ROBERT HALIFAX, AN OXFORD CALCULATOR OF SHADOWS[1]

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Abstract

In his commentary on Lombard's *Sentences*, question 1, Robert Halifax OFM presents a remarkably original and inventive optical argument. It compares two pairs of luminous and opaque bodies with two shadow cones until the luminous bodies reach the zenith. In placing two moving human beings into the shadow cones whose moral evolution parallels the size of the shadows, Halifax creates an unprecedented shadow theater equipped with mathematics and theorems of motion from Thomas Bradwardine's *Treatise on Proportions.* This paper is a first attempt at analyzing this imaginary experiment and the mathematics of the infinite it implies. It also shows that optics had new aims through its connexion with the theorems of motion of the Oxford Calculators.

Keywords

Proportions; Motion; Calculators; Optics; Astronomy; Thomas Bradwardine; Robert Halifax; Commentaries on the *Sentences*

Resumen

En su *Comentario a las Sentencias de Pedro Lombardo*, cuestión 1, Robert Halifax OFM presenta un argumento óptico notablemente original e inventivo. Compara dos pares de cuerpos luminosos y opacos con dos conos de sombra hasta que los cuerpos luminosos alcanzan el cenit. Al situar en los conos de sombra a dos seres humanos en movimiento cuya evolución moral es

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[1] This paper is a revised version of the talk I gave at Munich. I remain utterly convinced that optics and astronomy were essential for the development of the Oxford Calculators' theorem of mean speed. I thank Keith Snedegar, Monika Michałowska, György Geréby, Lukáš Lička and Luke DeWeese for insight, enthusiasm or friendship shared over Halifax. The Issue Editor and the Reviewers of this paper have also to be acknowledged.

paralela al tamaño de las sombras, Halifax crea un teatro de sombras sin precedentes, dotado con la matemática y los teoremas del movimiento derivados del *Tratado de las Proporciones* de Thomas Bradwardine. Este artículo es un primer intento de analizar este experimento imaginario y las matemáticas del infinito por él implicadas. Muestra además que la óptica ha tenido nuevos objetivos a través de su conexión con los teoremas del movimiento de los Calculadores de Oxford.

Palabras clave

Proporciones; movimiento; calculadores; óptica; astronomía; Thomas Bradwardine; Robert Halifax; Comentarios de las *Sentencias*

In an essay written forty years ago, John Murdoch and Edith Sylla characterized Thomas Bradwardine's *Treatise on Proportions* as enacting a "rather dramatic change" on the science of motion at Oxford in the 1330s. This change can be viewed on two levels. One level is formal: the *Treatise*, dated to 1328, analyzed motion outside the context in which medieval discussions about motion usually took place, that is, in commenting on one of Aristotle's relevant texts. The other level concerns content: Bradwardine departed from Aristotle's calculation of velocity in proposing that velocities "vary arithmetically when the proportions of force to resistance determining these velocities vary geometrically."[2] The arguments in favor of the new calculation were drawn from a few concrete or imaginary physical cases and entailed the application of mathematics beyond physics to metaphysics, ethics, and theology. Also, fourteenth-century Oxford science of motion evolved within the context of disputational logic, which dominated many writings that Bradwardine's seminal treatise gave rise to.[3] In this paper, I will argue that a further discipline should be added, namely, the "science of perspective" or optics. In this field, the Oxford educated Franciscan Robert Halifax proposed arguments regarding the new calculation of motion, which are both remarkably original and inventive.

We possess only scarce information regarding Robert Halifax. We know that he became the fifty-sixth Franciscan lector at Cambridge around 1336. Before taking up his teaching position, he studied at Oxford and was licensed in theology. His university years were the most significant period for the Oxford Calculators, who contributed to or developed the method and theorems Bradwardine posited in the *Treatise on*

[2] John E. Murdoch and Edith D. Sylla, "The Science of Motion", in *Science in the Middle Ages*, edited by D. C. Lindberg (Chicago: Chicago University Press, 1978), 206-264, 224, 225, and 227.

[3] Edith D. Sylla, "The Oxford Calculators", in *The Cambridge History of Later Medieval Philosophy: From the Rediscovery of Aristotle to the Disintegration of Scholasticism, 1100–1600*, edited by N. Kretzmann, A. Kenny, J. Pinborg and E. Stump (Cambridge: Cambridge University Press, 1982), 540-563, esp. 542-543; and Daniel A. Di Liscia, "Perfections and Latitudes. The Development of the Calculators' Tradition and the Geometrisation of Metaphysics and Theology", in *Quantifying Aristotle. The Impact, Spread, and Decline of the* Calculatores *Tradition*, edited by D. A. Di Liscia and E. D. Sylla (Leiden: Brill, 2022), 278-327.

Proportions.[4] Despite this governing trend in natural philosophy, Halifax is not known to have left any writing in the field; so far, scholarship attributes to him only a philosophical dialogue between an Ockhamist and a Scotist, which remains of doubtful authorship, and a commentary on Peter Lombard's *Sentences* that he read at the University of Oxford in the early 1330s.[5] Extant in seventeen, more or less complete witnesses on the continent, Halifax's theological writing left a lively, long-lasting impact on masters at the Universities of Paris and Vienna until at least ca. 1420.[6]

This commentary proves not only of theological interest. Almost every argument in it contains an analogy from physical motion and change, and draws on proportional calculation, the mathematics of the infinite, or a sophism. One such argument is probably his most complex thought experience, mixing optics, geometry, astronomy, proportional calculation of motion, and ethics, which Halifax placed at the beginning of his commentary. While the argument remains a sophisticated hypothetical case, unique for calculating motion from the size of shadows, it aims at demonstrating the rather simple claim that divine justice functions according to arithmetical proportions. In what follows, I will inquire into the main part of this argument, which provides unique evidence for the history of the science of motion at the University of Oxford in the 1330s.

The argument "about shadows"[7]

Halifax's commentary on the *Sentences*, like many Oxford commentaries of the period, concerns only Books 1 and 2 of Lombard's work. It is enough to read only the

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[4] For a description of the group of scholars called Oxford Calculators and its members, see Sylla, "The Oxford Calculators", 540.

