ESSAYS IN PANEL STATIONARITY AND COINTEGRATION TESTS

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Abstract

This Ph.D thesis consists of two studies in the panel stationarity and cointegration tests. Three main chapters have been developed. The first two refer to the panel stationarity study. In the first chapter, a residual-based LM panel stationarity test with structural breaks is proposed, in which four models based on different break patterns are specified. For two of the models, a modified test which does not asymptotically depend on break location is also proposed. It is shown that both statistics after standardization have standard normal limiting distribution that is free of nuisance parameters. We derived the asymptotic moments of both statistics in closed form via characteristic functions. Monte Carlo simulations are conducted and the results show that the LM test has a good performance in finite samples but the modified test in the presence of autocorrelated residuals performs less satisfactory. We then provide, in the second chapter, an empirical application to 14 macroeconomic and financial variables of OECD countries for the LM test. To select the appropriate break type for each variable, we applied the BIC and AIC criteria for each country, therefore, different models are allowed across countries. A bootstrap procedure is employed to control for the existence of cross-sectional dependence in the data. We found strong evidence of stationarity once a structural break and cross-sectional dependence are accommodated.

In the third chapter, we move to the panel cointegration study. A panel cointegration test with the null hypothesis of cointegration is considered. This is an extension of Harris, Leybourne and McCabe (2005) (HLM thereafter) stationarity test. We distinguish three models depending on the specifications of the integrated variables. The test is advantageous to control for the cross-sectional dependence and serial correlation of unrestricted structure in the panel. Although the statistic is shown to have asymptotic standard normal null distribution, the simulation results indicate a size distortion when the panel dimension is relatively large due to the finite-sample estimation errors. To correct for this, the bias correction factors proposed in HLM (2005) are used in the test and the validity of bias correction factors is assessed in the panel cointegration context. Monte Carlo simulations suggest the test provides a good approximation in finite samples.

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Chapter 1

General Introduction

This Ph.D thesis is organized in 6 chapters.

Chapter 2 begins with a detailed literature review about various panel stationarity or unit root tests as well as panel cointegration tests, at the same time, focusing on some of the tests that mainly motivated the work in this thesis.

Chapter 3 continues with a contribution of this work to the literature by proposing a panel residual-based Lagrange Multiplier (LM thereafter) stationarity test allowing for possible structural breaks. Four models based on different break effects are developed. For two of the models, a modified test of which the asymptotic distribution does not depend on break location is also proposed. The tests with both *i.i.d* and serial correlated residuals are discussed. We allow for different breaking dates in each individual unit. Both tests are shown to have standard normal distributions after standardizing using the appropriate moments and applying for Central Limit

Theorem. The asymptotic distributions are derived using sequential limits, wherein $T \to \infty$ followed by $N \to \infty$. The exact moments of all corresponding models for both tests are derived via the characteristic function technique. We also analyze the case when the break location is unknown, in which the least-square method is used to estimate the break. The good performance of the LM test in finite-sample are confirmed through Monte Carlo simulations. Simulations of the modified test indicate less satisfactory results in the presence of serially correlated residuals.

In Chapter 4, as an illustration, we apply the LM test to the annual data of 14 macroeconomic and financial variables of OECD countries. Instead of following a visual inspection procedure or imposing the most general model specifications, we use BIC and AIC criteria to select the appropriate break type. Different types of models are allowed for across countries for each variable. It is found that different combinations of all the models rather than any single particular model are selected for 13 out of 14 variables. For the determination of the autoregressive lag length, a general to specific recursive procedure *tsig* where the lag length is determined as the last lag in the autoregression that has a significant t statistic at the 10% significance level and the procedure based on BIC criterion are applied for comparison. The bootstrap method is used to correct for the existence of cross-sectional dependence in the data and the bootstrap critical values are obtained from the empirical distributions. The reported results indicate strong evidence of stationarity for all the variables when a structural break and cross-sectional dependence is accommodated.

Chapter 5 of the thesis is concerned with a panel cointegration test with the null hypothesis of cointegration. This is an extension of the panel stationarity test of Harris, Leybourne and McCabe (2005) (HLM thereafter) which has the advantage of allowing for general form of cross-sectional dependence as well as serial correlations in the panel. We consider three models based on different components of the integrated variables. The statistic is shown to have the standard normal distribution under the null. However, the finite-sample results evaluated by simulations show large size distortions when the panel dimension is relatively large. To avoid this problem, which is due to the accumulated individual finite-sample estimation errors, we include the bias correction factors proposed in HLM in our cointegration test. It is assessed that the bias correction factors are still valid in the panel cointegration context. We also show that the asymptotic normality properties still hold for the statistic. Monte Carlo simulations suggest that the finite-sample performance improves significantly. Therefore, the test proves to be an adequate approximation for the finite-sample distribution.

Finally, Chapter 6 concludes with the main findings in the thesis.

Chapter 2

Literature Review

In this chapter we provide a literature review of nonstationary panel tests. This review is not intended to be exhaustive but concentrated on the recent panel unit root tests, stationarity tests and cointegration tests which are the main motivations of the contributions in Chapter 3, 4 and 5. This chapter is organized as follows. We begin with a background introduction to recent developments in panel unit root and cointegration analysis, and then, move to discuss some important panel unit and stationarity tests in Section 2.2. The review of panel cointegration tests is considered in Section 2.3.

2.1 Introduction

With the growing availability of cross-country data over time, a variety of important panel data sets have been constructed and widely used. Some of these panel data sets, like the Penn-World tables, cover different individuals, industries, and countries over long time periods and have been useful in assessing and comparing economic growth characteristics such as real per capita GDP growth. The analysis of unit roots and cointegration in panel data has been a fruitful area of study in recent years, with Levin and Lin (1992, 1993), Quah (1994), Breitung and Meyer (1994), Im *et al.* (1997, 2003), O'Connell (1998), Maddala and Wu (1999), Kao (1999), McCoskey and Kao (1998) and Pedroni (1999, 2004) being the most noticeable studies in this field. The motivation of using panel data in unit roots and cointegration tests is with the hope that combing information from both the time series and cross-sectional dimensions would increase the statistical power of the tests and provide more reliable empirical evidence than their univariate counterpart, especially in situations where the time series for the data may not be very long but very similar data may be available across a cross section of units such as countries, regions, firms or industries. This was supported by the application of, for example, panel unit root tests to real exchange rates, output and inflation. For the example of real exchange rates, the hypothesis that the real exchange rate is nonstationary cannot typically be rejected by the augmented Dickey-Fuller test based on the single time series. In contrast, panel unit root tests applied to a collection of industrialized countries generally find that real exchange rates are stationary, therefore suggesting empirical support to the purchasing power parity hypothesis.

Among the studies mentioned above, the unit root is the null hypothesis to be tested. Since the classical hypothesis is carried out in the way that ensures the null hypothesis is accepted unless there is strong evidence to the contrary, hence, acceptance of the null of a unit root does not necessarily imply the existence of a unit root. On the other hand, when the null of a unit root is rejected, we cannot conclude that the process is trend stationary, as the unit root tests considered may have power not only against a stationary process with a break but also against more general alternatives. Therefore, in order to decide by classical methods whether economic data are stationary or integrated, it would be useful to perform tests of the null hypothesis of stationarity, as well as tests of the null hypothesis of unit root. Recent panel stationarity tests have been proposed by, *inter alia*, Hadri (2000), Shin and Snell (2002, 2006).

Another line of research that has paralleled developments in nonstationarity analysis relates to the tests in the presence of structural changes. A large literature has addressed the interplay between structural changes and unit roots due to the fact that both classes of processes contain similar qualitative features. Most tests that attempt to distinguish between a unit root and a (trend) stationary process will favor the unit root model when the true process is subject to structural changes but is otherwise (trend) stationary within regimes specified by the break dates. Also, most tests trying to assess whether structural change is present will reject the null hypothesis of no structural change when the process has a unit root component but with constant model parameters. Since the pioneering work of Perron (1989), the importance of allowing for possible structural changes in testing for a unit root has attracted attention in the literature. It is now well accepted that ignoring the existing structural breaks is likely to cause misleading inferences, that is, a significant loss of power in the unit root tests and severe size distortion in stationarity tests. This problem exists in both panel and time series tests.

Recent developments in panel data analysis have focused attention on unit root and cointegration properties of variables observed over a relatively long span of time across a large number of cross section units. Such macro panels with large cross sectional dimension (N) and large time series dimension (T) have different characteristics and implications for theoretical and empirical analysis from the large N small T micro panels. For example, when T is large, there is an obvious need also to consider serial correlation patterns in the panel, more generally, including both short memory and persistent components. Also, with large T, a proper limit theory for the asymptotic analysis rather than the conventional methods for just large N is needed.

2.2 Panel Unit Root and Stationarity Tests

According to whether unit root or stationarity tests allow for potential correlations across the residuals of panel units, two generations of tests can be distinguished. The first generation is based on the cross-sectional independence hypothesis while the second generation of the tests relax this assumption and aim to exploit the co-movements in order to define new test statistics. We overview these two generations of tests in turn in the Subsection 2.2.1 and 2.2.2. As another important parallel development in panel nonstationarity tests, the panel stationarity tests allowing for structural breaks are discussed in Subsection 2.2.3.

2.2.1 The First Generation Tests

This subsection reviews the contributions of Levin and Lin (1992, 1993), Im *et al.* (1997, 2003), Maddala and Wu (1999), Choi (2001) and Hadri (2000). These are the most noteworthy studies in the first generation of panel unit root and stationarity tests where cross-sectional dependence is not accounted for.

Levin, Lin and Chu tests

Being one of the seminal contributions in the field of panel unit root test, Levin and Lin (1992, 1993) and later Levin, Lin and Chu (2002) suggest a panel unit root test with the null hypothesis of a unit root in each individual time series against the alternative that each individual is stationary. This was motivated by the problem that individual unit root tests have limited power against the alternative hypothesis. The structure of Levin, Lin and Chu test (LLC hereafter) can be summarized as follows:

$$\Delta y_{it} = \rho y_{it-1} + \sum_{L=1}^{p_i} \theta_{iL} \Delta y_{it-L} + \alpha_{mi} d_{mt} + \varepsilon_{it}, \ m = 1, 2, 3, \tag{2.1}$$

where d_{mt} are the deterministic variables and α_{mi} are the corresponding vector of coefficients for model $m = 1, 2, 3$. In particular, $d_{1t} = \varnothing$, $d_{2t} = 1$, $d_{3t} = (1, t)'$.

To determine the lag order p_i for each unit, first choose a maximum lag order $p_{\max}$ then use the $t-$statistic of θ_{iL} to decide if a smaller lag is preferred. After this, the regression of (2.1) is estimated by regressing first Δy_{it} and then y_{it-1} on the remaining variables in (2.1), obtaining the residuals $\hat{e}_{it}$ and $\hat{v}_{it-1}$ respectively, and standardizing them by $\tilde{e}_{it} = \hat{e}_{it}/\hat{\sigma}_{\varepsilon i}$ and $\tilde{v}_{it-1} = \hat{v}_{it-1}/\hat{\sigma}_{\varepsilon i}$. LLC then estimate the ratio of long-run to short-run standard deviations. Under the null hypothesis, the long-run variance of (2.1) can be estimated by

$$\hat{\sigma}_{yi}^2 = \frac{1}{T-1}\sum_{t=2}^{T}\Delta y_{it}^2 + 2\sum_{L=1}^{\bar{K}}\omega_{\bar{K}L}[\frac{1}{T-1}\sum_{t=2+L}^{T}\Delta y_{it}\Delta y_{it-L}],$$

where $\bar{K}$ is a truncation lag and $\omega_{\bar{K}L} = 1-(L/(\bar{K}+1))$ for a Bartlett kernel. For each i, the ratio of the long-run standard deviation to the innovation standard deviation is estimated by $\hat{s}_i = \hat{\sigma}_{yi}/\hat{\sigma}_{\varepsilon i}$ and the average standard deviation is calculated by $\hat{S}_N = N^{-1}\sum_{i=1}^{N}\hat{s}_i$. Finally, estimate the pooled regression

$$\tilde{e}_{it} = \rho\tilde{v}_{it-1} + \tilde{\varepsilon}_{it}$$

and compute the $t-$statistic $t_{\rho=0} = \hat{\rho}/\hat{\sigma}(\hat{\rho})$ where

$$\hat{\rho} = \sum_{i=1}^{N}\sum_{t=2+p_i}^{T}\tilde{v}_{it-1}\tilde{e}_{it}/\sum_{i=1}^{N}\sum_{t=2+p_i}^{T}\tilde{v}_{it-1}^2,$$

$$\hat{\sigma}(\hat{\rho}) = \hat{\sigma}_{\tilde{\varepsilon}}/[\sum_{i=1}^{N}\sum_{t=2+p_i}^{T}\tilde{v}_{it-1}^2]^{1/2},$$

$$\hat{\sigma}_{\tilde{\varepsilon}}^2 = \frac{1}{N\tilde{T}}\sum_{i=1}^{N}\sum_{t=2+p_i}^{T}(\tilde{e}_{it} - \hat{\rho}\tilde{v}_{it-1})^2,$$

$$\bar{p} = N^{-1}\sum_{i=1}^{N}p_i,\ \tilde{T} = T - \bar{p} - 1.$$

The LLC statistic is an adjusted version of $t_{\rho=0}$ above, which is given by

$$t_\rho^* = \frac{t_\rho - N\tilde{T}\hat{S}_N\hat{\sigma}_{\tilde{\varepsilon}}^{-2}\hat{\sigma}(\hat{\rho})\mu^*_{m\tilde{T}}}{\sigma^*_{m\tilde{T}}},$$

where the mean adjustments $\mu^*_{m\tilde{T}}$ and standard deviation adjustment $\sigma^*_{m\tilde{T}}$ are provided by LLC via simulations. LLC show that t_ρ^* is asymptotically distributed as $N(0,1)$. The Monte Carlo simulations indicate that the normal distribution provides a good approximation to the empirical distribution of the test statistic, even in relatively small samples. In addition, the panel unit root test provides dramatic improvements in power over separate unit root tests for each unit.

Im, Pesaran and Shin tests

The Levin, Lin and Chu test is restrictive in the sense that it requires ρ to be homogeneous across individual. As pointed out by Maddala and Wu (1999), this restriction is too strong to be upheld in some empirical cases. For instance, in testing the convergence in growth among countries, it does not make any sense to restrict every country to converge at the same rate if they do converge. To solve this problem, Im *et al.* (1997, 2003) (IPS) propose an alternative testing procedure based on averaging individual unit root test statistics which allows for a heterogeneous coefficient of y_{it-1}. IPS consider the model (2.1) and substitute ρ_i for ρ,

$$\Delta y_{it} = \rho_i y_{it-1} + \sum_{L=1}^{p_i} \theta_{iL}\Delta y_{it-L} + \alpha_{mi}d_{mt} + \varepsilon_{it}. \quad (2.2)$$

The null hypothesis is defined that each series in the panel contains a unit root, i.e., $H_0 : \rho_i = 0$ for all i, and the alternative hypothesis $H_1 : \rho_i < 0$ for $i = 1, ..., N_1$ and

$\rho_i = 0$ for $i = N_1 + 1, .., N.(0 < N_1 < N)$, which allows for some (but not all) of the individual series to have unit roots. Thus, instead of pooling the data, IPS use separate unit root tests for the N individuals. Their test is based on the augmented Dickey-Fuller statistics averaged across groups as

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_{\rho_i},$$

where t_{ρ_i} is the individual t-statistic for testing $H_0 : \rho_i = 0$ for all i. In case the lag order is always zero ($p_i = 0$ for all i), IPS provide simulated critical values for $\bar{t}$ for different N and T. For the general case where the lag order p_i is nonzero for some individuals, IPS show that a properly standardized $\bar{t}$ has an asymptotic $N(0,1)$ distribution. Hence

$$t_{IPS} = \frac{\sqrt{N}(\bar{t} - N^{-1}\sum_{i=1}^{N} E[t_{iT}|\rho_i = 0])}{\sqrt{N^{-1}\sum_{i=1}^{N} var[t_{iT}|\rho_i = 0]}} \Rightarrow N(0,1)$$

as $T \to \infty$ followed by $N \to \infty$ sequentially. Here, t_{ρ_i} is the individual $t-$statistic for testing $H_0 : \rho_i = 0$ for all i. IPS have computed the values of $E[t_{iT}|\rho_i = 0]$ and $var[t_{iT}|\rho_i = 0]$ for different T and $p_i's$ via simulations. The Monte Carlo simulations show that if a large enough lag order is selected for the underlying ADF regressions, then the small sample performance of the $\bar{t}$ test is generally better than LLC test.

Fisher-Type tests

As mentioned above, IPS use an average unit root based statistic test to overcome the homogeneity problem in the LLC test. As an alternative testing strategy, Maddala and Wu (1999) and Choi (2001) proposed a Fisher-type test which combines the $p-$values from unit root tests for each individual i to test for unit root in panel data. The heterogeneous model (2.2) is still considered, and the hypothesis remains the same as IPS. The idea of Fisher-type test is that if the pure time series unit root test statistics, e.g. ADF, are continuous, the corresponding p-values p_i are uniform (0,1) variables. Consequently, the statistic proposed by Maddala and Wu (1999) is defined as

$$p_{MW} = -2\sum_{i=1}^{N} \ln(p_i)$$

and has a Chi-square distribution with $2N$ degree of freedom as $T_i \to \infty$ and N is fixed. The attraction of the Fisher test over the IPS test is that it does not require a balanced panel; that is, the time series dimension (T_i) can be different for each i. Also, the Fisher test can use different lag lengths in the individual ADF regressions and can be applied to any other unit root test. For large N samples, Choi (2001) proposes a standardized statistic[1] corresponding to the standardized cross sectional average of individual $p-$values,

$$p_m = \frac{\sqrt{N}\{N^{-1}p_{MW} - E[-2\ln(p_i)]\}}{\sqrt{Var[-2\ln(p_i)]}} = \frac{1}{2\sqrt{N}}\sum_{i=1}^{N}(-2\ln(p_i) - 2).$$

[1]Since $E[-2\ln p_i] = 2$ and $Var[-2\ln p_i] = 4$.

It is shown that p_m converges to a standard normal distribution by applying the Lindberg-Lévy central limit theorem.

Hadri LM test

Contrary to the above tests, the main focus of Hadri (2000) is testing the null hypothesis of stationarity. As mentioned in 2.1, testing for the null of stationarity is more natural than the null of a unit root. It is also useful to perform a stationarity test to complement the unit root test. The fact that the stationarity tests are relatively few in the literature, motivated the work in Chapter 3 of this thesis. The Hadri (2000) test is an extension of the stationarity test developed by Kwiatkowski *et al.* (1992) (KPSS hereafter) in the time series context. A residual-based LM test is proposed for testing under the null that the time series for each cross section unit i, are stationary around a level or around a deterministic time trend, against the alternative that at least one unit has a unit root. Hadri (2000) considers the following two models:

$$y_{it} = r_{it} + \varepsilon_{it}, \tag{2.3}$$

and

$$y_{it} = r_{it} + \beta_i t + \varepsilon_{it}, \tag{2.4}$$

where r_{it} is a random walk,

$$r_{it} = r_{it-1} + u_{it}$$

ε_{it} and u_{it} are mutually independent normal distributions. Also, ε_{it} and u_{it} are *i.i.d* across i and over t, with $E[\varepsilon_{it}] = 0, E[\varepsilon_{it}^2] = \sigma_\varepsilon^2 > 0$, $E[u_{it}] = 0, E[u_{it}^2] = \sigma_u^2 \geq 0$,

$t = 1, ..., T$ and $i = 1, ..., N$. The null hypothesis of stationarity is simply $\sigma_u^2 = 0$. Since the $\varepsilon_{it}'s$ are assumed $i.i.d$, then under the null hypothesis y_{it} is stationary around a level in model (2.3) and trend stationary in model (2.4). More specifically, Hadri (2000) tests the null $\lambda = 0$ against the alternative $\lambda > 0$ where $\lambda = \frac{\sigma_u^2}{\sigma_\varepsilon^2}$. Thus $\lambda = 0$ ($\sigma_u^2 = 0$) means that y is stationary whereas $\lambda = \infty$ entails that y comprises a random walk.

Let $\hat{\varepsilon}_{it,u}$ and $\hat{\varepsilon}_{it,\tau}$ be the estimated OLS residuals from the regression of y_{it} on an intercept (for model (2.3)) and on an intercept and a linear trend (for model (2.4)), then the consistent estimators of the error variance for the appropriate regression, $\hat{\sigma}_{\varepsilon,u}^2$ and $\hat{\sigma}_{\varepsilon,\tau}^2$, after the correction for degrees of freedom, are given by

$$\hat{\sigma}_{\varepsilon,u}^2 = \frac{1}{N(T-1)} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it,u}^2, \text{ and } \hat{\sigma}_{\varepsilon,u}^2 = \frac{1}{N(T-2)} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it,\tau}^2.$$

The partial sum process of the residuals, $S_{it,k}$,can be written as

$$S_{it,k} = \sum_{j=1}^{t} \hat{\varepsilon}_{it,k}^2, \; k = u, \tau.$$

The Lagrange Multiplier (LM) statistic is

$$\hat{LM}_k = \frac{1}{\hat{\sigma}_{\varepsilon,k}^2} \frac{1}{NT^2} (\sum_{i=1}^{N} \sum_{t=1}^{T} S_{it,k}^2), \; k = u, \tau \tag{2.5}$$

Under the null of stationarity around a level or time trend, and using a sequential asymptotic limit theory in which $T \to \infty$ followed by $N \to \infty$, the statistic

$$Z_u = \frac{\sqrt{N} \left\{ \hat{LM}_u - E[\int_0^1 V(r)^2 dr] \right\}}{\sqrt{Var[\int_0^1 V(r)^2 dr]}}, \tag{2.6}$$

$$Z_\tau = \frac{\sqrt{N}\left\{\hat{LM_\tau} - E[\int_0^1 V_2(r)^2 dr]\right\}}{\sqrt{Var[\int_0^1 V_2(r)^2 dr]}}, \tag{2.7}$$

follow standard normal distribution. Here $V(r)$ is standard Brownian bridge and $V_2(r)$ is the second level Brownian bridge. The cumulants of the characteristic function of $\int_0^1 V(r)^2$ and $\int_0^1 V_2(r)^2$ give, respectively, the mean and variance of $\int_0^1 V(r)^2$ and $\int_0^1 V_2(r)^2$ in (2.6) and (2.7). Hadri (2000) derived the mean and variance which are $\frac{1}{6}$ and $\frac{1}{45}$ for model (2.3) and $\frac{1}{15}$ and $\frac{11}{6300}$ for model (2.4) respectively. In a Monte Carlo study, Hadri (2000) demonstrate good finite sample performances of both statistics Z_u and Z_τ.

2.2.2 The Second Generation Tests

It is important to note that one crucial assumption in the tests mentioned above is cross-sectional independence. However, in many macroeconomic applications using country or regional data, cross-sectional dependence is often found. Cross section dependence can arise due to a variety of factors, such as omitted observed common factors, spatial spillover effects, unobserved common factors, or general residual inter-dependence that could remain even when all the observed and unobserved common effects are taken into account. It is found that tests of the first generation suffer from size distortion and power loss when ignoring the existence of cross-sectional dependence, thereby causing deceptive inferences. In particular, O'Connell (1998) showed that the pooled tests will over reject the null hypothesis when the independence is violated, whether the null hypothesis is a unit root or stationarity. This is due to the fact that the total amount of independent information contained in the panel is reduced.

In response to the need for panel unit root tests allowing for cross-sectional dependence, various tests have been proposed including the works of Bai and Ng (2001, 2004), Phillips and Sul (2003a), Moon and Perron (2004a), Choi (2002), Moon, Perron and Phillips (2003), Chang (2002, 2004), Pesaran (2003) and more recently, Harris *et al.* (2005). These are the second generation of nonstationary panel tests and this category of tests is still under development given the diversity of the potential cross-sectional correlations. The main approaches dealing with the cross-sectional dependence in these tests include the factor structure approach, which is suggested in Bai and Ng (2001, 2004), Phillips and Sul (2003a), Moon and Perron (2004a), Choi (2002) and Pesaran (2003) and a second approach, which consists in imposing few or no restrictions on the residuals covariance matrix, was adopted by O'Connell (1998), Maddala and Wu (1999), Chang (2002, 2004) and Harris *et al.* (2005). In this subsection, we concentrate on tests using the second approach which constitute the main motivations of Chapter 4 and 5 of this thesis.

Tests Based on GLS Regressions

Consider a panel series $\{y_{it}\}$ that is generated by a simple AR(1) process,

$$y_{it} = \alpha_i + \beta y_{it-1} + u_{it},$$

$$or\ \Delta y_{i,t} = \alpha_i + \rho y_{i,t-1} + u_{it}, \tag{2.8}$$

where $\Delta y_{i,t} = y_{it} - y_{it-1}$ and $\rho = \beta - 1$. The residual vector $u_t = [u_{1t}, ..., u_{Nt}]'$ is $i.i.d$ with $E(u_t) = 0$, and the cross-sectional dependence is represented by a non-diagonal matrix $\Omega = E(u_t u_t')$, for all t. Since the model (2.8) can be written as a seemingly unrelated regression (SUR) system of,

$$\Delta y_t = \alpha + \rho y_{t-1} + u_t,$$

where Δy_t, y_{t-1}, u_t and α are $N \times 1$ vectors, O'Connell (1998) suggested to estimate the system by using a generalized least squares (GLS) estimator. Let $\hat{\Omega} = T^{-1} \sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$ denote the sample covariance matrix of the residual vector. The test of null hypothesis of unit root can be based on the GLS estimator of ρ given by

$$\hat{\rho} = \frac{\sum_{t=1}^{T} y_{t-1}' \hat{\Omega}^{-1} \Delta y_t}{\sum_{t=1}^{T} y_{t-1}' \hat{\Omega}^{-1} y_{t-1}}.$$

O'Connell (1998) assumes cross-sectional dependence with the form

$$\Omega = \begin{bmatrix} 1 & \omega & \cdots & \omega \\ \omega & 1 & \cdots & \omega \\ \vdots & \vdots & \vdots & \vdots \\ \omega & \omega & \cdots & 1 \end{bmatrix} \quad \text{with } \omega < 1,$$

that is, all the off-diagonal elements of the error covariance matrix are the same. Such a specification seems overly restrictive in many applications such as using regional data, where it is usually assumed that co-movements of economic variables between one region and another are usually observed because of various spillover effects. Another restriction in this study is that the GLS approach cannot be used if $T < N$, in which case the estimated covariance matrix $\hat{\Omega}$ is singular. Furthermore, Monte Carlo simulations suggest that for reasonable size properties of the GLS test, T must be substantially larger than N (e.g. Breitung and Das, 2005a).

