Chapter

Computing the Performance Parameters of the Markovian Queueing System FM/FM/1 In Transient State

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Abstract

In this chapter, we utilize L–R method to calculate the parameters of performance of fuzzy Markovian queueing system FM/FM/1 in transient regime. The technique of calculating used is the arithmetic of L–R fuzzy numbers restricted to secant approximations. The membership function has helped us to represent graphically the curves of fuzzy parameters of performance into the space in three dimensions in transient regime in fuzzy environment. An illustrative example is given in the medical field to show the relevance of this study in operational research and in particular in queueing systems.

Keywords: performance parameters, fuzzy Markovian queueing systems, transient state, L–R method

1. Introduction

Nowadays, in many areas of life, whether it is computer systems, communication systems, production systems, medical or health systems or any other system of daily life, the world is looking for the best quality of service and performance of systems.

Thus, Baynat [1] points out that "it is becoming inconceivable to build any system without first doing a performance analysis. This analysis is related to the prior knowledge of the performance parameters of the queueiting systems such as the average number of customers in the queue and in the system; as well as the average waiting time of customers in the queue and in the system."

The fundamental question posed in this chapter is: "Would the L–R method be able to compute the performance parameters of the fuzzy Markovian queueing system FM/FM/1 in transient regime?"

In the literature browsed in fuzzy mathematics, it is well shown that fuzzy queues are widely studied in steady state by Ning and Zhao [2], Ritha and Robert [3], Ritha and Menon [4].

Many researches on Markovian fuzzy queueing systems and scientific papers have been based on computing the performance parameters of Markovian fuzzy queueing systems in steady state by the method of sluggish alpha-cuts and the L–R method, see for example Li and Lee [5, 6]; Kao et al. [7]; Palpandi and Geetharamani [8]; Wang et al. [9]. But, the calculation of these parameters of the system under study in transient regime is a major preoccupation of operational researchers nowadays.

In this chapter, the novelty of our study is due to the fact that we have computed the performance parameters of the queueing system, in the transient regime and in a fuzzy environment where these parameters are time-dependent fuzzy functions, whereas for all the authors presented above, the performance parameters have been analyzed in steady state where the results obtained are real numbers.

L–R Fuzzy Mathematics plays an important role if it is widened to secant approximation. This arithmetic conducts to the same results as those obtained by the most used and well-known alpha-cut and interval arithmetic (see Mukeba; Dubois D. and Prade [10–12].)

To achieve this, our approach is broken down as follows: The second section will present the classical M/M/1 queueing model and give the performance parameters of the model in a transient state The third section will recall the notions of fuzzy set, fuzzy numbers, fuzzy number of L–R type, arithmetic of fuzzy numbers of L–R type and triangular fuzzy number. The fourth section will be devoted to fuzzy functions. The fifth section will give a description of the L–R method and the calculation procedure. The sixth section will deal with a numerical example that uses the L–R method in transient state. The seventh section will give the conclusion of this chapter.

2. Presentation of the queueing model M/M/1 et some performance parameters in transient state

2.1 Classical M/M/1 queueing system

Definition 1: A queueing system or M/M/1 queue is a Markovian process unfolding in L.D.P.(Life and Death Process) with birth and death rates defined respectively by:

$$\begin{cases} \lambda_n = \lambda, \lambda > 0 \\ \mu_n = \begin{cases} \mu & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases} \end{cases} \tag{1}$$

2.1.1 Assumptions (or characteristics) of the model

- Customer arrivals follow a Poisson distribution of parameter λ;
- Service times follow an expo-negative distribution of parameter μ;

- The system has a single server;
- Waiting room capacity is infinite;
- The system capacity is infinite;
- The service discipline is FIFO (first in first out) or PAPS (first come, first served).

2.2 Performance parameters of the M/M/1 queue in transient state

In the literature, it is well-known that a queue is stable if and only if (cfr. [13]): $\lambda < \mu$

- This condition makes it possible to determine the following performance parameters in transient state:

$$\tilde{N}_S(t) = \frac{\rho}{1-\rho}\left(1 - e^{-(\mu-\lambda)t}\right) \tag{2}$$

$$\tilde{T}_S(t) = \frac{\rho\left(1 - e^{-(\mu-\lambda)t}\right)}{\mu(1-\rho)\left[\rho + (1-\rho)e^{-(\mu-\lambda)t}\right]} \tag{3}$$

where $\tilde{N}_S(t)$ and $\tilde{T}_S(t)$ are, respectively, the average number of customers in the system and the average waiting time in the system at time $t(t \geq 0)$..

