On order policies with pre-specified order schedules for a perishable product in retail

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Abstract. This paper studies a retail inventory system for a perishable product, based on a practical setting in Dutch retail. The product has a fixed shelf life of three days upon delivery at the store and product demand has a weekly pattern, which is stationary over the weeks, but varies over the days of the week. Items of varying age occur in stock. However, in retail practice, the age-distribution is often unknown, which complicates order decisions. Depending on the type of product or the size of the supermarket, replenishment cycle lengths may vary. We study a situation where a store is replenished either three or four times a week on pre-specified days. The research aim is to find practical and efficient order policies that can deal with the lack of information about the age distribution of items in stock, considering mixed LIFO and FIFO withdrawal. Reducing potential waste goes along with cost minimization, while the retailer aims at meeting a cycle service level requirement. We present four new heuristics that do not require knowledge of the inventory age-distribution. A heuristic, based on a constant order quantity for each order moment, often generates least waste and lowest costs. However, this requires a few minutes of computation time. A new base stock policy appears second best.

Keywords: inventory · perishable product · order policy · retail · service level

1 Introduction

Supermarket managers often face a trade-off between risking to lose both revenue and goodwill, by not having products available when demand arises, and discarding surplus products due to outdating (Gruen et al. 2002). Food waste is primarily a result of retailer and consumer behavior (Parfitt et al., 2010). It occurs either through markdowns when products approach their end of shelf life or appear less appealing, or through disposal when products are no longer usable, sellable, or edible. In Europe the total food loss and waste is 31% of the initial production from which 6.1% occurs in the food processing, packaging and distribution (HLPE, 2014). Lebersorger and Schneider (2014) report a food loss rate of fruit and vegetables of 4.19% in an Austrian food retail company from September 2011 to August 2012. For dairy products the food loss rate was 1.14%, and for bread and pastry 2.84%.

Generally, the demand is influenced by product availability and freshness (Sebatjane and Adetunji, 2021). Availability of fresher items significantly affects consumer choice on where to shop (Wyman, 2013). So, retailers rather build up more stock than risk a stock-out (Thyberg and Tonjes, 2016). This is not without risk since waste represents a loss of business and a risk for already small margins (Cicatiello et al., 2017). Reducing the annual food waste will result in benefits for companies, consumers and the environment in terms of money, volume, energy and sustainability. Retailers are therefore very keen to implement strategies to reduce food waste. To illustrate this, members of the Consumer Goods Forum promised in 2015 to halve the food they waste by 2025 (Forum, 2015). This motivates research into order policies that may help to both prevent food waste and reduce costs for retailers. We study a competitive strategy of retailers that focuses on availability of fresh produce for fixed prices, without discounting, applying strict service level requirements. Here, waste prevention is the main objective, though waste reduction and cost minimization are equivalent. This differs from studies that focus on discounting and dynamic pricing (Herbon, 2017; Buisman et al., 2019; Fan et al., 2020), or adapting the shelf life depending on the order policy (Ketzenberg et al., 2018). These practices aim at waste reduction, rather than prevention.

The Dutch retailer case studied in this paper enhances a highly perishable product inventory system with a fixed shelf life of three days on delivery at the store, noticeable by its best-before or use-by date. The days of the week the supermarket is replenished, are determined by warehouse capacity. Sternbeck and Kuhn, (2014) and Holzapfel et al. (2016) showed that pre-specified reorder schedules have major advantages with respect to warehouse and transportation costs and scheduling workforce. Such reorder schedules imply that the retailer has items of different ages in stock and replenishment cycles may vary in length. In many practical retail situations, checkout systems only register the number of items sold, but not the product age. Consequently, the retailer is facing an order decision without knowledge of the age-distribution of the remaining items in stock. The observed total number of items in stock may be different from the inventory status according to the checkout system, due to damaged items and the occurrence of more waste than expected based on supply and demand data of the supermarket. This situation is also described by Pantsar (2019). Technically, there are options to obtain knowledge on the age distribution e.g. to use RIFD tags, or use barcodes for the due date of an item.

In retail, food waste is related to food inventory management practices, as well as to purchasing behavior of customers (Cicatiello et al., 2017). The outdating quantity is affected by the inventory withdrawal sequence. Retailers will stimulate FIFO depletion of inventory because of lower costs (Cohen and Prastacos, 1981). Nonetheless, consumers typically prefer fresher items, so they at least partly adopt LIFO depletion (Nahmias, 1982). Although a large part of literature only considers one of the extremes, a mixed FIFO-LIFO policy is more realistic in food retail (Janssen et al., 2016). The total item shortage is not influenced by the sequence of withdrawal. However, the number of items that expire is significantly affected (Cohen and Prastacos, 1981). LIFO generally leads to higher costs due to more outdating. In order to minimize waste, supermarkets prefer and stimulate customers to pick the oldest items first (FIFO, First In First Out), by putting those items in front on the shelf. However, practitioners in Dutch retail estimate that about 40% of the customers searches for the freshest items and picks according to LIFO (Last In First Out).

Customers may behave differently when confronted with an empty shelf. It is often assumed that customer demand is partially or completely backordered when items are not in stock (Bijvank and Vis, 2012; Gupta et al., 2020), but only about 15% of customers that face an out-of-stock will buy the product later at the same store (Gruen et al., 2002). Moreover, once a stock-out occurs, information about actual demand is lost and there may be a decline in goodwill, which is difficult to quantify (Bijvank and Vis, 2012). Rather than attempting to quantify all effects in monetary terms, an alternative approach is to assess the performance of an inventory system by means of a service level measurement (Minner and Transchel, 2010). Service level targets have become increasingly important in times where product availability influences retail competition. A cycle service level is most suitable for an inventory system with periodic review and lost sales once an out-of-stock situation occurs (Cachon and Terwiesch, 2013).