Tests Using Bootstrap Methods

As pointed out by Maddala and Wu (1999), when the cross-sectional independence asssumption is violated, the derived distributions for the first generation panel unit root tests are no longer valid. One way out of this problem is to use the bootstrap method to obtain the empirical distributions of the test statistics to make inferences. Maddala and Wu (1999) and Chang (2004) have suggested a bootstrap procedure that attempts to allow for a more general specification of the contemporaneous covariance matrix of the errors. By using the bootstrap method, Maddala and Wu (1999) obtained the empirical distributions of LL, IPS and Fisher-type tests. Chang (2004) considers a general framework in which each panel is driven by a heterogeneous linear process, approximated by a finite order autoregressive process. In order to take into account the dependence among the innovations, Chang suggests a unit root test based on the estimation on the entire system of N equations. The critical values are computed by a bootstrap method and the validity of using bootstrap method is assessed.

HLM (2005) Test

One recent paper worthy of particular mention is HLM (2005), which motivated the work in Chapter 5. As none of the approaches so far discussed attempt to deal with the problem of testing for stationarity when the structure of cross-sectional dependence and time series dynamics are both unknown, HLM (2005) constructed a panel stationarity test to overcome both problems. The statistic is flexible enough to allow for arbitrary unknown cross-sectional dependence according to which the series may be contemporaneously or cross-serially dependent, and also permits a wide range of heterogeneous stationary time series dynamics which including the ARMA class. The statistic is, in essence, the sum of the lag-k sample autocovariances across the panel, suitably studentized, where k is allowed to be a simple increasing function of the time dimension. By controlling k in such a way, they remove the need to explicitly model the time series dynamics of each series in the panel and, at the same time, the studentization automatically robustifies the statistic to the presence of any form of cross-sectional dependence. A more detailed review is provided in Chapter 5.

2.2.3 Tests with Structural Breaks

Since the appearance of Perron (1989), the literature of unit root and stationarity tests has shown a special attention to the possible presence of structural breaks. This development stimulated the tremendous parallel developments focusing on dealing with structural changes. In the time series context, Perron (1989), Zivot and Andrews

(1992), Banerjee *et al.* (1992), Perron and Vogelsang (1992), Schmidt and Phillips (1992), Amsler and Lee (1995) and Perron (1997) have contributed by providing unit root tests in the presence of structural breaks, whereas Lee *et al.* (1997), Kurozumi (2002), Busetti and Harvey (2001, 2003) focused on stationarity tests with structural breaks. It is suggested that unit root tests not considering the structural breaks are biased towards accepting the false unit root null hypothesis and stationarity tests ignoring the existing break are biased toward rejecting the null of stationarity in favor of the false alternative unit root hypothesis. However, to our knowledge so far, there is relatively few concern about the structural changes in panel data field, except Im *et al.* (2005) in panel unit root test and Carrion-i-Silvestre *et al.* (2005) (CBL hereafter) in panel stationarity context. In the following paragraphs, we provide a brief review of CBL (2005) tests as an example for panel stationarity tests with structural breaks.

CBL (2005) Test

Within the panel data framework, CBL proposed a test statistic for the null of panel stationarity with multiple structural breaks. Two models, each based on different break effects, were specified; these being level breaks in non-trend function (model 1) and both level and slope breaks in trend function (model 2). The general model considered is given by:

$$y_{i,t} = \alpha_{i,t} + \beta_i t + \varepsilon_{i,t},$$

$$\alpha_{i,t} = \sum_{k=1}^{m_i} \theta_{i,k} DU_{i,k,t} + \sum_{k=1}^{m_i} \gamma_{i,k} D(T_{b,k}^i)_t + \alpha_{i,t-1} + \nu_{i,t},$$

where $\nu_{i,t} \sim i.i.d(0, \sigma^2_{v,i})$, $\varepsilon_{i,t}$ is allowed to be serially correlated. In particular, an AR(1) process is specified in the data-generating process of simulations. $\{\nu_{i,t}\}$ and $\{\varepsilon_{i,t}\}$ are assumed to be mutually independent across i and over t. $D(T^i_{b,k})_t$ and $DU_{i,k,t}$ are defined as $D(T^i_{b,k})_t = 1$ for $t = T^i_{b,k} + 1$ and 0 elsewhere, and $DU_{i,k,t} = 1$ for $t > T^i_{b,k}$ and 0 elsewhere with $T^i_{b,k}$ denoting the kth date of break for the ith individual, $k = 1, ..., m_i$. The null hypothesis is specified as $\sigma^2_{v,i} = 0$ for all i, under which the CBL analysis can be summarized by the following equation,

$$y_{i,t} = \alpha_i + \sum_{k=1}^{m_i} \theta_{i,k} DU_{i,k,t} + \beta_i t + \sum_{k=1}^{m_i} \gamma_{i,k} D(T^i_{b,k})_t + \nu_{i,t}, \tag{2.9}$$

Hence, model 1 is obtained when $\beta_i = \gamma_{i,k} = 0$, and model 2 is defined if $\beta_i \neq 0$ and $\gamma_{i,k} \neq 0$, α_i is the initial value of $\alpha_{i,t}$.

The proposed statistic, which is based on Hadri (2000) LM test, is expressed as:

$$LM(\lambda) = N^{-1} \sum_{i=1}^{N} (\hat{\omega}_i^{-2} T^{-2} \sum_{t=1}^{T} \hat{S}^2_{i,t}), \tag{2.10}$$

where $\hat{S}^2_{i,t} = \sum_{j=1}^{t} \hat{\varepsilon}_{i,t}$ denotes the partial sum of OLS estimated residuals $\hat{\varepsilon}_{i,t}$. For each i, $\lambda_i = (\lambda_{i,1}, ..., \lambda_{i,m_i})' = (T^i_{b,1}/T, .., T^i_{b,m_i}/T)'$ indicates the locations of the breaks over T. Since autocorrelation is allowed in the residuals, $\hat{\omega}^2_i$ is a consistent long-run variance (LRV) estimate of $\hat{\varepsilon}_{i,t}$ for each i. To obtain a consistent $\hat{\omega}^2_i$, CBL (2005) uses a parametric method jointly with the boundary condition rule suggested by Sul *et al.* (2003) which is shown to be effective in avoiding inconsistency problems of the KPSS-type test. Using appropriate moments and applying for the Central Limit Theorem (CLT), the limiting distribution of the statistic (2.10) is shown to be standard normal, that is,

$$Z(\lambda) = \frac{\sqrt{N}(LM(\lambda) - \bar{\xi})}{\bar{\varsigma}} \Longrightarrow N(0,1),$$

with

$$\bar{\xi} = N^{-1}\sum_{i=1}^{N} \xi_i, \; \overline{\varsigma^2} = N^{-1}\sum_{i=1}^{N} \varsigma_i^2.$$

The asymptotic mean and variances for each individual have been provided in CBL (2005) as follows:

$$\xi_i = A\sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^2; \; \varsigma_i^2 = B\sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^4.$$

The values of A and B equal the values of moments in Hadri (2000), that is, for model 1, $A = \frac{1}{6}$, $B = \frac{1}{45}$; for model 2, $A = \frac{1}{15}$, $B = \frac{11}{6300}$.

In the situation where break dates are unknown, the SSR procedure is employed to estimate the break point, that is, the estimated break dates are obtained by minimizing the sum of squared residuals. To estimate multiple break dates, CBL (2005) proposed the method of Bai and Perron (1998) that computes the global minimization of the SSR, so all the break dates are estimated via minimizing the sequence of individual $SSR(T_{b,1}^i, ..., T_{b,m_i}^i)$ computed from (2.9)

$$(\hat{T}_{b,1}^i, ..., \hat{T}_{b,m_i}^i) = \arg \min_{T_{b,1}^i, ..., T_{b,m_i}^i} SSR(T_{b,1}^i, ..., T_{b,m_i}^i).$$

After all the possible break dates have been estimated, the suitable number of breaks is decided by sequential computation with pseudo F-type test for model 1 and information criteria for model 2. It is shown that the standard normal distribution property of $LM(\lambda)$ still holds when the estimated break locations are used to calculate the statistic $LM(\hat{\lambda})$.

2.3 Panel Cointegration Tests

Like the panel unit root tests, panel cointegration tests can be motivated by the search for more powerful tests than those obtained by applying individual time series cointegration tests. The literature on testing for cointegration in panels has so far taken two broad directions. The first consists of taking as the null hypothesis that of no cointegration and using residuals derived from the panel analogue of an Engle and Granger (1987) static regression to construct the test statistics and tabulate the distributions. The most general statement of this problem may be taken from Pedroni (1999, 2004). A related paper by Kao (1999) contains very similar and related analysis. The second route is to take as the null that of cointegration. This is the basis of the test proposed by McCoskey and Kao (1998). This too is a residual-based test and has as its analogue in the time series literature the tests of Harris and Inder (1994), Shin (1994), Leybourne and McCabe (1994) and Kwiatowski *et al.* (1992). More recently, Westerlund (2005b) extended the CUSUM test proposed in time series context by Xiao and Phillips (2002), and Xiao (1999) to test of the null of panel cointegration that allows for mixtures of cointegrated and spurious alternatives. We discuss both approaches in turn. As a preface, it should be noted that the asymptotic analysis of both approaches involves the use of sequential limit arguments. This involves allowing the time series dimension T to grow large first and then letting $N \to \infty$. Other tests include Larsson, Lyhagen and Lőthgren (2001), which presented a likelihood-based (LR) panel test of cointegrating rank in heterogeneous panel methods based on an average of the individual rank trace statistics developed by Johansen (1995).

2.3.1 Tests without Cross-Sectional Dependence

As in the panel unit root tests, the early versions of panel cointegration test are also based on the assumptions of cross-sectional independence in the panel. These tests are constructed taking either cointegration or no cointegration as the null hypothesis. We review these tests using Kao (1999) as our starting point.

Kao (1999) Test

Kao (1999) proposed a DF and an ADF-type of panel cointegration test based on the null hypothesis of no cointegration. For the panel regression model

$$y_{it} = \alpha_i + x'_{it}\beta + e_{it}$$

where y_{it} and x_{it} are $I(1)$ and noncointegrated, the DF-type test can be calculated from residuals

$$\hat{e}_{it} = \rho\hat{e}_{it-1} + v_{it},$$

with $\hat{e}_{it} = \tilde{y}_{it} - \tilde{x}'_{it}\hat{\beta}$ and $\tilde{y}_{it} = y_{it} - \bar{y}_i$. The null hypothesis can be written as $H_0 : \rho = 1$. The OLS estimate of ρ and the t-statistic are given as

$$\hat{\rho} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{e}_{it}\hat{e}_{it-1}}{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{e}_{it}^2},$$

$$t_\rho = \frac{(\hat{\rho}-1)\sqrt{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{e}_{it-1}^2}}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=2}^{T}(\hat{e}_{it}-\rho\hat{e}_{it-1})^2}.$$

Kao proposed four DF-type tests. DF_t^*, one of the tests for the cointegration with endogeneity between regressors and errors, is as follows

$$DF_t^* = \frac{t_\rho + \frac{\sqrt{6N}\hat{\sigma}_v}{2\hat{\sigma}_{0v}}}{\sqrt{\frac{\hat{\sigma}_{0v}^2}{2\hat{\sigma}_v^2} + \frac{3\hat{\sigma}_v^2}{10\hat{\sigma}_{0v}^2}}} \tag{2.11}$$

where $\hat{\sigma}_{0v}$ and $\hat{\sigma}_v$ are defined in Kao (1999). The ADF test can be estimated by running

$$\hat{e}_{it} = \rho\hat{e}_{it-1} + \sum_{j=1}^{p}\psi_j\Delta\hat{e}_{it-j} + v_{it}, \tag{2.12}$$

and the ADF test statistic can be constructed as (2.11) with t_ρ replaced by t_{ADF} which is the $t-$statistic of ρ in (2.12). It is shown that all the statistics have a standard normal distribution by the sequential limit theory.

McCoskey and Kao (1998) Test

Instead of testing for the null of no cointegration in panels, McCoskey and Kao (1998) is the first study which derived a residual-based test for the null of cointegration. It was pointed out that testing the null of cointegration rather than the null of no cointegration can be very appealing in applications where cointegration is predicted *a priori* by economic theory. McCoskey and Kao (1998)'s test is an extension of the Lagrange Multiplier (LM) test and a locally best invariant (LBI) test for an MA unit root in the time series literature. The model considered allows for varying slopes and intercepts,

$$y_{it} = \alpha_i + x'_{it}\beta_i + e_{it},$$

$$x_{it} = x_{it-1} + \varepsilon_{it},$$

$$e_{it} = \gamma_{it} + u_{it},$$

$$\gamma_{it} = \gamma_{it-1} + \theta u_{it}.$$

where $\{u_{it}\}$ are $i.i.d.N(0, \sigma_u^2)$, and the correlation is allowed for in the error processes $(u_{it}, \varepsilon'_{it})$. The long-run variance-covariance matrix of $\omega_t = (u_{it}, \varepsilon'_{it})'$ is defined as

$$\Omega = [\begin{matrix} \varpi_1^2 & \varpi_{12} \\ \varpi_{21} & \Omega_{22} \end{matrix}].$$

The null hypothesis of cointegration is equivalent to $\theta = 0$. The test statistic proposed by McCoskey and Kao (1998) is defined as follows:

$$LM = \frac{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^2}\sum_{t=1}^{T}S_{it}^2}{\hat{\varpi}_{1.2}^2},$$

with $\hat{\varpi}_{1.2}^2$ is a consistent estimator of

$$\hat{\varpi}_{1.2}^2 = \varpi_1^2 - \varpi_{12}\Omega_{22}^{-1}\varpi_{21}, \quad y_{i,k}^{\dagger} = y_{i,k} - \hat{\varpi}_{12}\Omega_{22}^{-1}\omega_{i,k}, \tag{2.13}$$

and

$$S_{i,t}^{\dagger} = \sum_{k=1}^{t}(y_{i,k}^{\dagger} - \alpha_i - \hat{\beta}_i^{\dagger\prime}x_{ik})$$

$\hat{\beta}_i^{\dagger\prime}$ is the fully-modified estimator (FM) of β_i. The asymptotic result for the test is

$$\sqrt{N}(LM - \mu_v) \Rightarrow N(0, \hat{\sigma}_v^2),$$

the values of the mean $\mu_v \equiv E(\int V^2)$ and the variance $\hat{\sigma}_v^2 \equiv Var(\int V^2)$ are computed through simulations. The limiting distribution of LM is then free of nuisance parameters and robust to heteroskedasticity.

Westerlund (2005b) Test

The model in this test is given by

$$y_{it} = X_{it}'\delta_i + u_{it},$$

where $X_{it} = (z_t', x_{it}')'$, $\delta_i = (\gamma_i, \beta_i)'$. The vector $x_{it} = x_{it-1} + v_{it}$ and z_t is a vector of deterministic components such that $z_t = \varnothing$, $z_t = 1$ and $z_t = (1,t)'$ are distinguished in three models. The error vector $\omega_t = (u_{it}, v_{it}')'$ also allows for serial correlation. Hence, to obtain efficient estimation, both FMOLS and DOLS estimator are employed. The hypothesis maintained in Westerlund (2005b) is the null that all the individuals of the panel are cointegrated which is specified as $H_0 : \psi = 0$ [2]and the alternative is that a non-empty subset is not cointegrated, that is, $H_1 : \psi > 0$. The statistic proposed is

$$CS_{NT} = \frac{1}{N}\sum_{i=1}^{N}(\max_{t=1,..,T}\frac{1}{\hat{\varpi}_{1.2}^2 T^{1/2}}|S_{it}^*|),$$

[2] ψ is defined as the ratio of N_1 individuals which possessing a unit root to the whole N individuals in the panel, formally, $N_1/N \rightarrow \psi$ as $N \rightarrow \infty$.

where $S^*_{it} = \sum_{j=1}^{t} \hat{u}^*_{ij}$ is the partial sum of the FOLS (or DOLS) residuals $\hat{u}^*_{it}$, and $\hat{\varpi}^2_{1.2}$ is defined in (2.13). After applying the Lindberg-Lévy central limit theorem, it is shown that the standardized CS_{NT} has a limiting normal distribution under the null that is free of nuisance parameters and it is robust to heteroskedasticity, that is,

$$Z_{NT} = \frac{N^{1/2}(CS_{NT} - \mu)}{\sigma} \Rightarrow N(0,1).$$

under the sequential limit theorem. The values of μ and σ^2 are obtained by means of simulations and provided in Table 1 in Westerlund (2005b). Monte Carlo simulation results suggest that the test has small-size distortions and reasonable size-adjusted power (See Westerlund (2005b)).

2.3.2 Tests with Cross-Sectional Dependence

Like in panel unit root and stationarity tests, the cross-sectional dependence is also an important feature in panel cointegration analysis. Although this problem has not been paid as much attention as in unit root tests discussed so far, some solutions have recently been obtained in the literature. We summarize in the following, the main panel cointegration tests dealing with cross-sectional dependece.

Tests using the Common Factor Approach

Bai and Kao (2005) study panel cointegration under cross-sectional dependence, which is characterized by a structural factor. It is indicated that a common factor approach to panel models with cross-sectional dependence is useful when both the time series and cross-sectional dimensions are large. The limiting distributions for the OLS and FM estimators have been derived and a continuous updated fully modified (CUP-FM) estimator is proposed. Banerjee and Carrion-i-Silvestre (2006) carried out a common factor structure approach to allow for dependence among the units in the panel. They adopted the factor model approach of Bai and Ng (2004) to generalize the degree of permissible cross-section dependency in order to allow for idiosyncratic responses to multiple common factors. An ADF-type test statistic is proposed. In Banerjee and Carrion-i-Silvestre's framework the usual single-equation definition of cointegration (stationary residuals in the cointegrating equation) is accepted if the null of non-stationarity is rejected both for the estimated common factor and the idiosyncratic residuals.

It is noted that one drawback that applies to all these approaches is that the limiting test distributions depend critically on the nuisance parameters associated with both the number of regressors and the deterministic specification of the cointegrated regression. Thus, there is not just one set of critical values, but one for each combination of regressors and deterministic specification. Moreover, since the asymptotic distribution is often a poor approximation in small samples, a new set of critical values is usually needed for each sample size.

In response to this problem, Westerlund and Edgerton (2006b) propose two statistics (t-test and coefficient test) to test the null of no cointegration and again use common factor approach to model the cross-sectional dependence. Their model is general to accommodate heteroskedastic and serially correlated errors, individual specific intercepts and time trends, cross-sectional dependence and an unknown break in both the intercept and slope of the cointegrated regression, which may be located different dates for different units. By using sequential limit arguments, they show that the tests have limiting normal distributions that are free of nuisance parameters under the null hypothesis. In particular, it is shown that the asymptotic null distributions are independent of both the structural break and the common factors. Moreover, since the null distributions are also independent of the regressors, there is only one set of critical values for all testing situations considered. The small-sample performance of the tests results suggest that the tests generally perform well with small size distortions and good power even in small samples. The t-test seems to have better size and power properties than the coefficient test.

Tests using Other Approaches

We noticed that based on the factor structure approach, all the above studies have to make some restrictive assumptions on the form of cross-sectional dependence. To allow for more general structure of this dependence, Westerlund (2006b) proposes a bootstrap procedure in a panel cointegration test which takes breaks into account in the deterministic components of the cointegration regression. Groen and Kleibergen (2003) made an attempt to relax the cross-sectional independence assumption and developed tests based on seemingly unrelated regressions. To allow for an instantaneous feedback between the different individuals in the panel, they used a maximum likelihood framework in which an unrestricted disturbance covariance matrix is allowed for within the panel. These related issues are addressed in Chapter 5 where we propose a residual-based panel cointegration test in which arbitrary forms of cross-sectional dependence can be controlled for.

Chapter 3

Panel Stationarity Tests with Structural Breaks

This chapter constitutes the first contribution of this thesis to the econometrics literature. As failure to consider potential structural breaks in panel unit root or stationarity test can cause misleading inferences, we develop two panel stationarity tests that allows for structural breaks; that is, an LM residual-based test and a modified test. The LM test can be used for all the models proposed whereas the modified test is only applicable to two models. The break date is allowed to be different across units in the panel. We derive the limiting distribution of both tests and provide the appropriate asymptotic moments that are used to standardize the statistics. We also discuss how to use the tests when the break location is unknown. Finally, we examine the finite sample properties of both tests via Monte Carlo simulations. The main results of this chapter of the thesis appear in Hadri and Rao (2006a).

3.1 Introduction

An upsurge of interest in nonstationary panel data models has been witnessed in the recent econometric literature. Since the seminal papers by Breitung and Meyer (1994), Quah (1994), Maddala and Wu (1999), Phillips and Moon (1999), Levin, Lin and Chu (2002), Im, Pesaran and Shin (2003), Hadri (2000) and Hadri and Larsson (2005), panel unit root and stationarity tests have been applied to a variety of key economic issues with the hope that the increased power of these tests, due to the exploitation of the cross-section dimension, would provide more compelling evidence. Banerjee (1999), Baltagi and Kao (2000), Baltagi (2001) and Breitung and Pesaran (2005) provide comprehensive surveys of the subject. Additionally, since the pioneering work of Perron (1989), which illustrates the need to allow for a structural break when testing for a unit root in economic time series, the problem of structural breaks in the level/slope of a series has proved to be of considerable interest in the unit root testing literature. Perron (1989) and Amsler and Lee (1995) have found that unit root tests are biased toward accepting the false unit root null hypothesis in the presence of a structural break. It is widely accepted that the failure to take structural breaks into account is likely to lead to a significant loss of power in unit root tests. Similarly, stationarity tests ignoring the existence of breaks diverge and thus are biased toward rejecting the null hypothesis of stationarity in favour of the false alternative of a unit root. This is due to severe size distortion caused by the presence of breaks (see, *inter alia,* Lee *et al.* (1997)). Kurozumi (2002), Lee and Strazicich (2001) and Busetti and Harvey (2001, 2003) have considered testing the null hypothesis of stationarity in the presence of a single break versus the alternative of a unit root in time series.

To our knowledge, hardly any attention has been paid to the presence of structural changes in panel data unit root and stationarity tests. The only exceptions are Im, Lee and Tieslau (2005) for unit root tests and Carrion-i-Silvestre, Del Barrio and López-Bazo (2005) (CBL thereafter) in the stationarity tests context. Im *et al.* (2005) have shown that the power loss problem still exists for panel unit root tests when ignoring the existence of a break. Our work in this chapter considers panel stationarity tests in the presence of a break. The differences between our panel tests and CBL are:

(a) we propose tests for models with a break in the level and no time trend (Model 0), with break in the level and a time trend without a break (Model 1), with a level without a break and a time trend with a break (Model 2), with a level and a time trend, both with a break (Model 3). CBL deal only with Model 0 and 3.

(b) CBL propose tests for the case where the break fraction vector is known and unknown; in this chapter we propose tests for all the four models for a known break point, an unknown break point and a modified statistic that does not depend on the location of the break point for all the four models.

(c) CBL use Hadri (2000) moments to evaluate the moments for their statistics corresponding to our Model 0 and 3; in this chapter we derive from first principles the moments for all four models via the characteristic functions. CBL were able to allow for multiple structural breaks by exploiting the structure of the asymptotic distributions of Model 0 and 3. However, the structure of the asymptotic distributions for Model 1 and 2 proposed here does not permit more than one structural break.

Hadri (2000) proposed a residual-based panel stationarity test based on the KPSS (Kwiatkowiski *et al.* (1992)) stationarity test. After standardizing this statistic by appropriate moments, a panel statistic with standard normal limiting distribution is obtained. Since the standard normal distribution is much easier to work with than nonconventional distributions encountered in the literature on testing for unit root or stationarity, this framework becomes very attractive in this respect. Here, we extend Hadri (2000) to the case where a structural break is considered.

In this chapter of the thesis, we focus mainly on testing the null hypothesis of stationarity, allowing for one structural break. We also allow for different breaking dates in each time series. This is attractive for practitioners when the individuals in the cross-section are affected by similar shocks while responding differently to them. The asymptotic distributions of the tests are derived under the null and are shown to be normally distributed. The asymptotic distributions are derived using sequential limits, wherein $T \to \infty$ followed by $N \to \infty$. Under the rate condition $N/T \to 0$, it can be shown following Phillips and Moon (1999) that the sequential results obtained imply joint convergence. In the joint limit theory, T and N are allowed to pass to infinity simultaneously. The drawback of sequential limits is that in certain cases, they can give asymptotic results which are misleading. This is not the case for joint limit. As noted by Phillips and Moon (1999), the rate condition indicates that the joint limit theory is going to be applicable to cases when N is moderate and T is allowed to be large.

