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Comparison between Methods for Estimating Lengths of G.V.F. Curves in Mild Circular Open Channel with Experimental Lengths

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***Abstract-* A gradually varied flow computation is important for the design of water structures. To design dimensions of channels, examining the depth of water in a gradually varied flow is required. In this study, experiments study were performed to evaluate different methods for calculating the lengths of gradually varied flow curves in mild slope circular open channel. The study was performed in a circular flume with an inner diameter equal to 24.40 cm with a mild slope equal to 0.00083 for different discharges from 2.38 l/sec to 14.08 l/sec. The objective of this study is to compare water surface profiles resulting from experimental measurements with the water surface profiles computed from graphical integration method), direct step method, and Runge-Kutta method for two gradually varied flow curves M3 and M2. The graphical integration method has a minimum percentage error of -0.18% and a maximum of -8.68% for M3 curves, while it has a minimum percentage error of 0.78% and a maximum of 8.37% for M2 curves. The direct step method has a minimum percentage error of ± 0.04% and a maximum of 9.34% for M3 curves while having a minimum percentage error of 0.91% and a maximum of 8.25 for curves M2 curves. For the Runge-Kutta method, the maximum error in water surface for all runs is -5.00% for M3 curves and 1.95% for M2 curves. The Runge-Kutta method perfectly fits in the circular channels.**

***Keywords: Gradually Varied Flow, Circular Channel, Mild Slope, Computation Flow Profile, GIM, DSM,* Runge-Kutta method**

I. INTRODUCTION

A Gradually Varied Flow (GVF) is a non-uniform flow in an open channel with gradual changes in its water surface elevation along with the channel distances due to changing in channel bed slope or existing of an obstruction. The estimation of the GVF profiles has been a major focus of research throughout the last century. Twelve possible curves were first classified and described by Woodward [1]. Posey [2] classified these types according to bed slope as H2, H3, M1, M 2, M3, C1, C3, S1, S2, S3, A2, and A3; where the letters are descriptive of the character of channel slope: H for horizontal slope, M for mild slope, C for critical slope, S for steep slope, and A for adverse slope, respectively; while the numerals 1, 2, and 3 are descriptive of the location of flow profiles in the corresponding regions with respect to critical depth and normal water depth. The water surface profiles for all slopes (12 curves) can be determined using the dynamic equation which is a differential equation. The main objective of computation is to determine the shape of the flow profile. Many attempts have been made either to solve the equation for a few special cases or to introduce assumptions that make the equation amendable to mathematical integration. Many studies have been performed on the GVF in different shapes of channels. The circular channels have been used in sewage collection and transportation systems. The main focus of this research is to solve the dynamic equation of the GVF in circular open channels.

Babbit [3] demonstrated approaches step by step to solve the backwater curves in circular channels. Bakhmeteff [4] proposed a direct integration method that applies to any channel shape. His approximate integration method requires dividing the channel length into short reaches. In an attempt to improve the Bakhmeteff method, Mononobe [5] introduced two assumptions for hydraulic exponents. By these assumptions, the effects of velocity change and friction head are taken into account integrally without the necessity of dividing the channel length into segments. Thus, the Mononobe method efforts a more direct and accurate computation procedure whereby results can be obtained without recourse to successive steps.

Later, Lee [6], Von Seggern [7], and Kiefer [8] suggested new assumptions which result in more satisfactory solutions. Von Seggern [7] introduced a new varied-flow function in addition to the function used by Bakhmeteff, hence, an additional table for the new function is necessary for his method. In Lee's method, however, no new function is required. Chow [9] developed an easily applicable method to all cross-sectional shapes and is accurate in most cases. Chow [9] attempted to apply his approach to circular channels using a numerical integration scheme. Chow [10] broadly classified; there are three methods of computation, namely graphical-integration method, the direct-integration method, and the step method.

Lin and Gray [11], Carnahan et al. [12] calculated the backwater curves by using the Runge-Kutta method. The Runge–Kutta methods are a family of implicit and explicit iterative methods, which include the well-known routine called the Euler Method, used in temporal discretization for the approximate solutions of ordinary differential equations. These methods were developed around 1900 by the German mathematician's Carl Runge and Wilhelm Kutta. Nalluri and Tomlinson [13] used the method of Keifer and Chu (1955) to introduce backwater curves in circular-shaped pipes. Hager [14] demonstrated an explicit approach to solving the backwater curves in circular sewers and drainage conduits. The GVF profile in a circular channel section is computed using the direct step method and the integration method by Zaghloul [15]. A computer simulation of GVF profiles in circular sections was studied by Zaghloul and Shahin, [16]. Unsteady (GVF) wave propagation in a circular pipe was simulated using the modified explicit and characteristics methods by Zaghloul [17]. Rashwan [18] deduced a dimensionless formula to calculate the lengths of curves for circular sections. An accurate approximation of the Froude number for circular channels was demonstrated by Vatankhah and Easa [19, 20], which is part of the (GVF) equation. The approximate F is used in the governing GVF equation to develop an exact analytical solution of this equation using the concept of simplest partial fractions. A comparison of the proposed and approximate solutions for backwater length shows that the error of the existing approximate solution could reach up to 30% for large normal flow depths. A semi-analytical approach for establishing the (GVF) profiles in circular channels through the application of a variable Manning coefficient was presented by Vatankhah [21]. Jan and Chen [22], Jan [23] used the Gaussian hyper geometric function to solve the equation of gradually varied flow in open channels. Niazkar et al. [23] studied two artificial intelligence (AI) models named the artificial neural network (ANN) and genetic programming (GP) were employed to estimate the length of six steady GVF profiles for the first time. According to the literature review, Niazkar et al. [24], the GVF can be solved by: (1) analytical (2) semi-analytical methods, (3) numerical schemes, (4) models of Artificial Intelligence (AI), and (5) methods of optimization.

Shahvand et al. [25] developed the Adomian decomposition method (ADM) as an analytical solution that can validate for modeling the gradually varied flow profiles in circular and parabolic channels than other numerical methods. This method was used to solve ordinary and partial differential equations, linear and nonlinear. It was further extensible to stochastic systems using the Ito integral.

R. Zeidan et al. [26] reported the use of the momentum equation to evaluate the utilization of a smooth inverted semicircular channel end developed to compute the discharge if the end depth is known.

A. Zahran et al. [27] employed fluent software to simulate the flow characteristics of 90° open channel junctions for two geometries using a three-dimensional turbulence model.

However, different solutions to the GVF equation, especially in circular channels, were rarely searched in the past due to their complex geometrical. In the present study, the outcomes of experimental work for M2 and M3 curves in the circular open channel were compared to three methods for solving the dynamic equation as 1) the graphical integration method 2) the direct step method and 3) the 4th order Runge-Kutta method to calculate the water-surface profiles and the lengths of curves.

II. METHODOLOGY

The water surface profiles for all slopes (12 curves) can be determined using the dynamic equation as:

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - F_r^2} \tag{1}$$

Where y is the water depth, x is a one-dimensional space coordinate defined as positive in the direction of flow, S_o is the bed slope, S_e is the energy slope, and F_r is the Froude number

The main objective of the computation for Eq. (1) is to determine the flow profile's shape. There are many methods of solving. Only three methods called (1) graphical integration method, (2) direct step method, and (3) the 4th order Rung Kutta method are used in this study to calculate the water-surface profiles in circular open channels.

A. Graphical Integration Method (GIM)

This method is approximated solution uses a graphical procedure to integrate the dynamic equation of a progressively changing flow as:

$$\frac{dx}{dy} = \frac{1 - \left(Z_c / Z\right)^2}{s_o\left(1 - \left(K_n / K\right)^2\right)} \tag{2}$$

Where Z_c is the section factor at critical depth $\left(Z_c = \sqrt{A_c^3 / T_c}\right)$, A_c is the flow area at critical depth, T_c is the top width at critical depth, Z is the section factor at GVF $\left(Z = \sqrt{A^3 / T}\right)$, A is the flow area at GVF, T is the top width at GVF, K_n is conveyance at normal depth $\left(K_n = A_n R_n^{2/3} / n\right)$, A_n is the flow area at normal depth, R_n is the hydraulic radius at normal depth, n is Manning roughness coefficient $\left(n = AR^{2/3} S_o^{1/2} / Q\right)$, Q is the discharge, R is the hydraulic radius at GVF $\left(R = A/P\right)$, P is the wetted perimeter, K is the conveyance at gradually flow $\left(K = AR^{2/3} / n\right)$

This approach can be used in a variety of situations. It holds for flow in prismatic and non-prismatic channels of any form and slope.

B. Direct Step Method (DSM)

This approach is a straightforward step-by-step procedure for resolving channels of any shape and bed slope. The distance between two water depths in gradually varied flow can be given as:

$$\Delta x = \frac{E_2 - E_1}{S_o - \bar{S}_e} \tag{3}$$

Where Δx is the length of flow profile between the two sections, E_2 is the specific energy at section 2 and E_1 is the specific energy at section 1, and $\bar{S}_e$ is the average value of energy slope.

The specific energy can be written as:

$$E = d\cos\theta + \frac{\alpha V^2}{2g} \tag{4}$$

Where E is the specific energy, d is the normal water depth to bed, θ is the bed slope of the channel, V is the mean velocity, α is the energy coefficient and g is the gravitational acceleration.

