

# Interpreting the relationship between emotions and understanding in mathematics: An operational approach applied to measurement with preservice elementary teachers

Verónica A. Quintanilla, Jesús Gallardo\*

Department of Didactics of Mathematics, Social Sciences and Experimental Sciences, Faculty of Education Sciences, University of Málaga, Bulevar Louis Pasteur, 25 (Campus de Teatinos), 29010 Málaga, Spain

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## ABSTRACT

This research explores the relationship between emotions and understanding in mathematics. In concrete, an interpretive model is proposed allowing to relate, operationally, the student's emotional experience with a functional view of their understanding, based on their uses of mathematical knowledge. The model includes a specific method for detecting the connections between students' emotions and their understanding during mathematical practices in the classroom. This method is applied in an empirical qualitative study with preservice elementary teachers involved in measurement problem solving in pairs. The study provides positive results on the influence of students' understanding on the generation of their different emotions during the mathematical activity performed. In the same way, the emotions provide plausible reasons that help to explain the students' mathematical understanding.

## 1. Introduction

The complex world of human emotions is a major focus of interest in mathematics education (Evans, 2006; Hannula, 2012a; Martínez-Sierra et al., 2019; Pepin & Roesken-Winter, 2015; Zan et al., 2006). In recent decades, there has been an increasing number of studies on how human emotions are related to cognition in mathematics. The perspective put forward today is that emotion and cognition are not separate but rather conceived as related entities (Chen & Leung, 2015; Marmur, 2019). They develop together within subjectivation processes linked to participation in social and cultural activities (Evans, 2006; Radford, 2015). According to this paradigm shift in the domain of the mind, cognition is essentially of an emotional nature; emotions are acknowledged as necessary for rational behaviour, forming part of a shared vision of the world (Hannula, 2006, 2012a; Radford, 2015; Schlöglmann, 2010).

In this contemporary vision, the challenges that remain are, among others, to integrate the psychological, expressive and physiological aspects linked to emotions within the same process; to relate the dual, conscious and unconscious origin of the emotions themselves; and to reconcile their genetic, innate and universal nature with their contingent character dependent on historical, cultural and social conditions (Hannula, 2012b; Sumpter, 2020). In the same way, emotion and cognition are intertwined and interact with each other (Else-Quest et al., 2008). Therefore, a continuing research challenge is to separate them for observational and practical purposes (Hannula, 2015). In mathematics education, specific models that would include conceptualisations of emotions linked to the specific issues under study have been called for (Ronen, 2020; Schlöglmann, 2010). An emphasis has also been put on the need for

\* Corresponding author.

E-mail addresses: [veronicaquintanilla@uma.es](mailto:veronicaquintanilla@uma.es) (V.A. Quintanilla), [gallardoromero@uma.es](mailto:gallardoromero@uma.es) (J. Gallardo).

qualitative methodologies in which students take part interpreting their own emotions – a hitherto uncommon approach in empirical studies (Di Martino & Zan, 2011; Evans et al., 2006; Satyam, 2020).

It is in this problematic context that the present study takes place, which addresses understanding in mathematics. As understanding is a cognitive phenomenon of a mental and internal nature, it can also be perceived as related to, and conditioned by, a characteristic emotional component (DeBellis & Goldin, 2006; Goldin, 2000; Op ’t Eynde et al., 2006). It is thus relevant to incorporate emotions into the study of understanding. Moreover, as widely recognised in mathematics education, emotions are of a dynamic nature (DeBellis & Goldin, 2006; Hannula, 2012b; McLeod, 1989; Op ’t Eynde et al., 2006; Radford, 2015). Therefore, it is a complex task to interpret emotions in the classroom and this challenge also aroused our research interest.

Our research addresses the following questions: How can we characterise and clarify in practice the close links between emotions and understanding in mathematics? How can we incorporate assessment methods in classroom mathematical activities that take into account the emotional aspects of students’ understanding? The specific aim is to advance an operative proposal that would allow to explore the relationship between emotion and understanding in mathematics. To do so, we used a developing model (*An Operative Model for Interpreting Understanding in Mathematics [OMIUM]*) that is based on the interpretation of students’ mathematical experience (Gallardo & Quintanilla, 2016, 2019; Gallardo et al., 2014; Quintanilla & Gallardo, 2021; Quintanilla, 2019). At a theoretical level, a dialectical approach is proposed, which allows us to give a systemic character to the emotional experience, in order to incorporate some aspects of major consolidated knowledge regarding emotions within the same common process. It also proposes a characterisation of the relationship between emotion and understanding based on students’ uses of mathematical knowledge while performing a task in the classroom. At the methodological level, a specific qualitative method is provided to observe and interpret students’ mathematical understanding through their different emotions. This method grants students a major role in the interpretation of their own understanding in an ordinary mathematics classroom context.

**2. Theoretical framework**

The theoretical framework proposed by the OMIUM has proved to be operational and effective in describing mathematical understanding in our research carried out over the last decade. However, the need to identify possible reasons for students’ mathematical understanding, and not just to describe what they understand, have led to incorporate emotional issues into our study. We therefore expanded our approach by conceptualizing the emotional experience and defining the relationship between emotions and

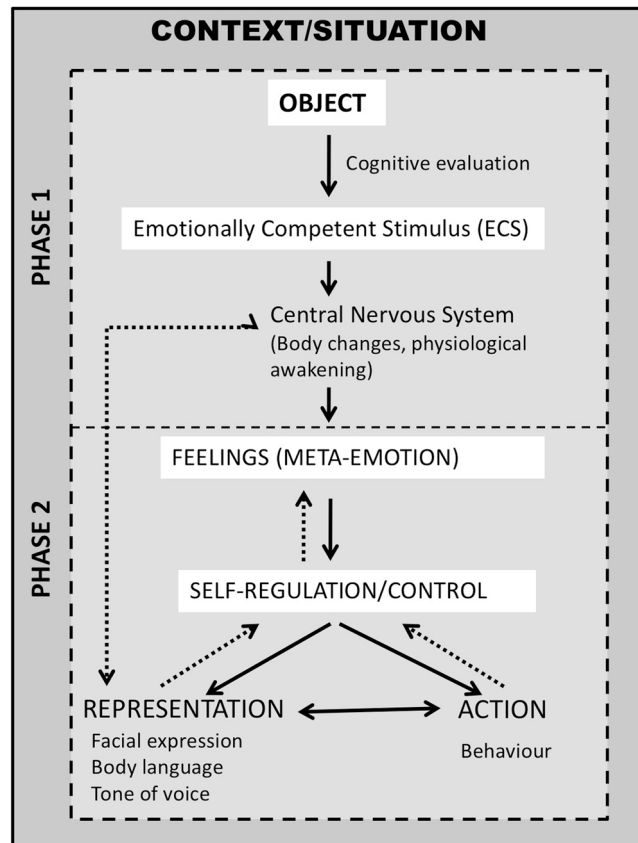


Fig. 1. Phases of emotional experience.

understanding in mathematics.

