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Peristaltic Flow for A Mixture of Blood and Gas Bubbles in the Vertical Inferior Mesenteric Vein

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Abstract: In this paper, the viscous blood concentration and gas bubbles flow in the peristaltic vertical inferior mesenteric vein (IMV) are studied. The mass, concentration, and Navier-Stokes equations introduce the mathematical simulation of the current problem in the case of long wavelength and low Reynolds number with an analytical solution. The results introduced that the gradient pressure of a mixture of blood flow with gas bubbles performs larger values than that of blood without gas bubbles. Moreover, the blood concentration, velocity of blood and gas bubbles are very sensitive to the small change in blood density ratio, amplitude ratio, and modified Grashof number.

Keywords: Peristaltic flow. Concentration of blood flow. Vertical inferior mesenteric vein. Gas bubbles flow velocity. Modified Grashof number. Pressure gradient.

1 Introduction

Peristaltic transport is the movement of bio-fluid by continuous waves such as muscle contraction and relaxation of the walls of physiological vessels as esophagus, stomach, intestines, usually in the ureters, blood vessels and other hollow tubes [7]. The blood flow inside the vessels, especially veins, is due to the rhythmic contraction and relaxation in the walls of veins, which is called peristalsis. This process of compression and relaxation causes blood to flow toward the heart.

Peristaltic force [3] is an organic siphon that uses episode wave-like gripping action that moves down a vessel and causes the vessel to be stuffed. Likewise, this mechanism also occurs in many applications, including bio-mechanical systems such as dialysis machines, heart-lung machines, finger and roller pumps, blood pump machines and even transport of harmful fluid in nuclear industries [7]. Recently, great attempts have been made to peristaltic flow study of Newtonian fluids with bubbles since a practical and basic usable constituent relationship for all fluids and flow is unavailable. Karapantsios et al. [5] studied the properties of bubbly flow (multiple bubbles flowing with the liquid) and the generating of a single bubble as this abundance of bubbles that is above some threshold is at the origin of DCS. Detection of bubbles in the blood stream is through measuring the gas fraction by In Vitro Embolic Detector (IVED) and an impedance spectroscopy technique. The in vitro phase demonstrated very good resolution and sensitivity to gas fraction and also bubble size changes in bubbly flows [5].

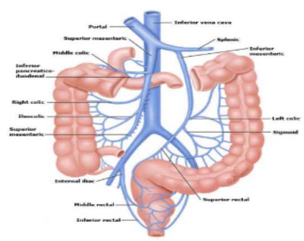

Fig.1. The sketch of colonic venous supply

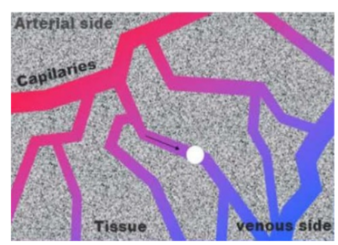

Fig. 2. The behavior of gas bubble inside the bio tissues

In the colon, there are a lot of gas bubbles that arise as a result of ingesting air either from inside the person or from blood diffusion, and these gas bubbles are (oxygen, nitrogen, hydrogen, carbon dioxide and methane). The volume of gas typically ranges from 100 to 200 cubic cm in the large intestine as shown in Fig.1 (6 to 12 cubic inches). Most of the oxygen is lost and there is a rise in the amount of carbon dioxide. In the colon, new gases are added which are produced by bacterial fermentation. As a result of food digestion, E. coli produces most of the gases, hydrogen is the main component. Part of this is absorbed by the blood and then released during respiration through the lungs. hydrogen sulfide, Methane, various sulfur-containing mercaptans and ammonia are other gas products. By a mechanism known as flatulence, excess colonic gases are

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gradually transmitted from the body. Due to their high sulfur content, some foods, such as beans, cabbage, onions, and cauliflower, are known to improve the production of gas [1, 4,11]. Also, there is an unusual condition involving gas bubbles within the colon wall known as Pneumatosis coli (PC) [2]. An arterial bubble joins a tissue capillary network in the initial stage of decompression. With the surrounding tissues, it exchanges gas and begins to grow. It could block the blood supply and cause ischemia [1,6,14,15] if it reaches a critical radius, as shown in Fig. 2.

Based on Fick's law, the rate of change of molar concentration of gas in the bubble equals the molar flux of gas through the bubble surface in the form

$$\frac{1}{\Re\,T}\frac{d}{dt}\left(\frac{4}{3}\pi R^3\rho_g\right) = 4\,\pi R^2 D_T\left(\frac{\partial C}{\partial r}\right)_{r=R} \tag{1}$$

where $\Re$ is Real gas constant, T is the blood temperature in IMV, ρ_g Gas bubble density and D_T is the blood diffusion coefficient in IMV

Colonic gases pass through the colon wall and then travel through the small mesenteric veins that combine to form the great mesenteric veins; the inferior mesenteric vein that pours blood into the splenic vein or the superior mesenteric vein that joins the splenic vein. Finally, they drain blood vertically into the portal vein that connects to the liver as shown in Fig.1 [2].