The plan of this chapter is as follows. In Section 3.2, we outline panel data models that accommodate a structural break. Section 3.3 describes the LM statistics and their limiting distributions when a structural break is present. A modified test that does not depend on the location of the break point under the null hypothesis is also proposed in this Section. These tests are investigated when the break date is known. The cases where the break date is unknown are dealt with in Section 3.4. Section 3.5 presents simulation results. which suggest that most of our asymptotic results are good approximations to the finite sample distributions. Section 3.6 concludes the chapter. Formal proofs are presented in the Appendix.

We use standard notation $\xrightarrow{p}$ to indicate convergence in probability and $\Rightarrow$ for convergence in distribution.

3.2 Models and Assumptions

We consider the following four models:

$$\text{Model 0:} \quad y_{it} = \alpha_i + r_{it} + \delta_i D_{it} + \epsilon_{it}, \tag{3.1}$$

$$\text{Model 1:} \quad y_{it} = \alpha_i + r_{it} + \delta_i D_{it} + \beta_i t + \epsilon_{it}, \tag{3.2}$$

$$\text{Model 2:} \quad y_{it} = \alpha_i + r_{it} + \beta_i t + \gamma_i DT_{it} + \epsilon_{it}, \tag{3.3}$$

$$\text{Model 3:} \quad y_{it} = \alpha_i + r_{it} + \delta_i D_{it} + \beta_i t + \gamma_i DT_{it} + \epsilon_{it}, \tag{3.4}$$

with

$$r_{it} = r_{it-1} + u_{it}, \tag{3.5}$$

where y_{it}, $i = 1, ..., N$ and $t = 1, ..., T$ are the observed series for which we wish to test stationarity. For all i, $\alpha_i's$, $\beta_i's$, $\delta_i's$ and $\gamma_i's$ are unknown parameters, r_{it} is a random walk with initial values $r_{i0} = 0$ $\forall i$ without loss of generality, as constant terms are already included as defined above. (See Abadir (1993) and Abadir and Hadri (2000) for the importance of initial values in autoregressive models.)

Assumption 3.1. *The u_{it} are iid variates with $E(u_{it}) = 0$, $Var(u_{it}) = \sigma^2_{u,i} \geq 0$. $\{\epsilon_{it}\}$ and $\{u_{it}\}$ are mutually independent across the two dimensions of the panel data.*

Assumption 3.2. *The disturbance term $\{\epsilon_{it}\}$, for $i = 1, ..., N$ and $t = 1, .., T$, satisfies the following assumptions:*

(i) $E(\epsilon_{it}) = 0$ for all i and t;

(ii) $\sup_t E|\epsilon_{it}|^{\psi+\varepsilon} < \infty$ for some $\psi > 2$ and $\varepsilon > 0$;

(iii) $\{\epsilon_{it}\}$ is strong mixing with mixing coefficients C_h that satisfy

$\sum_{h=1}^{\infty} C_h^{1-2/\psi} < \infty$.

Assumption 3.2 allows for quite general forms of temporal dependence and heterogeneity over t.

Assumption 3.3. *Suppose that there is a one-time change in the structure that occurred at time $T_{B,i}$, where $T_{B,i} = \omega_i T$, $\omega_i \in (0,1)$ denotes the fraction of the break point to the sample for the ith individual. The dummy variables D_{it} and DT_{it} are defined as $D_{it} = 1$ if $t > T_{B,i}$, and 0 otherwise; $DT_{it} = t - T_{B,i}$ if $t > T_{B,i}$, and 0 otherwise.*

The above four models specify different effects that the break may cause on the deterministic parts of the models. Model 0 has a break in the level and no time trend, Model 3 allows for a break in the level and slope. Model 1 has a shift in the intercept and no break in the time trend. Model 2 considers no break in the level but a break in the slope.

The null hypothesis is given by

$$H_0\colon \sigma^2_{u,1} = \sigma^2_{u,2} = \cdots = \sigma^2_{u,N} = 0, \tag{3.6}$$

against the alternative

$$H_1\colon \sigma^2_{u,i} > 0, i = 1, 2, ...N_1;\ \ \sigma^2_{u,i} = 0,\ \ i = N_1{+}1, ..., N. \tag{3.7}$$

This alternative hypothesis allows for $\sigma^2_{u,i}$ to be heterogeneous across units and includes the homogeneous alternative, i.e., $\sigma^2_{u,i} = \sigma^2_u > 0$ for all i. It also permits some of the individual series to be stationary under the alternative. The consistency of the present panel stationarity tests is guaranteed as shown by Hadri and Larsson (2005) if the fraction of the individual processes possessing a unit root is different from zero under the alternative.

3.3 Test Statistics and Limiting Distributions

3.3.1 LM test

In this Section, we extend Hadri (2000) to allow for a structural break under the null against the alternative of a unit root. The statistic for individual time series is

$$\eta_{i,T,k}(\omega_i) = \frac{\sum_{t=1}^{T} S_{it}^2}{T^2 \widehat{\sigma}^2_{\epsilon,i}}, \tag{3.8}$$

with k = 0, 1, 2, 3 indicating the statistics for the four models considered in this chapter. $S_{it} = \sum_{j=1}^{t} \widehat{\epsilon}_{ij}$ is the partial sum process, and $\widehat{\sigma}^2_{\epsilon,i}$ is an estimator of the long-run variance (LRV) of ϵ_{it} where

$$\sigma^2_{\epsilon i} = \lim_{T\to\infty} T^{-1} E(S^2_{i,T}). \tag{3.9}$$

The procedure of computing the LRV will be discussed later. It was pointed out by Hadri (2000) that when there is heterogeneity across individuals in panel, the statistic of interest is the average of individual univariate KPSS stationarity tests. Hence, the panel statistic is given by

$$\widehat{LM}_{T,N,k}(\omega) = \frac{1}{N} \sum_{i=1}^{N} \eta_{i,T,k}(\omega_i). \tag{3.10}$$

The ω_i denotes that the statistic has been constructed for a specific value of the break point location and this value is allowed to be different across individuals. The test is clearly not invariant to the presence of a break even asymptotically. Under the null, $\widehat{\epsilon}_{ij}$ are OLS residuals from regressing y_{it} on the appropriate set of regressors from

(3.1) to (3.4) excluding the random walk. For example, the regressors in (3.4) are the intercept, dummy for the level, the time trend and the time trend dummy.

The next theorem states the asymptotic distributions of $\eta_{i,T,k}(\omega_i)$ statistic under the null hypothesis. See Kurozumi (2002) for the proof.

Theorem 3.1 *Let $\{y_{it}\}$ be generated by Model k, $k = 0, 1, 2, 3$, then, under the null hypothesis, for the ith individual in the panel, as $T \to \infty$,*

$$\eta_{i,T,k}(\omega_i) \Longrightarrow G_{i,k}(B), \tag{3.11}$$

where

$$\begin{aligned} G_{i,k}(B) &= \omega_i^2(\int_0^1 B_1(r^2)dr - X(B_1)'\Lambda^{-1}X(B_1)) \\ &\quad +(1-\omega_i^2)(\int_0^1 B_2(r^2)dr - X(B_2)'\Lambda^{-1}X(B_2)), \end{aligned} \tag{3.12}$$

When $k = 0$ and 3.

When $k = 1$ and 2, $G_{i,k}(B)$ is given by

$$G_{i,k}(B) = \int_0^1 B(r^2)dr - X(B)'\Lambda^{-1}X(B). \tag{3.13}$$

$B(.)$ is standard Brownian motion, $B_1(.), B_2(.)$ are independent Brownian motions and $X(B)$ denotes a functional of $B(.)$, where

for Model 0,

$$X(B) = \int_0^1 B(r)dr \text{ and } \Lambda = 1,$$

for Model 3,

$$X(B) = \begin{bmatrix} \int_0^1 B(r)dr \\ \int_0^1 rB(r)dr \end{bmatrix} \quad and \quad \Lambda = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix},$$

for Model 1,

$$X(B) = \begin{bmatrix} \int_0^1 B(r)dr \\ \int_{\omega_i}^1 B(r)dr \\ \int_0^1 rB(r)dr \end{bmatrix} \quad and \ \Lambda = \begin{bmatrix} 1 & 1-\omega_i & \frac{1}{2} \\ 1-\omega_i & 1-\omega_i & \frac{1-\omega_i^2}{2} \\ \frac{1}{2} & \frac{1-\omega_i^2}{2} & \frac{1}{3} \end{bmatrix},$$

for Model 2,

$$X(B) = \begin{bmatrix} \int_0^1 B(r)dr \\ \int_0^1 rB(r)dr \\ \int_{\omega_i}^1 (r-\omega_i)B(r)dr \end{bmatrix} \quad and \ \Lambda = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1-\omega_i^2}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{(1-\omega_i)^2(\omega_i+2)}{6} \\ \frac{1-\omega_i^2}{2} & \frac{(1-\omega_i)^2(\omega_i+2)}{6} & \frac{(1-\omega_i)^3}{3} \end{bmatrix}.$$

The characteristic function for Model 0 *and* 3 *can be expressed as*

$$\phi(\theta) = [D_k(2i\omega_i^2\theta)D_k(2i(1-\omega_i)^2\theta)]^{-1/2}, \ k = 0 \ and \ 3.$$

For Model 1 *and* 2 *the characteristic function is given by*

$$\phi(\theta) = [D_k(2i\theta)]^{-1/2}, \ k = 1 \ and \ 2.$$

The $D_k(.)$ *functionals corresponding to the four models are given in the Appendix 1.*

The asymptotic distributions of $\eta_{i,T,k}(\omega_i)$ depend on which model has generated $\{y_{it}\}$ and on the location of the break point. The panel stationarity test given in equation (3.10) is the average of $\eta_{i,T,k}(\omega_i)$. We denote

$$\xi_{i,k} = E(G_{i,k}(B))$$

$$\zeta_{i,k}^2 = Var(G_{i,k}(B))$$

Theorem 3.2 *Under Assumption* 3.1, 3.2 *and* 3.3, *as* $T \to \infty$ *followed by* $N \to \infty$ *and for* $k = 0, 1, 2, 3$, *the statistic defined by equation* (3.10) *under the null of stationarity with a break has the following limiting distribution:*

$$Z_k(\omega) = \frac{\sqrt{N}(\widehat{LM}_{T,N,k}(\omega) - \overline{\xi}_k)}{\overline{\zeta}_k} \Rightarrow N(0,1), \tag{3.14}$$

where $\overline{\xi}_k = \frac{1}{N}(\sum_{n=1}^{N} \xi_{i,k})$ *and* $\overline{\zeta_i^2} = \frac{1}{N}(\sum_{n=1}^{N} \zeta_{i,k}^2)$.

Proof. See Hadri (2000). ■

To find the mean $\xi_{i,k}$ and variance $\zeta_{i,k}^2$, we use the characteristic functions given above. These moments are provided in the following theorem. The derivations are given in Appendix 1.

Theorem 3.3 *Let* $\{y_{i,t}\}$ *be given by* (3.1), (3.2), (3.3), (3.4). *Under Assumption* 3.1, 3.2 *and* 3.3 *for the ith individual unit as* $T \to \infty$, *we have,*

for Model 0,

$$\xi_{i,0} = \frac{1}{6}(2\omega_i^2 - 2\omega_i + 1),$$

$$\zeta_{i,0}^2 = \frac{1}{45}\left(2\omega_i^4 - 4\omega_i^3 + 6\omega_i^2 - 4\omega_i + 1\right),$$

for Model 1,

$$\xi_{i,1} = \frac{15\omega_i^4 - 30\omega_i^3 + 25\omega_i^2 - 10\omega_i + 2}{30\left(3\omega_i^2 - 3\omega_i + 1\right)},$$

$$\zeta_{i,1}^2 = \frac{315\omega_i^8 - 1260\omega_i^7 + 2415\omega_i^6 - 2835\omega_i^5 + 2275\omega_i^4 - 1295\omega_i^3 + 495\omega_i^2 - 110\omega_i + 11}{6300(3\omega_i^2 - 3\omega_i + 1)^2},$$

for Model 2,

$$\xi_{i,2} = \frac{1}{30}(3\omega_i^2 - 3\omega_i + 2),$$

$$\zeta_{i,2}^2 = \frac{1}{6300}(3\omega_i^4 - 6\omega_i^3 + 36\omega_i^2 - 33\omega_i + 11),$$

for Model 3,

$$\xi_{i,3} = \frac{1}{15}(2\omega_i^2 - 2\omega_i + 1),$$

$$\zeta_{i,3}^2 = \frac{11}{6300}\left(2\omega_i^4 - 4\omega_i^3 + 6\omega_i^2 - 4\omega_i + 1\right).$$

Remark 3.1 *We observe that these moments are functions of the break fraction parameters. In addition, the corresponding asymptotic mean and variance when* $\omega = 0$ *or* 1, *as* $T \to \infty$ *are* 1/6 *and* 1/45, *respectively for Model* 0. *While for Model 1, 2 and 3, they are* 1/15 *and* 11/6300. *These values coincide with the moments suggested by Hadri (2000) when no break exists.*

Remark 3.2 *We note that for Model 0 and Model 3, our moments are the same as the ones derived in CBL when one structural break is specified. However, the method of derivation is different. We included Model 0 and Model 3 for completeness and coherency of our use of the characteristic functions for deriving moments of all the statistics.*

Remark 3.3 *After replacing the asymptotic moments of* $\widehat{LM}_{T,N,k}(\omega)$ *by their numerical values in the appropriate model, the statistic* $Z_k(\omega)$ *can be used to test the null hypothesis. This is a one-sided test and the inference is performed on the upper tail* *of the distribution. The null is rejected when the value of $Z_k(\omega)$ exceeds the critical value of the standard normal distribution.*

3.3.2 A Modified Test

We propose a modified test for Model 0 and 3, which avoids the dependence of the limiting distributions under the null hypothesis on the nuisance parameters. From Theorem 3.1, we note that, the limiting distributions for Model 0 and 3 are expressed as the weighted sum of two independent functionals, $G(B_1)$ and $G(B_2)$. As noticed by Busetti and Harvey (2001), this allows the test statistic $\eta^*_{i,T,k}(\omega_i)$ to be presented as the sum of two functions, one depending on the residuals for the period before the break point and the other on the residuals after the break point:

$$\eta^*_{i,T,k}(\omega_i) = \frac{\sum_{t=1}^{T_{B,i}}(\sum_{j=1}^{t}\widehat{\epsilon}_{i,j})^2}{T_{B,i}^2 \times \widehat{\sigma}_{\epsilon,i}^2} + \frac{\sum_{t=T_{B,i}+1}^{T}(\sum_{j=T_{B,i}+1}^{t}\widehat{\epsilon}_{i,j})^2}{(T-T_{B,i})^2 \times \widehat{\sigma}_{\epsilon,i}^2}, \tag{3.15}$$

where $k = 0$ and 3, $\widehat{\sigma}_{\epsilon,i}^2$ is defined by (3.9) which is the estimator of LRV of $\{\epsilon_{it}\}, t = 1, 2, .., T$. The asymptotic distribution of the statistic is given by

$$\begin{aligned}\eta^*_{i,T,k}(\omega_i) \implies G_{i,k}(B_1) + G_{i,k}(B_2) &= \int_0^1 B_1(r^2)dr - X(B_1)'\Lambda^{-1}X(B_1) \\ &+ \int_0^1 B_2(r^2)dr - X(B_2)'\Lambda^{-1}X(B_2),\end{aligned} \tag{3.16}$$

where $G(B_1)$ and $G(B_2)$ are two independent functionals of Brownian motions. These two functionals do not depend on the break parameter. Note that the modified test is not applicable to Model 1 and 2 because their test statistics cannot be expressed as the sum of independent functionals of Brownian motions (see Busetti and Harvey

(2001), Kurozumi (2002)). The characteristic function corresponding to the modified test of Model 0 and 3 is given by

$$\phi(\theta) = [D_{k,M}(2i\theta)]^{-1}, \quad k = 0 \text{ and } 3.$$

The expressions of $D_{k,M}(.)$ for these models are provided in the Appendix. The limiting distribution in Theorem 3.2 still holds except that the asymptotic moments are derived via the characteristic functions corresponding to the statistic $\eta^{*}_{i,T,k}(\omega_i)$ which are different from the those corresponding to the statistic $\eta_{i,T,k}(\omega_i)$. The asymptotic distribution and moments of the modified test are given by the following theorem:

Theorem 3.4 *Let $\{y_{i,t}\}$ be given by (3.1) and (3.4). For the modified test, under Assumption 3.1, 3.2 and 3.3, as $T \to \infty$ followed by $N \to \infty$ and for $k = 0, 3$, the statistic defined by equation (3.15) under the null has the following distribution:*

$$Z^{*}_{k}(\omega) = \frac{\sqrt{N}(\frac{1}{N}\sum_{i=1}^{N}\eta^{*}_{i,T,k}(\omega_i) - \overline{\xi}^{*}_{k})}{\overline{\zeta}^{*}_{k}} \Rightarrow N(0,1), \tag{3.17}$$

where $\overline{\xi}^{*}_{k} = \frac{1}{N}(\sum_{n=1}^{N}\xi^{*}_{i,k})$ and $\overline{\zeta^{*2}_{k}} = \frac{1}{N}(\sum_{n=1}^{N}\zeta^{*2}_{i,k})$. For the ith individual unit as $T \to \infty$, we have

$$\textit{for Model } 0, \;\; \xi^{*}_{i,0} = \tfrac{1}{3} \textit{ and } \zeta^{*2}_{i,0} = \tfrac{2}{45}$$

and

$$\textit{for Model } 3, \;\; \xi^{*}_{i,3} = \tfrac{2}{15} \textit{ and } \zeta^{*2}_{i,3} = \tfrac{11}{3150}.$$

Proof. Given in Appendix 1. ■

Remark 3.4 *It should be noted that under the null hypothesis, although the limiting distribution of the modified statistic is free of break point location, the statistic itself still depends on it.*

Remark 3.5 *As will be shown in our Monte Carlo experiments in Section 3.5, the modified test in the presence of autocorrelated errors is found to be more distorted and less powerful than the LM test.*

3.4 Testing for Stationarity with an Unknown Break Date

The statistics in previous sections are based on the assumption that the break point is exogenous and known. However, in many empirical applications, we rarely know the break date *a priori*, with the consequences that the break date has to be estimated. In this Section, we deal with the case which allows for a structural break at an unknown date.

Several procedures have been proposed in the literature to determine an unknown break point endogenously. One practice is to choose the break date that gives the most favorable result for the null of stationarity with a break, as in Lee and Strazicich (2001) and Busetti and Harvey (2001). This means that the estimate of ω_i is obtained as the value that minimizes the statistic under the null,

$$\hat{\omega}_i = \arg \min_{0<\omega_i<1} (S(\omega_i)),$$

where $S(\omega_i)$ denotes the corresponding test statistic. Under their assumptions, they argue that the minimum functional will provide a consistent estimation of the true break fraction and therefore that the test statistic will converge to the same asymptotic distribution as the one derived when the break point is known. However, it was shown that the distribution also depends on the magnitude of the break. Hence, shrinking structural change has to be assumed.

Another approach was suggested by Bai (1994, 1997) and Kurozumi (2002). They consider using the estimate of the break date that minimizes the sum of squared residuals (SSR) from the relevant regression under the null hypothesis, that is,

$$\hat{\omega}_i = \arg \min_{0<\omega_i<1} (SSR(\omega_i)).$$

It is shown that this method provides consistent estimates of the break point without having to impose any shrinking break assumption. In this chapter, we apply this proposal to our panel stationarity test. Since we allow for different break locations across individuals in panels, we need to detect the break in each one of the individual time series. Therefore, after $\hat{\omega}_i$, $i = 1, ..., N$ are obtained, we only need to replace ω_i by $\hat{\omega}_i$ in (3.14) or (3.17) and hence obtain

$$Z_k(\widehat{\omega}) = \frac{\sqrt{N}(\widehat{LM}_{T,N,k}(\widehat{\omega}) - \bar{\xi}_k)}{\bar{\zeta}_k}, \tag{3.18}$$

where $\overline{\xi}_k$ and $\overline{\zeta}_k$ follow the same definition as in Theorem (3.3) or (3.4). Since $\widehat{\omega} \xrightarrow{p} \omega$ as $T \to \infty$, we obtain the same limiting distribution for our statistics as in Theorem (3.3) or (3.4). Thus, we perform the hypothesis testing as if the estimated break point were known.

Remark 3.6 *As in CBL, for Model 0 and 3, we can allow for the number of breaks and their positions to differ across individuals for models with known and unknown breaks. This can be extended to the modified test.*

Finally, we can state the following general remarks:

Remark 3.7 *As in Hadri and Larsson (2005), it is easy to show, using simulations, that the power of the tests increases as the proportion of unit roots increases under the alternative.*

Remark 3.8 *In the likely case where there is cross-sectional dependency, one readily available method to be used to correct for it is the bootstrap as shown, inter alia, by Maddala and Wu (1999), Wu and Wu (2001) and Chang (2004).*

Remark 3.9 *Following Phillips and Moon (1999), under the rate condition $N/T \to 0$, the sequential results obtained above imply joint convergence.*

3.5 Finite Sample Properties

In this section, we conducted Monte Carlo simulations to investigate the finite sample properties of our proposed statistics. All simulation results are based on 10000 replications (See Hadri and Phillips (1999) for the importance of the number of replications in simulations) and we use the critical value of 1.645 (5% significance level). It is evident that the distributions of our statistics under the null do not depend on $\alpha_i s$, $\delta_i s$, $\beta_i s$, $\gamma_i s$ and $\sigma^2_{\epsilon,i} s$. The data-generating processes (DGP) under the null hypothesis are given by equation (3.1)-(3.4) with $\alpha_i \sim U[0, 10]$, $\delta_i \sim U[0, 10]$, $\beta_i \sim U[0, 2]$ and $\gamma_i \sim U[0, 5]$, where $U[.]$ denotes the uniform distributions. Sample size is given as the combination of different N and T. For each sample size, the parameters of the DGP are generated once and fixed in all the replications. The break fraction is randomly generated as $\omega_i \sim U[0.10, 0.90]$ with a 10% trimming at both ends of the time series. The results for both *i.i.d* and autocorrelated errors are investigated.

3.5.1 Results of Models with *i.i.d* Errors

In the case of *i.i.d* errors in the models, we assume $\sigma^2_{\epsilon,i} = 1$ for simplicity. The simulations are carried out for the sample size of $T = \{50, 100, 150, 200\}$ and $N = \{15, 25, 50, 100\}$. We first focus on the situation when the break date is known. Under the assumption of *i.i.d* errors, the size of the test depends on T, N. It is shown in Table 3.1 that the empirical size of the LM test proposed is quite close to the nominal one. When the sample size increases, it becomes more accurate when T is relatively larger than N as expected. This is also true for the modified test, the results are summarized in Table 3.2.

Table 3.1: **Size of LM Test with Known Break Date (i.i.d Errors)**

	Model 0			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0613	0.0566	0.0507	0.0528
$T = 100$	0.0635	0.0544	0.0589	0.0553
$T = 150$	0.0592	0.0594	0.0599	0.0508
$T = 200$	0.0593	0.0642	0.0595	0.0545
	Model 1			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0491	0.0508	0.0451	0.0429
$T = 100$	0.0523	0.0513	0.0558	0.0547
$T = 150$	0.0598	0.0562	0.0512	0.0497
$T = 200$	0.0600	0.0588	0.0537	0.0541
	Model 2			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0580	0.0479	0.0428	0.0487
$T = 100$	0.0535	0.0508	0.0558	0.0496
$T = 150$	0.0606	0.0549	0.0561	0.0487
$T = 200$	0.0556	0.0553	0.0505	0.0532
	Model 3			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0514	0.0487	0.0394	0.0411
$T = 100$	0.0535	0.0527	0.0534	0.0510
$T = 150$	0.0584	0.0544	0.0569	0.0504
$T = 200$	0.0590	0.0608	0.0521	0.0527

Table 3.2: Size of the Modified Test with Known Break Date (i.i.d Errors)

Model 0				
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0523	0.0487	0.0471	0.0450
$T = 100$	0.0562	0.0549	0.0501	0.0461
$T = 150$	0.0588	0.0545	0.0546	0.0515
$T = 200$	0.0573	0.0586	0.0552	0.0495
Model 3				
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0342	0.0248	0.0216	0.0166
$T = 100$	0.0420	0.0437	0.0382	0.0359
$T = 150$	0.0556	0.0513	0.0484	0.0458
$T = 200$	0.0562	0.0520	0.0534	0.0494

Table 3.3 shows the power results of both test statistics with a known break. Under the alternative hypothesis, we allow for different proportions of unit root processes ($M = N_1/N$) in the panel. To save space, we only report the simulations under the condition that all the cross sections follow a unit root; that is, $M = 1$. As a function of T and N, the power also changes with different λ ($\lambda = \frac{\sigma_u^2}{\sigma_\epsilon^2}$). We recall that $\lambda = 0$ ($\sigma_u^2 = 0$) means that y is stationary whereas $\lambda = \infty$ implies that y comprises a random walk. By varying the value of λ we can see how the power of the test changes as we approach the two polar cases (stationary *versus* nonstationary y). We set $\lambda = 0.001$ and 0.003 in our simulations. In general, the power of the test increase monotonically as T or N, or both, get larger and increases with λ for all T and N. The power of the modified test is presented in Table 3.4. For Model 0 and Model 3, the modified test is relatively less powerful than the corresponding LM test when the sample size is small.

