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Bipolar Valued Intutionistic Multi Fuzzy Normal Subnear-Ring of a Near-Ring

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Abstract: In this paper, bipolar valued intutionistic multi fuzzy normal subnear-ring of a near-ring is introduced and some theorems are stated and proved.

Keywords: Bipolar valued fuzzy subset, Bipolar valued multi fuzzy subset, Bipolar valued intutionistic multi fuzzy subnear-ring, Bipolar valued intutionistic multi fuzzy normal subnear-ring, Product and strongest bipolar valued intutionistic multi fuzzy subnear-ring.

1. Introduction

In 1965, Zadeh [9] introduced the notion of a fuzzy subset of a universal set. Zhang [10, 11] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [3, 4]. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [4]. Anitha et al. [1, 2] introduced the bipolar valued fuzzy subgroup. Shyamala and Shanthi [7] have introduced the bipolar valued multi fuzzy subgroups of a group. Yasodara and Sathappan [8] defined the bipolar valued multi fuzzy subsemirings of a semi ring. Bipolar valued multi fuzzy subnear-ring of a near-ring has been introduced by Muthukumaran and Anandh [5]. In this paper, the concept of bipolar valued intuitionistic multi fuzzy normal subnear-ring of a near-ring is introduced and established some results.

2. Preliminaries

Definition 2.1 ([10]). A bipolar valued fuzzy set (BVFS) $\mathcal{B}$ in X is defined as an object of the form $\mathcal{B} = \{\langle x, \mathcal{B}^+(u), \mathcal{B}^-(u)\rangle / x \in X\}$, where $\mathcal{B}^+ : X \to [0,1]$ and $\mathcal{B}^- : X \to [-1,0]$. The positive membership degree $\mathcal{B}^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

Definition 2.2. A bipolar valued Intuitionistic fuzzy set (BVIFS) $\mathcal{B}$ in X is defined as an object of the form $\mathcal{B} = \{\langle x, \mathcal{B}^+(\alpha(x),\beta(x)), \mathcal{B}^-(\alpha(x),\beta(x)),\rangle / x \in X\}$, where $\mathcal{B}^+ : X \to [0,1]$ and $\mathcal{B}^- : X \to [-1,0]$. The positive membership degree $\mathcal{B}^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set

Definition 2.3. A bipolar valued Intuitionistic multi fuzzy set (BVIMFS) A in X is defined as an object of the form

$$\mathcal{B}=\left\{\left\langle \begin{matrix} x, \mathscr{B}_1^+(\alpha(x),\beta(x))\ \mathscr{B}_2^+(\alpha(x),\beta(x)) \dots\dots\dots\dots \mathscr{B}_n^+(\alpha(x),\beta(x)), \\ \mathscr{B}_1^-(\alpha(x),\beta(x))\ \mathscr{B}_2^-(\alpha(x),\beta(x)) \dots\dots\dots\dots \mathscr{B}_n^-(\alpha(x),\beta(x)) \end{matrix}\right\rangle / x \in X\right\},$$

where $\mathscr{B}_i^+ : X \to [0,1]$ and $\mathscr{B}_i^- : X \to [-1,0]$ for all i. The positive membership degree $\mathscr{B}_i^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathscr{B}_i^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

Definition 2.4. Let $\mathcal{R}$ be a near ring. A BVIMFS $\mathcal{B}$ is said to be a bipolar valued intuitionistic multi-fuzzy subnear-ring $\mathcal{R}$ (BVIMFSNR) if the following conditions are satisfied, for all i

$i)\ \mathscr{B}_i^+(x-y) \geq \min\{\mathscr{B}_i^+(\alpha(x)-\alpha(y)), \mathscr{B}_i^+(\beta(x)-\beta(y))\}$

$ii)\ \mathscr{B}_i^+(xy) \geq \min\{\mathscr{B}_i^+(\alpha(x),\beta(x)), \mathscr{B}_i^+(\alpha(y),\beta(y))\}$

$iii)\ \mathscr{B}_i^-(x-y) \leq \max\{\mathscr{B}_i^-(\alpha(x)-\alpha(y)), \mathscr{B}_i^-(\beta(x)-\beta(y))\}$