### 2.1. Emotional experience in mathematics

Our theoretical approach on the relationship between emotions and understanding focuses on the construct emotional experience, a two-phase cyclical process that we endow with a systemic and dynamic character. This conceptualisation allows us to integrate in the same process the contributions on emotions from philosophy, psychology, neuroscience, and mathematics education.

In general terms, we take into account the view adopted by Damasio (1994, 2003) and we perceive emotion as a complex set of chemical and neuronal responses to an external or internal stimulus. So, each emotion forms a distinctive pattern of actions or movements (emotional responses), some of which are visible and recognisable to an external observer. Emotions and emotional responses are part of the overall process we call the emotional experience.

Specifically, any emotional experience begins with an unconscious phase (Fig. 1) where the person, conditioned by the context, conducts a cognitive assessment (Damasio, 1994; Kagan, 1978), in order to establish whether a certain object (physical or mental) or event (real, evoked or imaginary) can become an emotionally competent stimulus (ECS) (Damasio, 2003). This stimulus, natural or acquired, has the ability to trigger a particular emotion. It always results from a value judgment made by the individual's cognitive system based on innate genetic or learned social and cultural patterns (LeDoux, 1996; Mandler, 1989; Nussbaum, 2001). If the initial cognitive assessment identifies an ECS, various physiological responses are triggered in the body, that are both imperceptible (for example, adrenaline secretion) or appreciable by those who experience them (for example, increased heart rate or respiratory changes) (Damasio, 2003; Eagleman, 2011; Kagan, 1978; LeDoux, 1996). This is when the specific emotion appears. It also usually manifests itself through emotional responses in the form of facial expressions, body language, tone of voice and verbal utterances that are recognisable by external observers (Ekman, 1993, 1999). In the mathematics classroom, students are continually subject to stimuli that can be regarded as emotionally competent. The stimuli come from the elements of mathematical knowledge themselves, from the problems associated with these elements, from teaching practices, from the student's history in mathematics or from the classroom's context and social norms. They generally arise from the past and present mathematical activity individually or jointly performed by students with their classmates or teacher. The stimuli can also be modified and evolve with experience.

In a second phase of the emotional experience (Fig. 1), awareness of various physiological changes triggers new thoughts on the subject relating to the initial object or situation that generated them and to the organism's general state itself. This is how feeling arises as a mental representation of emotion (Damasio, 2003; Sumpter, 2020). Feelings prolong the impact and effects of emotions and, when evaluated cognitively, enable generating new emotions within a process that is dynamic and cyclical (meta-emotion). In addition, they predispose the person to action by consciously creating adapted responses (Brown & Reid, 2006; Buck, 1999; Cobb et al., 1989; Damasio, 1994; Ekman, 1999; Hannula, 2006; Lang, 1985; Nussbaum, 2001). The latter launches specific actions associated with a particular emotion in a given context, as a result of a decision-making process that Lazarus and Lazarus (1994) links with the self-regulating and self-controlling facet of the emotion itself. This facet enables the management of the emotion's external representations according to the person's interests as well as its associated social and cultural norms; finally, it allows generating the voluntary behaviours that the individual considers appropriate in each situation (Nussbaum, 2001). These processes directly influence the student's different actions in the mathematics classroom. The emotions generated during the mathematical activity are also stored in the student's emotional memory and can explain their subsequent behaviour while he/she is solving problems or learning mathematics in general (LeDoux, 1996; McCulloch, 2011).

Our characterisation in phases allows us to conceive emotional experience not only as a particular experience – similar to that pointed out by Evans et al. (2006) or Martínez-Sierra et al. (2019) – but as a cyclical process composed of cognitive evaluations and associated responses that generate emotions and feelings. We interpret emotions, for their part, as the essential dynamic processes of regulation, that occurs in the first phase of the emotional experience. Their emergence is directly related to decision-making that triggers specific actions in the second phase. Therefore, emotions participate in the rational processes directly, allowing reason to transform itself into specific actions. Emotions are cognitive in nature because they are endowed with meaning by reason (Hannula, 2006). They stem from a cognitive evaluation of the different particular situations faced by students, as a result of some perceptual discrepancy or equally a cognitive one (Cobb et al., 1989; Mandler, 1989; Martínez-Sierra et al., 2019).

### 2.2. The role of emotions in mathematical understanding

From a functional point of view, it can be said that students manifest a certain understanding of a specific mathematical knowledge when, faced with situations they voluntarily decide to address, they elaborate and produce adapted responses they are satisfied with and in which they make a significant (free, conscious and intentional) use of this knowledge (Duffin & Simpson, 2000; Gallardo et al., 2014). Mathematical knowledge is not always used in the same way and their characteristics establish in each case the different conditioning requirements of its intended use. In addition, the students faced with mathematical situations need to identify mathematical knowledge that can be used in them, in some of their possible forms, as a means of resolution, as well as to decide about which mathematical knowledge to use, and in what way, among the previously identified possibilities. The latter involves analysing the situation, interpreting the available information, determining the advisability of intervening and acting accordingly by producing a response. The specific mathematical knowledge in question is used in this response. Finally, the student assesses the intervention in terms of its effectiveness and adequacy with regard to the experienced interaction situation, deciding to end the intervention or to pursue it by repeating certain steps of the process again.

The actions deployed in the concrete situation, including the uses of mathematical knowledge, shape the mathematical activity

itself and are directly related to a decision process where emotions have a fundamental role (Damasio, 2003). This is because emotions play an active part in the initial cognitive imbalance caused by the situation. They also actively intervene in the subsequent decision as to which specific mathematical knowledge are the most relevant to resolve the task in a given context. The student’s various accumulated emotional experiences, resulting from their experiences in the mathematics classroom, directly influence their future decisions, actions and uses in the classroom (Satyam, 2020). Therefore, the experiences have a direct impact on the development of their mathematical understanding.

In short, we assume understanding based on the uses of mathematical knowledge made by students during mathematical activity. In order to decide which uses will be brought into play, we need know the situation, the options for action-responses and the consequences of these uses (Damasio, 1994). In the decision-making processes during mathematical problem solving we also recognize the influence of emotions (Reinup, 2009). It is in this sense that we attribute an emotional character to the understanding of mathematical knowledge: we recognise the existence of cognitive and emotional processes that act in an interdependent way when making decisions about the uses of knowledge during mathematical activity in the classroom. This is how we established the close connection between emotion and understanding, by acknowledging the existence of mental processes that are strongly linked to the emotions underlying the decisions about the uses of mathematical knowledge and which account for the student’s understanding.

2.3. Interpretation of mathematical understanding through emotions

Over the last years, we have been developing an interpretive method, which we call *the hermeneutic circle of understanding in mathematics* (Gallardo & Quintanilla, 2019; Quintanilla & Gallardo, 2021). With this method we seek to interpret the student’s mathematical experience, that is, how the student acts and uses knowledge during his or her mathematical activity in the classroom. The method allows us to simultaneously interpret both the emotional traces that accompany and motivate actions and the traces of understanding displayed by students when they are dealing with problematic situations. Emotions become recognisable when external observers interpret them through their different external representations. As described below, the hermeneutic circle allows us to identify and relate these representations through the various semiotic, phenomenon-epistemological and dialogical planes included in their interpretive trajectory (Fig. 2).