3. Fuzzy sets, fuzzy numbers, alpha-cuts and interval arithmetic, L–R fuzzy numbers, arithmetic of L–R fuzzy numbers, triangular fuzzy numbers

3.1 Fuzzy sets

Definition 2: Let E be a classical set or a universe. A fuzzy subset $\tilde{A}$ (or a fuzzy set $\tilde{A}$) in E is defined by the function $\eta_{\tilde{A}}$, called membership function of $\tilde{A}$, from E to the real unit interval [0,1]. $\eta_{\tilde{A}}(x)$ is called the grade or the membership degree of x, $\forall x \in \tilde{A}$ (cf [11]).

Definition 3: Let $\tilde{A}$ fuzzy set on E. The $\alpha-$cut of $\tilde{A}$ denoted $\tilde{A}_\alpha$ the support $\sup(\tilde{A})$, the height $h(\tilde{A})$ and the core $(\tilde{A})$ are crisp sets defined as follows:

$$\tilde{A}_\alpha = \{x \in E : \eta_{\tilde{A}}(x) \geq \alpha\} \tag{4}$$

$$\sup(\tilde{A}) = \{x \in E : \eta_{\tilde{A}}(x) > 0\} \tag{5}$$

$$h(\tilde{A}) = \max\{\eta_{\tilde{A}}(x) : x \in E\} \tag{6}$$

$$\text{core}(\tilde{A}) = \{x \in E : \eta_{\tilde{A}}(x) = 1\} \tag{7}$$

Definition 4: A fuzzy set on a universe is said to be *normal* if:

$$h(\tilde{A}) = 1 \tag{8}$$

that is, $\exists m \in \tilde{A} : \eta_{\tilde{A}}(m) = 1$. In these conditions, m is called *modal value* of the fuzzy set $\tilde{A}$.

Definition 5: A fuzzy set $\tilde{A}$ on the universe $E = \mathbb{R}$ is said to be *convex* iff:

$$\forall x, y \in \tilde{A}, \forall \lambda \in [0, 1] : \eta_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq \min\{\eta_{\tilde{A}}(x), \eta_{\tilde{A}}(y)\} \tag{9}$$

3.2 Fuzzy numbers

Definition 6: A fuzzy set $\tilde{A}$ on a universe E is called a fuzzy number if it satisfies the following conditions:

1. $E = \mathbb{R}$

2. $\tilde{A}$ is normal

3. $\tilde{A}$ is convex

4. The membership function $\eta_{\tilde{A}}$ is piecewise continuous

3.3 Fuzzy numbers of type L: R of L: R type

Definition 7: A fuzzy set $\tilde{A}$ is said to be of L–R type if there exists three reals $m, a > 0, b > 0$ and two continuous and decreasing positive functions L and R from $\mathbb{R}$ in [0,1] such that: L(0) = R(0) =1

$$L(1) = 0, \quad \text{or} \quad L(x) > 0, \text{with} \lim_{x \to \infty} L(x) = 0 \tag{10}$$

$$R(1) = 0, \quad \text{or} \quad R(x) > 0, \text{with} \lim_{x \to \infty} R(x) = 0 \tag{11}$$

$$\eta_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{m - x}{a}\right) & \text{if } x \in [m - a, m] \\ R\left(\dfrac{m - x}{b}\right) & \text{if } x \in [m, m + b] \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The L–R representation of a fuzzy number $\tilde{A}$ is $\tilde{A} = \langle m, a, b \rangle_{L-R}$, m is called the modal value of $\tilde{A}$. a and b are called respectively the *left spread* and *right spread* of $\tilde{A}$.

By convention, $\langle m,0,0 \rangle_{L-R}$ is the ordinary real number m; called also fuzzy singleton. The support of $\tilde{A}$ is the open interval: $\sup(\tilde{A}) =]m - a, m] \cup [m, m + b[=]m - a, m + b[$.