In general, all veins carry non-oxygenated blood to heart against gravity, so the valves are present in their anatomical structure to prevent the blood from flowing backward in the direction of gravity, thereby permitting unidirectional flow that enhances venous return [4,14].

The existence of Mesenteric venous gas bubbles which go in the direction of blood flow stream caused many health problems such as ischemia, decrease venous return, mainly air embolism as a result of blocking the vein [2]. Recently, many models of fluid flow will be simplified as in [1]. Moreover, the growth of vapour and gas bubbles is studied by Mohammadein et al [6-8] and Srinivasan et al. [12-13]. The basis of our physical model builds on the blood flow in the vertical mesenteric vein. Moreover, the mathematical model is built based on Mohammadein model [7]. Recently there are some models of proposed for simplest solution of problems in fluid mechanics [9-10].

In this paper, the viscous blood concentration and gas bubbles flow in the peristaltic vertical inferior mesenteric vein are studied. The mass, concentration, and Navier-Stokes equations [10] introduce the mathematical simulation of the current problem in the case of long wavelength and low Reynolds number with an analytical solution. The results introduced that the gradient pressure of a mixture of blood flow with gas bubbles performs larger values than that of blood without gas bubbles. Moreover, the blood concentration, velocity of blood [3] and gas bubbles are very sensitive to the small change in blood density ratio, amplitude ratio and modified Grashof number.

In section 1, An introduction to Peristaltic mixture blood flow in a vertical inferior mesenteric vein is illustrated. In section 2. The physical and mathematical model of blood flow in a mesenteric vein is considered in the case of long wavelength only. Authors focus on the study of the mixture of blood and gas bubbles flowing in the vertical great mesenteric veins. The analytical solutions are obtained for concentration distribution of blood, blood velocity and change of pressure under the effect of modified Grashof number, initial concentration, and blood viscosity. In section 3. The discussion of graphs and results is shown in detail. In section 4. The concluded remarks are introduced

2. Analysis

Consider the incompressible and viscous Newtonian blood flow in the vertical inferior mesenteric vein with peristaltic motion.

The mathematical simulation assumes that the blood flows in the vertical inferior mesenteric vein represented in the cylindrical polar coordinates with z measured as along the axis of vein and r is in the radial direction with a sinusoidal wave of small amplitude traveling down its peristaltic wall as in Fig.3. The wall of the vein is given by the following equation

$$H(\bar{z}) = a + b\sin\left(\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t})\right) \tag{2}$$

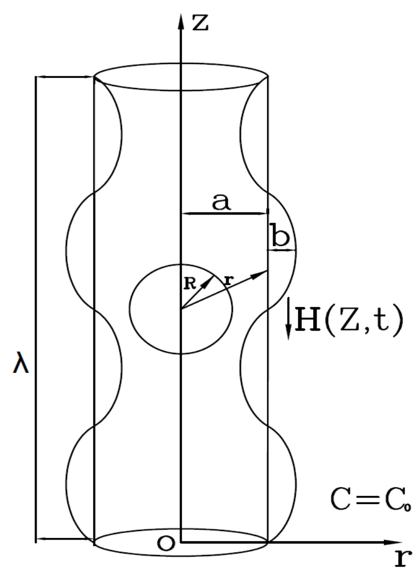

Fig. 3. The sketch of mathematical Problem

Where a is the average radius of the original undisturbed vein, b is the amplitude of the wave, λ is the wavelength and c is the wave speed of wall. Let the velocity components u and w is the radial and axial directions, respectively.

Introducing a wave frame $(\bar{r}, \bar{z})$ moving with velocity c away from the fixed frame $(\bar{R}, \bar{Z})$, the transformations

$$\bar{r} = \bar{R} - c\,\bar{t}, \quad \bar{z} = \bar{Z}, \quad \bar{u} = \bar{U} - c, \ \bar{w} = \bar{W},$$
$$\bar{p}(\bar{r},\bar{z}) = \bar{P}(\bar{\bar{R}},\bar{Z},\bar{t}),$$
$$\bar{c}(\bar{r},\bar{z}) = \bar{C}(\bar{R},\bar{Z},\bar{t}),$$

Where $(\bar{u},\bar{w})$ are the velocity components in the moving frame.

