Closure Structures as fixed points of some Galois connections *

Manuel Ojeda-Hernández[1][0000−0003−4785−6802], Inma P. Cabrera[1][0000−0001−5129−0085], Pablo Cordero[1][0000−0002−5506−6467], and Emilio Muñoz-Velasco[1][0000−0002−0117−4219]

Universidad de Málaga, Andalucía Tech, Málaga, Spain
{manuojeda,ipcabrera,pcordero,ejmunoz}@uma.es

The starting point of this work is [2], where the fuzzy powerset of a fuzzy lattice A, the set of isotone mappings on A and the set of isotone total relations on A were proved to be related by three fuzzy Galois connections such that fuzzy closure systems, fuzzy closure operators and the so called strong fuzzy closure relations are fixed points. The final part of that paper studied the commutativity of the diagrams formed by these mappings.

The next step would be to include closure systems as crisp sets in this problem. Since the powerset of A is a partially ordered set, this addition might be done in two main ways, either we consider the 1-cut of the preposets and study the crisp problem, or consider the "fuzzification" of the crisp order relation.

The restriction of the results in [2] to the 1-cut behave properly within the new paradigm and we focus on the definition of the two fuzzy Galois connections between $(2^A, \subseteq)$ and $(\mathrm{Ext}(L^A), S)$, and $(2^A, \subseteq)$ and $(\mathrm{Isot}(A^A), \preceq)$, where $\preceq$ is the pointwise order. These two conjectures hold substituting the set of all fuzzy sets by the set of all extensional fuzzy sets, which is not a strong restriction since every fuzzy closure system is an extensional set. This study was carried out in [1], which is currently submitted to journal. The study of the commutativity of the whole diagram of fuzzy Galois connections is still an open problem. Some partial solutions to this problem are the restriction to the fuzzy closure structures and the use of a Heyting algebra as the underlying residuated lattice.

References

1. M. Ojeda-Hernández, I. P. Cabrera, P. Cordero, and E. Muñoz-Velasco. Fuzzy closure structures as formal concepts II. *Submitted to journal*.
2. M. Ojeda-Hernández, I. P. Cabrera, P. Cordero, and E. Muñoz-Velasco. Fuzzy closure structures as formal concepts. *Fuzzy Sets and Systems*, 2022. doi:10.1016/j.fss.2022.12.014.

* This research is partially supported by the FPU19/01467 (MCIU) internship and the research project PID2021-127870OB-I00 (MCIU/AEI/FEDER, UE)