From the Definition (8) and the expression (12) of $\eta_{\tilde{A}}$, the support of $\tilde{A}$ is determined by the open following interval:

$$\sup(\tilde{A}) =]m - a, m] \cup [m, m + b[=]m - a, m + b[\tag{13}$$

3.4 Arithmetic of fuzzy numbers of L: R Type

3.4.1 Addition and subtraction of fuzzy numbers of L: R type

According to [10], if there exists two fuzzy numbers of the same L–R type. $\tilde{A} = \langle m, a, b \rangle_{L-R}$ and $\tilde{B} = \langle n, c, d \rangle_{L-R}$; then their sum and their difference are also fuzzy numbers of L–R type given respectively by:

$$\tilde{A} \oplus \tilde{B} = \langle m + n, a + c, b + d \rangle_{L-R} \tag{14}$$

$$\tilde{A} \ominus \tilde{B} = \langle m - n, a + c, b + d \rangle_{L-R} \tag{15}$$

3.4.2 Multiplication and division

According to [12], if there exist two fuzzy numbers of the same L–R type. $\tilde{A} = \langle m, a, b \rangle_{L-R}$ and $\tilde{B} = \langle n, c, d \rangle_{L-R}$; then:

$$\tilde{A} \odot \tilde{B} \approx \langle mn, mc, +na - ac, md + nb + bd \rangle_{L-R} \tag{16}$$

$$\frac{\tilde{A}}{\tilde{B}} = \frac{\langle m, a, b \rangle_{L-R}}{\langle n, c, d \rangle_{L-R}} \approx \left\langle \frac{m}{n}, \frac{md}{n(n+d)} + \frac{a}{n} - \frac{ad}{n(n+d)}, \frac{mc}{n(n-c)} + \frac{b}{n} + \frac{bc}{n(n-c)} \right\rangle_{L-R} \tag{17}$$

The product and the quotient of two numbers of the same type L–R are obtained by the secant approximation of Hanss [14], whose kernel and the support, for the quotient are given by:

$$\ker\left(\frac{\tilde{A}}{\tilde{B}}\right) = \frac{m}{n} \tag{18}$$

$$supp\left(\frac{\tilde{A}}{\tilde{B}}\right) = \left]\frac{m-a}{n+d}, \frac{m+b}{n-c}\right[\tag{19}$$

3.5 Fuzzy triangular numbers

Definition 8: A fuzzy number $\tilde{A}$ is said to be a *fuzzy triangular number* iff there exists three real numbers $a < b < c$ such that:

$$\eta_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) & \text{if } a \le x \le b \\ \left(\frac{c-x}{c-b}\right) & \text{if } b \le x \le c \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Remark 1.

a. Every fuzzy triangular number is noted by: $\tilde{A} = (a, b, c)$ or $\tilde{A} = (a|b|c)$

b. Every fuzzy triangular number of L–R type and the fuzzy L–R representation of $\tilde{A}$ is:

$$\tilde{A} = (a, b, c) = \langle b, b - a, c - b \rangle_{L-R} \tag{21}$$

4. Fuzzy function

In this section, let us define, in general, a fuzzy function of one real variable as follows:

$$\tilde{\psi} : \mathcal{D} \to \mathcal{F}(\mathbb{R}) : x \mapsto \tilde{\psi}(x) \tag{22}$$

where $\mathcal{F}(\mathbb{R})$ the set of fuzzy functions defined on $\mathbb{R}$.

a. *Alpha-cut of* $\tilde{\psi}(x)$

The α-cup representation of $\tilde{\psi}(x)$ is:

$$\tilde{\psi}(x)_\alpha = \tilde{\psi}(x, \alpha) = \left[\tilde{\psi}^L(x, \alpha), \tilde{\psi}^U(x, \alpha)\right], \alpha \in [0, 1] \tag{23}$$

b. *Kernel of* $\tilde{\psi}(x)$

The kernel of $\tilde{\psi}(x)$, also called modal of $\tilde{\psi}(x)$ is defined by:

$$\ker\left(\tilde{\psi}(x)_{\alpha=1}\right) = \tilde{\psi}^L(x, 1) = \tilde{\psi}^U(x, 1) \tag{24}$$

c. *Support of* $\tilde{\psi}(x)$

The support of $\tilde{\psi}(x)$ is defined by:

$$supp\left(\tilde{\psi}(x)_{\alpha=0}\right) = \tilde{\psi}(x, 0) = \left[\tilde{\psi}^L(x, 0), \tilde{\psi}^U(x, 0)\right] \tag{25}$$

d. *Membership function of* $\tilde{\psi}(x)$

The membership function of the fuzzy function $\tilde{\psi}(x)$ is defined by:

$$\eta_{\tilde{\psi}(x)} = \begin{cases} \left(\tilde{\psi}^L\right)^{-1}(x, \varsigma_x) & \text{if } \tilde{\psi}^{\mathrm{L}}(x, 0) \le \varsigma_{\mathrm{x}} \le \tilde{\psi}^{\mathrm{L}}(x, 1) \\ \left(\tilde{\psi}^U\right)^{-1}(x, \varsigma_x) & \text{if } \tilde{\psi}^{\mathrm{U}}(x, 1) \le \varsigma_{\mathrm{x}} \le \tilde{\psi}^{\mathrm{L}}(x, 0) \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