2.3.1. Semiotic plane

We present understanding as a student’s essential ability which is expressed in social practices and which can be publicly interpreted (Font et al., 2013). That is, mathematical understanding is communicable and includes interpretable traces in its external manifestation. On the semiotic plane, interpretation is circumscribed to visible mathematical activity and to the use made of the system of mathematical signs within this activity. Basically, interpreting entails transferring oneself into the semiotic environment created by these practices and observable mathematical productions (Sáenz-Ludlow & Zellweger, 2012). The objective here is to identify and delimit, among all that is observed and recorded of the student’s mathematical activity, the traces of understanding that could be considered indicators of some typified use given to mathematical knowledge. The range of observable evidence of the learner’s

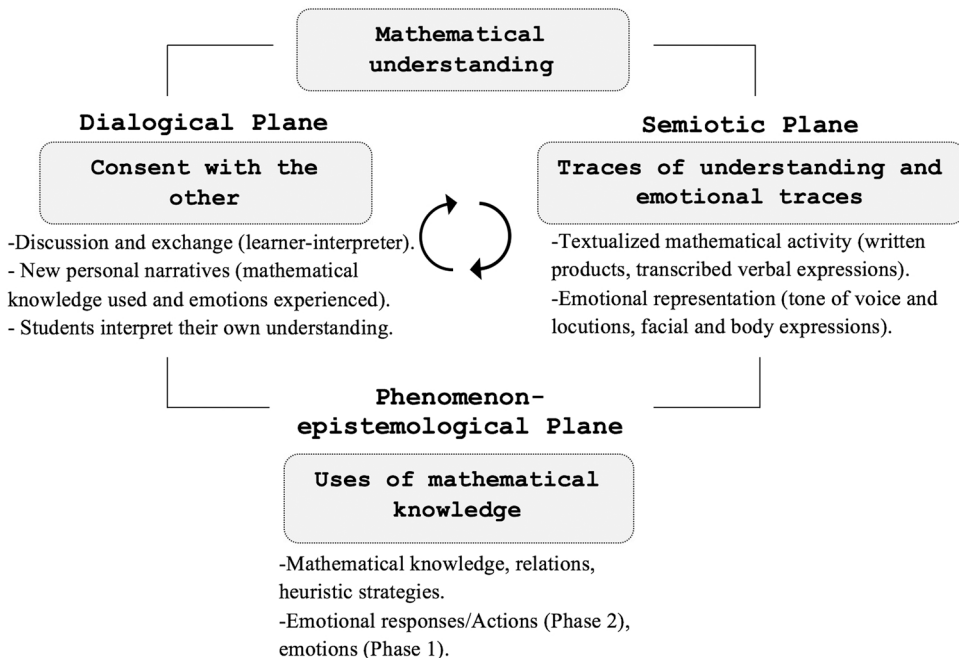


Fig. 2. Hermeneutic circle of understanding in mathematics.

mathematical activity is recorded, using different systems of semiotic representation. The resulting written record is necessary to be able to then detect and characterise genuine traces of understanding. The latter will indeed allow subsequently delimiting the various uses of mathematical knowledge. On this plane, emotions are characterised using different representation systems that inform us of what is communicated and how it is communicated: (a) the verbal system (tone of voice and locutions) and (b) the kinesthetic system (facial and body expressions).

2.3.2. Phenomenon-epistemological plane

In the decision that justifies the use of mathematical knowledge there is always a mental exercise of deliberation and choice of alternatives, linked to an intention and a certain conviction that the action is possible and pertinent. It will be the intentional use of mathematical knowledge by the student, as a form of observable and interpretable action, that accounts for his mathematical understanding. Therefore, interpretation is directed here to the externalization and characterization of the uses of mathematical knowledge that emerge from the traces of understanding. The phenomenon-epistemological plane contributes to directing the

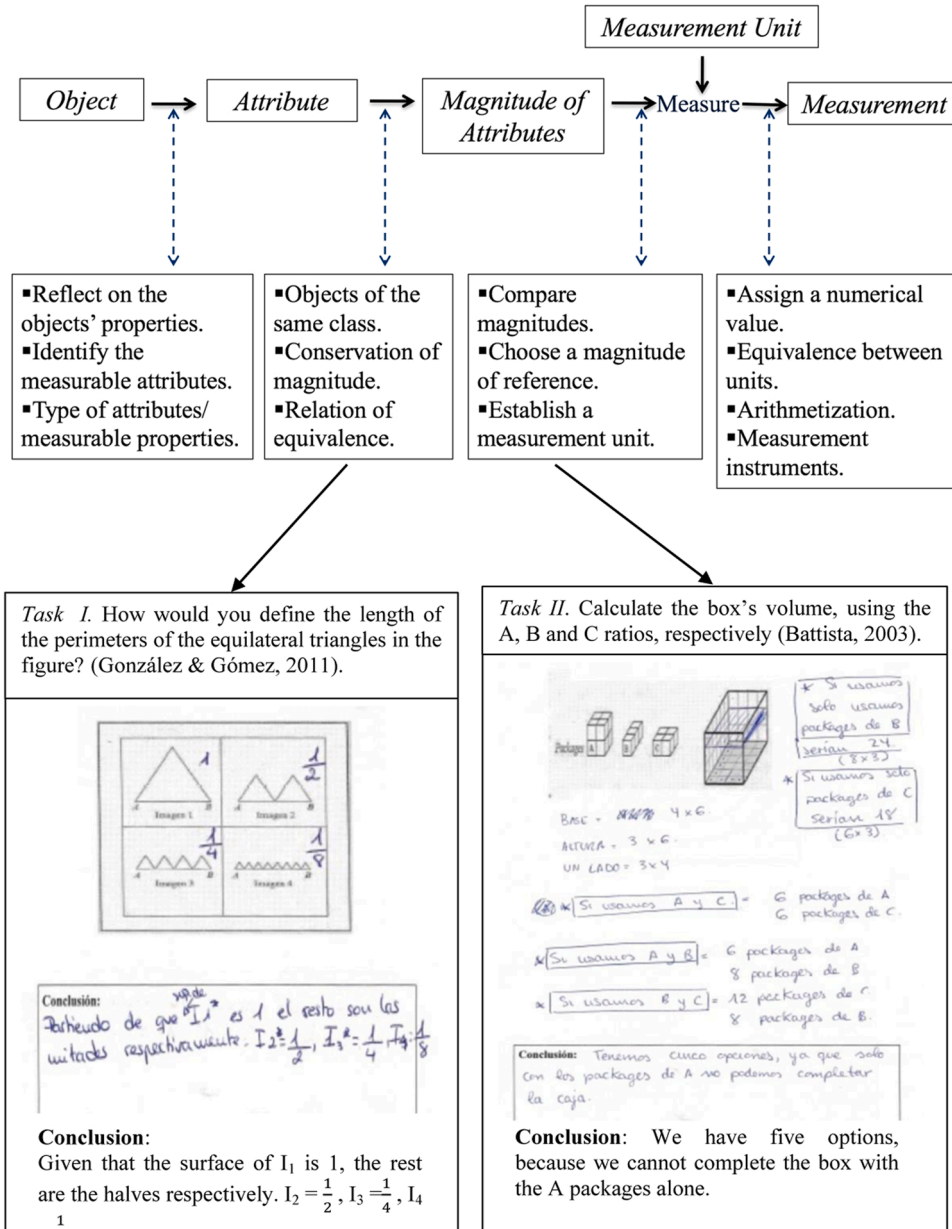


Fig. 3. Examples of tasks characterised according to mathematical concepts and processes that are solved using attributes of measurement. Written productions of Luisa and Martin (Task I), and María and Laura (Task II) (Battista (2003)).