The mathematical model [7] of the physical problem in the wave frame is described by the conservation equations of mass, momentum, and concentration distribution as follows:

Mass equation

$$\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\,\bar{u}) + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \qquad (3)$$

Navier-Stokes equations

$$\rho\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right) = -\frac{\partial\bar{P}}{\partial\bar{r}} + \eta\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{u}}{\partial\bar{r}}\right) - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right\} \qquad (4)$$

$$\rho\left(\bar{u}\frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right) = -\frac{\partial\bar{P}}{\partial\bar{z}} + \eta\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{w}}{\partial\bar{r}}\right) + \frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right\} + \rho g\alpha(\bar{C} - C_0) \qquad (5)$$

Concentration equation

$$\bar{u}\frac{\partial\bar{C}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{C}}{\partial\bar{z}} = D_T\left(\frac{\partial^2\bar{C}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{C}}{\partial\bar{r}} + \frac{\partial^2\bar{C}}{\partial\bar{z}^2}\right) \qquad (6)$$

where P is the pressure, η is the viscosity of the blood, C is the concentration of blood in the vertical IMV, g is the gravitational acceleration, α is the concentration coefficient of volumetric expansion, D_T is the coefficient of mass diffusivity, and ρ is the density of the blood.

Introducing the dimensionless variables as follows

$$r = \frac{\bar{r}}{a}, \ z = \frac{\bar{z}}{\lambda}, u = \frac{\bar{u}}{c\delta}, w = \frac{\bar{w}}{c}, \delta = \frac{a}{\lambda}, H = \frac{h}{a}, \quad h = 1 + e\sin(2\pi z),$$

$$e = \frac{b}{a}, \ G_c = \frac{\rho g\alpha a^2 C_0}{\eta c}, p = \frac{a^2}{c\lambda\eta}\bar{p}, \ \Phi = \frac{C - C_0}{C_0}, R_e = \frac{\rho c a}{\eta} \qquad (7)$$

where, δ is the wave number, R_e is the Reynolds number, e is the amplitude ratio, G_C is the modified Grashof number and Φ is the blood concentration distribution. Substituting from relations (7) into the equations (3-6), then

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial z} = 0, \qquad (8)$$

$$R_e\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial r} + \delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \delta^2\frac{\partial^2 u}{\partial z^2}\right\}, \qquad (9)$$

$$R_e\delta\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \delta^2\frac{\partial^2 w}{\partial z^2} + G_c\,\Phi,, \qquad (10)$$

$$c\,a\,\delta\left(u\frac{\partial\Phi}{\partial r} + w\frac{\partial\Phi}{\partial z}\right) = D_T\left(\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \delta^2\frac{\partial^2\Phi}{\partial z^2}\right), \qquad (11)$$

The above system is called the non-dimensional system. The physical problem is solved in case of large wavelength ($\delta \ll 1$) and the Reynolds number is quite small ($R_e \to 0$), then the equations (9-11) become

$$\frac{\partial P}{\partial r} = 0 \qquad (12)$$

$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + G_c\,\Phi \qquad (13)$$

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} = 0 \qquad (14)$$

The blood flow in the vertical inferior mesenteric vein is studied from the point of view of the existence of gas bubbles. The mixture of blood and gas bubbles in a vertical inferior mesenteric vein is formulated by the system equations (12-14) in terms of pressure, blood flow velocity, and concentration. This system is studied under the following dimensionless boundary conditions.

$$At \quad r = R_0, \quad \frac{\partial w}{\partial r} = 0 \qquad (15)$$

$$At \quad r = h, \quad w = A_0 \qquad (16)$$

$$At \quad r = R_0 \quad \frac{\partial\phi}{\partial r} = -\frac{\dot{R}_0\rho_g}{\Re\,T\,D_T C_0} \qquad (17)$$

$$At \quad r = h, \quad \Phi = \Phi_0 \qquad (18)$$

Where Φ_0 initial blood concentration distribution at the wall of IMV. The solution of concentration equation (14) under the above boundary conditions (17-18) has the form

$$\Phi(r,z) = C_1 Ln\,r + C_2,$$

Or

$$\Phi(r,z) = \Phi_0 + C_1 Ln\,\frac{r}{h}, \qquad (19)$$

$$C_1 = -\frac{R_0\dot{R}_0\rho_g}{\Re\,T\,D_T C_0}, \text{ and } C_2 = \Phi_0 - C_1\,Ln\,h \qquad (20)$$

Substituting by Eqn. (19) into the Eqn. (13) and solving Eqn. (13) with the boundary conditions (15-16), then the blood velocity becomes

$$w(r,z) = \frac{\left(\frac{dP}{dz}\right)}{4}r^2 - G_C\left(\frac{r^2}{4}\Phi_0 + C_1\left(\frac{r^2}{4}Ln\,\frac{r}{h} - \frac{r^2}{4}\right)\right) + C_9\text{Ln}\,r + C_{10} \qquad (21)$$