[5] Earlier Franciscans sources suggested that Halifax studied theology at Paris, a thesis that recent scholarship has rejected. William Courtenay dates Halifax's lecture on the *Sentences* to around 1336-1338, whereas Emden indicates the rather earlier date of 1332. See William J. Courtenay, "Some Notes on Robert of Halifax, OFM", *Franciscan Studies* 33 (1977): 135-142; and Alfred B. Emden, *A Biographical Register of the University of Oxford to A.D. 1500* (Oxford: Clarendon Press, 1958), II, 850-851. See also, Alfred B. Emden, *A Biographical Register of the University of Cambridge to 1500* (Cambridge: Cambridge University Press, 1963), 280.

[6] Murdoch investigated a few elements of Halifax's influence at Paris. See John E. Murdoch, "*Subtilitates Anglicanae* in Fourteenth-Century Paris: John of Mirecourt and Peter Ceffons", in *Machaut's World. Science and Art in the Fourteenth Century*, edited by M. P. Cosman and B. Chandler (New York: New York Academy of Sciences, 1978), 51-86. For Halifax's influence at Vienna, see Edit A. Lukács, "Robert Halifax on the Middle Act of the Will", forthcoming.

[7] The passages from Robert Halifax's commentary on the *Sentences*, Question 1 quoted in this paper are based on transcriptions from two witnesses: Paris, Bibliothèque Nationale de France, Lat. 15880, fol. 21^{rb}-23ra, and Vatican, Vat. Lat. 1111, fol. 13va-14^{rb}. Orthograph has been rendered standard. I indicate additions in angle brackets, and corrections in square brackets. All translations from the Latin are mine. The manuscript transmission of Halifax's commentary is complex. In case of the argument in question, the different manuscripts attest to variant readings, which give way to different interpretations of the optical experiment.

titles of the nine questions that compose it to notice that the acts of human will were of utmost importance and interest to Halifax:

> Question 1: Whether the commensuration of reward to merit and of punishment to sin, which can be recognized through theological study of Scripture, is justly ordered by God.
>
> Question 2: Whether, through the practice of studying theological truths, a theologian can attain a greater knowledge than the knowledge of faith.
>
> Question 3: Whether the science that a theologian can have through the practice of studying theological truths is practical or theoretical.
>
> Question 4: Whether between enjoyment and use, there is a middle act of the will that is neither enjoyment nor use.
>
> Question 5: Whether any act of the will can be suddenly produced by the will.
>
> Question 6: Whether the will is free with respect to any of its acts and objects.
>
> Question 7: Whether only the divine essence is an intensively infinite perfection.
>
> Question 8: Whether the blessed angels make progress in merit.
>
> Question 9: Whether every act of the will, if chosen in disagreement with one's erroneous conscience, would be without merit.[8]

Question 1, probably read as an introductory lecture, focuses on commensuration.[9] As Halifax posits it, commensuration is established from theological studies of the Scripture, yet its just character has to be proved. This seems easy to do with mathematics, especially with the mathematics of proportions, a science to which commensuration was not unfamiliar. In Bradwardine's formulation, commensuration was a relationship between commensurable or rational quantities according to a common, exact measure.[10]

[8] Raymond Edwards, "Themes and Personalities in *Sentences* Commentaries at Oxford in the 1330's", in *Mediaeval Commentaries on the* Sentences *of Peter Lombard: Current Research*, edited by G. R. Evans (Leiden: Brill, 2002), I 378-393 and 381-382. For the Latin title of the questions, see Courtenay, "Some Notes", 141. Together with Monika Michałowska, I am currently working on a critical edition of questions 5 and 6 from Halifax's *Sentences* commentary.

[9] "Quest. 1 [Principium I (?)]: Utrum commensuratio praemii ad meritum et poenae ad peccatum, quae per studium theologiae ex Scriptura potest cognosci, sit iuste a Deo ordinata", BNF, Lat. 15880, fol. 1ra.

[10] "'Communicative', 'commensurable', or 'rational' quantities are those for which there exists a common measure which measures them exactly", Thomas of Bradwardine, *His Tractatus de proportionibus: Its Significance for the Development of Mathematical Physics*, edited and translated by H. L. Crosby, Jr. (Madison, WI: The University of Wisconsin Press, 1955), 66-67. For Aristotle, distributive justice was proportional. See *Nicomachean Ethics* 5.3 (1131a4-6). For more concrete cases of commensuration calculated in Halifax's Question 1, see n. 38, 39, 42 and 43.

Halifax presents a long series of *dubia*, articles, and arguments in favour of a justly ordered commensuration. The last argument in the series has a specific scope: it aims at demonstrating that, through arithmetical compensation, first more and then less intensely virtuous moral beings achieve the same reward as continuously evolving ones. The argument is imaginary;[11] it starts with the following premises:

> Let us posit two opaque bodies that are equal in quantity and have the same shape. And I take two luminous bodies, which are bigger than these opaque bodies; they are equal in size and both have the same figure. Now let one luminous body be placed next to one of the opaque bodies at a certain distance, in a medium that can shed light. In the same way, let the other luminous body be placed next to the other opaque body at the same distance and a medium entirely similar to the first one. It is clear that these two opaque bodies cause two shadows of equal size, which have the same conical shape.[12]

None of the manuscripts have figures to represent the optical experiment. Thus, Halifax's audience was supposed to be equipped with the knowledge required for understanding an argument of this complexity without visual support. This figure shall represent the argument at this stage:

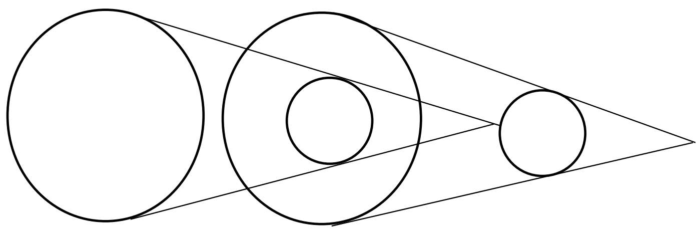

Figure 1: Reconstruction

The bigger circles represent the luminous bodies, the smaller circles the opaque bodies with their cones of shadows.

The schema corresponds to twice the astronomical case of the shadow the Earth casts within sunlight: this is the classical example astronomical optics proposes for conical shadows that can only be cast by bigger spheres on smaller spheres. In his argument, Halifax did not define the shape of neither the luminous, nor the opaque

[11] On the role of imaginary or thought experiments by the Oxford Calculators see Sylla, "The Oxford Calculators".

