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Analysis

Orbital debris and the market for satellites[☆]

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ABSTRACT

This paper studies the economic consequences of orbital debris for commercial outer-space activities. Spacecraft launches and other outer-space human activities produce pollution (i.e., orbital debris), which represent a hazardous negative externality increasing the risk of collision and the destruction of satellites. We regard outer space as a global common resource, where firms operating satellites maximize profits and do not internalize the social cost of orbital pollution. We develop a dynamic investment model for satellites and simulate the calibrated model to estimate how debris affects the optimal quantity of satellites and launches, and the number of satellites destroyed by collisions. We find that the optimal quantity of satellites is a negative function of the amount of debris. The paper derives a simple expression for the maximum number of satellites to prevent the Kessler syndrome. For the baseline calibration of the model, the estimated threshold for the maximum number of satellites in orbit is about 72,000. The model is simulated to study the effects of a decline in the launch cost and the increasing number of satellites per launch.

1. Introduction

Although the human exploration and economic exploitation of the outer space are quite recent (the first successful launch of a human-made spacecraft occurred in 1957), a number of market failures are arising at rocket speed as the commercial, military, and scientific activities in space are continuously expanding. Outer space is an example of an extra-terrestrial common resource. No agent (national government or international organization) has authority over the property rights of space, with the exception of spacecraft ownership; therefore, human activities in space, including commercial ones, are not subject to any centralized regulation or property rights. In this (non-)regulatory environment, outer space exhibits the characteristics of a global common resource and hence is subject to comparable economic failures to other international commons on the Earth (i.e., fisheries in international waters, the atmosphere, or Antarctica). A large number of countries has signed several treaties and principles regarding human activities in the outer space. The first international agreement that has established a list of basic principles to regulate human activity outside planet Earth is the Outer Space Treaty (OST).[1] However, the principles instituted in the OST represent a set of basic rules with a very limited scope to which countries pursuing activities in outer space are subject, and it cannot be considered as a fully operational regulatory framework but one in which the “first come, first serves” principle dominates. The consequences of this lack of regulation were clearly expressed by Hardin (1968), who pointed out that “ *Freedom in a commons brings ruin to all*”. Nevertheless, Ostrom (2010) demonstrated the possibility of avoiding the “tragedy of the commons” without privatization or government regulation. However, Weinzierl et al. (2016) show that the conditions for the kind of cooperation proposed by Ostrom are absent in the space.

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[1] The exact denomination of the OST is the “Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, including the Moon and Other Celestial Bodies”. The OST enumerates the basic principles for human activity in outer space to be followed by nations (not private companies as the Treaty was signed in 1967, when only governments had the technology and financial resources to access space), including freedom of access and exploration, no sovereignty, and peaceful purposes. The basic principles of the OST were later extended by other additional four treaties, together with other agreements and conventions on more specific issues, including orbital debris. The full list of treaties can be found in https://www.unoosa.org/oosa/en/ourwork/spacelaw/treaties.html. These treaties and agreements have been ratified by a large number of states, except the Moon agreement, which has been ratified by only 18 states to this day.

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The economic analysis of outer space is attracting increasing attention from scholars, although it is still too early to refer to a new research field as outer space economics. The foundations for the economics of outer space were constructed by O'Neill (1977) and Sandler and Shulze (1981). See Weinzierl (2018) for a review of the development of the space economy. O'Neill (1977) was the first to study the feasibility of space human colonies from an economic perspective. The path-breaking contribution by Sandler and Shulze (1981) enumerated and studied a number of economic issues related to commercial and other activities in outer space, including broadband, rights over the geostationary orbit, and the risk of collision. An earlier work by Snow (1975) developed a model for communication satellite capacity. Although the first human activities in outer space were carried out by nations, given the initial technological and financial barriers, private companies are progressively gaining importance. Indeed, a large variety of industries are expected to be developed in the near future, additional to the industries that are already well established (broadcasting and communications services, positioning services, Earth observation, etc.), other than military and scientific activities. Commercial activities in space generate around $300 billion in annual revenues (Weinzierl, 2018). Industries such as space manufacturing of special goods for customers on Earth using microgravity, vacuum and extreme temperatures (Patel, 2019), in-space manufacturing and maintenance and repair services (Prater et al., 2018), asteroid mining (Ross, 2002), and space tourism (Peeters, 2010), among others, are expected to be developed in the foreseeable future, further congesting outer space and generating more space pollution.