Assuming $\alpha = 1$ and $\theta = 0$.

$$E = y + \frac{V^2}{2g} \quad (5)$$

When the Manning formula is used, the friction slope is expressed by:

$$S_e = \frac{n^2 Q^2}{A^2 R^{4/3}} \quad (6)$$

C. Standard Fourth-Order Runge-Kutta method (SRK)

The basic differential equation of GVF, dynamic equation, can be expressed as:

$$\frac{dy}{dx} = F(y) \quad (7)$$

Where $F(y)$ is equal to $\left(F(y) = S_o - S_e / 1 - \left(Q^2 T / g A^3\right)\right)$ and is a function of (y)only for a given S_o , n , Q and channel geometry

If the first-degree ordinary differential equation can be expressed in the form of Eq. (7), the magnitude of y at the neighboring point x can be approximated numerically from known values of y_1 at x_1 (Figure 1) using the Runge-Kutta (Carnahan et al. [12]) method as follows:

$$y_2 = y_1 + 1/6 \begin{pmatrix} \Delta x.F(y_1) + 2\Delta x.F(y_1 + \Delta x.F(y_1)/2) + \\ 2\Delta x.F(y_1 + \Delta x.F(y_1 + \Delta x.F(y_1)/2)/2) + \\ \Delta x.F(y_1 + \Delta x.F(y_1 + \Delta x.F(y_1)/2)) \end{pmatrix} \quad (8)$$

Where y_2 is the approximated value of y at the neighboring point x_2 , y_1 is the approximated value of y at the neighboring point x_1 , Δx is the incremental distance in x direction, and $F(\mathrm{y}_1)$ is functions of y_1

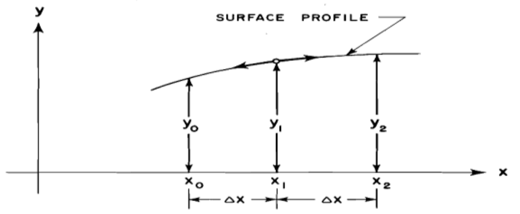

Fig. 1: Schematic for Runge-Kutta method

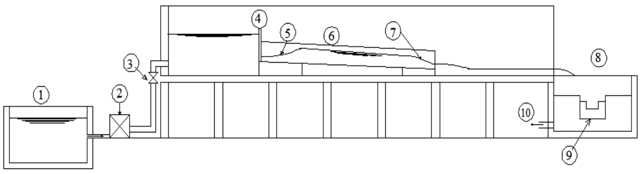

Fig. 2: Schematic diagram for flume

1- Constant water tank
2- 10 HP Pump
3- Control valve
4- Upstream sluice gate
5- M3 curve
6- Perspex pipe with a mild slope
7- M2 curve
8- Inner collected tank
9- Rectangular notch
10- Return pipe

III. EXPERIMENTAL WORKS

The tests were carried out in a flume, as indicated in Figure (2). It is divided into three parts. The first one is used for removing eddies and giving a uniform flow condition. The second part of the flume is a circular pipe with internal diameter (d) equals (0.244m) and mild slope equal to 0.00083 by dropping the end of flume 0.5 cm down the inlet where the flume length is 6.0 m long. The third part is escaping water above a rectangular notch to measure the discharge. A vertical moving gate was made in front of the circular pipe to control the level of water inside the pipe to develop gradually varied flow curves M2 and M3. The discharge measured was varied from min = 2.38 l/sec to max =14.08 l/s. The water depths in the circular pipe can be measured using piezometers installed in the centerline of the circular pipe. The water depths for M3 and M2 curves are recorded for different discharges Q . The recorded discharges are shown in Table (I). The discharges (Q)were computed by using the equation obtained from the weir calibration as:

$$Q = 399.3006 H^{1.3976} \quad (9)$$

Where (Q) is the discharge, (cm^3/sec) and (H) is the corresponding water head above the notch crest, (cm)

Experimental Procedures:

1- Open the valve to allow the flow to pass through the experimental flume, when the flow become uniform, measure the height of water above the weir to compute the discharge;
2- Use the upstream gate to create a free hydraulic jump and the water surface profiles within the flume;
3- M3 is created after the upstream gate and before the hydraulic jump; and
4- M2 water surface profile formed along the channel between the hydraulic jump and the free fall at downstream end of channel

TABLE I
EXPERIMENTAL MEASUREMENTS MAIN VARIABLES

Run No.	**H (cm)**	**Q (cm^3/sec)**	**y_n (cm)**
1	**3.59**	**2382.87**	**5.60**
2	**4.05**	**2820.20**	**6.00**
3	**4.19**	**2957.40**	**6.40**
4	**4.32**	**3086.40**	**6.50**
5	**4.61**	**3379.80**	**6.80**
6	**4.91**	**3691.10**	**7.20**
7	**5.19**	**3988.60**	**7.50**
8	**5.32**	**4128.90**	**8.00**
9	**5.98**	**4862.00**	**7.70**
10	**6.15**	**5056.00**	**8.20**
11	**6.44**	**5392.60**	**8.60**
12	**6.73**	**5735.00**	**9.30**
13	**7.52**	**6697.30**	**9.50**
14	**7.75**	**6985.34**	**10.00**
15	**8.20**	**7571.50**	**10.50**
16	**8.44**	**7869.00**	**10.30**
17	**8.79**	**8329.50**	**11.00**
18	**9.16**	**8823.50**	**11.30**
19	**9.47**	**9243.70**	**12.00**
20	**9.82**	**9724.70**	**13.00**
21	**9.97**	**9933.00**	**12.00**
22	**10.05**	**10044.50**	**12.30**
23	**10.48**	**10650.20**	**12.70**
24	**10.91**	**11265.80**	**14.20**
25	**11.32**	**11861.94**	**14.50**
26	**11.52**	**12155.80**	**14.10**
27	**11.82**	**12600.00**	**14.00**
28	**12.36**	**13412.80**	**14.50**
29	**12.67**	**13884.80**	**15.10**
30	**12.80**	**14084.34**	**15.30**

IV. RESULTS, ANALYSIS AND DISCUSSION

The normal depths (y_n) were read during the experimental work for each run by piezometer tubes, where is found after the hydraulic jump and continues for a certain distance until the curve M2.

The beginning water depth (y_s) and the end water depth (y_e) of M2 and M3 curves can be calculated as follows:

For M3 curves, the initial depths (y_s) were recorded after 5.00 cm from the upstream gate and the final depths, (y_e) are about the initial depths, (y_1) of hydraulic jumps that related to the sequent depth, $(y_2 = y_n)$ which are computed by trial and error from in a dimensionless specific force as:

$$F^* = \frac{(Y_c)^4}{\frac{4}{3}(Y)^{3/2}\left(1-\frac{1}{4}(Y)-\frac{4}{25}(Y)^2\right)} + \frac{2}{3}\left(Y-(Y)^2\right)^{3/2} - \frac{4}{3}(Y)^{3/2}\left(1-\frac{1}{4}(Y)-\frac{4}{25}(Y)^2\right)\left(\frac{1}{2}-Y\right) \qquad (10)$$

Where Y_c = relative critical water depth $(Y_c = y_c/d)$

For M2 curves, the initial depths (y_s) were recorded after the uniform flow (y_n) and the final depths (y_e) were recorded before the brink.

The critical depths were calculated using the following equation:

$$Q^2/g = A_c^3/T_c \qquad (11)$$

Using the Manning formula to calculate the Manning roughness coefficient (n) and by trial and error, modified values of the Manning coefficient (n^*) were used. It gives values for the lengths of the curves as close as possible to the real lengths obtained from experimental work.

The main parameter for water surface profiles for GVF was measured and computed according to Table (II).

TABLE II
CALCULATED MAIN VARIABLES

Run No.	y_c (cm)	n	n*	Run No.	y_c (cm)	n	n*
1	3.82	0.0101	0.00690	16	7.04	0.0099	0.00673
2	4.16	0.0098	0.00670	17	7.25	0.0105	0.00715
3	4.27	0.0107	0.00710	18	7.47	0.0104	0.00708
4	4.36	0.0105	0.00720	19	7.65	0.0110	0.00749
5	4.57	0.0105	0.00700	20	7.85	0.0120	0.00815
6	4.78	0.0108	0.00730	21	7.94	0.0102	0.00697
7	4.97	0.0108	0.00730	22	7.99	0.0106	0.00719
8	5.06	0.0118	0.00800	23	8.23	0.0105	0.00716
9	5.50	0.0093	0.00630	24	8.47	0.0119	0.00810
10	5.61	0.0101	0.00690	25	8.70	0.0117	0.00795
11	5.80	0.0103	0.00704	26	8.82	0.0109	0.00740
12	5.98	0.0113	0.00766	27	8.98	0.0104	0.00709
13	6.48	0.0100	0.00680	28	9.28	0.0103	0.00703
14	6.62	0.0106	0.00720	29	9.44	0.0106	0.00721
15	6.90	0.0106	0.00724	30	9.52	0.0107	0.00700

A. Water surface profiles for M3 curves

For M3 curves, Eq. (2), Eq. (3), and Eq. (8) are used to calculate the lengths of the curves and water surfaces profiles using the GIM, DSM, and SRK method. The lengths of curves (x) versus the depths of water (y) for M3 curves for the three mathematical methods and experimental results are presented in Figure (3). According to Figure (3), the lengths of curves for EXP, GIM, and DSM are close to each other.

The percentage error for the calculated length of the M3 curve deviated than the measured one is, Table (III):

$$\text{Error} = \frac{L_{me} - L_{cal}}{L_{me}} \times 100 \qquad (12)$$

Where L_{me} is the measured length from the laboratory, and L_{cal} is the calculated length of the curve.