$$\eta_{\tilde{\psi}(x)} = \begin{cases} \dfrac{\varsigma_x - \tilde{\psi}^L(x,)}{\tilde{\psi}^L(x, 1) - \tilde{\psi}^L(x, 0)} & \text{if } \tilde{\psi}^{\mathrm{L}}(x, 0) \le \varsigma_{\mathrm{x}} \le \tilde{\psi}^{\mathrm{L}}(x, 1) \\ \dfrac{\tilde{\psi}^U(x, 0) - \varsigma_x}{\tilde{\psi}^L(x, 0) - \tilde{\psi}^L(x, 1)} & \text{if } \tilde{\psi}^{\mathrm{U}}(x, 1) \le \varsigma_{\mathrm{x}} \le \tilde{\psi}^{\mathrm{L}}(x, 0) \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

5. Description of the L: R method and procedure of computing

5.1 Description of L: R method

Let us consider the classical Markovian queue M/M/1 defined in Section (2.2) and assume that the arrival rate λ and service rate μ are triangular fuzzy numbers denoted

$\tilde{\lambda} = (\lambda_1|\lambda_2|\lambda_3)$ and $\tilde{\mu} = (\mu_1|\mu_2|\mu_3)$, respectively. In this case, these rates are imprecise (or fuzzy) and also make the performance measures of the transient fuzzy functions and, we note:

$$\tilde{\psi}(t) = \tilde{f}\left(t, \tilde{\lambda}, \tilde{\mu}\right) \tag{28}$$

Where t is a real variable called time and $\tilde{\lambda}$ and $\tilde{\mu}$ are fuzzy variables. In this case, the queueing model becomes a fuzzy Markovian queue FM/FM/1, where FM is a fuzzy exponential distribution.

To determine the fuzzy performance measure $\tilde{\psi}(t)$, the L–R method proceeds as follows:

5.2 Procedure

Determine the L–R expressions of the fuzzy rates $\tilde{\lambda}$ and $\tilde{\mu}$ and substitute them:

$$\tilde{\psi}(t) = \tilde{f}\left(t, \tilde{\lambda}, \tilde{\mu}\right) \tag{29}$$

Apply the arithmetic of fuzzy numbers of (14)–(17) in (28) and we find:

$$\tilde{\psi}(t) = \langle m(t), \varphi(t), \omega(t) \rangle_{L-R} \tag{30}$$

Where $m(t)$ is the modal function of $\tilde{\psi}(t)$ (or the kernel of $\tilde{\psi}(t)$) and where $\varphi(t)$ and $\omega(t)$ represent respectively the *left spread* and *right spread* of $\tilde{A}$..

The support of $\tilde{\psi}(t)$ is:

$$supp(\tilde{\psi}(t)) =]m(t) - \varphi(t), m(t) + \omega(t)[\tag{31}$$

And its kernel (or modal) is:

$$\ker(\tilde{\psi}(t)) = m(t) \tag{32}$$

6. Numerical example

6.1 Statement

In a referral Hospital, an ophthalmologist doctor consults patients on odd days each week from $10h^{00}$ to $13h^{30}$. The patients arrive there following a Poisson distribution with parameter $\tilde{\lambda}$ and the doctor's consultation following an expo-negative distribution with parameter μ The fuzzy parameters $\tilde{\lambda}$ and μ are such that $\frac{\tilde{\lambda}}{\tilde{\mu}}$ is approximately 0.4. We note that $\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}}$ is the fuzzy traffic intensity. We further warn that this traffic intensity is a triangular fuzzy number and is denoted by $\tilde{\rho} = (0.3|0.4|0.5)$.

6.2 Questions

- Determine the following transient performance measures:
- A1. The average number of patients in the system.

- A2. The average time of stay of patients in the system.
- B. Give the graphical representation of these performance measures.