Where,

$$C_9 = -\frac{\left(\frac{dP}{dz}\right)}{2}{R_0}^2 + C_{91},$$

$$C_{91} = G_C\left(\frac{{R_0}^2}{2}\Phi_0 + C_1\left(\frac{{R_0}^2}{2}Ln\left(\frac{R_0}{h}\right) - \frac{{R_0}^2}{4}\right)\right),$$

$$C_{10} = A_0 - \frac{\left(\frac{dP}{dz}\right)h^2}{4} + G_C\left(\frac{h^2}{4}\Phi_0 - \frac{h^2}{4}C_1\right) - C_9\,Ln(h) \quad (22)$$

And the mixture of blood velocity and dissolved gas bubbles in the vertical inferior mesenteric vein becomes

$$w(r,z) = A_0 + \left(\frac{dP}{dz}\right)\left(\frac{r^2-h^2}{4} - \frac{R_0^2}{2}Ln\left(\frac{r}{h}\right)\right) + C_{91}Ln\left(\frac{r}{h}\right) + G_C\left(\frac{h^2-r^2}{4}\Phi_0 + C_1\left(\frac{r^2-h^2}{4} - \frac{r^2}{4}Ln\left(\frac{r}{h}\right)\right)\right) \quad (23)$$

The dimensionless volume flow rate "q" through vein is given by

$$q_m = 2\int_{R_0}^{h} wrdr \quad (24)$$

The pressure gradient in the vertical IMV for the mixture of blood and gas bubbles on basis of equations (23-24) has the form

$$\frac{dP}{dz} = \frac{1}{c_{44}}\left\{\frac{q}{2} - A_0\left(h^2 - R_0^{\,2}\right) + c_{91}\left(\frac{(h^2-R_0^{\,2})}{4} + \frac{R_0^{\,2}}{2}Ln\left(\frac{R_0}{h}\right)\right) + G_c\left\{\left(\frac{(h^4-R_0^{\,4})}{16} - \frac{h^4}{8} + \frac{h^2R_0^{\,2}}{4}\right)\Phi_0 + C_1\left(\frac{h^4}{8} - \frac{h^2R_0^{\,2}}{8} - \frac{5\,(h^4-R_0^{\,4})}{64} - \frac{R_0^{\,4}}{4}Ln\left(\frac{R_0}{h}\right)\right)\right\}\right\} \quad (25)$$

Where,

$$c_{44} = \frac{h^2R_0^{\,2}}{4} - \frac{3R_0^{\,4}}{16} - \frac{h^4}{16} + \frac{R_0^{\,4}}{4}Ln\left(\frac{R_0}{h}\right).$$

On the basis of continuity Eqn. (3) in a cylindrical coordinate, the bubble velocity can be written as

$$w(r,t) = \frac{\epsilon\,R\,\dot{R}}{r}. \quad (26)$$

Where $\epsilon = 1 - \frac{\rho_g}{\rho_b}$ is the density ratio. From equations (23) and (25-26), the velocity of gas bubble in the vertical IMV has the form

$$\dot{R}(R,t) = \frac{1}{\epsilon}\left\{A_0 + \frac{dP}{dz}\left(\frac{R^2-h^2}{4} - \frac{R_0^2}{2}Ln\left(\frac{r}{h}\right)\right) + C_{91}Ln\left(\frac{R}{h}\right) + G_C\left(\frac{h^2-R^2}{4}\Phi_0 + C_1\left(\frac{R^2-h^2}{4} - \frac{R^2}{4}Ln\left(\frac{R}{h}\right)\right)\right)\right\}. \quad (27)$$

We can also determine the stream function by using $w = \frac{1}{r}\frac{\partial\psi}{\partial r}$ and $\psi = 0$ at $r = 0$, then the stream function will be in the form

$$\psi(r,z) = \frac{r^4}{16}\left(\frac{dP}{dz}\right) - G_C\left(\frac{r^4}{16}\Phi_0 + C_1\left(\frac{r^4}{4}Ln\left(\frac{r}{h}\right) - \frac{r^4}{8}\right)\right) + C_9\left(\frac{r^2}{2}Ln(r) - \frac{r^2}{4}\right) + \frac{r^2}{2}C_{10}, \quad (28)$$

3. Discussion of Results

The physical problem is described by mass, momentum, and concentration equations (3-6) respectively for a Newtonian viscous fluid flow through the vertical inferior mesenteric vein with gas bubbles process. The problem is solved analytically to obtain the velocity and concentration distribution of the blood flow under the effect of some physical parameters such as, modified Grashof number G_c, initial concentration of blood C_0, density ratio ϵ, and amplitude ratio e. The non- linear system (3-6) is transformed into another nondimensional system (8-11) with the existence of non-dimensional numbers like Grashof number. The system (12-14) is obtained for the long wavelength $(\delta \to 0)$. The concentration distribution of blood flow in the vertical inferior mesenteric vein with two-phase density is obtained by relation (19). The blood flow velocity is given by Eqn. (21) Finally, the bubble velocity in terms of bubble radius is given by Eqn. (27).

