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ANALYTICS OF TURBOMACHINE GEOMETRY

Michel Sabourin
Département de génie mécanique, École de technologie supérieure, Montréal, Canada
cc-michel.sabourin@etsmtl.ca

Abstract— For any type of reaction turbomachine, the blade geometry has the same topology and can be modeled by a single mathematical representation. This canonical representation of a blade has been implemented in the Beltrami workbench (BW) which is fully integrated into the FreeCAD modeler. The result is a geometric representation embedded in FreeCAD that allows the blade to be considered as a parametric object that can be analyzed and modified.

Before any complex fluid simulation, analysis of the parametric representation of the blade profile can be useful to detect any deviation from healthy fluid behavior. This analysis can be carried out during the design but in addition, it can be carried out on an existing blade parameterized by a reverse engineering process.

One effect of the use in the design of such a tool is to control the angular deviation of the fluid in a progressive manner by avoiding successions of accelerations-decelerations which are not desirable and could cause losses, cavitation, vibes. This can result in a significant gain in performance and reliability.

Keywords: Hydraulic turbine; Turbine design; Blade design; Reverse engineering; Turbine analytics; FreeCAD; Beltrami

I. INTRODUCTION

Euler's relationship [1] relates the performance of the turbomachine to the angular deviation of the flow. As a hypothesis, it is assumed that the angles of the flow are the same as those of the blade geometry. Thus, it will be possible to control the geometric construction in order to link it to the expected performance.

The profile design method presented here is applicable to any type of reaction turbomachine blading, fixed or rotary: hydraulic turbine, wind turbine, fan, compressor, turbo-fan, propeller, pump, steam turbine, gas turbine, etc. It aims to define the canonical shape of the geometry of any blade.

The analysis of the three-dimensional geometry of the blades is greatly facilitated using two orthogonal transformations. One corresponding to the flow rate and the second to specific energy.

The references Moulin [2], Miller [3], Rossgatterer [4] and several other authors describe the development of the transformations that we take up here.

For several decades, a multitude of software have been developed for this purpose. Even recently, developments have been made to integrate it into the most advanced CAD software, see De Koning [5], Siddappaji [6]. However, this software is not public, it is private or commercial. Therefore, there was a need for open source, free and publicly available software, which motivated the development of the Beltrami workbench (BW) in FreeCAD. “FreeCAD is a free open-source parametric 3D modeler made primarily to design real-life objects of any size. Parametric modeling allows you to easily modify your design by going back into your model history and changing its parameters” [7]. BW is fully integrated in FreeCAD and can be activated directly in the menu. BW is written in Python language and is stored on GitHub [8].

BW can design a blade profile for a specific project. In this case, there will be iterations between the geometric definition and the evaluation method: performance calculated by numerical simulation or measured on a test bench. It can be used to reverse-engineer a blade, obtain the canonical shape of the airfoil and thus determine blade angles and other geometric parameters. Also, it can be used to analyze and then define corrections to be made to an existing profile to improve its performance.

In the following paragraphs, the analytical model and its implementation in the numerical model are presented. Next, a case study involving reverse engineering of an existing turbine is used to demonstrate geometric-based analysis capabilities within BW.

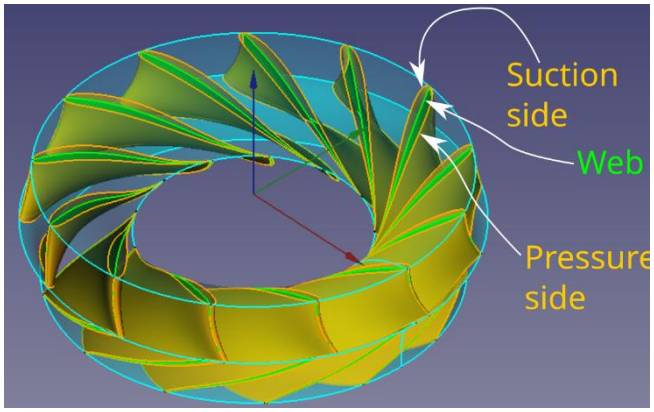
II. THE ANALYTICS MODEL

Two main virtual orthogonal planes are the basis of the method. These planes result from conformal mapping and projection.

A. The bijection between the 3D domain and the pair of virtual 2D planes

The first virtual plan involves the projection, in a cylindrical system of coordinates, of the blade on the meridian plane. The second is a kind of blade flattening which allows to express the angular evolution in two dimensions in a linear cascade.