Table (III) depicts percentages of the estimated length of the curves in error ranges for different GVF profiles.

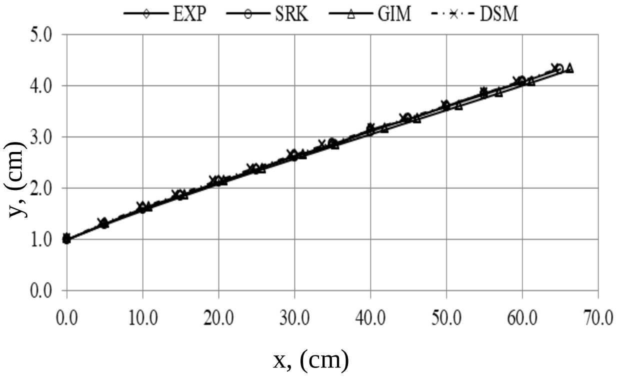

Fig. 3: Water surface profile for M3 curves, run No. 15

TABLE III
CALCULATED LENGTHS FOR GVF M3 CURVES

Run No.	The lengths for curves M_3				
	EXP (cm)	GIM (cm)	%Error	DSM (cm)	%Error
1	35.00	35.35	-1.00	31.73	9.34
2	45.00	45.52	-1.16	42.85	4.78
3	35.00	36.04	-2.97	33.81	3.40
4	35.00	36.41	-4.03	33.36	4.69
5	40.00	42.99	-7.48	39.73	0.68
6	40.00	43.47	-8.68	39.74	0.65
7	40.00	42.69	-6.73	39.02	2.45
8	35.00	37.07	-5.91	34.20	2.29
9	70.00	71.24	-1.77	69.75	0.36
10	60.00	61.25	-2.08	59.22	1.30
11	55.00	56.41	-2.56	59.22	-7.67
12	45.00	46.28	-2.84	44.50	1.11
13	60.00	61.41	-2.35	59.38	1.03
14	60.00	60.50	-0.83	58.59	2.35
15	65.00	66.29	-1.98	64.32	1.05
16	75.00	76.48	-1.97	74.97	0.04
17	75.00	76.25	-1.67	74.54	0.61
18	75.00	76.17	-1.56	74.48	0.69
19	65.00	66.55	-2.38	64.92	0.12
20	50.00	51.21	-2.42	48.95	2.10
21	80.00	81.70	-2.13	80.56	-0.70
22	75.00	77.14	-2.85	75.60	-0.80
23	80.00	81.56	-1.95	80.03	-0.04
24	60.00	60.11	-0.18	58.27	2.88
25	60.00	61.33	-2.22	59.72	0.47
26	75.00	77.29	-3.05	75.23	-0.31
27	90.00	91.81	-2.01	90.61	-0.68
28	90.00	91.95	-2.17	90.87	-0.97
29	85.00	89.22	-4.96	85.60	-0.71
30	90.00	90.45	-0.50	89.50	0.56

Table (III) shows that the error for calculating M3 curves using EXP and GIM leads to having a minimum error of -0.18% and a maximum of -8.68%. While the deviation between EXP and DSM leads to having a minimum percentage error of ± 0.04% and a maximum of 9.34%. Also, Table (III) can be determined which method have the least error rate in the largest number of runs for M3 curves. GIM is considering the closest method to EXP. While the GIM is with the least percentage error to eight runs, DSM is with percentage error to twenty-two runs.

B. *Water surface profiles for M2 curves*

For M2 curves, Eq. (2), Eq. (3), and Eq. (8) are used to calculate the lengths of the curves and water surfaces profiles using the GIM, DSM, and SRK method. The lengths of curves (x) versus the depth of water (y) for M2 curves for three mathematical methods and experimental results are illustrated in Figure (4).

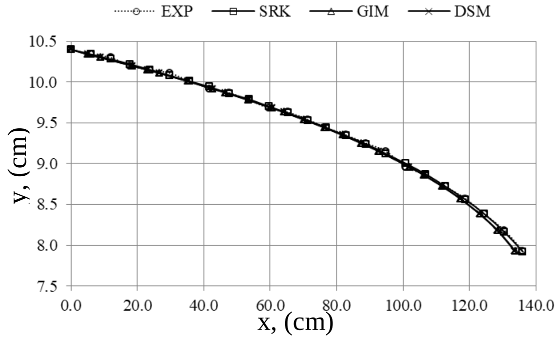

Fig. 4: Water surface profile for M2 curves, run No. 15

According to Figure (4), the lengths of curves for EXP, GIM, and DSM are close to each other.

TABLE IV
COMPARISON FOR LENGTHS FOR M2 CURVES

Run No.	The lengths for curves M2				
	EXP	GIM	%Error	DSM	%Error
1	65.10	59.65	8.37	62.94	3.32
2	65.10	64.30	1.23	62.14	4.55
3	82.60	79.36	3.92	79.02	4.33
4	82.60	77.77	5.85	77.64	6.00
5	88.60	84.90	4.18	84.53	4.59
6	94.40	90.87	3.74	90.31	4.33
7	100.40	95.57	4.81	94.89	5.49
8	118.20	114.12	3.45	113.65	3.85
9	82.60	79.10	4.24	78.58	4.87
10	100.40	95.88	4.50	95.30	5.08
11	100.40	92.36	8.01	92.12	8.25
12	136.00	131.25	3.49	130.59	3.98
13	117.80	115.60	1.87	115.27	2.15
14	136.00	132.36	2.68	131.91	3.01
15	148.00	142.63	3.63	142.09	3.99
16	118.20	115.63	2.17	115.35	2.41
17	153.80	150.24	2.31	149.87	2.56
18	153.80	152.22	1.03	151.93	1.22
19	183.40	180.11	1.79	179.72	2.01
20	229.20	219.80	4.10	219.42	4.27
21	165.60	163.41	1.32	163.17	1.47
22	177.40	176.02	0.78	175.78	0.91
23	189.40	185.86	1.87	185.45	2.09
24	254.10	251.15	1.16	250.79	1.30
25	248.30	244.69	1.45	244.24	1.64
26	234.20	229.88	1.84	227.58	2.83
27	217.50	214.60	1.33	214.19	1.52
28	234.20	224.05	4.33	223.33	4.64
29	248.30	244.00	1.73	248.14	0.06
30	279.20	276.86	0.84	276.61	0.93

The comparison between EXP and GIM leads to having a minimum percentage error of 0.78% and a maximum of 8.37%. The comparison between EXP and DSM leads to having a minimum percentage error of 0.91% and a maximum of 8.25%.

DSM and GIM are considered closest to each other. GIM is with the least percentage error to twenty-eight runs. DSM is with percentage error to two runs.

C. *GVF M3 curves using Runge-Kutta method*

To evaluate that the Runge-Kutta method is applicable for calculating the profiles of gradually varied flow profiles for arbitrary geometric configurations. The method is used to calculate the water depths and compare them with the experimental results using the same distances and compare the water depths. Calculations of the water depths were made by the Runge-Kutta method using values of experimental and were compared with the results obtained using the Graphical Integration technique and direct step method. A comparison of the results obtained using recorded distances is given in Table (V). The percentage error for the calculated depth of water than the measured one is:

$$\text{Error} = \frac{y_{me} - y_{cal}}{y_{me}} \times 100 \qquad (15)$$

Where y_{me} is the measured depth from the experimental work, and y_{cal} is the calculated water depth by SRK

Tables (V) show that the minimum error is 0.00% and the maximum error is -5.00%. The maximum error is limited to -5.00%, indicating the high accuracy of the Runge-Kutta method in accurately calculating the water surface profile with the same length of the M3 curves from the experiment.

D. *GVF M2 curves using Runge-Kutta method*

The comparison between depths of water for GVF by EXP and SRK method showed that the difference in depths is very small and the curves are almost identical.

The comparison of water surfaces profile for M2 curves between SRK and EXP are tabulated in Table (VI). Tables (VI) show that the minimum error is 0.00% and the maximum error is 1.95%. The maximum error is limited to 1.95%, indicating the high accuracy of the Runge-Kutta method in accurately calculating the water surface profile with the same length of the M2 curves.

V. CONCLUSIONS

In this paper, the GIM, DSM, and SRK methods were used to obtain and draw water surface profiles in circular mild channels for M2 and M3 gradually varied flow curves. The results of the presented methods were compared with those obtained by the experimental gradually varied flow in a circular channel with inner diameter $d = 24.40$ cm and with mild slope $S_o = 0.00083$ for thirty runs with discharge values ranging from $Q_{\min} = 2.38$ L/sec to $Q_{\max} = 14.08$ L/sec. By comparing the lengths of M3 and M2 curves from GIM, DSM, and SRK methods with experimental results, the following was obtained:

a. The graphical integration method has a minimum percentage error of -0.18% and a maximum of -8.68% for

M3 curves, while it has a minimum percentage error of 0.78% and a maximum of 8.37% for M2 curves,

b. The direct step method has a minimum percentage error of ± 0.04% and a maximum of 9.34% for M3 curves while having a minimum percentage error of 0.91% and a maximum of 8.25 for curves M2 curves.

c. For the Runge-Kutta method, the maximum error in water surface for all runs is -5.00% for M3 curves and 1.95% for M2 curves.

It was observed that in the same approximation of the three methods the results of the water surface profile in the circular channels were in good agreement with the experimental results.

The Runge-Kutta method perfectly fits in the circular channels. Therefore, the Runge-Kutta can draw water surface profiles and obtain water depths at different points in the circular channels more accurately than the GIM and DSM.