Table 3.3: Power of LM Test with Known Break Date (i.i.d Errors)

			Model 0		
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1179	0.1467	0.1822	0.2330
0.001	$T=100$	0.4186	0.5219	0.7638	0.9373
	$T=150$	0.7577	0.8984	0.9921	1.0000
	$T=200$	0.9580	0.9944	1.0000	1.0000
	$T=50$	0.3001	0.4135	0.5774	0.7624
0.003	$T=100$	0.8830	0.9646	0.9990	1.0000
	$T=150$	0.9935	0.9998	1.0000	1.0000
	$T=200$	1.0000	1.0000	1.0000	1.0000
			Model 1		
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.0666	0.0745	0.0823	0.0984
0.001	$T=100$	0.3037	0.2375	0.3043	0.3180
	$T=150$	0.4400	0.4383	0.6780	0.9396
	$T=200$	0.6274	0.8807	0.9856	0.9994
	$T=50$	0.1139	0.1634	0.2011	0.3308
0.003	$T=100$	0.8362	0.7603	0.8956	0.9374
	$T=150$	0.9603	0.9715	0.9998	1.0000
	$T=200$	0.9977	1.0000	1.0000	1.0000
			Model 2		
λ		$N=15$	$N=25$	$N=50$	$N=100$
0.001	$T=50$	0.0704	0.0904	0.0746	0.0864
	$T=100$	0.2156	0.1748	0.1955	0.2444
	$T=150$	0.3739	0.2489	0.6103	0.8641
	$T=200$	0.6110	0.7590	0.9824	0.9971
	$T=50$	0.1253	0.2103	0.1610	0.2172
0.003	$T=100$	0.6398	0.5804	0.6580	0.8334
	$T=150$	0.9214	0.7792	0.9988	1.0000
	$T=200$	0.9951	0.9999	1.0000	1.0000
			Model 3		
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.0670	0.0763	0.0598	0.0708
0.001	$T=100$	0.1874	0.1700	0.1729	0.1712
	$T=150$	0.2622	0.2098	0.5371	0.7945
	$T=200$	0.5306	0.6198	0.9494	0.9805
	$T=50$	0.1149	0.1661	0.1163	0.1666
0.003	$T=100$	0.5504	0.5547	0.5864	0.6385
	$T=150$	0.7728	0.6870	0.9955	0.9999
	$T=200$	0.9865	0.9977	1.0000	1.0000

Table 3.4: Power of the Modified Test with Known Break Date (i.i.d Errors)

λ		Model 0 $N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0925	0.1088	0.1028	0.2108
0.001	$T = 100$	0.4986	0.3751	0.5347	0.8329
	$T = 150$	0.5585	0.9666	0.9575	1.0000
	$T = 200$	0.9649	0.9945	1.0000	1.0000
	$T = 50$	0.2119	0.3223	0.3145	0.7921
0.003	$T = 100$	0.9859	0.9519	0.9962	1.0000
	$T = 150$	0.9943	1.0000	1.0000	1.0000
	$T = 200$	1.0000	1.0000	1.0000	1.0000

λ		Model 3 $N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0465	0.0397	0.0318	0.0274
0.001	$T = 100$	0.1187	0.1211	0.1174	0.1242
	$T = 150$	0.2060	0.1666	0.3698	0.5903
	$T = 200$	0.4155	0.4193	0.8317	0.9307
	$T = 50$	0.0685	0.0775	0.0560	0.0587
0.003	$T = 100$	0.3626	0.4111	0.4275	0.5012
	$T = 150$	0.6587	0.5871	0.9669	0.9989
	$T = 200$	0.9644	0.9770	1.0000	1.0000

We now move forward to investigate the performance of the statistics with an unknown break point. Tables 3.5 and 3.6 report the size results of the two tests respectively. Overall, in both tests there is a size distortion in small samples although this distortion disappears as T grows. It suggests that the LM test has relatively better performance especially when the sample size is small. However, for a fixed T, the size distortions appear to increase with N, this is due to the accumulation of errors in the estimation of break fractions, and any small-size distortion in each univariate series can cummlate in a panel setting. The size of models with an estimated break date is found to be more distorted than that when break point is assumed to be known.

Table 3.5: Size of LM Test with Unknown Break Date (i.i.d Errors)

	Model 0			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0475	0.0394	0.0394	0.0318
$T = 100$	0.0617	0.0392	0.0393	0.0368
$T = 150$	0.0563	0.0471	0.0524	0.0363
$T = 200$	0.0526	0.0602	0.0442	0.0396
	Model 1			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0243	0.0322	0.0176	0.0087
$T = 100$	0.0411	0.0257	0.0252	0.0206
$T = 150$	0.0432	0.0416	0.0303	0.0174
$T = 200$	0.0456	0.0501	0.0437	0.0217
	Model 2			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0258	0.0187	0.0089	0.0054
$T = 100$	0.0326	0.0264	0.0196	0.0231
$T = 150$	0.0492	0.0278	0.0251	0.0162
$T = 200$	0.0533	0.0398	0.0314	0.0287
	Model 3			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0465	0.0373	0.0327	0.0298
$T = 100$	0.0518	0.0486	0.0445	0.0484
$T = 150$	0.0514	0.0509	0.0544	0.0481
$T = 200$	0.0587	0.0606	0.0549	0.0466

Table 3.6: Size of the Modified Test with Unknown Break Date (i.i.d Errors)

	Model 0			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0313	0.0317	0.0287	0.0237
$T = 100$	0.0517	0.0384	0.0356	0.0285
$T = 150$	0.0497	0.0418	0.0440	0.0344
$T = 200$	0.0498	0.0517	0.0439	0.0310
	Model 3			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0285	0.0202	0.0135	0.0096
$T = 100$	0.0448	0.0397	0.0304	0.0341
$T = 150$	0.0463	0.0410	0.0421	0.0373
$T = 200$	0.0495	0.0570	0.0481	0.0438

The power results of the tests with an estimated break date is shown in Tables 3.7 and 3.8. It is also true that the power increases when the sample size becomes larger for both tests and, additionally, that the power increases with λ for all T and N. Compared with Tables 3.3 and 3.4, we found that when the break date is estimated, the power decreases relatively in certain cases. This applies to both the LM and the modified test. This is due to the fact that the estimation of the break point induces power losses. Again, the modified test is less powerful than the corresponding LM test, as in the known break case.

Table 3.7: Power of LM Test with Unknown Break Date (i.i.d Errors)

		Model 0			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
0.001	$T = 50$	0.0891	0.0999	0.0871	0.2200
	$T = 100$	0.4391	0.5982	0.6634	0.7834
	$T = 150$	0.5708	0.8805	0.9587	1.0000
	$T = 200$	0.9287	0.9668	1.0000	1.0000
0.003	$T = 50$	0.2275	0.3466	0.3155	0.8211
	$T = 100$	0.9586	0.9962	0.9996	1.0000
	$T = 150$	0.9952	1.0000	1.0000	1.0000
	$T = 200$	1.0000	1.0000	1.0000	1.0000
		Model 1			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
0.001	$T = 50$	0.0490	0.0388	0.0507	0.0252
	$T = 100$	0.3151	0.1671	0.1626	0.1875
	$T = 150$	0.4283	0.2560	0.4685	0.6389
	$T = 200$	0.4773	0.8264	0.9894	0.9867
0.003	$T = 50$	0.0747	0.0879	0.1477	0.1185
	$T = 100$	0.8860	0.6208	0.6417	0.8703
	$T = 150$	0.9671	0.8253	0.9937	0.9999
	$T = 200$	0.9888	1.0000	1.0000	1.0000
		Model 2			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
0.001	$T = 50$	0.0320	0.0342	0.0228	0.0131
	$T = 100$	0.0867	0.1042	0.0977	0.1036
	$T = 150$	0.2342	0.2036	0.3549	0.6011
	$T = 200$	0.5814	0.7334	0.8898	0.9511
0.003	$T = 50$	0.0521	0.0897	0.0469	0.0550
	$T = 100$	0.3457	0.4465	0.4517	0.6541
	$T = 150$	0.7632	0.7535	0.9728	1.0000
	$T = 200$	0.9944	0.9998	1.0000	1.0000
		Model 3			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
0.001	$T = 50$	0.0510	0.0564	0.0600	0.0530
	$T = 100$	0.1193	0.2238	0.5740	0.1866
	$T = 150$	0.2690	0.2297	0.4301	0.8370
	$T = 200$	0.4950	0.6422	0.9337	0.9699
0.003	$T = 50$	0.0777	0.1203	0.1256	0.1275
	$T = 100$	0.3187	0.7121	0.5231	0.7144
	$T = 150$	0.8022	0.7385	0.9804	1.0000
	$T = 200$	0.9764	0.9986	1.0000	1.0000

Table 3.8: **Power of the Modified Test with Unknown Break Date (i.i.d Errors)**

		Model 0			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0744	0.0659	0.0586	0.1127
0.001	$T = 100$	0.4183	0.2955	0.5521	0.5243
	$T = 150$	0.7190	0.9043	0.9371	0.9997
	$T = 200$	0.7014	0.9990	1.0000	1.0000
	$T = 50$	0.1810	0.1945	0.2467	0.6601
0.003	$T = 100$	0.9669	0.9262	0.9976	0.9989
	$T = 150$	0.9998	1.0000	1.0000	1.0000
	$T = 200$	0.9992	1.0000	1.0000	1.0000

		Model 3			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0274	0.0288	0.0193	0.0154
0.001	$T = 100$	0.0806	0.0879	0.0829	0.1682
	$T = 150$	0.1994	0.2433	0.3203	0.6073
	$T = 200$	0.2261	0.4364	0.7229	0.9019
	$T = 50$	0.0404	0.0399	0.0364	0.0348
0.003	$T = 100$	0.1881	0.2767	0.2928	0.7265
	$T = 150$	0.6769	0.8204	0.9569	0.9999
	$T = 200$	0.7509	0.9846	1.0000	1.0000

3.5.2 Results of Models with Autocorrelated Errors

We consider a stationary AR(1) process, where

$$\epsilon_{it} = \rho_i \epsilon_{it-1} + v_{it}. \tag{3.19}$$

with $v_{it} \sim i.i.d\ N(0,1)$ and $\rho_i \sim U[0.1, 0.9]$. Note that the autoregressive parameter is assumed to differ across individuals both under the null and the alternative hypothesis. The statistical procedures now require that T should be sufficiently large to estimate adequately the long-run variance. We increase the sample size T up to 500. The long-run variance of ϵ_{it} can be parametrically estimated by $\widehat{\sigma}^2_{\epsilon i} = \widehat{\sigma}^2_{vi}/(1-\widehat{\rho}_i)^2$, where $\widehat{\rho}_i$ and $\widehat{\sigma}^2_{vi}$ are least squares estimates of the coefficient and the error variances respectively in equation (3.19). $\widehat{\sigma}^2_{vi}$ denotes the LRV of the estimated residuals in (3.19) which can be estimated using the Quadratic Spectral window Heteroskedasticity and Autocorrelation Consistent (HAC) estimator. To avoid the problem of inconsistency of the statistics, we use the boundary condition rule which has been suggested by Sul *et al.* (2005) to obtain the long-run variance estimate:

$$\widehat{\sigma}^2_{\epsilon i} = \min\{T\widehat{\sigma}^2_{vi}, \frac{\widehat{\sigma}^2_{vi}}{(1-\widehat{\rho}_i)^2}\}.$$

The size and power results with autocorrelated errors are presented in Tables 3.9-3.16. In general, the empirical size of LM test is close to the nominal one as T and N increases (but with T larger than N as expected) with both known break and unknown break. For the power of the test, λ is set to be 0.01 and 1. The power of the LM test appear to grow monotonically as T or N, or both get larger and increases with λ for all T and N. However, the modified test shows severe size distortions and suffer from loss of power in the presence of serially correlated errors. Lastly, it should be noted that in general Model 0 and 3 perform relatively better than Model 1 and 2.

Table 3.9: **Size of LM Test with Known Break Date (AR(1) Errors)**

		Model 0		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0340	0.0313	0.0403	0.0561
$T = 100$	0.0460	0.0411	0.0420	0.0223
$T = 200$	0.0590	0.0471	0.0451	0.0363
$T = 500$	0.0588	0.0596	0.0512	0.0501
		Model 1		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0556	0.0625	0.0867	0.1991
$T = 100$	0.0433	0.0350	0.0598	0.0396
$T = 200$	0.0524	0.0477	0.0418	0.0327
$T = 500$	0.0575	0.0536	0.0486	0.0427
		Model 2		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0493	0.0438	0.0745	0.1973
$T = 100$	0.0514	0.0457	0.0602	0.0316
$T = 200$	0.0582	0.0512	0.0477	0.0418
$T = 500$	0.0547	0.0549	0.0452	0.0524
		Model 3		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.1581	0.2602	0.4395	0.6570
$T = 100$	0.0940	0.1050	0.0932	0.1685
$T = 200$	0.0611	0.0563	0.0673	0.0843
$T = 500$	0.0622	0.0641	0.0614	0.0562

Table 3.10: Size of the Modified Test with Known Break Date (AR(1) Errors)

	Model 0			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0040	0.0022	0.0009	0.0001
$T = 100$	0.0151	0.0097	0.0016	0.0003
$T = 200$	0.0197	0.0111	0.0086	0.0063
$T = 500$	0.0405	0.0261	0.0304	0.0170
	Model 3			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0083	0.0060	0.0079	0.0005
$T = 100$	0.0099	0.0062	0.0006	0.0000
$T = 200$	0.0125	0.0038	0.0029	0.0023
$T = 500$	0.0324	0.0122	0.0243	0.0097

Table 3.11: Power of LM Test with Known Break Date (AR(1) Errors)

		Model 0			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1222	0.3936	0.3738	0.6916
0.01	$T=100$	0.7662	0.8173	0.9655	1.0000
	$T=200$	0.9999	0.9999	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.6466	0.8968	0.9957	1.0000
1	$T=100$	0.9706	0.9986	1.0000	1.0000
	$T=200$	0.9994	1.0000	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
		Model 1			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1158	0.2295	0.2488	0.4638
0.01	$T=100$	0.3646	0.4169	0.5836	0.9528
	$T=200$	0.9753	0.9891	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.4235	0.6972	0.8240	0.9961
1	$T=100$	0.8599	0.9256	0.9968	1.0000
	$T=200$	0.9904	0.9992	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
		Model 2			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.0719	0.2023	0.1884	0.3946
0.01	$T=100$	0.3104	0.3064	0.4616	0.8731
	$T=200$	0.9699	0.9704	0.9999	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.1975	0.3976	0.5714	0.9435
1	$T=100$	0.7146	0.8629	0.9770	1.0000
	$T=200$	0.9905	0.9969	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
		Model 3			
		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1876	0.4201	0.5360	0.8602
0.01	$T=100$	0.3249	0.3546	0.5616	0.9053
	$T=200$	0.9554	0.9387	0.9998	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.4822	0.7590	0.9566	1.0000
1	$T=100$	0.8484	0.9676	0.9987	1.0000
	$T=200$	0.9974	0.9994	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000

Table 3.12: Power of the Modified Test with Known Break Date (AR(1) Errors)

λ		$N=15$	$N=25$	$N=50$	$N=100$
Model 0					
0.01	$T=50$	0.0179	0.0259	0.0142	0.0022
	$T=100$	0.7182	0.6693	0.9455	0.9748
	$T=200$	0.9757	1.0000	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
1	$T=50$	0.0962	0.1774	0.2706	0.3052
	$T=100$	0.6886	0.8010	0.9956	1.0000
	$T=200$	0.9954	0.9996	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
Model 3					
0.01	$T=50$	0.0105	0.0208	0.0273	0.0119
	$T=100$	0.1555	0.0508	0.0204	0.0942
	$T=200$	0.4418	0.4719	0.9864	0.9996
	$T=500$	1.0000	1.0000	1.0000	1.0000
1	$T=50$	0.0049	0.0029	0.0010	0.0000
	$T=100$	0.0989	0.0212	0.2414	0.0693
	$T=200$	0.2210	0.8143	0.7793	0.9948
	$T=500$	0.9799	1.0000	1.0000	1.0000

Table 3.13: Size of LM Test with Unknown Break Date (AR(1) Errors)

	Model 0			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0307	0.0465	0.0231	0.0310
$T = 100$	0.0310	0.0208	0.0254	0.0363
$T = 150$	0.0435	0.0437	0.0362	0.0222
$T = 200$	0.0530	0.0640	0.0558	0.0411
	Model 1			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0342	0.0967	0.0597	0.0796
$T = 100$	0.0286	0.0295	0.0153	0.0080
$T = 150$	0.0473	0.0191	0.0168	0.0183
$T = 200$	0.0270	0.0298	0.0286	0.0135
	Model 2			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0108	0.0409	0.0090	0.0074
$T = 100$	0.0105	0.0071	0.0051	0.0029
$T = 200$	0.0201	0.0172	0.0071	0.0071
$T = 500$	0.0331	0.0188	0.0230	0.0082
	Model 3			
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.1578	0.2021	0.4301	0.6234
$T = 100$	0.0786	0.0974	0.1266	0.1537
$T = 200$	0.0679	0.0793	0.0754	0.0950
$T = 500$	0.0621	0.0562	0.0660	0.0642

Table 3.14: Size of the Modified Test with Unknown Break Date (AR(1) Errors)

		Model 0		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0038	0.0010	0.0024	0.0002
$T = 100$	0.0054	0.0056	0.0002	0.0000
$T = 200$	0.0118	0.0138	0.0022	0.0018
$T = 500$	0.0244	0.0358	0.0164	0.0190
		Model 3		
	$N = 15$	$N = 25$	$N = 50$	$N = 100$
$T = 50$	0.0109	0.0216	0.0203	0.0033
$T = 100$	0.0086	0.0047	0.0007	0.0001
$T = 200$	0.0180	0.0085	0.0074	0.0016
$T = 500$	0.0257	0.0217	0.0216	0.0120

Table 3.15: **Power of LM Test with Unknown Break Date (AR(1) Errors)**

		Model 0			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1048	0.1565	0.3116	0.5516
0.01	$T=100$	0.5922	0.6594	0.9794	0.9972
	$T=200$	0.9989	0.9998	1.0000	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.2910	0.6100	0.6294	0.8831
1	$T=100$	0.4678	0.7486	0.9607	0.9995
	$T=200$	0.8114	0.9669	0.9987	1.0000
	$T=500$	0.9915	0.9999	1.0000	1.0000
		Model 1			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.0597	0.0533	0.0960	0.1476
0.01	$T=100$	0.1476	0.2843	0.4444	0.8894
	$T=200$	0.9456	0.9880	1.0000	0.9999
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.1565	0.1931	0.3849	0.7979
1	$T=100$	0.2501	0.5308	0.8917	0.9566
	$T=200$	0.8083	0.9180	0.9975	1.0000
	$T=500$	0.9830	0.9930	1.0000	1.0000
		Model 2			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.0154	0.0300	0.0152	0.0320
0.01	$T=100$	0.0915	0.1269	0.1932	0.5374
	$T=200$	0.9268	0.9758	0.9998	0.9991
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.0210	0.0322	0.0546	0.0386
1	$T=100$	0.1524	0.1606	0.3276	0.8614
	$T=200$	0.7420	0.7352	0.9788	1.0000
	$T=500$	0.9812	0.999	1.0000	1.0000
		Model 3			
λ		$N=15$	$N=25$	$N=50$	$N=100$
	$T=50$	0.1967	0.2947	0.4843	0.7708
0.01	$T=100$	0.2445	0.3963	0.6859	0.9112
	$T=200$	0.7856	0.9861	0.9995	1.0000
	$T=500$	1.0000	1.0000	1.0000	1.0000
	$T=50$	0.3714	0.6742	0.8670	0.9812
1	$T=100$	0.8054	0.9280	0.9982	1.0000
	$T=200$	0.9956	0.9946	1.0000	1.0000
	$T=500$	0.9998	1.0000	1.0000	1.0000

Table 3.16: Power of the Modified Test with Unknown Break Date (AR(1) Errors)

		Model 0			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0206	0.0182	0.0046	0.0050
0.01	$T = 100$	0.4930	0.3100	0.7240	0.7748
	$T = 200$	0.9586	0.9996	1.0000	1.0000
	$T = 500$	1.0000	1.0000	1.0000	1.0000
	$T = 50$	0.0648	0.0468	0.0474	0.1048
1	$T = 100$	0.2140	0.2698	0.3674	0.6404
	$T = 200$	0.6338	0.7866	0.9838	1.0000
	$T = 500$	1.0000	1.0000	1.0000	1.0000
		Model 3			
λ		$N = 15$	$N = 25$	$N = 50$	$N = 100$
	$T = 50$	0.0144	0.0073	0.0169	0.0071
0.01	$T = 100$	0.0175	0.0384	0.0504	0.0469
	$T = 200$	0.4344	0.8709	0.9614	0.9939
	$T = 500$	1.0000	1.0000	1.0000	1.0000
	$T = 50$	0.0142	0.0180	0.0080	0.0086
1	$T = 100$	0.1418	0.0296	0.5846	0.5520
	$T = 200$	0.6780	0.7668	0.9520	0.9936
	$T = 500$	0.9816	0.9966	1.0000	1.0000

3.6 Conclusion

This chapter extends the panel data stationarity test of Hadri (2000) to the case where a structural break is taken into account. Four models with different break effects under the null hypothesis are specified. We derived analytically the asymptotic moments of our proposed statistics for all the models via the characteristic functions. We also provide the moments for a modified statistic whose limiting distribution is independent of the break location. The situation where the break location has to be estimated is also investigated. CBL have shown that for Model 0 and 3, we can allow for the number of breaks and their positions to differ across individuals for models with known and unknown breaks; their results can easily be extended to the modified statistic. For Model 1 and 2 it is not possible to allow for more than one break. The asymptotic distributions of all the statistics proposed are derived under the null hypothesis and are shown to be normally distributed. Finally, we show by simulations that our suggested LM test has good performance in finite samples with both *i.i.d* and autocorrelated errors, but we find that the performance of the modified test in the presence of autocorrelated errors is less satisfactory.

Chapter 4

Application to Nelson and Plosser Data for OECD Countries

The objective of this chapter is to provide an empirical application of the LM test described in Chapter 3. For this purpose, we use data of 14 macroeconomic and financial variables observed for the OECD countries since 50's. These variables are the same as those considered by Nelson and Plosser (1982) for the U.S. We propose to use Schwarz Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC) to select the type of break for all the variables. We also show how to correct for the cross-sectional dependence by using the Bootstrap procedure. Our results clearly support that the null of stationarity with structural break is not rejected for all the variables. The main results of this chapter of the thesis appear in Hadri and Rao (2006b).

4.1 Introduction

Since the seminal work of Nelson and Plosser (1982), 14 macroeconomic and financial variables data have been analyzed in various studies, particularly in relation to testing the potential existence of unit roots. Nelson and Plosser (1982) applied the ADF test and were unable to reject the unit root hypothesis for 13 out of the 14 variables examined. They concluded that these series behave more like random walks than like transitory deviations from a steadily growing trend. Kwiatkowski *et al.* (1992) developed a test that reverses the null and the alternative hypotheses. The application of their test to the Nelson and Plosser data resulted in nonrejection of the stationary null hypothesis for 6 out of the 14 series. Perron (1989) questioned the ability of unit root tests to distinguish between unit root and stationary processes that contain segmented or shifted trends. He was the first to show that not taking a break into account causes a significant distortion in the unit root tests. He found, when allowing for a break, that the unit root null could be rejected in 10 out of 13 cases for the Nelson and Plosser (1982) data. Perron (1989) assumed the break date to be known. Authors like Christiano (1992), Banerjee *et al.*(1992), and Zivot and Andrews (1992) argued that it is more appropriate to allow for the break point to be endogenously determined. Zivot and Andrews (1992), applying an extension of the Perron model in which the break point is endogenously determined, rejected the unit root null at the 5% significance level for only 4 out of 13 series. Sen (2003a) applied unit root test to the mixed break model which allowed for a simultaneous break in both the intercept and slope of the trend function, and found the evidence for the original Nelson–Plosser series that the unit root null is rejected for all series except the GNP deflator, consumer prices, velocity and the interest rate. Followed this, Sen (2004) uses the extended Nelson-Plosser data which was first analyzed by Schotman and van Dijk (1991) to test the presence of a unit root in each time series. He also used the mixed break model and found that the unit root null is rejected for 9 out of 14 variables including real GNP, nominal GNP, real per capita GNP, industrial production, employment, GNP deflator, nominal wages, interest rate and common stock prices. These results are less evident against the unit root hypothesis compared to those using the original Nelson and Plosser data in Sen (2003a). The test results also suggested that the slope break should be included in above 9 variables except industrial production, employment and GNP deflator. Chang *et al.* (2001) applied a bootstrap procedure for the covariates augmented Dickey-Fuller (CADF) unit root test to the 14 macroeconomic time series in the extended Nelson-Plosser data set for the post-1929 samples. The results obtained in this study showed that the unit root null hypothesis is rejected for all the series in the data set.

It is often argued that single time series unit root tests have low power with short spans of data and therefore failure to reject the unit root null should be treated with caution. One response to this criticism has been the development of panel unit root and stationarity tests, such as Levin, Lin, and Chu (2002); Im, Pesaran and Shin (2003); Maddala and Wu (1999); Hadri (2000); Hadri and Larsson (2005). They demonstrated that even for relatively short panels the power of the tests can be greatly improved. Hurlin (2004) applies the first three of the aforementioned panel unit root techniques to the data of 14 macroeconomic and financial variables observed for OECD countries. These variables are the same as those considered by Nelson and Plosser (1982) for the United States except that GDP related variables rather than GNP related are used. Hurlin (2004) was unable to reject the null hypothesis of unit root for most of the variables. Rapach (2002) applied four different panel unit root tests and found strong evidence for the nonstationarity of real GDP and real GDP per capita. Different from this result, Hegwood and Papell (2006) used both univariate and panel methods to the same data set and concluded that there is strong evidence against the unit root null in favor of stationarity with structural changes in either the slope or in both the intercept and the slope of both the variables.

