ        --  THE VALUE OF THE FUNCTION TO BE MINIMIZED
C
C     MASK(J)     --  NONZERO IF X(J) IS TO BE HELD FIXED
C
C     X(J)        --  THE J-TH PARAMETER
C
C     XMAX(J)     --  THE UPPER LIMIT ON X(J)
C
C     XMIN(J)     --  THE LOWER LIMIT ON X(J)
C
C     DELTX(J)    --  THE INITIAL STEP SIZE FOR X(J)
C
C     DELMIN(J)   --  THE LOWER LIMIT (CONVERGENCE TOLERANCE)
C                        ON THE STEP SIZE FOR X(J)
C
```

```
C     ERR(J,K)    --  RETURNS THE ERROR MATRIX IF -MATRX-
C                        IS NONZERO
C                        (ERR IS ALSO USED FOR SCRATCH STORAGE)
C
C     NFMAX       --  THE MAXIMUM NUMBER OF FUNCTION
C                        EVALUATIONS
C
C     NFLAT       --  NONZERO IF THE SEARCH IS TO TERMINATE WHEN
C                        ALL TRIAL STEPS GIVE IDENTICAL FUNCTION
C                        VALUES.  THE RECOMMENDED VALUE OF NFLAT
C                        IS USUALLY NFLAT=1 .
C
C     JVARY       --  STEPIT SETS JVARY NONZERO IF X(JVARY) IS
C                        THE ONLY X(J) THAT HAS CHANGED SINCE
C                        THE LAST CALL TO FUNK
C                        (THIS CAN BE USED TO SPEED UP FUNK)
C
C     NXTRA       --  USED BY SUBROUTINE SIMPLEX BUT NOT BY
C                        STEPIT
C
C     KFLAG       --  RETURNED .GT. ZERO FOR A NORMAL EXIT,
C                        RETURNED .LT. ZERO FOR AN ABNORMAL EXIT
C
C     NOREP       --  RETURNED .GT. ZERO IF THE FUNCTION WAS NOT
C                        REPRODUCIBLE
C
C     KERFL       --  RETURNED .LT. ZERO IF SUBROUTINE STERR
C                        TERMINATED ABNORMALLY
C
C     KW          --  THE LOGICAL UNIT NUMBER OF THE PRINTER
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  THE FOLLOWING EXTERNAL STATEMENT IS REQUIRED BY SOME
C  COMPILERS (WATFIV, FOR EXAMPLE) AND IT IS FORBIDDEN BY SOME
C  OTHERS (MODCOMP II, FOR EXAMPLE).
C
      EXTERNAL FUNK
C
C  STEPIT SHOULD USUALLY BE RUN USING A FLOATING POINT
C  PRECISION OF AT LEAST TEN SIGNIFICANT DIGITS.  ON MOST
C  COMPUTERS, THIS REQUIRES THE USE OF DOUBLE PRECISION.
C
      DOUBLE PRECISION X,XMAX,XMIN,DELTX,DELMIN,ERR,FOBJ,
     *   VEC,DLX,XS,FSTORE,DX,SALVO,XOSC,FOSC,ARG,STCUT,ACK,
     *   FACUP
      DOUBLE PRECISION RZERO,XPLUS,
     *   FSAVE,FBEST,XSAVE,ABSDX,FPREV,DENOM,DEL,DXZ,DXU,DFZ,
     *   DFU,ABSVEC,SUMV,CINDER,COXCOM,COSIN,STEPS,ZSQRT,
     *   DSQRT
C
      INTEGER J,JFLAT,JFLMIN,JOCK,JUMP,JVARY,JX,K,KERFL,KFLAG,
```

```
     *    KL,KW,MASK,MATRX,MINOSC,MAXOSC,MAXSTP,NACK,NACTIV,
     *    NAH,NCIRC,NEQUAL,NF,NFLAT,NFMAX,NFSAV,NGATE,NGIANT,
     *    NONZER,NOREP,NOSC,NOUT,NRETRY,NSSW,NSTEPS,NT,NTRACE,
     *    NV,NXTRA,NZIP
C
C  THE DIMENSIONS OF ALL VECTORS AND MATRICES
C  (AS OPPOSED TO ARRAYS) ARE NV, EXCEPT FOR ....
C     ERR(NV,MAXOSC),XOSC(NV,MAXOSC),FOSC(MAXOSC).
C  IF ERRORS ARE TO BE CALCULATED BY SUBROUTINE STERR, HOWEVER,
C  THEN ERR MUST BE DIMENSIONED AT LEAST AS LARGE AS
C     ERR(NV,MAX(NV,MAXOSC)) .
C
      DIMENSION VEC(20),FSTORE(20),SALVO(20),JFLAT(20)
      DIMENSION XOSC(20,5),FOSC(5)
C
C  USER COMMON.....
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *    DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *    NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
C  INTERNAL STEPIT COMMON.....
      COMMON /STORK/ DX(20),XS(20),DLX(20),NACTIV,NSSW
C
C  THE ONLY SUBROUTINES CALLED ARE FUNK, STBEG, STERR, AND
C  DATSW.
C
C  STEPIT TERMINATES IF SENSE SWITCH NUMBER NSSW IS ON.
C  THE STATEMENT    CALL DATSW(NSSW,JUMP)    RETURNS JUMP=1 IF
C  SENSE SWITCH NUMBER NSSW IS ON, AND JUMP=2 IF IT IS OFF.
C  IF NO SENSE SWITCH IS TO BE USED, SUPPLY A DUMMY SUBROUTINE
C  FOR DATSW.
C
C  SET THE LIBRARY FUNCTION FOR SINGLE PRECISION (SQRT) OR FOR
C  DOUBLE PRECISION (DSQRT).  NO OTHER FUNCTIONS ARE USED,
C  EITHER EXTERNAL OR INTRINSIC, EXCEPT THE ROUTINE INVOKED BY
C  REAL**INTEGER .
C
C     ZSQRT(ARG)=SQRT(ARG)
      ZSQRT(ARG)=DSQRT(ARG)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  CALL STBEG TO SET DEFAULT VALUES AND PRINT INITIAL OUTPUT.
C
      CALL STBEG (FUNK)
C
C  FSAVE IS USED TO CHECK THE REPRODUCIBILITY OF FOBJ.
C
      FSAVE=FOBJ
C
C  SET FIXED QUANTITIES.
C
```

```
C  MAXSTP = LOG2(MAXIMUM NUMBER OF STEPS)
C
      MAXSTP=3
C
C  FACUP ...  IF MORE THAN FACUP STEPS ARE TAKEN, THE STEP SIZE
C             IS INCREASED
C
      FACUP=4.0D0
C
C  ACK = RATIO OF STEP SIZE INCREASE
C
      ACK=2.0D0
C
C  STCUT = RATIO OF STEP SIZE DECREASE
C
      STCUT=10.0D0
C
C  MAXOSC = MAXIMUM DEPTH OF SEARCH FOR ZIGZAGGING
C
      MAXOSC=5
C
C  MINOSC = MINIMUM PERIOD OF ZIGZAGGING SEARCH
C
      MINOSC=2
C
      RZERO=0.0D0
C
C  NO REAL OR DOUBLE PRECISION CONSTANTS ARE USED BEYOND THIS
C  POINT IN THIS SUBROUTINE.
C
      KERFL=0
C
C  JOCK IS A FLAG USED IN SETTING JVARY.
C
      JOCK=1
C
C  JUMP IS A FLAG SET BY SUBROUTINE DATSW.
C
      JUMP=2
C
C  NOSC = CURRENT DEPTH OF ZIGZAGGING INFORMATION
C
      NOSC=0
C
C  FBEST = BEST VALUE OF FOBJ FOUND SO FAR
C
      FBEST=FOBJ
C
C  DX(J) = CURRENT STEP SIZE FOR X(J)
C
      DO 10 J=1,NV
         DX(J)=DELTX(J)
```