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TABLE V

CALCULATED LENGTHS FOR GVF M3 CURVES USING RUNGE-KUTTA METHOD AND EXPERIMENTAL WATER SURFACE

Run No.	**1**			**2**			**3**			**4**			**5**		
***Hal, x* (cm)**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**
	EXP	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***	
0.0	0.50	0.50	0.00	0.70	0.70	0.00	0.80	0.80	0.00	0.60	0.60	0.00	0.60	0.60	0.00
5.0	0.80	0.84	-5.00	1.00	0.99	1.00	1.10	1.11	-0.91	0.95	0.95	0.00	0.90	0.93	-3.33
10.0	1.11	1.13	-1.80	1.30	1.26	3.08	1.41	1.40	0.71	1.26	1.26	0.00	1.21	1.22	-0.83
15.0	1.41	1.40	0.71	1.50	1.51	-0.67	1.61	1.67	-3.73	1.51	1.54	-1.99	1.51	1.50	0.66
20.0	1.62	1.66	-2.47	1.70	1.74	-2.35	1.92	1.94	-1.04	1.82	1.82	0.00	1.77	1.76	0.56
25.0	1.92	1.92	0.00	2.00	1.98	1.00	2.17	2.19	-0.92	2.07	2.08	-0.48	2.02	2.01	0.50
30.0	2.13	2.17	-1.88	2.20	2.21	-0.45	2.43	2.45	-0.82	2.33	2.35	-0.86	2.23	2.26	-1.35
35.0	2.38	2.42	-1.68	2.40	2.44	-1.67	2.73	2.71	0.73	2.63	2.61	0.76	2.53	2.55	-0.79
40.0				2.70	2.68	0.74							2.84	2.85	-0.35
45.0				2.90	2.93	-1.03									

Run No.	**6**			**7**			**8**			**9**			**10**		
***Hal, x* (cm)**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**	**water depth**		**% error**
	EXP	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***		***EXP***	***SRK***	
0.0	0.60	0.60	0.00	0.60	0.60	0.00	0.80	0.80	0.00	1.00	1.00	0.00	0.90	0.90	0.00
5.0	0.95	0.96	-1.05	0.95	0.96	-1.05	1.20	1.20	0.00	1.20	1.23	-2.50	1.20	1.19	0.83
10.0	1.31	1.28	2.29	1.31	1.28	2.29	1.51	1.55	-2.65	1.46	1.45	0.68	1.46	1.45	0.68
15.0	1.51	1.57	-3.97	1.51	1.57	-3.97	1.86	1.88	-1.08	1.66	1.67	-0.60	1.71	1.71	0.00
20.0	1.82	1.85	-1.65	1.82	1.85	-1.65	2.22	2.21	0.45	1.87	1.87	0.00	1.92	1.95	-1.56
25.0	2.12	2.12	0.00	2.12	2.13	-0.47	2.52	2.52	0.00	2.07	2.07	0.00	2.17	2.19	-0.92
30.0	2.38	2.38	0.00	2.38	2.39	-0.42	2.83	2.83	0.00	2.28	2.27	0.44	2.43	2.42	0.41
35.0	2.63	2.70	-2.66	2.73	2.71	0.73	3.23	3.22	0.31	2.43	2.46	-1.23	2.63	2.65	-0.76
40.0	3.04	3.00	1.32	3.00	3.00	0.00				2.64	2.65	-0.38	2.84	2.87	-1.06
45.0										2.84	2.84	0.00	3.09	3.10	-0.32
50.0										3.05	3.04	0.33	3.35	3.33	0.60

55.0										3.22	3.23	-0.31	3.55	3.57	-0.56
60.0										3.41	3.43	-0.59	3.81	3.82	-0.26
65.0										3.66	3.63	0.82			
70.0										3.82	3.84	-0.52			

Run No.	11			12			13			14			15		
Hal, x	water depth		%	water depth		%	water depth		%	water depth		%	water depth		%
(cm)	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error
0.0	1.00	1.00	0.00	1.10	1.10	0.00	0.90	0.90	0.00	1.00	1.00	0.00	1.00	1.00	0.00
5.0	1.30	1.29	0.77	1.40	1.43	-2.14	1.20	1.18	1.67	1.30	1.30	0.00	1.30	1.31	-0.77
10.0	1.56	1.56	0.00	1.76	1.74	1.14	1.46	1.44	1.37	1.61	1.58	1.86	1.61	1.59	1.24
15.0	1.81	1.82	-0.55	2.01	2.04	-1.49	1.66	1.68	-1.20	1.86	1.85	0.54	1.86	1.86	0.00
20.0	2.07	2.07	0.00	2.32	2.33	-0.43	1.92	1.92	0.00	2.12	2.11	0.47	2.12	2.12	0.00
25.0	2.32	2.31	0.43	2.62	2.61	0.38	2.17	2.15	0.92	2.37	2.36	0.42	2.37	2.37	0.00
30.0	2.53	2.55	-0.79	2.88	2.88	0.00	2.38	2.37	0.42	2.63	2.61	0.76	2.63	2.62	0.38
35.0	2.78	2.78	0.00	3.13	3.16	-0.96	2.58	2.59	-0.39	2.83	2.85	-0.71	2.83	2.86	-1.06
40.0	3.04	3.02	0.66	3.44	3.44	0.00	2.79	2.81	-0.72	3.09	3.09	0.00	3.14	3.11	0.96
45.0	3.24	3.26	-0.62	3.74	3.72	0.53	3.04	3.02	0.66	3.34	3.33	0.30	3.34	3.35	-0.30
50.0	3.50	3.50	0.00				3.25	3.24	0.31	3.55	3.57	-0.56	3.60	3.59	0.28
55.0	3.75	3.74	0.27				3.45	3.45	0.00	3.80	3.81	-0.26	3.85	3.83	0.52
60.0							3.66	3.67	-0.27	4.01	4.00	0.25	4.06	4.07	-0.25
65.0													4.31	4.32	-0.23

Run No.	16			17			18			19			20		
Hal, x	water depth		%	water depth		%	water depth		%	water depth		%	water depth		%
(cm)	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error
0.0	1.10	1.10	0.00	1.10	1.10	0.00	1.10	1.10	0.00	1.20	1.20	0.00	1.30	1.30	0.00
5.0	1.35	1.36	-0.74	1.40	1.39	0.71	1.40	1.39	0.71	1.50	1.51	-0.67	1.65	1.66	-0.61
10.0	1.61	1.60	0.62	1.66	1.66	0.00	1.66	1.65	0.60	1.81	1.80	0.55	2.01	2.00	0.50
15.0	1.86	1.84	1.08	1.91	1.92	-0.52	1.91	1.91	0.00	2.11	2.08	1.42	2.31	2.32	-0.43
20.0	2.07	2.06	0.48	2.17	2.18	-0.46	2.17	2.16	0.46	2.32	2.35	-1.29	2.62	2.63	-0.38
25.0	2.27	2.28	-0.44	2.42	2.42	0.00	2.42	2.40	0.83	2.62	2.62	0.00	2.92	2.93	-0.34
30.0	2.53	2.50	1.19	2.63	2.66	-1.14	2.63	2.63	0.00	2.88	2.88	0.00	3.23	3.23	0.00
35.0	2.73	2.71	0.73	2.93	2.90	1.02	2.88	2.86	0.69	3.13	3.13	0.00	3.33	3.35	-0.60
40.0	2.94	2.92	0.68	3.14	3.13	0.32	3.09	3.09	0.00	3.39	3.38	0.29	3.84	3.82	0.52
45.0	3.14	3.13	0.32	3.34	3.36	-0.60	3.34	3.32	0.60	3.64	3.63	0.27	4.14	4.12	0.48
50.0	3.35	3.33	0.60	3.55	3.59	-1.13	3.55	3.54	0.28	3.90	3.88	0.51	4.40	4.42	-0.45
55.0	3.55	3.54	0.28	3.85	3.83	0.52	3.75	3.77	-0.53	4.15	4.14	0.24			
60.0	3.76	3.75	0.27	4.06	4.06	0.00	4.01	4.00	0.25	4.36	4.39	-0.69			
65.0	3.96	3.96	0.00	4.31	4.30	0.23	4.21	4.23	-0.48	4.66	4.65	0.21			
70.0	4.17	4.17	0.00	4.57	4.54	0.66	4.47	4.46	0.22						
75.0	4.37	4.39	-0.46	4.77	4.79	-0.42	4.67	4.70	-0.64						