The question is how to generate order policies and corresponding parameter values for this retail situation in a limited computation time. The situation is characterized by pre-specified reorder schedules, varying replenishment cycle length and a service level requirement, when the age-distribution of the inventory is unknown to the decision maker. Demand has a weekly (seasonal) pattern, which is stationary over the weeks. Moreover, we focus on a shelf life of three days upon delivery, a lead time of one day and mixed LIFO-FIFO depletion.

This paper is organized as follows. Section 2 embeds our question in literature. Section 3 describes the retail situation, a stochastic dynamic model of the situation and approaches to determine order policy parameter values. Section 4 shows the results of numerical experiments based on practical data for the approaches. A discussion of findings and conclusions can be found in Sections 5 and 6.

2. Literature

Along this line the issue of finding a suitable order policy for highly perishable products is addressed, to incentivise food waste prevention (Thyberg and Tonjes, 2016). Early studies were mainly based on dynamic programming. Because of the complexity of finding exact optimal policies with this method, (Nahmias, 1975) already identified the need to develop heuristics. The most well-known and commonly used types of policies are base-stock policies (BSP) and constant order policies (COP), where every order is of the same fixed size. A base-stock policy entails ordering up to a specified inventory level, and is therefore responsive to fluctuations in demand. This is especially beneficial in case of stochastic demand (Cachon and Terwiesch, 2013). For perishable products, the structure of an optimal replenishment policy is complex as the simple BSP is not optimal. Whereas COPs lead to smoother order quantities, they entail the risk of building up too much stock. BSPs on the other hand, may not order enough when many items are about to expire at the same time (Haijema and Minner, 2016). Minner and Transchel (2010) showed that under stationary demand, short shelf live and LIFO withdrawal, COPs actually perform reasonably well. Nevertheless, the authors also indicated that if demand is non-stationary, hybrid versions of order policies may be required. As of today, there is no common understanding on which policy to use under what circumstances (Haijema and Minner, 2016).

Research has shown that considering age-information about the products in stock enhances the accuracy of replenishment decisions (Tekin et al., 2001; Haijema et al., 2007). Incorporation of this type of information can be obtained in several ways. First of all, some models divide the inventory into 'old' and 'new' parts, and base order decisions on the relative levels. Balugani et al. (2019) investigated a periodic inventory system for products with a fixed shelf life and intermittent demand. Their study distinguishes items that will versus items that will not expire during the review period. A fill rate service level constraint and a FIFO issuing policy are considered. Chen et al. (2021) propose an inventory strategy with a single adjustment plan for an expedited order or a return plan for products, such as blood platelets, with a FIFO issuing policy. Secondly, BSPs with weighted stock levels (BSP − WS) lead to larger order quantities when more products are about to expire. However, it has to be noted that BSP − WS is actually a special case of the division between old and newer inventory. Additionally, some models modify the order quantity with estimates of waste (BSP − EW) (Pauls-Worm et al., 2014; Haijema and Minner, 2019). A somewhat different approach was adopted by (Tekin et al., 2001). In the proposed policy, an order is placed when the inventory level drops to a reorder level, or when a specific amount of time elapsed, whichever occurs first. This time element represents the product-age threshold (Tekin et al., 2001). Gutierrez-Alcoba et al. (2017) introduced two heuristics to determine the optimal order quantity for perishable products with a fixed shelf life and non-stationary demand. In their approach, the expected value of the inventory for different product ages is computed, while considering penalty cost in case of out-of-stock.

Broekmeulen and Van Donselaar (2009) proposed an EWA Expected Withdrawal of Aging heuristic for both complete FIFO as well as complete LIFO withdrawal in which they consider the full age-distribution to be known to the decision maker. They assume non-stationary demand during the week which is stationary over the weeks. An (R, s, n, Q) policy, with fixed review period R, reorder point s, batch-size Q and order multiplier n, is corrected for the estimated amount of waste. Moreover, a constant safety stock was implemented, which may be undesirable in case of non-stationary demand. In turn, Duan and Liao (2013) explicitly included a fill rate constraint, and assumed less information about inventory ages. They proposed a policy based on an old inventory ratio (OIR), relative to the total inventory on hand. First, the order quantity is determined according to a classical BSP. Subsequently, the ratio of old items compared to overall stock is determined. If this amount exceeds a specified threshold, an additional replenishment is triggered. This is supposed to be a relatively easy policy to implement by practitioners, with better results than the EWA heuristic. However, optimization of an extra parameter, the threshold value, is required.

Kiil et al. (2018) intend to incorporate shelf life information in "a setting closer to the reality of today's grocery retailers", with pre-specified reorder schedules rather than fixed review periods. The result is a refined version of the EWA heuristic: EWAss. The safety stock assumption in the original EWA is modified adding expected waste quantities resulting in larger stock. With EWAss, the safety stock either equals the expected waste, or the safety stock required for demand uncertainty, instead of both. Kiil et al. (2018) assess a cycle service level, and they assume a mixed FIFO/LIFO withdrawal, with 90% FIFO. Both EWA and EWAss assume the age-distribution information is registered by grocery stores. With this assumption, Hendrix et al. (2023) show that if orders can be placed on a daily base, the optimal order quantity can be derived. This results in sufficient product-availability and less waste than using a heuristic like EWA and EWAss.

Concluding, there is a challenge in deriving easily applicable order policies in practise for perishable products in case of pre-specified reorder schedules combined with an unknown age-distribution of the items in stock. This study focuses on a situation with non-stationary demand and mixed LIFO-FIFO withdrawal.