interpretation to external references, i.e., focusing on the acting and doing, beyond the literal observable record (Brown, 2001; Morgan, 2014). Once the textualized mathematical activity has been semiotically analysed, this plane validates our functional proposal to seek evidence of the students' mathematical understanding in the uses they make of mathematical knowledge. The following analyses serve as a reference to orient this latter quest: (a) the phenomenon-epistemological analysis of the specific mathematical knowledge, object of understanding, or of the problematic situation raised; and (b), the phenomenological analysis of the student's emerging emotional system during mathematical practice.

### 2.3.3. Dialogical plane

The interpretation of mathematical understanding requires the participation of the student himself as a mediator between what he has previously realized (the observable record of his mathematical activity) and the interpreter who seeks to specify what he understands, how and why he understands it. We suggest then to continue the interpretive process with the search for a reciprocal conformity between the student and his interpreter (researcher, teacher, classmates) about the conclusions on the uses of mathematical knowledge obtained in the previous planes. The dialogical plane provides a common environment conducive to discourse, critical discussion and required exchanges to ultimately reach consent with the other (Gallardo & Quintanilla, 2016; Llewellyn, 2012; Radford, 2015). At this stage, students are directly and substantially involved, together with the interpreter, in the interpretive processes of their own mathematical understanding. The search for consent also allows us to compare information regarding the emotions that the student displayed during the episode's previous phases. Interpreting emotions on this plane requires that the protagonist of the emotional experience elaborate new personal narratives.

## 3. Methodology

We seek to contrast the effectiveness of our interpretive method in practice. For this purpose, we applied it in an empirical study to interpret students' mathematical understanding based on the various emotions they manifested during their mathematical activity in the classroom. Our proposal incorporates various instruments for data collection and result analysis strategies on successive interpretive planes (semiotic, phenomenon-epistemological and dialogical), characteristic of qualitative methodology recognised and used in empirical research on emotions in mathematics education (Evans, 2006; Op 't Eynde et al., 2006; Pepin & Roesken-Winter, 2015).

### 3.1. Participants and classroom context

Participants included 20 volunteer preservice elementary teachers in their fourth year of their *Degree in Primary Education* at the University of Málaga (Spain), who were studying the subject *Didactics of Measurement* during the second semester of the 2017–2018 academic year. The study's main researcher was the teacher of the subject herself. The participants were organised in pairs and they participated in the different activities proposed in their usual classroom, following the normal class schedule and with the rest of their classmates. The empirical study unfolded over nine weeks, between the months of March and May 2018, during the subject's two weekly hours of practice.

### 3.2. Mathematical tasks

The pairs of participants were presented with five measurement tasks to be undertaken during classroom practice. The selection was made taking each representative task of the different phases proper to the process of mathematical foundation of measurement, according to the proposal of González and Gómez (2011) adopted in the subject: Identification of attributes, conservation and comparison of an attribute's magnitude, choice of measurement units, use of measurement instruments, and arithmetization (Fig. 3). The tasks were of a non-equivalent nature, and their solving allowed delimiting the preservice teachers' understanding of measurement. In this paper, we illustrate the study using the records generated for two of these tasks by two separate pairs of participants: (a) Luisa and Martin, and (b) María and Laura. We chose these pairs because they offer us different scenarios in which to contrast the efficiency of our interpretive proposal. In them, the interactions between preservice teachers and their behaviors during task resolution are different, which allows us to identify a greater variety of observable relationships between their emotions and their mathematical understanding.

Applying the hermeneutic circle requires conducting a phenomenon-epistemological analysis of the tasks. This analysis can then be used as a prior reference to interpret the understanding on the semiotic and phenomenon-epistemological planes. In this concrete case,

**Table 1**  
Phenomenon-epistemological analysis of the tasks.

	Task I	Task II
Mathematical knowledge	Measurable attributes, magnitude of attributes, equilateral triangle, length, perimeter, conservation of length, equivalence of figures, similar triangles.	Cube, rectangular prism, volume, unit volume, capacity, submultiples, conservation of volume, unit comparison.
Relations	Homologous sides, similarity of triangles, length and perimeter; the length of the triangle side of image $i$ is twice the length of the homologous side of the triangle in image $i+1$ , geometry-measure.	Area and volume, distinct units, spatial structure, spatial coordination, geometry-measurement-arithmetic.
Heuristic strategies	Observe, classify, order, compare, visualise, look for regularities.	Compose and decompose, compare, fill, visualise, use analogies, count, identify units, structure sets of units.

Task I focused on the measurement of equilateral triangle perimeters. The solution required comparing the length and number of sides of the triangles in each image with those of the next image and relating the perimeters of the four groups of triangles included in each image. To do this, it was necessary to identify the figures as similar triangles and to recognise the length as an attribute. To conclude that all perimeters are equal, it is necessary to use comparison and conservation strategies regarding an attribute’s magnitude by means of geometric figure equivalence. Task II centred on volume measurement. The solution mainly involved visualising the unit’s structure, replicating or iterating that unit to cover the box’s volume and relating the total size with the number of units used in the measure. Unit A, unlike the others, does not allow to cover the box an integer number of times, an added difficulty that invites a reflection on the possibility of dividing the unit fixed in advance. [Table 1](#) brings together the essential elements of mathematical knowledge, relationships and heuristic strategies called upon to solve the given tasks.

### 3.3. Phases and instruments

Each episode was conducted over two consecutive phases in which we used different data collection instruments.

*Phase I. Solving mathematical tasks together.* Each pair of preservice teachers solved the five given tasks collaboratively, and attempted to find common strategies, procedures and results. During this process, we expected each student’s emotions to interact with the cognitive processes linked to mathematical problem solving ([Cobb et al., 1989](#); [Di Martino & Zan, 2011](#)). In this phase, we identified and characterised the uses given to the mathematical knowledge displayed during the task resolution (on the semiotic and phenomenon-epistemological planes of the hermeneutic circle). All conducted mathematical activity was recorded in audio and video. The generated observable record was composed of written productions, dialogue with transcribed verbal expressions, and external representations of the participants’ various emotional experiences: tone of voice, facial expressions and body language ([Cobb et al., 1989](#); [Evans, 2006](#); [Furinghetti & Morselli, 2009](#); [Schlögmann, 2002](#)). Data collection from multiple sources and in different formats is supported by a range of studies on emotions in mathematics education ([Di Martino & Zan, 2011](#); [Else-Quest et al., 2008](#); [Hannula, 2006](#); [Op ’t Eynde et al., 2006](#)).