```
   10    CONTINUE
C
      IF(KFLAG.LT.0) GO TO 760
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  VARY THE PARAMETERS, ONE AT A TIME.
C  THIS IS THE STARTING POINT USED EACH TIME THE STEP SIZE IS
C  REDUCED OR A SUCCESSFUL GIANT STEP IS COMPLETED.
C
C  NCIRC = NUMBER OF CONSECUTIVE X(JX) WITHOUT SIZABLE
C          CHANGES
C
   20 NCIRC=0
C
C  NZIP = NUMBER OF CONSECUTIVE CYCLES WITHOUT A GIANT STEP
C
      NZIP=0
C
C  MAIN DO LOOP FOR CYCLING THROUGH THE VARIABLES....
C  THE FIRST TRIAL STEP WITH EACH VARIABLE IS SEPARATE.
C
C  NACK = NUMBER OF ACTIVE X(JX) CYCLED THROUGH
C
   30 NACK=0
C
      DO 540 JX=1,NV
C
C  NOUT = NUMBER OF TRIAL POINTS OUT OF BOUNDS
C         (USED IN SETTING JFLAT(JX))
C
         NOUT=0
C
C  NEQUAL = NUMBER OF TRIAL POINTS WITH FOBJ .EQ. FBEST
C         (USED IN SETTING JFLAG(JX))
C
         NEQUAL=0
C
C  JFLAT(JX) WILL BE NONZERO IF CHANGING X(JX) DID NOT CHANGE
C  FOBJ.
C
         JFLAT(JX)=0
C
C  VEC(J) = CURRENT VECTOR OF NUMBER OF STEPS IN X(J)
C
         VEC(JX)=RZERO
C
C  DLX(JX) = CHANGE IN X(JX)
C
         DLX(JX)=RZERO
C
         IF(MASK(JX).EQ.0) GO TO 40
```

```
        JFLAT(JX)=1
        GO TO 530
C
   40   NACK=NACK+1
        ABSDX=DX(JX)
        IF(ABSDX.LT.RZERO) ABSDX=-ABSDX
C
C  CHECK THAT DX(JX) IS NOT NEGLIGIBLE.
C
        XSAVE=X(JX)
        XPLUS=XSAVE+DX(JX)
        IF(XPLUS.EQ.XSAVE) GO TO 50
        XPLUS=XSAVE-DX(JX)
        IF(XPLUS.NE.XSAVE) GO TO 60
C
C  DX(JX) IS NEGLIGIBLE COMPARED TO X(JX) , SO THERE IS NO
C  REASON TO STEP X(JX) .
C
   50   JFLAT(JX)=2
        GO TO 140
C
C  STEP X(JX).
C
   60   X(JX)=XSAVE+DX(JX)
        JVARY=0
        IF(JOCK.LE.0) GO TO 70
        JOCK=0
        JVARY=JX
C
   70   IF(X(JX).GE.XMIN(JX) .AND. X(JX).LE.XMAX(JX)) GO TO 80
        NOUT=1
        GO TO 90
C
   80   CALL FUNK
        NF=NF+1
        JVARY=JX
        FPREV=FOBJ
        IF(FOBJ.LT.FBEST) GO TO 170
        IF(FOBJ.EQ.FBEST) NEQUAL=1
C
C  STEP X(JX) THE OTHER WAY.
C
   90   XPLUS=X(JX)
        X(JX)=XSAVE-DX(JX)
        IF(X(JX).GE.XMIN(JX) .AND. X(JX).LE.XMAX(JX))
     *    GO TO 100
        NOUT=NOUT+1
        GO TO 110
C
  100   CALL FUNK
        NF=NF+1
        JVARY=JX
```

```
      IF(FOBJ.LT.FBEST) GO TO 160
      IF(FOBJ.EQ.FBEST) NEQUAL=NEQUAL+1
C
110   IF(NEQUAL.EQ.2 .OR. (NOUT.EQ.1 .AND. NEQUAL.EQ.1 .AND.
     *   (XSAVE.EQ.XMIN(JX) .OR. XSAVE.EQ.XMAX(JX))))
     *   GO TO 130
      IF(NOUT.GT.0) GO TO 140
C
C PERFORM QUADRATIC INTERPOLATION.
C
      DENOM=(FOBJ-FBEST)-(FBEST-FPREV)
      IF(DENOM.LE.RZERO) GO TO 140
      DLX(JX)=-DX(JX)*(FOBJ-FPREV)/(DENOM+DENOM)
      VEC(JX)=DLX(JX)/ABSDX
      X(JX)=XSAVE+DLX(JX)
      IF(X(JX).EQ.XSAVE) GO TO 120
      CALL FUNK
      NF=NF+1
      IF(FOBJ.GE.FBEST) GO TO 120
      FBEST=FOBJ
      JOCK=1
      GO TO 150
C
120   DLX(JX)=RZERO
      VEC(JX)=RZERO
      GO TO 140
C
C BOTH TRIAL POINTS HAD FOBJ .EQ. FBEST , OR ELSE
C ONE TRIAL POINT DID AND THE BASE POINT WAS ON A CONSTRAINT.
C
130   JFLAT(JX)=1
C
C WE WERE UNABLE TO IMPROVE FOBJ BY VARYING THIS X(JX) .
C RETREAT TO THE BASE POINT.
C
140   X(JX)=XSAVE
C
150   NCIRC=NCIRC+1
      IF(NCIRC.GE.NACTIV) GO TO 570
      GO TO 210
C
C FLIP DX(JX) FOR MORE EFFICIENT OPERATION NEXT TIME.
C
160   DX(JX)=-DX(JX)
C
C A LOWER VALUE OF FOBJ HAS BEEN FOUND.
C TAKE A STEP, INCREASE THE STEP SIZE, AND REPEAT AS LONG AS
C FOBJ DECREASES, UP TO MAXSTP TIMES.
C
170   NCIRC=0
      NSTEPS=0
      DEL=DX(JX)
```

```
C
  180    FPREV=FBEST
         FBEST=FOBJ
         VEC(JX)=VEC(JX)+DEL/ABSDX
         DLX(JX)=DLX(JX)+DEL
         NSTEPS=NSTEPS+1
         IF(NSTEPS.GE.MAXSTP) GO TO 190
         DEL=ACK*DEL
         XPLUS=XSAVE
         XSAVE=X(JX)
         X(JX)=XSAVE+DEL
         IF(X(JX).LT.XMIN(JX) .OR. X(JX).GT.XMAX(JX)) GO TO 200
         CALL FUNK
         NF=NF+1
         IF(FOBJ.LT.FBEST) GO TO 180
C
C  PERFORM QUADRATIC INTERPOLATION.
C
         DXZ=XSAVE-XPLUS
         DXU=X(JX)-XSAVE
         DFZ=FBEST-FPREV
         DFU=FOBJ-FBEST
         DENOM=DFZ*DXU-DFU*DXZ
         IF(DENOM.EQ.RZERO) GO TO 200
         DEL=(DFZ*DXU**2+DFU*DXZ**2)/(DENOM+DENOM)
         X(JX)=XSAVE+DEL
         IF(X(JX).EQ.XSAVE) GO TO 210
         CALL FUNK
         NF=NF+1
         IF(FOBJ.GE.FBEST) GO TO 200
         FBEST=FOBJ
         DLX(JX)=DLX(JX)+DEL
         VEC(JX)=VEC(JX)+DEL/ABSDX
C
  190    JOCK=1
         GO TO 210
C
C  RETREAT TO THE BEST KNOWN POINT.
C
  200    X(JX)=XSAVE
C
C  CHECK WHETHER THE STEP SIZE SHOULD BE INCREASED.
C
  210    IF((NZIP.LE.0 .AND. NACK.LE.1) .OR.
     *      VEC(JX).EQ.RZERO) GO TO 530
         ABSVEC=VEC(JX)
         IF(ABSVEC.LT.RZERO) ABSVEC=-ABSVEC
         IF(ABSVEC.LT.FACUP) GO TO 250
C
C  INCREASE THE STEP SIZE.
C
         DX(JX)=DX(JX)*ACK
```