6.3 Solution

A careful reading of our example reveals that it is a fuzzy Markovian queue noted FM/FM/1 with a single server and infinite capacity. The fuzzy traffic intensity being about 0.4 implies that the fuzzy rates $\tilde{\lambda}$ and $\tilde{\mu}$ are about 2 and 5, respectively. By assumption, since the fuzzy traffic intensity $\tilde{\rho}$ is a triangular fuzzy number, the rates $\tilde{\lambda}$ and $\tilde{\mu}$ are also triangular fuzzy numbers and can be written (cf. Remark 1, item a):

$$\tilde{\lambda} = (1|2|3) \quad \text{and} \quad \tilde{\mu} = (4|5|6)$$

Thus, in fuzzy model, the rates $\tilde{\lambda}$ and $\tilde{\mu}$ are fuzzy variables and the performance measures $\tilde{N}_S$ and $\tilde{T}_S$ are fuzzy time functions defined by:

$$\tilde{N}_S(t) = \tilde{f}_1(t, \tilde{\lambda}, \tilde{\mu}) = \frac{\tilde{\lambda}\left(1 - \exp\left(-\left(\tilde{\mu} - \tilde{\lambda}\right)t\right)\right)}{\tilde{\mu} - \tilde{\lambda}} \tag{33}$$

$$\tilde{T}_S(t) = \tilde{f}_2(t, \tilde{\lambda}, \tilde{\mu}) = \frac{\tilde{\lambda} - \tilde{\lambda}\exp\left(-\left(\tilde{\mu} - \tilde{\lambda}\right)\right)}{\tilde{\mu} - \tilde{\lambda}\left[\tilde{\lambda} + \left(\tilde{\mu} - \tilde{\lambda}\right)\exp\left(-\left(\tilde{\mu} - \tilde{\lambda}\right)t\right)\right]} \tag{34}$$

To evaluate these performance parameters by L–R method, we proceed as follows:

i. Let us determine the L–R expressions of fuzzy rates:

$\tilde{\lambda} = (1|2|3)$ and $\tilde{\mu} = (4|5|6)$ and we have according to (21):

$$\tilde{\lambda} = \langle 2,1,1\rangle_{L-R} \quad \text{and} \quad \tilde{\lambda} = \langle 5,1,1\rangle_{L-R} \tag{35}$$

ii. Let us substitute the expressions of (35) into (33) and (34), and use the operations in (14)–(17) to obtain successively:

$$\tilde{N}_S(t) = \frac{\tilde{\lambda}\left(1 - \exp\left(-\left(\tilde{\mu} - \tilde{\lambda}\right)t\right)\right)}{\tilde{\mu} - \tilde{\lambda}}$$

$$= \frac{\langle 2,1,1\rangle_{L-R} - \langle 2,1,1\rangle_{L-R}\exp(-X)}{\langle 5,1,1\rangle_{L-R} - \langle 2,1,1\rangle_{L-R}} \quad \text{with} \quad X = -\langle 3,2,2\rangle_{L-R}t$$

$$= \frac{\langle 2 - 2\exp(X), 1 + \exp(X), 1 + \exp(X)\rangle_{L-R}}{\langle 3,2,2\rangle_{L-R}}$$

$$\approx \left\langle \frac{2 - 2e^X}{3}, \frac{\left(2 - 2e^X\right)X2}{3(3+2)} + \frac{1 + e^X}{3} - \frac{\left(1 + e^X\right)X2}{3(3+2)}, \frac{\left(2 - 2e^X\right)X2}{3(3+2)} + \frac{1 + e^X}{3} + \frac{\left(2 - 2e^X\right)X2}{3(3+2)} \right\rangle_{L-R}$$

$$= \left\langle \frac{2 - 2e^X}{3}, \frac{7 - e^X}{15}, \frac{7 - e^X}{3}, \right\rangle_{L-R}$$

$$= \left\langle 0.67 - 0.67e^{-3t}, 0.47 - 0.1e^{-3t}, 2.3 - 0.3e^{-3t} \right\rangle_{L-R}, \quad X = -3t$$

since $X = -\langle 3,2,2\rangle t_{L-R} = -3t$ when we change the L–R writing of X to the α-cut writing, with $\alpha = 0 (t \geq 0)$

$$\tilde{T}_S(t) = \frac{\tilde{\lambda} - \tilde{\lambda}\exp(-(\tilde{\mu} - \tilde{\lambda}))}{\tilde{\mu} - \tilde{\lambda}[\tilde{\lambda} + (\tilde{\mu} - \tilde{\lambda})\exp(-(\tilde{\mu} - \tilde{\lambda})t)]}$$

$$= \frac{\langle 2,1,1\rangle_{L-R} - \langle 2,1,1\rangle_{L-R}\exp(-X)}{\langle 3,2,2\rangle_{L-R}[\langle 2,1,1\rangle_{L-R} - \langle 3,2,2\rangle_{L-R}\exp(-X)]} \quad \text{with} \quad X = -\langle 3,2,2\rangle_{L-R} = -3t$$