*Phase II. Reaching consent with the other.* A semi-structured conversational interview was conducted individually ([DeBellis & Goldin, 2006](#); [Furinghetti & Morselli, 2009](#); [Hannula, 2006](#); [McCulloch, 2011](#)). Each student was asked for a verbal narrative about the mathematical knowledge used and the emotions displayed during the task’s resolution. The researcher presented an interpretation of the student’s mathematical activity based on the results obtained in the previous phase. The main purpose of this phase was to deepen the emotional component of each participant’s mathematical understanding through mutual recognition. During the conversation, we clarified information that was considered relevant but insufficient, we identified possible inconsistencies in the students’ performance and we showed them alternative possibilities that differed from their own. In so doing, we were seeking an agreement with the student regarding the uses given to mathematical knowledge and their experienced emotions (on the dialogical plane). Each conducted interview was audio-recorded and we used the transcription to generate the second written records employed for obtaining data.

### 3.4. Data analysis and interpretation

On the circle’s semiotic and phenomenon-epistemological planes, we sought to identify visible traces of the participants’ mathematical understanding and using them as a basis, to describe the uses given to the different specific mathematical knowledge put into play. To do this, we used the previous phenomenon-epistemological analysis of the tasks as a reference ([Table 1](#)). In the same way, we sought to delimit each participant’s range of experienced emotions based on their emotional responses and associated specific actions and to establish their links to the mathematical understanding they demonstrated. While we did not attempt to be exhaustive, we did seek representativeness in our phenomenological analysis. [Table 2](#) illustrates the components and attributes composing the emotional system that we took as an initial reference, based on Ekman’s proposal (1993, 1999) regarding the relationship between emotional responses and emotions. We also take into account the characterisations of emotions put forward by authors as [Else-Quest et al. \(2008\)](#) and [Martínez-Sierra et al. \(2019\)](#).

We are aware that different emotions can share the same external representations and that certain emotional responses are not exclusive to some emotions ([Damasio, 1994](#)). For example, a person may smile when experiencing joy or embarrassment, but it is the triggering situation, and the context in which it occurs, that can help us to link smiling with joy or embarrassment, especially if the protagonists share the same response patterns and behavioral codes because they belong to a common society and cultural environment ([Hannula, 2006](#); [Nussbaum, 2001](#)). Our empirical study takes place in a shared environment where interaction is favored by the closeness and complicity that we maintain daily with the students in the mathematics classroom. We also rely on this context of mutual trust, where we favor discourse, discussion and exchange, to finally decide on the emotions that are linked to certain emotional responses.

**Table 2**  
Phenomenological analysis of the emotional experience.

Emotional responses /Actions (Phase 2)	Emotions (Phase 1)
Abandonment, time off task, blockage, tension.	Disappointment, disgust, anger, frustration.
Overwhelming, restlessness.	Worry, uncertainty, fear, shyness.
Eureka.	Joy, surprise.
Security, tranquillity.	Relief, confidence, empathy, satisfaction.

With respect to the circle’s dialogical plane, we concretised the protagonists’ emotional experience during the episode, we established their links to the uses given to mathematical knowledge, and we structured the conclusions relating to the students’ understanding of measurement.

**4. Results**

*4.1. The case of Luisa and Martin*

*4.1.1. Semiotic and phenomenon-epistemological planes*

Luisa and Martin undertook the solving of Task I together (Table 3). The episode’s observable record provides evidence of the uses given to mathematical knowledge and its relationship with the participants’ emotional system. In such evidence, we are seeking to identify: (a) features associated with knowledge, relationships, and strategies applied during task resolution, and (b) facial expressions, bodily expressions, tone of voice, and exclamations associated with emotional experiences. In the case of the latter, we identify the external representations resulting from the second phase of the emotional experience that allow us to identify the emotion generated during the first phase.

Luisa decided to use specific knowledge and proposed a solution based on comparing the figures formed by each image’s triangles. However, she confused length with surface and tried to define the surface areas instead of calculating the perimeters (line 3). As a strategy, she assigned the value 1 to the first image’s triangle surface area and used it as a reference to measure the surfaces of the other figures (lines 3, 5 and 7). She concluded that each figure’s surface was half that of the previous one (lines 9 and 11). Luisa considered Martin’s approaches, verbally expressing her interest in them at the beginning of the episode (lines 1 and 2). Her facial and body expressions indicated security and tranquility (second phase of an emotional experience). We linked these emotional responses to confidence (emotion, first phase) that led her to develop an own resolution strategy from her partner’s initial suggestion (lines 3–8). She also presented and shared her strategy and solution with satisfaction (lines 3–7), a new emotion that emerged from the cognitive evaluation of her latest actions and emotional responses (first phase of a new emotional experience). It is possible that the positive emotions experienced by Luisa during the dialogue made her not sufficiently aware of the different attributes that each considered, Martin talking about length and Luisa talking about surface, thus expressing an ultimate tranquillity (emotional response, second phase) that denoted satisfaction (emotion, first phase) (lines 8 and 9). Martin, on the other hand, recognised length and linked the perimeter to the sum of the triangle’s sides. This allowed him to advance a correct solution based on the comparison and conservation of length through the equivalence of triangles (line 2). All these knowledge elements related to the proposed task. In the end, however, Martin accepted Luisa’s solution (lines 4, 6 and 10) and did not insist on his own proposal, perhaps out of shyness (emotion, first phase). Yet Martin did somewhat doubt, showing some insecurity (emotional response, second phase), the relevance of his partner’s strategy (line 8). It seems that Martin was striving to reach an agreement with Luisa by seeking a joint solution. As a result of the discrepancy between his knowledge and Luisa’s during the cognitive assessment in the first phase, his facial and body expressions indicated tension throughout the episode (lines 4 and 7), an emotional response in the second phase that we link to worry as an emotion triggered in the first phase of the emotional experience. Finally, it is likely that Martin sought to relieve this emotion by accepting his partner’s proposal. Table 4 summarises the phenomenon-epistemological analysis of Luisa’s and Martin’s observable record.

*4.1.2. Dialogic plane*

In order to reach consent with Luisa, we sought to obtain more information about her decision to consider the surface area, her relationship with the ideas presented by her partner, and how the confidence shown during the resolution influenced her to consider the solution as correct. The following excerpt was drawn from this discussion (Table 5).