```
C
C  RESCALE THE NUMBERS OF STEPS STORED IN VEC(JX) AND
C  ERR(JX,*) .
C
         VEC(JX)=VEC(JX)/ACK
         IF(NOSC.LE.0) GO TO 230
C
         DO 220 J=1,NOSC
            ERR(JX,J)=ERR(JX,J)/ACK
  220       CONTINUE
C
  230    IF(NTRACE.GE.1) WRITE(KW,240)JX,DX(JX)
  240    FORMAT(/' STEP SIZE',I3,' INCREASED TO ',1PG12.4)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  STEP ALONG A RESULTANT DIRECTION, IF POSSIBLE.
C
  250    IF(NZIP.LE.0) GO TO 530
         NONZER=0
         SUMV=RZERO
C
         DO 260 J=1,NV
            IF(VEC(J).NE.RZERO) NONZER=NONZER+1
            SUMV=SUMV+VEC(J)**2
  260       CONTINUE
C
         IF(NONZER.LT.2) GO TO 530
         IF(SUMV.LE.RZERO) GO TO 380
C
C  GIANT STEPS WILL BE ATTEMPTED.
C  FIRST, CHECK FOR POSSIBLE GIGANTIC STEPS.
C
         IF(MAXOSC.LT.1) GO TO 380
C
C  ZIGZAGGING SEARCH SECTION......
C
C  KL = POINTER FOR ZIGZAGGING CHECK.
C
         KL=1
C
C  STORE ZIGZAGGING INFORMATION.
C
         NOSC=NOSC+1
         IF(NOSC.LE.MAXOSC) GO TO 290
         NOSC=MAXOSC
         IF(NOSC.EQ.1) GO TO 290
C
C  THE QUEUE OF ZIGZAGGING INFORMATION IS FULL.
C  PUSH IT DOWN, THROWING AWAY THE OLDEST ITEM.
C
         DO 280 K=2,NOSC
```

```
            FOSC(K-1)=FOSC(K)
C
            DO 270 J=1,NV
               XOSC(J,K-1)=XOSC(J,K)
               ERR(J,K-1)=ERR(J,K)
  270          CONTINUE
C
  280       CONTINUE
C
C  ADD THE NEW ITEM TO THE QUEUE.
C
  290    SUMV=ZSQRT(SUMV)
C
         DO 300 J=1,NV
            XOSC(J,NOSC)=X(J)
            ERR(J,NOSC)=VEC(J)/SUMV
  300       CONTINUE
C
         FOSC(NOSC)=FBEST
         IF(NOSC.LT.2) GO TO 380
C
C  SEARCH FOR A PREVIOUS SUCCESSFUL GIANT STEP IN A DIRECTION
C  MORE NEARLY PARALLEL TO THE DIRECTION OF THE PROPOSED STEP
C  THAN WAS THE IMMEDIATELY PREVIOUS STEP.
C  THIS MAY MEAN THAT THE DIRECTIONS OF THE GIANT STEPS ZIGZAG.
C  IF SO, TRY GIGANTIC (ZIGZAG) STEPS OF DECREASING PERIOD,
C  THEN TRY ORDINARY GIANT STEPS.
C  SINCE THE DIRECTIONS ARE GIVEN AS NUMBERS OF STEPS, THIS
C  PROCEDURE IS SCALE-INDEPENDENT.
C
         COXCOM=RZERO
C
         DO 310 J=1,NV
            COXCOM=COXCOM+ERR(J,NOSC)*ERR(J,NOSC-1)
  310       CONTINUE
C
         NAH=NOSC-MINOSC
C
  320    IF(KL.GT.NAH) GO TO 380
C
         DO 340 K=KL,NAH
C
C  NRETRY = NUMBER OF ZIGZAGGING PERIODS YET TO BE TESTED.
C
            NRETRY=NAH-K
            COSIN=RZERO
C
            DO 330 J=1,NV
               COSIN=COSIN+ERR(J,NOSC)*ERR(J,K)
  330          CONTINUE
C
            IF(K.GE.NOSC-1 .OR.
```

```
     *          (COSIN.GT.RZERO .AND. COSIN.GT.COXCOM))
     *          GO TO 350
  340        CONTINUE
C
          GO TO 380
C
C  ZIGZAGGING HAS BEEN DETECTED.
C  ATTEMPT TO TAKE GIGANTIC STEPS.
C
  350     KL=K+1
          NT=NOSC-K
          IF(NTRACE.GE.1) WRITE(KW,360)NT,COXCOM,COSIN
  360     FORMAT(/' ********     GIGANTIC STEP WITH PERIOD ',I2,
     *       ' BEING ATTEMPTED.'/6X,'COXCOM, COSIN = ',1PG12.4,
     *       G12.4)
C
          DO 370 J=1,NV
C
C  SALVO SAVES DLX DURING GIGANTIC STEPS.
C
             SALVO(J)=DLX(J)
             DLX(J)=X(J)-XOSC(J,K)
  370        CONTINUE
C
          FPREV=FOSC(K)
          GO TO 390
C
C  SIMON SAYS, TAKE AS MANY GIANT STEPS AS POSSIBLE.
C
  380     FPREV=FSTORE(JX)
C
C  NRETRY = -1 IF A GIANT STEP IS BEING ATTEMPTED
C
          NRETRY=-1
C
C  NGIANT = NUMBER OF GIANT OR GIGANTIC STEPS COMPLETED
C
  390     NGIANT=0
          NFSAV=NF
C
  400     DO 410 J=1,NV
             XS(J)=X(J)
             IF(MASK(J).NE.0) GO TO 410
             X(J)=X(J)+DLX(J)
             IF(X(J).GT.XMAX(J)) X(J)=XMAX(J)
             IF(X(J).LT.XMIN(J)) X(J)=XMIN(J)
  410        CONTINUE
C
          JOCK=0
          JVARY=0
          CALL FUNK
          NF=NF+1
```

```
         IF(FOBJ.GE.FBEST) GO TO 470
         FPREV=FBEST
         FBEST=FOBJ
C
         DO 420 J=1,NV
            DLX(J)=DLX(J)*ACK
  420       CONTINUE
C
         NGIANT=NGIANT+1
         IF(NTRACE.LT.1) GO TO 460
         IF(NGIANT.GT.1) GO TO 450
         WRITE(KW,430)(VEC(J),J=1,JX)
  430    FORMAT(/' NO. OF STEPS = ',1PG10.2,4G10.2/
     *      (16X,5G10.2))
         WRITE(KW,440)FPREV,NFSAV,(XS(J),J=1,NV)
  440    FORMAT(/' FOBJ =',1PG15.7,7X,' NF =',I7/
     *      ' X =',5G15.7/(4X,5G15.7))
  450    WRITE(KW,440)FOBJ,NF,(X(J),J=1,NV)
C
  460    CONTINUE
         GO TO 400
C
  470    IF(NGIANT.LE.0) GO TO 500
C
C  PERFORM QUADRATIC INTERPOLATION.
C
         DENOM=ACK*(FPREV-FBEST)-(FBEST-FOBJ)
         IF(DENOM.LE.RZERO) GO TO 500
         CINDER=((FPREV-FBEST)*ACK+(FBEST-FOBJ)/ACK)/
     *      (DENOM+DENOM)
C
         DO 480 J=1,NV
            IF(MASK(J).NE.0) GO TO 480
            X(J)=XS(J)+CINDER*DLX(J)
            IF(X(J).GT.XMAX(J)) X(J)=XMAX(J)
            IF(X(J).LT.XMIN(J)) X(J)=XMIN(J)
  480       CONTINUE
C
         JOCK=0
         JVARY=0
         CALL FUNK
         NF=NF+1
         IF(FOBJ.GE.FBEST) GO TO 500
C
         FBEST=FOBJ
         JOCK=1
         STEPS=NGIANT
         STEPS=STEPS+CINDER
         IF(NTRACE.GE.1) WRITE(KW,490)FBEST,STEPS,(X(J),J=1,NV)
  490    FORMAT(/' FOBJ =',1PG15.7,' AFTER',G10.2,
     *      ' GIANT STEPS.'//' X =',5G15.7/(4X,5G15.7))
         GO TO 640
```