[12] "Ponantur duo corpora opaca aequalia in quantitate et eiusdem figurae. Et capio duo corpora lucida maiora hiis opacis, et sint aequalia et eiusdem figurae inter se. Et ponatur unum lucidum iuxta unum opacum in certa distantia in tali medio quod possit illuminare. Et eodem modo ponatur aliud lucidum iuxta aliquod opacum et in aequali distantia et consimili medio sicut primum. Ab istis corporibus opacis causantur duae umbrae aequales concurrentes in cono, manifestum est", BNF, Lat. 15880, fol. 21[rb].

bodies, yet, his conclusions will imply that the opaque bodies are in fact flat shields. This matter of fact is corroborated in another argument, in which Halifax does not assume the same spherical shape for the Earth he assumes for the Sun.[13]

In the geometrical and optical settings he initially stated, Halifax adds two human beings into the cones of shadows: "I take two humans who are equal in merit at the beginning of hour *(a)*. Let the first be placed in the cone of one shadow, and the second in the cone of the other."[14] Next, luminous bodies start an ascent and the human beings a moral life:

> Let one opaque body be *a* and the other *b*. I want opaque body *a* to start to diminish at the beginning of that hour, and to diminish continuously such that its total quantity disappears and ceases to be by the end of that hour. And I want the luminous body positioned next to *a* to start to ascend at the beginning of that hour to the point directly over body *a* that is called its zenith, and to ascend such that by the end of that hour, it is at that point. And I mainly want luminous body *a'* to move precisely in the same way for that hour to the point directly over opaque body *b*, such that by the end of that hour, it is at that point. After the first instant of that hour, these shadows were ever shorter than they were before, and the shadow caused by body *a* was ever shorter for the entire hour than the other. Then, I want one of the men to move continuously with the shadow of the body such that he shall be ever in the cone of that shadow, and the other man to be in the cone of the other shadow. And [I want] them to merit by two acts according to the same proportion wherewith they move, and wherewith the shadows are shortened.[15]

[13] The length of the Earth's shadow was calculated first by Ptolemy, then Kepler focused on it: Ptolemy, *Almagest* 5.9; and Raz Chen-Morris, *Measuring Shadows: Kepler's Optics of Invisibility* (University Park, PA: Penn State University Press, 2016), 40-44. On the three different types of shadows spherical objects can cast see Lukáš Lička, "Shadows in Medieval Optics, Practical Geometry, and Astronomy: On a *Perspectiva* Ascribed to Thomas Bradwardine", *Early Science and Medicine* 27 (2022): 195-198 and 198, n. 51. In his other argument, Halifax writes: "Et ad probationem dico quod si supponatur quod sit corpus sphericum illuminosum, puta sol, positum in medio infinito secundum imaginationem intensivum lumen lucens ut est aer, et quod ponatur iuxta illud corpus opacum minoris quantitatis, puta terra...", Vat. Lat. 1111, fol. 69[vb]. On this thought experiment, see also n. 37.

[14] "Capio duos homines aequales in merito in principio *a* horae. Et ponatur unus in cono unius umbrae, et alius in cono alterius", BNF, Lat. 15880, fol. 21[ra].

[15] "Et sit unum corpus opacum *a* et aliud *b*. Et volo quod *a* corpus opacum incipiat diminui in principio illius horae, et sic diminuatur continue quod quantitas sua tota corrumpatur et desinat esse in fine illius horae. Et volo cum hoc quod corpus lucidum iuxta *a* positum incipiat in principio illius horae ascendere [corrected from *descendere*] usque ad illum punctum directe supra *a* corpus qui dicitur chemb, et ascendat sic quod in fine illius horae sit in illo puncto. Et volo principaliter quod *a* corpus lucidum moveatur praecise per illam horam eodem modo ad punctum illum directum supra *b* opacum ita quod <sit> in illius puncti fine horae. Post primum instans illius horae erant istae umbrae semper breviores quam prius erant, et umbra causata ab *a* corpore semper erat per totam horam brevior alia. Volo tunc quod unus homo continue moveatur cum

Initially, every fact and figure is identical. When the luminous bodies begin to move, the figures become different, and subject to comparison or, more adequately, commensuration. One opaque figure continuously shrinks, changing both the size of the shadow it casts and the motion of the body placed in it, while the other opaque body remains the same, its shadow changing "naturally" as the luminous body rises over it toward the zenith, with the mobile body moving with continuous motion in it. This figure actualizes the previous figure:

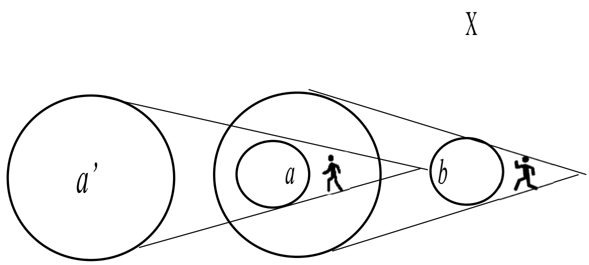

Figure 2: Reconstruction

This figure completes Figure 1 with the letters *a'*, *a* and *b*, and the human beings that move in the shadows of the opaque bodies. X corresponds to the zenith toward which the bigger circles accomplish their motion.

Different kinds of motions are involved in the argument. The two luminous bodies move with circular motion the quarter circle from the horizon to the zenith. Opaque body *a* is subject to diminution. The human beings that move in the shadows are subject to the motion of alteration in the ethical sense (they earn rewards or pains), whereas Halifax says them to move with local motion. Of the identical facts, some become identical again by the end of motion, when the shrinking figure *a* and the shadow of both figures disappear. In that moment, motion ceases again.

We must note that the analogy between physical motion and moral change corresponds to Richard Kilvington's understanding of ethics: Halifax's merit and demerit are, as Kilvington's virtues and vices, physical "things"; therefore, motion of change—increase or decrease—applies to them. Kilvington was the first among the

umbra corporis ita quod semper sit in cono illius umbrae et alius in cono alterius umbrae, et quod mereantur duobus actibus secundum eandem proportionem secundum quam moventur localiter et secundum quam umbrae istae abbreviantur", BNF, Lat. 15880, fol. 21ra. The letters *a* and *b* have a threefold meaning I distinguished in the main text with the help of different diacritic signs; they refer to: 1) the period of change (one hour *(a)*); 2) the opaque bodies and the human beings placed into their shadows (*a* and *b*); 3) the luminous body placed next to opaque body *a* (*a'*).