One of the issues that has attracted attention from academics in different disciplines is the market failure leading to the generation of space debris.[2] Debris is a type of space pollution than could have dramatic consequences for commercial and other activities in outer space (Liou and Johnson, 2006). Launching satellites and carrying out other operations in orbit generate debris that can collide with operational artificial satellites, with fatal consequences in some cases. Even small debris with little mass can have catastrophic consequences for the affected spacecraft due to high velocities. On the other hand, space debris is self-propagating, as collisions between pieces of debris create more debris. This is the so-called "Kessler syndrome" representing a scenario of collisions in cascade (Kessler and Cour-Palais, 1978). Debris is generated from different sources, including parts of launch vehicles and rocket bodies, non-functional satellites, the breaking-up of satellites and rocket bodies, and even tools lost by astronauts, but also by natural sources.[3]

Seminal papers studying the economic consequences of orbit debris are Adilov, Alexander and Cunningham (2015, 2018) and Macauley (2015). Adilov et al. (2015) developed a Salop-type model (Salop, 1979), for comparing the optimal number of launches in a decentralized versus a centralized market. They found that the numbers of satellites and launches are higher than the social optimum as firms do not take into account the negative externality of debris generated by their activities in space. Given that the negative externality affects all firms, there is under-investment in debris mitigation technologies. Adilov et al. (2018) used a net present value approach to determine that the threshold level of debris for economic viability is lower than the "Kessler syndrome" level identified by Kessler and Cour-Palais (1978). They found an initial positive relationship between launches and debris, to replace satellites destroyed, the relationship being negative after a threshold level of debris is reached. Macauley (2015) presented different technological strategies to mitigate debris generation and/or collision risk, including maneuvering capability, graveyarding capability and shielding. Klima et al. (2016) used a game theory approach whereby spacefaring agencies have the option of implementing costly active debris removal interventions that benefit all spacefaring agents or waiting for other agents carry out the work. Grzelka and Wagner (2019) developed a model containing property rights and instruments to incentivize ex ante increases in satellite quality, and collective or individual debris take-back interventions. Béal et al. (2020) compared the non-cooperative Nash equilibrium with a tax on launches to finance debris mitigation, with the welfare optimal traffic under a centralized tax. They found that, under a centralized tax, the traffic is increased and the debris mitigation cost is reduced compared with the non-cooperative scenario. Rouillon (2020) considered a model with a constant rate of satellite launches and concluded that the number of satellites is an inverted-U shape function of the launch rate. Rao et al. (2020) developed a model with infinity-lived satellites to study the implications of Pigouvian taxation consisting of an international orbital-use fee. Adilov et al. (2020) simulated the quantity of orbital debris under different policies, including a launch tax, voluntary debris mitigation, and active debris removal policies. Guyot and Rouillon (2022) study an environment where satellite operators make choices about the design of satellites while in-orbit servicing firms supply efforts to remove space debris. Finally, Bernhard et al. (2023) use a dynamic game to assess the impact of satellite mega-constellations on space debris and explore alternative taxation schemes for financing active debris removal policies.

The contribution of this paper to the literature is twofold, focusing on two key issues not previously addressed by the literature, that can be summarized in the following two research questions. First, what is the cost of the negative externality created by orbital debris? Second, what is the maximum number of satellites to prevent the Kessler syndrome? With the aim at answering the previous two research questions, this paper develops a model based on the standard neoclassical dynamic investment model to explore the consequences of orbital debris for the optimal number of satellites and launches, and the implications of launch cost declines and an increasing number of satellites per launch. It is assumed a perfect competitive environment in which firms maximize profits by choosing the optimal number of satellites launches. Given the characteristics of the outer-space market, in which there is no supervisory authority, the optimal number of satellites depends on the risk of destruction through collision with debris. The model is solved for a decentralized economy, in which the negative externality arising from debris is not internalized by firms, under two alternative scenarios representing different stages of space exploration. First, we consider an initial stage in human outer-space activities in which the amount of debris is small enough that the probability of collision is practically zero. In this scenario, firms maximize the sum of discounted profits without considering any externalities provoked by their activity. Second, we consider a second stage in which the amount of debris starts to be large enough to lead to a non-negligible probability of collision and in which this risky environment is incorporated into the firm's maximization problem. However, pollution in space shows a crucial

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[2] As defined by NASA, "orbital debris is any human-made object in orbit that no longer serves a useful purpose, including spacecraft fragments and retired satellites".