```
C
  500     DO 510 J=1,NV
            IF(NRETRY.GE.0) DLX(J)=SALVO(J)
            X(J)=XS(J)
  510       CONTINUE
C
          IF(NTRACE.GE.1) WRITE(KW,520)FBEST,NGIANT,
     *      (X(J),J=1,NV)
  520     FORMAT(/' FOBJ =',1PG15.7,' AFTER',I3,
     *      ' GIANT STEP(S).'/' X =',5G15.7/(4X,5G15.7))
C
          IF(NGIANT.GT.0) GO TO 640
C
          IF(NRETRY.GT.0) GO TO 320
C
C  IF ALL GIGANTIC STEPS WERE UNSUCCESSFUL, TRY A GIANT STEP.
C
          IF(NRETRY.EQ.0) GO TO 380
C
C  AN UNSUCCESSFUL GIANT STEP HAS OCCURRED.
C  DELETE ITS ZIGZAGGING INFORMATION.
C
          NOSC=NOSC-1
          IF(NOSC.LT.0) NOSC=0
C
C  COMPLETE THE MAIN DO LOOP.
C
C  FSTORE(JX) SAVES FBEST FOR INTERPOLATION IN GIANT STEPS.
C
  530     FSTORE(JX)=FBEST
C
C  RETURN IF THE SENSE SWITCH IS ON.
C
          CALL DATSW (NSSW,JUMP)
          IF(JUMP.LE.1) GO TO 710
C
          IF(NF.GT.NFMAX) GO TO 690
C
  540     CONTINUE
C
C  THIS IS THE END OF THE MAIN DO LOOP.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  ANOTHER CYCLE THROUGH THE VARIABLES HAS BEEN COMPLETED.
C  PRINT ANOTHER LINE OF TRACES.
C
      IF(NTRACE.GE.1) WRITE(KW,430)(VEC(J),J=1,NV)
      IF(NZIP.NE.0 .OR. NTRACE.LT.1) GO TO 560
      WRITE(KW,440)FBEST,NF,(X(J),J=1,NV)
      WRITE(KW,550)
  550 FORMAT(' ')
```

```
C
  560 NZIP=NZIP+1
      GO TO 30
C
C  ESTABLISH THE CURRENT BEST KNOWN POINT AS THE BASE POINT.
C
  570 FSTORE(JX)=FBEST
C
C  PRINT THE REMAINING TRACES.
C
      IF(NTRACE.LT.1) GO TO 580
      WRITE(KW,430)(VEC(J),J=1,JX)
      WRITE(KW,440)FBEST,NF,(X(J),J=1,NV)
C
C  DECREASE THE SIZE OF THE STEPS FOR ALL VARIABLES, AND
C  CHECK WHETHER OR NOT ALL ABS(DX(J)) .LE. DELMIN(J) .
C
  580 NGATE=1
C
      DO 600 J=1,NV
         IF(MASK(J).NE.0) GO TO 590
         ABSDX=DX(J)
         IF(ABSDX.LT.RZERO) ABSDX=-ABSDX
         IF(ABSDX.GT.DELMIN(J)) NGATE=0
  590    DX(J)=DX(J)/STCUT
  600    CONTINUE
C
      IF(NGATE.EQ.1) GO TO 650
C
C  CHECK THE JFLAT(J) .
C
      IF(NFLAT.LE.0) GO TO 620
      JFLMIN=5
C
      DO 610 J=1,NV
         IF(MASK(J).EQ.0 .AND. JFLAT(J).LT.JFLMIN)
     *      JFLMIN=JFLAT(J)
  610    CONTINUE
C
      IF(JFLMIN.GE.1) GO TO 670
C
  620 IF(NTRACE.GE.1) WRITE(KW,630)(DX(J),J=1,NV)
  630 FORMAT(/36(' *')//' STEP SIZES REDUCED TO....'/
     *   (1X,1PG12.4,4G12.4))
C
C  RETURN IF THE SENSE SWITCH IS ON.
C
      CALL DATSW (NSSW,JUMP)
      IF(JUMP.LE.1) GO TO 710
C
      IF(NF.GT.NFMAX) GO TO 690
C
```

```
C  SEARCH SOME MORE.
C
  640 CONTINUE
      GO TO 20
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  THE MINIMIZATION IS FINISHED.
C
  650 KFLAG=1
      IF(NTRACE.GE.0) WRITE(KW,660)
  660 FORMAT(//' TERMINATED WHEN THE STEP SIZES',
     *    ' BECAME AS SMALL AS THE DELMIN(J).')
      GO TO 730
C
  670 KFLAG=2
      IF(NTRACE.GE.0) WRITE(KW,680)
  680 FORMAT(//' TERMINATED WHEN THE FUNCTION VALUES',
     *    ' AT ALL TRIAL POINTS WERE IDENTICAL.')
      GO TO 730
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
  690 KFLAG=-2
      IF(NTRACE.GE.-1) WRITE(KW,700)NFMAX
  700 FORMAT(//' ABNORMAL TERMINATION....',
     *    '    MORE THAN NFMAX =',I8,
     *    ' CALLS TO THE FOBJ SUBROUTINE.')
      GO TO 740
C
  710 KFLAG=-3
      IF(NTRACE.GE.-1) WRITE(KW,720)NSSW
  720 FORMAT(//' ABNORMAL TERMINATION....  TERMINATED BY',
     *    ' OPERATOR VIA SENSE SWITCH ',I2)
      GO TO 740
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
  730 IF(NTRACE.LT.0) GO TO 760
  740 IF(NTRACE.LT.-1) GO TO 760
      WRITE(KW,750)(DX(J),J=1,NV)
  750 FORMAT(//' CURRENT STEP SIZES....'//(1X,1PG12.4,4G12.4))
C
C  CALL FUNK WITH THE BEST SET OF X(J) .
C
  760 JVARY=0
      CALL FUNK
      NF=NF+1
      IF(FBEST.LE.FSAVE .AND. FOBJ.EQ.FBEST) GO TO 780
      NOREP=NOREP+2
      IF(NTRACE.GE.-1) WRITE(KW,770)NF,FSAVE,FBEST,FOBJ
  770 FORMAT(//' WARNING....  FOBJ IS NOT A REPRODUCIBLE',
```

```
     *    ' FUNCTION OF X(J).',7X,' NF = ',I5//1X,1PG23.15,
     *    2G23.15)
C
  780 IF(NTRACE.GE.0) WRITE(KW,790)NF,FOBJ,(X(J),J=1,NV)
  790 FORMAT(//1X,I6,' FUNCTION COMPUTATIONS'//
     *    ' FINAL VALUE OF FOBJ =',1PG23.15//
     *    9X,' FINAL VALUES OF X(J)....'//(1X,5G15.7))
      IF(KFLAG.LT.0) GO TO 800
C
      IF(MATRX.LT.70 .OR. MATRX.GT.130) GO TO 800
C
C  CALL STERR TO COMPUTE AN APPROXIMATE ERROR MATRIX.
C
      CALL STERR (FUNK)
C
C  THIS IS THE ONLY RETURN STATEMENT IN THIS SUBROUTINE....
C
  800 RETURN
C
C  END STEPIT
C
      END
C
C
C
      SUBROUTINE STBEG (FUNK)
C
C  STBEG 1.5           DECEMBER 1991
C
C  A.N.S.I. STANDARD FORTRAN 77
C
C  COPYRIGHT (C) 1965, 1975, 1991 J. P. CHANDLER
C     COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY
C
C  STBEG SETS DEFAULT VALUES AND PRINTS INITIAL OUTPUT FOR
C  STEPIT.  THE CALL TO STBEG IS THE FIRST EXECUTABLE STATEMENT
C  IN STEPIT, TO FACILITATE OVERLAYING IF NECESSARY.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INPUT QUANTITIES.....   FUNK,X,XMAX,XMIN,DELTX,DELMIN,NV,
C                             NTRACE,MATRX,MASK,NFMAX,NFLAT,KW
C
C  OUTPUT QUANTITIES....   NSSW,NACTIV,NF,KFLAG,NOREP,
C                               AND SOMETIMES X,XMAX,XMIN,DELTX,
C                               DELMIN
C
      DOUBLE PRECISION X,XMAX,XMIN,DELTX,DELMIN,ERR,FOBJ,DX,XS,
     *    DLX
      DOUBLE PRECISION HUGE,DELDF,RZERO,UNITR,RTEN,RELACC,
     *    XPLUS,FSAVE
C
```

