Chapter

It All Adds Up: Connecting Home and School through Family Math

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Abstract

Considered a core component of children's foundational cognitive development, early mathematics experiences can support children's long-term academic success. Teachers and families alike share the common goal of wanting children to succeed developmentally, socially, and academically. Given the importance of early mathematics to academic success in all subjects, children need and deserve to build a robust knowledge of early math concepts in their earliest years. In this chapter, we consider the approach of the Young Mathematicians (YM) project at EDC, which for the past ten years, has partnered with families, teachers, and early childhood programs in richly diverse communities with large populations of students of color, linguistically minoritized students, and students living in poverty, to support math learning across home and school environments. We illustrate some of our fun early learning games that engage teachers and families alike and are freely available in multiple languages for anyone to use. We discuss how our close collaboration with families and teachers has informed our approach to equity and report on some of the positive results from our research. Finally, we reflect on ways we can all improve how we are partnering with families and teachers to create equitable and supportive learning communities.

Keywords: math, games, equity, family engagement, learning community, families, teachers, family math, mathematics education, growth mindset, early math

1. Introduction

Opportunities to engage in early math learning is an equity issue with lasting consequences, as math learning *before* kindergarten entry strongly impacts and predicts future success in school [1]. All children deserve to be inspired by math and to reach their full potential, but for many, differences in math knowledge are evident at kindergarten entry, favoring children who have greater access to economic resources [2, 3]. This results in persistent educational learning gaps, as children who start kindergarten behind in mathematics may struggle to catch up to their peers [3, 4]. Addressing this challenge requires investing in early childhood programs and supporting families as education partners [5] as early intervention with preschoolers could help to narrow this gap and have important longer-term implications [6].

All children, no matter where they live, should have what they need to learn and develop well. Some children are born into communities where resources are

abundant, but others may not be. In our work, we prioritize the communities that are under-resourced, so all families have the information they need to launch their children on a path to success. When children have opportunities to engage in meaningful mathematical interactions it can support their cognitive development and foster the skills and behaviors they need to engage in learning, such as problem-solving, puzzling, and persevering. The very skills they will need to successfully navigate life and work in the twenty-first century.

1.1 Importance of math learning

Early mathematics knowledge is considered a critical component of young children's foundational cognitive skills [7] and the early years provide a pivotal opportunity to tap into children's curiosity and motivation to learn from interactions that are enriched with mathematics [8–10]. Accumulating research evidence indicates that children's early math knowledge predicts their future academic outcomes and success in school, [1, 11–13], on high stakes standards-based math tests in middle school [14], and through high school [15]. Importantly, early math skills predict later literacy skills [16] and are an even stronger predictor of later outcomes than early reading skills [1]. In fact, supporting kindergarteners' early mathematics skills builds a foundation not only for advanced mathematical knowledge but also for achievement in reading [17] and science and engineering [13, 18]. Indeed, knowing how to bolster children's mathematics learning at home and in classroom settings, through games and other developmentally appropriate activities, is of great interest to families, educators, early childhood leaders, and policymakers [19].

1.2 Development of early math knowledge

From the time they are born, children are intrinsically mathematical [20], naturally engaging in mathematical ways of thinking in the areas of number, geometry, measurement, early algebraic reasoning, and data analysis [21, 22]. Developing early mathematical competencies is a complex process that begins well before children enter kindergarten [22] and children need adult support to build and extend their math knowledge [23]. Early childhood teachers and parents[1] alike typically recognize verbal counting as an important mathematical skill that is related to later school success [24], but it is mastery of the specific counting principles that is critical to laying the foundation for broader mathematical thinking. For instance, when counting a set of objects, children need to (a) tag one object with one count word without skipping or double counting (one-to-one correspondence), (b) be able to recite number words in the correct order, and (c) know that the last word reached when counting a set represents the whole set (cardinal principle) [25] and children need opportunities to engage in challenging counting activities to develop their understanding of the principles that underlie meaningful counting [26]. Although these principles may seem intuitive to adults, the cardinal principle is a major milestone for preschoolers that leads to new numerical competencies [27, 28] but developing that skill often requires support from both parents and teachers. Given the long-term positive impact that

[1] For the sake of brevity, we sometimes use the word *parent* to refer to children's primary caregivers, but we recognize families come in many configurations, and the primary caregivers may be grandparents, aunts, uncles, older siblings, other family members, or guardians.

early math knowledge has on children's future academic success [1], children need and deserve to build their knowledge of math concepts in their early years.

1.3 Math talk to support children's learning

Unfortunately, differences are evident in preschoolers' understanding of number and early math concepts [6], which has a cascading effect, as these differences in children's math knowledge predict their math achievement through the elementary school years [12, 29]. A potential explanation for these differences in children's math knowledge is the variability that children experience in their learning environments both at home [30–32] and in preschool classrooms [33]. To build and extend early foundational math skills, children need lots of opportunities to engage in mathematical thinking, in playful ways, that is supported by contingent discussion and "math talk" that allows time for children to think [34]. This is critical as children's mathematical language emerges early, and family engagement in math talk during the early years supports growth in their math skills [35]. Indeed, accumulating evidence suggests that the amount of math talk children experience at home [29] and in preschool [36] and the content of the math talk [37] are correlated with later math skills in elementary school. For instance, young children's number knowledge is related to the amount of family math talk about numbers, starting when children are toddlers [29, 38]. Math talk about key early math concepts such as cardinality [39], spatial reasoning [37, 40, 41], and talking to preschoolers about advanced number concepts [42] have all been shown to predict children's advanced number skills.

