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Development and validation of a software application to analyze thermal and kinematic multimodels of Stirling engines

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ABSTRACT

The work developed presents, for the first time, a tool to analyze all the thermodynamic models used in the study and development of Stirling engines: isothermal, ideal adiabatic and adiabatic with losses, combined adiabatic thermodynamic with finite speed (CAFS), thermodynamic with finite speed (FST), ideal polytropic and polytropic with losses (PSVL), allowing a comparative study of them.

This software (ASCE-UMA), designed and implemented in a Matlab GUI® allows to obtain the operating parameters of these engines, calculating the thermodynamic parameters, power output and efficiency. Additionally, the thermodynamic models can be evaluated with different mechanical configurations, for which different drive mechanisms are implemented: Sinusoidal, Alfa Ross yoke types, Alfa Ross V yoke, Beta rhombic type and free piston Stirling engine (FPSE). Thermoacoustic and other, models could be analyzed by virtue of their similarity of movement with some of the implemented models. In the same way, ASCE-UMA allows the study of various exchanger configurations, as well as various regenerator models. The versatility of ASCE-UMA allows the development analysis of all the fundamental elements of a new prototype as well as the analysis of experimental data by performing a customized and detailed calculation. To test the effectiveness of ASCE-UMA, its performance is verified by analyzing Ross Yoke D-90 models and a GM GPU-3 engine.

This is a tool that allows to analyze and comparing the different models and the different existing mechanisms for the multiple configurations of Stirling engines in an easy and intuitive application with a high-quality graphical interface.

1. Introduction

Computational models for Stirling engines development have been used for many years. Obviously, at the beginning, the models could only reach basic aspects and their similarity to reality was not very high. Nowadays models can analyze in detail the real operation of Stirling engines with virtually unlimited detail, providing equipment with adequate processing power is available.

Simulation usually focuses on the thermodynamic processes occurring in the engine using CFD software, including three-

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dimensional analysis of the physicochemical processes.

However, elementary or simple models can provide quick results that are sufficiently valid to give a basic idea of the process and behavior of the model without the need to invest extensive time and computational resources. These models may be the most suitable in initial design processes of Stirling engines.

Depending on the required objectives, analysis tools using more or less detailed mathematical models may be a more useful tool than detailed simulation using CFD programs [1].

Starting from simple mathematical models, valid results can be obtained for a first analysis of the feasibility of the proposed solution, all with little time and computational resources.

In the same way, these simple models allow a basic analysis without the need to define very detailed aspects of the machine to be evaluated, with the consequent agility in the decision of the fundamental aspects of the model. The more detailed aspects can be defined *a posteriori* once the basic aspects have been decided with an elementary analysis.

If the simple or basic models are correctly set out, they can provide a good approximation to the operating data of the real machine. In others words, a basic model does not have to be a bad approximation to the experimental reality.

After this elementary analysis and once the corresponding configuration has been decided, a detailed computational analysis can be performed using CFD, which will provide details of the engine behavior at a much higher level and will help the final development in a different way.

Among these details are usually the temperature and pressure gradients generated in the more geometrically complex areas of the engine. Using these detailed CFD models, it is much more feasible to perform studies that allow the detailed development of a prototype or the improvement of an existing Stirling engine [1].

One of the advantages of simple models is the required computation time. This time depends greatly on the model in question, but in general it will always be a considerable saving over CFD simulation. In the case of CFD models, they usually imply a vast knowledge of the thermodynamic processes that occur in the Stirling engine, while basic models can be approached with basic knowledge of thermodynamics, which is why they are usually more interesting for uses ranging from teaching applications to previous studies in prototype development projects.

In general, it cannot be said that the results of a CFD model are always the closest to the real behavior of a Stirling engine. In all cases, It is will necessary to validate the studies (basic or detailed) with experimental results.

When a study of a Stirling machine is considered, different tool will be required to optimize the resources invested depending on the required objectives. This results in the coexistence of different study methods for the analysis of these machines, from simple EXCEL worksheets for very elementary calculations to more complex applications.

There is no single approach to modelling Stirling-type machines that seems to be the best, or even appropriate, for all common simulation tasks. This is probably the main reason for the current diversity of co-existing methods used to model Stirling machines today.

There are many methods and programs for simulating Stirling engines, being some of them, namely SNAPpro, PROSA and Sage, commercially available. Other simulation softwares are internal developments of universities and institutions (such as NASA), or are the intellectual property of private individuals and companies. Several of the programs have been described in greater or lesser detail in the published literature, but none of them are freely available.

Analysing all the existing models for the study of Stirling engines is practically impossible in the context of a paper such as the present one.

Martini, in 1978 [2], made a detailed review of numerous existing methods in his text Handbook of Stirling Engine Design, concluding that even his extensive review was incomplete. It is obvious that from that date to the present day the different applications developed are impossible to mention them in detail.

Reviews of many of these methods, with comparisons between them, can be found in the works of Ash and Heames (1981) [3] and Urieli (1983) [4].

The proposed modelling makes it possible to calculate the power output, efficiency and engine operating parameters of the wide range of Stirling engine types available on the market. Through a simple and intuitive calculation interface, it allows the calculation of single or double-acting engines, engines with cylinder coupling: Alpha, betha or gamma, piston-coupled engines: sliding crank drive, rhombic drive, swashplate drive, Ross Rocker drive, Ringbom type. In addition, according to a classification with respect to their gas coupling, it allows the calculation of conventional engines or free piston Stirling engines. Other models could be analyzed by virtue of their similarity of movement with some of the implemented models.

It also allows the use of alternative calculation methods for elements such as regenerator, heater or cooler, as well as inputting the results of other calculation methods into the program. In this way, power, engine efficiency and alternative operating parameters can be determined.

2. Thermodynamic modelling methods

This section identifies and details the characteristics of the different calculation methods used, as well as their thermodynamic and mathematical definition.

2.1. Ideal isothermal analysis

The usual basic model is the isothermal model [5].

Most important consideration of this model is that the temperature of the hot source is the same as that of the heater and the expansion zone. Likewise, the temperature of the cold source is the same as that of the cooler and the compression zone.

With this isothermal simplification, a basic analysis of the pressure and volume variations of the working gas in the system can be carried out. It also allows an elementary study of the influence of the drive mechanisms. Schmidt's initial study considered a sinusoidal variation of the volumes, as this is the simplest basic movement.

The distribution of the various engine spaces is simplified into 5 domains [5] connected in series: a heater (*h*), a cooler (*k*), expansion and compression spaces (*e,c*) and a regenerator (*r*). Each of them is treated as a homogeneous domain with an absolute temperature (*T*), a pressure (*P*), a volume (*V*) and an instantaneous mass (*m*), for each domain the corresponding suffixes (*h,k,e,c,r*) are used.

When considering the main isothermal domains, it is assumed that all heat exchangers are ideal (efficiency = 1). The regenerator would also be included in them and the whole would have a temperature distribution between T_h and T_k.

The initial consideration is the continuity of the gas mass in the set of domains, will be:

$$m = m_c + m_k + m_r + m_h + m_e \tag{1}$$

Substituting the ideal gas law given by:

$$m = \frac{P \bullet V}{R \bullet T} \tag{2}$$

Is obtained:

$$m = P \bullet \frac{\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}\right)}{R} \tag{3}$$

Assuming a linear temperature distribution in the regenerator, it can be seen that the effective temperature of the regenerator T_r will be:

$$T_r = \frac{T_h - T_k}{Ln\left(\frac{T_h}{T_k}\right)} \tag{4}$$

The total mass of gas in the regenerator m_r of the empty space V_r is given by:

$$m_r = \int_0^{V_r} \rho \bullet dV_r \tag{5}$$

Where ρ is the density of the gas. Now with the equation of state of the ideal gas and using a free area flow constant A_r, we have:
se tiene:

$$P = \rho \bullet R \bullet T \tag{6}$$

$$dV_r = A_r \bullet dx \tag{7}$$

$$V_r = A_r \bullet L_r \tag{8}$$

Substituting for P, V_r and dV_r in the above equation:

$$m_r = \frac{V_r \bullet p}{R} \int_0^{L_r} \frac{1}{(T_h - T_k) \bullet x + T_k \bullet L_r} \bullet dx \tag{9}$$

Integrating and simplifying:

$$m_r = \frac{V_r \bullet p}{R} \bullet \frac{ln\left(T_h/T_k\right)}{(T_h - T_k)} \tag{10}$$

The average effective gas temperature in the regenerator T_r is defined in terms of the ideal gas equation:

$$m_r = \frac{V_r \bullet p}{R \bullet T_r} \tag{11}$$

Comparing the two previous equations and clearing T_r:

$$T_r = \frac{(T_h - T_k)}{\ln\left(\frac{T_h}{T_k}\right)} \tag{12}$$

This equation gives the effective average of the temperature of the regenerator (T_r) as a function of T_h and T_k.

Therefore, by equation (3), considering the volume variations V_c and V_e, the above equation can be solved for the pressure p as a function of V_c and V_e

$$P = \frac{m \bullet R}{\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r \bullet ln\left(\frac{T_h}{T_k}\right)}{(T_h - T_k)} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right)} \tag{13}$$

The work done in a complete cycle can be obtained by integrating $P{\cdot}dV$.

$$W = W_e + W_c = \oint PdV_c + \oint PdV_e = \oint \left(\frac{dV_c}{d\theta} + \frac{dV_e}{d\theta}\right) \tag{14}$$

If a typical differential element (Fig. 1), the basic unit of heat transfer, is analyzed. There is a mass flow m_i' at temperature T_i, which is the enthalpy carrier. At the output the conditions will be T_o and m_o. Considering the derivative operator d, one will have, for example, $dm/d\theta$, for a mass m and a cycle angle of θ.

Applying the principle of conservation of the generalized energy for the working fluid:

Heat transfer in cell	+	Net enthalpy by convection	=	Work done	+	Increase in internal energy of the cell

Thus, with the application of the principle of conservation of energy, obtain:

$$\delta Q + \left(c_p \bullet T_i \bullet m_i' - c_p \bullet T_o \bullet m_o'\right) = \delta W + c_v \bullet d(m \bullet T) \tag{15}$$

Equation (15) shows, for steady flow, the energy balance equation, where the potential and kinetic energy terms have not been considered, assuming that their influence is minimal.

Considering that in the isothermal model $T_i = T_o = T$, i.e. the temperature in the expansion and compression domains has a constant value, as in the cooler and the heater. In the same way that the mass flow difference (m_i'- m_o') is the mass accumulation rate in a differential element dm, equation (15) is transformed into equations (16) and (17):

$$\delta Q + c_p \bullet T \bullet dm = \delta W + c_v \bullet T \bullet dm \tag{16}$$

$$\delta Q = \delta W + R \bullet T \bullet dm \tag{17}$$

Considering that the ideal gas constant presents the value $R = c_p$ - c_v.

Performing the integration over the cycle of the transferred heat δQ, the net heat of the working cycle Q is obtained. Considering that the mass (m) in each of the domains is constant when the equilibrium state is reached and therefore, in this state, there is no mass exchange. Taking this into account, the above equation in each of the domains applied to the cycle is as follows:

$$Q_c = W_c \tag{18}$$

$$Q_e = W_e \tag{19}$$

Something similar occurs in the heat exchange domains, domains in which work has been produced:

$$W_k = 0 \tag{20}$$

$$W_h = 0 \tag{21}$$

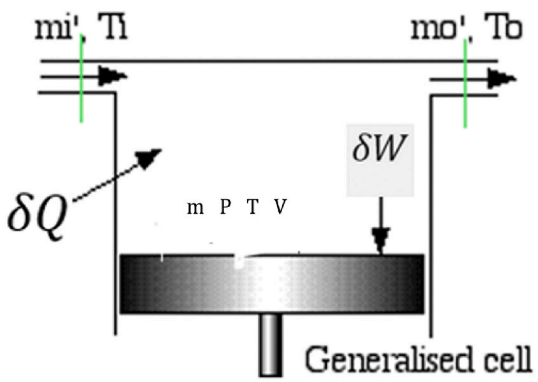

Fig. 1. Generalized calculation cell in isothermal ideal model [5].

If the regenerator is considered ideal $Q_r = 0$. This means that there is no heat exchange outside the domain and heat is only exchanged inside the space between the domain and the gas.