```
      INTEGER J,JUMP,JVARY,KERFL,KFLAG,KTYPE,KW,MASK,MATRX,
     *   NACTIV,NF,NFLAT,NFMAX,NOREP,NSSW,NTRACE,NV,NVMAX,
     *   NXTRA
C
C  USER COMMON.....
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
C  INTERNAL STEPIT COMMON.....
      COMMON /STORK/ DX(20),XS(20),DLX(20),NACTIV,NSSW
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  SET FIXED QUANTITIES ....
C
C  KTYPE = LOGICAL UNIT NUMBER OF THE CONSOLE TYPEWRITER, IF
C          ANY (IRRELEVANT IF A DUMMY DATSW IS USED)
C
      KTYPE=1
C
C  NSSW = SENSE SWITCH NUMBER FOR TERMINATION BY OPERATOR
C         (IRRELEVANT IF A DUMMY DATSW IS USED)
C
      NSSW=6
C
C  HUGE = A LARGE REAL NUMBER, THE DEFAULT VALUE FOR XMAX AND
C         FOR (-XMIN)
C
      HUGE=1.0D35
C
C  NVMAX = MAXIMUM VALUE OF NV
C
      NVMAX=20
C
C  DELDF = DEFAULT VALUE FOR DELTX(J)
C
      DELDF=0.01D0
C
      RZERO=0.0D0
      UNITR=1.0D0
      RTEN=10.0D0
C
C  NO REAL OR DOUBLE PRECISION CONSTANTS ARE USED BEYOND THIS
C  POINT IN THIS SUBROUTINE.
C
      KFLAG=0
      NOREP=0
      KERFL=0
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
```

```
C  CHECK SOME INPUT QUANTITIES, AND SET THEM TO DEFAULT VALUES
C  IF DESIRED.
C
      IF(NV.GE.1 .AND. NV.LE.NVMAX) GO TO 20
C
      WRITE(KW,10)NV,NVMAX
   10 FORMAT(///' ******  FATAL ERROR IN SUBROUTINE STEPIT...'
     *   //6X,'NV =',I14,6X,'NVMAX =',I14)
      STOP
C
C  MAKE SURE THAT THE SENSE SWITCH IS OFF.
C
   20 JUMP=2
      CALL DATSW (NSSW,JUMP)
      IF(JUMP.GE.2) GO TO 50
C
C  THIS IS THE ONLY USAGE OF THE CONSOLE TYPEWRITER.
C
      WRITE(KTYPE,30)NSSW
   30 FORMAT(/' TURN OFF SENSE SWITCH ',I2//' ')
   40 CALL DATSW (NSSW,JUMP)
      IF(JUMP.LE.1) GO TO 40
C
C  COMPUTE RELACC, THE RELATIVE PRECISION OF THE MACHINE AND
C  ARITHMETIC BEING USED.
C  RELACC IS USED IN SETTING DELMIN(J) TO A DEFAULT VALUE.
C
   50 RELACC=UNITR
   60 RELACC=RELACC/RTEN
      XPLUS=UNITR+RELACC
      IF(XPLUS.GT.UNITR) GO TO 60
C
C  NACTIV = NUMBER OF ACTIVE X(J)
C
      NACTIV=0
C
      DO 110 J=1,NV
         IF(MASK(J).NE.0) GO TO 110
         NACTIV=NACTIV+1
C
C   CHECK THAT DELTX(J) IS NOT NEGLIGIBLE.
C
         IF(DELTX(J).EQ.RZERO) GO TO 70
         XPLUS=X(J)+DELTX(J)
         IF(XPLUS.EQ.X(J)) GO TO 70
         XPLUS=X(J)-DELTX(J)
         IF(XPLUS.NE.X(J)) GO TO 90
C
C  DELTX(J) IS NEGLIGIBLE COMPARED TO X(J) .  RESET DELTX(J) .
C
   70    IF(X(J).EQ.RZERO) GO TO 80
         DELTX(J)=DELDF*X(J)
```

```
         GO TO 90
C
   80    DELTX(J)=DELDF
C
   90    IF(DELMIN(J).EQ.RZERO) DELMIN(J)=DELTX(J)*RELACC
         IF(DELMIN(J).LT.RZERO) DELMIN(J)=-DELMIN(J)
C
         IF(XMAX(J).GT.XMIN(J)) GO TO 100
         XMAX(J)=HUGE
         XMIN(J)=-HUGE
C
  100    IF(X(J).GT.XMAX(J)) X(J)=XMAX(J)
         IF(X(J).LT.XMIN(J)) X(J)=XMIN(J)
  110    CONTINUE
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      IF(NTRACE.LT.0) GO TO 200
      WRITE(KW,120)
  120 FORMAT('1SUBROUTINE STEPIT'//' INITIAL VALUES....'/' ')
      WRITE(KW,130)(MASK(J),J=1,NV)
  130 FORMAT(/' MASK  =',I7,4I13/(2X,5I13))
      WRITE(KW,140)(X(J),J=1,NV)
  140 FORMAT(/' X     =',1PG13.5,4G13.5/(8X,5G13.5))
      WRITE(KW,150)(XMAX(J),J=1,NV)
  150 FORMAT(/' XMAX  =',1PG13.5,4G13.5/(8X,5G13.5))
      WRITE(KW,160)(XMIN(J),J=1,NV)
  160 FORMAT(/' XMIN  =',1PG13.5,4G13.5/(8X,5G13.5))
      WRITE(KW,170)(DELTX(J),J=1,NV)
  170 FORMAT(/' DELTX =',1PG13.5,4G13.5/(8X,5G13.5))
      WRITE(KW,180)(DELMIN(J),J=1,NV)
  180 FORMAT(/' DELMIN=',1PG13.5,4G13.5/(8X,5G13.5))
      WRITE(KW,190)NV,NACTIV,MATRX,NFMAX,NFLAT,RELACC
  190 FORMAT(//1X,I3,' VARIABLES,',I3,' ACTIVE.',8X,' MATRX =',
     *   I4//' NFMAX =',I8,8X,' NFLAT =',I2,9X,' RELACC =',
     *   G11.4)
C
  200 JVARY=0
      CALL FUNK
      FSAVE=FOBJ
      CALL FUNK
C
      IF(NTRACE.GE.0) WRITE(KW,210)FOBJ
  210 FORMAT(//' FOBJ =',1PG18.10)
C
C  NF = NUMBER OF CALLS TO SUBROUTINE FUNK SO FAR
C
      NF=2
C
      IF(FOBJ.EQ.FSAVE) GO TO 230
      NOREP=1
      IF(NTRACE.GE.-1) WRITE(KW,220)NF,FSAVE,FOBJ
```