Figure 1. 3D cascade of blades with suction and pressure sides and the medial virtual surface: the web.

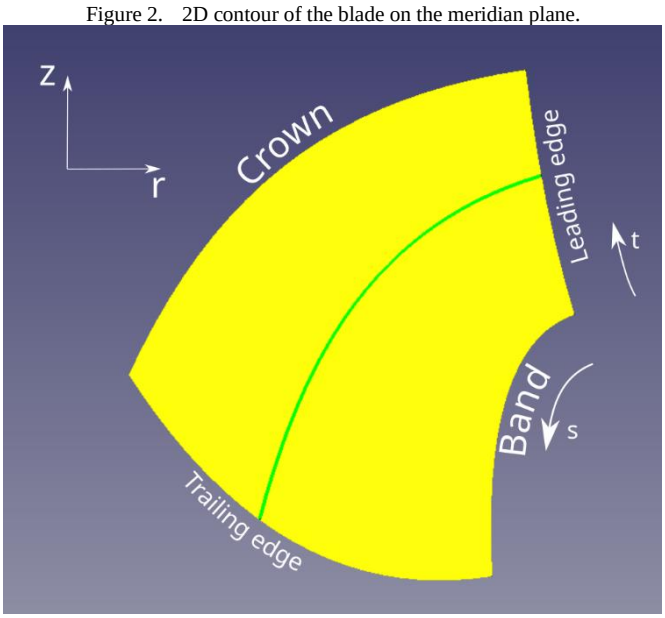


This development is based on the following concepts presented in Fig. 1. The cascade is made up of several blades, each having a pressure and a suction side. A virtual medial surface, here called "web", can be obtained therefrom.

B. *The representation of the flow in the virtual meridian plane*

The entire development of the method is based on the representation of the flow in the meridian plane (Fig. 2) to which other planes will be further associated.

Figure 2. 2D contour of the blade on the meridian plane.

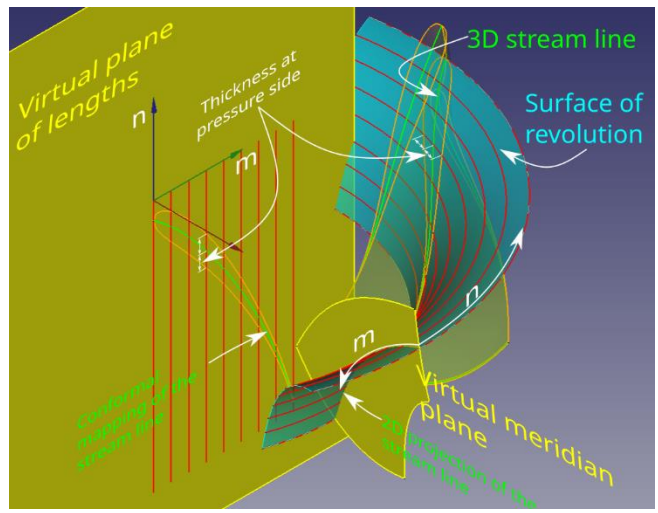


For the transformation on the meridian plane, we retain and keep intact the *r* and *z* values of the 3D domain presented in a cylindrical frame as follows: $(r,z) = f(r, \theta, z)$. In this meridian plane, the blading is bounded by the intersection of the hydraulic face with the crown (hub) defining a curve at iso-$t=1$, the hydraulic side of the band (shroud) with iso-$t = 0$, the leading edge at iso-$s = 0$, the trailing edge at iso-$s = 1$. These borders form a virtual square, in the meridian plane, on which one can interpolate. Each normalized parameters (s,t) of this tile varies between *0* and *1*, with the origin $(0,0)$ located at the intersection of the leading edge with the band. The abscissa *s* is in the flow direction and the ordinate *t* in the transverse direction.

On the meridian plane, between the crown and the band, it is possible, by interpolation, to generate two-dimensional curves at specified iso-*t*, resulting from the trajectories of the fluid, transiting from the leading edge to the trailing edge. These iso-*t* curves are approximately parallel to each other and to the crown and the band. For a curve simulating a streamline, therefore at constant *t*, one can express as a function of *s* all the geometric coordinates of all the domains and transforms. Thus, at a given s we find the coordinates of the point $(r, theta, z)$ in three dimensions and thus the coordinates (r, z) on the meridian plane. For example, the green curve on Fig. 2 is at $t=0,6$ for a variation of *s* from *0* to *1*.

