BOUND STATES OF THE VECTOR FIELD WITH A VORTEX IN THE ABELIAN HIGGS MODEL*

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It is pointed out that the vector particle of the 3+1 dimensional Abelian Higgs model can form bound states with a vortex. The modes of the vector field bound by the vortex are effectively represented by a two-dimensional Proca field living on the vortex. We also notice that a heavy-string limit of the vortex yields a bosonic string with internal degrees of freedom given by the two-dimensional Proca field.

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1.Introduction

The Abelian Higgs model is well-known for its very interesting features. Apart from being a textbook example of the Higgs phenomenon, it contains sectors with nonzero topological charge which is the winding number of the Higgs field. The ground state field configuration in the sector with the unit topological charge is given by a static, straight-linear vortex, [1,2]. The vortices have been observed in superconductors of the II-type. It has been shown by Nielsen and Olesen [2] that the vortex can be regarded as a field-theoretical prototype of a relativistic string.

In the following we shall consider the single-vortex sector of the model. This topological sector is defined by the requirements that the winding number for the Higgs field Φ is $+1$, and that the total energy per unit length of the vortex is finite. We shall consider the case of the static, straight-linear vortex. Generic fields Φ and A_μ in the single-vortex sector (A_μ is the Abelian gauge field), are given by sums of the proper fields of the vortex, denoted by $\Phi^{(0)}, A_\mu^{(0)}$, and of perturbations $\Phi^{(1)}, A_\mu^{(1)}$. The vortex fields $\Phi^{(0)}, A_\mu^{(0)}$ obey

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the classical field equations and they minimize the energy per unit length of the vortex. Thus, any perturbation of the vortex increases the energy (except for the translational and rotational zero-modes).

In general, we expect that any localized perturbation of the vortex will disperse very quickly – that is,its magnitude will decrease while its spatial size will increase in all directions with time – because all fields in the model have nonzero mass, due to the Higgs mechanism. However, in the presence of the vortex this reasoning is not complete. The point is that inside of the vortex tube the Higgs mechanism does not operate fully. The proper Higgs field of the vortex $\Phi^{(0)}$ vanishes on the center line of the vortex, and it exponentially approaches its vacuum value v in the directions perpendicular to the vortex. Thus, outside of the vortex the standard Higgs mechanism works and the vector field acquires a nonzero mass $m_A^2 \sim v^2$, but within the vortex tube the Higgs field is small and the would be mass term for the vector field vanishes on the center line of the vortex. Therefore, one may expect that those modes of the vector field which are localized on the vortex and propagate along it, can have special properties.

In fact, in the present paper we show that there exist particular small perturbations of the vortex which do not disperse in the directions perpendicular to the vortex. These perturbations are given by certain modes of the $A_0^{(1)}, A_3^{(1)}$ fields. We shall show that they can be regarded as the two-dimensional massive vector field (the Proca field) living on the vortex. The mass m_l^2 of the Proca field is smaller than the mass m_A^2 of the vector field. Therefore, one can say that the modes of the gauge field living on the vortex describe the vector meson of the mass m_A^2 captured by the vortex to form a bound state. Of course, the total energy is higher than the energy of the vortex. Thus, the particular perturbations we have found should be regarded as excitations of the vortex.

Our considerations are restricted to a particular kind of the small perturbations of the vortex. An analysis of all perturbations does not seem to be possible at the moment — let us recall the fact that even the vortex solution itself is quite cumbersome [3].

The plan of our paper is the following. In Section 2 we fix our notation and we recall the basic facts about the vortex solution. In Section 3 we describe the small excitations of the vortex. Section 4 is devoted to a discussion. In particular. we consider there the heavy-string limit of the vortex.

