Journal of Advanced Zoology

ISSN: 0253-7214

Volume 44 Issue S-5 Year 2023 Page 108:117

Multi – Fuzzy Ideals of Γ - Near Ring

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Article History

Received: 06 June 2023

Revised: 05 Sept 2023

Accepted: 07 Oct 2023

Abstract

Multi – fuzzy set theory is an extension of fuzzy set theory. In this paper, we define the multi fuzzy ideals of Γ - near ring. Also, the notion of anti multi fuzzy ideals of a Γ - near ring is introduced and investigated some related properties. This concept of multi fuzzy ideals of a Γ - near ring is a generalization of the concept of fuzzy ideals in Γ - near rings. Also, we define the multi-level subsets and multi anti level subsets of a multi fuzzy sub Γ - near ring of a Γ- near ring. In this paper we define the multi level subset and multi anti level subset of AUB. The purpose of this study establishes the algebra of multi fuzzy Γ- near ring.

Keywords: *Fuzzy set, multi fuzzy set, fuzzy sub Γ - near ring, multi fuzzy sub Γ - near ring, multi – fuzzy ideals, multi anti fuzzy ideals, Cartesian product of multi fuzzy sets*

1. Introduction

In 1965, Zadeh [1] proposed the notion of fuzzy set. Later A. Rosenfeld [5] developed fuzzy groups in 1971. The concept of ring, a generalization of a ring in algebra was introduced and studied first by Nobusawa [14] in 1964 and generalization by Barnes [15] in1966. A generalization of both the concepts near ring and the ring namely near ring was introduced by Bh. Satyanarayana [10,11,12], in 1999.They developed theoretically some important concepts in near ring. Later the authors S. Ragamai, Y. Bhargavi, T. Eswarlal [17] developed theory of fuzzy and L fuzzy ideals of near rings. Many authors developed concepts of fuzzy theory and applications in various fields. Many extensions and generalizations of Zadeh's fuzzy set theory are developed so far. But fuzzy set is not enough to study some reality problems. Characterization problems like complete colour characterization of colour images, taste recognition of food items, decision making problems with multi aspects etc. cannot completely be characterized by a single membership function of Zadeh's fuzzy sets. Some of these problems can completely be characterized by multi-membership functions of suitable multi-fuzzy sets. To consider such situations Yager defined a fuzzy bag to be a crisp bag of X x [0,1] in 1986. Miyamoto [16] later redefined it as fuzzy multi sets in 2000. Further studied concept of multi fuzzy sets by Sabu Sabestain [2,3,4] and re defined multi fuzzy sets is a generalisation of theories of fuzzy sets, fuzzy sets and intuitionistic fuzzy sets. K.Hemabala and B.Srinivas kumar[18,19,20] established algebraic properties of neutrosophic multi fuzzy sets. In this paper, we define the multi fuzzy sets of Γ - near ring and verified union and intersection of multi fuzzy ideals in Γ - near ring. Also, the notion of multi fuzzy ideals of Cartesian product and verified multi anti fuzzy ideals of Γ – near ring. We introduced multi anti level fuzziness.

Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition: Let be a non empty set and be a fuzzy set over is defined by = { } where [0,1].

Definition: Let X be a non empty set. A multi – fuzzy set in X is defined as a set of ordered sequence

= {(,,………….): } where : X→ [0,1] for all Where …………. one can append any number of zeros at the right end of a finite sequence of the membership values of x.

Remarks:

1. If the sequences of the membership functions have only k- terms (finite number of terms), i is called the dimension of A.
2. The set of all multi – fuzzy sets in X of dimension k is denoted by Mi FS(X)
3. The multi fuzzy membership function μA is a function from X to [0,1]i such that for all x in X,

μA (x) = (μ1(x) , μ2(x)………μi(x))

4. For the sake of simplicity, we denote the multi fuzzy set

A = {(x, μ1(x),μ2(x)……..μi(x)) : x ϵ X} as A = (μ1, μ2………..μi)

Definition: Let k be a positive integer and let A and B in Mk FS(X),

where A = (A1, A2,……Ai) and B = (B1, B2,…….. Bi), then we have the following relations and operations

1. A B if and only if An ≤ Bn for all i =1,2……i.
2. A = B if and only if An = Bn for all i = 1,2,…….i.
3. A ∪ B = (A1 ∪ B1, …….An ∪ Bn) = {(x, max (A1(x),B1(x)),…………max (An(x),Bn(x)) : x ϵ X}
4. A ∩ B = (A1∩B1,………..An ∩Bn) ={(x, min (A1(x),B1(x))……min (An(x),Bn(x)) : x ϵ X}
5. The multi – fuzzy complement of multi fuzzy set A is