1.4 Families and teachers need support in early math

Children also learn more math when they experience more math interactions at home [30, 43, 44] and in the classroom [45, 46]. Yet many families, including highly educated ones, report that they do not feel confident in their ability to support their child's math learning [47]. A growing body of evidence suggests that when parents interact around mathematics and provide their children with more mathematics-related activities and talk, children have higher mathematics outcomes regardless of their family's level of income or education [2, 29, 30, 48, 49]. However, families often do not have access to the types of early math experiences that are developmentally appropriate for preschool children [30, 50] and lack access to the network of supports, resources, and knowledge that are necessary to foster early math development effectively [2]. Importantly, families have highlighted their child's teacher as a key resource that could provide them with more information on what they can do to support their child's math learning [47]. Teachers could provide that bridge for families, which is critically important, as parent-child activities that go beyond counting, such as comparing amounts of items and adding or subtracting objects, can help children acquire more advanced mathematics knowledge [51].

However, early childhood teachers also find it challenging to translate developmentally appropriate teaching strategies into challenging math activities [52–54]. Even in the classroom, preschool teachers often limit their instruction to basic aspects of numbers and counting, and perhaps shapes [55], and spend more time on math concepts that may not be sufficiently challenging [56]. In fact, relatively little time is spent on math learning activities in preschool classrooms at all [33] even though preschoolers who experience more math-related interactions in the classroom and are exposed to more math activities, have higher math achievement [33, 57]. Indeed,

variations in preschool instruction [58] and kindergarten instruction [59] are related to children's math learning. In particular, the percentage of time teachers actively engage children in math, build upon their mathematical ideas, and facilitate children's responses predict gains in their math knowledge [60]. This underscores the need for teachers to understand children's mathematical thinking and use this knowledge not only to guide their teaching [61] but also to support families doing math at home.

2. Young Mathematicians

To enhance opportunities for all children, regardless of background, children need to be provided with learning opportunities that meet their diverse needs [62], and teachers and families need specific tasks to help them see and understand children's mathematical thinking, along with the support and resources to use this information. To address this need, we created the Young Mathematicians (YM) program at EDC, which has partnered with families and early childhood programs to support mathematics learning across home and school environments. YM is grounded in Bronfenbrenner's (1986) Ecological Systems theory [63, 64], which posits that children simultaneously grow and develop within different ecosystems, from the most intimate family and home ecological system, moving outward to the larger school system, and then to the most expansive syst'em: society and culture. Each system inevitably interacts with and influences each other in every aspect of the child's life. Young Mathematicians aim to capitalize on the interconnectedness of children's environments, infusing each level of the ecosystem with positive attitudes toward mathematics and opportunities for children to engage with high-quality early mathematics practices. For the past ten years, we have been working with teachers and families from Head Start[2] programs in richly diverse communities with large populations of students of color, linguistically minoritized students, and students living in poverty. Together with these communities, we developed over 55 mathematics games for young children to play at home, at school, or during family play and learning events.

The YM family engagement approach to supporting family math is built on empirical evidence that suggests four important parameters are key for family math interventions to successfully support children's mathematics learning. The first is that a particularly powerful strategy for promoting children's school readiness skills (regardless of content strand), is for schools to provide families with information about the kinds of activities they can do at home that complement children's school-based learning [57, 65]. The second is that family math interventions need to provide parents with concrete examples of the mathematics that preschoolers can learn through these daily activities and should illustrate the similarities between early mathematics and language [66]. The third parameter is that families who have a better understanding of early mathematical development may implement more mathematics activities, and therefore family math interventions should focus on developing families' knowledge of young children's mathematical thinking [2]. Finally, early

[2] The federal government funds Head Start programs through the U.S. Department of Health and Human Services, Administration for Children and Families. Eligible participants include 3–5 year old children whose families meet the HHS Poverty Guidelines. Based on the 2023 poverty guidelines [https://aspe.hhs.gov/topics/poverty-economic-mobility/poverty-guidelines], a family of 4 will be eligible for Head Start if their income is at or below $30,000.

mathematics interventions should take adults' attitudes toward mathematics into consideration as math anxiety can dampen children's mathematics outcomes [67].

Building on these principles, YM uses games and problem-solving activities (math minibooks) to support young children's foundational mathematics development. YM helps the adults in children's lives to overcome negative feelings about math by offering ideas for how mathematics can be infused into their lives in easy, fun, and playful ways. The materials also provide resources that families can use to expose their children to mathematics-related activities that promote number talk through games, activities, minibooks, and text messages with everyday math ideas. Notably, YM's family engagement approach to family math has helped to underscore for educators the importance of engaging families to support children's learning. Teachers have worked hard to discover new ways of engaging families in early math learning opportunities and in the words of one Head Start teacher who participated in YM:

> *We know children's learning begins in the home, families are fundamental in shaping children's interest and skills in math. So, in my opinion we can give families ingredients, and motivation to support their young children's mathematical development effectively. Families can also support children's math development by providing environments that are rich in learning. Families can teach children to see and name small quantities, count, add, subtract, and point out shapes.*