C. *The transformation from the 3D domain to the virtual plane of lengths*

Figure 3. The virtual plane of lengths built with (m,n) coordinates.



As an intermediate step before obtaining the cascade virtual plane, a transformation gives the virtual plane of lengths. The medial 3D surface, the blading web, is assumed to guide the flow uniformly in the 3D domain and is therefore the focus of the design. From a streamline identified at a normalized coordinate *t*, a surface of revolution is created where any point can be located with two coordinates $(m,n)=g(r, \theta, z)$ and which performs a flattening to be represented on the virtual plane of lengths.

The *m*-coordinate in the meridian plane evolves in the direction of flow from the leading edge, see equation (1).

$$m(s) = \int_0^s \sqrt{\left(\frac{dr}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2} \tag{1}$$

The *n*-coordinate in the direction perpendicular to the meridian plane and in a cylindrical system is expressed in equation (2).

$$n(s) = r(s)\,\theta(s) \tag{2}$$

The coordinates *(m,n)* are in millimeters and give real dimensions measurable in the 3D domain. The point of the leading edge on the streamline is observed to be at a specified constant *t* and is inferred from the coordinates *(r, θ, z)* of the streamline in three dimensions.

D. *The transformation of the virtual plane of lengths to the cascade plane*

While the virtual plane of lengths is useful for representing lengths, it is not practical for tracking the evolution of angles in the volume of flow. We must therefore request a transformation of the coordinates.

By dividing the coordinates *(m, n)* by *r* we get the following new variables *(u,v)* given in equations (3) and (4).

$$u(s) = \int_0^s \frac{\frac{dm}{ds}}{r} ds = \int_0^m \frac{1}{r} dm \tag{3}$$

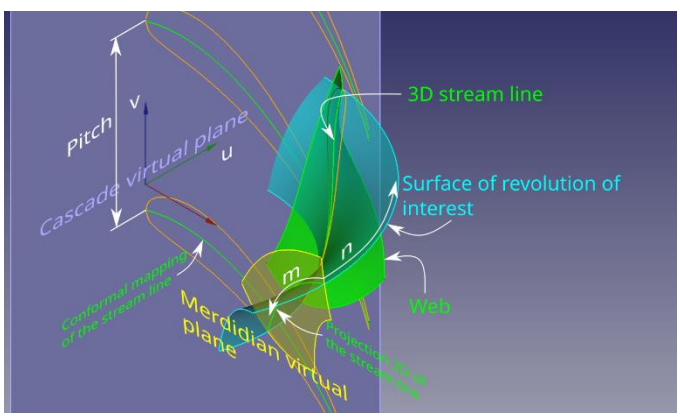
for u(0) = 0 and

$$v(s) = \theta(s) \tag{4}$$

Therefore, the evolution of incidence angle α is defined in equation (5).

$$\tan(\alpha) = \frac{rd\theta}{dm} = \frac{d\theta}{dm/r} = \frac{dv}{du} \tag{5}$$

Figure 4. The cascade virtual plane built with (u,v) coordinates.



The *(u,v)* coordinates of the virtual cascade plane are dimensionless. To represent them in FreeCAD, we multiplied *u* and *v* by 1000 to obtain millimeters, such dimensions are comparable to real entities as shown on Fig.4.

As it is a periodic linear cascade, we can obtain the neighboring blades by calculating the pitch from Z_r, the number of blades in the runner, the equation (6) is for the upper neighbor and equation (7) is for the lower.

$$v_{+1}(s) = \theta(s) + \frac{2\pi}{Z_r} \tag{6}$$

$$v_{-1}(s) = \theta(s) - \frac{2\pi}{Z_r} \tag{7}$$

This blades cascade in 2 dimensions *(u,v)* with linear periodicity is representative of the angular field over the entire domain represented by the surface of revolution and makes it possible to analyze the distribution of the angles.