C(A) = {x, C(A1(x)), C(A2(x)),…….C(An(x)): x ϵ X}

where C(An(x)) is the complement of An(x) for all n = 1 to i

C(An(x)) = 1-An(x)

Definition: A non – empty set N with two binary operations '+'(addition) and '.'(multiplication) is called a near ring if it satisfies the following axioms

1 (N, +) is a group

2 (N,.) is a semi group

3 (x + y) . z = x . z + y. z for all x, y, z ϵ N

Precisely speaking it is a right near – ring, because it satisfies the right distributive law. We will use the word "near- ring" to mean "right near ring". We denote x y instead of x. y. Moreover, a near ring N is said to be a zero – symmetric if r.0=0 for all rϵ N, where 0 is the additive identity in N

Definition: Let (R, +) be a group and Γ be a non – empty set then R is said to be a Γ - near ring if there exists a mapping Rx Γ x R R (the image of (x, α, y) is denoted by (x α y) satisfying the following

Conditions

1. (x + y) α z= x α z + y α z

2. (x α y) β z=x α (y β z)

For all x, y, z ϵR and α, β ϵ Γ

2.6 Definition: Let R be a Γ - near ring A normal subgroup (I,+) of (R,+)is called

1. A left ideal if x α (y +i)-x α yϵ I, for all x, yϵ R, αϵ Γ ,i ϵI

2. A right ideal if i α x ϵ I for all x ϵ R, αϵ Γ, i ϵ I

3. An ideal if it is both a left ideal and a right ideal of R

A Γ - near ring R said to be a zero symmetric if a α 0 = 0 for all a ϵ R and αϵ Γ where 0 is the additive identity in R

Definition: A subset M of a Γ- near ring R is said to a sub Γ - near ring if there exist a mapping

M x Γ x M M such that

1. (M, +) be a subgroup of (R, +)

2. (x+y) γ z = x γ z+y γ z for every x,y,z ϵ M and γ ϵ Γ

3. (x γ y) z = x γ (y z) for every x,y,z ϵ M and γ, ϵ Γ

Definition: Let R be a Γ - near ring. A fuzzy set of R is a function A: R [0,1]. Let A be a fuzzy set of R. For α ϵ [0,1] the set Aα = {x ϵ R: A(x) ≥ α} is called a level subset of A

Definition: A fuzzy subset A of a Γ - near ring R is said to be a fuzzy Γ - near ring of R if it satisfies the following conditions

1. A(x-y) ≥ min{A(x), A(y)} for all x,y ϵ R

2. A(xαy) ≥ min{A(x), A(y)} for all x,y ϵ R and α ϵ Γ

Definition: A fuzzy Γ - near ring A of R is called a fuzzy ideal if it satisfies the following conditions:

1. A(y+x-y) ≥ A(x) for all x,y,z ϵ R

2. A(xαy) ≥ A(y) for all x,y ϵ R and α ϵ Γ

3. A(xα(z+y)-xαy) ≥ A(z) for all x,y,z ϵ R and α ϵ Γ

Note: If A is a fuzzy ideal of Γ - near ring R then A(0) ≥ A(x) for all x ϵ R

Definition: Let k be a positive integer and A and B be two multi fuzzy sets of dimension K on Γ- near ring R then Cartesian product of multi fuzzy sets of A and B is defined by

AxB = {(x,y), min (A1(x) ,B1(y)),……..min(Ak(x),Bk(y))/(x ,y) ϵ RXR}

Multi Fuzzy Ideals Of Γ - near ring

Definition: Let be Γ - near ring, a multi fuzzy set of is called a multi fuzzy

Γ - near ring if it satisfies the following conditions

1. (-) ≥ min{), ( )} for all , ϵ

   ,……. ≥ ,….)}

2. ( ) ≥ min{ ( ), ( )} for all , ϵ and ϵ Γ

   ,……. ≥ ,……….)}

Where the membership sequence of x and y is defined as a non increasing sequence of

membership values of x and y.

+	0	1	2	3	0	0	1	2	3	1	0	1	2	3
0	0	1	2	3	0	0	0	0	0	0	0	0	0	0
1	1	0	3	2	1	0	1	1	1	1	0	0	0	0

2	2	3	1	0	2	0	2	2	2	2	0	0	0	0
3	3	2	0	1	3	0	3	3	3	3	0	0	0	0

Then $\mathcal{R}$ is a Γ - near ring. Define a multi fuzzy subset $A: \mathcal{R} \rightarrow [0,1]$by

A(0) = {0.8,0.7,0.6}, A(1) = {0.7,0.6,0.5}, A(2) = {0.6,0.5,0.4}, A(3) = {0.3,0.2,0.1}
Clearly A is a multi fuzzy Γ - near ring of $\mathcal{R}$.