2. Reminder of the Abelian Higgs model

Let us recall the Lagrangian of the Abelian Higgs model,

$$L = (\partial^\mu - iqA^\mu)\Phi^*(\partial_\mu + iqA_\mu)\Phi - \tfrac{1}{4}\lambda\left(\Phi^*\Phi - \frac{2\mu^2}{\lambda}\right)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

and the corresponding Euler–Lagrange equations

$$(\partial^\mu + iqA^\mu)(\partial_\mu + iqA_\mu)\Phi + \tfrac{1}{2}\lambda\Phi\left(\Phi^*\Phi - \frac{2\mu^2}{\lambda}\right) = 0, \tag{2}$$

$$\partial_\nu F^{\mu\nu} = iq(\Phi\partial^\mu\Phi^* - \Phi^*\partial^\mu\Phi) + 2q^2A^\mu\Phi^*\Phi. \tag{3}$$

The gauge invariant energy-momentum tensor has the form

$$\begin{aligned} T^{\mu\nu} = {} & (\partial^\nu + iqA^\nu)\Phi(\partial^\mu - iqA^\mu)\Phi^* + (\partial^\mu + iqA^\mu)\Phi(\partial^\nu - iqA^\nu)\Phi^* \\ & -g^{\mu\nu}(\partial_\sigma + iqA_\sigma)\Phi(\partial^\sigma - iqA^\sigma)\Phi^* + g^{\mu\nu}\frac{\lambda}{4}\left(\Phi^*\Phi - \frac{2\mu^2}{\lambda}\right)^2 \\ & +\tfrac{1}{4}g^{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma} - F^{\mu\sigma}F^\nu{}_\sigma. \end{aligned} \tag{4}$$

In the topologically trivial sector one can remove the phase of the Φ field (the Goldstone field) by an appropriate nonsingular gauge transformation. Then, the field Φ becomes real. Next, one introduces a new Higgs field χ,

$$\Phi(x) = v + \chi(x), \quad v = \sqrt{\frac{2\mu^2}{\lambda}},$$

which has the mass $m_\chi^2 = 2\mu^2$. The gauge field absorbs the Goldstone field and becomes massive, $m_A^2 = 4q^2\mu^2/\lambda$.

Now, let us turn to the single-vortex sector. The static, straight-linear vortex lying on the x^3-axis is a solution of equations (2), (3) of the following form [1,2]

$$A_0^{(0)} = A_3^{(0)} = 0\,, \quad A_1^{(0)} = -\frac{x^2}{\rho}H(\rho)\,, \quad A_2^{(0)} = \frac{x^1}{\rho}H(\rho), \tag{5}$$

$$\Phi^{(0)} = \sqrt{\frac{2\mu^2}{\lambda}}e^{i\theta}F(\rho), \tag{6}$$

where $\rho = \sqrt{(x^1)^2 + (x^2)^2}$ is the radius in the (x^1, x^2) plane, and $\theta = \arctan(x^2/x^1)$ is the azimuthal angle.

Energy per unit length of the vortex is given by the formula

$$E_{\text{vortex}} = \int dx^1dx^2T^{00}.$$

It can be written in the following form [4]

$$\begin{aligned} E_{\text{vortex}} = {} & \frac{4\pi\mu^2}{\lambda} + \int dx^1dx^2\left(\frac{1}{2}\left(H' + \frac{H}{\rho} + \frac{2\mu^2q}{\lambda}\left(1 - F^2\right)\right)^2\right. \\ & \left. +\frac{2\mu^2}{\lambda}\left(F' - qF\left(H + \frac{1}{q\rho}\right)\right)^2 + \left(\lambda - 2q^2\right)\frac{\mu^4}{\lambda^2}\left(1 - F^2\right)^2\right), \end{aligned} \tag{7}$$

where the superscript " ' " denotes the derivative with respect to ρ. The functions $H(\rho), F(\rho)$ have the following asymptotic behaviour. For $\rho \to 0$

$$H(\rho) \approx b_0\rho\,, \quad F(\rho) \approx b_1\rho. \tag{8}$$

For ρ sufficiently large

$$H(\rho) \approx -\frac{1}{q\rho} + \frac{c_0}{\sqrt{\rho}}e^{-m_A\rho}\,, \quad F(\rho) \approx 1 - c_1 e^{-M\rho}\,, \tag{9}$$

where [5]

$$M = \min(2m_A, m_\chi).$$

The vortices are observed for $\lambda > 2q^2$. This condition is equivalent to $m_\chi^2 > m_A^2$.