The isothermal model can be considered as an unrealistic model for Stirling engines, because of the optimization considerations in the heat exchanges in the different domains and its total isolation with the environment.

2.2. Ideal adiabatic analysis

The isotherm condition tends to be adiabatic in real engines, with the heat exchangers performing the net heat transfer. For this reason, the adiabatic model is usually considered as the theoretical model for the Stirling cycle (Fig. 2).

From the temperature distribution diagram, it is observed that, in the expansion and compression spaces (T_e and T_c) are not constant, they depend on the expansion and compression processes taking place in the engine. Therefore, the conditions that occur in the interface zones *ck* (Compression-Cooler) determine that the temperature T_{ck} and T_{he} are respectively:

$$\text{If } m_{ck}' > 0 \text{ then } T_{ck} = T_c \tag{22}$$

$$\text{If } m_{he}' > 0 \text{ then } T_{he} = T_c \tag{23}$$

The total mass (M) remains constant in the ideal model, i.e. no leakage of gases from the engine domain occurs. For this reason, there is no pressure drop due to leakage either. Therefore, P does not carry a suffix, it is the instantaneous pressure in the whole system.

The work W performed by the cycle is a function of the different volume variations in the corresponding domains V_c and V_e, and the heat transfer, Q_k and Q_h, are performed in the hot and cold domains of the system. In the same way, the regenerator is considered adiabatic with heat transfer occurring within the domain from the regenerator to the gas and vice versa.

To develop the established equation, the equations of state and conservation of energy are used for each of the domains. The equation of continuity of masses of the whole system is the one that allows to relate the resulting equations (Eq. 1).

Let us first consider the energy equation applied to a generalized cell, either a work domain or a heat transfer domain. Inside the domain the enthalpy transfer occurs through mass flux m'_i and temperature T_i and outside the domain by mass flux m'_O and temperature T_O. The derivative operator is denoted by d ($dm/d\theta$, is the derivative of the mass, where θ is the cycle angle).

Applying the principle of conservation of the generalized energy for the working fluid, mathematically this statement is transformed in eq. (15).

The working fluid can be considered as an ideal gas. This consideration is appropriate for Stirling engines because the processes of the working fluid are far away from its critical point. Each domain will have an equation of state and will have the expressions reflected in equation (2).

As already mentioned, the constant mass is considered and, the analysis starts with that condition Eq. (1). Substituting in each cell the law of ideal gases obtains M (Eq. (3)).

Deriving the equation of the mass balance equation:

$$dm_c + dm_k + dm_r + dm_h + dm_e = 0 \tag{24}$$

For all the domains of the heat exchanger the equation of state will be (eq. (25)). Considering that the temperatures and volumes are constant:

$$\frac{dm}{m} = \frac{dP}{P} \tag{25}$$

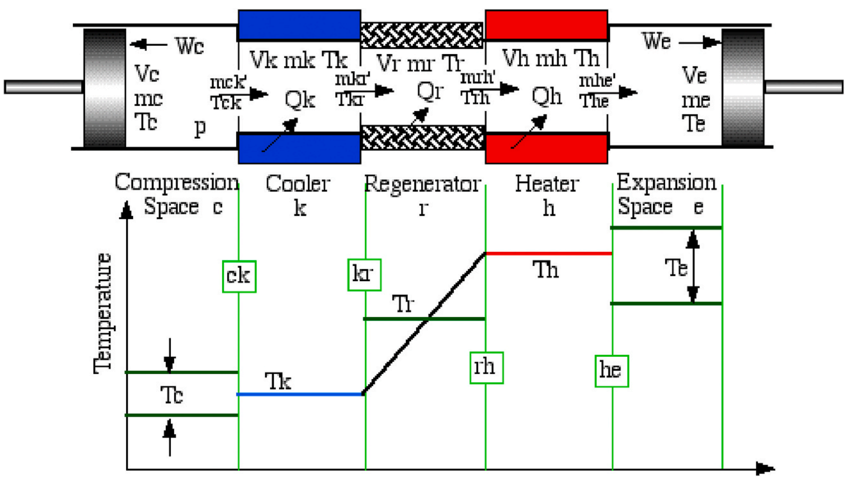

Fig. 2. Scheme elements for adiabatic engine analysis Stirling [5].

$$dm = \frac{dP \bullet m}{P} = \frac{(dP/R) \bullet V}{T} \tag{26}$$

Substituting in the mass balance equation:

$$dm_c + dm_e + \left(\frac{dP}{R}\right) \bullet \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) = 0 \tag{27}$$

The compression process is considered adiabatic (dQ_c=0) to cancel the terms d_{mc} and d_{me} and to obtain an explicit equation of dP. If the energy equation is applied in this space:

$$-c_p \bullet T_{ck} \bullet m'_{ck} = \delta W_c + c_v \bullet (m_c \bullet T_c) \tag{28}$$

From continuity considerations, the gas accumulation rate dm_c presents the same value as the incoming gas mass given by $-m'_{ck}$. The work W_c can be obtained from dV_c:

$$c_p \bullet T_{ck} \bullet dm_c = P \bullet dV_c + c_v \bullet d(m_c \bullet T_c) \tag{29}$$

Considering that P-$V_c = m_c$ - R - T, $c_p/c_v = \gamma$ and c_p - $c_v = R$, the following expression is obtained:

$$dm_c = \frac{\left(P \bullet dV_c + V_c \bullet \frac{dP}{\gamma}\right)}{(R \bullet T_{ck})} \tag{30}$$

For the expansion space:

$$dm_e = \frac{\left(P \bullet dV_e + V_e \bullet \frac{dP}{\gamma}\right)}{(R \bullet T_{he})} \tag{31}$$

Introducing dm_c and dm_e:

$$dP = \frac{-\gamma \bullet P \bullet \left(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}}\right)}{\left(\frac{V_c}{T_{ck}} + \gamma \bullet \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) + \frac{V_e}{T_{he}}\right)} \tag{32}$$

Now we can obtain the ratios dT_c and dT_e:

$$dT_c = T_c \bullet \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}\right) \tag{33}$$

$$dT_e = T_e \bullet \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e}\right) \tag{34}$$

Applying the energy equation in the different domains of the exchanger (dW=0 and T are constant) and using the equation of state of each domain of the exchanger $d_m = d_p$-m/P = $(dP/R$ -$V/T)$:

$$\delta Q + \left(c_p \bullet T_i \bullet m'_i - c_p \bullet T_o \bullet m'_o\right) = c_v \bullet T \bullet dm = V \bullet dP \bullet \frac{c_v}{R} \tag{35}$$

Thus, for the three domains that make up the exchanger, the heat flow is:

$$\delta Q_k = V_k \bullet dP \bullet \frac{cv}{R} - c_p \bullet \left(T_{ck} \bullet m'_{ck} - T_{kr} \bullet m'_{kr}\right) \tag{36}$$

$$\delta Q_r = V_r \bullet dP \bullet \frac{c_v}{R} - c_p \bullet \left(T_{kr} \bullet m'_{kr} - T_{rh} \bullet m'_{rh}\right) \tag{37}$$

$$\delta Q_h = V_h \bullet dP \bullet \frac{c_v}{R} - c_p \bullet \left(T_{rh} \bullet m'_{rh} - T_{he} \bullet m'_{he}\right) \tag{38}$$

The regenerator is considered ideal and heat exchangers isothermal: T_{rh}=T_h and T_{kr}=T_k.

The work done in the expansion and compression domains will be:

$$W = W_c + W_e \tag{39}$$

$$\delta W = \delta W_c + \delta W_e \tag{40}$$

$$\delta W_c = P \bullet dV_c \tag{41}$$

$$\delta W_e = P \bullet dV_e \tag{42}$$

2.3. Non-ideal adiabatic model

To contemplate a more real model, losses are included in an uncoupled way, trying to bring the mathematical modelling closer to the reality of the engine's operation. For this purpose, energy losses due to pressure drops in the heat exchangers and energy losses due to external conductivity are considered. Prior to these sections, the necessary scaling parameters are defined.

2.3.1. Scale parameters

2.3.1.1. Hydraulic diameter (d). This parameter represents the ratio of the two fundamental dimensional parameters of an exchanger: the wetted area A_{wg} and the empty volume V:

$$d = \frac{4 \bullet V}{A_{wg}} \tag{43}$$

For the flow in a circular tube (or a circular tube bundle) the Hydraulic Diameter is four times the inside diameter of the tube.

2.3.1.2. Reynolds Number (Re). The Re depends on the inertia forces and viscosity forces; its value determines the laminar or turbulent regime.

$$Re = \frac{\rho \bullet u \bullet d}{\mu} \tag{44}$$

Stanton Number (St)

This is obtained as:

$$St = \frac{h_c}{\rho \bullet u \bullet c_p} \tag{45}$$

2.3.1.3. Number of Transfer Units (NTU). Another commonly used parameter called "Number of Transfer Units", or NTU, c can be defined from the energy balance equation:

$$NTU = \frac{h_c \bullet A_{wg}}{c_p \bullet \rho \bullet u \bullet A} = St \bullet \frac{A_{wg}}{A} \tag{46}$$

2.3.1.4. Prandtl Number (Pr). Pr can be obtained as a function of the kinematic viscosity ν:

$$\nu = \mu / \rho \ \left(m^2 / s\right) \tag{47}$$

And the also known as moment diffusivity, to thermal diffusivity α_t:

$$\alpha_t = \frac{k}{\rho \bullet c_p} \ \left(m^2 / s\right) \tag{48}$$

Which represents the ratio between the viscous and thermal layers. Considering the typical temperatures and gases used in Stirling engines, Pr can be considered with a value of 0.7.

2.3.1.5. Nusselt Number (Nu). It is obtained as:

$$Nu = \frac{h_c \bullet d}{k} \tag{49}$$

$$Nu = St \bullet Pr \bullet Re \tag{50}$$

2.3.2. Pressure drop through the regenerator (dP_R)

Many studies can be found to model and calculate the pressure loss in the regenerator. Considering the most current calculation models used in scientific publications [6–12], for the calculation we will use the expression:

$$dP_R = \frac{2 \bullet f_r \bullet \mu \bullet V_r \bullet G \bullet l_r}{m_r \bullet d_r^2} \tag{51}$$

$$f_r = 54 + 1,43 \bullet Re^{0,78} \tag{52}$$

2.3.3. Pressure drop through the heater (dP_H)

Similarly, to obtain the pressure loss of load through the heat exchanger on the hot side, using the most current bibliography found that recommends for the calculation of the same [6–23].

$$dP_H = \frac{2 \bullet f_r \bullet \mu \bullet V_h \bullet G \bullet l_h}{m_h \bullet d_h^2} \tag{53}$$

$$f_r = 0,0791 \bullet Re^{0,75} \tag{54}$$

2.3.4. Pressure drop through the cooler (dP_K)

In order to calculate it, he proposed method is based on the most current bibliography recommended [6–23].

$$dP_H = \frac{2 \bullet f_r \bullet \mu \bullet V_h \bullet G \bullet l_h}{m_h \bullet d_h^2} \tag{55}$$

$$f_r = 0,0791 \bullet Re^{0,75} \tag{56}$$

2.3.5. Total pressure drop and pumping power lost (dP)

The total pressure losses in the heater, cooler and regenerator will be the total pressure loss:

$$dP = dP_R + dP_H + dP_K \tag{57}$$

From the total pressure drop, the power loss δW_{loss} in the heat exchangers is obtained:

$$\delta W_{loss} = dP \bullet dV \tag{58}$$

$$Pumping_{loss} = \left(\int (dP \bullet dV) \right) \bullet \mathrm{f} \tag{59}$$

2.3.6. Energy losses due to external conductivity

Equation (61) defines the expression to obtain the efficiency of a heat exchanger or regenerator:

$$\varepsilon = 1 - e^{-NTU} \tag{60}$$

Where ε is the efficiency of the exchanger and NTU is the “Number of Transfer Units".