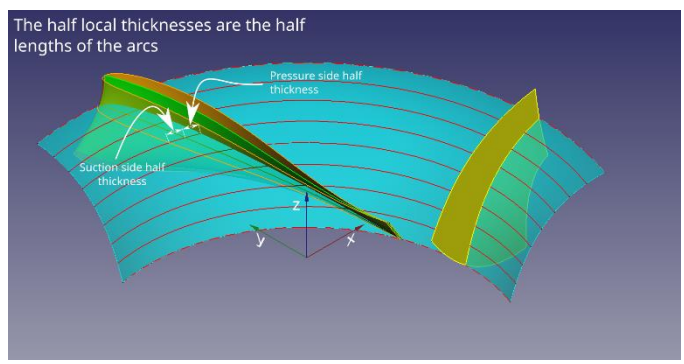
E. *Deconstruction of the 3D profile to obtain the web and the thickness law*

The medial surface or web is obtained by an operation of arithmetic average of the pressure and suction sides in the cylindrical coordinate system.

Thus, for a point on the pressure side (subscript *p)*, characterized by the coordinates *(r,* θ_p*, z)*, it is necessary to find the equivalent point at the same coordinates, *r* and *z* or *m*, on the suction side (subscript *b*). We then determine the coordinates θ_b of this point on the suction side which is equivalent to $n_b = r\theta_b$. Therefore, the web (subscript *a*) angular position of this point can be calculated from equation (8).

$$n_a = r\frac{\theta_b + \theta_p}{2} = r\theta_a \tag{8}$$

Figure 5. Determination of the web surface from suction and pressure sides.



The method to achieve this for the whole surface depends on the canonical shape of the surfaces and can be handled in a three-dimensional geometric modeler like FreeCAD. This is the subject of the next paragraph.

III. THE DIGITAL MODEL

A. *Implementation in the Beltrami workbench*

All the science of the hydraulic engineer will be to define the boundaries of the contour in the meridian plane and the evolution of the angles between the leading edge and the trailing edge. He generally builds on past experience and makes fine adjustments by iterating from geometry to Computational Fluid Dynamics (CFD) and to solid finite element analysis (strength of materials, vibrations).

The canonical definition of the blade is based on a series of points. It requires adjusting the discretized points density to the curvature but is easily managed in FreeCAD.

B. *The user interface*

The principle of use of the Beltrami workbench (BW) is based on the modification of an existing geometry. An integrated user interface is composed of a group of entities dedicated to this task. It uses:

1. A Feature Python "Parametres" [9] which defines and contains the canonical shape of the blade featuring 4 variables for basic data structure like points density and runner rotation direction;
2. The "Meridien" sketch defines the contour of the blade in the virtual meridian plane, it is made of 4 cubic b-splines for a total of 12 control points, Fig.8;
3. the "Tableau_pilote" spreadsheet, Fig. 9, gives the information on the blade distribution of angles, its angular position and its thickness law constructed over the web.

In BW, there is also other groups of information used to build the blade. This information cannot be modified directly but is useful to analyze the blade.

IV. A CASE STUDY: THE REVERSE ENGINEERING OF AN EXISTING BLADE

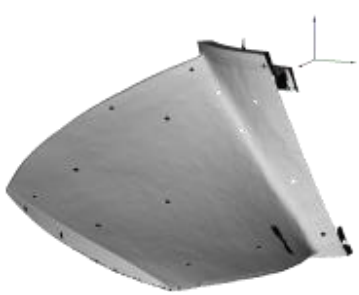
Although the main goal of the BW is to design a blade, this case study demonstrates how to model and assess an existing blade in order to determine its defects and imagine potential local modification that could improve it. A video demonstrates the complete process [10] also described below.

A. *Building the parametric representation of the blade*

1) *The starting point: the stl file of the existing blade.*

This is the most common and useful result of a 3D scan of an existing blade. It must include references to locate the axis of rotation. The *stl* file can be read directly by FreeCAD.

Figure 6. The FreeCAD representation of the *stl* file containing the points measured on the existing blade.



2) *The definition of the 2D contour of the existing blade*

It is preferable to start from a blade profile close to the shape of the existing blade. The BW installation directory contains sample blade profiles useful for this purpose. After loading the appropriate profile from an existing file, the rotation direction can be set in "Parametres". The next task is to adapt the "Meridien" sketch to the existing blade. FreeCAD can create a 360 degrees revolution of all the points contained in the stl file, this locates the overall contour of the blade to which the sketch can be adjusted manually by pulling on control points.

Figure 7. Sketch of de meridian plane in FreeCAD allowing to adjust B-spline 12 control points in order to fit the projection of the existing blade in cylindrical frame.