2.1 Why games?

In order to meet the needs of both families and teachers, we took the approach of designing and testing math games that could be played at home and at school. Math games provide a "hook" for teachers and families—playing a game is more interactive, fun, and developmentally appropriate than, for example, pulling out a worksheet. Play and other informal activities are important contexts in which children develop interests in mathematics, develop their skills, and extend conceptual understanding [45]. Adults can enhance children's exploration and learning through guided play—co-playing with children, asking open-ended questions, and exploring materials with them [68]. Math games can therefore promote adult–child co-play with mathematical ideas and foster more math talk in families and in the classroom [68, 69]. When adults and children interact around board games, mazes, and connect-the-dots activities, this helps to promote children's mathematics learning [10, 30, 51]. Games also provide playful learning opportunities that are fun, yet challenging [70] balancing difficulty and skill level, thus fostering motivation and engagement among young children [71]. Additionally, gameplay can promote self-regulation skills through following rules and taking turns, while also offering opportunities for children to practice their skills in communication, empathy, and conflict resolution [71].

For parents, particularly those who may be intimidated by the prospect of "doing math" or simply are not familiar with early math learning and development [47], the game context is also more approachable, as many families welcome incorporating games as part of their family routines and see games as a natural way to play and interact together. The game context can also support adults to engage in mathematics in playful ways, as many adults have had negative experiences with mathematics and may feel some math anxiety that leads them to avoid math. These feelings about math may be barriers to mathematics engagement in the home and may even negatively affect their children's mathematics achievement [72, 73]. Math games can provide families with specific concrete examples of the type of mathematics activities that are

developmentally appropriate and fun, which is something that parents say that they want [30, 74] suggesting that a potentially powerful intervention for engaging both teachers and families in mathematics is for children to participate in playful, engaging, and developmentally appropriate mathematics games at home and at school.

Indeed, in our research, we found that adding a family math component to a game-based classroom intervention resulted in positive impacts on preschoolers' mathematics knowledge and was an effective low-threshold intervention that helped to foster early math competencies [57]. For example, some of our games include games with cards and dice. Dice games encourage children to systematically repeat simple counting and adding procedures [75] and card games provide information about number symbols and number words [76] and magnitude comparison [77, 78]. Engaging with children in game-based learning also provides opportunities for adults to observe children's choices and strategies and then provide children with feedback about specific mathematics concepts. The structure of games also helps to support adult caregivers in co-playing while implementing developmentally appropriate instructional strategies. Games are also a fun and developmentally appropriate alternative to the worksheets or flashcards that many caregivers may turn to when they are unsure how to best support children's math learning.

2.2 YM math games

Together with the Head Start teachers, families, family engagement specialists, librarians, and other educators and partners over the past ten years, we developed over 55 mathematics games for young children to play at home, at school, or during family play and learn events. Resources and instructions in English, Spanish, and Portuguese, including videos and written documentation, are freely available on www.ym.edc.org. The written directions and videos explain how to engage children in the math games and introduce the mathematics concepts that are addressed by each game. The directions encourage players to adapt the games and modify them to their liking. Below we describe two of the games that specifically focus on developing children's knowledge of number.

2.2.1 Numbers, Numbers, 1, 2, 3

A favorite game for families and teachers to play is *Numbers, Numbers, 1, 2, 3* because all you need to play with are the fingers on your hands. In this game, children practice counting, knowing how many in all (cardinality), seeing how many immediately (subitizing), and composing and decomposing numbers (see Video 1 in Video Materials). The adult starts by holding their hands behind their back and chanting, "Numbers, numbers, one, two, three, how many fingers do you see?" The adult might show three fingers (one on one hand and two on the other hand). Children might know right away that there are three fingers or they might need to count the fingers—one, two, three. Or they might start counting at "1" but then start over again on the second hand counting "1, 2". The adult can then help them count across both hands "1, 2, 3." Once children figure out how many, you can play again. Children catch onto this game very quickly and typically want to take over being the one showing the fingers on their hands. As children gain more practice, adults can challenge them with numbers up to ten. And later, they can borrow a friend's hands and make numbers up to 20. For a video of how to play the game and an overview of the math for Numbers, Numbers, 1,2,3 go to: https://go.edc.org/Numbers123 (see also Video 1 in Video Materials) [79].

2.2.2 Roll Two: a game with cards and dice

The game *Roll Two* is a developmentally appropriate variation on the game *Shut the Box,* which is rumored to have origins dating back hundreds of years as a popular pub game in England. Our version is appropriate for preschoolers and families and uses two regular six-sided dice and 13 cards printed with numerals 0–12 with the corresponding dice pattern underneath (see **Figure 1**).

Children start by putting a set of cards in order from 1 to 12 (a more challenging variation is 0 to 12, less challenging variations are 1 to 6 or 1 to 3). Adults can learn what children know by watching them arrange the cards: *Do they put them in order from left to right? Do they recognize the numerals and know the order they go in? Do they count the dots on the cards to check which has more and which has fewer? Do they immediately leave a space between four and six to place the five card*. By simply setting up the game, adults learn what children know about numeral recognition, subitizing, and creating a number path. Then, children roll two dice and can turn over cards that match either: one of the rolled numbers; both rolled numbers; or the sum of the two rolled numbers (see **Figure 2**). The goal is to turn over all of the cards. Children can play alone or play against another player to see who can turn over all their cards first. (See also Video 2 in Video Materials for an overview of the math and instructions on how to play the game Roll 2 or go to: https://go.edc.org/Roll2) [80].