From the basic equations of convective heat transfer, it is obtained:

$$\delta \mathrm{Q} = h_c \bullet A_{wg} \bullet (T_w - T) \tag{61}$$

Thus:

$$Q_K = Q_k - Q_{rloss} = \frac{h_k \bullet A_{wgk} \bullet (T_{wk} - T_k)}{f} \tag{62}$$

$$Q_H = Q_h + Q_{rloss} = \frac{h_h \bullet A_{wgh} \bullet (T_{wh} - T_h)}{f} \tag{63}$$

The suffix k to the cooler, and the suffix h refers to the heater. With these conditions T_k and T_h can be defined as:

$$T_k = \frac{T_{wk} - (Q_k - Q_{rloss}) \bullet f}{h_k \bullet A_{wgk}} \tag{64}$$

$$T_h = \frac{T_{wh} - (Q_h + Q_{rloss}) \bullet f}{h_h \bullet A_{wgh}} \tag{65}$$

2.3.7. Regenerative effectiveness (ε)

The effectiveness of the regenerator considered in terms of the temperature profile of the cold and hot gas streams with respect to the regenerator matrix.

$$\varepsilon = \frac{NTU}{(1 + NTU)} \tag{66}$$

Once the efficiency of the regenerator is known and evaluated, the energy loss in the exchanger can be obtained, Q_{rloss}, as:

$$Q_{rloss} = (1 - \varepsilon) \bullet (Q_{r\ max} - Q_r\ min) \tag{67}$$

Where Q_{rmin} and Q_{rmax} are the minimum and maximum values of heat calculated in the regenerator respectively.

Once the value of the power lost in the exchanger is known, the value of the real thermal energy in the cooler and heater domains can be calculated,

$$Q_H = Q_h + Q_{rloss} \tag{68}$$

$$Q_K = Q_k - Q_{rloss} \tag{69}$$

2.3.8. Evaluation of the output work and performance

The output work of the engine in the Non-Ideal Adiabatic model (*nia*), is the subtraction of the work obtained without considering losses, *W*, which will be equal to the Ideal Adiabatic Model, minus the subtraction of the load losses in the exchangers and regenerator, δW_{loss}:

$$W_{nia} = W - \delta W_{loss} \tag{70}$$

Engine performance will be:

$$\eta_{nia} = \frac{W_{nia}}{Q_h} \tag{71}$$

2.4. Finite time thermodynamics analysis (FTT)

Different studies with the FTT approach can be found for the calculation of Stirling engine performance [24–41].
According to these models, the heat transferred in the regenerator (Q_r) will be:

$$Q_r = n \bullet c_v \bullet \varepsilon_r \bullet (T_1 - T_2) \tag{72}$$

In the same way the heat loss can be obtained with eq. (73) [23–39].

$$\Delta Q_r = \mathrm{n} \bullet c_v \bullet (1 - \varepsilon_r) \bullet (T_1 - T_2) \tag{73}$$

Considering that the heat transfer is irreversible, the time of these processes is considered [24]. The temperature in such processes can be obtained from eq. (74) [25,40].

$$\frac{dT}{dt} = \pm m_i \tag{74}$$

Where *M* is the regenerative time constant (proportionality constant), depends on the characteristics of the fluid and is independent of the temperature difference of ± sign. For heating $\mathrm{i} = 1$ and for cooling $i = 2$ [27,28].

$$t_3 = \frac{T_1 - T_2}{M_1} \tag{75}$$

$$t_4 = \frac{T_1 - T_2}{M_2} \tag{76}$$

$$t_{re} = t_3 + t_4 \tag{77}$$

The quantities of heat absorbed and dissipated can be obtained from the following equations [26,27].

$$Q_1 = \left(h_{HC} \bullet (T_H - T_1) + h_{HR}\left(T_H^4 - T_1^4\right)\right) \bullet t_1 = nRT_1 \bullet ln\,\lambda + n \bullet c_v \bullet (1 - \varepsilon_R) \bullet (T_1 - T_2) \tag{78}$$

$$Q_2 = h_{LC} \bullet (T_2 - T_L) \bullet t_2 = nRT_1 \bullet ln\,\lambda + n \bullet c_v \bullet (1 - \varepsilon_R) \bullet (T_1 - T_2) \tag{79}$$

So, have:

$$\lambda = \frac{V_1}{V_2} \tag{80}$$

It must be take into account that there is a heat loss between the heat sink and the heat source, which depends on the time and temperature difference, eq. (81) [25–42].

$$Q_0 = k_0 \bullet (T_H - T_L) \bullet t \tag{81}$$

Q_H (heat absorbed by the heat source) and Q_L (heat released at the heat sink) will correspond to:

$$Q_H = Q_1 + Q_0 \tag{82}$$

$$Q_L = Q_2 + Q_0 \tag{83}$$

Using (75) to (83), the cyclic period *t* can be obtained according to the equation:

$$t = t_1 + t_2 + t_r \times \begin{array}{c} t = \dfrac{nRT_1 \bullet ln\,\lambda + n \bullet c_v \bullet (1-\varepsilon_r) \bullet (T_1 - T_2)}{\left(h_{HC} \bullet (T_H - T_1) + h_{HR}\left(T_H^4 - T_1^4\right)\right)} + \\ + \dfrac{nRT_1 \bullet ln\,\lambda + n \bullet c_v \bullet (1-\varepsilon_r) \bullet (T_1 - T_2)}{h_{LC} \bullet (T_2 - T_L)} + \left(\dfrac{1}{m_1} + \dfrac{1}{m_2}\right) \bullet (T_1 - T_2) \end{array} \tag{84}$$

By including the cyclic period of the Stirling engine, the power (*p*), the thermal efficiency η_t and output of the motor are determined by the following expressions:

$$p = \frac{W}{t} = \frac{Q_H - Q_L}{t} \tag{85}$$

$$\eta_t = \frac{W}{t} = \frac{Q_H - Q_L}{t} \tag{86}$$

$$p = \frac{T_1 - T_2}{\frac{T_1 + m \bullet (T_1 - T_2)}{h_{HC} \bullet (T_H - T_1) + h_{HR} \bullet \left(T_H^4 - T_1^4\right)} + \frac{T_2 + m \bullet (T_1 - T_2)}{h_{LC} \bullet (T_2 - T_L)} + F_1 \bullet (T_1 - T_2)} \tag{87}$$

$$\eta_t = \frac{T_1 - T_2}{T_1 + m \bullet (T_1 - T_2) + [k_0 \bullet (T_H - T_L)]} \cdot \frac{1}{\left[\frac{T_1 + m \bullet (T_1 - T_2)}{h_{HC} \bullet (T_H - T_1) + h_{HR} \bullet \left(T_H^4 - T_1^4\right)} + \frac{T_2 + m \bullet (T_1 - T_2)}{h_{LC} \bullet (T_2 - T_L)} + F_1 \bullet (T_1 - T_2)\right]} \tag{88}$$

Where:

$$m = \frac{c_v \bullet (1 - \varepsilon_R)}{R \bullet ln\,\lambda} \tag{89}$$

$$F_1 = \frac{1}{nR \bullet ln\,\lambda}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \tag{90}$$

2.5. The combines adiabatic-finite speed thermal approach (CAFS)

CAFS is an analysis based on the combination of finite-velocity and adiabatic thermodynamics. The method also considers pressure throttling in the regenerator and heat exchangers, in addition to finite piston velocity and the effects of mechanical piston friction. This results in higher irreversibility and less useful power.

In FST (finite speed thermodynamics), the finite piston speed losses consider the piston velocity and the average molecular velocity eq. (91).

$$\Delta P_w = \frac{1}{2} \cdot \left(P_c \cdot \frac{a.w_c}{c_c} + P_e \cdot \frac{a.w_e}{c_c}\right) \tag{91}$$

Where *P* is the instantaneous pressure, *c* is the mean molecular velocity, *w* is the piston velocity and the subscripts *c* and *e* represent compression and expansion. The values of *a* and *c* are obtained as indicated in eq. (92)and 93. [43]:

$$a = \sqrt{3 \bullet \gamma} \tag{92}$$

$$c = \sqrt{3 \bullet R \bullet T} \tag{93}$$

Where γ is the ratio between the specific heats and *R* is the gas constant. The pressure loss, which produces loss of useful power, resulting from the mechanical friction of the elements will be [44]:

$$\Delta P_f = \frac{(0,94 + 0,0045 \bullet w) \bullet 10^5}{3 \bullet \mu} \bullet \left(1 - \frac{1}{r_v}\right) \tag{94}$$

Where *w* the linear velocity of the piston, r_v is the compression ratio and μ is a constant that depends on r_v [44]:

$$\mu = 1 - \frac{1}{3 \bullet r_v} \tag{95}$$

The power losses due to the phenomena referenced above, will be:

$$\delta W_{FST} = P_m \bullet \left(\pm \frac{a \bullet w}{c} \pm \frac{f \bullet \Delta P_f}{P_m}\right) \bullet dV \tag{96}$$

Where P_m represents the instantaneous mean effective pressure, f represents the engine rotation frequency and V is the instantaneous engine volume. The sign "-" would correspond to expansion and "+" to compression. According to Petrescu et al. [44–48], another expression for the pressure drop in the regenerator is included, using ΔP_{throtl} to substitute in equation (96). The total work loss due to FST, mechanical friction and throttling process will be:

$$\delta W_{CAFS\_total} = \delta W_{FST} + (\Delta P_{throtl} \bullet dV) \tag{97}$$

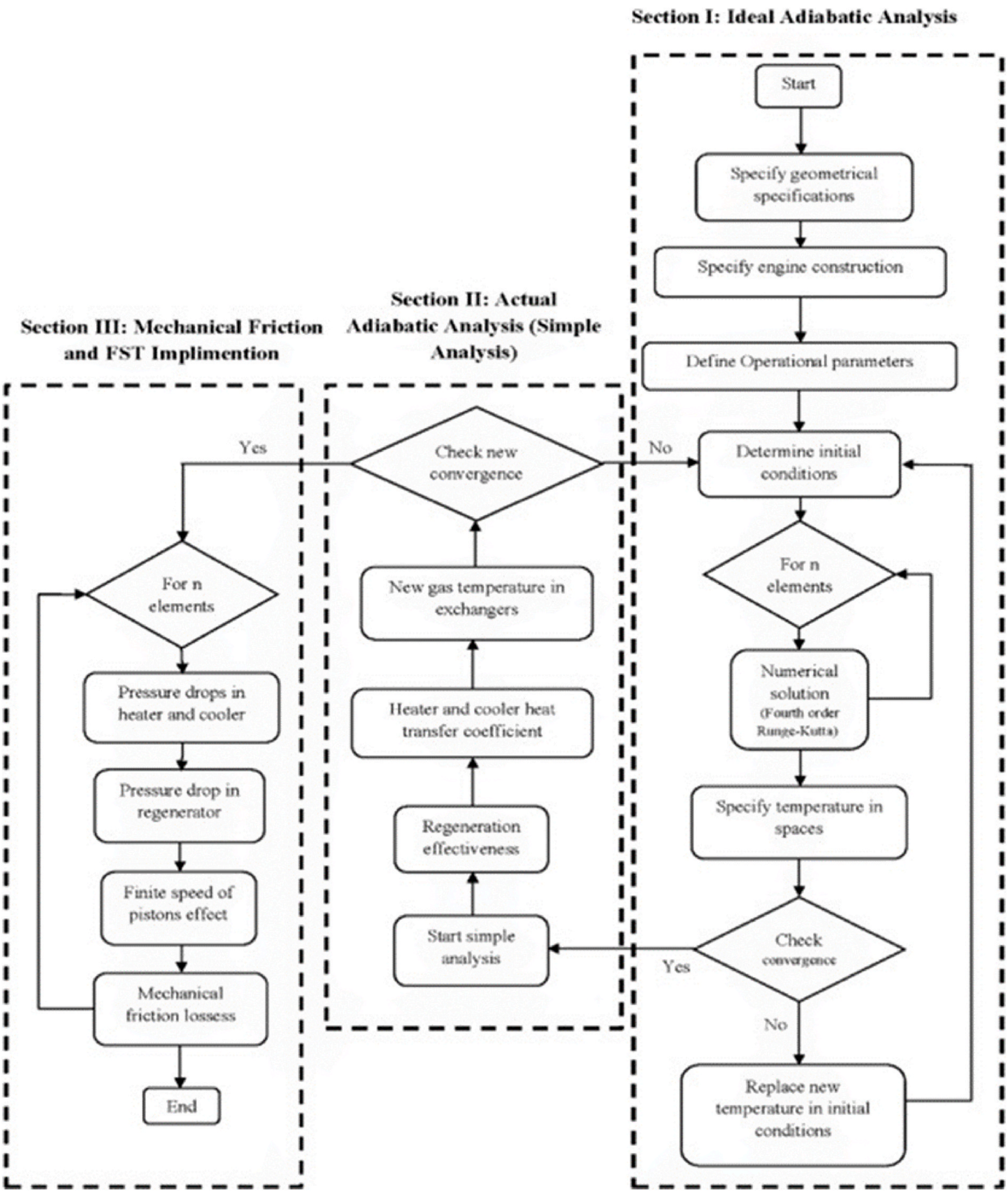

Fig. 3. Thermal approach process schema through CAFS [50].