3. Modelling the retail situation

To be able to model the stochastic dynamics for this problem, it is necessary to first identify the underlying characteristics of the practical situation. These characteristics are discussed in Section 3.1 and the model in Section 3.2. The order schedule is given by pre-specified days of the week, which results in a limited number of possible replenishment schedules that is discussed in Section 3.3. The approaches to determine order policies are presented in Section 3.4.

3.1 Retail situation description

In this study, a period t in the model is a day at the store, from opening until closing time. In the retail practice of perishable products, mostly the order quantity Q_t of today is delivered the next day, so the lead time is 1 day. We aim to develop and evaluate order policies that do not need age-information of the items in stock. In the model, the sequence of events is as follows:

1. Store opening
2. Delivery of quantity Q_{t-1} if $Q_{t-1} > 0$
3. Ordering of quantity Q_t if reordering is allowed according to the schedule
4. Demand during the day from a mixed FIFO and LIFO withdrawal, aging of remaining items in stock and disposal of wasted items, at store closure

At the moment of the order decision, the previous order has arrived, so there is no outstanding order. The order quantity is based on the on-hand inventory and the expected demand during the replenishment cycle. At the end of day t, the inventory level I_{bt} is realized for items of all ages b. So, items that are delivered on day t in quantity Q_{t-1}, have age $b = 1$ at the end of day t. Items with an age reaching the shelf life $b = M$, are waste and removed from the shelf at the end of the day. In case of waste, the purchasing cost is lost. The time horizon T is 7 days, where $t = 1$ is Monday. The inventory at the end of Sunday ($T = 7$) transfers to Monday morning.

The demand is independently Poisson distributed with expectation μ_t for day t where we consider values obtained from a practical case. When demand is higher than the inventory level, sales are lost and the inventory level will be zero. Moreover, we consider a mixed LIFO-FIFO withdrawal, where the number of customers buying LIFO is binomially distributed. Section 3.2 presents the stochastic evaluation model for this problem.

Table 1. Used symbols

Indices	
t:	Day of the week, $t = 1,..,7$, $T = 7$
b:	Age of the item in stock, $b = 1,..,M$, $M = 3$
r:	Replenishment cycle length
L:	Lead time, $L = 1$
Data	
c:	Purchasing cost per item
μ_t:	Expected demand day t
d_t:	Random demand day t, Poisson distributed
$F_{rt}(\,)$:	Cumulative distribution function of demand $d_{t+1}+..+d_{t+r}$
α:	Service level requirement as probability
λ:	Probability a client selects according to LIFO
$\hat{S}_{L+r,t}$:	Basic order-up-to level
$\hat{Q}_{rt}$.:	Basic order quantity, $\hat{Q}_{rt} = F_{rt}^{-1}(\alpha)$
Variables	
Q_t:	Quantity ordered on day t
I_{bt}:	Number of items in stock of age b at the end of day t
S:	Order-up-to level
Sal:	Order-up-to level S after lead time

3.2 Stochastic evaluation model

In the discussion about the model with the stakeholders, we realized that in fact traditional concepts from inventory control like inventory holding cost, reorder cost and even the salvage value of the waste were not relevant for this practical situation. This means that in this situation, minimization of cost and minimization of waste coincide. The general reorder decision problem can be formulated as a stochastic optimization model that minimizes purchasing cost.

$$\text{Min}\ \{E(TC) = \sum_{t=1}^{T} E(cQ_t)\} \tag{1}$$

Let $(x)^+ = \max\{x, 0\}$. The inventory balance for the total inventory of all ages is given by:

$$\sum_{b=1}^{M} I_{bt} = \left(\sum_{b=1}^{M-1} I_{b,t-1} + Q_{t-1} - d_t\right)^+ \qquad t = 1,..,T \tag{2}$$

where $t - 1 = 0$ corresponds to $T = 7$ in our case. Period t starts with the inventory levels at the end of period $t - 1$ of ages $b = 1,.., M - 1$, since items of age M are waste. The starting inventory is increased by the delivery Q_{t-1} minus the demand in period t, giving the end inventory.
The service level requirement is modelled as

$$P\left(d_t \leq \sum_{b=1}^{M-1} I_{b,t-1} + Q_{t-1}\right) \geq \alpha \qquad t = 1,..,T \tag{3}$$

The probability that demand is met from available inventory should be at least α. This type of service level is known as α-service level or cycle service level (CSL). We apply a minimal service level

constraint as studied by Chen and Krass (2001), where for every day, on average, the service level has to be met. A long-run average service level constraint is easier to meet, but can cause under-achievement of the service level on a specific day. This is undesirable in a retail situation.
Equations (4) and (5) model the inventory levels in case of only LIFO withdrawal, where demand is fulfilled first by the freshest items before the older items.

$$I_{1t} = (Q_{t-1} - d_t)^+ \qquad t = 1,..,T \qquad (4)$$
$$I_{bt} = \left(I_{b-1,t-1} - \left(d_t - Q_{t-1} - \textstyle\sum_{j=1}^{b-2} I_{j,t-1}\right)^+\right)^+ \qquad t = 1,..,T; b = 2,..,M \qquad (5)$$

The distribution of demand into LIFO and FIFO follows a binomial distribution, with $0 \leq \lambda \leq 1$ the fraction of customers that choose the items according to LIFO. This means that the dynamics of Equations (4) and (5) is first followed for (binomially drawn) λ% of the customers and then for (1-λ)% of the customers it follows the FIFO dynamics of equations (6) and (7), where demand is fulfilled first by the oldest items before the fresher items.