Run No.	21			22			23			24			25		
Hal, x	water depth		%	water depth		%	water depth		%	water depth		%	water depth		%
(cm)	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error
0.0	1.40	1.40	0.00	1.20	1.20	0.00	1.20	1.20	0.00	1.20	1.20	0.00	1.30	1.30	0.00
5.0	1.65	1.66	-0.61	1.50	1.49	0.67	1.50	1.48	1.33	1.55	1.56	-0.65	1.65	1.64	0.61
10.0	1.91	1.91	0.00	1.76	1.76	0.00	1.76	1.75	0.57	1.91	1.90	0.52	1.96	1.96	0.00
15.0	2.16	2.15	0.46	2.01	2.02	-0.50	2.01	2.01	0.00	2.21	2.22	-0.45	2.26	2.27	-0.44
20.0	2.37	2.38	-0.42	2.24	2.27	-1.34	2.27	2.26	0.44	2.52	2.53	-0.40	2.57	2.57	0.00
25.0	2.62	2.61	0.38	2.52	2.51	0.40	2.52	2.50	0.79	2.82	2.83	-0.35	2.87	2.86	0.35
30.0	2.83	2.83	0.00	2.73	2.75	-0.73	2.73	2.74	-0.37	3.13	3.13	0.00	3.13	3.14	-0.32
35.0	3.03	3.05	-0.66	2.98	2.99	-0.34	2.98	2.97	0.34	3.43	3.42	0.29	3.43	3.42	0.29
40.0	3.29	3.27	0.61	3.24	3.22	0.62	3.19	3.20	-0.31	3.74	3.71	0.80	3.69	3.70	-0.27
45.0	3.49	3.49	0.00	3.44	3.45	-0.29	3.44	3.43	0.29	3.99	4.00	-0.25	3.99	3.98	0.25
50.0	3.70	3.70	0.00	3.70	3.68	0.54	3.65	3.66	-0.27	4.25	4.28	-0.71	4.25	4.25	0.00
55.0	3.90	3.92	-0.51	3.90	3.91	-0.26	3.90	3.89	0.26	4.55	4.57	-0.44	4.55	4.53	0.44
60.0	4.16	4.14	0.48	4.16	4.14	0.48	4.11	4.11	0.00	4.80	4.87	-1.46	4.81	4.80	0.21
65.0	4.36	4.36	0.00	4.36	4.38	-0.46	4.31	4.34	-0.70						
70.0	4.57	4.58	-0.22	4.57	4.61	-0.88	4.57	4.57	0.00						
75.0	4.82	4.80	0.41	4.87	4.85	0.41	4.82	4.81	0.21						
80.0	5.03	5.03	0.00				5.03	5.04	-0.20						

Run No.	26			27			28			29			30		
Hal, x	water depth		%	water depth		%	water depth		%	water depth		%	water depth		%
(cm)	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error	*EXP*	*SRK*	error
0.0	1.30	1.30	0.00	1.40	1.40	0.00	1.50	1.50	0.00	1.60	1.60	0.00	1.70	1.70	0.00
5.0	1.60	1.60	0.00	1.65	1.67	-1.21	1.75	1.76	-0.57	1.85	1.87	-1.08	1.90	1.95	-2.63
10.0	1.86	1.88	-1.08	1.91	1.92	-0.52	2.01	2.01	0.00	2.11	2.13	-0.95	2.20	2.19	0.45
15.0	2.16	2.15	0.46	2.16	2.17	-0.46	2.26	2.25	0.44	2.36	2.38	-0.85	2.40	2.42	-0.83
20.0	2.42	2.42	0.00	2.42	2.41	0.41	2.47	2.48	-0.40	2.62	2.62	0.00	2.70	2.65	1.85
25.0	2.67	2.67	0.00	2.62	2.64	-0.76	2.72	2.71	0.37	2.87	2.86	0.35	2.90	2.87	1.03
30.0	2.93	2.93	0.00	2.88	2.87	0.35	2.93	2.93	0.00	3.08	3.09	-0.32	3.10	3.09	0.32
35.0	3.18	3.17	0.31	3.08	3.10	-0.65	3.18	3.16	0.63	3.33	3.33	0.00	3.30	3.31	-0.30
40.0	3.44	3.42	0.58	3.29	3.32	-0.91	3.39	3.37	0.59	3.54	3.56	-0.56	3.50	3.53	-0.86
45.0	3.64	3.66	-0.55	3.54	3.55	-0.28	3.59	3.59	0.00	3.79	3.78	0.26	3.75	3.74	0.27
50.0	3.90	3.90	0.00	3.75	3.77	-0.53	3.80	3.81	-0.26	4.00	4.01	-0.25	4.00	3.95	1.25
55.0	4.15	4.14	0.24	3.95	3.98	-0.76	4.00	4.02	-0.50	4.25	4.24	0.24	4.15	4.16	-0.24
60.0	4.36	4.38	-0.46	4.21	4.20	0.24	4.26	4.24	0.47	4.46	4.46	0.00	4.38	4.38	0.00
65.0	4.61	4.63	-0.43	4.41	4.42	-0.23	4.46	4.45	0.22	4.66	4.69	-0.64	4.60	4.59	0.22
70.0	4.87	4.87	0.00	4.67	4.64	0.64	4.67	4.67	0.00	4.92	4.92	0.00	4.80	4.80	0.00
75.0	5.12	5.12	0.00	4.87	4.87	0.00	4.87	4.88	-0.21	5.17	5.14	0.58	5.00	5.01	-0.20
80.0				5.08	5.09	-0.20	5.08	5.10	-0.39	5.38	5.38	0.00	5.20	5.23	-0.58
85.0				5.33	5.32	0.19	5.33	5.32	0.19	5.58	5.60	-0.36	5.50	5.45	0.91
90.0				5.54	5.55	-0.18	5.54	5.54	0.00				5.60	5.67	-1.25

TABLE VI

CALCULATED LENGTHS FOR GVF M2 CURVES USING RUNGE-KUTTA METHOD AND EXPERIMENTAL WATER SURFACE

Run No.	1			Run No.	2			Run No.	3			Run No.	4		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	5.50	5.50	0.00	0.0	5.90	5.90	0.00	0.0	6.30	6.30	0.00	0.0	6.40	6.40	0.00
6.0	5.45	5.43	0.37	6.0	5.85	5.83	0.34	6.0	6.24	6.24	0.00	6.0	6.34	6.34	0.00
12.0	5.35	5.36	-0.19	12.0	5.75	5.76	-0.17	11.5	6.19	6.18	0.16	11.5	6.29	6.27	0.32
18.0	5.26	5.29	-0.57	18.0	5.66	5.68	-0.35	17.5	6.10	6.11	-0.16	17.5	6.20	6.20	0.00
24.0	5.21	5.20	0.19	24.0	5.61	5.60	0.18	23.5	6.05	6.04	0.17	23.5	6.15	6.13	0.33
29.9	5.12	5.12	0.00	29.9	5.52	5.51	0.18	29.5	5.95	5.97	-0.34	29.5	6.05	6.04	0.17
35.9	5.02	5.01	0.20	35.9	5.42	5.41	0.18	35.5	5.86	5.89	-0.51	35.5	5.96	5.95	0.17
41.7	4.88	4.90	-0.41	41.7	5.28	5.30	-0.38	41.5	5.81	5.80	0.17	41.5	5.86	5.86	0.00
47.7	4.78	4.77	0.21	47.7	5.18	5.17	0.19	47.4	5.67	5.70	-0.53	47.4	5.77	5.75	0.35
53.2	4.63	4.62	0.22	53.2	4.98	5.02	-0.80	53.4	5.57	5.59	-0.36	53.4	5.62	5.63	-0.18
59.3	4.39	4.41	-0.46	59.3	4.79	4.82	-0.63	59.2	5.48	5.48	0.00	59.2	5.48	5.50	-0.36
65.1	4.09	4.10	-0.24	65.1	4.54	4.55	-0.22	65.2	5.33	5.34	-0.19	65.2	5.33	5.34	-0.19
								70.7	5.18	5.18	0.00	70.7	5.18	5.15	0.58
								76.8	4.99	4.97	0.40	76.8	4.89	4.88	0.20
								82.6	4.69	4.68	0.21	82.6	4.39	4.41	-0.46

Run No.	5			Run No.	6			Run No.	7			Run No.	8		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	6.70	6.70	0.00	0.0	7.10	7.10	0.00	0.0	7.40	7.40	0.00	0.0	7.90	7.90	0.00
6.0	6.63	6.64	-0.15	6.0	7.03	7.04	-0.14	6.0	7.33	7.34	-0.14	6.0	7.80	7.84	-0.51
11.5	6.59	6.58	0.15	11.8	6.98	6.98	0.00	11.8	7.28	7.27	0.14	12.0	7.77	7.78	-0.13
17.5	6.49	6.51	-0.31	17.8	6.94	6.91	0.43	17.8	7.23	7.21	0.28	17.8	7.72	7.72	0.00
23.5	6.45	6.44	0.16	23.3	6.84	6.84	0.00	23.3	7.14	7.14	0.00	23.8	7.68	7.65	0.39
29.5	6.35	6.37	-0.31	29.3	6.75	6.77	-0.30	29.3	7.04	7.07	-0.43	29.8	7.58	7.58	0.00
35.5	6.30	6.29	0.16	35.3	6.65	6.68	-0.45	35.3	6.95	6.99	-0.58	35.6	7.53	7.51	0.27
41.5	6.21	6.20	0.16	41.3	6.60	6.60	0.00	41.3	6.90	6.90	0.00	41.6	7.44	7.44	0.00
47.4	6.11	6.11	0.00	47.3	6.51	6.51	0.00	47.3	6.80	6.81	-0.15	47.1	7.39	7.37	0.27
53.4	5.97	6.00	-0.50	53.3	6.41	6.40	0.16	53.3	6.71	6.71	0.00	53.1	7.25	7.28	-0.41
59.2	5.87	5.89	-0.34	59.2	6.27	6.29	-0.32	59.2	6.61	6.61	0.00	59.1	7.20	7.19	0.14
65.2	5.78	5.76	0.35	65.2	6.17	6.17	0.00	65.2	6.47	6.49	-0.31	65.1	7.10	7.10	0.00
71.2	5.58	5.61	-0.54	71.0	6.08	6.04	0.66	71.0	6.37	6.36	0.16	71.1	6.96	6.99	-0.43
76.7	5.43	5.44	-0.18	77.0	5.88	5.87	0.17	77.0	6.23	6.21	0.32	77.1	6.86	6.88	-0.29
82.8	5.19	5.21	-0.39	82.5	5.68	5.69	-0.18	82.5	5.98	6.03	-0.84	83.0	6.77	6.76	0.15
88.6	4.89	4.86	0.61	88.6	5.44	5.43	0.18	88.6	5.83	5.83	0.00	89.0	6.62	6.62	0.00
				94.4	5.00	5.04	-0.80	94.4	5.54	5.53	0.18	94.8	6.48	6.47	0.15
								100.4	5.04	5.03	0.20	100.8	6.28	6.29	-0.16
												106.3	6.08	6.08	0.00
												112.4	5.79	5.80	-0.17
												118.2	5.37	5.36	0.19