Example: Let $\mathcal{R}$ be the set of the 2x2 matrices over the set of integers and $I_{2x2} \in \Gamma$, Then $\mathcal{R}$ is a Γ near ring, Define a multi fuzzy Γ - near ring of $\mathcal{R}$ as

$$A(x) = \begin{cases} (0.2,0.2,0.1) & if\ x \in \begin{pmatrix} p & 0 \\ q & 0 \end{pmatrix} \\ (0.8,0.7,0.5) & otherwise \end{cases}$$

Clearly A is a multi fuzzy Γ - near ring of $\mathcal{R}$.

Definition: Let A and B are two multi fuzzy Γ-near ring of $\mathcal{R}$ then A+B can be defined by $(A+B)(x) = \sup(\min(A(y), B(z))), \quad x = y+z$

= 0, otherwise

i.e $(\mu_A^1(x)+ \mu_B^1(x) \ldots \ldots \mu_A^i(x)+ \mu_B^i(x))=sup\{min(\mu_A^1(y), \mu_B^1(z)), \ldots\ldots\ min(\mu_A^i(y), \mu_B^i(z))\}$

Example: From the example 3.2 we define multi fuzzy Γ-near rings A and B by

A(x) = (0.8,0.7), x=0

= (0.2,0.1), otherwise

B(x) = (0.9,0.8), x=0

= (0.1,0.1), otherwise

By simple calculation shows that

$$(A+B)(x) = (0.8,0.7), \qquad x = 0$$

= (0.1,0.1), 0therwise

Proposition: Let A and B are two multi fuzzy Γ-near ring of $\mathcal{R}$ then A+B is also multi fuzzy Γ-near ring of $\mathcal{R}$.

Algebraic properties of multi fuzzy Γ-near ring

Consider the 3.2 we have A(0) = {0.8,0.7,0.6}, A(1) = {0.7,0.6,0.5}, A(2) = {0.6,0.5,0.4}, A(3) = {0.3,0.2,0.1}

Also we define B(0) ={0.6,0.5,0.4}, B(1) ={0.6,0.6,0.4}, B(2) ={0.7,0.6,0.4}, B(3) ={0.8,0.7,0.6}

Clearly $A \cap B$ is a multi fuzzy Γ-near ring of $\mathcal{R}$ but $A \cup B$ not a multi fuzzy Γ-near ring.

Since,

$$(A \cup B)(3-1) \ngeq \min\{(A \cup B)(3), (A \cup B)(1)\}$$

$(0.7,0.6,0.4) \ngeq min\{(0.8,0.7,0.6),(0.7,0.6,0.5)\}$

$(0.7,0.6,0.4) \ngeq (0.7,0.6,0.5)$

We observed from the example if $A \subseteq B$ or $B \subseteq A$ then $A \cup B$ is a multi fuzzy Γ-near ring

Theorem: Let A and B are two multi fuzzy Γ-near ring of $\mathcal{R}$. Then $A \cap B$ is a multi fuzzy Γ-near ring of $\mathcal{R}$.

Proof:

Let A and B are two multi fuzzy Γ-near rings of $\mathcal{R}$.

Let $x, y \in \mathcal{R}$ and $\tau \in \Gamma$

1. $(A \cap B)(x - y) = \min\{A(x - y), B(x - y)\}$

$\geq \min\{\min\{A(x), A(y)\}, \min\{B(x), B(y)\}\}$

$\geq \min\{\min\{A(x), B(x)\}, \min\{A(y), B(y)\}\}$

$\geq \min\{(A \cap B)(x), (A \cap B)(y)\}$

2. $(A \cap B)(x \tau y) = \min\{A(x \tau y), B(x \tau y)\}$

$\geq \min\{\min\{A(x), A(y)\}, \min\{B(x), B(y)\}\}$

$\geq \min\{\min\{A(x), B(x)\}, \min\{A(y), B(y)\}\}$

$\geq \min\{(A \cap B)(x), (A \cap B)(y)\}$

Theorem : Let A and B are two multi fuzzy Γ-near ring of $\mathcal{R}$. Then $A \cup B$ is a multi fuzzy Γ-near ring of $\mathcal{R}$ if $A \subseteq B$ or $B \subseteq A$.