$$I_{bt} = \left(I_{b-1,t-1} - \left(d_t - \textstyle\sum_{j=b}^{M-1} I_{j,\,t-1}\right)^+\right)^+ \qquad t = 1,..,T; b = 2,..,M \qquad (6)$$

describe the levels of waste and the older items in stock, whereas

$$I_{1t} = Q_{t-1} - \left(d_t - \textstyle\sum_{b=1}^{M-1} I_{b,t-1}\right)^+ \qquad t = 1,..,T \qquad (7)$$

give the freshest items in stock. Finally, the model keeps track of nonnegativity and the balance at the end of the week, where the final order will be the first delivery on Monday the next week.

$$Q_0 = Q_T \qquad (8)$$
$$Q_t \geq 0 \qquad t = 1,..,T \qquad (9)$$
$$I_{bt} \geq 0 \qquad t = 1,..,T; b = 1,..,M \qquad (10)$$

The last order quantity Q_T of the time horizon equals the first delivery Q_0 of the time horizon (9). The inventory levels of items of all ages are nonnegative (10), this implies that if demand exceeds available inventory, excess demand is lost and inventory levels are zero.

3.3 Replenishment schedules

In the retail situation under study, ordering takes place on predetermined days of the week according to an order schedule. The use of a base-stock policy is popular, because it is easy to apply in practice and has more flexibility in the order quantity than a constant order policy. In a base-stock policy (BSP), an order-up-to level S determines the order quantity Q. We call such a policy a YS policy. Let $Y_t = 1$ when there is an order on day t and $Y_t = 0$ when no order takes place. Given the order schedule Y, values for day dependent order-up-to levels S_t should be found for each day t of the week. The order quantity in terms of an order-up-to policy is determined by

$$Q_t = \left(S_t - Q_{t-1} - \textstyle\sum_{b=1}^{M-1} I_{b,t-1}\right)^+, \qquad (11)$$

depending on the amount of stock that is still acceptable for use. Given a target α-service level, the value S_t for which the in-stock probability is α for Poisson distributed demand is a value $S \in \{0,1,2,3,\dots\}$, such that

$$P(d \leq S) = e^{-\mu} \sum_{i=0}^{S} \frac{\mu^i}{i!} \geq \alpha. \tag{12}$$

Standard textbooks like (Chopra and Meindl, 2016) describe the derivation of the order-up-to level in a periodic review system taking demand during lead time L and the replenishment cycle $r \leq M$ into account. The Poisson distribution allows us to calculate order-up-to levels for all possible replenishment cycle lengths r and lead time L by summing expected demand over the days. We call these order-up-to levels the basic order-up-to levels $\hat{S}_{L+r,t}$ for ordering for r periods in period t as described in (Hendrix et al., 2015). The values for S can easily be derived using an Excel or Matlab search routine.

Two issues make this analysis less appropriate for the case we study. First, the described concept to include the lead time demand in the order-up-to level is based on a backlogging situation, which is not reflecting a retail stock-out situation. When at the start or during period t a stock-out occurs, the order quantity Q_t will be higher than necessary. Secondly, in a perishable inventory situation, waste may occur during the replenishment cycle. In case of lost sales during the lead time, one can determine the order quantity based on the following reasoning. Let $F_{rt}(.)$ be the cumulative distribution function of demand $d_{t+1}+..,+d_{t+r}$ during the replenishment cycle of length r, then

$$\hat{Q}_{rt} = F_{rt}^{-1}(\alpha) \tag{13}$$

gives the amount that should at least be in stock at the beginning of the next day. In an out-of-stock situation in period t, this quantity is exactly the amount to be ordered. Figure 1 shows the time frame of the used symbols.

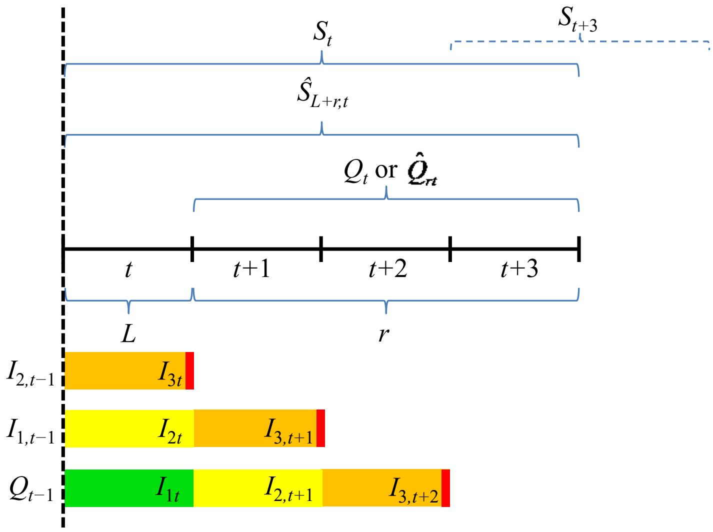

Fig 1. Time frame of used symbols and possible age-distribution for a replenishment cycle of $r = 3$ periods considering $L = 1$

Consider a starting inventory $\sum_{b=1}^{M-1} I_{b,t-1} + Q_{t-1} = 0$, minimizing the order quantity fulfilling the chance constraint (3) (i.e., $Q_t \geq \hat{Q}_{rt}$.) implies that the optimal order quantity is $Q_t = \hat{Q}_{rt}$.
In case waste occurs during the replenishment cycle, basic order-up-to level $\hat{S}_{r+1,t}$ is too low, so we should correct the order quantity for the expected waste. The challenge is how to determine this amount. The delivery schedule and consequently order timing is determined by the warehouse, based on the weekly demand pattern and lead time. Stakeholder information shows that if the supermarket is not daily replenished, delivery takes place on Monday and/or Tuesday, Thursday and/or Friday, and Saturday.

The 9 resulting suitable order schedules are listed in Table 2; five schedules with four order moments and four schedules with three order moments. Order policies for daily ordering are discussed in (Hendrix et al.,2023).