While this seems like a simple game, there is a lot of math and a lot of strategy involved. For example, say a child rolls a 3 and a 2. First, they are practicing recognizing dots on the die (subitizing) and connecting that quantity of dots to the numerals 3 and 2 on the cards. They are also practicing combining (or adding) 3 and 2 to make 5. Children may have to count 1, 2, 3, 4, 5 one-by-one to know that these together make 5 or they may be able to recognize 3 and 2 immediately on the dice and be able to count on from

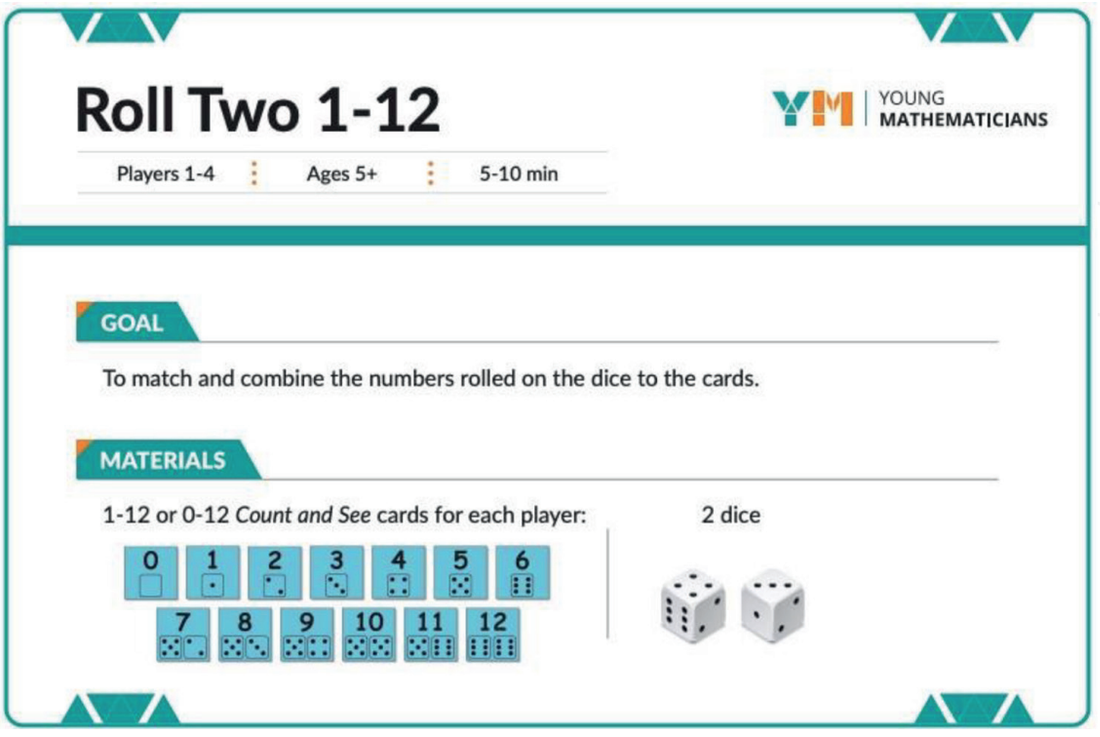

Figure 1.
Materials for Roll Two 1–12.

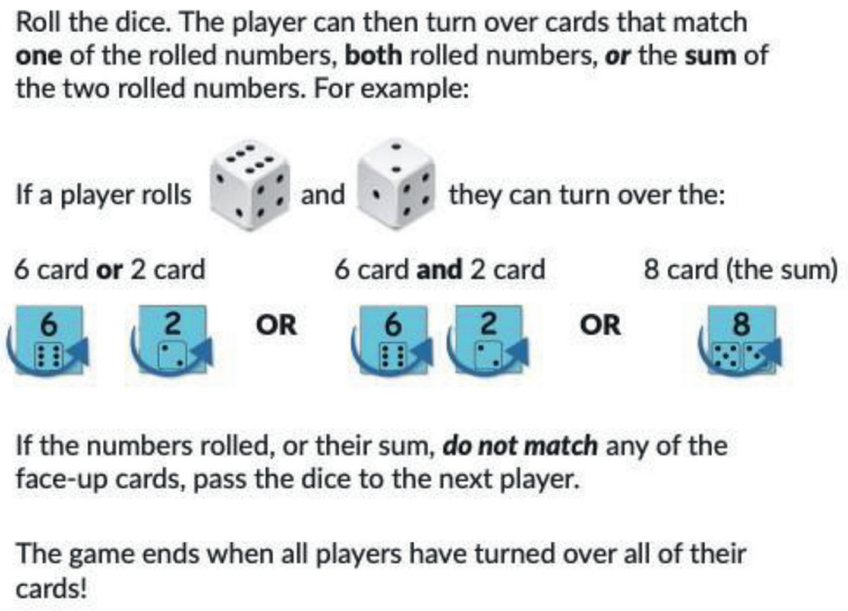

Figure 2.
Roll Two directions.