This chapter contributes to the existing unit root literature by applying a panel stationarity test incorporating structural break and cross-sectional dependence, to a set of Nelson-Plosser-type data. Perron (1989) used visual inspection to decide which break model to adopt. For 11 variables he considered that the "crash model" (corresponds to Model 1 in Chapter 3) is the appropriate model and only common stock prices and real wages are specified by the most general model. Following Perron's (1989) lead, most papers, including Zivot and Andrews (1992), Lumsdaine and Papell (1997) and Sen (2003a) tended to accept this classification. However, Montañés *et al.* (2005) argued that although some of these strategies may well lead us to a proper inference, it can nevertheless be argued that the selection of the most general model might not lead to such an inference, or even to an accurate estimation of the time of the break. Therefore, there is a need to find the most appropriate type of break to be used in the sense of being congruent to the data. In this chapter, we follow Montañés *et al.* (2005) and use AIC and BIC criteria to choose the appropriate break type. We found that the null of stationarity with a break cannot be rejected for 4 out of 14 variables when cross-sectional dependence is not accounted for. We then correct for cross-sectional dependence using the bootstrap method. In this last case, our results indicate the nonrejection of the null of stationarity for all the variables. We also conduct a small Monte Carlo simulation to evaluate the small-sample performance of the bootstrap test. The results suggest that the test is undersized in small samples. Therefore, the empirical inferences based on the bootstrap test have to be treated with caution and need further research.

The rest of the chapter is structured as follows. Section 4.2 gives some descriptions of the data. The test results of the 14 panel variables are presented in Section 4.3. Section 4.4 concludes this chapter.

4.2 Data

To illustrate how to apply the proposed LM panel test in real world[1], we consider the data of 14 macroeconomic and financial series used by Hurlin (2004) which include measures of output spending, money, prices and interest rates. The variables are real GDP, nominal GDP, industrial production, the unemployment rate, GDP deflator, consumer prices, wages, real wages, employment, common stock prices, money stock, velocity, bond yield and real per capita GDP. These are the same series considered in Nelson and Plosser (1982) for the U.S., with the only difference being that GDP (and GDP per capita, real GDP) rather than GNP (and GNP per capita, real GNP) are considered in this chapter due to the data availability. These annual data start from 1952 to 1971 and end from 2000 to 2003. Except for bond yield and common stock prices, the number of countries ranges from 18 to 25. The lists of countries and data sources are presented in Appendix 2. All the series have been transformed to natural logs except bond yield which is analyzed in level form.

4.3 Panel LM Test Results

In this section, we show how to implement the LM test based on the following main steps.

[1]Since it has been shown that the modified test does not perform well in finite samples when the model residuals are autocorrelated, we only discuss the LM test here.

4.3.1 Selecting Break Type

For each variable, the first step is to find which model to use. Perron (1989) used mainly visual inspection to choose a model. Subsequent authors followed his lead. In this chapter, we follow the idea of Montañés *et al.* (2005) and use the Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (BIC) to find the appropriate break type model for the series. The criteria are given by

$$BIC_{i,k} = \ln \tilde{\sigma}^2_{i,k} + r_{i,k}\frac{\ln T}{T},$$

$$AIC_{i,k} = \ln \tilde{\sigma}^2_{i,k} + \frac{2r_{i,k}}{T},$$

where $\tilde{\sigma}^2_{i,k} = SSR_{i,k}/T$, with $SSR_{i,k}$ being the sum of squared residuals of the ith individual and the kth model. $r_{i,k}$ is the number of regressors used to model the ith individual and k indicates the break model used. T is the sample size. It is noted that in order to compute the values of $BIC_{i,k}$ and $AIC_{i,k}$, we need firstly to estimate break locations for each unit and for each model, the break date $\hat{T}_{B,i,k}$, $i = 1, ..., N$ and $k = 0, 1, 2, 3$, by minimizing SSR, as described in Chapter 3. We then, for each unit, choose the break type model which minimizes the $BIC_{i,k}$ and $AIC_{i,k}$ for the four models[2]. Then the model with the lowest value of $BIC_{i,k}$ and $AIC_{i,k}$ is preferred. Hence, we allow for the break dates and the models to be different across countries. The results obtained show that for each variable different break models are chosen

[2]We also compute the $AIC_{i,k}$ and $BIC_{i,k}$ values for another two models, i.e, the level and time trend models proposed in Hadri (2000) without any break. This can allow for the possibilities that it might be the case that no break exists at all in some of the series. In our data, only for the variable of real GDP per capita in the country of U.S., the time trend model has been selected as being the most suitable by BIC. Others all tend to choose the model with break.

within each panel, except for the wages variable, in which the only model selected is Model 2 across all the countries. For the variables GDP deflator, consumer prices, wages and money stock, both selection criteria select consistent models for all the countries, while for the other variables, the model selected for some countries differ between using BIC and AIC. Table 4.1 gives, in column 2 and 6, the index of selected model using BIC and AIC criteria respectively for the variable of velocity in each country. The corresponding estimated break dates are presented in column 3 of the Table. The model selection results for other variables are presented in Appendix 3. It seems that the break type selection carried out in Perron's (1989) is not supported by our data using either of the two criteria for most of the countries. The estimated break points vary between 1970's and 1990's. Since each country can correspond differently to the similar shocks, the breaks could be caused by the oil price shocks that occurred during the 1970's.

4.3.2 Correcting for Serial Correlations

The following step is to correct for possible serial correlation via the estimation and use of the long run variance. This has been described in Chapter 3. First we specify an $AR(p)$ autoregressive process for the prewhitening:

$$\hat{\epsilon}_{it} = \rho_{i,1}\hat{\epsilon}_{it-1} + \rho_{i,2}\hat{\epsilon}_{it-2} + \ldots.. + \rho_{i,p_i}\hat{\epsilon}_{it-p_i} + v_{it}, \tag{4.1}$$

where $\hat{\epsilon}_{it}$ are obtained after estimation of the chosen break type model. Then, the long-run variance estimate of $\widehat{\sigma}^2_{\epsilon i}$ is obtained as in Sul *et al.* (2005) together with

Table 4.1: Model Selection Results for Velocity

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	2	1983	0	1	2	1983	0	1
Austria	2	1987	3	1	2	1987	3	1
Belgium	1	1987	1	1	1	1987	1	1
Canada	3	1990	4	1	3	1990	4	1
Switzerland	1	1971	2	2	1	1971	2	2
Spain	3	1977	1	3	3	1977	1	3
Finland	0	1985	1	1	0	1985	1	1
U.K.	1	1976	1	1	1	1976	1	1
Greece	2	1990	1	4	2	1990	1	4
Ireland	3	1990	1	1	3	1990	1	1
Iceland	2	1977	3	1	3	1979	1	1
Japan	1	1992	3	4	3	1987	3	4
Netherlands	3	1983	4	1	3	1983	4	1
Norway	0	1987	2	2	0	1987	2	2
New Zealand	2	1978	3	3	2	1978	3	3
Portugal	0	1971	4	4	0	1971	4	4
Sweden	1	1985	3	2	1	1985	4	2
US	1	1993	1	1	1	1993	1	1

the boundary condition rule in order to avoid inconsistent estimation of the long run variance $\widehat{\sigma}^2_{\epsilon i}$:

$$\widehat{\sigma}^2_{\epsilon i} = \min\{T\widehat{\sigma}^2_{vi}, \frac{\widehat{\sigma}^2_{vi}}{(1-\widehat{\rho}_i(1))^2}\},$$

where $\widehat{\rho}_i(1) = \widehat{\rho}_{i,1} + \widehat{\rho}_{i,2} + \cdots + \widehat{\rho}_{i,p_i}$ is the sum of all the autoregressive coefficient estimates from the fitted regression (4.1).

For the determination of the lag length of the autoregression p_i, two methods are applied because the results may be sensitive to the criterion employed. The first is a general to specific recursive procedure where the lag length is determined as the last lag in the autoregression that has a significant t statistic at the 10% significance level. This procedure, denoted as *tsig*, has been proposed by Campbell and Perron (1991). We denote the lag length of the autoregressions resulting from this procedure as *tsig_p*. Starting with an upper bound *pmax* which we prespecify to be 4 considering our short panels, if the last included lag is significant, choose $p = pmax$, if not, reduce p by one until the last lag becomes significant. If no lag is significant, set $p = 0$. We use a two-sided 10% test based on the asymptotic normal distribution to assess the significance of the last lags. The second method is based on the BIC criterion with *BIC_p* is used to indicate the lag length decided by this method. The *pmax* is also set to 4. The autoregressive lag length results obtained by both methods for each country in the velocity panel are shown in column 4, 5, 8 and 9 of Table 4.1. Generally, it seems the *tsig* procedure tends to choose larger lag length than BIC criterion does for most of the countries. Then we proceed to compute the panel LM test statistic for each variable, the statistic values of using BIC and AIC break type selection methods are reported in column 4 and 7 of Table 4.2 and 4.3. For each table, Panel I shows the results obtained from using BIC lag length BIC_p, while Panel II reports the results with *tsig* lag length $tsig\_p$. It is found that with lag length BIC_p, the null of stationarity with a break is rejected for 10 out of 14 variables using either the BIC or AIC criterion in selecting the model. The exceptions are the variables of real GDP, industrial production, consumer prices and real wages, in that the null hypothesis cannot be rejected at 5% significance level. In contrast, when the lag length is decided by *tsig*, the null of stationarity with a break is rejected for all the variables if the model is selected by BIC and we can reject the null for 13 out of 14 variables when AIC is employed.

4.3.3 Controlling for Cross-Sectional Dependence

It is widely known that for many macroeconomic and financial variables, it is inappropriate to assume that the cross section units are independent due to the existence of strong inter-economy linkages. O'Connell (1998) showed that the pooled tests will over reject the null hypothesis when the cross-sectional independence is violated, whether the null hypothesis is a unit root or stationarity. Banerjee, Marcellino and Osbat (2001, 2004) argued against the use of panel unit root tests because of potential cross-country cointegration realationships. Therefore, it is imperative in applications involving panels to account for the possibility of cross-sectional dependence. This

Table 4.2: **Panel LM Test Results with Models Selected by BIC**

Series	N	T	Panel I. BIC_p statistic value	Bootstrap critical values		Panel II. $tsig\_p$ statistic value	Bootstrap critical values	
				10%	5%		10%	5%
Real GDP	25	41	0.8069	9.2775	10.2960	4.4951	14.5165	15.6496
Nominal GDP	25	41	14.5507	18.9118	20.2010	17.0089	22.9950	24.7330
Industrial production	24	43	0.7675	7.4556	8.2684	3.8926	11.9057	12.8734
Unemployment rate	23	39	3.4007	8.3987	9.2509	6.1478	11.2573	12.1278
GDP deflator	24	41	9.9205	29.9090	35.0720	11.4330	32.2021	37.0624
consumer prices	21	52	1.5181	4.8312	6.0229	1.6744	8.2853	10.1732
Wages	20	33	9.2450	19.2640	21.0339	10.5856	21.2065	22.8012
Real wages	20	33	1.4054	8.3639	9.3098	4.6552	12.2300	13.2460
Employment	23	39	1.8249	9.6469	10.5817	4.5749	13.7457	14.7699
Common stock prices	11	36	5.1937	20.3801	22.9987	6.8292	24.9544	27.9798
Money stock	19	30	11.1681	24.1045	26.5873	13.5167	26.9725	29.2902
Velocity	18	30	1.9005	8.7801	9.9989	2.9948	11.4211	12.6881
Bond yield	13	47	1.8970	7.2038	8.4475	4.5140	11.7152	12.7329
Real per capita GDP	25	36	1.7287	10.5462	11.4832	7.2665	16.2731	17.6862

Table 4.3: **Panel LM Test Results with Models Selected by AIC**

Series	N	T	Panel I. BIC_p statistic value	Panel I. Bootstrap critical values 10%	Panel I. Bootstrap critical values 5%	Panel II. $tsig\_p$ statistic value	Panel II. Bootstrap critical values 10%	Panel II. Bootstrap critical values 5%
Real GDP	25	41	1.5187	9.7414	10.8979	4.9803	15.0324	16.2387
Nominal GDP	25	41	12.1994	17.9656	19.4199	14.9900	22.4700	24.0810
Industrial production	24	43	1.0405	7.8332	8.7389	4.2167	12.6047	13.5534
Unemployment rate	23	39	4.1070	10.3925	11.4359	6.8598	13.8238	14.9521
GDP deflator	24	41	9.4189	29.2424	34.8569	10.4712	31.7382	36.8721
consumer prices	21	52	1.5209	3.8635	4.8692	1.5834	5.6669	6.7166
Wages	20	33	9.2453	19.2640	21.0339	10.5856	21.2065	22.8013
Real wages	20	33	1.0420	8.1785	9.1274	3.8208	11.9130	12.9689
Employment	23	39	1.9754	9.5741	10.4725	4.5077	13.4516	14.4826
Common stock prices	11	36	6.3912	22.6963	25.4480	8.4423	27.7762	31.0812
Money stock	19	30	11.1682	24.1045	26.5873	13.5167	26.9710	29.2901
Velocity	18	30	2.1861	9.2025	10.4863	2.9012	11.5617	12.8206
Bond yield	13	47	1.7859	7.2001	8.5132	4.4913	11.9764	13.0924
Real per capita GDP	25	36	1.9212	10.6413	11.6526	7.2929	17.3261	18.6768

correlation is indicated by examining the covariance correlations between the countries' variables. Table 4.4 presents the cross-sectional correlations of the different countries for variable of velocity. It clearly indicates the existence of cross-sectional dependence[3]. It should be noted that the asymptotic distribution of the LM statistics proposed in Hadri and Rao (2006a) holds under the assumption of cross-sectional independence. If this assumption is violated, the panel LM test depends on various nuisance parameters associated with the cross-sectional dependence and therefore the normal limiting distribution result is no longer valid.

To account for the possible presence of cross-sectional dependence, we use the bootstrap approach, which makes inference viable even under very general forms of cross-sectional dependence. The bootstrap method used here proceeds as follows:

1. Compute the residuals $\widehat{\epsilon}_{it}s$ from the regression of the appropriate break model.

2. Obtain the $\widehat{v}'_{it}s$ from (4.1) which are grouped in a $N \times T$ matrix. This is to correct for the serial correlation. After this, the $\widehat{v}'_{it}s$ are not serially correlated over time but are potentially cross-sectionally dependent. We let

$$\hat{v} = \begin{bmatrix} \hat{v}_{11} & \cdots & \hat{v}_{1t} & \cdots & \hat{v}_{1T} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{v}_{i1} & \cdots & \hat{v}_{it} & \cdots & \hat{v}_{iT} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{v}_{N1} & \cdots & \hat{v}_{Nt} & \cdots & \hat{v}_{NT} \end{bmatrix}$$

[3]The cross-sectional correlations for other variables are available upon request.

Table 4.4: Cross-Sectional Correlation of the Disturbances in Velocity

	AUS	AUT	BEL	CAN	SWI	SPA	FIN	U.K	GRE	IRE	ICE	JPN	NET	NOR	NZL	PTG	SWE	U.S.
AUS	1																	
AUT	-0.08	1																
BEL	0.34	-0.35	1															
CAN	0.12	0.27	0.14	1														
SWI	-0.33	0.18	-0.04	0.13	1													
SPA	0.30	-0.20	-0.02	0.21	-0.25	1												
FIN	0.19	0.17	0.05	0.47	-0.22	0.42	1											
U.K	0.26	-0.21	0.02	-0.06	-0.10	0.54	0.14	1										
GRE	-0.31	0.26	-0.13	-0.15	0.11	0.25	-0.06	0.25	1									
IRE	0.04	-0.03	0.10	0.07	0.11	-0.31	-0.01	-0.47	-0.57	1								
ICE	0.15	-0.21	-0.033	0.08	-0.33	0.67	0.18	0.67	0.29	-0.43	1							
JPN	0.30	-0.13	0.209	0.06	-0.13	0.52	0.21	0.54	0.57	-0.48	0.53	1						
NET	0.36	-0.30	0.294	-0.09	-0.27	0.06	0.01	0.56	-0.25	0.04	0.35	0.05	1					
NOR	0.03	-0.25	0.159	0.33	-0.01	0.20	0.36	0.12	-0.26	-0.12	0.20	-0.08	-0.03	1				
NZL	-0.11	-0.26	0.328	-0.02	0.12	-0.45	-0.26	-0.28	-0.38	0.24	-0.23	-0.35	-0.08	0.25	1			
PTG	-0.11	0.200	-0.246	0.19	-0.17	-0.13	-0.05	-0.53	-0.14	0.22	-0.04	-0.21	-0.47	-0.13	0.21	1		
SWE	-0.07	0.01	0.086	0.09	-0.26	0.013	-0.24	-0.37	0.06	0.20	0.01	0.01	-0.29	-0.24	0.34	0.62	1	
U.S.	-0.02	-0.17	0.352	-0.24	-0.02	0.27	-0.18	0.28	0.46	-0.56	0.42	0.47	0.14	-0.08	-0.14	-0.20	-0.12	1

Notes: AUS=Australia, AUT=Austria, BEL=Belgium, CAN=Canada, SWI=Switzerland, SPA=Spain, FIN=Finland, GRE=Greece, IRE=Ireland, ICE=Iceland, JPN=Japan, NET=Netherlands, NOR=Norway, NZL=New Zealand, PTG=Portugal, SWE=Sweden

After obtaining $\hat{v}$, we compute the matrix of centered residuals

$$[\tilde{v}_{it}] = \left[\hat{v}_{it} - T^{-1}\sum_{t=1}^{T}\hat{v}_{it}\right].$$

3. Generate the bootstrap innovations $\tilde{v}^*$ by resampling from $\tilde{v}$ with replacement. In order to preserve the contemporaneous cross-sectional dependence, we randomly select T columns from $\hat{v}$ with replacement. (See Maddala and Wu (1999)). Note that in order for the columns to be interchangeable, it is important that the $v_{it}'s$ are not autocorrelated over time.

4. Generate $[\hat{\epsilon}_{it}^*]$ recursively from $[\tilde{v}_t^*]$ as

$$\hat{\epsilon}_{it}^* = \hat{\rho}_{i,1}\hat{\epsilon}_{it-1}^* + \hat{\rho}_{i,2}\hat{\epsilon}_{it-2}^* + \ldots.. + \hat{\rho}_{i,p}\hat{\epsilon}_{it-p}^* + \tilde{v}_{it}^*. \tag{4.2}$$

where $(\hat{\rho}_{i,1}, \hat{\rho}_{i,2}, \ldots\hat{\rho}_{i,p})$ are the coefficients estimates from the fitted regression (4.1). It is necessary to initialize (4.2) to obtain bootstrap samples for $[\hat{\epsilon}_{it}^*]$. Although this is unimportant asymptotically, it may affect the finite sample performance of the bootstrap. We follow Chang (2004) by choosing zeros for initial values then generate $T + 100$ values for $[\hat{\epsilon}_{it}^*]$ and discard the first 100 values. This can minimizes the effects of initial values. Finally, obtain y_{it}^* by adding $\hat{\epsilon}_{it}^*$ to the deterministic term with selected break date $\hat{T}_{B,i}$ in (3.2) to (3.4).

5. Compute the standardized panel statistic Z^* for each bootstrap sample $[y_{it}^*]$ using 5,000 replications in total. This allows us to obtain the empirical distribution of the statistic and hence the bootstrap critical values.

The bootstrap critical values at 10% and 5% significance level are reported in related columns of Table 4.2 and 4.3. We now examine the results in the presence of cross-sectional dependence using bootstrap critical values. The strong evidence is found that the null of stationarity with a break is rejected for all the variables when either BIC or AIC criterion is used to select the break type and either procedure to decide the autoregressive lag length.

4.3.4 Results

It is concluded that if we do not control for the cross-sectional dependence, evidence implying the rejection of stationarity for the variables of nominal GDP, GDP deflator, consumer prices, employment, common stock prices, velocity, bond yield and real GDP per capita. is consistent with the findings reported in, for example, Hurlin (2004) who used the same panel data set but did not allow for possible structural breaks in all the models. However, when the cross-sectional dependence is considered, our results overwhelmingly indicate that all the variables can be well described as stationary with structural breaks. This provides consistent but stronger stationarity evidence than those of Sen (2003a) and Sen (2004) which use the original time series Nelson-Plosser data and the extended Nelson-Plosser data respectively. In contrast, our results differ from those of Hurlin (2004), in which he found a general nonrejection of a unit root for most of the variables when the cross-sectional dependence is taken into account. Our results therefore illustrate the importance of incorporating structural change in panel unit roots and stationarity tests and accounting for cross-sectional dependence where it exists.

4.3.5 Size of the bootstrap test

The bootstrap methods have been used widely with the attempt to control for the cross-sectional dependence with more general form, for example, in Maddala and Wu (1999) and Chang (2004). Although bootstrapping often provides better finite sample critical values for test statistics than first-order asymptotic theory does, bootstrap values are still approximations and are not exact. To assess the finite sample performance of the bootstrap test that we proposed, we perform a small set Monte Carlo experiment based on the DGP as follows:

$$y_{it} = x'_{it} z_i + \epsilon_{it},$$

$$\epsilon_{it} = \beta_i \epsilon_{it-1} + v_{it}.$$

In generating the panel series y_{it}, we include both intercept and time trend as well as structral breaks in the deterministic term x_{it}. The disturbances ϵ_{it} is assumed to follow a stationary AR(1) process where $\beta_i \sim U[0.1, 0.9]$. We also allow for cross-sectional dependence of general form in v_{it}, which is generated by

$$v_{it} = P\varepsilon_t, \text{ where } \varepsilon_t \sim N(0_{N\times 1}, I_N).$$

where P is Cholesky decomposition of the cross-sectional correlations matrix Σ. The off diagonal elements in Σ are the pair-wise correlations ρ_{ij}. Therefore, ϵ_{it} contains both serial correlation and cross-sectional dependence. We consider the value of $\rho_{ij} = \{0, 0.1, 0.9\}$ in the simulations.

To analyze the size of the bootstrap tests, we consider the above DGP under the null of stationarity with a break. We use different combinations of sample size, that is, $T = \{30, 50, 100, 500\}, N = \{10, 25, 50\}$. The first two values of T correspond to the range of sample size for the data we used in this chapter while the other two values are used to examine the performance of the bootstrap test in relatively large samples. The first two values of N correspond to the number of coutries inculded in our data sets. The nominal significance level α is set to 0.05.

We present the size of the bootstrap panel test in Table 4.5, based on 1000 replications. In each replication, we follow the bootstrap procedures described in Subsection 4.3.3 and repeat 1000 times. The bootstrap critical values are obtained from the empirical distributions. We found that the result is undersized when the sample is small. However, it seems that the size results improve as the sample size goes larger.

The above size results indicated that the bootstrap method in this case needs to be used with caution when the sample size is small. Based on this bootstrap method, the consequences for the LM test is that one would reject too infrequently the stationarity null hypothesis. Therefore, the empirical inferences drawn using this bootstrap method needs further investigation in future research.

Table 4.5: Size of the Bootstrap Test

	$\rho_{ij} = 0$		
	$N = 10$	$N = 25$	$N = 50$
$T = 30$	0.0050	0.0030	0.0020
$T = 50$	0.0160	0.0130	0.0090
$T = 100$	0.0320	0.0240	0.0180
$T = 500$	0.0480	0.0500	0.0410
	$\rho_{ij} = 0.1$		
	$N = 10$	$N = 25$	$N = 50$
$T = 30$	0.0020	0.0010	0.0010
$T = 50$	0.0190	0.0180	0.0100
$T = 100$	0.0310	0.0250	0.0210
$T = 500$	0.0480	0.0420	0.0360
	$\rho_{ij} = 0.9$		
	$N = 10$	$N = 25$	$N = 50$
$T = 30$	0.0050	0.0021	0.0032
$T = 50$	0.0230	0.0270	0.0170
$T = 100$	0.0360	0.0390	0.0320
$T = 500$	0.0450	0.0450	0.0400

4.4 Conclusion

In this chapter, we applied the LM panel stationarity test which allows for a structural break as proposed in Chapter 3, to 14 macroeconomic and financial variables of OECD countries. We used a model selection procedure based on BIC and AIC criteria to choose the appropriate break type for each individual series. Different break types are allowed across units for each variable panel. To determine the autoregressive lag length, we use both a general to specific recursive procedure ($tsig$) and the BIC criterion. The results suggest that if we do not control for cross-sectional dependence, with lag length BIC_p, the null of stationarity with a break is rejected for 10 out of 14 variables. The exceptions are as follows: real GDP, industrial production, consumer prices and real wages. In contrast, when the lag length is decided by $tsig$, the null of stationarity with a break is rejected for all the variables suggested by BIC and for 13 out of 14 variables when AIC is employed. We then take the cross-sectional dependence into account by using the bootstrap critical values. The results indicated strong evidence that the null of stationarity with structural break is not rejected for all the variables. However, it is pointed out that the evidence based on the bootstrap method needs further investigation since the bootstrap test does not seem to perform well when the sample size is small.

Chapter 5

Assessing the Bias Correction Factors in Panel Cointegration Tests

In this chapter we are interested in extending the panel stationarity test of HLM (2005) to a panel cointegration test. The test is based on the null hypothesis of cointegration against the alternative of no cointegration. As simulation results suggest that the test is biased in finite samples, we propose a bias correction factors. We also show that the bias correction factors is valid in the models containing $I(1)$ regressors. Finally, another Monte Carlo experiment is conducted to investigate the finite sample properties of the corrected test. The main results of this chapter of the thesis appear in McCabe and Rao (2007).