Table 2. Pre-specified order schedules from practice

Schedule	Reorder days
mwfs	Monday – Wednesday – Friday – Sunday
mtfs	Monday – Thursday – Friday – Sunday
wtfs	Wednesday – Thursday – Friday – Sunday
mwtf	Monday – Wednesday – Thursday – Friday
ttfs	Tuesday - Thursday – Friday – Sunday
wfs	Wednesday - Friday – Sunday
mwf	Monday – Wednesday – Friday
mtf	Monday – Thursday – Friday
Tfs	Tuesday – Friday. – Sunday

Finding good order policies for non-daily ordering with pre-specified order schedules requires an approach that considers the varying replenishment cycle length.

3.4 Approaches to determine order policy parameter values

The aim of this investigation is to develop and investigate order policies that are suitable for use in practice with pre-specified order schedules, without requiring information on the age-distribution of the inventory. Four approaches are described. The approaches vary in accuracy, calculation time and practical applicability. We compare the approaches numerically in Section 4.

YQSEW (Expected Waste) approach: For this approach based on SEW in (Hendrix et al 2023), the basic order-up-to level $\hat{S}_{r+1,t}$ is used. The actual order quantity Q_t is calculated using Eq. (11) corrected by an expected waste estimate considering a mixed expected LIFO − FIFO demand and the inventory dynamics. For pre-specified reorder moments, the previous replenishment cycle might be of length $r = M$, such that the starting inventory in stock equals zero for sure. The appropriate order quantity is given by Eq. (13). When at the start or during period t a stock-out occurs, the order quantity Q_t will be higher than necessary. A practical way to deal with the lost sales during lead time is to take as order quantity $Q_t = \hat{Q}_{rt}$ if $Q_{t-1} + \sum_{b=1}^{M-1} I_{b,t-1} - \mu_t \leq 0$.

YQS augmented heuristic: This approach based on SEW in (Hendrix et al 2023) also uses basic order-up-to level $\hat{S}_{r+1,t}$. The actual order quantity Q_t is calculated using Eq. (11), but in case the replenishment cycle is of length $r = M$, or when at the start or during period t a stock-out occurs, the order quantity is taken according to Eq. (13). In a simulation-optimisation approach, if in a simulation the average CSL is below the target on one or more days t, the order-up-to level of the previous order moment of the minimum average CSL value is augmented by one unit. The new order-up-to levels S_t are input in a new simulation run, until for all days the target CSL is met, resulting in a vector (S_1, S_2,.., S_7) of order-up-to levels that meets the CSL requirement.

YQSal incremented basic order-up-to level: Textbooks like Chopra and Meindl (2016) focus on an order-up-to level S_t that covers the lead time and the upcoming replenishment cycle. The order quantity is determined using the available inventory at the beginning of the day (Eq. (11)). Alternatively, consider the value for S_t to cover only the replenishment cycle after lead time. We call this order-up-to level S after lead time: Sal, which provides a lower safety stock level than using the previously defined order-up-to level $\hat{S}_{r+1,t}$. The basic $S\hat{a}l_t$ is determined by Eq. (13), so $S\hat{a}l_t = \hat{Q}_{rt}$. The order quantity is determined based on the anticipated inventory level at the end of the day, where the expected demand is subtracted from the inventory level at the beginning of the day. The corresponding order quantity is given by

$$Q_t = \begin{cases} S\hat{a}l_t \text{if } r = M \\ \left(S\hat{a}l_t - \left\lfloor \left(Q_{t-1} + \sum_{b=1}^{M-1} I_{b,t-1} - \mu_t\right)^+ \right\rfloor\right)^+ \text{ if } r < M \end{cases} \tag{19}$$

Although the basic order-up-to levels $S\hat{a}l_t$ may lead to acceptable performance, the parameter setting may not be optimal and might violate the service level requirement due to waste during the replenishment cycle. Using a simulation-optimisation procedure, we varied the levels of $S\hat{a}l_t$ adding values in the range {0,..,5} if $r < M$, resulting in incremented order-up-to levels *Sal*, that are corrected for waste during the replenishment cycle.

Order schedules including three order moments require an enumeration of at most of $6^3 = 216$ vectors *Sal* to be tested. This number becomes $6^2 = 36$, if one of the order quantities is fixed. Order schedules with four order moments have at most $6^4 = 1296$ combinations to simulated.
The simulation-optimization procedure evaluates performance of a vector of order-up-to levels $S\hat{a}l_t$ on attained service levels and related costs. Functions $\widehat{TC}(Sal)$ (20) and $\widehat{csl}(Sal)$ (21) return the estimators of total costs and reached service levels respectively based on demand realisations *d*. Aiming at α = 0.90, a sample size of $N = 5000$ gives a rule of thumb accuracy of about 0.005 for estimators $\widehat{TC}$and $\widehat{csl}$ (Hendrix et al., 2015).
TC_i in equation (20) measures simulated costs of week *i*, given a vector *Sal*. This is simulated for *N* weeks and averaged accordingly to obtain estimator $\widehat{TC}$.

$$\widehat{TC}(Sal) = \frac{1}{N}\sum_{i=1}^{N} TC_i(Sal) \tag{20}$$

Attained service levels are retrieved in a similar way. The indicator function $\gamma_t(Sal, d)$ equals one when all demand during period *t* could be met from stock, and zero if a stock-out occurs.

$$\gamma_t(Sal, d) = \begin{cases} 1 & \text{if } d \leq \sum_{b=1}^{M-1} I_{bt} - Q_{t-1} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

This translates the service level constraint into

$$\widehat{csl}_t(Sal) = P\left(d \leq \sum_{b=1}^{M-1} I_{bt} - Q_{t-1}\right) = E\, \gamma_t\,(Sal, d) \tag{22}$$

Now, *N* sample paths are tested to estimate this probability. The average service level is

$$\widehat{csl}_t(Sal) = \frac{1}{N}\sum_{i=1}^{N} \gamma_t(Sal, d_i) \tag{23}$$

The minimum cost order-up-to level vector *Sal* that meets the service level criterion is chosen.