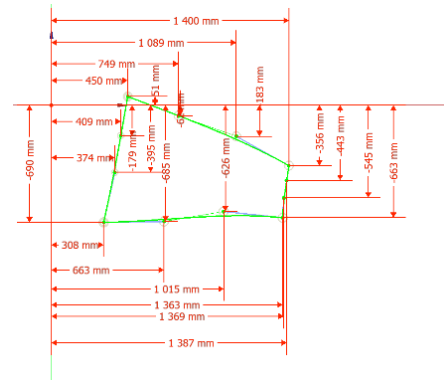
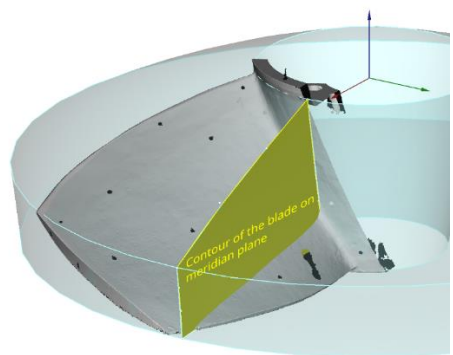
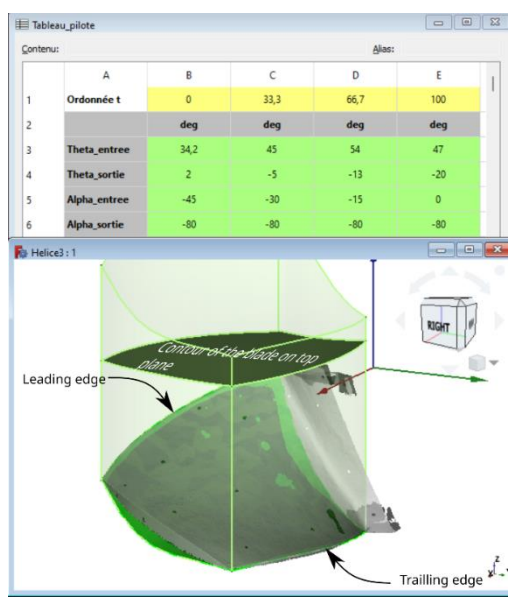


Figure 8. 2D projection of the existing blade on the meridian plane.



3) *The fitting of the 3D contour of the existing blade*

Figure 9. Fitting of the web within blade contour using "*Tableau_pilote*" spreadsheet variables "*Theta_entree*" and "*Theta_sortie*".



	A	B	C	D	E
1	Ordonnée t	0	33,3	66,7	100
2		deg	deg	deg	deg
3	Theta_entree	34,2	45	54	47
4	Theta_sortie	2	-5	-13	-20
5	Alpha_entree	-45	-30	-15	0
6	Alpha_sortie	-80	-80	-80	-80

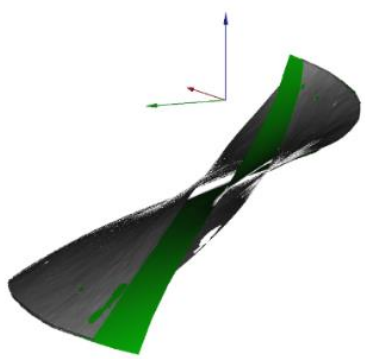
In Fig.9, the variables from the “Tableau_pilote” spreadsheet on green background are used to adjust the web in 3D. The variables “Theta_entree” and “Theta_sortie” represent the angular position of the leading edge and the trailing edge, θ_e and θ_s. All spreadsheet variables are expressed in terms of the *t*-coordinate in the meridian plane. For graphic reasons, here *t*-coordinate changes from 0 to 100 mm, from the outer diameter to diameter of the hub. The four values define the control points of a mathematical b-spline.

At this stage, the 3D web resulting from the parametric model when adjusting the 2D contour is not yet in the middle of the existing blade. This will be performed in the next step.

4) The fitting of the web to the medial surface of the existing blade

This is achieved by modifying the variables of the “*Tableau_pilote*” spreadsheet: "*Alpha_entree*" (α_e) for leading edge angle of the blade and "*Alpha_sortie*" (α_s) for trailing edge angle. The goal is to place the web surface in the middle by trial and error until satisfied. To observe the result inside the blade, some cuts can be made in area of interest, Fig.10.

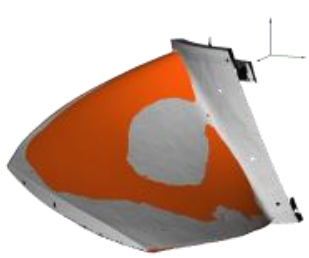
Figure 10. The web in green is now fitted to be in the middle of the existing blade.