5.1 Introduction

In recent years, a considerable amount of attention has been paid in the literature to testing for the presence of cointegrating relationships among integrated variables in panel data. Recent surveys of the growing literature on nonstationary panel models include Banerjee (1999), Baltagi and Kao (2000), Phillips and Moon (2000), and more recently Breitung and Pesaran (2006). The literature concerned with the development of panel cointegration tests has taken two broad directions. The first one is to take as the null hypothesis that of no cointegration. This is based on the cross-sectional average of time series cointegration test statistics, including DF and ADF tests (see Kao (1999) and Pedroni (2004)). Tests within this category are almost exclusively based on the methodology of Engle and Granger (1987) whereby a statistic is employed to test for the existence of a unit root in the residuals of a static spurious regression. The second approach consists of taking as the null hypothesis that of cointegration. This is the basis of the panel cointegration tests proposed by McCoskey and Kao (1998). As the primary interest usually lies in the cointegration hypothesis, it is often argued that the null of cointegration rather than the null of no cointegration would be more appealing in empirical applications where cointegration is predicted *a priori* by economic theory.

Existing panel cointegration analysis often assumes cross-sectional independence across units in the panel. This assumption is convenient since it allows the application of the Central Limit Theorem over cross sections in order to achieve standard normality for the limiting distribution of the relevant statistics. However, this assumption is unlikely to be fulfilled by economic variables, which exhibit strong linkages due to the inter-connected nature of the global economy. To overcome this limitation, the second generation statistics attempt to control for cross-sectional dependence. A variety of panel unit root and stationarity tests have been developed to allow for different forms of cross-sectional dependence. For example, Chang (2002) suggested that nonlinear instructmental variables estimators should be used to eliminate the effects of cross-sectional dependence when testing for unit roots in panel data. Bai and Ng (2004), Moon and Perron (2004) and Phillips and Sul (2003a) used factor models for nonstationarity data. Harris, Leybourne and McCabe (2005) (HLM thereafter) proposed a new panel-based test of stationarity that allows for arbitrary cross-sectional dependence as well as unknown serial correlation in the panel. However, there are few studies dealing with cross-sectional dependence in panel cointegration tests. Among these, the most widely used approach is that based on the common factor structure, for example, in Bai and Kao (2005), Banerjee and Carrion-i-Silvestre (2006), Westerlund and Edgerton (2006b). It is argued that using the common factor approach to correct for the cross-sectional dependence, one has to make some restrictive assumptions regarding the form of cross-sectional dependence. Other approaches, such as Westerlund (2006b), adopted bootstrap procedure, while Groen and Kleibergen (2003) based their approach on seemingly unrelated regressions (SUR).

To this end, we propose a very simple residual-based test of the null hypothesis of panel cointegration that allows for arbitrary cross-sectional dependence in the panels. This is an extension of HLM (2005) panel stationarity test. The proposed test is shown to be straightforward and easy to implement. It has a limiting normal distribution under the null hypothesis that is free of nuisance parameters and it is robust to heteroskedasticity.

The remainder of the chapter is organized as follows. Section 5.2 give a brief review of the HLM (2005) test. Section 5.3 sets out the model and describes the proposed statistic as a panel cointegration test. The validity of the bias correction factors in the panel cointegration context is also investigated in this section. Section 5.4 conducts Monte Carlo simulations to investigate the finite sample properties of the test. Section 5.5 concludes.

5.2 HLM (2005) Test

HLM (2005) proposed a new panel-based test of stationarity that allows for arbitrary cross-sectional dependence as well as serial correlation. The statistic is in essence the sum of the lag-k sample autocovariances across the panel, suitably studentized, where k is allowed to be a simple increasing function of the time dimension. By controlling k in such a way, they remove the need to explicitly model the time series dynamics of each series in the panel and, at the same time, the studentization automatically robustifies the statistic to the presence of any form of cross-sectional dependence. The statistic is found to have a standard normal limiting null distribution under quite general linear process assumptions. In order to illustrate this, we consider a model given by

$$y_{i,t} = \beta_i' X_{i,t} + u_{i,t}, \tag{5.1}$$

$$u_{i,t} = \phi_i u_{i,t-1} + \varsigma_{i,t},$$

where $X_{i,t}$ denotes deterministic terms which allow for a wide range of deterministic regression functions including constants, linear and polynomial trends, structural breaks and various other models. The disturbance $\varsigma_{i,t}$ follows the stationary linear process assumption of HML (2003) which allows for cross-sectional correlation of any form between the series in the panel. When testing for the null hypothesis of joint stationarity, it is specified that

$$H_0 : |\phi_i| < 1 \text{ for all } i,$$

against the unit root alternative,

$$H_1 : \phi_i = 1 \text{ for at least one } i,$$

the sum of the individual test statistics is considered,

$$\tilde{C}_k = \sum_{i=1}^{N} \tilde{C}_{i,k}, \tag{5.2}$$

where $\tilde{C}_{i,k}$ is the corresponding kth-order autocovariance of $\tilde{z}_{i,t}$,

$$\tilde{C}_{i,k} = (T-k)^{-1/2} \sum_{t=k+1}^{T} \tilde{u}_{i,t}\tilde{u}_{i,t-k}, \tag{5.3}$$

$\tilde{u}_{i,t} = \hat{u}_{i,t}/s_i$ denotes the standardized residuals of $\hat{u}_{i,t}$, which are the OLS estimated residuals from the regression (5.1), s_i is the sample standard deviation of $\hat{u}_{i,t}$. It is shown that $\tilde{C}_k$, when suitably studentized, has an asymptotic standard normal distribution under the null and is consistent under the alternative, that is,

$$\hat{S}_k = \frac{\tilde{C}_k}{\hat{\omega}\{\tilde{a}_{k,t}\}} \Rightarrow N(0,1), \tag{5.4}$$

where $\tilde{C}_k = (T-k)^{-1/2}\sum_{t=k+1}^{T}\tilde{a}_{k,t}$ with the definition of $\tilde{a}_{k,t} = \sum_{i=1}^{N}\tilde{u}_{i,t}\tilde{u}_{i,t-k}$, and $\hat{\omega}^2\{\tilde{a}_{k,t}\}$ is the long-run variance estimator (LRV) of $\{\tilde{a}_{k,t}\}$ given by,

$$\begin{aligned}\hat{\omega}^2\{\tilde{a}_{k,t}\} &= \hat{\gamma}_0\{\tilde{a}_{k,t}\} + 2\sum_{j=1}^{l}(1-\frac{j}{l+1})\hat{\gamma}_j\{\tilde{a}_{k,t}\}, \\ \hat{\gamma}_j\{\tilde{a}_{k,t}\} &= T^{-1}\sum_{t=j+k+1}^{T}\tilde{a}_{k,t}\tilde{a}_{k,t-j}.\end{aligned} \tag{5.5}$$

Since $\hat{S}_k$ depends on $\tilde{u}_{i,t}$ and $\tilde{u}_{i,t-k}$ only through the cross-sectional sum, that is, $\tilde{a}_{k,t} = \sum_{i=1}^{N}\tilde{u}_{i,t}\tilde{u}_{i,t-k}$, any valid estimate of the LRV of $\{\tilde{a}_{k,t}\}$ will automatically correct for any pattern of cross-sectional dependence in $\tilde{u}_{i,t}$. It is shown that when the panel dimension is not relatively small, individual finite-sample errors that arise from the estimation of the regression models combine in the construction of (5.2), generating significant effect on the finite-sample null distribution of $\hat{S}_k$. To correct for this problem, a bias correction term has been proposed as follows. For a given i,

$$\tilde{c}_i = tr[(T^{-1}\sum_{t=1}^{T}X_{i,t}X_{i,t}')^{-1}\hat{\Omega}\{X_{i,t}\tilde{u}_{i,t}\}], \tag{5.6}$$

where

$$\hat{\Omega}\{X_{i,t}\tilde{u}_{i,t}\} = \hat{\Gamma}_0\{X_{i,t}\tilde{u}_{i,t}\} + \sum_{j=1}^{l}(1-\frac{j}{l+1})(\hat{\Gamma}_j\{X_{i,t}\tilde{u}_{i,t}\} + \hat{\Gamma}_j\{X_{i,t}\tilde{u}_{i,t}\}'). \tag{5.7}$$

and

$$\hat{\Gamma}_j\{X_{i,t}\tilde{u}_{i,t}\} = T^{-1}\sum_{t=j+1}^{T}(X_{i,t}\tilde{u}_{i,t})(X_{i,t-j}\tilde{u}_{i,t-j})'.$$

Therefore, the adjusted version of statistic is

$$\tilde{S}_k = \frac{\tilde{C}_k + \tilde{c}}{\hat{\omega}\{\tilde{a}_{k,t}\}}, \tag{5.8}$$

where $\tilde{c} = (T-k)^{-1/2}\sum_{i=1}^{N}\tilde{c}_i$, $\tilde{c}_i$ is given by (5.6), and $\tilde{C}_k = (T-k)^{-1/2}\sum_{t=k+1}^{T}\tilde{a}_{k,t}$. The null hypothesis of stationarity is rejected if the value of $\tilde{S}_k$ is greater than the upper-tail critical value of the standard normal distribution.

5.3 Panel Cointegration Test

In this section, we extend the HLM (2005) test and propose a statistic that provides a panel cointegration test dealing with possible cross-sectional dependence of an unknown structure.

5.3.1 Models

Consider a panel series $\{y_{i,t}\}$, which is observable for $i = 1, ..., N$ cross-sectional and $t = 1, ..., T$ time series observations. The data generating process (DGP) for $y_{i,t}$ is given by

$$y_{i,t} = \beta_i' X_{i,t} + u_{i,t}, \tag{5.9}$$

$$u_{i,t} = \phi_i u_{i,t-1} + \varsigma_{i,t}, \tag{5.10}$$

where $X_{i,t} = (z_{i,t}', x_{i,t}')'$ is a vector of regressors and $\beta_i = (\gamma_i, \delta_i)'$ is a conformable vector of parameters. The vector $x_{i,t} = x_{i,t-1} + v_{i,t}$ is a $k-$ dimensional vector of $I(1)$ processes, $v_{i,t} \sim i.i.d\ N(0,1)$ and is generated independently from $u_{i,t}$, z_{it} is a vector of deterministic components with $z_{i,t} = (1, t)'$, so both individual-specific constants and time trends are included in the model. The vectors γ_i and δ_i are conformable with respect to $z_{i,t}$ and $x_{i,t}$. Depending on the component of integrated regressors, we distinguish between three models in this chapter. Model 1 refers to the case where regressor $x_{i,t}$ following an independently generated $I(1)$ process. Model 2 arises in the case where $X_{i,t} = (z'_{i,t}, x'_{1i,t}, x'_{2i,t})'$, $x'_{1i,t}$ and $x'_{2i,t}$ are cointegrated. In Model 3, $X_{i,t} = (z'_{i,t}, x'_{i,t})'$, since $x_{i,t}$ can be written as $x_{i,t} = (x'_{1,t}, ...x'_{N,t})'$, Model 3 imposes cointegration relationships among $x_{i,t}$ across i.

The regression disturbances $u_{i,t}$ in (5.9) are assumed to be serially correlated and to follow an $AR(1)$ process in (5.10). In addition, the autoregressive coefficients ϕ_i is allowed to differ across i. The disturbance term $\varsigma_{i,t}$ is $I(0)$, $\varsigma_{i,t}$ and $\varsigma_{j,t}$ may be correlated for any i and j. Therefore, we allow for both serial correlation and cross-sectional dependence in the panel. The series $\{u_{i,t}\}$ is stationary when $y_{i,t}$ and $x_{i,t}$ are cointegrated. Thus, testing for the null hypothesis that $\{y_{i,t}\}$ and $\{x_{i,t}\}$ are cointegrated is equivalent to testing the regression residuals $u_{i,t}$ for stationarity using (5.10). So we wish to test

$$H_0 : |\phi_i| < 1 \text{ for all } i = 1, ..., N,$$

against

$$H_1 : \phi_i = 1 \text{ for } i = 1, ..., N_1 \quad \text{and} \quad |\phi_i| < 1 \text{ for } i = N_1 + 1, ..., N,$$

where we require that $N_1/N \rightarrow \lambda$ as $N \rightarrow \infty$, $\lambda \in (0, 1]$. This alternative hypothesis allows ϕ_i to differ across the units, and it is assumed that the fraction of spurious individuals is non-empty.

For a single set of residuals $\hat{u}_{i,t}$, from (5.9), it is proved in Theorem 1 of McCabe *et al.* (2006) that when suitably studentized, the statistic

$$\hat{C}_{i,k} = (T-k)^{-1/2} \sum_{t=k+1}^{T} \hat{u}_{i,t}\hat{u}_{i,t-k}$$

is asymptotically normal under the null hypothesis of cointegration and is consistent under the no cointegration alternative[1]. Therefore, in a panel of N time series $\hat{u}_{i,t}$, the sum of individual test statistics after suitable studentization will still have the property of asymptotic standard normal null distribution. This can be summarized in the following theorem.

Theorem 5.1 *Assume $\varsigma_t = (\varsigma_{1,t},\ldots,\varsigma_{N,t})'$ follows the stationary linear process assumption LP of HML (2003), $X_{i,t}$ defined in (5.9), and $k = O(T^{1/2})$, then as $T \to \infty$ with N fixed: Under H_0,*

$$\begin{aligned} \hat{S}_k &= \frac{\tilde{C}_k}{\hat{\omega}\{\tilde{a}_{k,t}\}} = \frac{\sum_{i=1}^{N} \tilde{C}_{i,k}}{\hat{\omega}\{\sum_{i=1}^{N} \tilde{u}_{i,t}\tilde{u}_{i,t-k}\}} \\ &= \frac{\sum_{i=1}^{N}((T-k)^{-1/2}\sum_{t=k+1}^{T} \tilde{u}_{i,t}\tilde{u}_{i,t-k})}{\hat{\omega}\{\sum_{i=1}^{N} \tilde{u}_{i,t}\tilde{u}_{i,t-k}\}} \Rightarrow N(0,1); \end{aligned}$$

Under H_1, the distribution of $\hat{S}_k$ diverges to $+\infty$. $\tilde{u}_{i,t}$ here is standardized $\hat{u}_{i,t}$.

Therefore, the asymptotic normality result continues to hold when $\hat{S}_k$ is calculated using residuals from regression models with both deterministic and $I(1)$ regressors in our models.

[1]This null of cointegration includes stationary cointegration and also heteroskedastic cointegration. It therefore encompasses our setup as a special case, so the proof is omitted.

The size results for the statistic in the three models are shown in Tables 5.1-5.3. The details of DGP are described in Section 5.4. In each model, we distinguish between three cases according to the deterministic component of the regression (5.1). The cases are:

(a) no deterministic component, $z_{i,t} = \varnothing$;

(b) individual-specific intercept as the deterministic component, $z_{i,t} = 1$;

(c) both intercepts and time trends in the deterministic term, $z_{i,t} = (1, t)'$.

We found that in all three models, the results are strongly undersized in finite samples, although they tend to converge asymptotically with T increases, but with very low rates. We also find that the more complicated the deterministic term, the greater size distortion of the results. A possible solution to the problem is to include the bias correction factors.

Table 5.1: Size of $\hat{S}_k$ for Model 1–without the Bias Correction Factors

	Results with $z_{i,t} = \varnothing$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0060	0.0080	0.0032	0.0046
$T = 100$	0.0200	0.0150	0.0160	0.0122
$T = 150$	0.0258	0.0208	0.0178	0.0172
$T = 300$	0.0312	0.0264	0.0258	0.0204

	Results with $z_{i,t} = 1$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0006	0.0004	0.0004	0.0008
$T = 100$	0.0048	0.0022	0.0020	0.0008
$T = 150$	0.0062	0.0092	0.0042	0.0056
$T = 300$	0.0144	0.0096	0.0118	0.0084

	Results with $z_{i,t} = (1, t)'$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0004	0.0002	0.0006	0.0002
$T = 100$	0.0018	0.0008	0.0006	0.0002
$T = 150$	0.0004	0.0004	0.0016	0.0026
$T = 300$	0.0084	0.0048	0.0078	0.0030

Table 5.2: Size of $\hat{S}_k$ for Model 2–without the Bias Correction Factors

	Results with $z_{i,t} = \varnothing$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0076	0.0068	0.0066	0.0072
$T = 100$	0.0220	0.0138	0.0132	0.0130
$T = 150$	0.0242	0.0252	0.0196	0.0144
$T = 300$	0.0330	0.0262	0.0212	0.0240

	Results with $z_{i,t} = 1$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0014	0.0014	0.0016	0.0008
$T = 100$	0.0074	0.0042	0.0022	0.0012
$T = 150$	0.0090	0.0058	0.0042	0.0040
$T = 300$	0.0180	0.0106	0.0112	0.0090

	Results with $z_{i,t} = (1, t)'$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0014	0.0008	0.0002	0.0002
$T = 100$	0.0032	0.0010	0.0006	0.0004
$T = 150$	0.0020	0.0022	0.0012	0.0014
$T = 300$	0.0118	0.0060	0.0068	0.0034

5.3.2 Bias Correction Factors

After some manipulation, the term $\tilde{C}_{i,k}$ in the numerator of statistic $\hat{S}_k$ which is defined as $\tilde{C}_{i,k} = (T-k)^{-1/2}\sum_{t=k+1}^{T}\tilde{u}_{i,t}\tilde{u}_{i,t-k}$ can be expressed as follows:

$$\begin{aligned}\tilde{C}_{i,k} &= \frac{1}{s_i^2}T^{-1/2}\sum_{t=k+1}^{T}u_{i,t}u_{i,t-k} \\ &\quad -T^{-1/2}[\frac{1}{s_i^2}\sum_{t=1}^{T}u_{i,t}X_{i,t}'\left(\sum_{t=1}^{T}X_{i,t}X_{i,t}'\right)^{-1}\sum_{t=1}^{T}X_{i,t}u_{i,t}] \\ &\quad +o_p(T^{-1/2}).\end{aligned}$$

Table 5.3: Size of $\hat{S}_k$ for Model 3–without the Bias Correction Factors

		Results with $z_{i,t} = \varnothing$			Results with $z_{i,t} = 1$			Results with $z_{i,t} = (1,t)'$		
		$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$
$T = 50$	$N = 10$	0.0072	0.0076	0.0064	0.0010	0.0012	0.0030	0.0012	0.0016	0.0018
	$N = 20$	0.0094	0.0064	0.0054	0.0016	0.0008	0.0012	0.0004	0.0002	0.0004
	$N = 30$	0.0054	0.0076	0.0058	0.0018	0.0020	0.0010	0.0016	0.0010	0.0006
	$N = 40$	0.0034	0.0064	0.0020	0.0002	0.0014	0.0002	0.0000	0.0002	0.0002
$T = 100$	$N = 10$	0.0204	0.0206	0.0176	0.0068	0.0072	0.0040	0.0036	0.0036	0.0018
	$N = 20$	0.0120	0.0156	0.0162	0.0058	0.0032	0.0040	0.0024	0.0006	0.0012
	$N = 30$	0.0166	0.0130	0.0172	0.0024	0.0050	0.0052	0.0006	0.0028	0.0024
	$N = 40$	0.0124	0.0134	0.0112	0.0016	0.0050	0.0018	0.0004	0.0028	0.0008
$T = 150$	$N = 10$	0.0198	0.0216	0.0244	0.0066	0.0070	0.0144	0.0028	0.0018	0.0090
	$N = 20$	0.0168	0.0178	0.0184	0.0036	0.0074	0.0050	0.0008	0.0036	0.0024
	$N = 30$	0.0186	0.0210	0.0142	0.0060	0.0032	0.0046	0.0020	0.0016	0.0024
	$N = 40$	0.0152	0.0206	0.0168	0.0026	0.0048	0.0056	0.0006	0.0006	0.0028
$T = 300$	$N = 10$	0.0284	0.0278	0.0274	0.0096	0.0130	0.0124	0.0032	0.0064	0.0050
	$N = 20$	0.0264	0.0254	0.0270	0.0096	0.0098	0.0140	0.0036	0.0044	0.0090
	$N = 30$	0.0234	0.0246	0.0216	0.0064	0.0128	0.0084	0.0026	0.0086	0.0036
	$N = 40$	0.0208	0.0216	0.0250	0.0074	0.0066	0.0114	0.0020	0.0028	0.0060

It is the term in the square brackets that induces finite sample estimation errors into each individual statistic and therefore amplifies the finite-sample bias effects for the panel statistic. It is clear that this term is also "self scaling"; that is, it is always $O_P(1)$ since the term $\sum_{t=1}^{T} X_{i,t}u_{i,t}$ is scaled by $\left(\sum_{t=1}^{T} X_{i,t}X_{i,t}^{'}\right)^{-1/2}$. The bias can be corrected by adding back an estimate of the term in square brackets, i.e.,

$$\tilde{c}_i = \frac{1}{s_i^2} tr[(T^{-1}\sum_{t=1}^{T} X_{i,t}X_{i,t}^{'})^{-1}\hat{\Omega}\{X_{i,t}\hat{u}_{i,t}\}],$$

where the long-run variance $\hat{\Omega}\{X_{i,t}\hat{u}_{i,t}\}$ is defined the same way as in (5.7) using a Bartlett lag window. Note that the component terms in $\tilde{c}_i$ are not $O_P(1)$ in the case where we have $I(1)$ regressors, in this case $T^{-1}\sum_{t=1}^{T} X_{i,t}X_{i,t}^{'}$ and $\hat{\Omega}\{X_{i,t}\hat{u}_{i,t}\}$ diverge; the scalings shown are those appropriate for the stationary case and as used in HLM (2005). Of course, the components could be rescaled in order to ensure that they are $O_P(1)$ in any particular model but, in any event, the ratio itself is correctly scaled and $O_P(1)$. In fact, $\tilde{c}_i$ will converge in distribution to a constant. This can be confirmed by the experimental evidence shown in Table 5.4.

The bias correction factors in the panel are obtained by

$$\tilde{c} = (T-k)^{-1/2}\sum_{i=1}^{N} \tilde{c}_i,$$

Therefore, in presence of $I(1)$ regressors,

$$\tilde{S}_k = \hat{\omega}\{\tilde{a}_{k,t}\}^{-1} \times (\tilde{C}_k + \sum_{i=1}^{N} T^{-1/2}\tilde{c}_i) = \hat{\omega}\{\tilde{a}_{k,t}\}^{-1} \times \tilde{C}_k + O_p(T^{-1/2}) \Rightarrow N(0,1),$$

so the inclusion of the bias correction factors does not affect the asymptotic normality properties of the statistic under the null distribution.

Table 5.4: Mean and Standard Deviation of the Replication Values in the Bias Correction Factor $\tilde{c}_i$

	$\phi_i = 0$		$\phi_i = 0.6$	
$(\mathrm{T}^{-2}\sum_{t=1}^{T} X_{i,t}X'_{i,t})$	*mean*	*std*	*mean*	*std*
$T = 100$	1.4773036	1.8450115	1.4773036	1.8450115
$T = 300$	0.83719171	0.93143816	0.83719171	0.93143816
$T = 500$	0.68770456	0.81752674	0.68770456	0.81752674
$T = 1000$	0.61873308	0.69817775	0.61873308	0.69817775
$T = 2000$	0.53003425	0.59936319	0.53003425	0.59936319
$T = 5000$	0.50551657	0.59097990	0.50551657	0.59097990
$T = 10000$	0.47855500	0.54112761	0.47855500	0.54112761
$T = 20000$	0.49751576	0.56343655	0.49751576	0.56343655

$T^{-1}\hat{\Omega}\{X_{i,t}\hat{u}_{i,t}\}$	*mean*	*std*	*mean*	*std*
$T = 100$	1.2433938	1.7285694	6.415871	9.23445
$T = 300$	0.78223118	0.98948336	4.3055475	5.5386032
$T = 500$	0.66042056	0.83986539	3.7259723	4.7998032
$T = 1000$	0.59609856	0.72268599	3.4499925	4.2144502
$T = 2000$	0.52206243	0.60804253	3.0779064	3.5957363
$T = 5000$	0.80352738	0.60211083	3.0156032	3.6138115
$T = 10000$	0.47647269	0.55227418	2.8839885	3.3487586
$T = 20000$	0.49352083	0.56143118	3.0147288	3.4313817

$\tilde{\mathbf{c}}_i$	*mean*	*std*	*mean*	*std*
$T = 100$	0.08189	0.4138	4.1232	2.3997
$T = 300$	0.9017	0.3190	4.9233	1.9208
$T = 500$	0.9422	0.2841	5.2762	1.7316
$T = 1000$	0.9515	0.2249	5.4840	1.3928
$T = 2000$	0.9804	0.1909	5.7680	1.1980
$T = 5000$	0.9924	0.1301	5.9388	0.8210
$T = 10000$	0.9921	0.1028	6.0030	0.6490
$T = 20000$	0.9917	0.0804	6.0573	0.5101

5.4 Monte Carlo Simulations

In this section, we report simulation results for all the models proposed to evaluate the finite-sample properties of the panel cointegration test considered in this chapter. We consider $N = \{10, 20, 30, 40\}$ and $T = \{50, 100, 150, 300\}$ for the sample size. The number of replications in all cases is set to 5,000 and the nominal size is set at the 5% level. The random seed is fixed in each simulation. The DGP is as follows:

$$y_{i,t} = \beta_i' X_{i,t} + u_{i,t}, \tag{5.11}$$

$$u_{i,t} = \phi_i u_{i,t-1} + \varsigma_{i,t}.$$

Under the null of cointegration, the autoregressive coefficients ϕ_i is generated uniformly distributed between 0.1 and 0.9. Since we allow for arbitrary cross-sectional dependence in $\varsigma_t = (\varsigma_{1,t},..., \varsigma_{N,t})'$, the DGP for $\varsigma_{i,t}$ that is used in simulations is as follows:

$$\varsigma_t = P\varepsilon_t, \text{ where } \varepsilon_t \sim N(0_{N\times 1}, I_N).$$

where P is Cholesky decomposition of the cross-sectional correlations matrix Σ. The off diagonal elements in Σ are uniformly generated between 0.1 and 0.9. Therefore, $u_{i,t}$ contains both serial correlation and cross-sectional dependence. The residual $\hat{u}_{i,t} = y_{i,t} - \hat{\beta}_i' X_{i,t}$ is estimated by OLS from the regression (5.11). The regressor $X_{i,t} = (z_{i,t}', x_{i,t}')'$ includes both an intercept and time trend in $z_{i,t}$, as well as a integrated variable $x_{i,t}$. For Model 1, $x_{i,t}$ is generated as N independent random walks. In Model 2, both $x_{1i,t}$ and $x_{2i,t}$ are integrated of order one. First, $x_{1i,t}$ is generated as

N random walk processes, then $x_{2i,t}$ is generated as cointegrated with $x_{1i,t}$, with the cointegration vector of $(1, -\alpha_i)'$, where $\alpha_i \sim U(0, 5)$. In Model 3, the cointegration relationships exist in $x_{i,t}$ across units i. We assume there is r cointegration vectors so that rank$= r$. We consider $r = \{0.1N, 0.5N, 0.9N\}$ throughout all the simulations for Model 3. For the choice of k and l, we follow HLM (2005) and use $k = [(3T)^{1/2}]$, $l = [12(T/100)^{1/4}]$. Tables 5.5–5.7 present the size results for all the models using the statistic $\tilde{S}_k$. It is found that the undersize problem has been corrected, and that the size results after correction are now close to the nominal level, especially when T is large but N is small.