Oxford Calculators to apply a physical approach to ethics, and the only one, whose methodologically developed approach came down to us in a commentary on Aristotle's *Nicomachean Ethics*.[16] While Halifax's adhesion to Kilvington's new approach underlines his proximity with the Oxford Calculators, his argument is outstanding even in this context. It unifies perfection and imperfection, the motion in the celestial spheres, that is the circular motion of a planet, and motion in the inner region of the universe, "within which all was subject to continual alteration, growth and decay", that is the moral evolution of human beings.[17]

By the end of this peculiar experiment, the shadows disappear because the luminous body reaches a peak, the zenith. The zenith was a concept possibly more familiar in optical than in astronomical treatises at Oxford.[18] It appears in the works of Robert Grosseteste and Roger Bacon, followed by John Peckham and the astronomer Richard of Wallingford. More interestingly, the Oxford optical tradition devoted special interest not only to the zenith, but also to its mean degree: "finding the height of an object, when the solar altitude is 45°" was one of the specific aims optics was tasked with.[19] Reaching 45° also had an important implication for the proportions of the shadows, to which I shall come back below.

At this point of the argument, Halifax draws two conclusions, one scientific, the other theological: (1) In the first conclusion, he enunciates a theorem[20] valid for natural sciences and physical motions, which has no immediate theological relevance. This theorem could find its place in any work on physics: "Two moving bodies move precisely at the same time through two equal magnitudes, and one of them moves continuously faster than the other for the whole time; and yet, by the end of that time, an altogether equal space will have been traversed by each of them."[21] (2) In the second

[16] Monika Michałowska, "Kilvington's Use of Physical and Logical Arguments in Ethical Dilemmas", *Documenti e studi sulla tradizione filosofica medievale* 22 (2011): 467-494 and 470-471; Richard Kilvington, *Quaestiones super libros Ethicorum*, edited by M. Michałowska (Leiden: Brill, 2016).

[17] John D. North, *Stars, Minds and Fate. Essays in Ancient and Medieval Cosmology* (London: The Hambledon Press, 1989), 312.

[18] Follow http:-www.dmlbs.ox.ac.uk/web/dmlbs.html while citing the *DMLBS*, and https://logeion.uchicago.edu/lexidium while searching the corpus (28.6.2022).

[19] See Lička, "Shadows in Medieval Optics", 207.

[20] For the word 'theorem' and and its use about the mathematics of proportions in Thomas Bradwardine's *De causa Dei*, see Edit A. Lukács, "Calculations in Thomas Bradwardine's *De causa Dei*, Book I", in *Quantifying Aristotle*, 117.

[21] "Duo mobilia in eodem tempore praecise moventur per duas magnitudines aequales, et unum illorum continue per totum tempus movetur velocius alio, et tamen in fine temporis ab utroque illorum erit aequale <spatium> omnino pertransitum", BNF, Lat. 15880, fol. 21ra. This theorem seems to be a reformulation of Thomas Bradwardine's theorem 9 in chapter 3 of his *Treatise on Proportions*: "An object may fall in the same medium both faster, slower, and equally with some other object that is lighter than itself", Bradwardine, *Tractatus de proportionibus*, 115. This analogy would explain why BNF, Lat. 15880 has first *descendere*, although a descent would

conclusion, Halifax states that the theorem works analogically in theology: Equal human beings eliciting unequally meritorious acts can gain equal merits. Even though Halifax separates natural science from theology, he further states that the latter functions like the former: theology reflects natural science and not the reverse; hence, natural science is primary; theological speculation, derivative.

Next, Halifax proposes a proof of the argument that is based on a definition and a sophism related to the nature of space and the quicker motion. The proof is centred on the velocity of the moving body that follows the shadow of body *a*, a uniformly increasing motion. This mobile body moves continuously faster and traverses more space than the other mobile. Yet, what does being quicker mean? To define it, Halifax quotes one of Aristotle's texts debated by the Calculators in the context of the proportional calculation of motion:

> The mobile following the shadow of body *a* shall move continuously quicker for the whole hour according to the definition of quickness and slowness that the Philosopher gives in *Physics* 4. For the quicker is that which traverses more space in the same time, or an equal space in less time, or more <space> in less time. But at any part of that hour and continuously for the whole hour, the moving body *b*, which follows the shadow of body *a*, has traversed more space. Therefore, it moves faster, because it traverses more space.[22]

At the beginning of the argument, Halifax defined the same time frame for the two motions; therefore, he has to keep to the first definition of the quicker:

> Always after the first instant of that time, the shadow of body *a* becomes shorter, and consequently, in every instant, its cone was less distant from the *terminus ad quem* and more distant from the *terminus a quo* than the cone of the other shadow, and, thus, the mobile body extant in the other cone. And yet, by the end of the time, they will have traversed equal space, because the shadows will disappear in the same instant, namely, in the last instant of that hour, when the luminous bodies are at the points directly above the opaque bodies, therefore both mobile bodies will be in the place where the opaque body was. And thus, I have proved my point, that in the end they have traversed

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constitute a case for the nadir, the opposite of the zenith. See n. 15. As we shall see, Halifax will continue to implicitly use this part of Bradwardine's *Treatise*.

[22] "Mobile sequens umbram *a* corporis per totam horam continue movetur velocius per definitionem velocitatis et tarditatis quem dat Philosophus 4 *Physicorum*. Nam velocius est quod maius spatium in eodem tempore, vel aequale spatium in minori tempore, vel maius in minori tempore pertransit. Sed in qualibet parte illius horae et continue per totam horam *b* mobile sequens umbram *a* corporis pertransit plus de spatio, ergo velocius movetur quod autem plus pertransit de spatio", BNF, Lat. 15880, fol. 21va. Cf. Aristotle, *Physics* 4.10 [218b15–18]. See also Clagett's discussion of related Aristotelian definitions and the nature of the continuum: Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison, WI: The University of Wisconsin Press, 1959), 176-179, esp. 178.