5) The construction of pressure and suction sides by adding the thickness law

The same trial and error method is used to adjust spreadsheet variables dedicated to the definition of the thickness law which is applied over the web to obtain the suction and pressure sides.

Figure 11. Pressure and suction sides are built with the thickness law applied over the web surface.



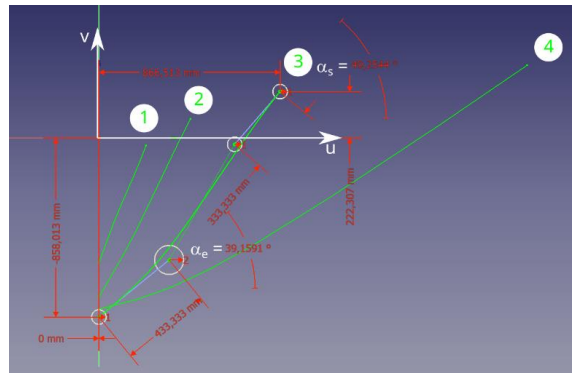
B. Blade analysis

BW uses graphical representation of the spreadsheet variables in order to build the web surface and suction and pressure sides. The goal of reverse engineering was to find these variables to adjust the parametric definition of the blade to the existing one. Therefore, it will be interesting to look at the graphical evolution of variables. Few examples are presented in the following paragraphs.

1) The deflection of the flow by the blade shape

The "Plan_Cascade" group created by BW has for each streamline a sketch showing the deviation as it is perceived by the flow. These streamlines are given by the shape of the web located in the middle of the blade. This evolution from the leading to the trailing edge must be monotonic and smooth. The rate of deviation must be as continuous as possible.

Figure 12. The 4 streamlines in 2D on cascade plane numbered from outer diameter to hub diameter.

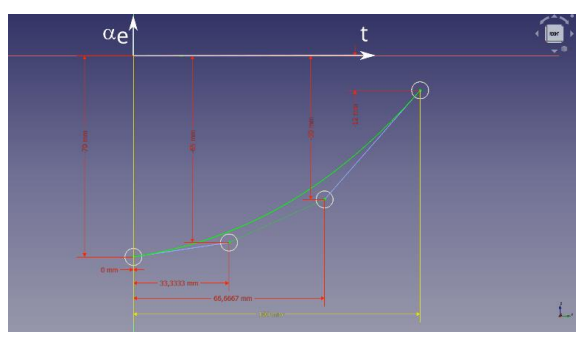


On Fig. 12, 4 streamlines are used to build the web of the blade. They are numbered from outer diameter to hub diameter. The sketch of the third one shows the parameters involved. The *(u,v)*-coordinate system is defined by equations (3) and (4). However, for graphic reasons it is multiplied by 1000 and is shown with mm units.

Geometric analysis can be performed by examining at regularity of the streamlines and how they change. For example, streamline 3 shows a small inflexion in the curve that can be questioned. To take the analysis further, we will need Computional Fluid Dynamics and Finite Element Analysis.

In BW, the “Pilote” group associates each variable of the “Tableau_pilote” spreadsheet with a sketch showing the evolution of the variable in the *t*-coordinates, from the outer diameter (shroud) to the hub diameter.

Figure 13. Evolution of the blade angle of the blade at inlet from outer diameter to hub diameter.

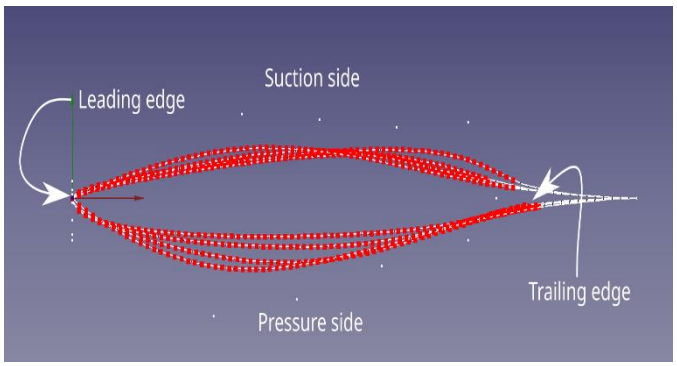


In Fig. 13, the graphical representation of *t*-coordinates is from 0 to 100 mm, from the outer diameter (shroud) to the hub diameter and the ordinate is the blade angle at inlet in degrees represented in mm. We observe a smooth evolution of the angle.