The gradient pressure of blood with gas bubbles is given by Eqn. (27). Fixed values of physical parameters for the mathematical simulation of present problem tabulated.

$$\rho_g = 1.37\,kg.m^{-3}, \rho_b = 1060\,kg.m^{-3}, r_{vein} = 1\text{mm},$$

$$\Re = 8.3144N.m/mol\,\text{K}, T = 37C,$$

$$D_T = 2 * 10^{-8}m^2.s^{-1}, z = 0.10m,,$$

The input data; in Mathematica program which are used in calculations take the form as in the following Table.

Symbol	q	A_0	Φ_0	R_0	$\dot{R}_0$
Value	0.05	1	2	10^{-5}	0.8

The mathematical simulation of a present problem in the form of figures is as follows:

The Concentration Φ in terms of r for different values of initial concentration C_0, amplitude ratio e and gas density ρ_g is shown in Figs. 4-6 respectively. It is observed that the concentration Φ is inversely proportional with initial concentration C_0 and amplitude ratio e. The high blood concentration has more mass, the less Amplitude of wave for a peristaltic vein. 0n contrary, it is proportional with gas density ρ_g for different gases in the veins.the gas bubble with high density, its mass increases, concentration increases. The blood flow velocity in the vertical IMV w (r, t) in terms of r for different values of modified Grashof number G_C, initial concentration C_0 and amplitude ratio e is shown by Figs. 7-9 respectively. It is observed that the blood velocity is proportional with all values of Grashof number and amplitude ratio e. If there is no peristalsis, zero amplitude ratio, the blood flows slowly. The more amplitude of peristaltic vein, the faster blood flow moves and overcomes gravity. But the blood velocity is inversely proportional with initial concentration $C_0$and viscosity of blood η.

The gas bubble flow velocity in the vertical vein $\dot{R}$(R, t) in terms of r for different values of modified Grashof number G_C ,amplitude ratio e and initial concentration C_0 is shown by Figs. 10-12 respectively. It is observed that the bubble velocity is proportional with all values of Grashof number and amplitude ratio e. It is inversely with initial concentration C_0.This is the same proportionality of blood velocity because gas bubble and blood are in the same flow.

The gradient pressure of blood and gas bubbles as shown in Figs. 13-14 respectively. It is observed that the gradient pressure of a mixture of blood with gas bubbles is proportional with modified Grashof number G_C and inversely proportional with blood viscosity η.The more viscous blood needs more pressure to push it.

It is observed that the pressure gradient is directly proportional with the initial concentration of blood C_0 at the peaks and troughs of the sin wave function. More blood concentration needs more pressure from peristaltic wall of vein.

The effect of trapping can be discussed as shown in Figs. 15, where the streamlines are plotted for different physical parameters of C_0. In Fig. 15, it is clear that the size of trapped bolus decreases by the increasing C_0.

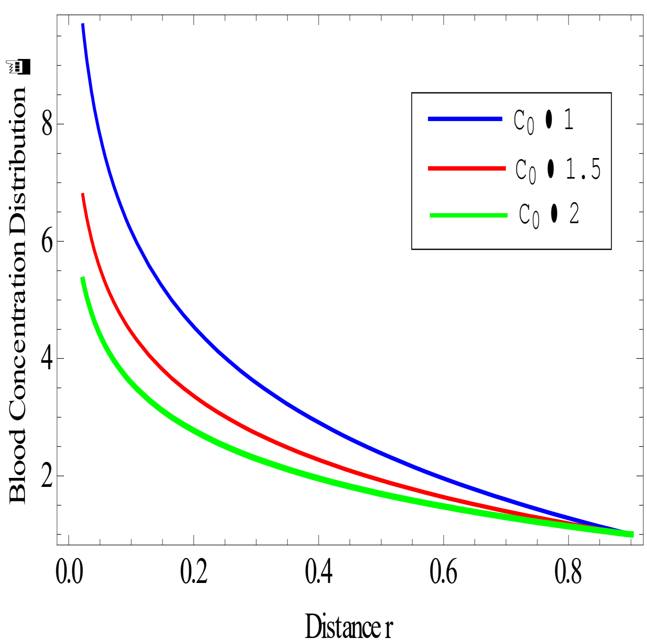

Fig. 4. The concentration of a mixture of blood in the in vert. IMV for different values of “C_0”