3—saying 3, 4, 5. With practice, children will be able to quickly add 3 and 2 in their heads and know 5. When playing this game, there are natural opportunities for adults to support children's learning such as encouraging them to use using the counting on strategy. As children gain more practice, they can add a rule that they are also allowed to subtract their roll—if they roll doubles, such as a 4 and a 4—they can subtract 4 from 4 and turn over their 0 card. When introducing subtraction in this game, we always like to add the caveat that children might venture into negative numbers—and that's okay! It can be tempting for adults to say that you cannot take away a bigger number from a smaller number—that you cannot take away 5 from 2—but, of course, you can. It is negative 3. In our materials, we encourage adults not to try to explain negative numbers to preschoolers but also not give children the misconception that it is not possible to subtract a bigger number from a smaller number. We suggest that adults acknowledge that you can take away 5 from 2 and that it will give you a negative number—negative numbers are very interesting and something children will learn about when they are older.

3. Young Mathematicians program of research

The Young Mathematicians (YM) program began in 2013 with a study to explore how math games could promote mathematics learning and persistence for preschool children attending Head Start programs. After successfully developing and testing a set of seven math games with over thirty classrooms in three Head Start programs and hearing from teachers that who wished families had access to these games, we expanded the work in 2015 by adding a family math component. The new intervention included both a classroom math and a family math component.

3.1 Adding family math to the equation

To build evidence for the importance of including a family math component in a classroom mathematics intervention, we evaluated two preschool math intervention

conditions and their effects on child outcomes, relative to a practice-as-usual condition: a classroom math (CM) intervention and a classroom math plus family math (CM + FM) intervention (for more details see [57]). Using math games has been key to our approach for creating inclusive classroom and family math practices and thus, the classroom math intervention included math games and playful instructional materials, and a relatively light touch professional development (PD) course for Head Start teachers. The family math (CM + FM) intervention included the same classroom math PD and games but added family math games and resources (such as bilingual math minibooks) that teachers could send home with children. Both conditions were "ecologically valid" such that they were implemented under naturalistic conditions, implemented by the teacher with the whole class, and aimed to support more than one mathematics skill.

We conducted a clustered randomized control trial with 573 children from 66 classrooms in three Head Start programs across two states in the Northeastern United States and found that in mixed-age (3- to 5-years) Head Start classrooms, the classroom plus family math intervention was significantly associated with spring mathematics scores relative to practice as usual (effect size of d = .20) but the classroom math intervention (without the family math component) was not significantly associated with outcomes. This finding underscores the value of combining a family-engagement component with a classroom mathematics intervention and adds to the evidence of the key role that family engagement plays in promoting children's math learning. In addition, this finding suggests that the combined intervention (CM + FM) has potential as an effective means to fill a gap in early childhood instructional practice especially because it can be implemented at scale without substantial investments in a specific mathematics curriculum, PD, or coaching support. In fact, interventions that can be qualified as "ecologically valid" are limited, and knowing under which conditions these interventions can be considered effective is essential [81].

YM's family math component provides children with two touchpoints for the mathematics—within the classroom, and at home—thus allowing the child to become the expert who "teaches" their parent how to play the games they learned at school. Parents report that seeing their children competent and confident about math is a great source of pride. As part of our family math community partnership work, we have expanded upon these touchpoints by adding more places within the community for families and children to encounter positive experiences engaging with mathematics ideas.

Highlights from our findings showed that

- For older preschoolers, 50 months and up, a classroom math plus family math intervention had a significant positive impact on children's mathematics learning compared to practice-as-usual classrooms (effect size of .36).

- The number of math games played (regardless of intervention condition) was significantly associated with higher mathematics scores.

- The classroom plus family math intervention had a significant positive impact on teachers' *instructional support* as measured by the CLASS preK [82] (effect size of .79)

3.2 Forming a family math learning community

Based on the success of adding a family math component to the YM classroom intervention, in 2019, we received funding to create a family math learning community

in Worcester, Massachusetts. Worcester is a unique and richly diverse city and holds the triple distinction of being the second largest city in New England, a leading Gateway City, and the leading refugee resettlement community, with a decades-long history of welcoming refugees from around the world. This project was one of two Family Math Roadmap Implementation Project Learning Community Grants funded for two years (2019–2021) by the Heising-Simons Foundation and the Overdeck Family Foundation (for more details see [83]). This family math learning community focused on scaling up the implementation of the YM classroom and family math program, adding culturally relevant resources, and promoting sustainability by coordinating across Head Start, the public library, a family partnership program, and the pre-service teacher program at the local community college to ultimately close the opportunity gap in early mathematics. In partnership with these community organizations, we aimed to align young children's learning experiences across the community and establish a "web of opportunity" by linking home, school, and the broader community, and promoting positive attitudes toward math. A key goal of the family math learning community has been to increase access and equity in early mathematics learning by putting families first and promoting the belief that math is for everyone.