[3] The main cause of in-orbit explosions is related to residual fuel that remains in the tanks of rockets' upper stages or derelict satellites abandoned in orbit. The extreme conditions in outer space quickly cause mechanisms and devices to deteriorate, leading to leaks mixing fuel components, which provoke accidental explosions that break-up rocket bodies and other spacecraft, and generate a large number of fragments that travel around the initial orbit at hyper-velocity (above 10,000 kilometer per hour). Besides such accidental break-ups, spacecraft interceptions by surface-launched missiles have been a major contributor in the recent past. Four countries, the US, Russia, China and India have conducted direct-ascend anti-satellite tests. A single event, the intentional destruction of the Chinese Feng-Yun 1C satellite by a missile in January 2007, increased the trackable space debris population by 30% (OECD, 2020). Most debris (around 85%) is at a Low Earth Orbit (LEO) altitude (below 2000 kilometers), with peak concentration around an altitude of 700–900 km (NASA, 2020). Nevertheless, the environment in the outer space is in continuous transformation. For instance, new launches of satellite constellations (such as the Starlink network) will transform significantly the environment at low Earth orbit (LEO).

difference from pollution on Earth: space junk has a negative impact on firms' stock of capital assets.

We find that debris has a negative impact on the equilibrium quantity of satellites compared with an ideal outer-space environment with no debris. The parameters of the model are calibrated to the present conditions observed in outer space. We measure forgone satellite services as the cost of this negative externality. The estimated relationship between launches and debris resulting from the calibrated model is negative. We derive a simple expression for the maximum number of satellites that can be inserted into orbit to prevent the Kessler syndrome. This threshold value for the number of satellites is a function of the physical parameters, resulting in a value of around 72,000 satellites for the baseline calibration. Nevertheless, this figure should be taken with caution, as this is an average number for the different orbits and it depends on the base line calibration of the model. Orbits have different level of congestion, available volumetric space, debris concentration, atmospheric drag, satellites of different volume and mass, etc. The calibrated model is used to carry out several simulations to investigate the consequences of a reduction in the launch costs, and an increase in the number of satellites per launch. As expected, as the launch cost decreases or the number of satellites per launch increases, the optimal number of satellites increases up to the threshold for the Kessler syndrome. Increased activity in the space also causes an increase in the amount of debris and the number of satellites destroyed by collision, reducing the economic benefits of the per satellite launch cost reduction. The social cost of debris is estimated at around $11.5 billions annually.

The structure of the rest of the paper is as follows. Section 2 presents a simple economic model for satellites based on the standard dynamic investment model extended to consider the negative externality from orbital debris. Section 3 calibrates the physical and economic parameters of the model. Section 4 uses the calibrated model to simulate alternative scenarios and estimate the cost of the externality from debris. Finally, Section 5 concludes.

2. An economic model for satellites

We consider a market for satellite services (communications, broadcasting, Earth observation, weather and climate monitoring, geographical positioning, etc.). Although a large number of human-made objects in orbit are military or scientific-purpose spacecraft, we focus on commercial activity as outer space is moving from a government-run to a private firm-managed but state regulated environment (see Weinzierl and Acocella, 2016; Weinzierl, 2018; BryceTech, 2022).[4] The incipient exploration of outer space, given the initial technological and cost barriers, was conducted by states, but, in the last decades, the private sector has expanded significantly, and new types of business with a higher private presence are expected to be developed in the future. We assume a competitive market in which infinite-lived space-operating firms maximize the sum of discounted profits from satellite services.[5] To increase the number of satellites in orbit, additional investment in launches is necessary. We assume that the launching cost includes all the costs of manufacturing a satellite and the launching vehicle, the cost of the launch, and the operating cost during the life-span of the satellite.[6]

Human activity in space provokes a negative externality (a kind of pollution) related to launches and other activities in orbit. This pollution takes the form of human-produced space debris (or junk). Debris poses a danger to operating satellites as they can be damaged or destroyed by collisions. The model considers the average probability of destruction of a satellite through a collision with space debris. This reduces the expected profits from launching a satellite. The destruction risk depends on the probability of a hit times the amount of debris. Therefore, in each period, the number of operational satellites can be reduced in the figure resulting from the destruction risk times the number of satellites. The average probability of collision used in this paper is an important simplification assumption in the model, as this probability varies with orbital altitude and the size of the objects. On the other hand, probability of collision also depends whether the debris at risk of collision is trackable or not. Contrary to other types of pollution on Earth, pollution in outer space presents two main distinct characteristics. First, this type of outer-space pollution has a direct negative affect on the stock of capital assets of firms in this market. Second, pollution is self-propagating, which could result in an explosive path that makes human activities in the Earth's orbits impractical (the Kessler syndrome).