3. The small perturbations of the vortex

Now, let us investigate small perturbations of the vortex solution. We shall use the following notation.

$$\Phi(x) \equiv \sqrt{\frac{2\mu^2}{\lambda}}\Psi(x)\,, \quad (A_\mu) \equiv (A_\alpha, A_i),$$

$$(\partial_\alpha) \equiv (\partial_0, \partial_3)\,, \quad (\partial_i) \equiv (\partial_1, \partial_2)\,, \tag{10}$$

where $\alpha = 0, 3$ and $i = 1, 2$. We shall also use the two-dimensional Laplacian

$$\triangle \equiv (\partial/\partial x^1)^2 + (\partial/\partial x^2)^2.$$

For the vortex solution $A_\alpha^{(0)} = 0$. The vortex solution obeys the Lorentz gauge condition $\partial_\mu A^{(0)\mu} = 0$. We do not impose this gauge condition on all fields A_μ because the gauge is already fixed by the form (5), (6) of the vortex solution. In order to obtain equations for the small perturbations we substitute in Eqs (2), (3)

$$\Psi = \Psi^{(0)} + \psi\,, \quad A^i = A^{(0)i} + a^i,$$

and we retain only the terms linear in ψ, a^i and A^α. We obtain the following equations for ψ, a^i, A^α.

$$-\partial_\beta\partial^\beta A^\alpha + \triangle A^\alpha + \partial^\alpha(\partial_\beta A^\beta + \partial_k a^k) = \frac{4q^2\mu^2}{\lambda}\Psi^{(0)*}\Psi^{(0)}A^\alpha$$
$$+\frac{2iq\mu^2}{\lambda}\left(\Psi^{(0)}\partial^\alpha\psi^* - \Psi^{(0)*}\partial^\alpha\psi\right)\,, \tag{11}$$

$$- \partial_\beta \partial^\beta a^i + \triangle a^i + \partial^i \left(\partial_\beta A^\beta + \partial_k a^k\right) = \frac{4q^2\mu^2}{\lambda} \Psi^{(0)*}\Psi^{(0)} a^i$$
$$+ \frac{4q^2\mu^2}{\lambda} A^{(0)i} \left(\Psi^{(0)*}\psi + \Psi^{(0)}\psi^*\right)$$
$$+ \frac{2iq\mu^2}{\lambda} \left(\Psi^{(0)}\partial^i\psi^* + \psi\partial^i\Psi^{(0)*} - \Psi^{(0)*}\partial^i\psi - \psi^*\partial^i\Psi^{(0)}\right), \tag{12}$$
$$- \partial_\alpha\partial^\alpha\psi + \left(\partial_k + iqA_k^{(0)}\right)\left(\partial_k + iqA_k^{(0)}\right)\psi = iq\left(\partial_\alpha A^\alpha + \partial_i a^i\right)\Psi^{(0)}$$
$$+ 2iqa^k\left(\partial_k + iqA_k^{(0)}\right)\Psi^{(0)} + \mu^2\left(2\Psi^{(0)*}\Psi^{(0)} - 1\right)\psi + \mu^2\left(\Psi^{(0)}\right)^2\psi^* . \tag{13}$$

The system of equations (11) — (13) is quite complicated. Full analysis of the spectrum of the small perturbations is out of our reach. We shall only consider particular perturbations which describe the vector field bound by the vortex. We shall assume that the fields of the vortex are not changed in the linear approximation, *i.e.*

$$\psi = 0, \qquad a^i = 0. \tag{14}$$

Let us note that this assumption excludes the zero modes from our considerations. It follows from equation (13) that

$$\partial_\alpha A^\alpha = 0 . \tag{15}$$

Then equation (12) is also satisfied , while equation (11) reduces to

$$-\partial_\beta\partial^\beta A^\alpha + \triangle A^\alpha = \frac{4q^2\mu^2}{\lambda} A^\alpha F^2(\rho) . \tag{16}$$

Equation (16) can be analysed further with the help of separation of variables. We separate (x^0, x^3) from (x^1, x^2) :

$$A^\alpha = W^\alpha(x^0, x^3)\phi(x^1, x^2) .$$

Then, it follows from equation (16) that W^α, ϕ obey the following equations

$$-\partial_\beta\partial^\beta W^\alpha = cW^\alpha , \tag{17}$$
$$-\triangle\phi + \frac{4q^2\mu^2}{\lambda} F^2(\rho)\phi = c\phi , \tag{18}$$

where c is the separation constant.