Table 5.5: Size of $\tilde{S}_k$ for Model 1

	Results with $z_{i,t} = \varnothing$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.0994	0.1276	0.1288	0.1280
$T = 100$	0.1046	0.1040	0.1084	0.1062
$T = 150$	0.0916	0.0694	0.0844	0.0766
$T = 300$	0.0692	0.0668	0.0594	0.0664

	Results with $z_{i,t} = 1$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.1134	0.1036	0.0932	0.1036
$T = 100$	0.0876	0.0972	0.1002	0.0968
$T = 150$	0.0852	0.0854	0.0862	0.0834
$T = 300$	0.0654	0.0696	0.0648	0.0712

	Results with $z_{i,t} = (1, t)'$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.1252	0.1176	0.1048	0.1152
$T = 100$	0.0752	0.0950	0.0954	0.1018
$T = 150$	0.0940	0.0836	0.0806	0.0886
$T = 300$	0.0690	0.0736	0.0740	0.0720

Table 5.6: Size of $\tilde{S}_k$ for Model 2

	Results with $z_{i,t} = \varnothing$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.1594	0.2162	0.1998	0.2196
$T = 100$	0.1158	0.1346	0.1426	0.1468
$T = 150$	0.0844	0.1186	0.1054	0.1224
$T = 300$	0.0954	0.0844	0.0814	0.0924

	Results with $z_{i,t} = 1$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.1634	0.1678	0.1836	0.1950
$T = 100$	0.1320	0.1428	0.1324	0.1488
$T = 150$	0.1036	0.1138	0.1198	0.1210
$T = 300$	0.0804	0.0822	0.0850	0.0902

	Results with $z_{i,t} = (1,t)'$			
	$N = 10$	$N = 20$	$N = 30$	$N = 40$
$T = 50$	0.1640	0.1434	0.1852	0.1942
$T = 100$	0.1472	0.1452	0.1244	0.1498
$T = 150$	0.1224	0.1112	0.1310	0.1254
$T = 300$	0.0768	0.0830	0.0954	0.0996

For the power of the test, we set $\phi_i = 1$ for $i = 1, ..., \lambda N$ and $\phi_i \sim U[0.1, 0.9]$ for $i = \lambda N + 1, ..., N$, where $\lambda = \{0.1, 0.2, 0.5\}$. Tables 5.8-5.10 report the scaled power for all the models and Table 5.11 also shows the non-scaled power for Model 3 where $\lambda = 1$[2]. As expected, the power grows with T and N, and also increases with the signal to noise ratio λ. In particular, in Model 3, the power decreases with the cointegration rank r. This pattern is more obvious indicated by the results in Table 5.11.

[2]To save space, we only report the power results with the most general deterministic component included, that is, both intercept and time trend. For the other two cases, the results are similar although the power is relatively larger.

Table 5.7: Size of $\tilde{S}_k$ for Model 3

		Results with $z_{i,t} = \varnothing$			Results with $z_{i,t} = 1$			Results with $z_{i,t} = (1,t)'$		
		$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$
$T = 50$	$N = 10$	0.1026	0.1026	0.0924	0.1152	0.1158	0.1120	0.1194	0.1174	0.1164
	$N = 20$	0.1348	0.1382	0.1084	0.1152	0.1244	0.1136	0.1096	0.1164	0.1280
	$N = 30$	0.1140	0.1236	0.1384	0.1306	0.1384	0.1272	0.1468	0.1440	0.1140
	$N = 40$	0.1448	0.1224	0.1272	0.1122	0.1244	0.0972	0.1066	0.1306	0.0878
$T = 100$	$N = 10$	0.0804	0.0762	0.0728	0.0870	0.0964	0.0764	0.0932	0.1076	0.0790
	$N = 20$	0.0794	0.1052	0.0868	0.0944	0.0970	0.0912	0.1006	0.0928	0.0924
	$N = 30$	0.0944	0.0784	0.0892	0.0986	0.0892	0.1006	0.0926	0.1044	0.1128
	$N = 40$	0.0922	0.0814	0.0848	0.0888	0.0968	0.0914	0.0878	0.1074	0.0920
$T = 150$	$N = 10$	0.0678	0.0796	0.0650	0.0752	0.0788	0.0794	0.0804	0.0806	0.0936
	$N = 20$	0.0844	0.0692	0.0754	0.0740	0.0880	0.0786	0.0706	0.1050	0.0828
	$N = 30$	0.0800	0.0884	0.0650	0.0816	0.0862	0.0840	0.0828	0.0834	0.1006
	$N = 40$	0.0864	0.0852	0.0724	0.0806	0.0838	0.0860	0.0764	0.0870	0.0954
$T = 300$	$N = 10$	0.0640	0.0636	0.0608	0.0634	0.0660	0.0596	0.0650	0.0668	0.0628
	$N = 20$	0.0620	0.0644	0.0588	0.0630	0.0662	0.0692	0.0662	0.0654	0.0804
	$N = 30$	0.0692	0.0582	0.0634	0.0652	0.0696	0.0592	0.0654	0.0788	0.0654
	$N = 40$	0.0684	0.0616	0.0640	0.0684	0.0606	0.0704	0.0720	0.0672	0.0760

Table 5.8: Power of $\tilde{S}_k$ for Model 1

		$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.5$
$T = 50$	$N = 10$	0.1064	0.0910	0.0652
	$N = 20$	0.0938	0.0720	0.0308
	$N = 30$	0.0670	0.0566	0.0290
	$N = 40$	0.0886	0.0682	0.0280
$T = 100$	$N = 10$	0.0766	0.0834	0.1004
	$N = 20$	0.1026	0.1000	0.1076
	$N = 30$	0.0912	0.0950	0.1026
	$N = 40$	0.0936	0.1036	0.1108
$T = 150$	$N = 10$	0.1068	0.1406	0.2454
	$N = 20$	0.1546	0.2088	0.3106
	$N = 30$	0.1380	0.1920	0.3244
	$N = 40$	0.1466	0.2044	0.3250
$T = 300$	$N = 10$	0.1998	0.3284	0.6592
	$N = 20$	0.2262	0.4302	0.8026
	$N = 30$	0.2792	0.5048	0.8702
	$N = 40$	0.2538	0.5064	0.8902

Table 5.9: Power of $\tilde{S}_k$ for Model 2

		$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.5$
$T = 50$	$N = 10$	0.1036	0.1128	0.0774
	$N = 20$	0.0838	0.0824	0.0516
	$N = 30$	0.1110	0.0978	0.0670
	$N = 40$	0.1154	0.0964	0.0628
$T = 100$	$N = 10$	0.1532	0.1910	0.3054
	$N = 20$	0.1596	0.2084	0.3542
	$N = 30$	0.1584	0.2226	0.4170
	$N = 40$	0.1654	0.2170	0.4082
$T = 150$	$N = 10$	0.2008	0.3262	0.5842
	$N = 20$	0.2170	0.3236	0.6666
	$N = 30$	0.2420	0.4140	0.7618
	$N = 40$	0.2398	0.4062	0.7714
$T = 300$	$N = 10$	0.2708	0.5326	0.9000
	$N = 20$	0.3436	0.6612	0.9774
	$N = 30$	0.4608	0.8014	0.9920
	$N = 40$	0.4474	0.7854	0.9952

Table 5.10: Power of $\tilde{S}_k$ for Model 3

		$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.5$		
		$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$	$r = 0.1N$	$r = 0.5N$	$r = 0.9N$
$T = 50$	$N = 10$	0.0942	0.1012	0.1010	0.0862	0.0848	0.0850	0.0578	0.0612	0.0566
	$N = 20$	0.0890	0.1042	0.1066	0.0698	0.0854	0.0850	0.0310	0.0346	0.0426
	$N = 30$	0.1236	0.1178	0.0776	0.1032	0.0854	0.0730	0.0392	0.0330	0.0310
	$N = 40$	0.0796	0.1104	0.0734	0.0880	0.0922	0.0600	0.0342	0.0360	0.0324
$T = 100$	$N = 10$	0.1008	0.1184	0.0876	0.1266	0.1252	0.0952	0.1342	0.1274	0.1176
	$N = 20$	0.1074	0.0980	0.1106	0.1072	0.0970	0.1146	0.1132	0.1102	0.1330
	$N = 30$	0.1010	0.1208	0.1190	0.1220	0.1186	0.1216	0.1170	0.1260	0.1292
	$N = 40$	0.0942	0.1198	0.0992	0.1022	0.1218	0.1036	0.1116	0.1172	0.1262
$T = 150$	$N = 10$	0.1126	0.1198	0.1328	0.1476	0.1682	0.1758	0.2352	0.2672	0.2600
	$N = 20$	0.1160	0.1540	0.1326	0.1660	0.2014	0.1748	0.2940	0.3052	0.2878
	$N = 30$	0.1346	0.1274	0.1620	0.1934	0.1778	0.1994	0.3086	0.2918	0.3172
	$N = 40$	0.1290	0.1402	0.1502	0.1942	0.1944	0.2108	0.3252	0.3128	0.2990
$T = 300$	$N = 10$	0.1732	0.1886	0.1970	0.3058	0.3284	0.3474	0.6258	0.6466	0.6792
	$N = 20$	0.2234	0.2290	0.2660	0.4292	0.4206	0.4600	0.7964	0.7870	0.7864
	$N = 30$	0.2422	0.2896	0.2168	0.5178	0.5294	0.4642	0.8752	0.8522	0.7988
	$N = 40$	0.2442	0.2520	0.2988	0.5214	0.4652	0.5282	0.8874	0.8430	0.8482

Table 5.11: Non-Scaled Power of $\tilde{S}_k$ for Model 3 ($\lambda = 1$)

		$r = 0.1N$	$r = 0.5N$	$r = 0.9N$
$T = 50$	$N = 10$	0.0196	0.0168	0.0186
	$N = 20$	0.0106	0.0098	0.0110
	$N = 30$	0.0056	0.0070	0.0078
	$N = 40$	0.0052	0.0050	0.0052
$T = 100$	$N = 10$	0.1160	0.1186	0.1142
	$N = 20$	0.0992	0.1174	0.1198
	$N = 30$	0.0934	0.1004	0.1024
	$N = 40$	0.0884	0.0988	0.1118
$T = 150$	$N = 10$	0.3426	0.3488	0.3400
	$N = 20$	0.4038	0.3970	0.3880
	$N = 30$	0.4228	0.4246	0.4104
	$N = 40$	0.4376	0.4256	0.4192
$T = 300$	$N = 10$	0.8864	0.8586	0.8450
	$N = 20$	0.9544	0.9474	0.9340
	$N = 30$	0.9710	0.9566	0.9480
	$N = 40$	0.9782	0.9656	0.9686

5.5 Conclusion

In this chapter, we propose a residual-based panel cointegration test with the null hypothesis of cointegration against the alternative of no cointegration. It is an extension of panel stationarity test proposed in HLM (2005). This test is advantageous since it can solve the problem of arbitrary cross-sectional dependence in the panel disturbances and also can simultaneously correct for the serial correlation. The limit of theory of this test is based on large T and N fixed in the panel. We consider three models in the chapter. Model 1 contains a regressor which includes one $I(1)$ process as well as intercept and time trend. In Model 2, we include two cointegrated $I(1)$ regressors, and also fit both intercept and time trend in the deterministic term. In Model 3, we regress on intercept, time trend and one $I(1)$ regressor which have cross-unit cointegration across i in the panel. The panel cointegration test statistics are shown to have standard normal distribution under the null and to be consistent under the alternative hypothesis. However, a size distortion problem is found to exist in finite samples due to the individual estimation errors accumulated over N. To correct for the bias, we incorporate the bias correction factors proposed in HLM (2005) into the panel cointegration test. The validity of the bias correction factors when the model contains $I(1)$ regressors is assessed in this chapter. It is shown that the factors does not affect the asymptotic normality properties of the statistic. Therefore, the corrected statistic which includes the bias correction factors can be applied in cases with $I(0)$ and/or $I(1)$ variables. It is a one-sided test and the null is rejected if the value of the statistic is greater than the appropriate upper-tail standard normal critical value. Finally, we evaluate the finite sample properties of the corrected statistic for all three models through Monte Carlo simulations. Our reported results suggest an adequate performance of the test in finite samples.

Chapter 6

Conclusion

This thesis begins in Chapter 2 with a review of the recent econometric literature on the panel unit root, stationarity and cointegration tests.

Chapter 3 extends Hadri (2000) test to the case where a structural break is taken into account by proposing a panel residual-based LM stationarity test allowing for possible structural breaks. We set out four models based on different break patterns. For two of the models, a modified test of which the asymptotic distribution does not depend on break location is also proposed. The tests with both *i.i.d* and serially correlated residuals are discussed and we considered an $AR(1)$ process for the autocorrelated residuals. Both tests have been shown to have standard normal distributions after standardizing using the appropriate moments and applying the Central Limit Theorem. The sequential limits wherein $T \to \infty$ followed by $N \to \infty$ are used to derive the asymptotic distributions. The exact moments of all corresponding models for both tests are derived. In the situation where the break location is unknown, we obtain the estimated break locations by minimizing the sum of squared residuals (SSR) from the relevant model regressions under the null hypothesis. We have shown how one can allow for the positions of breaks to differ across individuals for models with known and unknown breaks. Finally, we showed by simulations that our suggested LM test has satisfactory performance in finite samples with both *i.i.d* and autocorrelated errors, but we found that the performance of the modified test in the presence of autocorrelated errors is less satisfactory.

In Chapter 4, we applied the LM test to the data of 14 macroeconomic and financial variables observed for the OECD countries since 1950's. These variables are the same as those considered in Nelson and Plosser (1982) for the U.S., with the only difference that GDP related variables rather than GNP related ones are considered. Instead of following the usual visual inspection approach, or imposing the most general model specifications, we used a model selection procedure based on the BIC and AIC criteria to choose the appropriate break type for each individual. Different break types are allowed across units in the panel. It is found that different combinations of all the models rather than any single particular model are selected for 13 out of 14 variables. Before correcting for serial correlations, two methods are adopted to determine the autoregressive lag length; that is, a general to specific recursive procedure (*tsig*) and the application of the BIC criterion. If the cross-sectional dependence is not considered, our results suggest that when using the autoregressive lag length suggested by BIC_p, the null of stationarity with a break is not rejected for real GDP, industrial production, consumer prices and real wages, using either BIC or AIC criteria in selecting the model. In contrast, when the lag length is decided by *tsig*, the null of stationarity with a break is rejected for all the variables based on the models suggested by BIC, and the rejection is supported for 13 out of 14 variables when AIC is employed to select models. We then take the cross-sectional dependence into account via the bootstrap method. The results thus obtained strongly indicated that the null of stationarity with a structural break is not rejected for all the variables. This finding, after we allowed for possible structural changes in panel series, provides more convincing evidence of the stationarity than was revealed in the previous studies of Hurlin (2004) and Sen (2003a, 2004). However, we pointed out that the evidence based on the bootstrap method should be treated with caution and needs further research due to the fact that the bootstrap test has size distortions in small samples.

Chapter 5 of this thesis is focused on the panel cointegration study. In this chapter, we propose a residual-based panel cointegration test with the null hypothesis of cointegration against the alternative of no cointegration. This test is an extension of the stationarity test proposed in HLM (2005). This test is advantageous since it can control for cross-sectional dependence as well as for serial correlation with unknown structures. The limiting theory of this test is based on large T and N fixed in the panel. We considered three models in this chapter. Model 1 contains one integrated regressor, plus an intercept and a time trend. In Model 2, we include two cointegrated $I(1)$ variables, plus both an intercept and a time trend in the deterministic term. In Model 3, we consider an intercept, time trend and one $I(1)$ regressor which has cross-unit cointegration across i in the panel. Although the proposed statistic for the panel cointegration test is shown to be standard normally distributed under the null and consistent under the alternative hypothesis, simulation results indicate size distortion in finite sample due to the individual estimation errors accumulated over N and, moreover, that the bias can be severe if the panel dimension is relatively large. To overcome this problem, we applied the bias correction factors proposed by HLM (2005) in the panel cointegration test. The validity of the bias correction factors in the presence of the $I(1)$ regressors is assessed in this chapter. It is shown that the factors do not affect the asymptotic normality properties of the test under the null. Therefore, the corrected statistic can be applied for the cases which contain either $I(0)$ and/or $I(1)$ regressors. It is a one-sided test and the null is rejected if the value of the statistic is greater than the appropriate upper-tail standard normal critical value. Finally, we report simulation results of the corrected statistic for all three models. It is found that the test provides satisfactory finite sample properties.

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Appendices

We present now the appendices that correspond to the previous chapters of the thesis.

Appendix 1: Proofs of Theorems in Chapter 3

Proof. of Theorem **3.3** for Model 0. Under the null hypothesis of stationarity with a break, the characteristic function (c.f) of $G_{i,0}(B)$ for *ith* individual is given in Kurozumi (2002) as

$$\phi_i(\theta) = [D_0(2i\omega_i^2\theta)D_0(2i(1-\omega_i)^2\theta)]^{-1/2},$$

where $D_0(\lambda) = \frac{\sin\sqrt{\lambda}}{\sqrt{\lambda}}$. The cumulants are obtained by the expansion of $\log_k\{\phi_i(\theta)\}$ (*with* $k = 0, 1, 2, 3$ denotes Model 0, 1, 2 and 3 respectively) as power series in $i\theta$. The first, second, third and fourth cumulants give, respectively, the mean, the variance, the skewness and kurtosis of $G_{i,k}(B)$. For model 0 we obtain

$$\begin{aligned}
& \log_0\{\phi_i(\theta)\} \\
= \; & \frac{1}{6}(2\omega_i^2 - 2\omega_i + 1)\frac{i\theta}{1!} \\
& + \frac{1}{45}(1 - 4\omega_i + 6\omega_i^2 - 4\omega_i^3 + 2\omega_i^4)\frac{(i\theta)^2}{2!} \\
& + \frac{8}{945}(2\omega_i^2 - 2\omega_i + 1)(\omega_i^4 - 2\omega_i^3 + 5\omega_i^2 - 4\omega_i + 1)\frac{(i\theta)^3}{3!} \\
& + \frac{8}{1575}\left(1 - 8\omega_i + 28\omega_i^2 - 56\omega_i^3 + 70\omega_i^4 - 56\omega_i^5 + 28\omega_i^6 - 8\omega_i^7 + 2\omega_i^8\right)\frac{(i\theta)^4}{4!} \\
& + O((i\theta)^5).
\end{aligned}$$

Therefore, for Model 0, the mean and variance are:

$$\xi_{i,0} = \frac{1}{6}\left(1 - 2\omega_i + 2\omega_i^2\right),$$

$$\zeta_{i,0}^2 = \frac{1}{45}\left(1 - 4\omega_i + 6\omega_i^2 - 4\omega_i^3 + 2\omega_i^4\right).$$

■

Proof. of Theorem **3.3** for Model 1. In (3.2), the characteristic function(c.f) of $G_{i,1}(B)$ is

$$\phi_i(\theta) = [D_1(2i\theta)]^{-1/2},$$

with

$$\begin{aligned} D_1(\lambda) &= -12\frac{\sqrt{\lambda}\sin\sqrt{\lambda\omega_i^2}\sin\sqrt{\lambda(1-\omega_i)^2}}{\lambda^{5/2}\omega_i(1-\omega_i)\{1-3\omega_i(1-\omega_i)\}} \\ &+\frac{2\left(\sin\sqrt{\lambda}-\sin\sqrt{\lambda\omega_i^2}-\sin\sqrt{\lambda(1-\omega_i)^2}\right)}{\lambda^{5/2}\omega_i(1-\omega_i)\{1-3\omega_i(1-\omega_i)\}}. \end{aligned}$$

So the power series is described as

$$\begin{aligned} \log_1\{\phi_i(\theta)\} &= \frac{1}{30}\times(\frac{15\omega_i^4-30\omega_i^3+25\omega_i^2-10\omega_i+2}{3\omega_i^2-3\omega_i+1})\frac{i\theta}{1!} \\ &+\frac{1}{6300}\times(\frac{315\omega_i^8-1260\omega_i^7+2415\omega_i^6-2835\omega_i^5}{(3\omega_i^2-3\omega_i+1)^2} \\ &\frac{+2275\omega_i^4-1295\omega_i^3+495\omega_i^2-110\omega_i+11}{(3\omega_i^2-3\omega_i+1)^2})\frac{(i\theta)^2}{2!} \\ &+\frac{1}{94500}\times(\frac{1350\omega_i^{12}-8100\omega_i^{11}+24300\omega_i^{10}}{(3\omega_i^2-3\omega_i+1)^3} \\ &\frac{-47250\omega_i^9+66645\omega_i^8-72180\omega_i^7+60775\omega_i^6-39045\omega_i^5}{(3\omega_i^2-3\omega_i+1)^3} \\ &\frac{+18\,570\omega_i^4-6325\omega_i^3+1470\omega_i^2-210\omega_i+14}{(3\omega_i^2-3\omega_i+1)^3})\frac{(i\theta)^3}{3!} \\ &+\frac{1}{24255000}(\frac{509-10\,180\omega_i+96\,710\omega_i^2-578\,150\omega_i^3+2430\,235\omega_i^4}{(3\omega_i^2-3\omega_i+1)^4} \\ &\frac{-7603\,960\omega_i^5+18\,288\,725\omega_i^6-34\,420\,540\omega_i^7+51\,201\,535\omega_i^8}{(3\omega_i^2-3\omega_i+1)^4} \\ &\frac{-60\,668\,300\omega_i^9+57\,782\,725\omega_i^{10}-44\,629\,200\omega_i^{11}+27\,933\,675\omega_i^{12}}{(3\omega_i^2-3\omega_i+1)^4}) \\ &\frac{-13\,825\,350\omega_i^{13}+5093\,550\omega_i^{14}-1247\,400\omega_i^{15}+155\,925\omega_i^{16}}{(3\omega_i^2-3\omega_i+1)^4})\frac{(i\theta)^4}{4!} \end{aligned}$$

$$+O((i\theta)^5).$$

Hence, the mean and variance of $G_{i,1}(B)$ are given by:

$$\xi_{i,1} = \frac{15\omega_i^4 - 30\omega_i^3 + 25\omega_i^2 - 10\omega_i + 2}{30\left(3\omega_i^2 - 3\omega_i + 1\right)},$$

$$\zeta_{i,1}^2 = \frac{315\omega_i^8 - 1260\omega_i^7 + 2415\omega_i^6 - 2835\omega_i^5 + 2275\omega_i^4 - 1295\omega_i^3 + 495\omega_i^2 - 110\omega_i + 11}{6300\left(3\omega_i^2 - 3\omega_i + 1\right)^2}.$$

■

Proof. of Theorem **3.3** for Model 2. Under the null hypothesis of stationarity with a break, the c.f of $G_{i,2}(B)$ is

$$\phi_i(\theta) = [D_2(2i\theta)]^{-1/2},$$

where

$$D_2(\lambda) = \frac{D_I(\lambda) + D_{II}(\lambda) + D_{III}(\lambda)}{\lambda^{7/2}\omega_i^3(1-\omega_i)^3},$$

with

$$D_I(\lambda) = \lambda\omega_i(1-\omega_i)\sin\sqrt{\lambda},$$

$$\begin{aligned} D_{II}(\lambda) &= 2\{\sin\sqrt{\lambda\omega_i^2} + \sin\sqrt{\lambda(1-\omega_i)^2} - \sin\sqrt{\lambda} \\ &\quad -\lambda^{1/2}\left(\omega_i\cos\sqrt{\lambda\omega_i^2} + (1-\omega_i)\cos\sqrt{\lambda(1-\omega_i)^2}\right)\}, \end{aligned}$$

$$D_{III}(\lambda) = \lambda^{1/2}\left(\cos\sqrt{\lambda} + \cos\sqrt{\lambda\omega_i^2}\cos\sqrt{\lambda(1-\omega_i)^2}\right).$$