```
  220 FORMAT(//' WARNING....  FOBJ IS NOT A REPRODUCIBLE',
     *    ' FUNCTION OF X(J).',7X,' NF =',I6//5X,1PG23.16,
     *    G23.16)
C
  230 IF(NACTIV.GT.0) GO TO 250
      KFLAG=-1
      IF(NTRACE.GE.-1) WRITE(KW,240)
  240 FORMAT(///' ******  WARNING...  MASK(J).NE.0 FOR ALL J ,'
     *    ,' IN A CALL TO SUBROUTINE STEPIT.'//
     *    6X,'FOBJ WILL BE EVALUATED BUT NOT MINIMIZED.'//' ')
      GO TO 270
C
  250 IF(NTRACE.GE.0) WRITE(KW,260)
  260 FORMAT(///' BEGIN MINIMIZATION'//' ')
C
  270 RETURN
C
C  END STBEG
C
      END
C
C
C
      SUBROUTINE STERR (FUNK)
C
C  STERR 1.6          DECEMBER 1991
C
C  A.N.S.I. STANDARD FORTRAN 77
C
C  COPYRIGHT (C) 1965, 1975, 1991 J. P. CHANDLER
C     DEPARTMENT OF COMPUTER SCIENCE,
C     OKLAHOMA STATE UNIVERSITY
C
C  STERR IS CALLED BY STEPIT TO COMPUTE AN APPROXIMATE ERROR
C  MATRIX FOR A NONLINEAR FITTING PROBLEM.
C  THE VALUES COMPUTED ARE SOMETIMES POOR APPROXIMATIONS.
C  FOR EACH CLASS OF PROBLEMS, THE ERRORS SHOULD BE CHECKED
C  USING SUBROUTINE FIDO.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INPUT QUANTITIES.....  FUNK,KW,NSSW,DX,NF,X,NTRACE,NV
C
C  OUTPUT QUANTITIES....  NF,ERR,KERFL, AND SOMETIMES DX
C
C  SCRATCH STORAGE......  XS,DLX
C
C  THE DX(J) ARE THE STEP SIZES USED IN APPROXIMATING THE
C  SECOND PARTIAL DERIVATIVES OF FOBJ WITH RESPECT TO THE X(J)
C  BY FINITE DIFFERENCES.
C  ERR RETURNS THE ERROR MATRIX.
C  XMAX, XMIN, AND MASK ARE IGNORED IN STERR.
```

```
C  THE REAL FORMAT SPECIFICATIONS USED ARE G13.5, G16.8, AND
C  G23.16 .
C
      DOUBLE PRECISION X,XMAX,XMIN,DELTX,DELMIN,ERR,FOBJ,DX,XS,
     *   DLX,SECND,FBEST,RZERO,UNITR,RTWO,ABSERR,DENOM,RTEN,
     *   P,Q,ARG,ZSQRT,DSQRT,ABSX,DXFAC
C
      INTEGER J,JACTIV,JJ,JMU,JUMP,JVARY,K,KACTIV,KERFL,KFLAG,
     *   KK,KW,L,LL,M,MASK,MATRX,NACTIV,NEGDEF,NF,NFLAT,
     *   NFMAX,NOREP,NSSW,NTRACE,NV,NXTRA
C
      DIMENSION SECND(2,2)
C
C  USER COMMON.....
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
C
C  INTERNAL STEPIT COMMON.....
      COMMON /STORK/ DX(20),XS(20),DLX(20),NACTIV,NSSW
C
C  XS AND DLX ARE IN COMMON ONLY TO CONSERVE STORAGE.
C
C     ZSQRT(ARG)=SQRT(ARG)
      ZSQRT(ARG)=DSQRT(ARG)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      RZERO=0.0D0
      UNITR=1.0D0
      RTWO=2.0D0
      RTEN=10.0D0
C
C  NO REAL OR DOUBLE PRECISION CONSTANTS ARE USED BEYOND THIS
C  POINT, IN THIS SUBROUTINE.
C
      KERFL=0
C
C  SET THE STEP SIZES.
C
      DXFAC=RTEN**(100-MATRX)
C
      DO 10 K=1,NV
         XS(K)=X(K)
         IF(MASK(K).NE.0) GO TO 10
         ABSX=X(K)
         IF(ABSX.LT.RZERO) ABSX=-ABSX
         DX(K)=DXFAC*ABSX
         IF((ABSX+DX(K))-ABSX.GT.RZERO) DX(K)=(ABSX+DX(K))-ABSX
         IF(DX(K).LE.RZERO .OR. (ABSX+DX(K))-ABSX.LE.RZERO)
     *      DX(K)=DXFAC
 10      CONTINUE
```

```
C
      CALL FUNK
      NF=NF+1
      FBEST=FOBJ
C
      IF(NTRACE.LT.0) GO TO 40
      WRITE(KW,20)
   20 FORMAT('1SUBROUTINE STERR.'//' COMPUTE AN APPROXIMATE',
     *    ' ERROR MATRIX USING FINITE DIFFERENCES.'///
     *    ' INCREMENTS IN X(J) TO BE USED....')
      WRITE(KW,30)(DX(K),K=1,NV)
   30 FORMAT(/(1X,1PG13.5,4G13.5))
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  APPROXIMATE THE (SYMMETRIC) MATRIX OF SECOND PARTIAL
C  DERIVATIVES OF FOBJ WITH RESPECT TO THE X(J), USING DIVIDED
C  DIFFERENCES.
C  COMPUTE THE DIAGONAL PARTIALS FIRST.
C
   40 DO 60 J=1,NV
         ERR(J,J)=RZERO
         IF(MASK(J).NE.0) GO TO 60
         JVARY=0
C
         DO 50 K=1,2
            X(J)=XS(J)+DX(J)
            CALL FUNK
            NF=NF+1
            JVARY=J
            SECND(K,1)=FOBJ
            DX(J)=-DX(J)
   50       CONTINUE
C
         X(J)=XS(J)
         ERR(J,J)=((SECND(1,1)-FBEST)-(FBEST-SECND(2,1)))/
     *      DX(J)**2
   60    CONTINUE
C
C  COMPUTE THE OFF-DIAGONAL PARTIALS.
C
      IF(NV.LT.2) GO TO 110
C
      DO 100 J=2,NV
         JMU=J-1
C
         DO 90 K=1,JMU
            ERR(J,K)=RZERO
            IF(MASK(J).NE.0 .OR. MASK(K).NE.0) GO TO 90
C
            DO 80 L=1,2
               X(J)=XS(J)+DX(J)
```

```
              JVARY=0
C
              DO 70 M=1,2
                X(K)=XS(K)+DX(K)
                CALL FUNK
                NF=NF+1
                JVARY=K
                SECND(L,M)=FOBJ
                X(K)=XS(K)
                DX(K)=-DX(K)
   70           CONTINUE
C
              X(J)=XS(J)
              DX(J)=-DX(J)
C
C  RETURN IF THE SENSE SWITCH IS ON.
C
              JUMP=2
              CALL DATSW (NSSW,JUMP)
              IF(JUMP.EQ.2) GO TO 80
              KERFL=-1
              GO TO 470
   80         CONTINUE
C
            ERR(J,K)=((SECND(1,1)-SECND(1,2))-
     *                (SECND(2,1)-SECND(2,2)))/
     *               (RTWO*DX(J)*RTWO*DX(K))
   90       CONTINUE
C
  100     CONTINUE
C
C  THIS IS THE END OF THE DERIVATIVE COMPUTATION.
C
  110 IF(NTRACE.LT.0) GO TO 180
      WRITE(KW,120)
  120 FORMAT(///' MATRIX OF THE SECOND PARTIAL DERIVATIVES...')
      WRITE(KW,130)(K,K=1,NV)
  130 FORMAT(//11X,'K  =',I3,4I12/(6X,5I12))
      WRITE(KW,140)(MASK(K),K=1,NV)
  140 FORMAT(/6X,'MASK(K) =',I3,4I12/(6X,5I12))
      WRITE(KW,150)
  150 FORMAT(/'   J  MASK(J)')
C
      DO 170 J=1,NV
        WRITE(KW,160)J,MASK(J),(ERR(J,K),K=1,J)
  160   FORMAT(/1X,I3,I6,2X,1PG12.4,4G12.4/(12X,5G12.4))
  170   CONTINUE
C
C  PACK THE MATRIX OF SECOND DERIVATIVES.
C
  180 NACTIV=0
C
```