Our approach to forming a cohesive family math community has been to codesign with educators and families the math games, resources, professional development, and supports that can be used in different contexts across the community—at home with different age siblings, at library story times, during family playgroups, and in preschool classrooms. A particular goal of the family math community work has also been to engage Emergent Multilingual Learners (EMLs) and to support children and families who have historically been denied access and equitable opportunities to engage with high-quality math experiences. While there is strong evidence that connecting home and school environments can reap great learning benefits for children and their families, there is still a question as to what models work for whom and under what circumstances [81] and whether such models can be scaled to broaden access to more families. In an effort to illuminate the practices and benefits of a community-wide approach to family math, we conducted an external evaluation of the YM family math learning community.

3.2.1 External evaluation of family math learning community

The external evaluation of this family math learning community found that the partnership among the community-based organizations promoted an increased understanding of the importance of early math for families, with families showing an increased interest in, and knowledge of, early math. Families also increased in how comfortable they felt helping their children with early math while feeling that they had also improved in their ability to come up with fun math activities to do with their children. The partnership also promoted an increased understanding of the importance of math for educators, with educators showing an increased interest in early math. In addition, educators were more comfortable engaging young children in early math activities in the classroom. In some cases, educators' beliefs about early math improved, for instance, educators grew in their understanding that everyone can learn math and that young children are curious about math ideas. Educators also felt more comfortable supporting family math; they increased their confidence to help parents understand children's age-appropriate math skills and were more confident answering families' questions about early math activities. In addition, educators reported that they were more confident about knowing the best ways to share math

information with families and were more confident about how to connect families to resources that support children's math development. Finally, educators were more confident about knowing the best practices and having the right tools for engaging families in early math.

Together with our community partners, we have bolstered support and awareness of the importance of early math, provided professional development to early childhood educators, and connected with families to share information, support, and materials that complement what their children's educators are learning in professional development sessions. We continue to sustain and expand this work with a current grant from Heising-Simons.

In the 2022–2023 school year, we expanded the mathematics content addressed in the program and tested the YM classroom and family math intervention in over 30 classrooms in three additional Head Start programs. Analyses for this study are underway.

4. Learnings from the YM codesign process: developing family math resources with families and educators

As part of YM, we have been able to work in close collaboration with early childhood educators and families with young children. The goal has been to support teachers' instructional practice, teachers' family engagement around math, and families' math play at home. Together we have codesigned many additional games and the supports and resources that accompany them. For example, during professional learning sessions with educators and family math workshops with families, we debriefed which aspects of the family math program were working and which aspects could be improved. We brainstormed design changes together, then we modified the materials and brought these revised materials back to get feedback on the new versions. This process has been transformative in creating our current family math resources. These are some of the principles that were most important.

4.1 Games need to be adaptable to children of different ages

Children in Head Start classrooms range in age from 2.9 to almost six years old. Likewise, families have siblings of different ages. Families and teachers found that the games they played the most were the ones that they could play with children of different ages. In *Numbers, Numbers, 1, 2, 3*, adults could play with just one, two, or three fingers for younger children (ages two- or three-year-olds), but go up to 10 or even 20 with older children. Likewise, with *Roll Two*, children could play a variation with just one die and numbers one to six when they are younger. This meant that when the younger children ask, C*an I play too?*, parents and teachers were able to say, *Yes!*, and adapt the game so they were also successful.

4.2 Easy to access written game directions

Both families and teachers asked for home versions of classroom math games. Teachers asked for games they could easily send home to families, and families asked for fun games to play and incorporate into their family time. In addition, families and teachers asked for the games to be accompanied by easy-to-follow instructions in multiple languages. We redesigned our original instruction sheets to be visual with

images and icons to make them easier to follow and written with an informal voice and everyday language to reinforce the idea that the games are fun and easy to use.

4.3 Video directions

Going beyond written directions, teachers and families thought to create short game direction videos that can be watched on a mobile device and easily shared with friends and family. These "how to play" videos could quickly and easily illustrate the game rules and key math concepts the games addressed. We created these videos so that you only see the gameplay so that they can easily be voiced over or subtitled in different languages to increase their accessibility.

4.4 Math minibooks

We had initially prototyped math minibooks that were bilingual in English and Spanish on each page. These eight-page, easy-to-read books were printed in black and white and could be easily printed and assembled at home or by teachers to be sent home with children. The minibooks were designed to complement the math games and give families a fun way to help their children deepen their learning by reading the books together and practicing the math concepts at home. Families told us how much they liked having English and Spanish on the same page and this also helped teachers who could send the same resources home to families whose home language was either Spanish or English. While this was valued by teachers and families, they also asked for additional versions that included more languages. We worked to add Portuguese and Arabic to these resources.

4.5 Families want to learn about young children's mathematical development

At first, we thought that families would not want to learn as much about children's mathematical development as teachers need to know. But we were wrong. The families who participated in our studies were clear that they wanted to know about the research in early mathematics and children's learning trajectories. Therefore, all of the written and video game directions include information about the mathematics children are learning, including common misconceptions and ways to extend their learning.

4.6 Math lens

Teachers and families emphasized that the games were a launching point for them to begin to see the world through a math lens and to develop that math lens in their children. Teachers and families alike reported that children not only asked to play the YM math games, but they also began creating their own math games.

5. Creating equitable and supportive mathematics learning communities

When we approach our work with educators and families, we begin by honoring adults' mathematical identities, acknowledging math anxiety and the history of exclusion in mathematics, then focusing on the assets adults bring to creating positive identities for the children in their lives. Beginning our work this way is critical

for creating a positive math community because so many adults carry the baggage of negative experiences with mathematics. If we seek to interrupt this cycle for the next generation of learners, we must first address it with the adults who are their role models and teachers.