$iv)\ \mathscr{B}_i^-(xy) \geq \min\{\mathscr{B}_i^-(\alpha(x),\beta(x)), \mathscr{B}_i^-(\alpha(y),\beta(y))\}$, for all x,y $for\ all\ x,y \in \mathscr{R}$

Definition 2.5. Let $\mathcal{R}$ be a near ring. A bipolar valued intuitionistic multi fuzzy subnear–ring of $\mathcal{R}$ is said to be a bipolar valued intuitionistic multi fuzzy normal subnear –ring (BVIMFNSNR) of $\mathcal{R}$ if

$i)\ \mathcal{A}_i^+(\alpha(x+y),\beta(x+y)) = \mathcal{A}_i^+(\alpha(y+x),\beta(y+x)$

$ii)\ \mathcal{A}_i^-(\alpha(x+y),\beta(x+y)) = \mathcal{A}_i^-(\alpha(y+x),\beta(y+x))$

$iii)\ \mathcal{A}_i^+(\alpha(xy),\beta(xy)) = \mathcal{A}_i^+(\alpha(yx),\beta(yx))$

$iv)\ \mathcal{A}_i^-(\alpha(xy),\beta(xy)) = \mathcal{A}_i^-(\alpha(yx),\beta(yx))$ for all x,y in $\mathcal{R}$ and for all i.

Definition 2.6. Let $\mathcal{A} = (\mathcal{A}_1^+\mathcal{A}_2^+ \dots\dots\dots\dots \mathcal{A}_n^+, \mathcal{A}_1^-\mathcal{A}_2^- \dots\dots\dots\dots \mathcal{A}_n^-)$ be a bipolar valued intuitionistic multi fuzzy subset in a set $\mathcal{S}$, the strongest bipolar valued intuitionistic fuzzy relation on $\mathcal{S}$ that is bipolar valued intuitionistic multi fuzzy relation on $\mathcal{A}$ is

$$\mathcal{V} = \left\{\left\langle \begin{matrix} (\alpha(xy),\beta(xy)), \mathcal{V}_1^+(\alpha(xy),\beta(xy)) \dots\dots\dots\dots.. \mathcal{V}_n^+(\alpha(xy),\beta(xy)) \\ \mathcal{V}_1^-(\alpha(xy),\beta(xy)) \dots\dots\dots\dots.. \mathcal{V}_n^-(\alpha(xy),\beta(xy)) \end{matrix}\right\rangle / x,y \in \mathcal{S}\right\}$$

Where $\mathcal{V}_i^+(\alpha(xy),\beta(xy)) = \min\{\mathcal{A}_i^+(\alpha(xy),\beta(xy))\}$ and

$$\mathcal{V}_i^-(\alpha(xy),\beta(xy)) = \max\{\mathcal{A}_i^-(\alpha(xy),\beta(xy))\} \quad for\ all\ x,y \in \mathcal{S}\ and\ for\ all\ i$$

3. Properties

Definition 3.1. Let $\mathcal{R}$ be a near ring and $\mathcal{B}$ be an bipolar valued Intuitionistic multi fuzzy subset of a ring $\mathcal{R}$. Then $\mathcal{B}$ is called an bipolar valued intuitionistic multi fuzzy right ideal of $\mathcal{R}$ if $\mathcal{B}$ is an bipolar valued intuitionistic multi fuzzy sub near ring of $\mathcal{R}$ and satisfies for all $x,y \in \mathscr{R}$, $\mathrm{i} = 1,2,\ldots$.

i) $\mathscr{B}_i^+(x-y) \geq \min\{\mathscr{B}_i^+(\alpha(x-y),\beta(x-y))\}$

$\mathscr{B}_i^+(x-y)I \leq \max \mathrm{I}\{\mathscr{B}_i^+(\alpha(x-y),\beta(x-y))\}$

ii) $\mathscr{B}_i^-(x-y) \leq \max\{\mathscr{B}_i^-(\alpha(x-y),\beta(x-y))\}$

$\mathscr{B}_i^-(x-y)I \geq \min \mathrm{I}\{\mathscr{B}_i^-(\alpha(x-y),\beta(x-y))\}$

iii) $\mathscr{B}_i^+((x+i)y-xy) \geq \mathscr{B}_i^+(\mathrm{i})$ & $\mathscr{B}_i^+((x+i)y-xy)I \leq \mathscr{B}_i^+(\mathrm{i})\,\mathrm{I}$