Proof:

Let A and B are two multi fuzzy Γ-near ring of $\mathcal{R}$

Let $x, y \in \mathcal{R}$ and $\tau \in \Gamma$

Suppose $A \subseteq B$

1. $(A \cup B)(x - y) = \max\{A(x - y), B(x - y)\}$

$= A(x - y)$

$\geq \min\{A(x), A(y)\}$

$\geq \min\{\max\{\{A(x), B(x)\}, \max\{A(y), B(y)\}\}$ (since $A \subseteq B$)

$\geq \min\{(A \cup B)(x), (A \cup B)(y)\}$

2. $(A \cup B)(x \tau y) = \max\{A(x \tau y), B(x \tau y)\}$

$= A(x \tau y)$

$\geq \min\{A(x), B(y)\}$

$\geq \min\{\max\{\{A(x), B(x)\}, \max\{A(y), B(y)\}\}$

$\geq \min\{(A \cup B)(x), (A \cup B)(y)\}$

Similarly iif $B \subseteq A$ we get $A \cup B$ is a multi fuzzy Γ-near ring of $\mathcal{R}$.

Multi Fuzzy Ideals Of Γ - Near Ring

Definition: Let $\mathcal{R}$ be a Γ - near ring and A be a multi fuzzy set in $\mathcal{R}$ then A is said to be multi fuzzy left (resp. right) ideal of $\mathcal{R}$ if it satisfies the following conditions

1. $A(x - y) \geq \min\{A(x), A(y)\}$

$\{\mu_A^1(x-y), \mu_A^2(x-y), \ldots\ldots., \mu_A^i(x-y)\} \geq$

$\{min(\mu_A^1(x), \mu_A^1(y)), \ldots\ldots\ldots., min(\mu_A^i(x), \mu_A^i(y))\}$

2. $A(y + x - y) \geq A(x)$

$\{\mu_A^1(y + x - y), \mu_A^2(y + x - y), \ldots\ldots., \mu_A^i(y + x - y)\} \geq \{\mu_A^1(x),, \ldots\ldots\ldots., \mu_A^i(x)\}$

3. $A(m\tau(x + n) - m\tau\, n) \geq A(x)$ (resp.right $A(x\tau\, m) \geq A(x)$)

$\{\mu_A^1(m\tau(x + n) - m\tau\, n), \ldots\ldots., \mu_A^i(m\tau(x + n) - m\tau\, n)\} \geq \{\mu_A^1(x),, \ldots\ldots\ldots., \mu_A^i(x)\}$

(resp,right $\{\mu_A^1(x\tau\, m), \ldots\ldots., \mu_A^i(x\tau\, m)\} \geq \{\mu_A^1(x),, \ldots\ldots\ldots., \mu_A^i(x)\}$ for all $x, y, m, n \in \mathcal{R}$ and $\tau \in \Gamma$

A is called a multi fuzzy ideal of $\mathcal{R}$ if A is both left and right multi fuzzy ideal of $\mathcal{R}$

Example: Let $\mathcal{R}$ be the set of 2x2 matrices over the set of all integers and $\Gamma = I_{2x2}$. Then $\mathcal{R}$ is a Γ - near ring. Let A be multi fuzzy Γ - near ring defined by

A(x) = (0.8,0.7,0.6) if x is of the form $\begin{pmatrix} 0 & r \\ 0 & s \end{pmatrix}$

=(0.2,0.2,0.1) , otherwise

Then clearly A(x) is a multi fuzzy ideal of **$\mathcal{R}$.**

Example: Consider the additive group Z_6={0,1,2,3,4,5} and Γ={γ_1, γ_2} where

γ_1 = {0,1,0,0,0,0} γ_2 = {0,0,1,0,0,0} are given by

Y1	0	1	2	3	4	5	Y2	0	1	2	3	4	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	1	0	0	1	0	0	0
2	0	2	0	0	0	0	2	0	0	2	0	0	0
3	0	3	0	0	0	0	3	0	0	3	0	0	0
4	0	4	0	0	0	0	4	0	0	4	0	0	0
5	0	5	0	0	0	0	5	0	0	5	0	0	0

Then Z_6 is a Γ - near ring with zero symmetric.

Let A be a multi fuzzy Γ - near ring defined by

A(x) = (0.9,0.8,0.6), if x=0

= (0.2,0.1,0.1), otherwise

Then clearly A(x) is a multi fuzzy ideal of $\mathcal{R}$

Theorem: A multi fuzzy subset A of Γ - near ring $\mathcal{R}$ is a multi fuzzy left (resp. right) ideal of $\mathcal{R}$ if and only if each non empty multi level subset $A_{\alpha,}$ α ϵ[0,1] is left (resp. right) ideal of $\mathcal{R}$.

Proof: Since A is multi fuzzy left (resp. right) ideal of $\mathcal{R}$.
Now, we have to prove that A_α= {x ϵ X/A $(x) \geq$ α}is a left (resp. right) ideal of $\mathcal{R}$.