YQ reduced order quantity: Minner and Transchel (2010) showed that in certain situations, a COP performs reasonably well. Therefore, we also examine a *YQ* policy implying a fixed order quantity for each order moment. A sound starting point to determine optimal order quantities Q_t is the basic order-up-to level $S\hat{a}l_t = \hat{Q}_{rt}$ as described by Eq. (13). As these levels provide upper bounds, it is expected that the quantities need to be adjusted downwards in an optimisation step. A finite number of vectors is assessed in a similar way as followed by the *YQSal* approach. Now the value is adjusted downwards using values {-5 (occasionally -7),.., 0}.

4. Numerical evaluation

We evaluate the described order policy approaches. The design of experiments is described in Section 4.1, followed by the obtained results in Sections 4.2 and 4.3.

4.1 Design of experiments

All approaches are evaluated in a rolling horizon simulation of 10,000 weeks using pseudo random samples from the Poisson - and the binomial distribution. The expected demand μ_t varies during the week and is taken from observed data in a practical retail case regarding iceberg lettuce. The evaluated three demand patterns are shown in Table 3 and Figure 2; a base demand, double base demand and a pattern with higher peaks on Wednesday and Saturday. The target CSL is taken as 90%.

Table 3. Expected Poisson demand

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Periods t	1	2	3	4	5	6	7
Base μ_t	3.5	2.3	3.0	2.8	4.5	4.2	2.0
Double base μ_t	7.0	4.6	6.0	5.6	9.0	8.4	4.0
Peaks μ_t	2.6	2.9	4.4	2.0	3.6	8.5	5.9

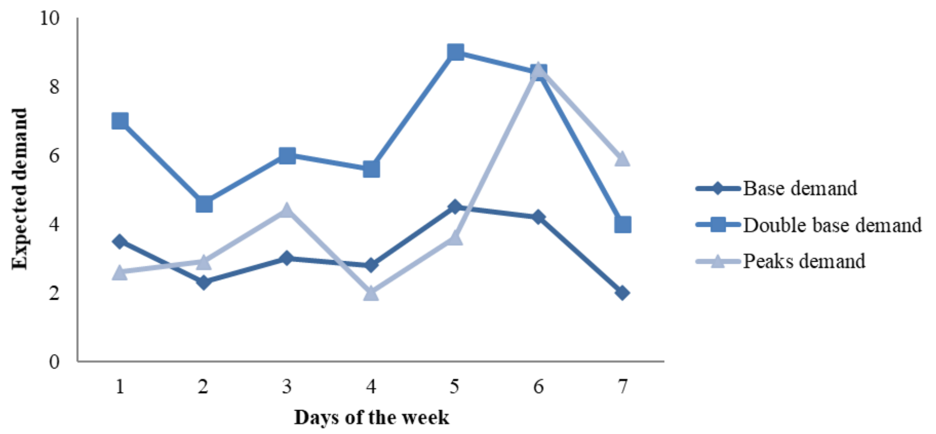

Fig. 2. Evaluated patterns of expected demand during the week, where day 1 corresponds to Monday

The variable purchasing cost is $c = 1$ per unit. As a result, the average total cost per week is equal to the average order quantity per week. The practitioners in retail we consulted, estimate a LIFO fraction of about 0.4 to be realistic. For the base demand pattern, we vary the fraction of LIFO demand $\lambda \in \{0, 0.4, 0.6\}$ for all schedules. To show the effect of partly LIFO demand on the order policies and the average amount of waste, also the situation of only FIFO withdrawal and a LIFO fraction of 0.6 are investigated for the base demand pattern. The other demand patterns are evaluated for a LIFO fraction of $\lambda = 0.4$, for all schedules.

4.2 Results

Tables 4 and 5 present the simulation results for the approaches for nine pre-specified reorder schedules. In Table 4, the LIFO fractions are varied for the base demand pattern. Table 5 gives the results for three demand patterns and a fixed LIFO fraction of $\lambda = 0.4$. For all approaches, a higher LIFO fraction leads to higher costs and waste. The results substantially vary per schedule. For example, for the *YQ reduced* approach and a LIFO fraction of $\lambda = 0.4$, schedule mtf (€31) is 14.8% more expensive than schedules mwfs and wtfs (€27) for the base demand. The results of schedule mtf are equal for all approaches for the base demand. This schedule has the highest costs, because it requires the highest order quantities, due to two replenishment cycles of a length equal to the maximum shelf life M. However, the schedule is robust for all LIFO fractions, where a higher LIFO fraction λ implies more waste and a lower service level. Table 7 shows the same effect for schedule mtf in case of double base demand. The *YQSEW* approach does not meet the CSL target with four out of five reorder schedules with four order moments, for all demand patterns. The *YQS augmented* and *YQSal incremented* policies generate seven times the same parameters, for base demand and $\lambda = 0.4$, and four times for a double base demand. For two other schedules, mtfs and wtfs, the results of those policies are very close. For the peaks demand pattern, all approaches give different solutions, except in schedules mtf and tfs. In general, in most schedules, a more expensive approach generates more waste, but provides also higher cycle service levels.