$\mathscr{B}_i^-((x+i)y-xy) \leq \mathscr{B}_i^-(\mathrm{i})$ & $\mathscr{B}_i^-((x+i)y-xy)I \geq \mathscr{B}_i^-(\mathrm{i})\mathrm{I}$

iv) $\mathscr{B}_i^+(y+x-y) \geq \mathscr{B}_i^+(\alpha(x),\beta(x))$ & $\mathscr{B}_i^+(y+x-y)I \leq \mathscr{B}_i^+(\alpha(y),\beta(y))I$

$\mathcal{B}_i^-(y+x-y) \leq \mathcal{B}_i^-(\alpha(x),\beta(x))$ & $\mathcal{B}_i^-(y+x-y)I \geq \mathcal{B}_i^-(\alpha(y),\beta(y))\,I$

Definition 3.2. Let $\mathcal{R}$ be a near ring and $\mathcal{B}$ be an bipolar valued Intuitionistic multi fuzzy subset of a ring $\mathcal{R}$. Then $\mathcal{B}$ is called an bipolar valued intuitionistic multi fuzzy left ideal of $\mathcal{R}$ if $\mathcal{B}$ is an bipolar valued intuitionistic multi fuzzy sub near ring of $\mathcal{R}$ and satisfies for all $x,y \in \mathscr{R}$, $\mathrm{i} = 1,2,\ldots$.

i) $\mathscr{B}_i^+(x-y) \geq \min\{\mathscr{B}_i^+(\alpha(x-y),\beta(x-y))\}$

$\mathrm{I}\,\mathscr{B}_i^+(x-y) \leq \max\{I\mathscr{B}_i^+(\alpha(x-y),\beta(x-y))\}$

ii) $\mathscr{B}_i^-(x-y) \leq \max\{\mathscr{B}_i^-(\alpha(x-y),\beta(x-y))\}$

$\mathrm{I}\,\mathscr{B}_i^-(x-y) \geq \min\{I\mathscr{B}_i^-(\alpha(x-y),\beta(x-y))\}$

iii) $\mathscr{B}_i^+(\alpha(xy),\beta(xy)) \geq \mathscr{B}_i^+(\beta(\mathrm{xy}))$ & $\mathrm{I}\mathscr{B}_i^+(\alpha(xy)) \leq I\mathscr{B}_i^+(\alpha(\mathrm{xy}))$

$\mathscr{B}_i^-(\alpha(xy)) \leq \mathscr{B}_i^-(\beta(\mathrm{xy}))$ & $\mathrm{I}\mathscr{B}_i^-(xy) \geq I\mathscr{B}_i^-(\alpha(\mathrm{xy}))$

iv) $\mathscr{B}_i^+(y+x-y) \geq \mathscr{B}_i^+(\alpha(x),\beta(x))$ & $I\,\mathscr{B}_i^+(y+x-y) \leq I\mathscr{B}_i^+(\alpha(y),\beta(y))$

$\mathcal{B}_i^-(y+x-y) \leq \mathcal{B}_i^-(\alpha(x),\beta(x))$ & $I\,\mathcal{B}_i^-(y+x-y) \geq I\mathcal{B}_i^-(\alpha(y),\beta(y))$

Proposition 3.3. If an bipolar valued intuitionistic multi fuzzy subsets of $\mathcal{R}$ satisfies the properties

$\mathscr{B}_i^+(x-y) \geq \min\{\mathscr{B}_i^+(\alpha(x-y),\beta(x-y))\}$, $\mathscr{B}_i^-(x-y) \leq \max\{\mathscr{B}_i^-(\alpha(x-y),\beta(x-y))\}$

then

i) $\mathcal{A}_i^+(0) \geq \mathcal{A}_i^+(\alpha(x))$ & $\mathcal{A}_i^-(0) \leq \mathcal{A}_i^-(\alpha(x))$

ii) $\mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(\alpha(x))$ & $\mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(\alpha(x))$

$x,y \in \mathscr{R}$ & $i = 1,2,\ldots$.