Let $x, y \in A_\alpha$, α = (α_1, α_2,…. α_i)

$A(x) \geq \alpha$, $A(y)$) $\geq \alpha$

$A^n(x) \geq \alpha_n$, $A^n(y) \geq \alpha_n$ for n=1,2,…i

1. $A(x\text{-}y) \geq \min\{A(x), A(y)\}$

$\geq$ {min{$A^1(x)$,$A^1(y)$},min{$A^2(x)$,$A^2(y)$}………….,min {$A^i(x)$, $A^i(y)$}}

$\geq$ {min(α_1, α_1),min(α_2, α_2),………min(α_i, α_i)}

$\geq$ (α_1,α_2,…….α_i)

= α

$\Rightarrow$ $x\text{-}y \in A_\alpha$

2. Let $x \in A_{\alpha,} y \in \mathcal{R} \Rightarrow A(x) \geq \alpha$

Moreover, $A(y + x - y) \geq A(x) \geq \alpha$

$\Rightarrow$ $y + x - y \in A_\alpha$

3. Let $x \in A_\alpha$, $m, n \in \mathcal{R}$, $\tau \in \Gamma \Rightarrow A(x) \geq \alpha$

Moreover,$A(m\tau(x+n)\text{-} m\tau n) \geq A(x) \geq \alpha$

$\Rightarrow m\tau(x+n)\text{-} m\tau n \in A_\alpha$

[resp. right

Let $x \in A_\alpha$, $m \in \mathcal{R}$, $\tau \in \Gamma \Rightarrow A(x) \geq \alpha$

$A(x\tau m) \geq A(h) \geq \alpha$

$\Rightarrow x\,\tau\, m \in A_\alpha$]

Therefore, $A_\alpha$is a left (resp. right) ideal of $\mathcal{R}$.

Conversely suppose that $A_\alpha$is a left (resp. right) ideal of $\mathcal{R}$

Now, we will prove that A is multi fuzzy left ideal of $\mathcal{R}$.

Let $x, y \in \mathcal{R}$, and $A\,(x\,) = \eta$, $A\,(y\,)= \beta$ where $\eta = (\,\eta_1, \eta_2, \ldots \eta_m)$, $\beta = (\,\beta_1, \beta_2, \ldots \beta_m)$.

Putting $\alpha = \min(\eta, \beta_,)$

Therefore $A\,(x\,) = \eta \geq \alpha$, $A\,(y\,) = \beta \geq \alpha$,

Let $x, y \in A$

$\Rightarrow x - y \in A_\alpha$

$\Rightarrow A(x - y) \geq \alpha = \min(\eta, \beta_,) = \min\{A\,(x), A\,(y)\}$

Again let $x, y, m, n \in \mathcal{R}$, $\tau \in \Gamma$. Let $A\,(x\,) = \alpha$.

Then $x \in A_\alpha$

$y + x - y$, $m\tau\,(x{+}n\,) - m\tau\, n$, $x\,\tau\, m \in A_\alpha$

$A(y + x - y) \geq \alpha$, $A\,(m\tau\,(x{+}n\,) - m\tau\, n) \geq \alpha$,$A\,(x\tau\, m) \geq \alpha$

$A\,(y + x - y) \geq A\,(h\,)$, $A\,(m\tau\,(x{+}n\,) - m\tau\, n) \geq A\,(x\,)$, $A\,(x\,\tau\, m) \geq A\,(x\,)$

$\therefore A$ is a multi fuzzy left (resp. right) ideal of $\mathcal{R}$.

Theorem: Let A and B are two multi fuzzy left (resp. right) ideals of a Γ-near ring of $\mathcal{R}$. Then $A \cup B$ is also a multi fuzzy left (resp. right) ideal of a Γ-near ring of $\mathcal{R}$ if $A \subseteq B$ or $B \subseteq A$.

Proof:

$A = (\mu_A^1, \mu_A^2 \ldots\ldots\ldots, \mu_A^i)$, $B = (\mu_B^1, \mu_B^2 \ldots\ldots\ldots, \mu_B^i)$

be two multi fuzzy ideals of a Γ – near ring $\mathcal{R}$

Let $x, y, m, n \in \mathcal{R}$ and $\tau \in \Gamma$

1. $(A \cup B)(x - y) = \max\{\mu_A^1(x-y), \mu_B^1(x-y)\}\ldots\ldots\max\{\mu_A^i(x-y), \mu_B^i(x-y)\}$

$\geq \max\{\min(\mu_A^1(x), \mu_A^1(y)), \min(\mu_B^1(x), \mu_B^1(y))\}\ldots\ldots\ldots\ldots$

$\max\{\min(\mu_A^i(x), \mu_A^i(y)), \min(\mu_B^i(x), \mu_B^i(y))\}$

$\geq \max\{\min(\mu_A^1(x), \mu_A^1(y), \mu_B^1(x), \mu_B^1(y))\}\ldots\ldots\ldots\ldots$