$$= \frac{\langle 2 - 2e^X, 1 + e^X, 1 + e^X\rangle_{L-R}}{\langle 3,2,2\rangle_{L-R}[\langle 2 - 3e^X, 1 + 2e^X, 1 + 2e^X\rangle_{L-R} e^X]}$$

$$= \frac{\langle 2 - 2e^X, 1 + e^X, 1 + e^X\rangle_{L-R}}{\langle 6 + 9e^X, 5 + 8e^X, 9 + 16e^X >_{L-R}}$$

$$\approx \left\langle \frac{2 - 2e^X}{6 + 9e^X}, \frac{(2 - 2e^X)(9 + 16e^X)}{(6 + 9)[6 + 9e^X + 9 + 16e^X]} + \frac{1 + e^X}{6 + 9e^X} - \frac{(1 + e^X)(9 + 16e^X)}{(6 + 9e^X)[6 + 9e^X - 5 - 16e^X]}, \right.$$

$$\left. \frac{(2 - 2e^X)(5 + 8e^X)}{(6 + 9e^X)[6 + 9e^X - 5 - 8e^X]} + \frac{1 + e^X}{6 + 9e^X} + \frac{(1 + e^X)(5 + 8e^X)}{(6 + 9e^X)[6 + 9e^X - 5 - 8e^X]} \right\rangle_{L-R}$$

$$= \left\langle \frac{2 - 2e^X}{6 + 9e^X}, \frac{24 + 29e^X - 23e^X}{(6 + 9e^X)(15 + 25e^X)}, \frac{16 + 21e^X - 7e^{2X}}{(6 + 9e^X)(1 + e^X)} \right\rangle_{L-R}$$

with $X = -\langle 3,2,2\rangle_{L-R} t = -3t$ and $t \geq 0$.

6.4 Supports and modals

$$\begin{aligned} supp(\tilde{N}_S(t)) &= \left](0.67 - 0.67e^X) - (0.47 - 0.1e^X), (0.67 - 0.67e^X) + (2.3 - 0.3e^X)\right[\\ &= \left]0.2 - 0.6e^X, 3 - e^X\right[, \quad X = -3t, t \geq 0 \end{aligned}$$

$$\begin{aligned} supp(\tilde{T}_S(t)) &= \left]\frac{2 - 2e^X}{6 + 9e^X} - \frac{24 + 29e^X - 23e^X}{(6 + 9e^X)(15 + 25e^X)}, \frac{2 - 2e^X}{6 + 9e^X} + \frac{16 + 21e^x - 7e^X}{(6 + 9e^X)(1 + e^X)}\right[\\ &= \left]\frac{1 - 3e^X}{15 + 25e^X}, \frac{3 - e^X}{1 + e^X}\right[, \quad X = -3t, t \geq 0 \end{aligned}$$

$$\ker(\tilde{N}_S(t)) = 0.67 - 0.67e^X, \quad \text{where} \quad X = -3t, t \geq 0$$

$$\ker(\tilde{T}_S(t)) = \frac{2 - 2e^X}{6 + 9e^X}, \quad X = -3t, t \geq 0$$

6.5 Graphical representations of $\tilde{N}_S(t)$ and $\tilde{T}_S(t)$

The graphical representation of the fuzzy functions $\tilde{N}_S(t)$ and $\tilde{T}_S(t)$ is made from their membership functions when their supports and modes are known. Referring to (27), we obtain:

- $$\eta_{\tilde{N}_{s(t)}}(t, x_t) = \begin{cases} \dfrac{x_t - \tilde{N}_s^L(t,0)}{\tilde{N}_s^L(t,1) - \tilde{N}_s^L(t,0)} & \text{if } \tilde{N}_s^L(t,0) \le x_t \le \tilde{N}_s^L(t,1), t \ge 0 \\ \dfrac{\tilde{N}_s^U(t,0) - x_t}{\tilde{N}_s^U(t,0) - \tilde{N}_s^U(t,1)} & \text{if } \tilde{N}_s^U(t,1) \le x_t \le \tilde{N}_s^U(t,0), t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \dfrac{x_t + 0.6e^{-3t} - 0.2}{0.5 - 0.1e^{-3t}} & \text{if } 0.2 - 0.6e^{-3t} \le x_t \le 0.67 - 0.67e^{-3t}, t \ge 0 \\ \dfrac{(3 - e^{-3t}) - x_t}{2.3 - 0.3e^{-3t}} & \text{if } 0.67 - 0.67e^{-3t} \le x_t \le 3 - e^{-3t}, t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