Equation (18), after dividing it by 2, has the form of two-dimensional Schroedinger equation for levels $\varepsilon \equiv c/2$ of a particle of unit mass in the potential well

$$V(\rho) = \frac{2q^2\mu^2}{\lambda} F^2(\rho)$$

($c = \hbar = 1$). The shape of the potential well is given by formulae (8), (9). Its height is $2q^2\mu^2/\lambda$ and the width is of the order M^{-1}. It is clear that the eigenvalues $\varepsilon_l = c_l/2$ are positive (we assume that $\lambda > 0$). Using well-known results for the potential well

$$V_1(\rho) = V_0\Theta(\rho - a)\,,$$

we can estimate the largest eigenvalue ε_1 corresponding to a bound state. For the potential well $V_1(\rho)$ the highest bound state has the energy [6,7]

$$\varepsilon_1 \approx V_0 - \frac{2}{a^2}\exp\left(-\frac{2}{a^2V_0}\right).$$

In our case

$$V_0 \approx \frac{2q^2\mu^2}{\lambda}, \quad a \approx \frac{1}{M}.$$

Any two-dimensional well, no matter how narrow or shallow it is, always has at least one bound state [6,7]. The strength of the potential well is characterized by the parameter $\xi \equiv a^2V_0$. In our case

$$\xi \approx M^{-2}2q^2\mu^2\lambda^{-1} \leq 0.5$$

if $\lambda \geq 2q^2$. States with nonzero angular momentum appear if $\xi \geq 2.88$, [7]. Thus, in our case the bound states are the s-states, and the corresponding wave functions depend only on ρ.

Let ϕ_l, $l = 1, 2, \ldots$, denote the wave functions of the bound states determined from equation (18). We may choose them to be real valued. They are orthogonal to each other and normalized to 1. Then, the general solution of equation (16) can be written in the form

$$A^\alpha = \sum_l W^\alpha_{(l)}(x^0, x^3)\,\phi_l(x^1, x^2)\,, \tag{19}$$

where $W^\alpha_{(l)}(x^\beta)$ obeys the equation

$$-\partial_\beta\partial^\beta W^\alpha_{(l)} = c_l W^\alpha_{(l)}\,, \tag{20}$$

with $c_l = 2\varepsilon_l$, and the Lorentz condition

$$\partial_\alpha W^\alpha_{(l)} = 0. \tag{21}$$

The modes of the gauge field which are given by formula (19) are localized on the vortex because the eigenfunctions ϕ_l exponentially vanish for $\rho > a$.

Using formulae (4), (14), (19) and the orthogonality of the eigenfunctions ϕ_l it is easy to compute the energy per unit length of the vortex. The result is

$$E = \int dx^1 dx^2 T^{00}$$
$$= E_{\text{vortex}} + \sum_l \left(\tfrac{1}{2} \left(\partial_0 W^3_{(l)} - \partial^3 W^0_{(l)} \right)^2 + \tfrac{1}{2} c_l \left((W^0_{(l)})^2 + (W^3_{(l)})^2 \right) \right), \quad (22)$$

where E_{vortex} is the contribution of the vortex alone (*i.e.* of the fields $A^{(0)}_\mu$, $\Phi^{(0)}$). We see that the contribution of the $W^\alpha_{(l)}$ field in formula (22) has the form of energy density of the two-dimensional, real-valued Proca field of the mass $m_l^2 = c_l$. The Lagrangian of such a field has the form

$$L_W = -\tfrac{1}{4}(\partial_\alpha W_\beta - \partial_\beta W_\alpha)^2 + \tfrac{1}{2} m_l^2 W^\alpha W_\alpha , \quad (23)$$

where $\alpha, \beta = 0, 3$. The Euler–Lagrange equations following from the Lagrangian (23) imply the Lorentz condition (21), and afterwards they are reduced to equation (20).