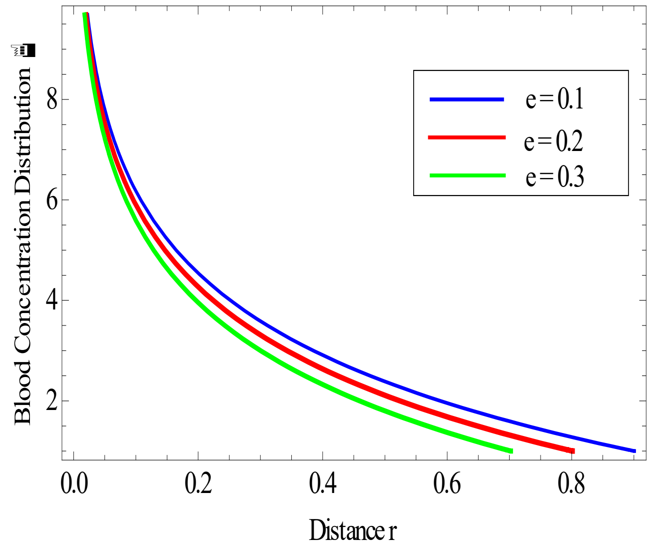

Fig. 5. The concentration of blood in vert. IMV for different values of “e”

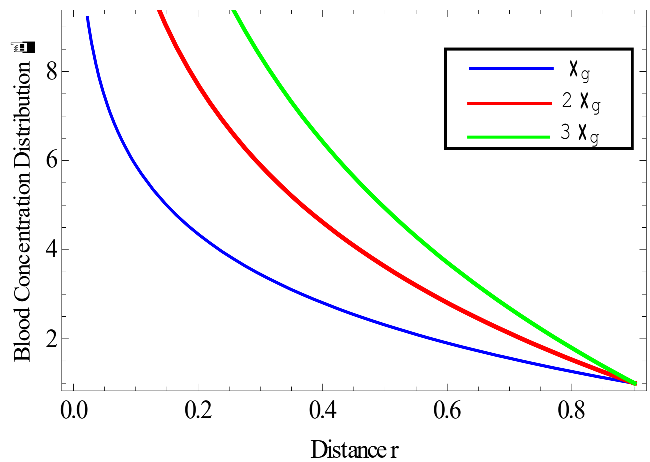

Fig. 6. The concentration of blood in vert. IMV for different values of “ρ_g”

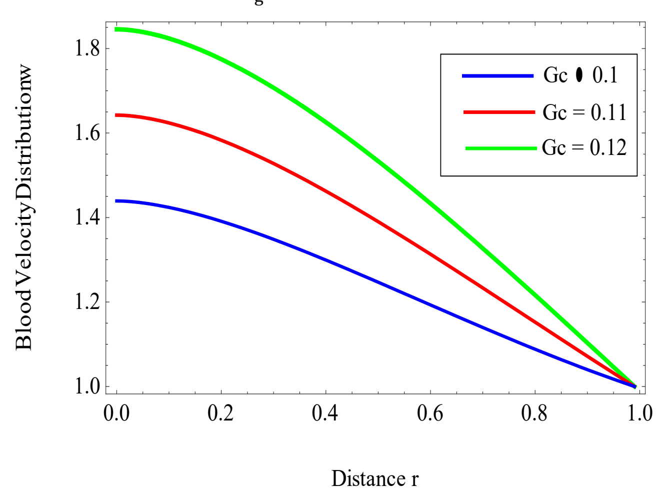

Fig. 7. Blood velocity in vert. IMV for different values of “G_C”

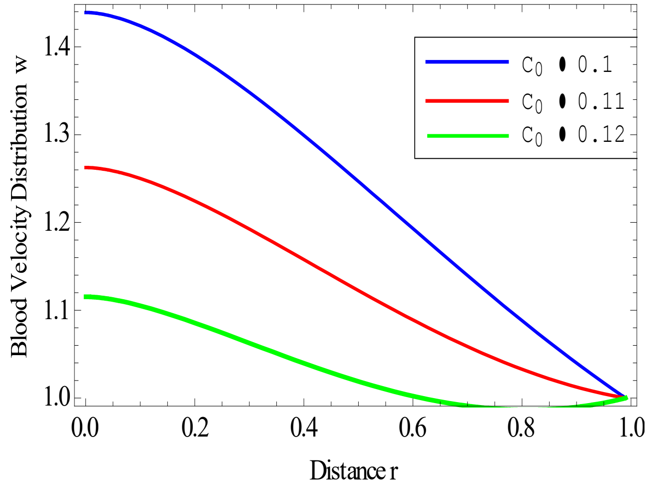

Fig. 8. Blood velocity in vert. IMV for different values of "C_0"