Run No.	9			Run No.	10			Run No.	11			Run No.	12		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	7.60	7.60	0.00	0.0	8.10	8.10	0.00	0.0	8.40	8.40	0.00	0.0	9.20	9.20	0.00
6.0	7.54	7.53	0.13	6.0	8.03	8.04	-0.12	6.0	8.33	8.33	0.00	5.8	9.15	9.14	0.11
11.5	7.49	7.47	0.27	12.0	7.98	7.97	0.13	12.0	8.28	8.26	0.24	11.8	9.10	9.08	0.22
17.5	7.40	7.39	0.14	17.8	7.93	7.90	0.38	17.8	8.18	8.19	-0.12	17.8	9.01	9.02	-0.11
23.5	7.30	7.31	-0.14	23.8	7.84	7.82	0.26	23.8	8.14	8.12	0.25	23.8	8.96	8.95	0.11
29.5	7.25	7.22	0.41	29.3	7.74	7.75	-0.13	29.3	8.04	8.04	0.00	29.8	8.92	8.88	0.45
35.5	7.11	7.13	-0.28	35.3	7.65	7.67	-0.26	35.3	7.95	7.96	-0.13	35.6	8.82	8.82	0.00
41.5	7.06	7.03	0.42	41.3	7.60	7.58	0.26	41.3	7.85	7.87	-0.25	41.6	8.73	8.74	-0.11
47.4	6.92	6.92	0.00	47.3	7.50	7.49	0.13	47.3	7.75	7.77	-0.26	47.6	8.68	8.67	0.12
53.4	6.77	6.80	-0.44	53.3	7.36	7.38	-0.27	53.3	7.66	7.67	-0.13	53.4	8.58	8.59	-0.12
59.2	6.68	6.67	0.15	59.3	7.26	7.27	-0.14	59.3	7.56	7.55	0.13	59.4	8.49	8.50	-0.12
65.2	6.48	6.51	-0.46	65.2	7.17	7.15	0.28	65.2	7.32	7.34	-0.27	64.9	8.44	8.42	0.24
70.7	6.33	6.32	0.16	71.2	6.97	7.02	-0.72	71.2	7.27	7.30	-0.41	70.9	8.35	8.33	0.24
76.8	6.09	6.06	0.49	77.0	6.88	6.87	0.15	77.0	7.18	7.14	0.56	76.9	8.25	8.23	0.24
82.6	5.64	5.65	-0.18	83.0	6.68	6.69	-0.15	83.0	6.98	6.96	0.29	82.9	8.15	8.12	0.37
				88.5	6.48	6.48	0.00	88.5	6.73	6.76	-0.45	88.9	8.01	8.00	0.12
				94.6	6.19	6.18	0.16	94.6	6.49	6.48	0.15	94.9	7.86	7.88	-0.25
				100.4	5.69	5.67	0.35	100.4	5.99	6.04	-0.83	100.8	7.72	7.74	-0.26
												106.8	7.57	7.59	-0.26
												112.6	7.43	7.42	0.13
												118.6	7.18	7.21	-0.42
												124.1	6.98	6.98	0.00
												130.2	6.64	6.63	0.15
												136.0	6.04	6.05	-0.17

Run No.	13			Run No.	14			Run No.	15			Run No.	16		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.00	9.40	9.40	0.00	0.0	9.90	9.90	0.00	0.0	10.40	10.40	0.00	0.0	10.11	10.10	0.10
6.00	9.31	9.34	-0.32	5.8	9.85	9.84	0.10	6.0	10.34	10.34	0.00	6.0	10.01	10.03	-0.20
11.80	9.27	9.27	0.00	11.8	9.80	9.78	0.20	12.0	10.30	10.28	0.19	12.0	9.97	9.96	0.10
17.80	9.22	9.21	0.11	17.8	9.71	9.71	0.00	17.8	10.20	10.22	-0.20	17.8	9.92	9.90	0.20
23.30	9.13	9.14	-0.11	23.8	9.66	9.65	0.10	23.8	10.15	10.15	0.00	23.8	9.83	9.82	0.10
29.30	9.08	9.06	0.22	29.8	9.57	9.58	-0.10	29.8	10.11	10.08	0.30	29.8	9.73	9.74	-0.10
35.30	8.98	8.99	-0.11	35.6	9.52	9.50	0.21	35.8	10.01	10.01	0.00	35.6	9.68	9.67	0.10
41.30	8.89	8.91	-0.22	41.6	9.43	9.43	0.00	41.8	9.92	9.94	-0.20	41.6	9.59	9.58	0.10
47.30	8.84	8.83	0.11	47.6	9.33	9.35	-0.21	47.6	9.87	9.86	0.10	47.1	9.49	9.50	-0.11
53.30	8.75	8.74	0.11	53.4	9.28	9.27	0.11	53.6	9.78	9.79	-0.10	53.1	9.40	9.41	-0.11
59.20	8.65	8.65	0.00	59.4	9.19	9.18	0.11	59.6	9.68	9.70	-0.21	59.1	9.30	9.31	-0.11
65.20	8.55	8.55	0.00	64.9	9.09	9.10	-0.11	65.4	9.63	9.62	0.10	65.1	9.20	9.20	0.00
71.00	8.46	8.44	0.24	70.9	9.05	9.01	0.44	71.4	9.54	9.53	0.10	71.1	9.11	9.09	0.22
77.00	8.31	8.33	-0.24	76.9	8.90	8.90	0.00	76.9	9.44	9.44	0.00	77.1	8.96	8.97	-0.11
82.50	8.22	8.20	0.24	82.9	8.80	8.80	0.00	82.9	9.35	9.34	0.11	83.0	8.82	8.84	-0.23
88.60	8.07	8.06	0.12	88.9	8.60	8.68	-0.93	88.9	9.25	9.24	0.11	89.0	8.67	8.69	-0.23
94.40	7.93	7.91	0.25	94.9	8.56	8.55	0.12	94.9	9.15	9.12	0.33	94.8	8.53	8.53	0.00
100.40	7.73	7.73	0.00	100.8	8.42	8.42	0.00	100.9	8.96	9.00	-0.45	100.8	8.33	8.34	-0.12
105.90	7.53	7.53	0.00	106.8	8.27	8.26	0.12	106.9	8.86	8.87	-0.11	106.3	8.13	8.13	0.00

112.00	7.24	7.25	-0.14	112.6	8.08	8.09	-0.12	112.8	8.72	8.72	0.00	112.4	7.84	7.85	-0.13
117.80	6.84	6.86	-0.29	118.6	7.88	7.89	-0.13	118.8	8.57	8.56	0.12	118.2	7.44	7.44	0.00
				124.1	7.68	7.67	0.13	124.6	8.38	8.38	0.00				
				130.2	7.34	7.34	0.00	130.6	8.18	8.16	0.24				
				136.0	6.84	6.84	0.00	136.1	7.93	7.92	0.13				
								142.2	7.59	7.56	0.40				
								148.0	6.99	6.97	0.29				