```
      DO 200 J=1,NV
         IF(MASK(J).NE.0) GO TO 200
         NACTIV=NACTIV+1
         KACTIV=0
C
         DO 190 K=1,J
            IF(MASK(K).NE.0) GO TO 190
            KACTIV=KACTIV+1
            ERR(NACTIV,KACTIV)=ERR(J,K)
            IF(ERR(J,K).EQ.RZERO) KERFL=1
  190       CONTINUE
C
  200    CONTINUE
C
      IF(KERFL.GE.1) WRITE(KW,210)
  210 FORMAT(//' THE ABOVE MATRIX CONTAINS ONE OR MORE',
     *    ' UNEXPECTED ZEROES.'/' PERHAPS A SMALLER VALUE OF',
     *    ' -MATRX- SHOULD BE TRIED,',
     *    ' TO SEE IF THEY ARE LEGITIMATE.')
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INVERT THE MATRIX OF SECOND PARTIAL DERIVATIVES USING THE
C  GAUSS-JORDAN METHOD
C  (F. L. BAUER AND C. REINSCH, P. 45 IN "LINEAR ALGEBRA"
C  BY J. H. WILKINSON AND C. REINSCH (SPRINGER-VERLAG, 1971)).
C  ONLY THE LOWER TRIANGLE OF ERR IS USED OR ALTERED.
C
C  NEGDEF = 1 IF THE MATRIX IS NEGATIVE DEFINITE
C
      NEGDEF=0
C
      DO 310 LL=1,NACTIV
         L=NACTIV+1-LL
         P=ERR(1,1)
         IF(P.LT.RZERO) GO TO 230
         IF(P.GT.RZERO) GO TO 240
         KERFL=-2
         WRITE(KW,220)
  220    FORMAT(//' A PIVOT ELEMENT OF THE MATRIX IS ZERO.',
     *       '  PERHAPS -MATRX- SHOULD BE DECREASED.')
         GO TO 470
C
  230    NEGDEF=1
C
  240    IF(NACTIV.LT.2) GO TO 290
C
         DO 280 K=2,NACTIV
            Q=ERR(K,1)
            IF(K.LE.L) GO TO 250
            XS(K)=Q/P
            GO TO 260
```