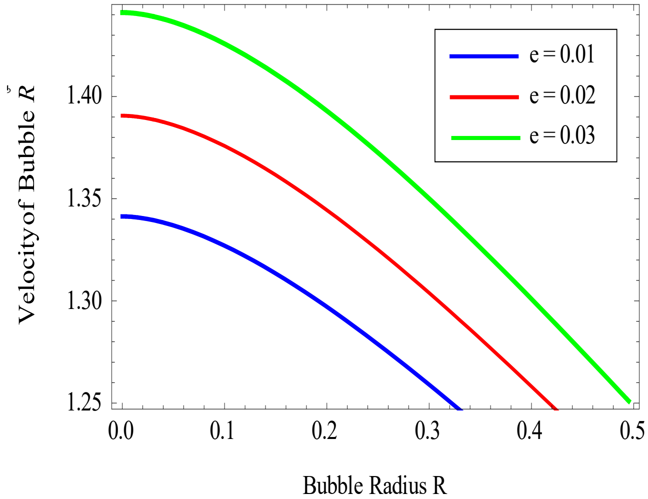

Fig. 11. Gas bubble velocity in vert. IMV for different values of "e"

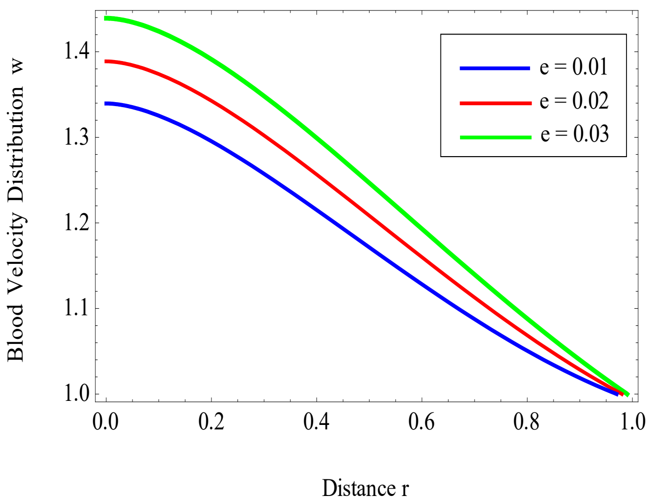

Fig. 9. Blood velocity in vert. IMV for different values of "e"

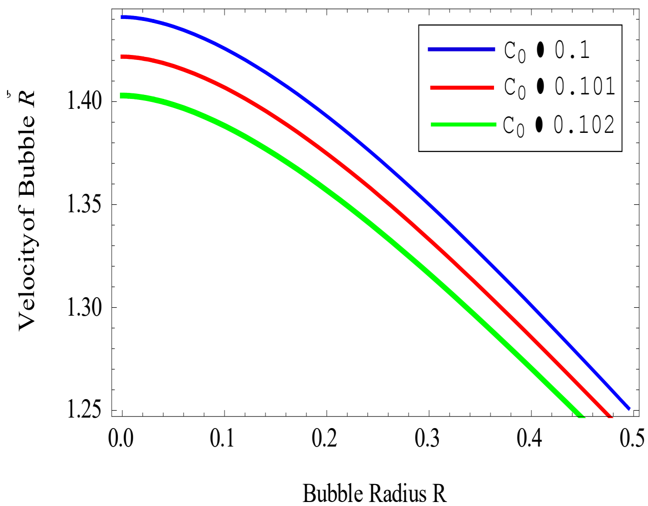

Fig. 12. Gas bubble velocity in vert. IMV for different values of "C_0"

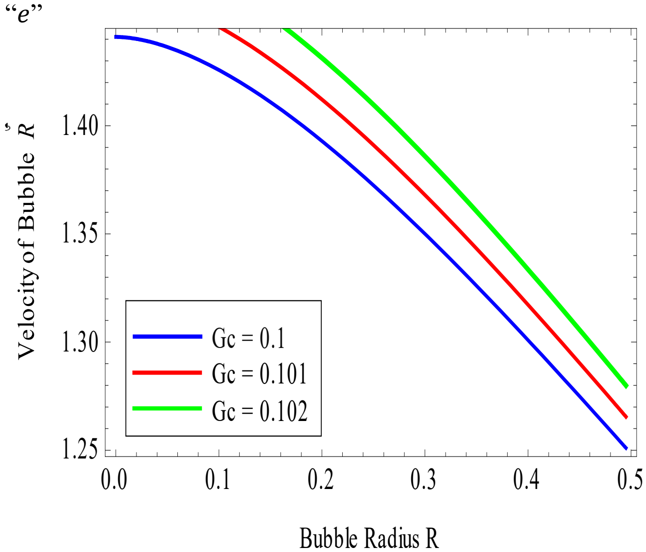

Fig. 10. Gas bubble Velocity in vert. IMV for different values of "G_C"