Fig. 3 represents the picture of the CAFS approach [49]. It shows the CAFS procedure in three sections. Section 1 deals with the ideal adiabatic work [22]. Section 2 introduces the regenerator efficiency and performs a simple analysis. To account for irreversibility the outlet temperature of this group links back to section 1, repeating the process until convergence is reached.

The regenerator efficiency is now determined by the expression [22]:

$$NTU = \frac{St \bullet L_r}{R_{H,r}} \tag{98}$$

In which *NTU* represents the number of transfer units in the regenerator, obtained from:

$$NTU = \frac{St \bullet l_r}{R_{H,r}} \tag{99}$$

Where l_r and $R_{H,r}$ correspond to the length of the regenerator and the hydraulic radius. The Stanton number *St*. The l_r and $R_{H,r}$ will be [49]:

$$St = 0,023 \bullet Re^{-0,2} \bullet Pr^{-0,6} \tag{100}$$

$$R_{H,r} = \frac{1}{4} \bullet D_{H,r} \tag{101}$$

$$D_{H,r} = \frac{4 \bullet \Pi}{\varphi \bullet (1 - \Pi)} \tag{102}$$

$$\varphi = \frac{A_{wg}}{V_{mesh}} \tag{103}$$

Where R_e and *Pr* represent the values in the regenerator of the Reynolds and Prandtl numbers. In addition, Φ and Π indicate the shape factor and porosity of the regenerator, V_{mesh} and A_{wg} are the wire volume and wetted area of the regenerator, respectively. For modelling the real regeneration performance, the gas temperature in the cooler and heater will be [22]:

$$T_h = T_{wh} - \frac{f \bullet [Q_h + Q_r \bullet (1 - \varepsilon)]}{h_h \bullet A_h} \tag{104}$$

$$T_k = T_{wk} - \frac{f \bullet [Q_k + Q_r \bullet (1 - \varepsilon)]}{h_k \bullet A_k} \tag{105}$$

The heat exchanger wall temperatures are assumed equal to the heat sink/source. The heat transfer coefficient to determine the enthalpy in equations (104) and (105), will be [22]:

$$h = \frac{0,0791 \bullet Re_m^{0,75} \bullet \mu \bullet c_p}{2 \bullet \mathrm{d} \bullet Pr} \tag{106}$$

The outlet temperature of section 2 impacts in section 1 and the process is repeated until the temperatures are convergent. Afterwards, the results of section 2 are used as inputs in section 3, as illustrated in Fig. 3. Throughout this stage, the work loss due to the throttling process in the cooler, the heater and the regenerator, which affects the output work of section 2, is calculated.

ΔP_{thrott} is calculated, for the estimation of work losses, according to eq. (107), for the different heat transfer points [51–55]:

$$\Delta P_{thrott} = C_f \bullet \frac{\rho}{2} \bullet \frac{L}{D_H} \bullet u_{max}^2 \tag{107}$$

The coefficient of friction in the refrigerator and the heater can be obtained through the following expression [56]:

$$C_f = 0,316 \bullet Re^{-0,25} \tag{108}$$

The value of C_f for the regenerator has been extensively studied [49,56,57] and from these studies there are different expressions to determine it [56,57].

To calculate the regenerator friction factor, the Gedeon equation has been used. The regenerator, for example of the GPU-3 engine, is composed of eight small interwined wire mesh regenerators, in cylindrical vessels, and placed close the central cylinder of the engine [43]. In addition, in Section 3 Fig. 3, the effect of finite piston velocity and mechanical friction are included, (eq. (92)). The algorithm starts with the prediction of the temperature for each $d\theta$ (section I of Fig. 3).

2.6. *Polytropic analysis of Stirling engine with various loss mechanisms (PSVL)*

The PSVL model proposes the analysis of the Stirling engine considering polytropic compression and expansion processes [58]. The adiabatic model is extended with the heat transfers between the engine parts and the working and boundary fluids.

In these calculation processes previously used, the expansion and compression processes were assumed as adiabatic [5,43–61] or isotherms [22–63].

Gas leakage to the crankcase and heat losses from the reciprocating motion (shuttle) are considered. Therefore, the ordinary differential equations of the adiabatic model are corrected with these losses, applying a new system of differential equations. The system is solved by the fourth order Runge-Kutta method.

In this model the influence of the Stirling engine losses are classified into three main groups (Fig. 4).

A first group of the loss mechanisms that compose it are: gas leakage, Shuttle effect and polytropic heat transfer. All of them are included in the ordinary differential equations.

The second group consists of heat exchanger losses and non-ideal heat transfer, which are evaluated differentially with the first part and used to modify the temperature of the engine spaces.

The third block includes the mechanical friction (of the main engine parts), the pressure loss due to the finite piston speed and the longitudinal heat conduction between the cooler and the heater that occurs due to their metallic connection through the regenerator wall. They are determined as separate loss terms that do not affect the temperature distribution of the engine spaces; therefore, they are calculated decoupled.

In the PSVL model, the engine is divided into five sections or calculation domains, including cooler (section k), heater (section h), expansion zone (section e), regeneration (section r) and compression zone (section c). The temperature in the exchanges of the cooler and the heater with respect to $d\theta$ and the heat transfer in the heat exchangers is not constant.

In the fundamental analysis of Stirling engines in this model, the Ordinary Differential Equations (ODE) of the energy conservation law are analyzed for each engine section, to consider polytropic heat transfer effect terms and Shuttle heat losses (Fig. 5).

The ODE of application of the first principle of thermodynamics can be expressed as follows:

$$\delta Q - \delta Q_{polytropic} - \delta Q_{shuttle} + \left(m_i \bullet c_{p,i} \bullet T_i - m_0 \bullet c_p \bullet T_0\right) = \delta W + c_v \bullet d(mT) \tag{109}$$

Where δQ is the heat transfer of the working fluid through the focus and δW is the net work. $\delta Q_{polytropic}$ and $\delta Q_{shuttle}$ are the polytropic heat losses of the engine section and the heat losses due to conduction (shuttle). $\delta Q_{shuttle}$ can be expressed as the following equation: [12]

$$\delta Q_{shuttle} = \frac{\pi \bullet S^2 \bullet k_g \bullet D_d}{8 \bullet J \bullet L_d} \bullet (T_e - T_c) \tag{110}$$

$\delta Q_{polytropic}$ represents the heat transfer from the different domains (or engine elements) to the environment and will be [61]:

$$Q_{polytropic} = m \bullet c_n \bullet (T_0 - T) \tag{111}$$

$$\delta Q_{polytropic} = c_n \bullet (T_0 - T) \bullet dm - m \bullet c_n dT \tag{112}$$

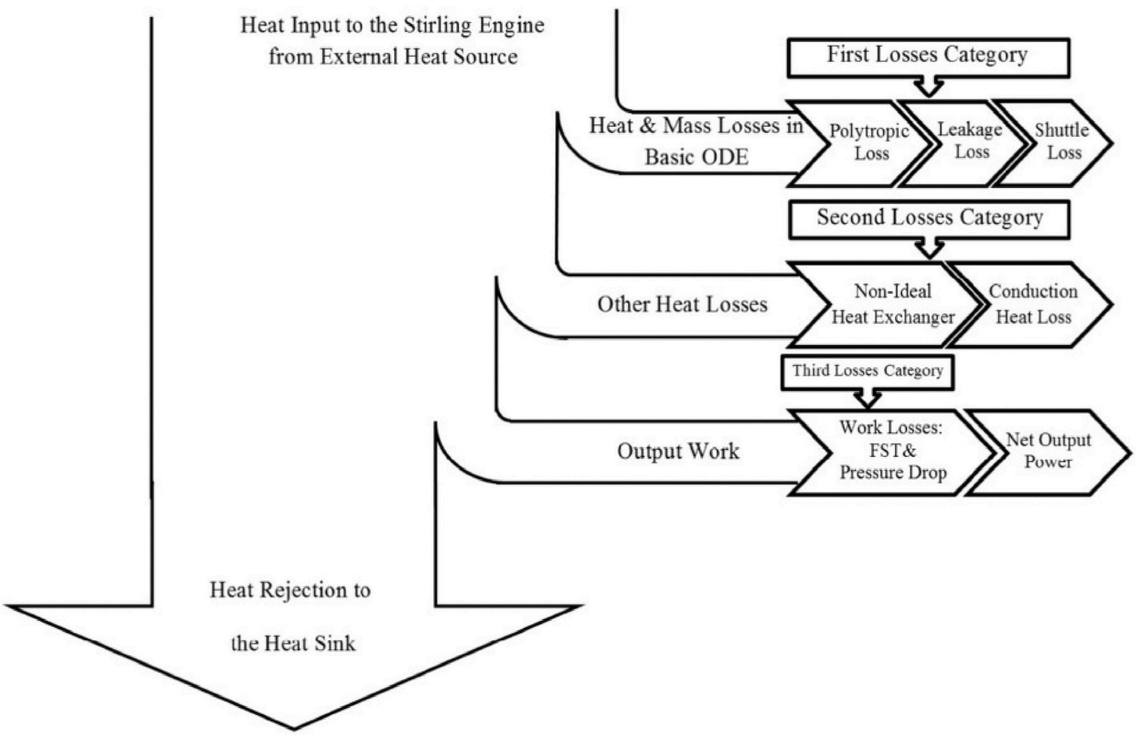

Fig. 4. Losses effects of Stirling engines in PSVL model [58].

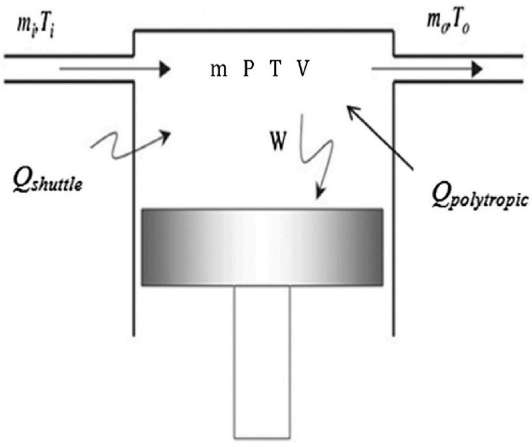

Fig. 5. Scheme of volume of control of component of the engine according to scheme PSVL [44,58].