Momentum per unit length of the vortex is given by the formulae

$$P^i = \int dx^1 dx^2 T^{0i} = 0, \quad i = 1, 2 , \quad (24)$$

$$P^3 = \int dx^1 dx^2 T^{03} = \sum_l c_l W^3_{(l)} W^0_{(l)} . \quad (25)$$

Again, this result agrees with the supposition that $W^\alpha_{(l)}$ is the Proca field — Lagrangian (23) gives the same formula for P^3. In order to prove that the energy and momentum densities for the Proca field obtained from Lagrangian (23) are equivalent to formulae given by (22), (25) we consider the integrated (over x^3) quantities and we use equations (20), (21).

Components of the angular momentum of the perturbed vortex are given by the formula

$$M^{ik} = \int dx^1 dx^2 (x^i T^{0k} - x^k T^{0i}) .$$

They do not vanish in general. The reason is that the perturbed vortex has nonzero electric field $E^i \equiv F_{0i} = -\partial_i A_0$, $i = 1, 2$, $E^3 \equiv F_{03} = \partial_0 A_3 - \partial_3 A_0$, and the magnetic field $-B^3 \equiv F_{12} = \partial_1 A^{(0)}_2 - \partial_2 A^{(0)}_1$, $-B^2 \equiv F_{31} = -\partial_1 A_3$, $-B^1 \equiv F_{23} = \partial_2 A_3$. It is easy to see that the angular momentum density of the electromagnetic field, given by the double cross-product $\vec{x} \times (\vec{E} \times \vec{B})$, does not vanish. Simple computation gives

$$M^{31} = M^{23} = 0 .$$

These components vanish because of the axial symmetry of the perturbed vortex. For the M^{12} component we obtain the following formula

$$M^{12} = 4\pi \sum_l W^0_{(l)} \int\limits_0^\infty d\rho\, \rho\, \phi_l \left(H' + \frac{H}{\rho} \right) .$$

In the derivation we have used integration by parts, and the fact that $H(\rho)$ obeys the equation

$$\left(H' + \frac{H}{\rho} \right)' = \frac{4q^2\mu^2}{\lambda} F^2 \left(H + \frac{1}{q\rho} \right) ,$$

which is equation (3) in the case of the unperturbed vortex. The formula for M^{12} given above might of course change its form if we pass from our approximate solution to the exact solution.

4. Remarks

(i) Our considerations have the obvious shortcoming that they are restricted to the small perturbations of the vortex. To go beyond the linear approximation to the equations of motion does not seem to be a simple task. Let us recall the discouraging fact that even the exact form of the vortex solution itself is not known as yet. The best result till now has the form of an infinite series obtained only for the particular case $\lambda = 2q^2$, [3]. On the other hand, symmetry of the problem is quite large, and it suggests a rather simple Ansatz for the exact form of the fields pertinent to our problem. We hope that on the basis of this Ansatz we can obtain a proof that the exact solution with the required properties exists. Work in this direction is in progress.

In the recent paper, [8], finite perturbations of the vortex exactly obeying the field equations (2), (3) have been found. These perturbations are different from the ones considered in our paper, because they are massless and because they modify the Higgs field and the A^i, $i = 1, 2$, fields already in the linear approximation.

(ii) It is a well-known fact that otherwise unrelated fields can be transformed into each other when considered in 1+1 dimensional space-time. In this vein, one may suspect that the classical Proca field in the two-dimensional case is equivalent to a scalar field, because the gauge condition (21) allows us to eliminate one of the two components of this field. More precisely, one could introduce the "potential" u,

$$W_\alpha = \epsilon_{\alpha\gamma} \partial^\gamma u .$$

Then the condition (21) is satisfied automatically, while equation (20) takes the form

$$\partial_\beta \partial^\beta \partial_\gamma u + c_l \partial_\gamma u = 0 .$$

This equation is equivalent to the equation

$$\partial_\beta \partial^\beta u + c_l \, u = b_0 , \tag{26}$$

where b_0 is an arbitrary constant. Equation (26) has the following general solution

$$u = \frac{b_0}{c_l} + h(x^\alpha) , \tag{27}$$

where $h(x^\alpha)$ obeys the ordinary wave equation

$$\partial_\beta \partial^\beta h + c_l \, h = 0 . \tag{28}$$

Thus, $h(x^\alpha)$ is the usual classical scalar field. On the other hand, the field u is not the usual classical scalar field, because from formula (27) we see that it contains one degree of freedom more than necessary, namely the b_0 mode. Therefore, the two-dimensional classical Proca field is not equivalent to the usual scalar field.