- $$\eta_{\tilde{T}_{s(t)}}(t, x_t) = \begin{cases} \dfrac{x_t - \tilde{T}_s^L(t,0)}{\tilde{T}_s^L(t,1) - \tilde{T}_s^L(t,0)} & \text{if } \tilde{T}_s^L(t,0) \le x_t \le \tilde{T}_s^L(t,1), t \ge 0 \\ \dfrac{\tilde{T}_s^U(t,0) - x_t}{\tilde{T}_s^U(t,0) - \tilde{T}_s^U(t,1)} & \text{if } \tilde{T}_s^U(t,1) \le x_t \le \tilde{T}_s^U(t,0), t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \left(\dfrac{90 + 285e^{-3t} + 225e^{-6t}}{24 + 29e^{-3t} - 23e^{-6t}}\right)x_t - \dfrac{6 - 9e^{-3t} - 27e^{-6t}}{24 + 29e^{-3t} - 23e^{-6t}} & \text{if } \dfrac{1 - 3e^{-3t}}{15 + 25e^{-3t}} \le x_t \le \dfrac{2 - 2e^{-3t}}{6 + 9e^{-3t}}, t \ge 0 \\ -\left(\dfrac{6 + 15e^{-3t} + 9e^{-6t}}{24 + 21e^{-3t} - 23e^{-6t}}\right)x_t + \dfrac{18 + 21e^{-3t} - 9e^{-6t}}{16 + 21e^{-3t} - 7e^{-6t}} & \text{if } \dfrac{2 - 2e^{-3t}}{6 + 9e^{-3t}} \le x_t \le \dfrac{3 - e^{-3t}}{1 + e^{-3t}}, t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

6.5.1 Graphics

6.5.2 Interpretation of results

Figure 1, shows that the support of the function $\tilde{N}_s(t)$ is an open interval from $\tilde{N}_s^L(t,0) = 0.2 - 0.6e^{-3t}$ to $\tilde{N}_s^U(t,0) = 3 - e^{-3t}$, $(t \ge 0)$ this means that the function average number of patients in the system is a fuzzy function.

It is impossible for the curve of the fuzzy function $\tilde{N}_s(t)$ to be below the curve of $\tilde{N}_s^L(t,0)$ nor above that of $\tilde{N}_s^U(t,0)$. The curve of its modal function $\tilde{N}_s^U(t,0) = 0.67 - 0.67e^{-3t}(t \ge 0)$, is the most likely curve for the function $\tilde{N}_s(t)$.

Similarly, **Figure 2**, indicates that the curve for the fuzzy function $\tilde{T}_s^L(t)$, the average time of patients' stay in the system, is approximately between the curves of equations $\tilde{T}_s^L(t,0) = \frac{1-3e^{-3t}}{15+25e^{-3t}}$ and $\tilde{T}_s^U(t,0) = \frac{3-3e^{-3t}}{1+e^{-3t}}$ $(t \ge 0)$. In other words, the curve of $\tilde{T}_s(t)$ cannot go below that of $\tilde{T}_s^L(t,0)$ nor can it go above that of $\tilde{T}_s^U(t,0)$.

In the classical, we can say that the average waiting time of patients in the system is approximately between 0.1 hours ($\simeq$ 6 min) and 3 hours ($\simeq$ 180 min).