In eqs. (111) and (112) T is the temperature of the engine domain or section and T_0 is the ambient temperature. The specific heat capacity (polytropic) is c_n, its value will be: [58]:

$$c_n = c_v \bullet \frac{n-k}{n-1} \tag{113}$$

The polytropic index is determined using the following expression:

$$P \bullet V^n = const. \tag{114}$$

Differentiating equation (114), is obtained:

$$d(P \bullet V^n) = d(const.) = 0 \tag{115}$$

$$P \bullet \left(n \bullet V^{n-1} \bullet dV\right) + V^n \bullet (dP) = 0 \tag{116}$$

The polytropic exponent for each case is determined:

$$n = \frac{V \bullet dP}{P \bullet dV} \tag{117}$$

The equation of state of the working fluid (considered ideal gas) will be eq (2). The differential of equation (118) will be:

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dm}{m} + \frac{dT}{T} \tag{118}$$

The total mass of the working fluid (*m)* will be:

$$m = m_c + m_k + m_r + m_h + m_e - m_{leak} \tag{119}$$

Where m_{leak} represents the mass loss to the crankcase. This can be determined as follows: [22]:

$$m_{leak} = \pi \bullet D \frac{P + P_{buffer}}{4 \bullet R \bullet T_g} \bullet \left(u_p \bullet J - \frac{J^3 \bullet P}{6 \bullet \mu} \bullet \frac{P - P_{buffer}}{L}\right) \tag{120}$$

$$m = \left\{ \frac{P \bullet \left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}\right)}{R} \right\} - \left\{ \pi \bullet D \bullet \frac{P + P_{buffer}}{4 \bullet R \bullet T_g} \left(u_p \bullet J - \frac{J^3 \bullet P}{6 \bullet \mu} \bullet \frac{P - P_{buffer}}{L}\right) \right\} \tag{121}$$

Through the aforementioned descriptions, equation (120) can be reordered as indicated below [58]:
Ec. 119 in differential form will be:

$$dm_c + dm_k + dm_r + dm_h + dm_e - dm_{leak} = 0 \tag{122}$$

In the heater, cooler and regenerator: dV/V 0 therefore, the mass differential equation (119), will be:

$$\left.\frac{dP}{P}\right|_{k,h,r} = \left.\frac{dm}{m}\right|_{k,h,r} \tag{123}$$

Thus:

$$dm_{k,h,r} = \left.\frac{m \bullet dP}{P}\right|_{k,h,r} = \left.\frac{V \bullet dP}{R \bullet T}\right|_{k,h,r} \tag{124}$$

Replacing equation (124) in equation (122) results in the following equation:

$$dm_c + dm_e - dm_{leak} + \frac{dP \bullet \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right)}{R} = 0 \tag{125}$$

In the compression process (polytropic), equation (109) will be [60]:

$$-c_n \bullet (T_c - T_0) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} - c_p \bullet m_{ck} \bullet T_{ck} = = \delta W_c + c_v \bullet d(m_c \bullet T_c) \tag{126}$$

Including $\delta W_c = P\ dV_c$ in equation (126), the equation for the mass change of the compression gas d_{mc} is obtained using some intermediate calculations and the Equation of State of the Ideal Gases as follows [58]:

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet dV_c + c_v \bullet d(m_c \bullet T_c)$$

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet dV_c + c_v \bullet d\left(\frac{P \bullet V_c}{R}\right)$$

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet \left(1 + \frac{c_v}{R}\right) \bullet dV_c + \frac{c_v}{R} \bullet V_c \bullet dP$$

$$\frac{c_n}{c_p} \bullet (T_0 - T_c) \bullet dm_c - \frac{m \bullet c_n}{c_p} \bullet dT_c + \frac{Q_{shuttle}}{c_p} + T_{ck} \bullet dm_c = \frac{P \bullet dV_c}{R} + \frac{c_v}{R \bullet c_p} \bullet V_c \bullet dP$$

$$\frac{c_n}{c_p} \bullet \frac{(T_0 - T_c)}{T_{ck}} \bullet dm_c - \frac{m \bullet c_n}{T_{ck} \bullet c_p} \bullet dT_c + \frac{Q_{shuttle}}{T_{ck} \bullet c_p} + dm_c = \frac{P \bullet dV_c}{R \bullet T_{ck}} + \frac{1}{R \bullet \gamma \bullet T_{ck}} \bullet V_c \bullet dP$$

$$dm_c \bullet \left[\frac{c_n}{c_p} \bullet \frac{(T_0 - T_c)}{T_{ck}} + 1\right] + \frac{c_n}{c_p} \bullet \frac{m_c}{T_{ck}} \bullet dT_c + \frac{Q_{shuttle}}{c_p \bullet T_{ck}} = \frac{P \bullet dV_c}{R \bullet T_{ck}} + \frac{V_c}{\gamma \bullet R \bullet T_{ck}} \bullet dP$$

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet dV_c + c_v \bullet d(m_c \bullet T_c)$$

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet dV_c + c_v \bullet d\left(\frac{P \bullet V_c}{R}\right)$$

$$c_n \bullet (T_0 - T_c) \bullet dm_c - m \bullet c_n \bullet dT_c + Q_{shuttle} + c_p \bullet T_{ck} \bullet dm_c = P \bullet \left(1 + \frac{c_v}{R}\right) \bullet dV_c + \frac{c_v}{R} \bullet V_c \bullet dP$$

$$\frac{c_n}{c_p} \bullet (T_0 - T_c) \bullet dm_c - \frac{m \bullet c_n}{c_p} \bullet dT_c + \frac{Q_{shuttle}}{c_p} + T_{ck} \bullet dm_c = \frac{P \bullet dV_c}{R} + \frac{c_v}{R \bullet c_p} \bullet V_c \bullet dP$$

$$\frac{c_n}{c_p} \bullet \frac{(T_0 - T_c)}{T_{ck}} \bullet dm_c - \frac{m \bullet c_n}{T_{ck} \bullet c_p} \bullet dT_c + \frac{Q_{shuttle}}{T_{ck} \bullet c_p} + dm_c = \frac{P \bullet dV_c}{R \bullet T_{ck}} + \frac{1}{R \bullet \gamma \bullet T_{ck}} \bullet V_c \bullet dP \tag{127}$$

$$dm_c \bullet \left[\frac{c_n}{c_p} \bullet \frac{(T_0 - T_c)}{T_{ck}} + 1\right] + \frac{c_n}{c_p} \bullet \frac{m_c}{T_{ck}} \bullet dT_c + \frac{Q_{shuttle}}{c_p \bullet T_{ck}} = \frac{P \bullet dV_c}{R \bullet T_{ck}} + \frac{V_c}{\gamma \bullet R \bullet T_{ck}} \bullet dP$$

$$dm_c = \frac{\left[\left(\frac{P \bullet dV_c + \frac{V_c}{\gamma} \bullet dP}{R \bullet T_{ck}}\right) - \left(\frac{c_{nc}}{c_p} \bullet \frac{m_c}{T_{ck}} dT_c\right) - \left(\frac{Q_{shuttle}}{c_p \bullet T_{ck}}\right)\right]}{\left[\left(\frac{c_{nc}}{c_p}\right) \bullet \left(\frac{T_0 - T_c}{T_{ck}}\right) + 1\right]}$$

In addition, it is known that $dm_c = \dot{m}_{ck}$ for the expansion domain, d_{me} will be:

$$dm_e = \frac{\left[\left(\frac{P \bullet dV_e + \frac{V_e}{\gamma} \bullet dP}{R \bullet T_{he}}\right) - \left(\frac{c_{ne}}{c_p} \bullet \frac{m_e}{T_{he}} dT_e\right) - \left(\frac{Q_{shuttle}}{c_p \bullet T_{he}}\right)\right]}{\left[\left(\frac{c_{ne}}{c_p}\right) \bullet \left(\frac{T_0 - T_e}{T_{he}}\right) + 1\right]} \tag{128}$$

Like $\dot{m}_{rh} = \dot{m}_{he} - dm_h$. substituting d_{mc} y and d_{me} from eqs. (127) and (128)) in equation (125) results in the following differential pressure formulation in the workspace [58]:

$$\left\{\frac{\left[\left(\frac{P\bullet dV_c+\frac{V_c}{\gamma}\bullet dP}{R\bullet T_{ck}}\right)-\left(\frac{c_{nc}}{c_p}\bullet\frac{m_c}{T_{ck}}dT_c\right)-\left(\frac{Q_{shuttle}}{c_p\bullet T_{ck}}\right)\right]}{\left[\left(\frac{c_{nc}}{c_p}\right)\bullet\left(\frac{T_0-T_c}{T_{ck}}\right)+1\right]}\right\}+\left\{\frac{\left[\left(\frac{P\bullet dV_c+\frac{V_e}{\gamma}\bullet dP}{R\bullet T_{he}}\right)-\left(\frac{C_{nc}}{c_p}\bullet\frac{m_e}{T_{he}}dT_e\right)-\left(\frac{Q_{shuttle}}{c_p\bullet T_{he}}\right)\right]}{\left[\left(\frac{c_{ne}}{C_p}\right)\bullet\left(\frac{T_0-T_e}{T_{he}}\right)+1\right]}\right\}+dm_{leak}+\frac{dP}{R}\left[\frac{V_k}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}\right]=0 \tag{129}$$

Finally, with proper generalization, the following expression can be obtained for *dP* [58]:

$$dP=\frac{-\gamma\bullet\left[\frac{\frac{P\bullet dV_e}{T_{he}}-\frac{R\bullet c_{ne}}{cp}\bullet\frac{m_e}{T_{he}}\bullet dT_e+\frac{R\bullet Q_{shuttle}}{cp\bullet T_{he}}}{B_1}+\frac{\frac{P\bullet dV_c}{T_{ck}}-\frac{R\bullet c_{nc}}{cp}\bullet\frac{m_c}{T_{ck}}\bullet dT_c+\frac{R\bullet Q_{shuttle}}{cp\bullet T_{ck}}}{B_1}\right]+R\bullet m_{leak}}{\frac{V_c}{T_{ck}\bullet B_1}+\frac{V_e}{T_{ck}\bullet B_2}+\gamma\bullet\left[\frac{V_k}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}\right]} \tag{130}$$

Where:

$$B_1=\left[\left(\frac{c_{ne}}{c_p}\right)\bullet\left(\frac{T_0-T_e}{T_{he}}\right)+1\right] \tag{131}$$

$$B_2=\left[\left(\frac{c_{nc}}{c_p}\right)\bullet\left(\frac{T_0-T_c}{T_{ck}}\right)+1\right] \tag{132}$$

In addition, differential temperature equations have the following form (see eq. (118)):

$$dT_c=T_c\bullet\left(\frac{dP}{P}+\frac{dV_c}{V_c}-\frac{dm_c}{m_c}\right) \tag{133}$$

$$dT_e=T_e\bullet\left(\frac{dP}{P}+\frac{dV_e}{V_e}-\frac{dm_e}{m_e}\right) \tag{134}$$

In the regenerator, cooler and heater, the heat transfer rate will be [58]:

$$\delta Q_k=\frac{V_k\bullet dP\bullet c_v}{R}-c_p\bullet(T_{ck}\bullet\dot{m}_{ck}-T_{kr}\bullet\dot{m}_{kr}) \tag{135}$$

$$\delta Q_h=\frac{V_h\bullet dP\bullet c_v}{R}-c_p\bullet(T_{he}\bullet\dot{m}_{he}-T_{he}\bullet\dot{m}_{he}) \tag{136}$$

$$\delta Q_r=\frac{V_r\bullet dP\bullet c_v}{R}-c_p\bullet(T_{kr}\bullet\dot{m}_{kr}-T_{rh}\bullet\dot{m}_{rh}) \tag{137}$$

In the same way, the output work developed by the power piston will be:

$$\delta W_c=P\bullet dV_c \tag{138}$$

$$\delta W_e=P\bullet dV_e \tag{139}$$

Allowing to calculate the net output work (W_{net}) and the thermal efficiency (η) as follows:

$$W_{net}=\int\delta W_e-\int\delta W_c \tag{140}$$

$$\eta=\frac{W_{net}}{Q_h} \tag{141}$$

3. Calculation

This section defines the procedure followed for the development of the software and the application of all equations and calculation procedures.

3.1. Input variables definition

The first work that has been done is a definition of the input variables that were needed to undertake the calculation of each of the possible situations. To do this, all the methods and calculation options that were desired were analyzed, based on the aforementioned numerical methods of analysis.

Once the input variables have been collected, they are arranged in a flow diagram incorporating the English term that defines the input variable, its translation into Spanish, as well as the nomenclature of the variable, following the following nomenclature.

3.2. Selection of calculation methods

The methods chosen for the calculation have been validated for several engines for which experimental or analytical results were available based on scientific Stirling engine test publications. Finally, all the models that had been studied have been implemented, these models are specified in Table 1.

Each of the above calculation methods has been implemented separately from the rest, the model or models that the user chooses in the program entry panel can be calculated and the data of the different models can be saved and compared once these are obtained.

3.3. Entry data storage

Once the input data is entered, the operation parameters are configured and the calculation methods are chosen, some preliminary calculations must be made to obtain variables from geometric parameters and the data is saved in the appropriate variables.

Once the geometrical configuration variables of the engine, hot and cold side heat exchangers, regenerator, operating parameters and calculation models that will be available in the program are recognized, the input panel is configured in MATLAB GUI, which presents the aspect shown in Fig. 6, where the input data blocks are identified.

The insertion and configuration of the possible types of calculation of each section can be seen in more detail in Fig. 7.

3.4. Calculation procedure

Having defined the input variables and designed the input panel of values, the calculation procedure to be followed by the program is developed. The summary of the general calculation procedure of the program can be seen in the flow diagram of Fig. 8.

3.5. Results panel

In this section, the output panels of values calculated by the designed program are detailed. For this, two results panels have been designed:

- Preliminary Results Panel

Table 1
Calculation models and characteristics.