> an altogether equal space, and yet one <mobile> moved for the whole time in every part of it faster than the other.[23]

This passage identifies continuously increasing motion as having a specific relationship to time and space: this kind of motion extends space and leaves time unchanged. The proof itself proceeds barely from definitions and linguistic construct (i.e., from a sophism), which means that it stands in the tradition of the Oxford Calculators, and in the mathematical tradition more generally.

Replies to the argument and other arguments

In further arguments of question 1, Halifax concentrates on the ethical implications of his optical experience. While drawing on mathematics and astronomy, he makes remarks mainly concerning the moral agents. In one of these remarks, he alludes to a common principle:

> And yet by the end of the time, <the two moral agents> will be equal in merit, which seems impossible and against the common principle, evident per se to every intellect that if you add unequals to equals, the things that result shall be unequal, which is per se known.[24]

The principle of unequals added to equals was used by the English mathematician and astronomer, Johannes de Sacro Bosco, in the context of the equinoxes in the thirteenth century. As we shall immediately see, equinoxes can also play a role in Halifax's experiment.

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[23] "... semper post primum instans illius temporis umbra *a* corporis fit brevior, et per consequens in omni instanti minus distabat conus illius a termino ad quem et magis a termino a quo quam ille conus alterius umbre, et per consequens mobile existens in cono alterius. Et tamen in fine temporis est aequale spatium pertransitum ab eis, quia in eodem instanti finientur illae umbrae, scilicet in ultimo instanti illius horae quando corpora luminosa sunt in punctis directe supra ista corpora opaca, ergo tunc utrumque mobile erint in loco ubi erat corpus opacum. Et habetur intentum quod in fine est aequale spatium omnino pertransitum ab eis, et tamen unum in toto tempore movebatur velocius alio et in qualibet parte illius", BNF, Lat. 15880, fol. 21[va]. The definition of motion with the *terminus a quo-terminus ad quem* pair singularly recalls Roger Bacon's definition of motion, on which see Irène Rosier-Catach, "Roger Bacon and Grammar", in *Roger Bacon and the Sciences: Commemorative Essays*, edited by J. Hackett (Leiden: Brill, 1997), 67-102.

[24] "Et tamen in fine temporis sunt omnino aequales in merito quod videtur esse impossibile et contra commune principium omni intellectui per se notum, quod si aequalibus inaequalia addas que resultant, erunt inaequalia, quod est per se notum", BNF, Lat. 15880, fol. 21[vb]. For the principle about equinoxes, see Johannes de Sacro Bosco, *De sphaera mundi* (Paris: Jean Petit, 1495), fol. 52[vb]. The nature of imaginary experience makes Halifax's argument to match several concrete physical cases.

Halifax expands on these mathematical considerations while introducing two new elements into his argument. (1) He emphasizes that there are contraries involved in the first motion, not only acceleration, but deceleration too:

> Concerning this argument, I say that the mobile body following the shadow of body *a* moves for one part of the time faster than the mobile following the shadow of the other body, and for the other part <of time> slower than before, because first, it moves much faster until a given point in space and a given instant in time, and from that instant, it moves slower until the end of the time. The cause of this is the different approximation of the shadow to the opaque body, which causes the shadow.[25]

(2) He encourages readers to consult the *Perspectiva* to find out the moment of change from slower to quicker in the case of mobile *b*: "But the point, at which the one, which first moved slower, starts to move faster, is to be found in a conclusion of the *Perspectiva*."[26] It is remarkable that Halifax only hints at the *Perspectiva* without giving a precise reference. Fortunately, contemporary Oxford works and their sources allow for a plausible identification, since only one point in space was singled out in concerns about the zenith, namely the already mentioned mean degree at 45°.

The trigonometrical approach in optics allowed for two types of shadows: the *umbra recta* stood for the horizontal shadow, and the *umbra iacens* for the vertical shadow. From sunrise until 45° elevation of the sun, the *umbra recta* decreased, while the *umbra iacens* increased. When 45° was reached, the pair of shadows, *umbra recta* and *umbra iacens*, were equal in size, and corresponded to the height of the object casting the shadow. After the sun has passed the 45° altitude, the former proportions of the shadows were inversed: The *umbra iacens* was longer than the *umbra recta*.[27] It seems that Halifax's mobile bodies obey these rules: Their motion changes, when the luminous

[25] "Ad istud argumentum dico quod mobile sequens umbram corporis *a* in aliqua parte temporis movetur velocius quam mobile sequens umbram alterius corporis, et in aliqua parte tardius, quia primo movetur multo <velocius> usque ad determinatum punctum in spatio et determinatum instans in tempore, et ab illo instanti movetur tardius usque ad finem temporis. Et causa est diversa appropinquatio umbrae ad corpus opacum ex quo causatur umbra", BNF, Lat. 15880, fol. 21[vb].

[26] "Sed in quo puncto illud quod prius tardius movebatur, incipiat velocius moveri, hoc potest haberi ex alia conclusione Perspectiva", BNF, Lat. 15880, 22[ra]. Halifax is possibly mentioning the (pseudo-)Bradwardinian treatise *Perspectiva cum sit una*; other treatises that include the inverse proportionality in the two kinds of shadows alluded to here as inverse motion are quoted in Lička, "Shadows in Medieval Optics", 209-210.

[27] Lička's transcription from a *Liber de umbris* gives the principle as follows: "Et sciendum umbram rectam sole oriente infinitam esse, iacente vero nullius quantitatis. Sole vero ascendente recta descrescit, iacens vero crescit. Si vero sol pervenerit usque ad altitudinem 45 graduum, erunt umbre equales. Si vero ascenderit ultra 45, fiet iacens maior recta. Et nota hec incrementa et decrementa umbrarum proporcionaliter esse; ut cum altera fuerit medietas sue mensure fixe, altera erit dupla sue mensure fixe.", Lička, "Shadows in Medieval Optics", 210, n. 91. Cf. the "mean motion" as "an angular distance measured from some base direction" in astronomy: North, *Stars, Minds and Fate*, 314.

bodies traverse the 45° altitude. The second period in time that starts then represents the following change in the motion of the mobile following opaque body *a*: "... First, it moves much faster until a given point in space and a given instant in time, and from that [point and instant], it moves slower until the end of the time."[28] While it moved fast with the *umbra recta* between 0° and 45°, it moves slower after the 45° were passed, when the *umbra recta* gets smaller, and vice versa.