(iii) One of the most interesting aspects of our work is that it has some implications for the stringlike limit of vortices. The idea that the relativistic strings can be obtained from vortices in a field-theoretical model has been put forward by Nielsen and Olesen, [2]. This idea has been elaborated on by Förster [9], and by Gervais and Sakita [10]. It has been shown that the action functional for the Abelian Higgs model in $3+1$ dimensional space-time, calculated for a thin vortex in a strong coupling limit, contains the action functional for Nambu–Goto string as the leading term. Recently, the relationship between relativistic strings and vortices has been investigated again, see, *e.g.* [11–14]. This time, the main problem addressed to is the form of the term which is next to the leading Nambu–Goto term. In all these attempts to relate strings to the vortices of the Abelian Higgs model only elementary bosonic strings with no internal degrees of freedom have appeared in the leading order.

In the paper [9] it has been shown that all excitations of the vortex decouple in the strong coupling limit. This limit is taken in such a way that $\mu^2 R^2, \lambda$ and q increase, but the quotients $\mu^2 R^2/\lambda$ and q^2/λ are kept constant. Here we consider a curved vortex, and R is the minimal value of the radius of curvature of the line on which the Higgs field vanishes. In this limit the vector particle and the Higgs particle have very large masses, while the energy of the vortex per unit length is kept constant — it is of the order μ^2/λ. This energy becomes the string tension. The vortex becomes very thin and it can be approximately regarded as a string. The string tension is a dimensionful quantity. Therefore, we cannot meaningfully say whether it is small or large. In order to obtain a simple dimensionless characteristics κ of the mass of a piece of the string we take the energy stored in the piece of the string of the length equal to the Compton wave length of the Higgs particle, and we divide it by the mass of this particle,

$$\kappa \sim \frac{\mu^2}{\lambda} \frac{1}{\mu} \frac{1}{\mu} = \frac{1}{\lambda}.$$

Thus, in the strong coupling limit κ is very small because λ is of the order $\mu^2 R^2$. Therefore, we may say that the string obtained in this limit is light. As argued by Förster, its internal degrees of freedom are practically frozen, because in order to excite them one needs a very large energy of the order μ which is hard to come by in a typical motion of the light string.

From a theoretical point of view, and also in view of possible applications in cosmology [15], it is also interesting to consider another type of the stringlike limit, namely the heavy-string limit. In this limit $\mu^2 R^2$ becomes very large, while λ and q^2 are kept constant. Then, the masses of the vector particle and of the Higgs particle again become very large, but this time the energy of the vortex per unit length is also of the order μ^2. The vortex again is very thin and can be regarded as a string. However, the relative mass of the piece of the string, given by $\kappa \sim \lambda^{-1}$, does not vanish in this limit. For $\lambda = 1$ the piece of the string is as heavy as the Higgs particle. The mass of the Proca particle m_l^2 is also of the order μ^2. Therefore, we expect that the degrees of freedom given by the Proca field will be excited during a typical motion of the curved vortex. The corresponding string will have internal degrees of freedom represented by the two-dimensional Proca field. For still more energetic motions of the heavy vortex one should not neglect emission of the Higgs and of the vector particles from the vortex. In this regime the string is coupled to certain modes of the Higgs and of the vector fields. Calculational details of the heavy-string limit of the vortex will be presented in a forthcoming paper.

Note added in proof: After this paper was written we have learned that the idea of the vector field propagation along the vortex has been outlined in the paper by R.L. Davis, *Nuc. Phys.* **B294**, 867 (1987).

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