Software ACES-UMA Name	General Characteristics Of The Model
Schmidt	Ideal model isotherm lossless
Ideal Adiabatic	Model that supposes Adiabatic evolutions in the spaces of compression and expansion of the engine
Adiabatic with Losses	Model that supposes Adiabatic evolutions in the engine spaces of expansion and compression and incorporates the calculation of losses: • Pressure losses in Cooler • Pressure losses in Heater • Pressure losses in Regenerator • Energy losses due to external conductivity in heat exchangers • Effectiveness of the Regenerator
CAFS	Model that supposes Adiabatic evolutions in the spaces of compression and expansion of the motor, as well as thermodynamics of finite speed for the calculation of losses due to the internal friction, mechanical friction and speed of the piston: • Pressure losses in Cooler • Pressure losses in Heater • Pressure losses in Regenerator • Energy losses due to external conductivity in heat exchangers • Effectiveness of the Regenerator • Losses of pressure due to mechanical friction in the engine parts • Pressure losses due to mechanical friction in the engine parts • Losses of pressure resulting from the speed of the piston
PSVL	Model that supposes Polytropic evolutions in the spaces of compression and expansion of the engine, as well as thermodynamics of finite speed for the calculation of losses due to internal friction, mechanical friction and piston speed: • Pressure losses in Cooler • Pressure losses in Heater • Pressure losses in Regenerator • Energy losses due to external conductivity in heat exchangers • Effectiveness of the Regenerator • Losses of pressure due to mechanical friction in the engine parts • Pressure losses due to mechanical friction in the engine parts • Losses of pressure resulting from the speed of the piston

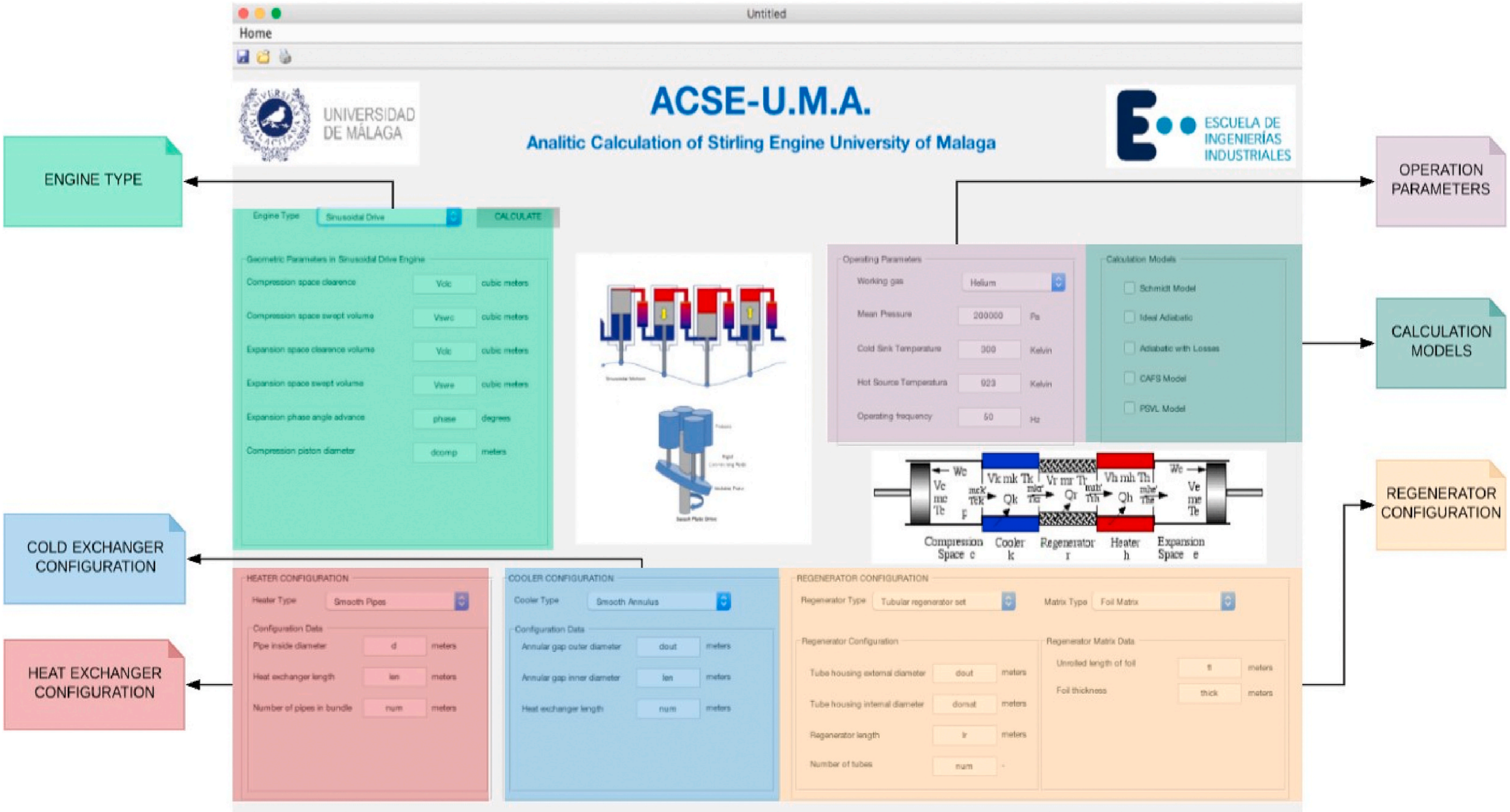

Fig. 6. Appearance and configuration of the data entry panel made in Matlab GUI®.

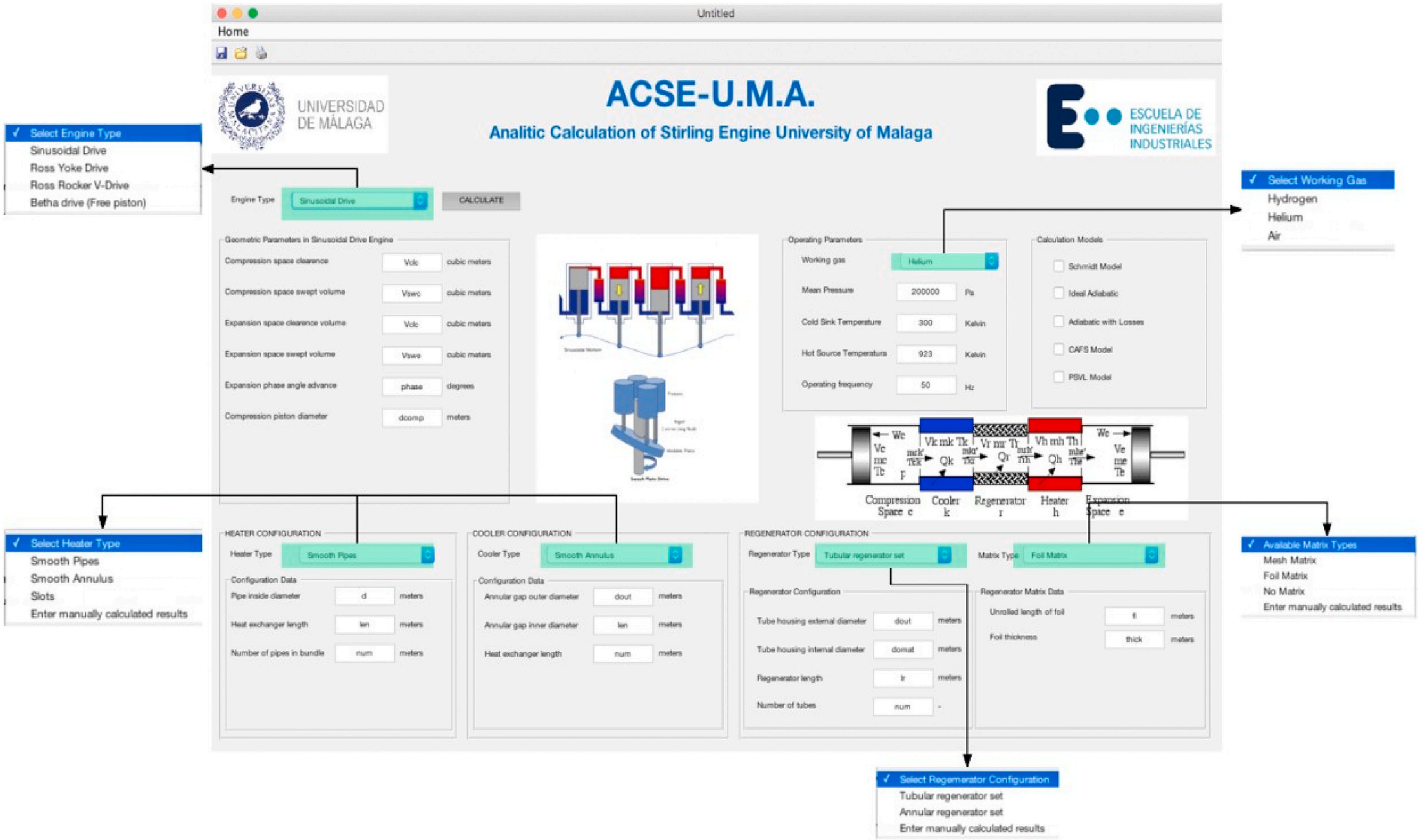

Fig. 7. Detail of possibilities of Calculation Configuration in the Data Entry Panel made in Matlab GUI®.

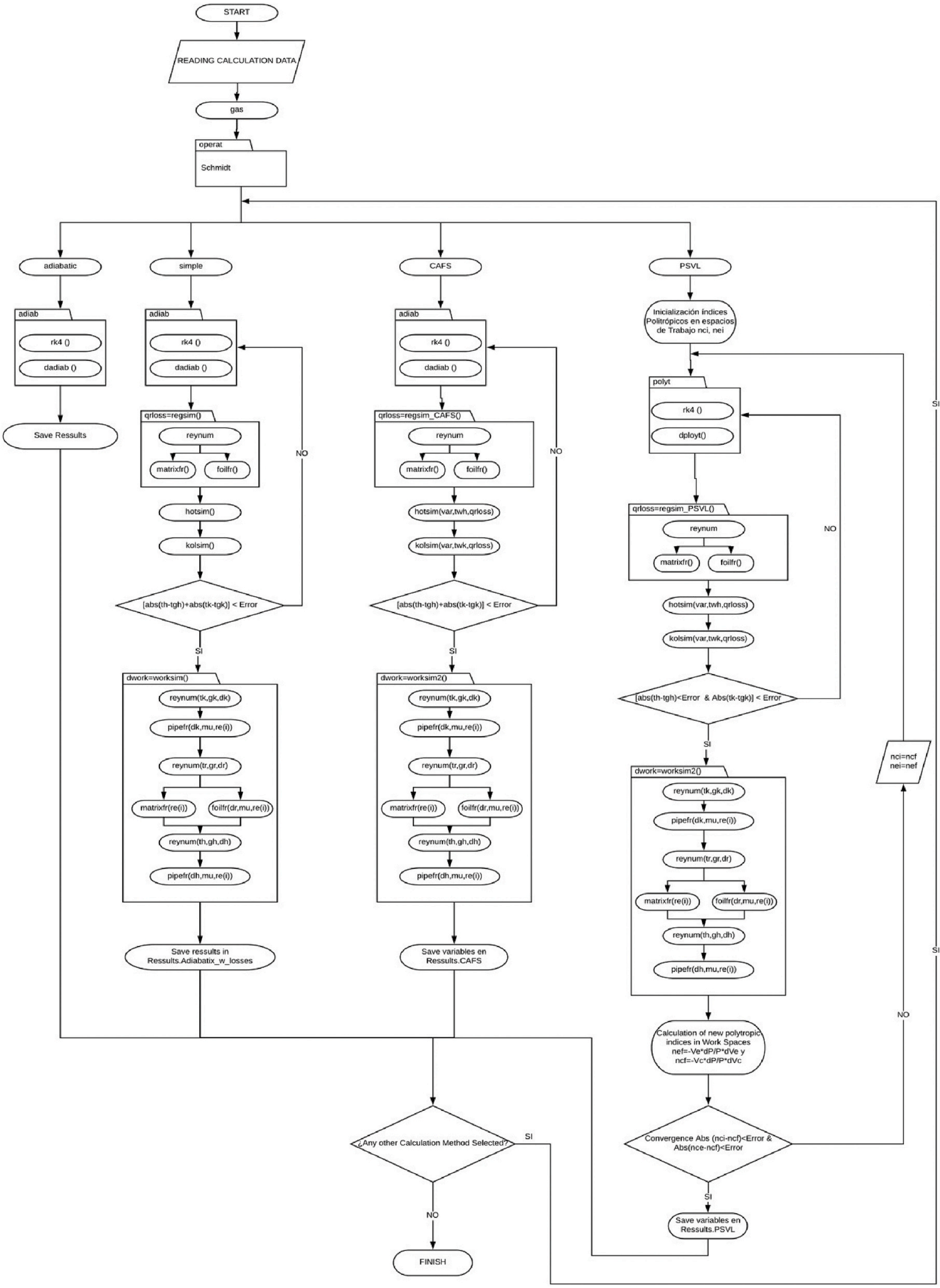

Fig. 8. Scheme of functions and calculation algorithm in Matlab GUI®.