With this inversion of the motion, Halifax significantly enriched his theorem, which in its first formulation read:

> Two moving bodies move precisely at the same time through two equal magnitudes, and one of them moves continuously quicker than the other for the whole time, and yet, by the end of that time, an altogether equal space will have been traversed by each of them.[29]

The second formulation gives a theorem, in which an arithmetical proportion of gain and loss in velocity beyond the mean degree allows for affirming the overall equality of velocity.[30] At this point, one can fully appreciate the dependence of Halifax's argument on both the new Oxford tradition about motion, and the old tradition, on which Bradwardine still relied, namely Gerard of Brussels's *Book on Motion*. Considerations of geometrically perfect, three-dimensional objects in motion, as well as their velocity at the midpoint are aspects that appear in both Gerard and Halifax, while they are absent from Bradwardine.[31] In the rest of the argument, and especially in the inferences that follow, Halifax repeatedly insists on the arithmetical nature of the proportions he established. With this, he stands in line with the optical tradition as it appears in the Oxford treatise called *Perspectiva cum sit una*, but underlines his

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[28] "... primo movetur multo <velocius> usque ad determinatum punctum in spatio et determinatum instans in tempore, et ab illo movetur tardius usque ad finem temporis", BNF, Lat. 15880, fol. 21^{vb}; see n. 25.

[29] "Duo mobilia in eodem tempore praecise moventur per duas magnitudines aequales, et unum illorum continue per totum tempus movetur velocius alio, et tamen in fine temporis ab utroque illorum est aequale spatium omnino pertransitum", BNF, Lat. 15880, fol. 21ra; see n. 21.

[30] According to M. Clagett, this principle is at the core of the theorem of uniform acceleration. See Clagett, *The Science of Mechanics*, 262-266.

[31] Halifax also considers, as Gerard does, the problem related to the circulation of a circular surface in its own plane. See BNF, Lat. 15880, fol. 21^{vb}. A closer comparison of Halifax's and Gerard's texts will be necessary to understand the nature of their interdependence. Also, as Murdoch and Sylla note, "Gerard's work consistently appears in medieval manuscripts together with other works on mathematics, statics, and optics, and not with questions or treatises in natural philosophy, something that was characteristic of fourteenth-century works on motion, even those were most mathematical in character", Murdoch and Sylla, "The Science of Motion", 222-223. Halifax's argument evidently tightens the link between Gerard and the Oxford Calculators.

difference toward Bradwardine, whose theorem of motion concerned geometrical proportions.[32]

As he discusses the inversion of proportions, Halifax switches to the causal explanation of the different motions in the shadows. This is an important issue because it returns to the beginning of the argument, and highlights the very reason of its complex setting, namely, the two shadows being part of the same shadow-cone: "If there were only one cause for the shortening of body *a*'s shadow, it would be shortened equally with the other shadow, but there are here two causes, equal to each other, for the diminishing of body *a*'s shadow, each of which is equal to the cause of the diminishing of the other body's shadow."[33] Halifax closes this issue with the diversity of proportions following the diversity of motions: "I say that these two causes of that one diminishing of the whole shadow do not make for a quicker diminishing of the whole shadow, only for a diversity of the proportions."[34] Bradwardine referred this principle to Averroes in his *Treatise on Proportions*, after he stated his theorem mentioned earlier:

> This is what Averroes intends when he says, in comment 71 on *Physics* 4: "...If, therefore, there are two movers and the things which they respectively move are equal, then the two motions are of equal speed. If the proportion is varied, the motion is also varied in that proportion.... The difference between motions with respect to slowness and fastness varies in accordance with the proportion between the two powers (namely, motive and resistive)."[35]

We recall that Halifax's proof of his argument relied on Aristotle's *Physics* 4; here, it ends with Averroes's commentary on the same passage.

After these considerations, Halifax calculates and discusses the implications the diverse mathematical proportions have for the moral agents.[36] This constitutes a

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[32] For the prevalence of arithmetical proportions in optics, see Lička, "Shadows in Medieval Optics", 207, 210. For Bradwardine's geometrical proportions, see Bradwardine, *His Tractatus*, 113.

[33] "Si non esset nisi una causa breviationis umbrae corporis *a,* aequaliter abbreviaretur cum umbra altera, sed iam sunt duae causae aequales inter se abbreviationis umbrae *a* corporis, quarum utraque est aequalis causa abbreviationis umbrae corporis *a*", Vat. Lat. 1111, fol. 14[ra].

[34] "Dico quod istae duae causae respectu abbreviationis unius umbrae non faciunt ad velocius abbreviationem totius umbrae, sed solum ad diversitatem proportionis", Vat. Lat. 1111, fol. 14[rb].

[35] Bradwardine, *His Tractatus*, 111.

[36] Here, I will give one example: "Et sic consimiliter respondetur ad illud argumentum de merito quod *a* plus meretur in aliqua parte *c* temporis quam *b* quia in prima parte proportionali. Sed *b* in omni parte post primam partem plus mereatur quam *a*, quia *b* acquisivit tantum de merito in tertia parte proportionali quam *a* acquisivit in secunda parte de novo. Sed tertia pars in duplo est brevior quam secunda pars, ergo *b* intensius meruit in secunda parte. Et sequitur propositum quod *a* per totum tempus non meretur intensius quam *b*. Antecedens probatur per casum, quia *a* habuit tantum de merito in fine primae partis quantum *b* habiturus fuit in fine secundae partis, et *a* habuit tantum in fine secundae partis quantum *b* fuit habiturus in fine tertio ut *b* 8. Si mensura meriti *a* in fine partis primae designata per 4 tantum habiturus est *b* in fine

definite turning point in his argument, for he introduces the infinite into the proportions. The infinite is mentioned only one time in Bradwardine's *Treatise on Proportions*.[37] Yet, Halifax informs it with a notion from Bradwardine's *Treatise*, namely composite bodies:

> Now a human being can commit a venial sin, for which he does not make satisfaction, and the same human being can commit a mortal sin, of which he does not repent. The same person then has a venial and a mortal sin. Let the mortal sin be *a* and the venial *b*, and I give the name *c* to both at the same time, likewise we give the name *knife* to signify the handle and the steel together. I do not want it to be united with the infinite otherwise than *per accidens*, as the Philosopher says in *Metaphysics* 5; not as if they would be something unified per se. ... If I posit them proportionally as before, *c* exceeds *a* with regard to the gravity of the sin, and thus one having *b* and *a*, which constitute *c*, it ascends heavier than if it had only *a*. This is evident per se.[38]

Halifax is concerned here with the commensuration or proportionality between the infinite and the finite. He posits two principles. One principle denies that the infinite can enter a substantial union with the finite. The other principle allows the finite entity to remain an element of the composite body one has to account for when

secundae partis, et sic mensura meriti *a* in fine partis secundae tantum habiturus est *b* in fine tertiae partis, ergo sicut *a* acquisivit in tertia. Et cum tertia pars est in duplo brevior quam secunda, ergo intensius meruit *b* in tertia quam *a*. Et sic potest argui de omnibus sequentibus", BNF, Lat. 15880, fol. 22ra.