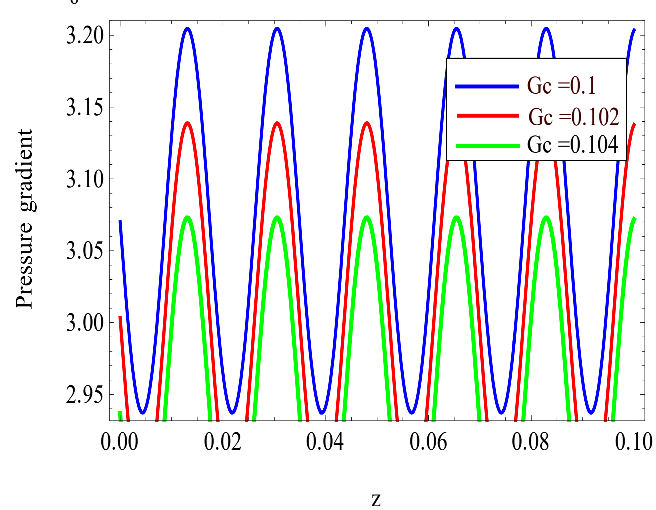

Fig. 13. The pressure gradient of mixture of blood and gas bubbles in vert. IMV for different values of "G_C" with fixed values $C_0 = 0.2, e = 0.01$

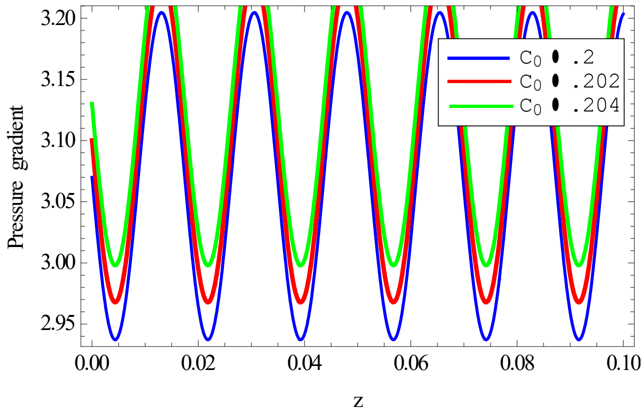

Fig.14. The pressure gradient of a mixture of blood and gas bubbles in vert. IMV for different values of "C_0" with fixed values $G_C = 0.1, e = 0.01$

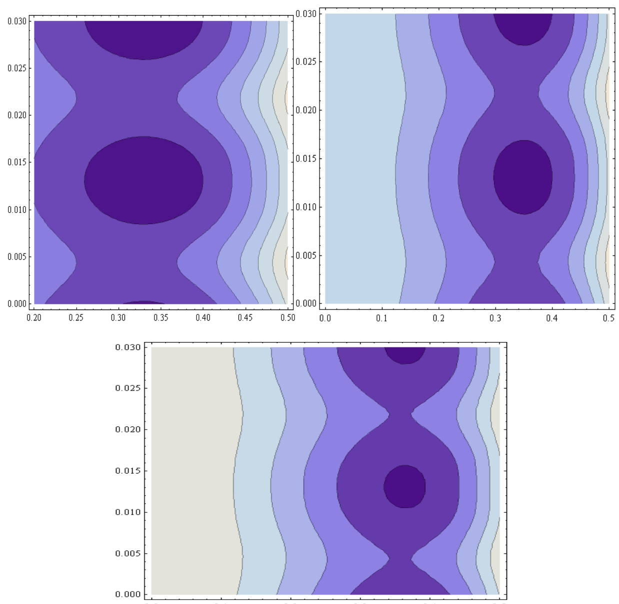

Fig. 15. Stream function for different values of concentration $C_0 = 0.5, 0.52, and\ 0.54$ respectively

4. Conclusions

The mixture of blood and gas bubbles vertical inferior mesenteric vein is studied under the effect of peristaltic motion with long wavelength and low Reynolds number. Other physical parameters, taking into account the effect of modified Grashof number, amplitude ratio, and initial concentration. The discussion of results concluded the following remarks:

1. The concentration is proportional with modified Grashof number, gas density, initial bubble radius and initial bubble velocity. On contrary, the concentration is inversely proportional with amplitude ratio, viscosity, and initial concentration.

2. The velocity of blood flow and gas bubbles are proportional with gas density, modified Grashof number and amplitude ratio and inversely blood viscosity in the vertical IMV.

3. The concentration, velocity of blood flow and gas bubbles are very sensitive to the small change in all physical parameters.

4. The gradient pressure of a mixture of blood and gas bubbles decreases by the increasing of modified Grashof number G_C and inversely with blood viscosity. Moreover, more blood concentration needs more pressure from the wall of vein to push it.

5. The streamlines are affected by limited values of Grashof number and initial concentration.

6. The concentration of blood Φ changes through the vertical IMV according to the contraction and relaxation of the peristaltic wall of vein and affected by amplitude ratio and vein length.

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Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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