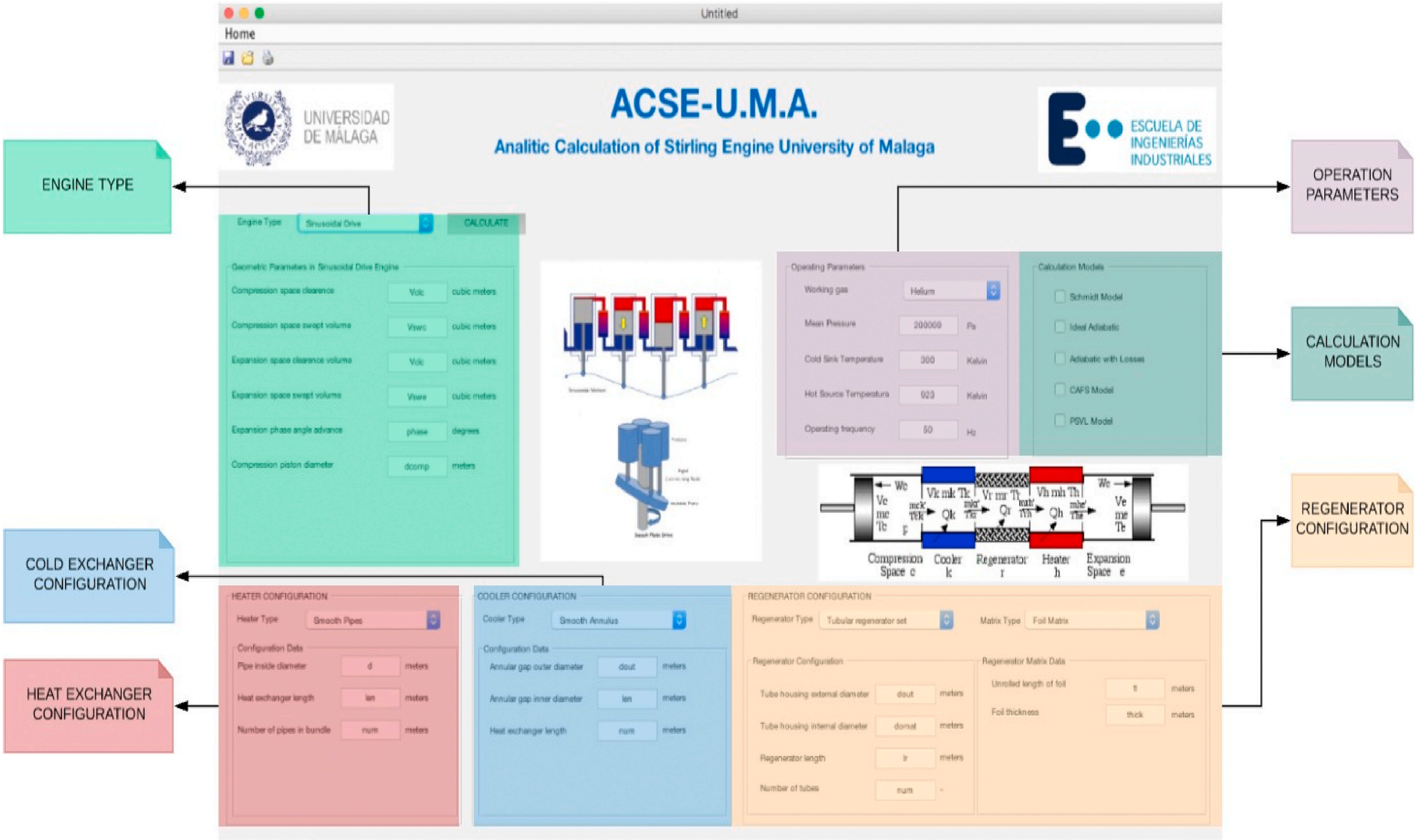

Fig. 9. Preliminary results panel.

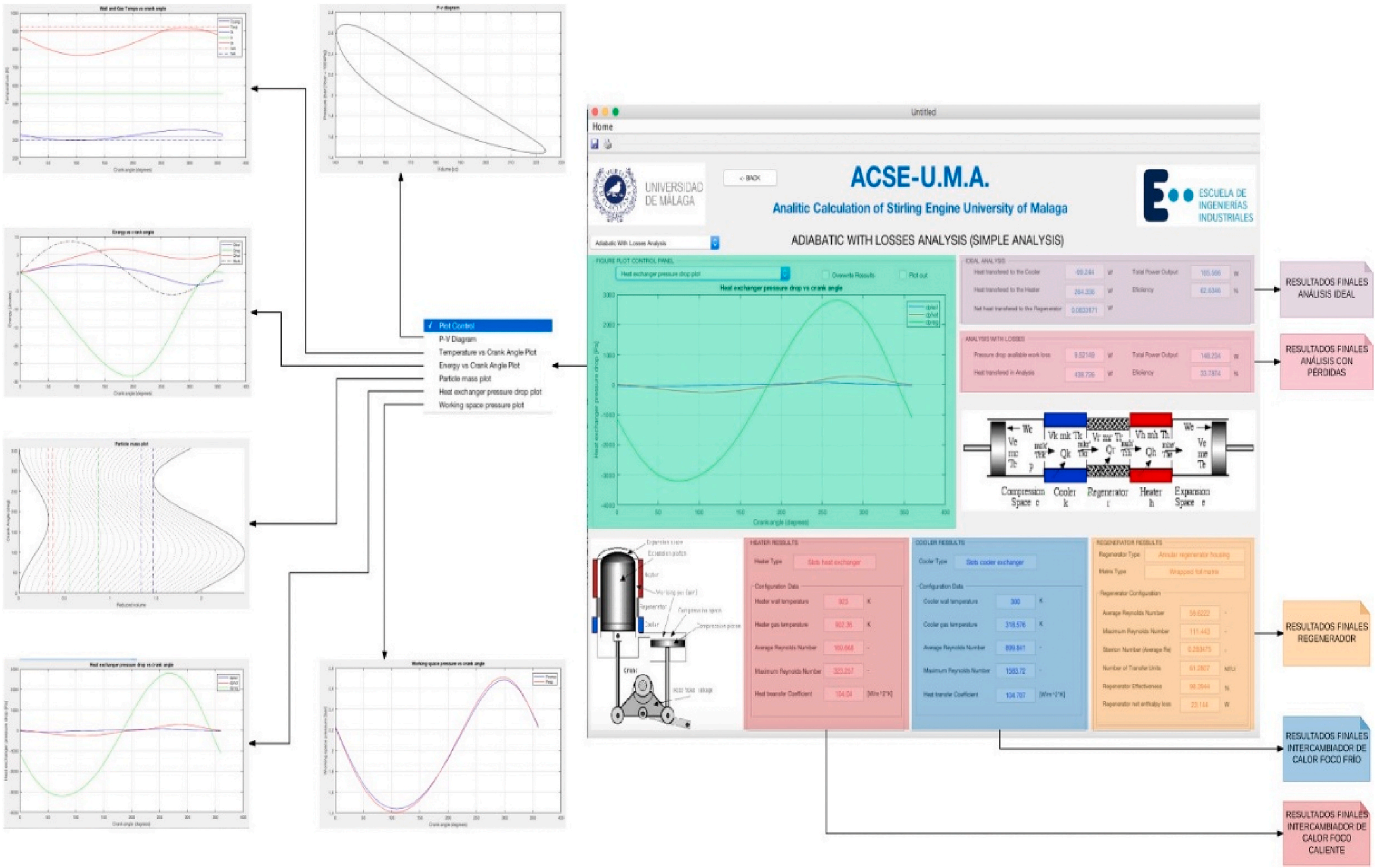

Fig. 10. Final results panel.

- Final Results Panel

3.5.1. Preliminary results panel

In the first instance, a panel has been designed, showing all the results obtained during the preliminary calculation. These calculations concern the first results from geometric data and operation, which must be performed to obtain: engine volumes, parameters in the hot, cold and regenerator exchangers, in addition to detailing the calculations obtained in the Schmidt isothermal model.

This panel therefore obeys a first degree of concreteness and is subdivided into the sections shown in Fig. 9.

3.5.2. Final results panel

This panel shows all the final results of the calculation, with a structure similar to the one that used for the preliminary calculations. Here, the ideal analysis is detailed against the analysis with losses In this, allowing to see the final results in the exchangers, as well as in the regenerator. There is an advanced representation menu, which allows the execution of graphic figures inside of the screen, as well as the comparison between different types of calculation models. The following blocks of data are presented in this panel, shown in Fig. 10.

4. Results and validation

A widely used in research literature engine has been chosen for the validation of mathematical modelling in the calculation of output power and thermal efficiency, such as the GPU-3 engine (Ground Power Unit 3). This engine developed in 1965 by General Motors Research Labs has a rhombic mechanism and a power of 7.5 kW. From this engine results of real power output tests are known, as well as its isothermal analysis from I. Urieli and D.M. Berchowitz results, published in 1984 [22].

4.1. Main features of the test

In Table 2 the parameters used for the motor test are summarized by the program.

4.2. Discussion

For the validation of engine modelling, the power output and efficiency of several numerical models published in research articles are analyzed [62]. These are compared with those obtained by modelling in the developed software, ACSE-UMA (Table 3).

Ideal Adiabatic Test: It is verified that the obtained results are practically identical; being exactly equal the power obtained by

Table 2
Test configuration characteristics of a Stirling GM GPU-3 engine [6].

Parameter	Value	Units
Engine configuration	Beta	–
Compression space diameter	69,9	*mm*
Expansion space diameter	69,9	*mm*
Compression space dead volume	26,88	cm^3
Expansion space dead volume	30,52	cm^3
Crank radius	13,8	*mm*
Heater temperature	977	*K*
Cooler Temperature	288	*K*
Mean effective pressure	4,13	*Mpa*
Working fluid	Helio	–
Frequency	41,7	Hz
Parameter	*Value*	*Units*
Heatertube's length	245,3	mm
Number of heater tubes	40	–
Heater tube's diameter	3,02	*mm*
Cooler tube's length	46,1	*mm*
Number of cooler tubes	312	–
Colertube's diameter	1,08	*mm*
Number of regenerators	8	–
Regenerator diameter	22,6	*mm*
Number of mesh layers	308	–
Number of wires per each mm	7,84	*mm*
Regenerator length	226	*mm*
Diameter wire	40	*mm*
Porosity	0,697	–
Eccentricity	20,8	*mm*
Stroke	31,2	*mm*
Displacer rod diameter	9,52	*mm*

ACSE-UMA and efficiency is practically equal.

Adiabatic Model with Losses: It is verified that the power is accurate, being the value of the Berchowitz e Urieli model of 6.70 kW and that obtained by ACSE-UMA of 6.51 kW. The efficiency of Berchowitz e Urieli is 51.50% and that obtained by ACSE-UMA is 44.90%. The new developed model is closer to reality; this may be because more modern equations are used to obtain losses than those used by Urieli and Berchowitz in Ref. [22].

Combined Adiabatic Model with Finite Velocity Thermodynamics (CAFS): The values obtained by ACSE-UMA are 4.02 kW, compared to 4.11 kW presented by similar bibliographic tests, while the efficiency obtained for this model by ACSE- UMA, is of 34.97% compared to 36.20%. These differences are negligible in terms of values, and may be due to the discretization used, of steps of 10° of rotation in the engine or to the given values of maximum error for the iterations to finish, which we remember has been established in a smaller difference of 1 °C. The results are good in relation to the bibliography consulted.