[37] In the *Treatise*, Bradwardine mentions the following case from Aristotle's *On Heavens* 1: "In the chapter on the "infinite", where the following two theorems are proved: (1) that the infinite cannot be moved by the finite, and (2) that the infinite cannot move the finite", Bradwardine, *His Tractatus*, 119. Halifax proposes an argument on this section in Bradwardine, which cannot be presented here, but see n. 13 and 44. Yet, Bradwardine calculates with the infinite in his *Sentences* commentary, questions 1 and 9: Jean-François Genest, "Les premiers écrits théologiques de Bradwardine: textes inédits et découvertes récentes", in *Mediaeval Commentaries on the Sentences of Peter Lombard*, edited by G. Evans (Leiden: Brill, 2002), 395-421, 397-398, and 408-409; and in *De causa Dei*: Lukács, "Calculations in Thomas Bradwardine's *De causa Dei*, Book I", 115-122. In none of these theological writings is Bradwardine's presentation as analytical as Halifax's.

[38] "Nam unus homo potest committere unum peccatum veniale pro quo non satisfacit, et idem homo potest comittere peccatum mortale de quo non penitet. Idem habet tunc peccatum veniale et mortale. Et sit mortale *a* et veniale *b*, et inpono hoc nomen *c* ad significandum utrumque simul sicut inponitur hoc nomen *cultellus* ad significandum manubrium et ferrum simul iuncta. Nec volo aliter quod sint agregata infinita quam unum per accidens ut loquitur Philosophus 5 *Metaphysicae*, non ut sint per se aliquod unum... Et hoc posito proportionaliter sicut prius, *c* excedit *a* in gravitate peccati et sic habens *b* et *a* quae sunt *c*, gravius ascendit quam habens *a* solum. Illud est per se notum", BNF, Lat. 15880, fol. 22^{rb-va}. Here and in the quotes below in n. 39, 42, and 43, *a* stands for a mortal sin, *b* for a venial sin, and *c* for the composite entity uniting *a* mortal and *b* venial sin. For Thomas Aquinas, a knife was not a non-composite artificial object. Richard Kilvington referred to it in his *Sophismata* in discussing composites, as Halifax does, but without defining its nature. See *The Sophismata of Richard Kilvington*, edited by N. Kretzmann and B. E. Kretzmann (Oxford: Oxford University Press, 1990), 56.

calculating the motion of the composite body. The finite is neither annihilated, nor overwhelmed by the infinite. As for the relationship of the infinite to the composite body, Halifax describes it in using a peculiar vocabulary:

> *C* exceeds *a* with a proportion of greater inequality, and yet it is less than a proportion of greater inequality denominated by a given number, since it is neither double, nor triple, nor quadruple, nor sesquialternate, etc.; and yet, it is greater than a proportion of equality and less than every proportion of greater inequality denominated by a given number. In the same way, there can be a proportion less than a proportion of equality, and yet greater than every proportion of lesser inequality denominated by a given number.[39]

This passage relies on chapter 2 of Bradwardine's *Treatise on Proportions* in which the language and properties of proportions are presented and defined. Halifax does not only adopt Bradwardine's language, but his approach too. Bradwardine extended the role of denomination to proportions between incommensurables, which are according to him "not immediately, but mediately denominated by a given number, for they are immediately denominated by a given proportion, which is, in turn, immediately denominated by a number."[40] Thus, to a certain extent, mathematics admitted the commensuration we defined according to Bradwardine in introducing Halifax's question of the finite and the infinite.

Yet, "from these two inferences follow many others that appear surprising to many people."[41] Halifax introduces with these words a series of nine inferences closing question 1. I would like to point out only two of these inferences, which explain the infinite in terms of mathematical excess. While Halifax used Aristotle and Averroes approvingly until now with regard to the calculation of excess, in the second inference, he explicitly rejects Averroes's approach:

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[39] "*C* excedit *a* proportione maioris inaequalitatis et tamen minor proportione maioris inaequalitatis designabili per numerum quia nec est dupla, nec tripla, nec quadrupla, nec sexquialtera et sic de aliis; et tamen est maior proportione aequalitatis et minor omni proportione maiori inaequalitatis [corrected from *aequalitatis*] designabili per numerum. Et eodem modo potest esse proportio minor proportione aequalitatis et tamen maior omni proportione minoris inaequalitatis designabili per numerum", BNF, Lat. 15880, fol. 22[va]. On these proportions, see Thomas Bradwardine, *Traité des rapports entre les rapidités dans les mouvements*, translated by S. Rommevaux (Paris: Les Belles Lettres, 2010), xxviii–xxxi.

[40] Quoted in John E. Murdoch, "The Medieval Language of Proportions: Elements of the Interaction with Greek Foundations and the Development of New Mathematical Techniques", in *Scientific Change: Historical Studies in the Intellectual, Social, and Technical Conditions for Scientific Discovery and Technical Invention, from Antiquity to the Present*, edited by A. C. Crombie (New York: Basic Books, 1963), 237-271 and 258-259.

[41] "Ex istis duabus conclusionibus sequuntur multae aliae quae apparent multis mirabiles", BNF, Lat. 15880, fol. 22[va]. Bradwardine also uses the same adjective in talking about motion caused by magnets. See Bradwardine, *His Tractatus*, 123.