$\max\{\min(\mu_A^i(x), \mu_A^i(y), \mu_B^i(x), \mu_B^i(y))\}$

$\geq \max\{\min(\mu_A^1(x), \mu_B^1(x), \mu_A^1(y), \mu_B^1(y))\}\ldots\ldots\ldots$

$\max\{\min(\mu_A^i(x), \mu_B^i(x), \mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{\max(\mu_A^1(x), \mu_B^1(x), \mu_A^1(y), \mu_B^1(y))\}\ldots\ldots\ldots$

$\min\{\max(\mu_A^i(x), \mu_B^i(x), \mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{\max(\mu_A^1(x), \mu_B^1(x)), \max(\mu_A^1(y), \mu_B^1(y))\}\ldots\ldots$

$\min\{\max(\mu_A^i(x), \mu_B^i(x)), \max(\mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{\max(\mu_A^1(x), \mu_B^1(x))\ldots\ldots\ldots\ldots\max(\mu_A^i(x), \mu_B^i(x)),$

$\max(\mu_A^1(y), \mu_B^1(y))\ldots\ldots\max(\mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{(A \cup B)(x), (A \cup B)(y)\}$

2. $A\,(y + x - y) \geq A\,(x)$, $B\,(y + x - y) \geq B\,(x)$

$(A \cup B)\,(y + x - y) = \max\{\mu_A^1(y + x - y), \mu_B^1(\kappa + h - \kappa)\}\ldots\ldots\ldots\ldots$

$\max\{\mu_A^i (y + x - y), \mu_B^i (y + x - y)\}$

$\geq \max\{\mu_A^1(x), \mu_B^1(x)\} \ldots\ldots\ldots \max\{\mu_A^i(x), \mu_B^i(x)\}$

$\geq (A \cup B)(x)$

3. $A(m\tau(x+n) - m\tau n) \geq A(x)$, $B(m\tau(x+n) - m\tau n) \geq B(x)$

$(A \cup B)(m\tau(x+n) - m\tau n)$

$= \max\{\mu_A^1(m\tau(x+n) - m\tau n), \mu_B^1(m\tau(x+n) - m\tau n)\} \ldots\ldots\ldots$

$\max\{\mu_A^i(m\tau(x+n) - m\tau n), \mu_B^i(m\tau(x+n) - m\tau n)\}$

$\geq \max\{\mu_A^1(x), \mu_B^1(x)\} \ldots\ldots\ldots\ldots\ldots \max\{\mu_A^i(x)), \mu_B^i(x))\}$

$\geq (A \cup B)(x)$

(Resp. right

$A(x\tau m) \geq A(x)$, $B(x\tau m) \geq B(x)$

$(A \cup B)(x\tau m) = \max\{\mu_A^1(x\tau m), \mu_B^1(x\tau m)\}, \ldots\ldots\ldots \max\{\mu_A^i(x\tau m), \mu_B^i(x\tau m)\}$

$\geq \max\{\mu_A^1(x), \mu_B^1(x)\} \ldots\ldots\ldots\ldots\ldots\ldots \max\{\mu_A^i(x)), \mu_B^i(x)\}$

$\geq (A \cup B)(x)$

$\therefore$ $A \cup B$ is a multi fuzzy left (resp. right) ideal of $\mathcal{R}$

Theorem : Let A and B are two multi fuzzy left (resp. right) ideals of a Γ-near ring of $\mathcal{R}$. Then $A \cap B$ is also a multi fuzzy left (resp. right) of a Γ-near ring of $\mathcal{R}$.

Proof:

$A = (\mu_A^1, \mu_A^2 \ldots\ldots\ldots, \mu_A^i)$, $\dot{B} = (\mu_B^1, \mu_B^2 \ldots\ldots\ldots, \mu_B^i)$ be two multi fuzzy ideals of a Γ – near ring $\mathcal{R}$

Let $x, y, m, n \in \mathcal{R}$ and $\tau \in \Gamma$

1. $(A \cap B)(x - y) = \min\{\mu_A^1(x - y), \mu_B^1(x - y)\} \ldots\ldots \min\{\mu_A^i(x - y), \mu_B^i(x - y)\}$

$\geq \min\{\min(\mu_A^1(x), \mu_A^1(y)), \min(\mu_B^1(x), \mu_B^1(y))\} \ldots\ldots\ldots\ldots$

$\min\{\min(\mu_A^i(x), \mu_A^i(y)), \min(\mu_B^i(x), \mu_B^i(y))\}$

$\geq \min\{\min(\mu_A^1(x), \mu_A^1(y), \mu_B^1(x), \mu_B^1(y))\} \ldots\ldots\ldots\ldots$