Ideal Polytropic Model: The efficiency results of the ACSE-UMA, in the polytropic model without considering losses, have been 59.80% efficiency and 7.68 kW. These are very close to the reference values of the literature of 60.36% efficiency and 7.73 kW power. These variations have been minimized by the calibration of the Buffer Pressure, which is a parameter that is not always known by reference to published tests, but can be calculated if the geometry of all the elements of the engine is known.

For the calculation, it has been started using as reference 100000 Pa, which Babaelahi and Sayyaadi recommend, in their modified PSVL model [58].

Polytropic Model with Losses (PSVL): The power obtained by ACSE-UMA is 3.15 kW, compared to the 3.14 kW of modified PSVL model presented by Babaelahi and Sayyaadi [60]. The efficiency is 23.54 compared to 24.44% of the source literature. Although the differences are minuscule, and practically insignificant, it is worth mentioning, as has been done previously, the importance of having parameter data such as the piston-cylinder clearance (parameter J), in order to make a calculation as real as possible. , since in addition this value, as well as the mentioned Buffer Pressure, influence tremendously in the final result and in the real possibilities of closing of the iterations.

Modelling of the third order: These results, which refer to highly complex methods and which require a high computational cost, have been incorporated, so that it is clear, as simpler methods such as those proposed in ACSE-UMA, can give results closer to the experimental results, with a much lower calculation power and in a much shorter time than complex methods.

5. Conclusions

A deep analysis, study and explanation of the current numerical methods that allow the simulation of Stirling engines has been conducted, developing the equations that govern these models and explaining the many variables and interrelationships between them.

With the use of computer applications and numerical simulations, it is possible to obtain satisfactory results for Stirling engines in low calculations times.

A computer application has been created, capable of simulating various types of Stirling engines under various calculation assumptions, as well as being programmed with enough flexibility so that it can have future versions and incorporate other calculation models, engine typologies and be updated according to the studies and investigations in this field.

In addition, the models have been validated, through the testing and verification of several engines that have found reference data in research literature.

This computer tool, together with the information base that complements this work, can contribute to the development of new Stirling engines or improvement the research on the existing ones.

It has been verified that the numerical methods for simulating Stirling engines, give satisfactory results for the calculation and optimization of the output power and thermal efficiency of the engines, as it has been verified in the validation by means of the test of a GPU-3 engine. In addition, these are obtained in a short time of calculation, which allows and makes feasible the possibility of introducing optimization methods.

In this regard it is worth mentioning that the best results are obtained by the polytropic method with losses (PSVL), which is important to note that it is necessary to know the buffer pressure that is lost by the swing of the piston. This in many occasions requires knowing experimental results to be able to adjust its value, as it has been proved in the trial of the aforementioned engine. In addition,

Table 3
Results obtained by ACSE-UMA compared to other models published in the trial of a GM GPU-3 engine.

Model	Other Results Of Previous Papers				Results in ACSE-UMA			
	Efficiency (%)	Power (kW)	Error absolute efficiency	Error relative potency	Efficiency (%)	Power (kW)	Error absolute efficiency	Error power
Ideal Adiabatic	62,30	8,30	41,00	176,67	63,92	8,30	42,62	176,67
Ideal Polytrophic	60,36	7,73	39,06	157,67	59,80	7,68	21,30	100,00
Adiabatic with losses: Berchowitz & Urieli [22]	52,50	6,70	31,20	123,33	44,90	6,51	23,60	117,00
CAFS Model [60]	36,20	4,11	14,90	37,00	34,97	4,02	13,67	32,00
PSVL [58]	24,44	3,03	3,14	1,00	23,54	3,15	2,24	5,00
Third order analysis [6]	42,00	4,26	20,70	42,00	–	–	–	–
Experimental Result [63]	21,30	3,00	–	–	–	–	–	–

the polytropic model requires more geometric data, which are not always available or found in the Stirling engine literature. It has been found that the sensitivity of the results other than the PSVL model to the parameters Pressure buffer (*P_buffer*) and to the parameter *J* measuring the annular space between piston and cylinder is very high. Therefore, if these values are unknown or cannot be estimated use of this model is not recommended.

The method that combines the adiabatic analysis with losses with finite speed thermodynamics (CAFS) yields good results without requiring the knowledge of as many geometric variables as for the polytropic method. It can be used when all the geometric data are not known or there is no possibility of accessing experimental data to adjust the buffer pressure of the motor, which is required in the polytropic model.

The adiabatic method with losses requires practically the same values as the CAFS method, requiring the latter only of the stroke, as additional data. For this reason, it is recommended to use the CAFS analysis against the adiabatic one, since for the same calculation time it yields better results.LI.

Also with respect to the polytropic model, it is necessary to emphasize the requirement to implement coherent values to initiate the iterations in the calculation of the polytropic coefficients. If these values are not chosen, the iterations may not be closed and consequently no results will be obtained. In this regard, it is recommended, based on the experience obtained, to initialize the polytropic coefficients for compression with a value n_ci = 1.2 and for the expansion with a value n_ei = 1.9. These values have been obtained from the analyses of multiple publications on PSVL tests in Stirling engines and have been adjusted for the developed model. While there have been multiple tests with various engines, it is worth mentioning that if the iterations were not closed, other initial polytropic indices should be tested for expansion and compression.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclature

Symbol	Meaning	Units
A_r	Free flow area	m^2
A_h	Heater transfer area	m^2
A_k	Cooler transfer area	m^2
A_{wg}	Wetted area	m^2
A_{wgh}	Heater wetted area	m^2
A_{wgk}	Cooler wetted area	m^2
c	Mean molecular speed	m/s
C_f	Friction coefficient	–
c_n	Heat capacity (polytropic)	J/kg K
c_p	Specific heat of the gas at constant pressure	J/kg K
c_v	Specific heat of the gas at constant volume	J/kg K
CAFS	The Combines Adiabatic-finite Speed thermal approach	–
d	Hydraulic diameter	m
d_h	Heater hydraulic diameter	m
d_k	Cooler hydraulic diameter	m
d_r	Regenerator hydraulic diameter	m
D	Piston diameter	m
D_d	Displacer diameter	m
f_r	Friction coefficient	–
f	Engine frequency	Hz
FST	Finite speed thermodynamics	–
FTT	Finite Time Thermodynamics Analysis	–
G	Mass flow rate of the working fluid	kg/s
h	Specific Enthalpy	J/kg
h_c	Convective heat transfer coefficient	W/m^2 K
h_{HC}	Convective heat transfer coefficient (hot end of the engine)	W/m^2 K
h_{HR}	Radiative heat transfer coefficient (hot end of the engine)	W/m^2 K
h_{LC}	Convective heat transfer coefficient (cold end of the engine)	W/m^2 K
h_{LR}	Radiative heat transfer coefficient (cold end of the engine)	W/m^2 K
h_h	Heat transfer coefficient in the heater	W/m^2 K
h_k	Heat transfer coefficient in the cooler	W/m^2 K
k	Thermal conductivity	W/m^2 K
k_0	Conductive thermal bridge loss coefficient	W/K s
k_g	Thermal conductivity of gas	W/m^2 K
J	Annular space between displacer and cylinder	m
L	Piston length	m
L_d	Displacer length	m
l_h	Heater length	m

(*continued on next page*)

(*continued*)

Symbol	Meaning	Units
l_k	Cooler length	m
l_r	Regenerator length	m
M	Regenerative time constant	K/s
m	Mass	kg
m_c	Mass stored in the compression space	kg
m_e	Mass stored in the expansion space	kg
m_h	Mass stored in the heating space	kg
m_k	Mass stored in the cooling space	kg
m_r	Mass stored in the regenerator	kg
m_{leak}	Loss of mass to crankcase in polytropic model	kg
m'_{ck}	Compression-cooling interface zone mass flow	kg
m'_{he}	Heater-expansion interface zone mass flow	kg
m'_{kr}	Compression-regenerator interface zone mass flow	kg
m'_i	Input mass	kg
m'_o	Output mass	kg
n	Polytropic index	–
η_t	Indicated efficiency	–
nia	Non-Ideal Adiabatic model	–
NTU	Number of Transfer Units	–
Nu	Nusselt number	–
p	Power	W
P	Gas pressure	Pa
p_c	Indicated compression power	W
p_e	Indicated expansion power	W
Pr	Prandtl number	–
dP	Total pressure drop	Pa
dP_H	Pressure drop through the heater	Pa
dP_K	Pressure drop through the cooler	Pa
P_m	Instantaneous mean effective pressure	Pa
dP_R	Pressure drop through regenerator	Pa
Pr	Prandtl number	–
ΔP_f	Friction pressure loss	Pa
ΔP_{throtl}	Pressure loss in the regenerator	Pa
Q	Heat	J
Q_c	Heat in compression space	J
Q_e	Heat in expansion space	J
Q_H	Total heat in heater space	J
Q_h	Net heat in heater space	J
Q_K	Total heat in cooler space	J
Q_k	Net heat in cooler space	J
Q_L	Heat released at the heat sink	J
Q_r	Regenerator heat exchange	J
Q_{rloss}	Lost energy in regenerator	J
r_v	Compression ratio	–
R	Ideal Gas Constant	J/kg K
$R_{H,r}$	Regenerator hydraulic radius	m
Re	Reynolds number	–
S	Stroke	m
St	Stanton number	–
t	Time	s
t_{re}	Regenerative time	s
T	Temperature	K
$tan\ \beta$	Ratio between thermodynamic and kinematic volume	–
T_c	Compression space gas temperature	K
T_{ck}	Compression-cooling interface zone gas temperature	K
T_e	Expansion space gas temperature	K
T_H	Regenerator high temperature	K
T_h	Heater space gas temperature	K
T_{he}	Heater-expansion interface zone gas temperature	K
T_i	Input temperature	K
T_k	Cooler space gas temperature	K
T_{kr}	Cooler-regenerator space gas temperature	K
T_L	Regenerator low temperature	K
T_o	Output temperature	K
T_r	Regenerator space gas temperature	K
T_{rh}	Regenerator-heater space gas temperature	K
T_w	Wall temperature	K
T_{wh}	Heater wall temperature	K
T_{wk}	Cooler wall temperature	K
T_0	Ambient temperature	K

(*continued on next page*)

(*continued*)

Symbol	Meaning	Units
u_p	Piston velocity	m/s
u_{max}	Maximum velocity of gas when moving into the heater	m/s
v	Swept volume ratio	–
V	Volume	m^3
V_c	Compression space volume	m^3
V_{clc}	Dead volume of compression space	m^3
V_{cle}	Dead volume of expansion space	m^3
V_e	Expansion space volume	m^3
V_h	Heater space volume	m^3
V_k	Cooler space volume	m^3
V_{mesh}	Wire volume	m^3
V_r	Regenerator volume	m^3
V_{rv}	Regenerator Void volume	m^3
V_{sw}	Swept volume of power piston	m^3
V_{swc}	Swept volume of compression piston or power piston	m^3
V_{swe}	Swept volume of expansion piston or displacer piston	m^3
w	Piston speed	m/s
W	Indicated work	J
W_c	Indicated compression work	J
W_e	Indicated expansion work	J
W_h	Indicated heater area work	J
W_k	Indicated cooler area work	J
W_{nia}	Net work of engine output non-ideal adiabatic analysis	J
x	Length	m
Δp	Pressure loss	Pa
δQ	Heat transfer	J
$\delta Q_{polytropic}$	Polytropic heat transfer	J
$\delta Q_{shuttle}$	Heat loss due to conduction	J
δW	Output work	J
δW_{loss}	Power loss	W
δW_{FST}	Power losses due to mechanical friction FST	W
α_t	Thermal diffusivity	m^2/s
γ	Adiabatic coefficient	
ϵ	Exchanger efficiency	–
ϵ_r	Regenerator efficiency	–
η_{nia}	Engine efficiency in nia	–
η_t	Thermal efficiency	–
λ	Volumetric ratio of the engine	–
Π	Regenerator porosity	–
Θ	Cycle angle	Rd
Φ	Regenerator shape factor	–
ρ	Density	kg/m^3
μ	Dynamic viscosity	$N\ s/m^2$
ν	Kinematic viscosity	m^2/s

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