Luisa identified and accepted her mistake since she acknowledged that she confused perimeter measurement with surface measurement, showing an understanding of both concepts, and providing a new correct answer to the task (lines 4 and 6). Nevertheless,

**Table 3**  
Solving Task I by Luisa and Martin.

Line	Participant	Utterance	Expression
1	Luisa	What do you think of the perimeters in this figure?	Luisa: Relaxed body and face, easy-going smile (2).
2	Martin	I think the perimeters are what’s on the outside. I mean this line here. So, what can I say about the perimeter measurement? They’re the same, right? ( <i>Firm Voice</i> )	
3	Luisa	Yes, in each. I believe that if this, for example, in Image 1 is equal to one, Image 2 is a half ( <i>firm voice</i> ).	
4	Martin	It’s a half (low voice). That’s right.	Martin: Tense body, tight lips, presses hand strongly against his face (4).
5	Luisa	Image 3 is one fourth and Image 4, an eighth.	
6	Martin	Yes, yes. That’s it.	Martin: Tense body, looking down, covers face with hand (7).
7	Luisa	Well that’s how I see it ( <i>loud and firm voice</i> ).	
8	Martin	What was I going to say? That I was having a look and they haven’t given a numerical value to anything.	Luisa: Relaxed face, upright torso, raised cheeks (8).
9	Luisa	Half of each image.	
10	Martin	In other words, Image 1 is worth one, all the others are halves.	
11	Luisa	Of the previous one.	



**Table 4**  
Phenomenon-epistemological analysis of Luisa’s and Martin’s observable record.

	Luisa	Martin
Mathematical knowledge	Surface and area, magnitude of attributes, unit of measurement, fraction.	Length, perimeter, magnitude of attributes, conservation of length.
Relations	Each figure’s surface measures half that of the previous one.	Length and perimeter, the length of each triangle’s side measures twice that of the triangle’s equivalent side in the next image.
Heuristic strategies	Search for regularities, sort, compare, visualise.	Observe, look for regularities, compare.
Emotional system	Emotional responses: security, tranquillity. Emotions: confidence, satisfaction.	Emotional responses: tension, insecurity. Emotions: shyness, worry.

**Table 5**  
Reaching consent with Luisa.

Line	Participant	Utterance
1	Researcher	You write: "Given that the surface of $I_1$ is 1."
2	Luisa	No, the perimeter of Image 1 is one, the rest are halves, respectively. That is, this perimeter ( <i>of a triangle in Image 2</i> ) is half this one ( <i>of the triangle in Image 1</i> ).
3	Researcher	How do you reach that conclusion?
4	Luisa	But if I look at it now, this part would be up here and this part down there. So, it wouldn’t be exactly half, it would be. equal. The perimeter would be the same because all that we’ve done is move two parts.
5	Researcher	And the conclusion you gave?
6	Luisa	It’s wrong. Of course, because it looks like half. I think I was thinking of it as the surface, not the perimeter.
7	Researcher	You consulted your partner. Did it seem important to you?
8	Luisa	Yes. When I did the exercise with Martin it was very clear to me. So, I tried to explain it as many times as I could.
9	Researcher	Martin clearly saw that it was the perimeter and he raised the question.
10	Luisa	Yes, he did, he talked about the perimeter, but. in my mind I knew what the perimeter was, but I kept seeing it as half.
11	Researcher	If you had revised your solution, would you have spotted your own confusion?
12	Luisa	I don’t think so. I saw it and said: this is half of the half. And when I started doing it, I kept seeing it in the same way and when we wrote the conclusion, I was still seeing it in the same way.
13	Researcher	You were sure of yourself and you were pleased with your solution.
14	Luisa	I was, yes.

despite this and her intention to help Martin (line 8), she continued to justify her initial ideas in order to explain her actions during the episode. We appreciate Luisa’s initiative to argue about her own actions, despite not explicitly recognizing the mathematical correctness of her partner’s initial proposal (line 10). Such a decision may have been motivated by the confidence shown in her own proposal, which also leads her to not feel the need to validate her solution (line 12). In any case, again we recognise the intervention of Luisa’s emotional system in these arguments, presenting new evidence of confidence and satisfaction, along with associated emotional responses such as being self-confident and feeling satisfied with her solution (lines 13 and 14).

In our dialogue with Martin, we sought to reach consent on his interpretation of Luisa’s resolution strategy, on the reason that prevented him from insisting on his own initial solution and on his emotional management following the change of proposal (Table 6).

Martin recognised that the objective of the exercise was to measure the perimeter and again identified the correct solution (lines from 1 to 4, and 10). We did not, therefore, perceive any limitation to his understanding. At first, he did not clarify the reason that led him to renounce his initial solution and adopt Luisa’s during the episode (lines from 4 to 6). The task can also be solved by comparing the lengths of the perimeters of each singular equilateral triangle in each of the four figures. For example, if the triangle in the first figure has a total perimeter of 1, then each equilateral triangle in the second figure has a perimeter of 0.5 with respect to the previous one, which means that the two equilateral triangles together also have a total perimeter of 1. Martin could have interpreted that Luisa was comparing the perimeters of each singular equilateral triangle in each figure, which would justify the use of the term half in the conversation (Table 3), and to conclude that Luisa’s and his answer were really the same just described differently (lines 9 and 10). In

**Table 6**  
Reaching consent with Martin.

Line	Participant	Utterance
1	Researcher	You looked at the task and it was very clear to you.
2	Martin	Yes.
3	Researcher	You told Luisa that the perimeters were equal. But what she saw was how the triangle surfaces were divided.
4	Martin	Ah! There was no sense in that. I was asked about the perimeter and I focused on the surface area.
5	Researcher	It’s as if you gave up on your solution, despite having the right answer.
6	Martin	Yes, maybe.
7	Researcher	What do you think happened?
8	Martin	Luisa didn’t agree with what I. She told me one thing and I said yes, understanding that she was understanding what I was understanding. And I said, okay, okay!
9	Researcher	But the answer you gave was different from the answer you would have given with your first approach.
10	Martin	I can tell you that all the perimeters are equal.

any case, his explanation points to a misunderstanding with Luisa (lines 7 and 8) and we also recognize his attempt to value his partner's proposal, to make sense of it and make it compatible with his own. This effort made by Martin we have already perceived during the episode through his utterances and external emotional responses and again it is now manifested in his final comments (lines from 8 to 10).

#### 4.2. The case of Laura and María

##### 4.2.1. Semiotic and phenomenon-epistemological planes

Laura and María jointly undertook to solve Task II (Table 7). The observable record once again provides us with evidence of the uses given to mathematical knowledge and its relationship with the protagonists' emotional system.

Initially, the students identified the separate use of the three units described in the problem to measure the box's volume. Laura employed the strategy of counting the number of complete A units needed to cover the prism (line 2). Unable to do so, both began to rethink the statement's initial conditions and to discuss the meaning of the term "respectively" (lines from 3–7). This term refers to the fact that the requested measurement must be performed up to three different times. Each time, with one of the units A, B and C independently. They overcame this difficulty by adopting a new strategy: that of combining the given units to calculate the volume. In their final conclusion, they proposed five different solutions that arose from the joint use of unit A with B and C and the use of units B and C independently (with the latter, they did succeed in covering the box) (lines from 8 to 12). The protagonists revealed different positive traces of an understanding of measurement during the episode. Specifically, these traces were: the identification of an attribute involved in the task; the comparison and conservation of magnitudes; the establishment of relationships between units, and the fact of combining them to perform the measurement. In parallel, they also showed understanding limitations as they did not detect the option of using unit submultiples. The latter would have allowed them to divide unit A and give a solution in accordance with the statement's requirements.