5.1 Mathematical identity: *how did you experience math as a young learner*?

We often begin our workshops with adults with a four-corners activity using these picture books: *The Little Engine That Could* by Watty Piper [84]*; Oh, the Thinks You Can Think!* by Dr. Suess [85]*; Alexander and the Terrible, Horrible, No Good, Very Bad Day* by Judith Viorst [86]*;* and *Know and Follow Rules* by Cheri J. Meiners [87] (see **Figure 3**). We ask them to reflect on their experiences as a young math learner and pick the book they most identify with. In each group, many adults identify with the book, *Alexander and the Terrible, Horrible, No Good, Very Bad Day*. Some will describe the stress of having to solve 50 multiplication problems in one minute and feeling inadequate every time they did not make it to the end or made a mistake somewhere on their paper. Others will describe playing *Around the World* and the pit in their stomach every time it was their turn to answer a question for their team. Fortunately, many (and often the same people) will have had positive experiences where they identify with the book *The Little Engine that Could*, and remember math teachers that believed in them, inspired them to keep trying, and stuck with them until they figured it out. Some people will identify with the book, *Oh the Thinks You Can Think*, and explain that they saw math as a puzzle and loved the feeling of figuring out how to solve the different puzzles and challenges. We believe that honoring adults' experiences as math learners is a critical place to start this work because we know that parents' and teachers' feelings about math, and particularly math anxiety, can be passed on to children [67, 72, 88, 89]. We discuss as a group how we can interrupt that cycle of math anxiety so that they can provide their children and the children they work with positive experiences.

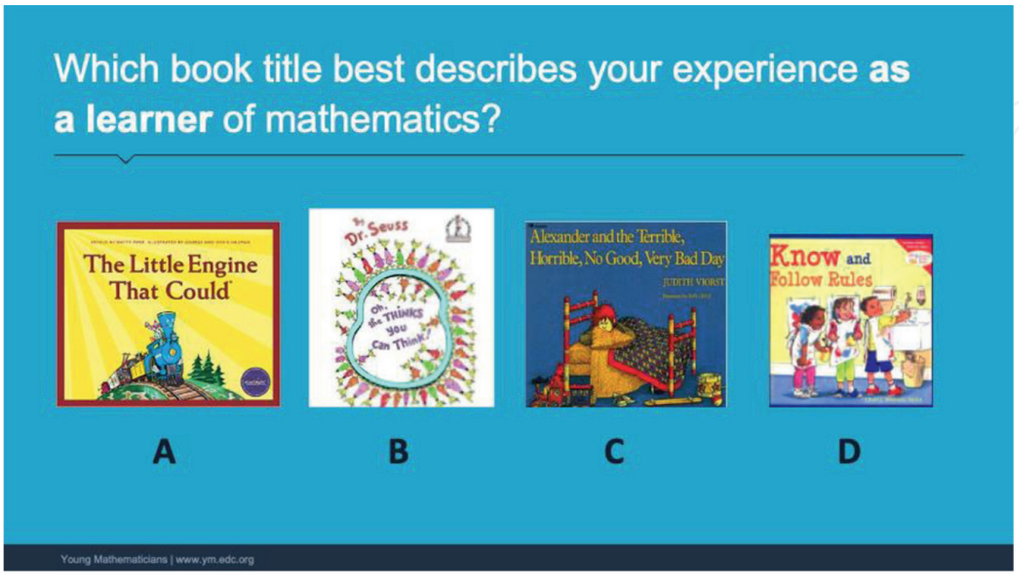

Figure 3.
Four corners activity.

5.2 Inclusive mathematics

We recognize that math education has an unjust history routed in institutional systems of oppression that have led to differences in learning opportunities and outcomes based on race, class, culture, language, and gender. Historically, mathematics, and particularly algebra, has been used as a gatekeeper that determines who is qualified for higher education and who is not [90]. Many adults hold the belief that some people are math people and other people are not. One of the ways we try to disrupt this conception is by doing open-ended mathematics together. For example, we often start a session by noticing the math we see in a photograph or a picture book. In this illustration (**Figure 4**), people often notice things like: two forks, two cups, six groups of three blueberries on toothpicks, the clock shows 6 o'clock, one red train car and three blue train cars, and many more mathematical ideas. By looking in picture books for mathematics, participants gain practice in noticing the math all around them and seeing how they are using math every day.