> The second inference is that not every <motion> exceeding another is divisible into an equal and an excess, which is against the Commentator in *Physics* 4, comment 74. The consequence is proved, since a mortal sin that is bigger than *c* would be smaller than *a*. Yet *c* is not divisible into an equal to *c* and into what exceeds *c*, since what would equal *c* is greater than *a*, since *c* is greater than *a*, and thus, it could be divided into an equal *a* and into what exceeds it; and thus, that by which it exceeds it would equal *b*, and thereby, the mortal part would be equal to the venial, the opposite of which has been said.[42]

This inference explains the infinite in terms of a composite mover through division, as we saw in the example of the knife. Another inference glosses the same relationship also in terms of excess, but this time through additions. This argument does not posit the infinite as negative, but in positive terms:

> The sixth inference is that any sin exceeds sin *a* only finitely, since not in its double proportion, and yet, through the finite addition of equal parts or the imperfect addition of unequal parts, it cannot be equal to itself. This is proved, since a mortal sin exceeds *a* only in a double proportion, and yet, *a* with the addition of one venial sin cannot be equal to itself, nor through the addition of two, three, or four equal parts, and so forth infinitely. Therefore, it cannot be equal to itself through the addition of equal, infinite other parts, as a syncategorematic infinite is created.[43]

Halifax again makes a significant contribution in applying the theorem and language of proportions Bradwardine presented in his *Treatise* to the syncategorematic infinite and in characterizing it as an ever-increasing series of venial sins. The fact that

[42] "Secunda conclusio est quod non omne excedens aliud est divisibile in aequale et excessum quod est contra Commentatorem, 4 *Physicorum*, commento 74. Et consequentia probatur quia peccatum mortale maius *c* esset minus *a*. *C* tamen non est divisibile in aequale *c* et in illud per quod excedit *c*, quia illud quod esset aequale *c*, esset maius *a*, cum *c* sit maius *a*, et tunc illud posset dividi in equale *a* et in illud per quod excedit. Et tunc illud per quod excederet, esset equale *b*, et ita pars mortalis esset aequale veniali, cuius oppositum dictum", BNF, Lat. 15880, fol. 22va. The fifth inference bespeaks the excess between different species, a topic that refers motion and proportions to their metaphysical background. On this background, see Sylvain Roudaut, *La mesure de l'être: Le problème de la quantification des formes au Moyen Âge (ca. 1250-1370)* (Leiden: Brill, 2022), 142-143.

[43] "Sexta conclusio est quod aliquod peccatum excedit *a* peccatum solum finite, quia nonnisi in duplo, et tamen per finitam additionem partium aequalium vel imperfectam additionem partium inaequalium non potest sibi aequari. Probatur, quia aliquod <peccatum> mortale excedit *a* solum in duplo, et tamen *a* cum additione unius venialis non potest sibi aequari, nec cum additione duorum aequalium, nec trium, nec quattuor, et sic in infinitum, ergo per infinitarum aliarum partium additionem aequalium non potest sibi aequari accidendo infinitum syncategorematice", BNF, Lat. 15880, fol. 22^{vb}. Halifax is said to have followed a contemporary Oxford theologian, Richard FitzRalph on the infinite, while some aspects of Gregory of Rimini's approval of the actual infinite fit Halifax's approach outlined here. See North, *Stars, Minds and Fate*, 243; and *De la théologie aux mathématiques. L'infini au XIV^{e} siècle*, edited by J. Biard and J. Celeyrette (Paris: Les Belles Lettres, 2005), 197-219.

the infinite in question is part of a specific theological case that human acting—and not divine existence—implies, is a further aspect of the complexity the Oxford Calculators had to deal with when thinking and calculating motion.

Conclusions

It is not easy to sort out the most significant novelty that Robert Halifax's argument provides. The experiment with shadows Halifax presents is the only optical experiment we know to apply the new Oxford method of calculating motion. The double setting of luminous and opaque bodies with shadow cones allows for the commensuration of different motions and the calculation of their velocity. Halifax applies the demonstration further to show that the proportional calculation of motion applies to Christian ethics, or, more simply, to the moral evolution of human beings. The preoccupation with the zenith, the change in the shadow's size at 45° altitude as a trigonometric premise and the applicability of this astronomical setting to a theological argument attest to the far-reaching context and implications that an optical experiment can have. In the same argument, Halifax provides inferences about the infinite. In these, he likewise uses proportional calculation, and emphasizes the theological and mathematical reality of the syncategorematic infinite.

From the first novelty, there is a rather simple, but significant historiographical conclusion to be posited. When, from the experiment with shadow cones, Halifax makes two inferences, the first concerns the philosophy of nature. In another section of his *Sentences* commentary, Halifax introduces another experiment, with the same pattern, intended to prove another theorem in Bradwardine's *Treatise on Proportions*.[44] Another experiment with the same pattern of shadow cones and decreasing bodies appears in one of the difficulties discussed in the treatise *De sex inconvenientibus* amid other theories of the Oxford Calculators.[45] Because at least two arguments about shadows, heavily indebted to the Franciscan optical tradition, were penned by Robert Halifax in his commentary on the *Sentences* and no one else, we shall assume that Robert Halifax was their author. If we define the Oxford Calculators as thinkers having contributed to or developed the method and theorems Bradwardine posited in the *Treatise on Proportions*, then we shall conclude that Robert Halifax was one of the Oxford Calculators.

Yet the greatest novelty Robert Halifax's argument implies concerns optics. In the growing series of aims medieval optics is said to have been tasked with– astronomy,

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[44] See n. 13 and 37.

[45] On this treatise, see Clagett, *The Science of Mechanics*, 216, 262, 263-265, and S. Rommevaux-Tani's works and her contribution to this issue. The argument the treatise *De sex inconvenientibus* refers to is not by Richard Kilvington, cf. Elżbieta Jung and Robert Podkoński, *Towards the Modern Theory of Motion: Oxford Calculators and the New Interpretation of Aristotle* (Łódż: Łódż University Press, 2020), 95.

practical geometry, and ethics–[46], Halifax adds a rather unexpected aim. With his argument, medieval optics accounted for quantitative change and proportional calculation of motion, thereby demonstrating theorems in mathematics and physics. This is a forceful, new approach we barely started considering.

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[46] For the first two see Lička, "Shadows in Medieval Optics", 217-223 and the literature quoted there; on the last, there is ample literature.

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