The impossibility of calculating the prism's volume using unit A generated worry and frustration in Laura and María (lines 2 and 3). Despite reinterpreting the statement and changing strategy, these emotions persisted during their quest for the different combinations (lines 7 and 8). Such emotions resulted from the discrepancy between the need to seek relief from their emotions and the certainty of having correctly interpreted the task's statement and not having a satisfactory strategy as a result of the cognitive assessment during the first phase of the emotional experience. It was only in the end, once they had agreed on five different solutions, that they showed relief and satisfaction, as an emotional response in the second phase, with respect to what they had accomplished (lines 11 and 14). During the episode, we also perceived the dynamic and unstable nature of the emotions as some manifested themselves almost simultaneously. For example, María presented uncertainty and worry within the same intervention (line 3) and Laura showed empathy towards her partner despite her own uncertainty (line 4). Table 8 summarises the main traces of understanding and emotion found during task's solving.

##### 4.2.2. Dialogical plane

We sought to reach consent with María regarding the difficulty of calculating the volume with unit A, the possibility of dividing this

**Table 7**  
Solving Task II by María and Laura.

Line	Participant	Utterance	Expression
1	María	Calculate the volume of the box. Ah okay, we have to put A first, then B, then C.	Laura: Body forward, looking down, tight lips (2).
2	Laura	So, with A. It would be one, two, three, four, five and six. But then it doesn't fit at the top, you know what I mean? Because they are. ( <i>low voice</i> ). I can't, a part is missing and it can't be measured. with A.	
3	María	You don't always have to use A. You have to alternate. Wait. ( <i>Sighs</i> ). The more I look at it, the more I get confused, because let's see.	María: Tense face and body, abrupt and sudden movements (3).
4	Laura	What should I put in the box? Only parts of A, only parts of B, only parts of C or can I? ( <i>Looks approvingly at her partner, moves her hands when speaking</i> )	Laura: Drooping eyes, looking down, she holds her head with one hand (7).
5	María	Mix the three of them.	
6	Laura	But they say respectively ( <i>with an emphasis</i> ).	
7	María	Respectively. Right. Because if we only use this one ( <i>unit A</i> ) we're going to have to use more. We won't fill the box because when we put another one in, it sticks out, you see? We can't close it.	
8	Laura	Yeah, that's why I suggest using A and C parts. With C we've got it, it would be complete. I mean, what do we want B for?	María: Eyes open, lips tight, staring at Laura's sheet (8).
9	María	Because with B you can too.	
10	Laura	Yup, so do we use them all or.? You need 6 pieces of C and you've got the figure.	Laura: Upright body, large smile, relaxed face and body (11).
11	Laura	And you don't need B. Unless. Look! ( <i>Eureka</i> ) Do you realise that with C and B it can be done on their own? With A, no. Let's put that in the conclusion.	
12	María	18 bits of C and 24 bits of B. Brilliant! I haven't thought so hard in my life!	María: Body upright, looking forward, relaxed smile (14).
13	Laura	So, we've got five options, haven't we? I don't know if this is what they are asking for, but we've got a conclusion.	
14	María	Well look, we've found all possible cases. It doesn't matter if they were alone or separate. We've done it both ways. That's cool!	
15	Laura	Yep, it really is.	

**Table 8**  
Phenomenon-epistemological analysis of María’s and Laura’s observable record.

	María	Laura
Mathematical knowledge	Unit of volume, conservation of volume, comparison of units, volume.	Unit of volume, volume, capacity, comparison of units, conservation of volume.
Relations	Between different units.	Between different units.
Heuristic strategies	Count, visualise, conjecture, compose, structure sets of units, fill.	Visualise, fill, count, identify units, structure sets of volume units, use analogies.
Emotional system	Emotional responses: Blockage, tension, tranquillity. Emotions: Uncertainty, worry, frustration, relief, satisfaction.	Emotional responses: Tranquillity, Eureka, tension, blockage. Emotions: Uncertainty, frustration, empathy, worry, surprise, relief.

unit, and the reasons for the change of interpretation and strategy during the task (Table 9).

María acknowledges that she did not clearly understand the statement, specifically the meaning of the word "respectively" (lines 2 and 3). She revised her strategy of filling the prism with unit A and concluded again that it was not possible to calculate the volume with this unit alone. She also used this fact as an argument to justify all the solutions they proposed (line 3). She did not consider using a submultiple of unit A, because the statement did not explicitly state that possibility (lines from 4 to 7). Although she agreed that the use of such a submultiple would allow the volume to be measured using this unit, she did not provide details on how to do so (line 7). On the other hand, María considered that reducing the pressure of the challenge helped her to resolve the task (line 9). In addition, she did not regard the resolution process as a negative experience, but rather as a game which she was ultimately satisfied with despite having doubts (lines 10 and 11). During the episode, we recorded a variety of traces of emotions experienced by María, but they were not explicitly reflected in the explanations given in this second phase (for example, frustration).

In our dialogue with Laura, we focused on the difficulties of the exclusive use of unit A, on the procedure applied and on the different solutions advanced (Table 10).

As in the case of María, Laura detected the difficulty of using only unit A. She explains that the discrepancy between their solution strategy and the statement’s true requirements was due to the interpretation of the term "respectively" (line 1). She did not contemplate either at this stage the possibility of using the unit’s submultiples (lines 1 and 3). Regarding her emotional experience, Laura acknowledged that she felt satisfied when she finished the task because she advanced an appropriate solution (line 5). She even perceived the representations associated with this emotion as a characteristic feature of her identity (line 7). The resolution process does not seem to have represented a negative experience for Laura, despite the other emotions (for example, uncertainty or worry) that surfaced during the episode.

**5. Discussion and conclusion**

The empirical study described herein shows that emotions intervene in the decision-making processes regarding the shared use of mathematical knowledge during classroom problem-solving (Brown & Reid, 2006; Marmur, 2019; McCulloch, 2011). In the first case, we considered that Luisa’s mathematical understanding, in terms of uses of knowledge, was conditioned by her emotions (Martínez-Sierra et al., 2019). Indeed, her emotions also defined the specific mathematical knowledge she implemented during the episode. Specifically, according to our interpretation, her decision to assume the mathematical suitability of her resolution strategy was due to her confidence in solving the problem and satisfaction with her own proposal. Martín, for their part, also revealed different links between his emotions and his mathematical understanding. At the beginning of the episode, his possible shyness perhaps prevented him from insisting on using his own proposal rather than his partner’s, which then conditioned his later performance and understanding in the task (Hannula, 2012b). We also understand that trying to reconcile different resolution proposals (either with equivalent or non-equivalent solutions) generated worry. Therefore, the fact of accepting Luisa’s understanding also ultimately determined his own emotional experience during the episode.