Run No.	17			Run No.	18			Run No.	19			Run No.	20		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	10.90	10.90	0.00	0.0	11.20	11.20	0.00	0.0	11.90	11.90	0.00	0.0	12.89	12.90	-0.08
5.8	10.84	10.84	0.00	5.8	11.14	11.14	0.00	6.0	11.82	11.84	-0.17	11.7	12.84	12.85	-0.08
11.8	10.79	10.78	0.09	11.8	11.09	11.08	0.09	11.8	11.77	11.78	-0.08	16.7	12.75	12.78	-0.24
17.8	10.70	10.72	-0.19	17.8	11.00	11.02	-0.18	17.8	11.73	11.72	0.09	24.7	12.65	12.68	-0.24
23.6	10.65	10.65	0.00	23.6	10.95	10.95	0.00	23.6	11.68	11.66	0.17	34.7	12.60	12.63	-0.24
29.6	10.60	10.58	0.19	29.6	10.90	10.88	0.18	29.6	11.58	11.60	-0.17	39.8	12.56	12.57	-0.08
35.6	10.51	10.51	0.00	35.6	10.81	10.81	0.00	35.4	11.54	11.54	0.00	45.8	12.51	12.51	0.00
41.6	10.46	10.44	0.19	41.6	10.76	10.74	0.19	41.4	11.49	11.47	0.17	51.8	12.47	12.45	0.16
47.6	10.37	10.37	0.00	47.6	10.67	10.67	0.00	47.4	11.40	11.40	0.00	57.6	12.37	12.38	-0.08
53.4	10.32	10.29	0.29	53.4	10.57	10.59	-0.19	53.2	11.30	11.33	-0.27	63.6	12.33	12.32	0.08
59.4	10.23	10.21	0.20	59.4	10.53	10.51	0.19	59.2	11.25	11.26	-0.09	69.4	12.23	12.25	-0.16
65.4	10.13	10.13	0.00	65.4	10.43	10.43	0.00	65.2	11.21	11.18	0.27	75.4	12.18	12.18	0.00
71.2	10.03	10.04	-0.10	71.2	10.33	10.34	-0.10	71.2	11.11	11.10	0.09	81.2	12.09	12.11	-0.17
77.2	9.94	9.95	-0.10	77.2	10.24	10.25	-0.10	77.2	11.02	11.02	0.00	87.2	12.04	12.04	0.00
82.7	9.84	9.86	-0.20	82.7	10.19	10.17	0.20	83.0	10.92	10.94	-0.18	93.2	11.95	11.97	-0.17
88.7	9.75	9.76	-0.10	88.7	10.05	10.07	-0.20	89.0	10.88	10.85	0.28	99.0	11.90	11.89	0.08
94.7	9.65	9.65	0.00	94.7	9.95	9.96	-0.10	95.0	10.78	10.76	0.19	105.0	11.80	11.81	-0.08
100.7	9.55	9.54	0.10	100.7	9.85	9.85	0.00	100.8	10.68	10.67	0.09	111.0	11.71	11.73	-0.17
106.7	9.41	9.41	0.00	106.7	9.71	9.73	-0.21	106.8	10.59	10.57	0.19	117.0	11.61	11.64	-0.26
112.7	9.26	9.28	-0.22	112.7	9.61	9.60	0.10	112.3	10.49	10.48	0.10	123.0	11.57	11.56	0.09
118.6	9.12	9.13	-0.11	118.6	9.47	9.46	0.11	118.3	10.35	10.37	-0.19	128.8	11.47	11.47	0.00
124.6	8.97	8.97	0.00	124.6	9.32	9.30	0.21	124.3	10.25	10.25	0.00	134.8	11.38	11.37	0.09
130.4	8.78	8.79	-0.11	130.4	9.13	9.13	0.00	130.3	10.15	10.12	0.30	140.8	11.28	11.27	0.09
136.4	8.58	8.58		136.4	8.98	8.94	0.45	136.3	9.96	9.99	-0.30	146.6	11.18	11.17	0.09
141.9	8.33	8.34		141.9	8.73	8.72	0.11	142.3	9.86	9.85	0.10	152.6	11.09	11.06	0.27
148.0	7.99	8.00		148.0	8.44	8.43	0.12	148.2	9.67	9.69	-0.21	158.1	10.94	10.96	-0.18
153.8	7.49	7.47		153.8	7.99	8.05	-0.75	154.2	9.52	9.52	0.00	164.1	10.80	10.84	-0.37
								160.0	9.33	9.33	0.00	170.1	10.65	10.71	-0.56
								166.0	9.08	9.10	-0.22	176.1	10.55	10.58	-0.28
								171.5	8.83	8.84	-0.11	182.1	10.41	10.43	-0.19
								177.6	8.49	8.48	0.12	188.1	10.26	10.27	-0.10
								183.4	7.94	7.94	0.00	194.1	10.12	10.11	0.10
												200.0	9.92	9.92	0.00
												205.8	9.73	9.71	0.21
												211.8	9.38	9.40	-0.21
												217.3	9.18	9.19	-0.11
												223.4	8.79	8.80	-0.11
												229.2	8.24	8.23	0.12

Run No.	21			Run No.	22			Run No.	23			Run No.	24		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	11.90	11.90	0.00	0.0	12.20	12.20	0.00	0.0	12.60	12.60	0.00	0.0	14.10	14.10	0.00
5.8	11.88	11.84	0.34	5.8	12.17	12.14	0.25	6.0	12.56	12.54	0.16	5.8	14.07	14.05	0.14
11.8	11.78	11.78	0.00	11.8	12.08	12.08	0.00	12.0	12.47	12.48	-0.08	10.5	14.03	14.00	0.21
17.6	11.69	11.72	-0.26	17.6	12.03	12.02	0.08	17.8	12.42	12.42	0.00	15.6	13.98	13.96	0.14
23.6	11.64	11.65	-0.09	23.6	11.98	11.96	0.17	23.8	12.38	12.36	0.16	19.9	13.93	13.92	0.07
29.6	11.60	11.59	0.09	29.4	11.89	11.89	0.00	29.6	12.28	12.30	-0.16	24.9	13.84	13.87	-0.22
35.4	11.50	11.52	-0.17	35.4	11.84	11.83	0.08	35.6	12.23	12.23	0.00	36.6	13.74	13.75	-0.07
41.4	11.45	11.45	0.00	41.4	11.75	11.76	-0.09	41.4	12.19	12.16	0.25	41.6	13.70	13.70	0.00
47.4	11.41	11.38	0.26	47.2	11.70	11.69	0.09	47.4	12.09	12.09	0.00	49.6	13.60	13.62	-0.15
53.4	11.31	11.30	0.09	53.2	11.60	11.62	-0.17	53.4	12.00	12.02	-0.17	59.6	13.50	13.52	-0.15
59.4	11.22	11.22	0.00	59.2	11.51	11.54	-0.26	59.2	11.95	11.95	0.00	64.7	13.46	13.46	0.00
65.2	11.17	11.15	0.18	65.2	11.46	11.47	-0.09	65.2	11.90	11.88	0.17	70.7	13.36	13.39	-0.22
71.2	11.08	11.06	0.18	71.2	11.42	11.39	0.26	71.2	11.81	11.80	0.08	76.7	13.32	13.33	-0.08
77.2	10.98	10.97	0.09	77.0	11.32	11.31	0.09	77.2	11.71	11.72	-0.09	82.5	13.27	13.26	0.08
83.0	10.88	10.89	-0.09	83.0	11.23	11.22	0.09	83.2	11.62	11.63	-0.09	88.5	13.18	13.19	-0.08
89.0	10.79	10.79	0.00	89.0	11.13	11.13	0.00	89.0	11.57	11.55	0.17	94.3	13.13	13.12	0.08
94.5	10.69	10.70	-0.09	94.0	11.03	11.04	-0.09	95.0	11.48	11.46	0.17	100.3	13.03	13.04	-0.08
100.5	10.60	10.60	0.00	100.8	10.94	10.95	-0.09	101.0	11.38	11.36	0.18	106.1	12.99	12.97	0.15
106.5	10.45	10.48	-0.29	106.3	10.84	10.85	-0.09	106.8	11.28	11.27	0.09	112.1	12.89	12.89	0.00
112.5	10.35	10.37	-0.19	112.3	10.75	10.74	0.09	112.8	11.19	11.17	0.18	118.1	12.80	12.81	-0.08
118.5	10.26	10.24	0.19	118.3	10.65	10.63	0.19	118.3	11.09	11.07	0.18	123.9	12.75	12.73	0.16
124.5	10.11	10.11	0.00	124.3	10.50	10.51	-0.10	124.3	10.95	10.95	0.00	129.9	12.65	12.65	0.00
130.4	9.97	9.96	0.10	130.3	10.36	10.38	-0.19	130.3	10.85	10.83	0.18	135.9	12.56	12.56	0.00
136.4	9.77	9.80	-0.31	136.3	10.26	10.25	0.10	136.3	10.70	10.70	0.00	141.9	12.46	12.47	-0.08
142.2	9.63	9.62	0.10	142.2	10.07	10.10	-0.30	142.3	10.56	10.57	-0.09	147.9	12.37	12.37	0.00
148.2	9.43	9.42	0.11	148.2	9.97	9.93	0.40	148.3	10.41	10.42	-0.10	153.7	12.27	12.28	-0.08
153.7	9.18	9.19	-0.11	154.0	9.78	9.76	0.20	154.2	10.27	10.26	0.10	159.7	12.18	12.18	0.00
159.8	8.89	8.88	0.11	160.0	9.58	9.55	0.31	160.2	10.07	10.08	-0.10	165.7	12.08	12.07	0.08
165.6	8.49	8.47	0.24	165.5	9.28	9.32	-0.43	166.0	9.88	9.88	0.00	171.5	11.93	11.97	-0.34
				171.6	8.99	9.02	-0.33	172.0	9.63	9.64	-0.10	177.5	11.84	11.85	-0.08
				177.4	8.59	8.62	-0.35	177.5	9.38	9.37	0.11	183.0	11.74	11.74	0.00
								183.6	8.99	8.99	0.00	189.0	11.60	11.62	-0.17
								189.4	8.39	8.37	0.24	195.0	11.45	11.48	-0.26
												201.0	11.35	11.34	0.09
												207.0	11.21	11.19	0.18
												213.0	11.01	11.02	-0.09
												218.9	10.82	10.84	-0.18
												224.9	10.62	10.65	-0.28
												230.7	10.43	10.43	0.00
												236.7	10.18	10.17	0.10
												242.2	9.88	9.89	-0.10
												248.3	9.48	9.49	-0.11
												254.1	8.89	8.92	-0.34