$\min\{\min(\mu_A^i(x), \mu_A^i(y), \mu_B^i(x), \mu_B^i(y))\}$

$\geq \min\{\min(\mu_A^1(x), \mu_B^1(x), \mu_A^1(y), \mu_B^1(y))\} \ldots\ldots\ldots$

$\min\{\min(\mu_A^i(x), \mu_B^i(x), \mu_A^i(y), \mu_B^i(y))$

$\geq \min\{\min(\mu_A^1(x), \mu_B^1(x)), \min(\mu_A^1(y), \mu_B^1(y))\} \ldots\ldots\ldots$

$\min\{\min(\mu_A^i(x), \mu_B^i(x)), \min(\mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{\min(\mu_A^1(x), \mu_B^1(x)) \ldots\ldots\ldots\ldots \min(\mu_A^i(x), \mu_B^i(x)),$

$\min(\mu_A^1(y), \mu_B^1(y)) \ldots\ldots\ldots \min(\mu_A^i(y), \mu_B^i(y))\}$

$\geq \min\{(A \cap B)(x), (A \cap B)(y)\}$.

2. $\mu_A^n(y + x - y) \geq \mu_A^n(x))$, $\mu_B^n(y + x - y) \geq \mu_B^n(x)$

$(A \cap B)(y + x - y) = \min\{\mu_A^1(y + x - y), \mu_B^1(y + x - y)\} \ldots\ldots\ldots$

$\min\{\mu_A^i(y + x - y), \mu_B^i(y + x - y)\}$

$\geq \min\{\mu_A^1(x), \mu_B^1(x)\} \ldots\ldots\ldots \min\{\mu_A^i(x), \mu_B^i(x)\}$

$\geq (A \cap B)(x)$

3. $\mu_A^n(m\tau(x+n) - m\tau n) \geq \mu_A^n(x)$, $\mu_B^n(m\tau(x+n) - m\tau n) \geq \mu_B^n(x)$

$(A \cap B)$ $(m\tau\ (x+n)\text{-}\ m\tau\ n)$ = min$\{\mu_A^1(m\tau\ (x+n)\text{-}\ m\tau\ n), \mu_B^1\ (m\tau\ (x+n)\text{-}$
$m\tau\ n)\}$………min$\{\mu_A^i\ (m\tau\ (x+n)\text{-}\ m\tau\ n), \mu_B^i\ (m\tau\ (x+n)\text{-}\ m\tau\ n)\}$
$\geq$ min$\{\mu_A^1(x), \mu_B^1\ (x)\}$……………min$\{\mu_A^i\ (x)), \mu_B^i\ (x))\}$
$\geq (A \cap B)$ (x)
(Resp. right
$\mu_A^n\ (x\ \tau\ m) \geq \mu_A^n\ (x)$, $\mu_B^n\ (x\ \tau\ m) \geq \mu_B^n\ \boldsymbol{(x)}$
$(A \cap B)$ $(x\ \tau\ m)$ = min$\{\mu_A^1\ (x\ \tau\ m), \mu_B^1\ (x\ \tau\ m)\}$,………min$\{\mu_A^i\ (x\ \tau\ m), \mu_B^i\ (\ (x\ \tau\ m)\}$
$\geq$ min$\{\mu_A^1(x), \mu_B^1\ (x)\}$................min$\{\mu_A^i\ (x)), \mu_B^i\ (x)\}$
$\geq (A \cap B)\ (x)$
$A \cap B$ is a multi fuzzy left (resp. right) ideal of $\mathcal{R}$.

Cartesian Product of Multi Fuzzy Γ - Near Ring

Definition: Let n be a positive integer and A and B be two multi fuzzy sets of dimension n in Γ - near rings $\mathcal{R}$ and S then Cartesian product of multi fuzzy sets of A and B is defined by

AxB = { $(x\ ,y)$, min $(\mu_A^1(x), \mu_B^1\ (y))$…….min $(\mu_A^i(x), \mu_B^i\ (y))/(\ x\ ,y) \in \mathcal{R}x\ S$ }

Theorem: Let A and B are two multi fuzzy left (resp. right) ideals of a Γ – near ring $\mathcal{R}$ and S then the Cartesian product AxB is also a multi fuzzy left (resp. right) ideal of $\mathcal{R}$x S.