Proof

We have that for any $x \in \mathscr{R}$, i=1,2,……..

i) $\mathcal{A}_i^+(0) = \mathcal{A}_i^+(\alpha(x) - \alpha(x))$

$\geq \min\{ \mathcal{A}_i^+(\alpha(x)), \mathcal{A}_i^+(\alpha(x)\}$

$= \mathcal{A}_i^+(\alpha(x)$

Hence $\mathcal{A}_i^+(0) \geq \mathcal{A}_i^+(\alpha(x))$

$\mathcal{A}_i^-(0) = \mathcal{A}_i^-(\alpha(x) - \alpha(x)))$

$\leq \max\{ \mathcal{A}_i^-(\alpha(x)), \mathcal{A}_i^-(\alpha(x)\}$

$= \mathcal{A}_i^-(\alpha(x)$

Hence $\mathcal{A}_i^-(0) \leq \mathcal{A}_i^-(\alpha(x))$

ii) $\mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(0 - \alpha(x))$

$\geq \min\{ \mathcal{A}_i^+(0), \mathcal{A}_i^+(\alpha(x)\}$

$= \mathcal{A}_i^+(\alpha(x)$

Hence $\mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(\alpha(x))$

$\mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(0 - \alpha(x))$

$\leq \max\{ \mathcal{A}_i^-(0), \mathcal{A}_i^-(\alpha(x)\}$

$= \mathcal{A}_i^-(\alpha(x)$

Hence $\mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(\alpha(x))$

Proposition 3.4.

If $\mathcal{B} = (\mathcal{B}_1^+, \mathcal{B}_2^+ \dots \dots .. \mathcal{B}_n^+, \mathcal{B}_1^-, - \cdots \dots .. \mathcal{B}_n^-)$ $and\ \mathcal{C} = (\mathcal{C}_1^+ \mathcal{C}_2^+ \dots \dots \dots . \mathcal{C}_n^+, \mathcal{C}_1^- \mathcal{C}_2^- \dots \dots \dots . \mathcal{C}_n^-)$ are two BVIMFNSNR with degree n of a near ring $\mathcal{R}$, then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of N.

Proof

Let $\mathcal{D} = \mathcal{B} \cap \mathcal{C}$, by two BVIMFSNRs with degree n of a near ring then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFSNR of N. Let $\mathcal{D}$ is a BVIMFSNR of the near ring N.

Let $u, v \in N$, $for\ all\ i$

$\mathcal{D}_i^+(u + v) = \min\left\{\mathcal{B}_i^+\left(u(x) + v(x)\right), u(y) + v(y)\right), \mathcal{C}_i^+\left(u(x) + v(x)\right), u(y) + v(y))\}$

$= \mathcal{D}_i^+(u + v)$ for every $u, v \in N$

$\mathcal{D}_i^+(uv) = \min\{\mathcal{B}_i^+(u(xy), v(xy)), \mathcal{C}_i^+\left(u(xy), v(xy)\right)\}$

$= \mathcal{D}_i^+(uv)$ for every $u, v \in N$

$\mathcal{D}_i^-(u + v) = \min\left\{\mathcal{B}_i^-\left(u(x) + v(x)\right), u(y) + v(y)\right), \mathcal{C}_i^-\left(u(x) + v(x)\right), u(y) + v(y))\}$

$= \mathcal{D}_i^-(u + v)$ for every $u, v \in N$

$\mathcal{D}_i^-(uv) = \min\{\mathcal{B}_i^-(u(xy), v(xy)), \mathcal{C}_i^-\left(u(xy), v(xy)\right)\}$

$= \mathcal{D}_i^-(uv)$ for every $u, v \in N$

$\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of the near ring N.

4. Conclusion

Bipolar valued multi fuzzy normal subnear-ring of a near-ring is defined and their properties are proved. We can extend these concepts into many algebraic systems.

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