```
C
  250         XS(K)=-Q/P
C
  260         DO 270 M=2,K
                 ERR(K-1,M-1)=ERR(K,M)+Q*XS(M)
  270            CONTINUE
C
  280         CONTINUE
C
  290      ERR(NACTIV,NACTIV)=UNITR/P
           IF(NACTIV.LT.2) GO TO 310
C
           DO 300 K=2,NACTIV
              ERR(NACTIV,K-1)=XS(K)
  300         CONTINUE
C
  310      CONTINUE
C
        IF(NEGDEF.LE.0) GO TO 330
C
        KERFL=-3
        WRITE(KW,320)
  320 FORMAT(//' THE ERROR MATRIX IS NEGATIVE DEFINITE.'/
     *     ' PERHAPS -MATRX- SHOULD BE INCREASED.')
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  UNPACK, DOUBLE, AND SYMMETRIZE THE INVERSE TO FORM THE
C  ERROR MATRIX.
C
  330 JACTIV=NACTIV
C
        DO 370 JJ=1,NV
           J=NV+1-JJ
           KACTIV=JACTIV
C
           DO 360 KK=1,J
              K=J+1-KK
              IF(MASK(J).EQ.0 .AND. MASK(K).EQ.0) GO TO 340
              ERR(J,K)=RZERO
              GO TO 350
C
  340         ERR(J,K)=ERR(JACTIV,KACTIV)*RTWO
              KACTIV=KACTIV-1
C
  350         ERR(K,J)=ERR(J,K)
  360         CONTINUE
C
           IF(MASK(J).EQ.0) JACTIV=JACTIV-1
  370      CONTINUE
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

```
C
C  PRINT THE STANDARD ERRORS AND THE CORRELATIONS, AND RETURN.
C
      IF(NTRACE.GE.0) WRITE(KW,380)
  380 FORMAT(///' APPROXIMATE STANDARD ERRORS....'///
     *    3X,'J',6X,'MASK(J)',9X,'X(J)',14X,'ERROR')
C
      DO 420 J=1,NV
         ABSERR=ERR(J,J)
         IF(ABSERR.EQ.RZERO) GO TO 400
         IF(ABSERR.LT.RZERO) ABSERR=-ABSERR
         ABSERR=ZSQRT(ABSERR)
         IF(MASK(J).NE.0) GO TO 400
         IF(ERR(J,J).GT.RZERO) GO TO 400
         IF(ERR(J,J).LT.RZERO) ABSERR=-ABSERR
         KERFL=-4
         WRITE(KW,390)ERR(J,J)
  390    FORMAT(//' NEGATIVE OR ZERO MEAN SQUARE',
     *      ' ERROR ENCOUNTERED....',3X,G16.8/
     *      ' PERHAPS -MATRX- SHOULD BE INCREASED.')
C
  400    IF(NTRACE.GE.0) WRITE(KW,410)J,MASK(J),X(J),ABSERR
  410    FORMAT(/1X,I3,I10,6X,1PG16.8,4X,G13.5)
         XS(J)=ABSERR
  420    CONTINUE
C
C  COMPUTE AND PRINT THE CORRELATIONS.
C
      IF(NTRACE.LT.0 .OR. NV.LT.2) GO TO 470
      WRITE(KW,430)
  430 FORMAT(///' LOWER TRIANGLE OF THE CORRELATION MATRIX....'
     *    )
      WRITE(KW,130)(K,K=1,NV)
      WRITE(KW,140)(MASK(K),K=1,NV)
      WRITE(KW,150)
C
      DO 460 J=1,NV
C
         DO 450 K=1,J
            DENOM=XS(J)*XS(K)
            IF(DENOM.NE.RZERO) GO TO 440
            DLX(K)=RZERO
            GO TO 450
C
  440       IF(DENOM.LT.RZERO) DENOM=-DENOM
            DLX(K)=ERR(J,K)/DENOM
  450       CONTINUE
C
         WRITE(KW,160)J,MASK(J),(DLX(K),K=1,J)
  460    CONTINUE
C
C  CALL FUNK AGAIN, SO THAT THE LAST CALL TO FUNK WILL HAVE
```

```
C  BEEN MADE WITH THE OPTIMAL VECTOR X(*).
C
  470 JVARY=0
      CALL FUNK
      NF=NF+1
      RETURN
C
C  END STERR
C
      END
      SUBROUTINE DATSW (NSSW,JUMP)
C
C  THIS IS A DUMMY VERSION OF SUBROUTINE DATSW, WITH ALL
C  SWITCHES PERMANENTLY OFF.
C
      INTEGER NSSW,JUMP
C
      JUMP=2
      RETURN
      END
C
C
C
      SUBROUTINE STSET
C
C  STSET 3.2          DECEMBER 1991
C
C  STSET SETS SOME INPUT QUANTITIES TO DEFAULT VALUES, FOR
C  SUBROUTINES STEPIT, SIMPLEX, MARQ, STP, MINF, OR KAUPE.
C
C  J. P. CHANDLER, DEPARTMENT OF COMPUTER SCIENCE,
C     OKLAHOMA STATE UNIVERSITY
C
C
C  USAGE.....
C
C  CALL STSET.
C  THEN SET SOME INPUT QUANTITIES (NV, AT LEAST) AND RESET ANY
C  OF THOSE SET IN STSET (BETTER VALUES OF X(J), ETC.) BEFORE
C  CALLING STEPIT OR SIMPLEX OR MARQ, ETC.
C
C
      DOUBLE PRECISION XMAX,XMIN,DELTX,DELMIN,ERR,FOBJ,FLAMBD,
     *   FNU,RELDIF,RELMIN,RZERO,HUGE,        X
C
      INTEGER JVARY,JX,KALCP,KERFL,KFLAG,KORDIF,KW,LEQU,MASK,
     *   MATRX,MAXIT,METHD,MAXSUB,MAXUPD,NFLAT,NFMAX,NOREP,
     *   NTRACE,NV,NVMAX,NXTRA
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),
     *   DELMIN(20),ERR(20,21),FOBJ,NV,NTRACE,MATRX,MASK(20),
     *   NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW,NF
```

```
C
      COMMON /NLLS4/ FLAMBD,FNU,RELDIF,RELMIN,METHD,KALCP,
     *   KORDIF,MAXIT,LEQU,MAXSUB,MAXUPD
C
C
      HUGE=1.0D30
C
      RZERO=0.0D0
C
C  KW = LOGICAL UNIT NUMBER OF THE PRINTER
C
      KW=6
C
C  NVMAX IS THE MAXIMUM PERMISSIBLE VALUE OF NV, GIVEN THE
C  PRESENT DIMENSIONS OF ARRAYS.
C  NVMAX IS THE DIMENSION OF THE ARRAYS X(*), XMAX(*), XMIN(*),
C  DELTX(*), DELMIN(*), AND MASK(*).  NVMAX IS ALSO THE FIRST
C  DIMENSION OF ERR(*,*).  THE SECOND DIMENSION OF ERR(*,*)
C  IS NVMAX+1.
C
      NVMAX=20
C
C  THE USER MUST SET NV AFTER CALLING STSET.
C
      NV=-1
C
      NTRACE=0
      NFMAX=1000000
      MAXIT=50
      MAXSUB=30
      METHD=1
      KALCP=0
      LEQU=0
      NFLAT=1
      MATRX=105
      NXTRA=0
      FLAMBD=1.0D0
      FNU=10.0D0
C
      KORDIF=1
      RELDIF=1.0D-8
      RELMIN=1.0D-6
C
C  FOR GREATER ACCURACY BUT LESS SPEED IN MARQ, SET...
C
C            KORDIF=2
C            RELDIF=0.4D-5
C            RELMIN=1.0D-9
C
      DO 10 JX=1,NVMAX
         X(JX)=RZERO
         XMAX(JX)=HUGE
```

```
          XMIN(JX)=-HUGE
          DELTX(JX)=RZERO
          DELMIN(JX)=RZERO
          MASK(JX)=0
   10     CONTINUE
C
      RETURN
C
C  END STSET
C
      END
C
      SUBROUTINE JACOBI(A,N,NP,D,V,NROT)
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (NMAX=100)
      DIMENSION A(NP,NP),D(NP),V(NP,NP),B(NMAX),Z(NMAX)
      DO 12 IP=1,N
        DO 11 IQ=1,N
          V(IP,IQ)=0.D0
11      CONTINUE
        V(IP,IP)=1.D0
12    CONTINUE
      DO 13 IP=1,N
        B(IP)=A(IP,IP)
        D(IP)=B(IP)
        Z(IP)=0.D0
13    CONTINUE
      NROT=0
      DO 24 I=1,50
        SM=0.D0
        DO 15 IP=1,N-1
          DO 14 IQ=IP+1,N
            SM=SM+DABS(A(IP,IQ))
14        CONTINUE
15      CONTINUE
        IF(SM.EQ.0.D0)RETURN
        IF(I.LT.4)THEN
          TRESH=2.D-1*SM/DBLE(N**2)
        ELSE
          TRESH=0.D0
        ENDIF
        DO 22 IP=1,N-1
          DO 21 IQ=IP+1,N
            G=1.0D2*DABS(A(IP,IQ))
            IF((I.GT.4).AND.(DABS(D(IP))+G.EQ.DABS(D(IP)))
     *         .AND.(DABS(D(IQ))+G.EQ.DABS(D(IQ))))THEN
              A(IP,IQ)=0.D0
            ELSE IF(DABS(A(IP,IQ)).GT.TRESH)THEN
              H=D(IQ)-D(IP)
              IF(DABS(H)+G.EQ.DABS(H))THEN
                T=A(IP,IQ)/H
              ELSE
```

```
                THETA=5.D-1*H/A(IP,IQ)
                T=1.D0/(DABS(THETA)+DSQRT(1.D0+THETA**2))
                IF(THETA.LT.0.D0)T=-T
              ENDIF
              C=1.D0/DSQRT(1.D0+T**2)
              S=T*C
              TAU=S/(1.D0+C)
              H=T*A(IP,IQ)
              Z(IP)=Z(IP)-H
              Z(IQ)=Z(IQ)+H
              D(IP)=D(IP)-H
              D(IQ)=D(IQ)+H
              A(IP,IQ)=0.D0
              DO 16 J=1,IP-1
                G=A(J,IP)
                H=A(J,IQ)
                A(J,IP)=G-S*(H+G*TAU)
                A(J,IQ)=H+S*(G-H*TAU)
16            CONTINUE
              DO 17 J=IP+1,IQ-1
                G=A(IP,J)
                H=A(J,IQ)
                A(IP,J)=G-S*(H+G*TAU)
                A(J,IQ)=H+S*(G-H*TAU)
17            CONTINUE
              DO 18 J=IQ+1,N
                G=A(IP,J)
                H=A(IQ,J)
                A(IP,J)=G-S*(H+G*TAU)
                A(IQ,J)=H+S*(G-H*TAU)
18            CONTINUE
              DO 19 J=1,N
                G=V(J,IP)
                H=V(J,IQ)
                V(J,IP)=G-S*(H+G*TAU)
                V(J,IQ)=H+S*(G-H*TAU)
19            CONTINUE
              NROT=NROT+1
            ENDIF
21        CONTINUE
22      CONTINUE
        DO 23 IP=1,N
          B(IP)=B(IP)+Z(IP)
          D(IP)=B(IP)
          Z(IP)=0.D0
23      CONTINUE
24    CONTINUE
      PAUSE '50 iterations should never happen'
      RETURN
      END
```

```
      SUBROUTINE EIGSRT(D,V,N,NP)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(NP),V(NP,NP)
      DO 13 I=1,N-1
        K=I
        P=D(I)
        DO 11 J=I+1,N
          IF(D(J).GE.P)THEN
            K=J
            P=D(J)
          ENDIF
11      CONTINUE
        IF(K.NE.I)THEN
          D(K)=D(I)
          D(I)=P
          DO 12 J=1,N
            P=V(J,I)
            V(J,I)=V(J,K)
            V(J,K)=P
12        CONTINUE
        ENDIF
13    CONTINUE
      RETURN
      END
```

APPENDIX J

COMPARISON ON THE EFFICIENCY OF THE STEPIT AND SIMPLEX PROCEDURES

Efficiency Comparison Between The STEPIT
And SIMPLEX Procedures

λ	Run	STEPIT		SIMPLEX	
		No. of Function Evaluations	Optimal Value	No. of Function Evaluations	Optimal Value
0.5	1	68	24.75260	113	24.75260
	2	66	25.51996	78	25.51996
	3	58	26.32169	71	26.32169
1.0	4	67	9.47687	84	9.48052
	5	60	9.80626	262	9.80626
	6	66	10.15975	109	10.15975
1.5	7	65	5.24416	74	5.24683
	8	67	5.43185	140	5.43185
	9	73	5.63550	71	5.63550
2.0	10	71	3.43492	74	3.43686
	11	66	3.55848	75	3.55848
	12	62	3.69307	91	3.69307
2.5	13	84	2.46351	75	2.46351
	14	78	2.55490	75	2.55490
	15	79	2.65439	79	2.65439
3.0	16	82	1.85983	93	1.85983
	17	74	1.93063	89	1.93063
	18	61	2.00813	73	2.00813

Note: Data are obtained from the optimal bivariate
EWMA principal component charts

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