Table 4. Results for the 9 pre-specified reorder schedules with base demand: LIFO fractions $\lambda = 0, 0.4, 0.6$

		YQSEW			YQS augmented			YQSal incremented			YQ reduced		
	λ	avgTC	avgTW	minSL	avgTC	avgTW	minSL	avgTC	avgTW	minSL	avgTC	avgTW	minSL
mwfs	0.0	26.51	4.55	0.92	26.09	4.32	0.92	25.52	3.86	0.91	26.00	4.28	0.90
	0.4	28.84	6.98	0.89	28.06	6.42	0.91	27.94	6.34	0.91	27.00	5.42	0.90
	0.6	29.76	8.04	0.83	30.03	8.39	0.90	30.20	8.52	0.91	29.00	7.25	0.93
mtfs	0.0	27.39	5.66	0.90	28.56	6.79	0.91	27.42	5.85	0.90	27.00	5.44	0.90
	0.4	27.93	6.36	0.84	28.97	7.27	0.92	27.87	6.33	0.90	28.00	6.44	0.90
	0.6	28.11	6.89	0.70	29.39	7.72	0.92	29.26	7.66	0.90	29.00	7.41	0.90
wtfs	0.0	28.22	6.38	0.94	28.08	6.30	0.92	28.08	6.30	0.92	28.00	6.24	0.93
	0.4	28.91	7.13	0.93	29.07	7.33	0.92	29.07	7.33	0.92	29.00	7.23	0.93
	0.6	29.77	7.99	0.93	30.21	8.45	0.93	30.21	8.45	0.93	30.00	8.20	0.93
mwtf	0.0	27.90	6.23	0.88	27.89	6.16	0.91	27.89	6.16	0.91	28.00	6.31	0.91
	0.4	28.40	6.85	0.84	28.36	6.75	0.90	28.36	6.75	0.90	28.00	6.42	0.90
	0.6	28.73	7.25	0.83	29.64	7.99	0.91	29.64	7.99	0.91	29.00	7.39	0.91
ttfs	0.0	25.20	3.41	0.92	25.53	3.76	0.92	25.08	3.40	0.90	25.00	3.38	0.90
	0.4	26.93	5.34	0.84	27.88	6.26	0.91	27.88	6.26	0.91	27.00	5.37	0.91
	0.6	28.29	6.88	0.78	30.71	9.03	0.92	30.10	8.43	0.91	29.00	7.27	0.90
wfs	0.0	28.87	6.93	0.93	27.39	5.63	0.90	28.30	6.45	0.93	28.00	6.21	0.92
	0.4	30.32	8.40	0.94	30.08	8.22	0.93	30.08	8.22	0.93	29.00	7.24	0.91
	0.6	32.07	10.11	0.94	30.46	8.69	0.91	30.46	8.69	0.91	31.00	9.12	0.93
mwf	0.0	29.42	7.46	0.91	27.95	6.16	0.91	27.95	6.16	0.91	28.00	6.24	0.91
	0.4	31.05	9.12	0.91	29.47	7.73	0.91	29.47	7.73	0.91	29.00	7.28	0.91
	0.6	32.02	10.09	0.91	30.85	9.07	0.91	30.85	9.07	0.91	30.00	8.24	0.91
mtf	0.0	31.00	9.18	0.91	31.00	9.18	0.91	31.00	9.18	0.91	31.00	9.18	0.91
	0.4	31.00	9.20	0.92	31.00	9.20	0.92	31.00	9.20	0.92	31.00	9.20	0.92
	0.6	31.00	9.22	0.92	31.00	9.22	0.92	31.00	9.22	0.92	31.00	9.22	0.92
tfs	0.0	28.18	6.37	0.91	28.29	6.49	0.91	28.29	6.49	0.91	28.00	6.26	0.91
	0.4	29.57	7.78	0.91	29.92	8.17	0.91	29.92	8.17	0.91	29.00	7.30	0.91
	0.6	31.18	9.33	0.91	30.24	8.58	0.91	31.14	9.40	0.91	30.00	8.26	0.91

Table 5. Results for the 9 pre-specified reorder schedules for 3 demand patterns: LIFO fraction λ = 0.4