Run No.	25			Run No.	26			Run No.	27			Run No.	28		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*			*EXP*	*SRK*	
0.0	14.30	14.30	0.00	0.0	14.00	14.00	0.00	0.0	13.90	13.90	0.00	0.0	14.40	14.40	0.00
4.7	14.28	14.26	0.14	5.0	13.94	13.96	-0.14	5.0	13.85	13.86	-0.07	5.0	14.34	14.35	-0.07
9.8	14.23	14.21	0.14	16.7	13.89	13.91	-0.14	13.0	13.80	13.81	-0.07	16.7	14.29	14.30	-0.07
14.1	14.18	14.17	0.07	21.7	13.85	13.86	-0.07	23.0	13.75	13.77	-0.15	21.7	14.25	14.26	-0.07
19.1	14.14	14.12	0.14	29.7	13.85	13.82	0.22	28.1	13.71	13.72	-0.07	29.7	14.20	14.21	-0.07
30.8	13.99	14.00	-0.07	39.7	13.70	13.70	0.00	34.1	13.61	13.60	0.07	39.7	14.15	14.16	-0.07
35.8	13.95	13.95	0.00	44.8	13.66	13.65	0.07	40.1	13.52	13.54	-0.15	44.8	14.11	14.11	0.00
43.8	13.85	13.87	-0.14	50.8	13.56	13.56	0.00	45.9	13.47	13.46	0.07	50.8	14.01	13.99	0.14
53.8	13.75	13.76	-0.07	56.8	13.47	13.45	0.15	51.9	13.38	13.34	0.30	56.8	13.92	13.93	-0.07
58.9	13.71	13.70	0.07	62.6	13.40	13.40	0.00	57.7	13.28	13.28	0.00	62.6	13.87	13.84	0.22
64.9	13.61	13.64	-0.22	68.6	13.33	13.33	0.00	63.7	13.23	13.21	0.15	68.6	13.73	13.73	0.00
70.9	13.57	13.57	0.00	74.4	13.28	13.26	0.15	69.5	13.14	13.14	0.00	74.4	13.68	13.67	0.07
76.7	13.47	13.50	-0.22	80.4	13.18	13.19	-0.08	75.5	13.09	13.07	0.15	80.4	13.58	13.60	-0.15
82.7	13.43	13.43	0.00	86.2	13.09	13.11	-0.15	81.0	13.00	12.99	0.08	86.2	13.49	13.52	-0.22
88.5	13.33	13.35	-0.15	92.2	13.04	13.04	0.00	87.3	12.90	12.91	-0.08	92.2	13.44	13.45	-0.07
94.5	13.28	13.28	0.00	98.2	12.95	12.96	-0.08	93.3	12.85	12.83	0.16	98.2	13.35	13.37	-0.15
100.3	13.19	13.20	-0.08	104.0	12.90	12.88	0.16	99.3	12.76	12.75	0.08	104.0	13.30	13.29	0.08
106.3	13.14	13.12	0.15	110.0	12.80	12.80	0.00	105.3	12.66	12.66	0.00	110.0	13.20	13.21	-0.08
112.3	13.30	13.04	1.95	116.0	12.71	12.72	-0.08	111.3	12.57	12.57	0.00	116.0	13.11	13.12	-0.08
118.1	12.95	12.96	-0.08	122.0	12.61	12.63	-0.16	117.1	12.47	12.48	-0.08	122.0	13.01	13.04	-0.23
124.1	12.90	12.87	0.23	128.0	12.52	12.54	-0.16	123.1	12.38	12.39	-0.08	128.0	12.92	12.94	-0.15
130.1	12.81	12.79	0.16	133.8	12.47	12.45	0.16	129.1	12.28	12.28	0.00	133.8	12.82	12.85	-0.23
136.1	12.66	12.69	-0.24	139.8	12.38	12.35	0.24	134.9	12.18	12.18	0.00	139.8	12.73	12.75	-0.16
142.1	12.57	12.60	-0.24	145.8	12.23	12.25	-0.16	140.9	12.09	12.07	0.17	145.8	12.63	12.65	-0.16
147.9	12.52	12.50	0.16	151.6	12.13	12.15	-0.16	146.4	11.94	11.96	-0.17	151.6	12.53	12.54	-0.08
153.9	12.43	12.40	0.24	157.6	12.04	12.03	0.08	152.4	11.85	11.84	0.08	157.6	12.44	12.43	0.08
159.9	12.28	12.29	-0.08	163.1	11.94	11.93	0.08	158.4	11.70	11.72	-0.17	163.1	12.29	12.32	-0.24
165.7	12.18	12.18	0.00	169.1	11.80	11.81	-0.08	164.6	11.60	11.58	0.17	169.1	12.20	12.20	0.00
171.7	12.09	12.07	0.17	175.1	11.70	11.68	0.17	170.4	11.41	11.43	-0.18	175.1	12.10	12.08	0.17
177.2	11.94	11.95	-0.08	181.1	11.55	11.54	0.09	176.4	11.26	11.27	-0.09	181.1	11.95	11.94	0.08
183.2	11.85	11.83	0.17	187.1	11.41	11.39	0.18	182.3	11.12	11.10	0.18	187.1	11.76	11.79	-0.26
189.2	11.70	11.69	0.09	193.1	11.26	11.23	0.27	188.3	10.92	10.91	0.09	193.1	11.61	11.63	-0.17
195.2	11.55	11.54	0.09	199.0	11.07	11.06	0.09	194.1	10.68	10.69	-0.09	199.0	11.47	11.46	0.09
201.2	11.41	11.39	0.18	205.0	10.87	10.87	0.00	200.1	10.43	10.44	-0.10	205.0	11.24	11.26	-0.18
207.2	11.21	11.22	-0.09	210.8	10.68	10.67	0.09	205.6	10.13	10.14	-0.10	210.8	11.08	11.05	0.27
213.1	11.02	11.04	-0.18	216.8	10.43	10.42	0.10	211.7	9.74	9.73	0.10	216.8	10.78	10.80	-0.19
219.1	10.82	10.83	-0.09	222.3	10.13	10.15	-0.20	217.5	9.09	9.07	0.22	222.3	10.48	10.51	-0.29
224.9	10.63	10.61	0.19	228.4	9.79	9.78	0.10					228.4	10.09	10.11	-0.20
230.9	10.33	10.35	-0.19	234.2	9.24	9.23	0.11					234.2	9.19	9.20	-0.11
236.4	10.03	10.05	-0.20												
242.5	9.64	9.63	0.10												
248.3	8.99	8.98	0.11												

Run No.	29			Run No.	30		
Hal, x (cm)	water depth		% error	*Hal, x* (cm)	water depth		% error
	EXP	*SRK*			*EXP*	*SRK*	
0.0	15.00	15.00	0.00	0.0	15.20	15.20	0.00
4.7	14.98	14.96	0.13	5.0	15.15	15.16	-0.07
9.8	14.93	14.91	0.13	10.0	15.10	15.11	-0.04
14.1	14.88	14.87	0.07	15.0	15.06	15.07	-0.08
19.1	14.84	14.82	0.13	20.0	15.01	15.02	-0.05
30.8	14.69	14.70	-0.07	25.0	14.97	14.98	-0.09
35.8	14.65	14.65	0.00	30.0	14.92	14.92	0.01
43.8	14.55	14.57	-0.14	35.0	14.88	14.88	-0.03
53.8	14.45	14.46	-0.07	40.0	14.83	14.83	-0.01
58.9	14.41	14.40	0.07	45.0	14.78	14.79	-0.05
64.9	14.36	14.33	0.21	50.0	14.74	14.74	-0.01
70.9	14.27	14.27	0.00	61.7	14.64	14.63	0.08
76.7	14.17	14.20	-0.21	66.7	14.60	14.58	0.11
82.7	14.13	14.13	0.00	74.7	14.50	14.50	0.00
88.5	14.08	14.06	0.14	84.7	14.40	14.40	0.03
94.5	13.98	13.98	0.00	89.8	14.36	14.34	0.13
100.3	13.89	13.91	-0.14	95.8	14.26	14.27	-0.05
106.3	13.84	13.83	0.07	101.8	14.22	14.21	0.05
112.3	13.75	13.75	0.00	107.6	14.12	14.14	-0.13
118.1	13.65	13.67	-0.15	113.6	14.08	14.07	0.04
124.1	13.60	13.58	0.15	119.4	13.98	14.00	-0.15
130.1	13.51	13.49	0.15	125.4	13.93	13.92	0.09
136.1	13.41	13.40	0.07	131.2	13.89	13.85	0.27
142.1	13.32	13.31	0.08	137.2	13.79	13.77	0.16
147.9	13.22	13.22	0.00	143.2	13.70	13.69	0.04
153.9	13.13	13.12	0.08	149.0	13.60	13.61	-0.07
159.9	13.03	13.01	0.15	155.0	13.50	13.53	-0.19
165.7	12.93	12.91	0.15	161.0	13.41	13.44	-0.24
171.7	12.79	12.79	0.00	167.0	13.36	13.35	0.10
177.2	12.69	12.68	0.08	173.0	13.27	13.26	0.05
183.2	12.55	12.56	-0.08	178.8	13.17	13.17	0.01
189.2	12.45	12.43	0.16	184.8	13.08	13.07	0.04
195.2	12.30	12.29	0.08	190.8	12.93	12.96	-0.24
201.2	12.16	12.14	0.16	196.6	12.88	12.86	0.18
207.2	11.96	11.98	-0.17	202.6	12.74	12.75	-0.09
213.1	11.82	11.81	0.08	208.1	12.64	12.64	0.02
219.1	11.62	11.62	0.00	214.1	12.50	12.51	-0.11
224.9	11.43	11.42	0.09	220.1	12.35	12.38	-0.24
230.9	11.18	11.18	0.00	226.1	12.25	12.24	0.11
236.4	10.93	10.92	0.09	232.1	12.11	12.10	0.07
242.5	10.59	10.57	0.19	238.1	11.96	11.94	0.19
248.3	10.09	10.11	-0.20	244.0	11.77	11.76	0.06
254.1	8.89	8.92	-0.34	250.0	11.57	11.57	0.01
				255.8	11.38	11.36	0.13
				261.8	11.13	11.12	0.08
				267.3	10.88	10.85	0.30
				273.4	10.49	10.47	0.17
				279.2	9.99	9.94	0.52