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Frequency response of Batchelor vortex

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We carried out a frequency response analysis of Batchelor vortex model in two different spatial configurations: punctual and annular jet. The theoretical base flow corresponds to the experimental setting of a wing model with airfoil NACA0012 for a chord-based Reynolds number Re_c=40000 and angle of attack of α=9°. We found that Batchelor model presents a gain in the annular jet configuration higher than the punctual jet for a pair of parameters k and ω_f. The results of this research work will be used to propose future candidates of active control.

1 Introduction

Aircraft generate wingtip vortices due to the finite length of their wingspans and the pressure difference between both sides of the wing model (Spalart, 2003). The generation of lift is associated with the presence of vortices. One of the best and widely extended theoretical descriptions of these trailing vortices corresponds to Batchelor's model (Batchelor, 1964) and its simplification called q-vortex. The stability of the q-vortex was studied by the pioneering work of Mayer and Powell (1992), among other studies that have been carried out up to the present day. These vortices are very stable in the range of Reynolds numbers (Re) and vortex intensity (q) corresponding to real aircraft and can, therefore, remain for a long time on airport runways (Jacquin and Pantano, 2002). This long-term presence on airport runways diminishes the number of take-off and landing operations.

2 Numerical methodology and results

The base flow has been obtained by adjusting Batchelor's model from 3D-2C PIV data. The experimental data has been divided into three areas: near (NF), intermediate (IF) and far-field (FF) since it provides a better understanding in terms of vorticity decay (Gutierrez-Castillo *et al.*, 2022).

We carried out the frequency response or the three-dimensional stability of the q-vortex, $[\boldsymbol{U}(r,\theta), P(r,\theta)]^T$, using two-dimensional simulations of the linear equations forced by a given out-of-plane (axial) wavenumber (Blanco-Rodríguez *et al.*, 2016). We will solve the equations in a rectangular periodic domain of size L_x and L_y and periodic boundary conditions. The cartesian base flow $[\boldsymbol{U}(x,y), P(x,y)]^T$ has infinitesimal three-dimensional perturbations for the velocity $\boldsymbol{u}(x,y,z) = (u_x, u_y, w)$ and pressure $p(x,y,z)$ governed by the forced incompressible linearised Navier-Stokes (LNS), which can be written as

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla\boldsymbol{U} + \boldsymbol{U}\cdot\nabla\boldsymbol{u}) = -\nabla p + \frac{1}{Re}\Delta\boldsymbol{u} + \mathbf{f}, \tag{2}$$

where $\mathbf{f}$ denotes a volumetric forcing function. We used forcing jets that act only in the streamwise direction ($\mathbf{f} = F_z(x,y,t)\,\mathbf{e}_z$) varying harmonically in time with a frequency ω_f as

$$\mathbf{f}(x,y,t) = W_f(x,y)\left(\mathrm{e}^{i\omega_f t} + c.c.\right)\mathbf{e}_z. \tag{3}$$

The spatial structure of the forcing has a general mathematical expression given by

$$W_f(x,y) = \eta\,\mathrm{e}^{-\beta\, d^2(x,y)}, \tag{4}$$

where η is the maximum value of the forcing and β is a parameter that controls the jet spreading. Two different spatial configurations are proposed in this work: an annular jet (AJ) centered in the vortex axis which is applied at $r = a_f$,

$$d^2(x,y) = (r-a_f)^2, \quad a_f = 2, \tag{5}$$

and an off-axis (θ_f) single-point injection (SPI) which is located at a distance a_f of the vortex center

$$d^2(x,y) = (x-x_c)^2 + (y-y_c)^2, \quad \theta_f = \pi/3, a_f = 2, \tag{6}$$

where $(x_c,\, y_c)$ is the vortex center.

Finally, we analyse the variation of the gain at large times, G_∞, for a constant k and ω_f. We define G_∞ as the value where $G(t)$ defined as

$$G(t;k,\omega) = \frac{\int_{\mathcal{D}} (u\,u^* + v\,v^* + w\,w^*)\,dx\,dy}{\int_{\mathcal{D}} W_f^2\,dx\,dy}. \tag{7}$$

reaches a steady value at $t \to 120$.

In the Lamb-Oseen vortex, if we excite with a function at low frequencies and low wavenumbers, resonance modes are expected (Blanco-Rodríguez *et al.*, 2016) for both configurations (AJ and SPI). However, this is not the case when we consider the presence of the axial component in the q-vortex. In other words, for this type of vortices, we must excite the perturbations at a higher frequency ($\omega_f = 1$) and, consequently, we do obtain significant gains for small axial wavenumber values as shown in figure 1 where $G_\infty \approx 300$. Furthermore, the system selects the mode $m = 0$, as one should expect from an axisymmetric excitation of the annular jet.

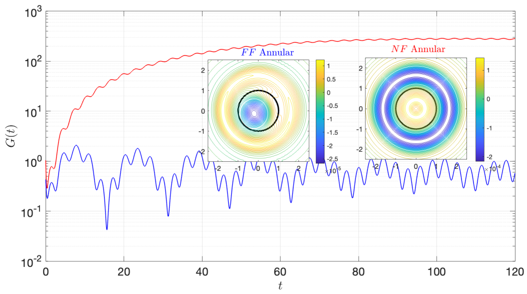

Figure 1: AJ configuration. Time evolution of the energy gain of Batchelor's experimental vortex in two areas ($q = 4$, $Re = 300$, NF) and ($q = 2.25$, $Re = 600$, FF) for the same wavenumber $\log_{10} k = -1$ and $\log_{10} \omega_f = 0$.

As in the annular case, we observe significant gains for the point excitation case for high frequencies and small axial wavenumber. Figure 2 also shows the combined effect of the Reynolds number and the vortex strength on the most unstable mode ($m = 0$ for the near field and m= 1 for the far field). Logically, smaller q values produce smaller energy gains.

3 Conclusions

In this work, we develop a stability analysis based on the frequency response of Batchelor vortex. We obtained the numerical base flow from experimental data. The Reynolds number based on the vortex core increases from 300 to 600, and the parameter q decreases from 4 to 2.25 as the vortex evolves spatially downstream. We observe two main changes concerning Lamb-Oseen vortex, i.e. by including the effect of axial velocity in the theoretical model. Firstly, the gains of Batchelor vortex are small for the same values of k and ω_f compared to the existing Lamb-Oseen vortex results. Secondly, and for a pair of values of k and ω_f, the annular configuration produces higher

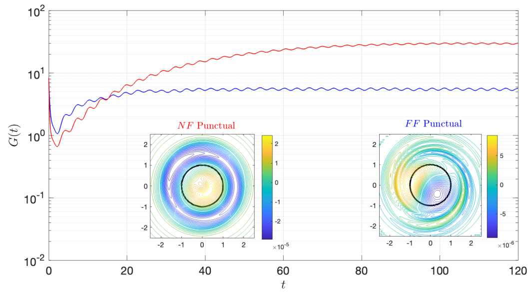

Figure 2: SPI configuration. Time evolution of the energy gain of Batchelor's experimental vortex in two areas ($q = 4$, $Re = 300$, NF) and ($q = 2.25$, $Re = 600$, FF) for the same wavenumber $\log_{10} k = -1$ and $\log_{10} \omega_f = 0$.

gains in the q-vortex compared to those obtained in Lamb Oseen vortex case.

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