Demand pattern		Base demand			Double base demand			Peaks demand		
	Approach	avgTC	avgTW	minSL	avgTC	avgTW	minSL	avgTC	avgTW	minSL
mwfs	YQSEW	28.84	6.98	0.89	50.05	6.38	0.84	36.27	7.13	0.81
	YQS augm	28.06	6.42	0.91	51.40	7.66	0.92	38.43	9.32	0.90
	YQSal	27.94	6.34	0.91	50.18	6.59	0.91	37.61	8.56	0.91
	YQ	27.00	5.42	0.90	50.00	6.36	0.91	36.00	6.97	0.90
mtfs	YQSEW	27.93	6.36	0.84	50.70	7.32	0.81	36.66	7.73	0.78
	YQS augm	28.97	7.27	0.92	52.36	8.69	0.91	39.51	10.30	0.92
	YQSal	27.87	6.33	0.90	52.26	8.63	0.90	38.27	9.22	0.91
	YQ	28.00	6.44	0.90	53.00	9.31	0.92	38.00	8.86	0.91
wtfs	YQSEW	28.91	7.13	0.93	53.10	9.26	0.91	39.93	10.50	0.94
	YQS augm	29.07	7.33	0.92	53.19	9.43	0.91	39.67	10.40	0.92
	YQSal	29.07	7.33	0.92	53.15	9.45	0.91	38.47	9.38	0.91
	YQ	29.00	7.23	0.93	53.00	9.26	0.91	38.00	8.82	0.90
mwtf	YQSEW	28.40	6.85	0.84	52.73	9.19	0.84	38.72	9.61	0.80
	YQS augm	28.36	6.75	0.90	52.80	9.09	0.91	38.62	9.34	0.93
	YQSal	28.36	6.75	0.90	52.80	9.09	0.91	37.45	8.32	0.91
	YQ	28.00	6.42	0.90	53.00	9.28	0.91	38.00	8.81	0.91
ttfs	YQSEW	26.93	5.34	0.84	49.49	5.78	0.87	37.08	7.87	0.84
	YQS augm	27.88	6.26	0.91	51.04	7.35	0.91	38.88	9.70	0.91
	YQSal	27.88	6.26	0.91	50.50	6.92	0.90	37.70	8.64	0.90
	YQ	27.00	5.37	0.91	50.00	6.35	0.90	36.00	6.84	0.90
wfs	YQSEW	30.32	8.40	0.94	54.24	10.25	0.94	40.10	10.62	0.94
	YQS augm	30.08	8.22	0.93	53.49	9.64	0.91	39.32	9.98	0.92
	YQSal	30.08	8.22	0.93	53.49	9.64	0.91	38.20	8.97	0.90
	YQ	29.00	7.24	0.91	53.00	9.21	0.90	38.00	8.71	0.90
mwf	YQSEW	31.05	9.12	0.91	54.64	10.94	0.93	39.38	9.92	0.93
	YQS augm	29.47	7.73	0.91	53.99	10.11	0.92	38.93	9.59	0.92
	YQSal	29.47	7.73	0.91	53.99	10.11	0.92	37.95	8.71	0.91
	YQ	29.00	7.28	0.91	54.00	10.13	0.92	38.00	8.78	0.91
mtf	YQSEW	31.00	9.20	0.92	56.00	12.20	0.90	40.00	10.69	0.91
	YQS augm	31.00	9.20	0.92	56.00	12.20	0.90	40.00	10.69	0.91
	YQSal	31.00	9.20	0.92	56.00	12.20	0.90	39.00	9.78	0.91
	YQ	31.00	9.20	0.92	56.00	12.20	0.90	39.00	9.78	0.91
tfs	YQSEW	29.57	7.78	0.91	55.01	11.01	0.92	41.07	11.60	0.91
	YQS augm	29.92	8.17	0.91	54.81	10.93	0.92	40.11	10.85	0.91
	YQSal	29.92	8.17	0.91	53.86	10.08	0.91	40.11	10.85	0.91
	YQ	29.00	7.30	0.91	54.00	10.16	0.92	38.00	8.78	0.91

4.3. Computational aspects

With respect to the computational aspects of the four described approaches to determine order policies, we found the following. The *YQSEW* approach offers easy and fast calculation rules to determine the order quantity. The determination of the parameters with the simulation-optimization approaches, *YQS augmented*, *YQSal incremented* and *YQ reduced* require far more computation time. Computing time is hard to compare among the implemented methods, because different software and processors were used. The computational speed also depends on the way of programming. For the investigated experiments, the *YQS augmented heuristic* needs 2.4 s to 15.3 s. There is no clear distinction in time between three or four order moments. These experiments were performed in Matlab, on an Intel Core i7-4770 CPU @ 3.40 GHz desktop processor.

YQSal incremented and *YQ reduced* have been programmed in Python on an Intel Xeon X3450 @ 2.67 Ghz server processor. For the pre-specified reorder schedules, the computation time of *YQSal* and *YQ* depends on the range needed to obtain satisfactory results and the number of *Sal* or *Q* values that can be varied (after a replenishment cycle of *M* days, the values are fixed). For all demand patterns, a range of 6 values was used. *YQSal* needs 2.3 to 14.8 min for four order moments, and 23.1 to 3.64 s for three order moments. *YQ* needs 11.9 min for four order moments and 116 s for three order moments.

5. Discussion

In the retail situation we describe, ordering takes place on pre-specified days of the week according to reorder schedules provided by stakeholders. This implies that fixed ordering cost and holding cost do not influence the decision on the order quantity. Therefore, these costs were not included in the model. In this paper, disposal costs or salvage values of wasted items are not considered. In case of waste, the purchasing cost is lost. For future research, a disposal cost or salvage value (negative cost) of wasted items may be included in the model, additional to the selling price of the product. Like in the newsvendor problem, considering a profit margin may lead to higher stock keeping and consequently to higher waste and service levels.

We assume that an order can contain any integer number of items. When products are ordered in batches, this will have a negative effect on costs and waste, if the service level requirement remains unchanged.

6. Conclusion

The research question of this paper deals with the development and investigation of order policies for a Dutch retail situation with pre-specified reorder schedules, varying replenishment cycle lengths and a cycle service level requirement, when the age-distribution of the inventory is unknown. We investigated a retail situation where a product has a fixed shelf life of three days upon delivery, demand is non-stationary during the week, but stationary over the weeks, with a mixed LIFO-FIFO depletion and a lead time of one day.

Table 6 gives an overview of the studied approaches and an indication of the required computation time to find the corresponding optimal parameter values.

Table 6. Overview of studied approaches with computation time indication

Approach | Characteristics	4 / 3 reorder days
YQSEW	< second
YQS augmented heuristic	seconds
YQSal incremented	minutes / seconds
YQ reduced	minutes

For all reorder schedules and demand patterns, the minimum amount of waste is realised by the lowest cost approach. This is a logical consequence in absence of the disposal cost or salvage value and not focusing on a profit margin.

All designed approaches can be implemented without knowledge of the age-distribution of items in stock. Overall, the *YQ reduced* approach generates the best policy parameters. *YQSal incremented* is second best and performs slightly better than the *YQS augmented heuristic*. Considering the computation time, it is up to the logistics department of a retail organisation whether the *YQ reduced* approach can be implemented. Potentially, computing time can be reduced by further programming investment or increase computing power. For a fast computation option, the *YQS augmented heuristic* can be a good alternative.

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