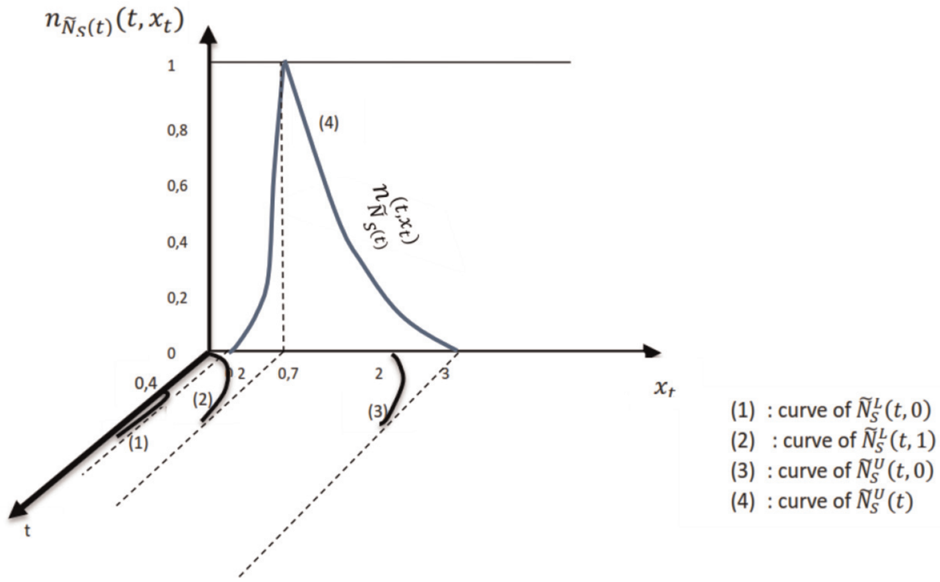

Figure 1.
Membership function of the fuzzy function $\tilde{N}_s(t)$ average number of patients in the system at a time $t(t \geq 0)$.

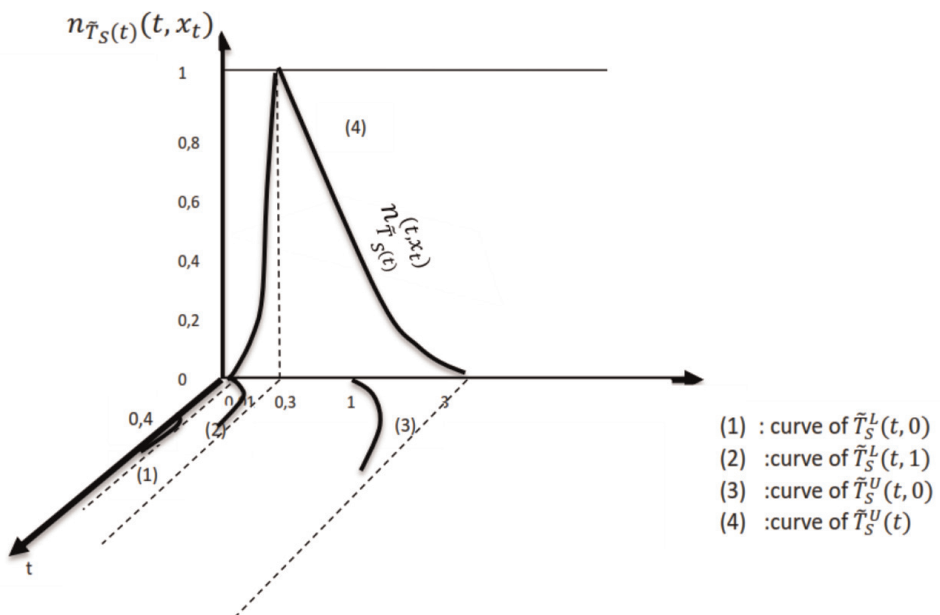

Figure 2.
Membership function of the fuzzy function $\tilde{T}_s(t)$ average time of patients' stay in the system at a time $t(t \geq 0)$.

This means in other words that the average patient waiting time in the system cannot go below 6 min nor above 180 min. The most likely mean patient waiting time in the system is 0.3 hours ($\simeq$ 18 min).

7. Conclusion

At the end of this chapter related to "the calculation of the performance parameters of the fuzzy Markovian queueing system FM/FM/1 in transient state by the L–R method," it was a question of evaluating these performance parameters of the queueing system considered in transient regime by means of a method called the L–R method based primarily on the arithmetic of the fuzzy numbers of type L–R restricted to the secant approximations.

The L–R method facilitated us to find, in transient state, the L–R representation of the performance parameters, their support and mode as well as their membership functions which allowed us to represent graphically the performance parameters of the queueing model in study in the space in three dimensions. This is the originality of this scientific work.

In this chapter, a fuzzy queue (or waiting system) analysis method called L–R method, essentially based on the fuzzy L–R arithmetic deduced from secant approximations has been studied. The L–R representation was used to find the performance measures of the studied model. Using this method, the fuzzy functions were computed for the fuzzy expectation model FM/FM/1 and the results are found in L–R representation. Under this representation, the fuzzy results give much more reliable information than the relaxed alpha-cuts method which will be the subject of a future research study. The solutions obtained show that this method has three major advantages: it is short, convenient and flexible compared to other methods used in this field.

We are sure that the L–R method can still help, to obtain results of other similar problems posed in this field in the framework of evaluating the performance parameters of fuzzy Markovian expectation models in transient regime.

An illustrative example was given in the medical field to show the relevance of this study in operational research and in particular in queueing.

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