5.3 Mathematical assets

The mathematical knowledge and experiences that children and their families bring have historically not been valued in school mathematics classrooms [92]. Yet, we know that there is rich mathematics happening at home and in communities [93], even though it may be implicit and harder to recognize [94]. In our work with educators and families, we begin to unpack the many ways we use mathematics in our daily lives and uncover how we can leverage these experiences to support children's learning. Families and educators asked for easy and accessible ways to incorporate math into their daily

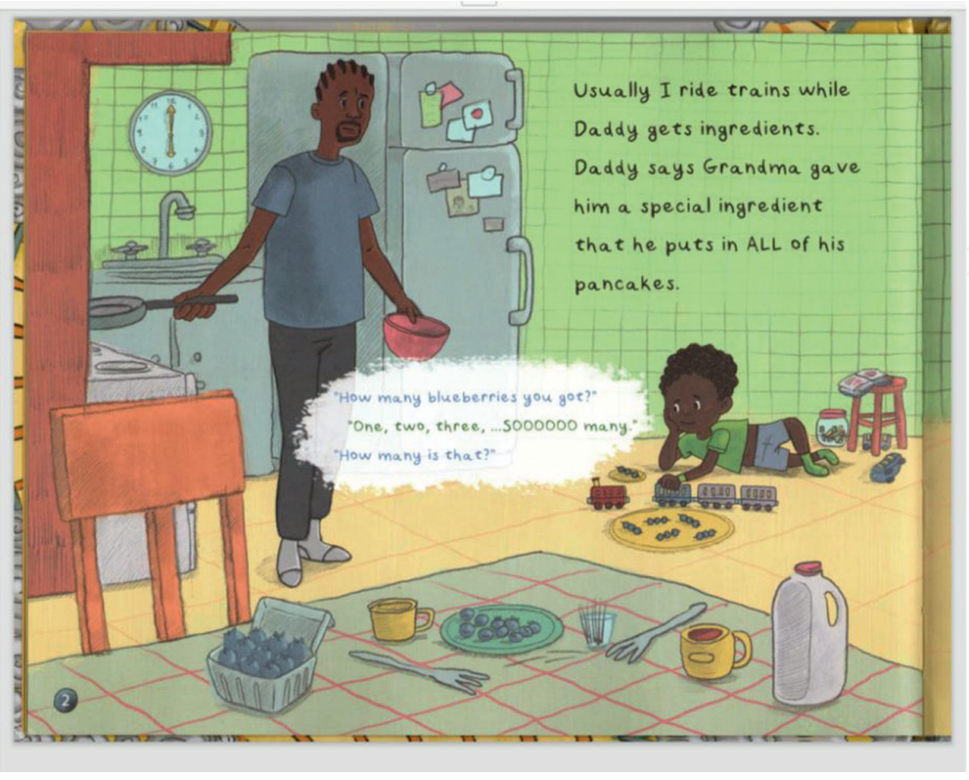

Figure 4.
Page from Sometimes We Do, written by Omowale Moses [91] and illustrated by Diego Chaves.

activities and conversations. Families were often surprised by how easy this could be and how much math could be found in their regular routines. Capitalizing on this, we identified common routines that families could "mathematize" easily, such as counting plates and forks while setting the table, sorting laundry, and comparing and contrasting the size and shape of common objects. By pointing out the math concepts embedded in these routines, families said that they suddenly "saw the math all around them."

5.4 Families and teachers as partners

A key component of creating an equitable learning community is for families and teachers to see each other as equal partners who are committed to helping children succeed developmentally, socially, and academically. Unfortunately, during our sessions, we address the core beliefs about family engagement and look for ways to create effective and respectful partnerships between families and teachers [95].

5.5 Improving family partnerships

Educators and families alike share the goal of wanting children to succeed developmentally, socially, and academically. However, to support family engagement, and especially for a family math learning community to be successful, stakeholders must find the right mix of strategies that will empower and engage families, while being sustainable for schools and other community educators. Everyone involved should be committed to continuous improvement and to adapting and revising plans while keeping a clear focus on the overall mission. By engaging young children with developmentally appropriate mathematics experiences and providing family math opportunities, teachers and educators can support families to build a firm foundation for their children's later academic success and contribute to addressing inequity in children's long-term educational outcomes.

6. Conclusion

In this chapter, we have described the Young Mathematicians program, which is designed to create a more equitable start for young children by supporting mathematics learning across home and school environments. We take a socioecological view of children's development and see the whole community—all the places where children and families live, learn, and play—as part of an opportunity web that supports children's learning. In close collaboration with educators and families, we have created over 55 freely available mathematics games for young children to play at home, at school, or during family play and learn events. By design, these games are accessible and engaging to a wide range of ages so that everyone who wants to play together—siblings of all ages, caregivers, and grandparents—has a way to join in.

In our work, we seek to ensure that all families, particularly those from historically underserved communities, have the resources and tools they need to create opportunities for their children to engage in meaningful early mathematics experiences. We know that educators and families alike share the common goal of wanting children to succeed and that for a family math learning community to be successful stakeholders must find the right mix of strategies that will empower and engage families while being sustainable for schools and other community educators. The YM approach seeks to provide tools and resources that can help all families with young children have the

information and support they need to give their children rich and developmentally appropriate early math opportunities.

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Author note

For the sake of brevity, we sometimes use the word parent to refer to children's primary caregivers, but we recognize families come in many configurations, and the primary caregivers may be grandparents, aunts, uncles, older siblings, other family members, or guardians.

Video materials

Video 1: Numbers, numbers, 1,2,3

For an overview of the math and video instructions of how to play the game see: https://go.edc.org/Numbers123 (follow the link for videos in Spanish, English, and Portuguese) [79].

Video 2: Roll two

For an overview of the math and video instructions of how to play the card game Roll 2 see: https://go.edc.org/Roll2 (follow the link for videos in Spanish, English, and Portuguese) [80].

It All Adds Up: Connecting Home and School through Family Math
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