The expansion as a power series is given by:

$$\begin{aligned}\log_2\{\phi_i(\theta)\} &= (\frac{3\omega_i^2 - 3\omega_i + 2}{30})\frac{i\theta}{1!} \\ &+\frac{1}{6300}(3\omega_i^4 - 6\omega_i^3 + 36\omega_i^2 - 33\omega_i + 11)\frac{(i\theta)^2}{2!} \\ &+\frac{1}{94500}(14 - 6\omega_i^6 + 18\omega_i^5 + 36\omega_i^4 - 102\omega_i^3 \\ &+117\omega_i^2 - 63\omega_i)\frac{(i\theta)^3}{3!} + \\ &\frac{1}{24255000}(509 - 3054\omega_i + 7989\omega_i^2 - 11\,352\omega_i^3 + 9114\omega_i^4 \\ &-3378\omega_i^5 - 120\omega_i^6 + 1068\omega_i^7 - 267\omega_i^8)\frac{(i\theta)^4}{4!} \\ &+O((i\theta)^5).\end{aligned}$$

The mean and variance of $G_{i,2}(B)$ are therefore:

$$\xi_{i,2} = \frac{3\omega_i^2 - 3\omega_i + 2}{30},$$

$$\zeta_{i,2}^2 = \frac{3\omega_i^4 - 6\omega_i^3 + 36\omega_i^2 - 33\omega_i + 11}{6300}.$$

■

Proof. of Theorem **3.3** for Model 3. The characteristic function for Model 3 is given by

$$\phi_i(\theta) = [D_3(2i\omega_i^2\theta)D_3(2i(1-\omega_i)^2\theta)]^{-1/2},$$

where $D_3(\lambda) = \frac{12}{\lambda^2}(2 - \sqrt{\lambda}\sin\sqrt{\lambda} - 2\cos\sqrt{\lambda})$. The Expansion as power series can be written as follows:

$$\begin{aligned}\log_3\{\phi_i(\theta)\} &= \frac{1}{15}(1-2\omega_i+2\omega_i^2)\frac{i\theta}{1!}\\ &+\frac{11}{6300}(1-4\omega_i+6\omega_i^2-4\omega_i^3+2\omega_i^4)\frac{(i\theta)^2}{2!}\\ &+\frac{1}{6750}(1-2\omega_i+2\omega_i^2)(\omega_i^4-2\omega_i^3+5\omega_i^2-4\omega_i+1)\frac{(i\theta)^3}{3!}\\ &+\frac{509}{24255000}(1-8\omega_i+28\omega_i^2-56\omega_i^3+70\omega_i^4\\ &-56\omega_i^5+28\omega_i^6-8\omega_i^7+2\omega_i^8)\frac{(i\theta)^4}{4!}\\ &+O((i\theta)^5).\end{aligned}$$

Hence, the mean and variance of $G_{i,3}(B)$ are

$$\xi_{i,3}=\frac{1}{15}(1-2\omega_i+2\omega_i^2),$$

$$\zeta_{i,3}^2=\frac{11}{6300}\left(1-4\omega_i+6\omega_i^2-4\omega_i^3+2\omega_i^4\right).$$

■

Proof. of Theorem **3.4.** In the modified tests, for both Model 0 and 3, the characteristic function can be expressed as

$$\phi_i(\theta)=[D_{k,M}(2i\theta)]^{-1},$$

where $D_{k,M}(\lambda)s$ are defined as in the Proof of Theorem 3.3 for Model 0 and Model 3 respectively. The expansion of $\log_k(\phi_i(\theta))$ as a power series of $i\theta$ gives

$$\log_0\{\phi_i(\theta)\}=\frac{1}{3}\frac{i\theta}{1!}+\frac{2}{45}\frac{(i\theta)^2}{2!}+\frac{16}{945}\frac{(i\theta)^3}{3!}+\frac{16}{1575}\frac{(i\theta)^4}{4!}+O((i\theta)^5),$$

and

$$\log_3\{\phi_i(\theta)\} = \frac{2}{15}\frac{i\theta}{1!} + \frac{11}{3150}\frac{(i\theta)^2}{2!} + \frac{1}{3375}\frac{(i\theta)^3}{3!} + \frac{509}{12127500}\frac{(i\theta)^4}{4!} + O((i\theta)^5).$$

Therefore, the mean and variance for the Modified Model 0, are $\frac{1}{3}$ and $\frac{2}{45}$, and for the modified Model 3, they are $\frac{2}{15}$ and $\frac{11}{3150}$ respectively. ■

Appendix 2: Data Details in Chapter 4

The data lists for the 14 series are:

1. Real GDP ($T = 41$, $N = 25$). Source: Economic Outlook, OECD. Code: GDPVD (gross domestic product, volume, at 2000 PPP, US$). Base 100 in 2000. The sample is balanced with 25 countries observed over the period 1963-2003. Excluded countries are Hungary, Korea, Czech Republic, Poland and the Slovak Republic.

2. Nominal GDP ($T = 41$, $N = 25$). Source: Economic Outlook, OECD. Code: GDPV (gross domestic product, volume, market prices). Base 100 in 2000. The sample is balanced with 25 countries observed over the period 1963-2003. Excluded countries are Hungary, Korea, Czech Republic, Poland and the Slovak Republic.

3. Real per Capita GDP ($T = 36$, $N = 25$). Source: World Development Indicators, World Bank. Code: NY.GDP.PCAP.KD (gross domestic product per capita, constant 1995 US$). Base 100 in 1995. The sample is balanced with 25 countries observed over the period 1965-2000. Excluded countries are Turkey, Germany, Czech Republic, Poland and the Slovak Republic.

4. Industrial Production ($T = 43$, $N = 24$). Source: International Financial Statistics, IMF, Washington. Code: line 61. Base 100 in 1995. The sample is balanced with 24 countries observed over the period 1960-2002. Excluded countries are Turkey, New Zealand, Czech Republic, Hungary, Poland and the Slovak Republic.

5. Employment ($T = 39$, $N = 23$). Source: Economic Outlook, OECD. Code: ET

(total employment). The sample is balanced with 23 countries observed over the period 1965-2003. Excluded countries are: Luxembourg, Mexico, the Netherlands, the Czech Republic, Hungary, Poland and the Slovak Republic.

6. Unemployment rate ($T = 39$, $N = 23$). Source: Economic Outlook, OECD. Code: UN (unemployment rate). The sample is balanced with 23 countries observed over the period 1965-2003. Excluded countries are: Luxembourg, Mexico, the Netherlands, the Czech Republic, Hungary, Poland and the Slovak Republic.

7. GDP Deflator ($T = 41$, $N = 24$). Source: World Development Indicators, World Bank. Code: NY.GDP.DEFL.ZS. Base 100 in 1995. The sample is balanced with 24 countries observed over the period 1960-2003. Excluded countries are: Canada, Germany, Turkey, Czech Republic, Poland and the Slovak Republic

8. Consumer prices ($T = 52$, $N = 22$). Source: International Financial Statistics, IMF, Washington. Code: line 64. Base 100 in 2000. The sample is balanced with 22 countries observed over the period 1952-2003. Excluded countries are: Germany, Turkey, Mexico, Korea, Czech Republic, Hungary, Poland and the Slovak Republic.

9. Wages ($T = 33$, $N = 20$). Source: Economic Outlook, OECD. Code: WR (wage rate of the business sector). Base 100 in 2000. The sample is balanced with 20 countries observed over the period 1971-2003. Excluded countries are: Switzerland, Czech Republic, Hungary, Korea, Luxembourg , Mexico, Norway, Poland, Turkey and the Slovak Republic.

10. Real Wages ($T = 33$, $N = 20$). Source: Economic Outlook, OECD. Code: WSRE (real compensation rate of the business sector). Base 100 in 2000. The sample is balanced with 20 countries observed over the period 1971-2003. Excluded countries are: Switzerland, Czech Republic, Hungary, Korea, Luxembourg , Mexico, Norway, Poland, Turkey and the Slovak Republic.

11. Money Stock ($T = 30$, $N = 19$). Source: Economic Outlook, OECD. Code: MONEYS (money supply, broad definition, M2 or M3). Base 100 in 1995. The sample is balanced with 20 countries observed over the period 1969-1998. Excluded countries are: Luxembourg, Italy, France, Denmark, Turkey, Mexico, Korea, Czech Republic, Hungary, Poland and the Slovak Republic.

12. Velocity ($T = 30$, $N = 18$). Source: Economic Outlook, OECD. Code: VLCTY (velocity of money). The sample is balanced with 18 countries observed over the period 1969-1998. Excluded countries are: Germany, Luxembourg, Italy, France, Denmark, Turkey, Mexico, Korea, Czech Republic, Hungary, Poland and the Slovak Republic.

13. Bond Yield ($T = 47$, $N = 13$). Source: International Financial Statistics, IMF, Washington. Code: line 61. The sample is balanced with 13 countries observed over the period 1952-2002. Excluded countries are: Portugal, Sweden, Ireland, Austria, Finland, Greece, Iceland, Japan, Luxembourg, Spain, Turkey, Mexico, Korea, Czech Republic, Hungary, Poland and the Slovak Republic.

14. Common stock prices ($T = 36$, $N = 11$). Source: Main Economic Indicators, OECD. Code: share prices. Base 100 in 2000. The sample is balanced with 11 countries observed over the period 1968-2003. Excluded countries are: Belgium, Czech Republic, Denmark, Finland, Greece, Hungary, Iceland, Italy, Korea, Mexico, Netherlands, Norway, Poland, Portugal, Spain, Turkey, Luxembourg, United Kingdom and the Slovak Republic.

Appendix 3: Model Selection Results in Chapter 4

Table 6.1: Model Selection Results for Real GDP

Countries	Model selected by BIC				Model selected by AIC			
	Model	TB	*tsig_p*	*BIC_p*	Model	TB	*tsig_p*	*BIC_p*
Australia	2	1971	2	1	3	1974	2	2
Austria	2	1975	4	1	2	1975	4	1
Belgium	2	1975	1	2	2	1975	1	2
Canada	2	1977	2	2	2	1977	2	2
Switzerland	3	1975	1	3	3	1975	1	3
Germany	3	1991	2	2	3	1991	2	2
Denmark	2	1970	1	2	2	1970	1	2
Spain	3	1978	3	1	3	1978	3	1
Finland	1	1991	1	1	1	1991	1	1
France	2	1975	4	2	2	1975	4	2
U.K.	1	1980	2	2	1	1980	2	2
Greece	2	1974	4	1	2	1974	4	1
Ireland	2	1995	2	1	2	1995	2	1
Iceland	2	1982	2	2	2	1982	2	2
Italy	2	1979	4	1	2	1979	4	1
Japan	2	1974	4	4	2	1974	4	4
Luxembourg	3	1981	1	1	3	1981	1	1
Mexico	3	1983	3	1	3	1983	3	1
Netherlands	2	1972	2	2	2	1992	2	2
Norway	2	1980	3	2	2	1980	3	2
New Zealand	3	1978	1	1	3	1978	1	1
Portugal	2	1974	4	4	2	1974	4	4
Sweden	2	1972	2	2	2	1972	2	2
Turkey	2	1976	1	1	3	1979	1	1
U.S.	2	1967	2	2	2	1967	2	2

Table 6.2: Model Selection Results for Nominal GDP

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	2	1989	2	2	2	1989	2	2
Austria	2	1982	4	2	2	1982	4	2
Belgium	2	1983	3	2	2	1983	3	2
Canada	2	1985	2	2	2	1985	2	2
Switzerland	2	1975	3	2	2	1975	3	2
Germany	2	1979	4	2	2	1979	4	2
Denmark	2	1985	4	4	2	1985	4	4
Spain	2	1987	4	4	2	1987	4	4
Finland	2	1986	2	2	2	1986	2	2
France	2	1987	2	2	2	1987	2	2
U.K.	2	1987	3	3	2	1987	3	3
Greece	2	1995	4	4	2	1995	4	4
Ireland	2	1984	4	4	3	1979	2	1
Iceland	2	1990	4	4	3	1983	1	1
Italy	2	1989	4	4	2	1989	4	4
Japan	2	1981	4	4	2	1981	4	4
Luxembourg	1	1973	3	3	1	1973	3	3
Mexico	1	1986	1	1	1	1986	1	1
Netherlands	2	1978	2	2	3	1978	2	2
Norway	2	1986	4	2	2	1986	4	2
New Zealand	2	1989	4	4	2	1989	4	4
Portugal	2	1993	4	4	2	1993	4	4
Sweden	2	1990	2	2	2	1990	2	2
Turkey	2	1979	3	2	2	1979	3	2
U.S.	2	1987	4	1	2	1987	4	1

Table 6.3: Model Selection Results for Industrial Production

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1975	1	2	3	1975	1	2
Austria	2	1973	2	2	3	1975	2	2
Belgium	3	1975	4	1	3	1975	4	1
Canada	2	1973	2	2	3	1975	2	2
Finland	2	1974	2	2	2	1974	2	2
Denmark	3	1974	1	1	3	1974	1	1
France	2	1974	4	1	2	1974	4	1
Germany	2	1973	2	2	3	1969	2	1
Greece	2	1978	4	1	2	1978	4	1
Iceland	1	1977	2	2	1	1977	2	2
Ireland	2	1993	1	1	2	1993	1	1
Italy	2	1974	4	1	2	1974	4	1
Japan	2	1973	4	1	2	1973	4	1
Luxembourg	1	1975	2	2	1	1975	2	2
Netherlands	2	1974	1	1	2	1974	1	1
Norway	2	1998	2	1	2	1998	2	1
Portugal	2	1983	2	2	2	1983	2	2
Spain	2	1974	4	4	2	1974	4	4
Sweden	2	1971	2	1	3	1976	1	1
Switzerland	3	1975	2	2	3	1975	2	2
U.K.	3	1980	2	2	3	1980	2	1
U.S.	2	1967	2	2	2	1967	2	2
Mexico	3	1983	3	3	3	1983	3	3
Korea	2	1980	1	1	3	1977	1	1

Table 6.4: Model Selection Results for Unemployment Rate

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	2	1983	4	2	2	1984	4	2
Austria	3	1982	4	1	3	1982	4	1
Belgium	2	1976	4	4	2	1984	4	4
Canada	2	1982	2	2	2	1984	2	2
Switzerland	3	1975	3	3	3	1975	3	3
Germany	2	1982	5	5	3	1982	4	4
Denmark	1	1975	1	1	0	1975	1	1
Spain	3	1980	4	1	3	1980	4	1
Finland	3	1992	2	2	3	1992	2	2
France	2	1984	5	5	2	1986	2	4
U.K.	3	1981	5	2	3	1981	2	2
Greece	3	1981	0	1	3	1981	0	1
Ireland	2	1992	3	3	2	1993	3	3
Iceland	1	1992	2	5	0	1989	2	2
Italy	2	1983	2	2	2	1998	2	2
Japan	3	1989	1	1	3	1989	1	1
Korea	3	1998	1	1	3	1998	1	1
Norway	3	1989	4	1	3	1989	4	1
New Zealand	2	1978	5	4	2	1991	4	4
Portugal	1	1976	4	4	1	1976	4	4
Sweden	3	1992	1	1	3	1992	1	1
Turkey	1	1970	3	1	1	1970	3	1
US	2	1984	2	2	3	1984	2	2

Table 6.5: Model Selection Results for GDP Deflator

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	2	1972	2	2	2	1972	2	2
Austria	3	1974	1	1	3	1974	1	1
Belgium	3	1974	1	1	3	1974	1	1
Finland	2	1972	2	2	2	1972	2	2
Denmark	2	1971	2	2	2	1971	2	2
France	3	1981	1	1	3	1981	1	1
Greece	2	1985	3	3	2	1985	3	3
Iceland	2	1980	2	2	2	1980	2	2
Ireland	3	1980	1	1	3	1980	1	1
Italy	2	1976	2	2	2	1976	2	2
Japan	2	1987	4	2	2	1987	4	2
Luxembourg	2	1969	3	1	2	1969	3	1
Netherlands	1	1976	1	1	1	1976	1	1
New Zealand	2	1986	1	1	1	1986	1	1
Norway	2	1972	2	2	2	1972	2	2
Portugal	2	1981	2	2	2	1981	2	2
Spain	2	1976	2	2	2	1976	2	2
Sweden	2	1974	2	2	2	1974	2	2
Switzerland	3	1991	1	1	3	1991	1	1
U.K.	2	1973	2	2	2	1973	2	2
U.S.	2	1971	3	2	3	1979	1	1
Mexico	2	1990	4	4	2	1990	4	4
Korea	2	1976	4	4	2	1976	4	4
Hungry	2	1991	2	2	2	1991	2	2

Table 6.6: Model Selection Results for Consumer Prices

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1976	1	1	3	1976	1	1
Austria	1	1975	1	1	1	1975	1	1
Belgium	3	1976	1	1	3	1975	1	1
Canada	1	1979	1	1	1	1979	1	1
Finland	3	1977	1	1	3	1977	1	1
Denmark	3	1980	1	1	3	1980	1	1
France	3	1980	1	1	3	1980	1	1
Greece	3	1971	4	2	3	1974	4	3
Ireland	1	1977	1	1	1	1977	1	1
Italy	1	1980	1	1	1	1980	1	1
Japan	3	1975	1	1	3	1975	1	1
Luxembourg	3	1975	1	1	3	1975	1	1
Netherlands	3	1975	1	1	3	1975	1	1
New Zealand	1	1979	1	1	1	1979	1	1
Norway	3	1981	1	1	3	1981	1	1
Portugal	3	1977	1	1	3	1977	1	1
Spain	3	1977	1	1	3	1978	1	1
Sweden	1	1979	1	1	1	1979	1	1
Switzerland	1	1973	1	1	1	1973	1	1
U.K.	3	1976	1	1	3	1976	1	1
U.S.	1	1979	1	1	1	1979	1	1

Table 6.7: Model Selection Results for Wages

Countries	Model selected by BIC				Model selected by AIC			
	Model	TB	$tsig\_p$	BIC_p	Model	TB	$tsig\_p$	BIC_p
Australia	2	1985	2	4	2	1985	2	4
Austria	2	1982	2	2	2	1982	2	2
Belgium	2	1982	3	4	2	1982	3	4
Canada	2	1985	2	2	2	1985	2	2
Germany	2	1982	1	1	2	1982	1	1
Denmark	2	1984	2	4	2	1984	2	4
Spain	2	1984	2	2	2	1984	2	2
Finland	2	1986	2	2	2	1986	2	2
France	2	1985	2	2	2	1985	2	2
U.K.	2	1983	2	2	2	1983	2	2
Greece	2	1992	4	2	2	1992	4	4
Ireland	2	1984	1	4	2	1984	1	2
Iceland	2	1989	2	2	2	1989	2	2
Italy	2	1986	2	2	2	1986	2	2
Japan	2	1980	2	4	2	1980	2	4
Netherlands	2	1979	2	2	2	1979	2	2
New Zealand	2	1989	2	2	2	1989	2	2
Portugal	2	1990	2	4	2	1989	2	4
Sweden	2	1990	2	2	2	1990	2	2
U.S.	2	1984	3	2	2	1984	3	2

Table 6.8: Model Selection Results for Real Wages

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	1	1986	3	4	3	1977	1	1
Austria	2	1979	2	2	2	1979	2	2
Belgium	2	1978	3	1	2	1978	3	1
Canada	3	1978	5	1	3	1978	4	1
Germany	2	1980	1	1	2	1980	1	1
Denmark	1	1988	4	1	1	1988	4	1
Spain	2	1981	2	2	2	1981	2	2
Finland	2	1991	5	2	2	1991	2	2
France	2	1980	3	3	2	1980	3	3
U.K.	2	1976	4	1	2	1976	4	1
Greece	2	1979	2	4	2	1979	2	4
Ireland	2	1980	1	1	2	1980	1	1
Iceland	2	1985	2	2	3	1983	2	2
Italy	2	1980	5	1	2	1980	2	1
Japan	2	1976	3	4	2	1976	3	4
Netherlands	2	1976	2	2	2	1976	2	2
New Zealand	3	1976	1	1	3	1976	1	1
Portugal	3	1979	1	1	3	1979	1	1
Sweden	1	1982	5	1	1	1982	1	1
U.S.	3	1998	2	2	3	1998	2	2

Table 6.9: **Model Selection Results for Employment**

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1975	2	2	3	1975	2	2
Austria	1	1982	3	3	1	1982	3	3
Belgium	3	1982	4	1	3	1982	4	1
Canada	2	1981	3	2	2	1981	3	2
Switzerland	1	1975	3	1	1	1975	3	1
Germany	1	1991	2	3	1	1991	2	3
Denmark	3	1985	3	3	3	1985	3	3
Spain	2	1995	2	2	2	1995	2	2
Finland	3	1992	0	1	3	1992	0	1
France	3	1993	1	1	3	1993	1	1
U.K.	2	1984	4	2	3	1987	2	2
Greece	2	1972	1	1	2	1972	1	1
Ireland	2	1994	2	2	2	1994	2	2
Iceland	3	1989	3	1	3	1989	3	1
Italy	3	1993	4	1	3	1993	4	1
Japan	3	1991	1	1	3	1991	1	1
Korea	2	1993	1	1	2	1993	1	1
Norway	1	1990	1	1	1	1990	1	1
New Zealand	3	1988	1	1	3	1988	1	1
Portugal	3	1974	2	2	3	1974	2	2
Sweden	1	1993	1	1	1	1993	1	1
Turkey	2	2000	1	1	2	2000	1	1
U.S.	2	1988	2	2	3	1991	2	2

Table 6.10: Model Selection Results for Common Stock Prices

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1979	1	1	3	1979	1	1
Austria	1	1989	1	1	3	1985	4	2
Canada	3	1979	4	1	3	1979	4	1
France	3	1984	4	4	3	1984	4	4
Germany	3	1984	4	2	3	1984	4	2
Ireland	1	1974	4	4	1	1974	4	4
Japan	3	1987	2	2	3	1987	2	2
New Zealand	1	1983	2	2	1	1983	2	2
Sweden	3	1983	3	3	3	1983	3	3
Switzerland	2	1983	5	4	2	1983	4	4
US	2	1978	2	2	2	1978	2	2

Table 6.11: Model Selection Results for Real GDP per Capita

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	1	1982	3	4	3	1991	1	1
Austria	2	1975	4	4	2	1975	4	4
Belgium	2	1975	1	1	2	1975	1	1
Canada	2	1978	2	2	2	1978	2	2
Finland	1	1991	4	1	1	1991	4	1
Denmark	3	1974	1	1	3	1974	4	1
France	2	1975	4	2	2	1975	4	2
Greece	2	1974	4	1	2	1974	4	1
Iceland	2	1982	2	4	2	1982	2	4
Ireland	2	1994	4	2	2	1994	4	2
Italy	2	1980	4	1	2	1980	4	1
Japan	2	1972	4	1	2	1972	4	1
Luxembourg	3	1981	1	1	3	1981	1	1
Netherlands	3	1981	1	1	3	1981	1	1
New Zealand	3	1977	4	4	3	1977	4	4
Norway	3	1988	1	1	3	1988	1	1
Portugal	3	1975	3	3	3	1975	3	3
Spain	3	1978	3	1	3	1978	3	1
Sweden	1	1992	1	1	1	1992	1	1
Switzerland	3	1975	2	2	3	1975	2	2
U.K.	1	1980	2	2	1	1980	2	2
U.S.	-1^a	−−	1	2	1	1985	2	2
Mexico	3	1983	3	1	3	1983	3	1
Korea	1	1998	1	1	1	1998	1	1
Hungry	2	1981	3	3	2	1981	3	3

Note: [a] denotes the time trend model in Hadri (2000) without any break is chosen.

Table 6.12: Model Selection Results for Money Stocks

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1989	4	4	3	1989	4	4
Austria	2	1983	1	1	2	1983	1	1
Belgium	2	1977	4	1	2	1977	4	1
Canada	2	1989	4	2	2	1989	4	2
Switzerland	1	1971	1	4	1	1971	1	4
Germany	2	1977	2	1	2	1977	2	1
Spain	2	1987	2	3	2	1987	2	3
Finland	2	1990	1	1	2	1990	1	1
U.K.	2	1991	3	4	2	1991	3	4
Greece	2	1991	4	1	2	1991	4	1
Ireland	2	1981	2	2	2	1981	2	2
Iceland	2	1990	3	3	2	1990	3	3
Japan	2	1980	4	4	2	1980	4	4
Netherlands	3	1983	4	4	3	1983	4	4
Norway	2	1988	1	1	2	1988	1	1
New Zealand	3	1985	0	1	3	1985	0	1
Portugal	2	1989	2	2	2	1989	2	2
Sweden	2	1984	4	1	2	1984	2	1
U.S.	2	1987	4	4	2	1987	4	4

Table 6.13: Model Selection Results for Bond Yield

Countries	Model selected by BIC				Model selected by AIC			
	Model	*TB*	*tsig_p*	*BIC_p*	Model	*TB*	*tsig_p*	*BIC_p*
Australia	3	1991	1	1	3	1991	1	1
Belgium	3	1980	4	2	3	1980	4	2
Canada	3	1980	4	1	3	1980	4	1
Denmark	3	1983	1	1	3	1983	1	1
France	3	1980	1	1	3	1980	1	1
Germany	2	1982	2	2	2	1982	2	2
Italy	3	1980	2	2	3	1980	2	2
Netherlands	3	1983	2	2	3	1983	2	2
New Zealand	3	1985	2	2	3	1985	2	2
Norway	3	1981	1	1	3	1981	1	1
Switzerland	2	1975	2	2	3	1976	1	1
U.K.	2	1982	2	2	2	1982	2	2
U.S.	3	1980	4	1	3	1980	4	1