2) The thickness laws

In BW, the group "Plan_Epaisseurs" has a sketch for each streamline. The relative rigidity of the curve prevents the reproduction of defects of the existing blade. It is therefore continuous. However, according to the rules imposed at the basis of BW, it would be necessary to have equality, as far as possible, between the thickness laws of the pressure and the suction sides, in order to preserve the web representing the average direction of flow.

Figure 14. For each streamline, thickness distribution with control points (white dots) and discretization dots (red).

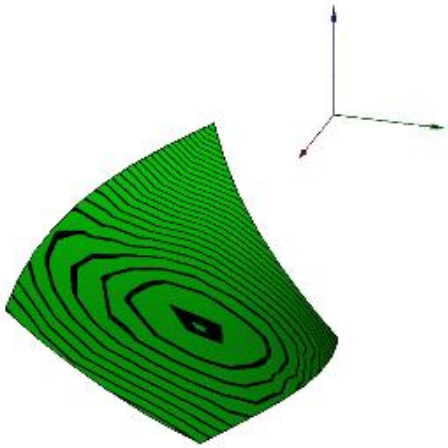


This is not what can be observed on Fig. 14. There is no symmetry probably due to lack of precision in the fabrication process.

3) Curvature analysis

Another workbench in FreeCAD is the Curves workbench. It provides a tool for analizing the curvature of surfaces by drawing lines reflected on them. It can be useful to detect defects. On Fig. 15, when applied to the web surface of the blade, we can see a smooth distribution of curvature, but the diagonally circled pattern can be questioned.

Figure 15. Reflection lines over the web surface showing a smooth pattern in diagonal with a peak that can be questionable.



V. Conclusion

The Beltrami workbench is a robust and user-friendly tool for designing and to analyzing turbomachinery blades. It provides parametric geometry perfectly embedded in the set of FreeCAD entities. It facilitates creation and modification of parametric drivers. The resulting blade geometry can be exported to other simulation software under all common communication protocols. It is free, open source and user friendly. Knowledge of blade analysis is fundamental to understanding the turbomachine behavior and performance.

References

[1] Sabourin M, "La relation d'Euler", page from web site SimTurb, https://michelsabourin.scenari-community.org/SimTurbTech/co/1_222_Euler.html.

[2] Moulin C., Wegner M., Eremeef R. et Phong V., La Houille Blanche, N°7-8 (Novembre 1977), pp. 617-627, "Méthodes de tracé des turbomachines hydrauliques". https://www.shf-lhb.org/articles/lhb/abs/1977/05/lhb1977045/lhb1977045.html

[3] Perry L. Miller IV, James H. Oliver, David P. Miller and Daniel L. Tweedt, National Aeronautics and Space Administration, Lewis Research Center, Cleveland, Ohio, "BladeCAD : An interactive Geometric Design Tool for Turbomachinery Blades", July 1996. https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19960047133.pdf

[4] Rossgatterer M., Jüttler B., Kapl M., Della Vecchia G., "Medial design of blades for hydroelectric turbines and ship propellers", Computer & Graphics, March 15, 2012.

[5] De Koning R.C.W, "Development of a Parametric 3D Turbomachinery Blade modeler", Master of Science Thesis, Delft University of Technology, August 28, 2015.

[6] Siddappaji Kiran, "Parametric 3D Blade Geometry Modeling Tool for Turbomachinery Systems", Master of Science Thesis, University of Cincinnati, October 5, 2012.

[7] https://www.freecad.org/ "FreeCAD, your own 3D parametric modeler".

[8] https://github.com/Simturb/Beltrami, "FreeCAD Beltrami Workbench".

[9] https://wiki.freecad.org/App_FeaturePython, "App FeaturePython".

[10] https://www.youtube.com/watch?v=qfiIO525jTI, "Comment transformer un relevé d'une pale existante en géométrie paramétrisée dans FreeCAD" with English subtitles.

WWW Ressources

SimTurb YouTube channel https://www.youtube.com/@simturb features more than 8 videos in Beltrami playlist .

The site SimTurb is dedicated to hydraulic turbines, it includes the complete theory on Beltrami workbench and the Beltrami user manual.

The Beltrami source code is on GitHub.