**Table 9**  
Reaching consent with María.

Line	Participant	Utterance
1	María	This time I got really confused with so many squares. I was filling the squares with each and as we did not understand exactly what the statement was asking for.
2	Researcher	Wasn’t the statement clear?
3	María	Respectively. we didn’t know if it was with A, with B and with C, separately, or mixing two of them. So, I thought: well, let’s try all options. With A, you couldn’t fill the box, because when you added it, it overflowed at the top. With the others, you could.
4	Researcher	And is there no way of measuring using A?
5	María	I don’t know, you can’t fit it in whole. it’s either not enough or too much.
6	Researcher	Could you cut A in half?
7	María	Hum. They don’t say that you can. If you could split A, then it could be completed with A alone.
8	Researcher	In this task, you seemed confused to me.
9	María	When I saw the word volume: Ugh, volume, what’s that? And I broke away a little from what the statement was asking. I took it as a game, I imagined putting the little pieces in the box, and that was it.
10	Researcher	But you started doubting again.
11	María	Yes, as always.

**Table 10**  
Reaching consent with Laura.

Line	Participant	Utterance
1	Laura	<i>(Laughs)</i> We wondered what "respectively" meant. We contemplated options with A, with B and with C and as we were trying each one, we concluded that you could not fill the whole box with A. Because obviously, its height is two and the height of the box is three. So, we thought: if we use A alone, we cannot complete the box, so we used A with C, which has indeed a low height and we could fill it. That led us to think that we could also fill it with B, because it has the same height as C. And from that point onwards we did a mishmash.
2	Researcher	But you solved it.
3	Laura	Yes, we solved it well and, we didn't know if it was the answer they were asking for, because maybe the statement meant only A, B and C. But we thought we might as well provide all the options.
4	Researcher	When you finished, you said: cool!
5	Laura	<i>(Laughs)</i> I don't know, it's probably the satisfaction. after having messed around to reach an answer and, in the end, it came out well. It's like. cool! We were not altogether lost.
6	Researcher	Were you pleased?
7	Laura	Well, I can't remember, but it's very typical of me, yes <i>(laughs)</i> . I always say something like that when I feel good, satisfied.

In the second case, where the interaction between the participants was greater, Laura and María began to resolve the task by properly interpreting the statement and defining a first feasible strategy. However, their limitations regarding the understanding of measurement, relative to the use of a unit's submultiples, prevented them from completing it satisfactorily. This latter occurrence generated emotions (uncertainty, worry) that they sought to overturn by configuring a new alternative strategy, based on the combination of different measurement units. That is, an initial discrepancy, a consequence of a certain understanding, caused initial emotions to emerge (Mandler, 1989). The latter then determined subsequent actions and uses of mathematical knowledge in the resolution. From here, a second discrepancy arose between the statement's original requirements and the adopted strategy. The new conflict between what must be done in the task and what the protagonists really intended to do continued to generate negative emotions, which they succeeded in transforming with the final given conclusion. Specifically, the worry they felt about the possibility of not having fulfilled the task's requirements, changed to satisfaction and relief when they were aware that their understanding led to multiple resolutions (Op 't Eynde et al., 2006; Satyam, 2020).

This interpretation of the relationship between emotion and understanding enables us to justify the students' use of mathematical knowledge, to provide evidence of the characteristics of their emotions and to identify reasons for their understanding. Incorporating emotions in the interpretive process thereby allowed us to obtain information not only on students' mathematical understanding, but also on why they understood such elements of knowledge in a certain way. The study also provided empirical evidence of the dynamic nature of emotions. Like authors such as Evans (2006), McCulloch (2011) or Radford (2015), we appreciated students' different emotional paths as a result of their assessments, where emotions varied according to the resolution of the mathematical problem addressed.

The configuration of theoretical frameworks that help to understand the role of emotions in mathematical learning still constitutes an ongoing goal in the field of mathematics education (Marmur, 2019; Ronen, 2020). The present study made the specific contribution of a model that enables an exploration of the understanding of mathematical knowledge through emotions. This model allowed us to establish direct connections between emotion and understanding. A bidirectional relationship was proposed between the student's emotional system and understanding through the uses given to mathematical knowledge. By adopting this perspective, we are acknowledging within the same process a number of assumptions and contributions which are representative of the main approaches to emotions in mathematics. In this way, we adhere to the recognition that emotions are not separate from or opposed to rational processes (Else-Quest et al., 2008). We also share the socio-constructivist perspective that emotions are social processes arising in specific contexts and dependent on the cultural characteristics of the situations in which they emerge (Evans, 2006; Nussbaum, 2001; Op 't Eynde et al., 2006; Radford, 2015). In addition, we recognise that emotions form during discursive practices, through interactions with others and via the self-positioning adopted in the relationships and that contribute to social identity construction in the classroom (Evans et al., 2006). Finally, from a holistic viewpoint, we acknowledge that emotions are functional and responsible for activating the tendency towards action, and they thus have a key role in human adaptation (Damasio, 1994; DeBellis & Goldin, 2006; Eagleman, 2011; Evans, 2006; Hannula, 2012b).

Another aim of our study has been to define procedures that allow to interpret, in practice, the relationship that have been acknowledged between emotion and cognition in mathematics. Following the methodological recommendations from different authors in our field (Cobb et al., 1989; Di Martino & Zan, 2011; Evans et al., 2006; Hannula, 2006; Leder, 2006; Schlöglmann, 2010), our proposal incorporated a qualitative method including various instruments and interpretation strategies. Written production and shared dialogues allowed us to obtain information about the uses given to mathematical knowledge and various accompanying emotional responses. The emotional activity's external representations also played a decisive role in the interpretation. Indeed, they gave access to complementary information that made it possible to explain visible actions and determine reasons for the decisions made. Thus, we integrated various cognitive, expressive and physiological aspects of the emotional component of mathematical understanding within the same interpretive process (Hannula, 2012b; Martínez-Sierra et al., 2019; Sumpter, 2020).

In short, we consider that our proposal represents a contribution to the conceptual framework of emotions in mathematics, at a theoretical and methodological level. The OMIUM proposes its own theoretical assumptions and a qualitative methodology, coherent with its principles, which uses different sources both for data collection and interpretation. Identifying the relationship between emotions and understanding provides a more complete assessment that is closer to students' actual mathematical understanding. In the

future, this will enable us to guide students' emotional responses towards learning with understanding.

We have presented an approach that enables exploring the understanding of mathematical knowledge through emotions. However, our analyses should be considered initial and exploratory, since it still has a highly subjective and interpretive component. In this study, we have evidenced alternative interpretations and the possibility of reaching different conclusions about the relationships that emerge between emotions and mathematical understanding. Ensuring agreement in interpreting the mathematical understanding evidenced by students during their interactions in the classroom is still an open question for us. It seems reasonable to us to seek ways of interpreting that are more valid and reliable, so, in future research, we will aim to make efforts to improve the effectiveness of our own interpretive method.

Emotions have not been studied in mathematics education with the same depth and breadth as other components of the affective domain, such as beliefs or attitudes (Quintanilla, 2019; Ronen, 2020; Satyam, 2020). Focusing on the key role of emotions in the development of understanding in mathematics, we also aspire to further elaborate an integrative framework of the various components of affect in mathematics, a major challenge recognised in our field of research.

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## Declaration of Interest Statement

None.

## Data Availability

The data that has been used is confidential.

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