Proof:

Let A and B are two multi fuzzy left (resp. right) ideals of a Γ – near ring $\mathcal{R}$ and S

Let (x_1,x_2),(y_1,y_2),(m_1,m_2) ,(n_1,n_2) Є $\mathcal{R}$ x S and τ Є Γ then

1. xB $((x_1,x_2)\text{-}(y_1,y_2))$
= (AxB)$(x_1\text{-}y_1, x_2\text{-}y_2)$
= min $\{\mu_A^1(x_1\text{-}y_1), \mu_B^1\ (x_2\text{-}y_2)\}$, min $\{\mu_A^2(x_1\text{-}y_1), \mu_B^2\ (x_2\text{-}y_2)\}$,…..
min $\{\mu_A^i(x_1\text{-}y_1), \mu_B^i\ (x_2\text{-}y_2)\}$
$\geq$ min{min$(\mu_A^1(x_1), \mu_A^1(y_1))$,min $((\mu_B^1\ (x_2), \mu_B^1\ (y_2))\}$……..…
min{min$(\mu_A^i(x_1), \mu_A^i(y_1))$,min$((\mu_B^i\ (x_2), \mu_B^i\ (y_2))\}$
$\geq$ min{min$(\mu_A^1\ (x_1), \mu_B^1\ \ (x_2))$,min$(\mu_A^1\ (y_1), \mu_B^1\ (y_2))$…………..
min$(\mu_A^i\ (x_1), \mu_B^i\ \ (x_2))$,min$(\mu_A^i(y_1), \mu_B^i\ (y_2))\}$
$\geq$ min{(min$(\mu_A^1\ (x_1), \mu_B^1\ \ (x_2))$………min$(\mu_A^i\ (x_1), \mu_B^i\ \ (x_2)))$,
(min$(\mu_A^1\ (y_1), \mu_B^1\ \ (y_2))$….min$(\mu_A^i(y_1), \mu_B^i\ (y_2))\}$
$\geq$ min$\{(A$x$B)\ (x_1,x_2),(A$x$B)(y_1,y_2)\}$

2. $(A$x$B)$ $((y_1,y_2)\ +(x_1,x_2)\text{-}(y_1,y_2))$
= AxB $(y_1+x_1\text{-}y_1,\ y_2+x_2\text{-}y_2)$
= min$\{\mu_A^1(y_1+x_1\text{-}y_1), \mu_B^1\ (y_2+x_2\text{-}y_2)\}$……… min$\{\mu_A^i(y_1+x_1\text{-}y_1), \mu_B^i\ \ (y_2+x_2\text{-}y_2)\}$
$\geq$ min $\{\mu_A^1(x_1),\ \mu_B^1\ (x_2)\}$…… min $\{\mu_A^i\ (x_1),\ \mu_B^i\ (x_2)\}$
$\geq (A$x$B)$ (x_1,x_2).

3. $(A$x$B)\{(m_1,m_2)\ \tau((x_1,x_2)+(n_1,n_2))\text{-}(m_1,m_2)\ \tau(n_1,n_2)\}$
= $(A$x$B)\{m_1\tau(x_1+n_1)\text{-}(m_1\tau n_1), m_2\tau(x_2+n_2)\text{-}(m_2\tau n_2)\}$
= min $\{\mu_A^1(m_1\tau\ (x_1+n_1)\text{-}(m_1\tau n_1)), \mu_B^1\ (m_2\tau\ (x_2+n_2)\text{-}(m_2\tau n_2)\}$………

min $\{\mu_A^i\ (m_1\tau\ (x_1+n_1)\text{-}(m_1\tau n_1)), \mu_B^i\ (m_2\tau\ (x_2+n_2)\text{-}(m_2\tau n_2)\}$

$\geq$ min $\{\mu_A^1(x_1), \mu_B^1\ (x_2)\} \ldots\ldots$ min$\{\mu_A^i(x_1), \mu_B^i\ (x_2)\}$

$\geq (A\text{x}B)\ (x_1, x_2)$.

[resp. right $(A\text{x}B)\{(x_1, x_2). \tau(m_1, m_2)) = (A\text{x}B)\{x_1\tau m_1, x_2\tau m_2\}$

$=$ min$\{\mu_A^1(x_1\tau m_1), \mu_B^1\ (x_2\tau m_2)\} \ldots\ldots$min$\{\mu_A^i(x_1\tau m_1), \mu_B^i\ \ (x_2\tau m_2)\}$

$\geq$ min $\{\mu_A^1(x_1), \mu_B^1\ (x_2)\} \ldots\ldots\ldots$min $\{\mu_A^i(x_1), \mu_B^i\ (x_2)\}$

$\geq (A\text{x}B)\ (x_1, x_2)$]

$\therefore A\text{x}B$ is also a multi fuzzy left (resp. right) ideal of $\mathcal{R}$ x S

4. Conclusion

This paper has focused on the multi fuzzy ideals of Γ- near ring R. Also, we enumerate multi level subsets and multi anti level subsets of R and verified some properties.

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