2.1. Model setup

We assume a competitive market with N identical firms launching and operating satellites. Infinity-lived firms operating in outer space maximize the sum of the discounted profits (the present value of future receipts, $V_{i,0}$), defined as $\Pi_{i,t}$, for $i = 1, \dots, N$.

$$\max V_{i,0} = E_t \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \Pi_{i,t} \tag{1}$$

where E_t is the expectation operator, $1/(1+r)$ is the discount factor, and r is the interest rate, which is assumed to be constant. Firm i profits are defined as,

$$\Pi_{i,t} = P_t s_{i,t} - c_i l_{i,t} \tag{2}$$

where P_t is the market price, $s_{i,t}$ is the number of satellites in orbit by firms i, $l_{i,t}$ is the number of launches by firms i, and c_i represents the cost per launch. For simplicity, we assume that c_i is exogenously given and that space operating firms have perfect-foresight. The total number of operational satellites in orbit is given by $S_t = N s_{i,t}$.[7] Satellites produces services for consumers on Earth. We assume that the technology function for satellite services producing revenues for the industry is given by $P_t S_t = A_t S_t^{\alpha}$. The parameter α $(0 < \alpha < 1)$ represents the elasticity of satellite services with respect to the quantity of satellites, which is assumed to be lower than one indicating the existence of decreasing returns given the demand for satellite services. A_t is a measure of productivity, representing technological change in the production of satellite services, which it is assumed to be exogenous and identical for all the firms.

Since firms are price-takers under perfect competition, the marginal income for an extra satellite equals the market price given by,

$$P_t = A_t S_t^{\alpha-1} \tag{3}$$

which is also the demand function.

The stock of operational satellites in orbit in period $t+1$ by firms i follows the law of motion,

$$s_{i,t+1} = (1-\delta_s)s_{i,t} + \eta l_{i,t} - x_{i,t} \tag{4}$$

where $0 < \delta_s < 1$ is the physical depreciation rate of satellites, and $x_{i,t}$ is the number of satellites of firm i destroyed by collision with debris in

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[4] The number of satellites operated by private-firms related to governments is increasing, but the overall budgets for space activities are still predominantly government, being the US government the leading operator. However, governmental decisions are difficult to be modeled as they are motivated by a set of non-economic factors other than profit-maximization.

[5] The assumption that economic agents are infinite-lived is standard in economics, meaning that, in each moment of time, economic agents take decisions considering their impact in the future.

[6] For the sake of simplicity it is assumed that operating cost is a fixed cost. It is true that in case of collision, lifespan is reduced and also the operating cost should be reduced. However, given that probability of collision is already small, we abstract for introducing such complexity into the model.

[7] Given the heterogeneity in the characteristics of satellites (i.e., mass and volume), the model considers the number of a representative satellite.

every period. Physical depreciation refers to the number of satellites that each period ends their operational lifetime. The parameter η represents the number of satellites per successful launch. Failures or accidental explosions of satellites during the launch are also included in this parameter. In practice, the value of this parameter is increasing over time, as micro-satellites' design and more powerful launch systems increase the number of satellites that can be inserted into orbit with the same launch rocket. Nevertheless, for simplicity, it is assumed that the quantity of satellites per launch is one in the baseline scenario (i.e., every satellite inserted into orbit is considered to be a launch).

Total number of launches are $L_t = N l_{i,t}$ and the total number of satellites destroyed by collision with debris is given by $X_t = N x_{i,t}$. Therefore, the total stock of operational satellites in orbit in period $t+1$ follows the law of motion,

$$S_{t+1} = (1-\delta_s)S_t + \eta L_t - X_t \tag{5}$$

We follow Farinella and Cordelli (1991), and assume that the collision rate is proportional to the product of debris and operating satellites. We also assume that, in the case of collision, the satellite is destroyed and that the collision creates a number of new pieces of debris. An alternative way of modeling the probability of collision was provided by Letizia et al. (2017). Here, we assume that the total quantity of destroyed satellites is given by,

$$X_t = \theta D_t S_t \tag{6}$$

where the term $\theta D_t S_t$ results in the number of satellites destroyed in every period by collisions with debris. It is assumed that collisions only occur between an operational satellite and a piece of debris.[8] This implies that the number of satellites of firm i destroyed by collision with debris is given by

$$x_{i,t} = \theta D_t s_{i,t} \tag{7}$$

The proportional parameter $\theta > 0$ represents the probability of collision of two objects in orbit. It is assumed that the probability of collision of a satellite is proportional to the quantity of debris, θD_t. When $\theta D_t = 1$, that is if the stock of debris is $D_t = 1/\theta$, the probability of collision is one. If the stock of debris reaches that value, all satellites are destroyed by collisions in the period. Adilov et al. (2018) considered that this reflects the "Kessler Syndrome" as defined by Kessler and Cour-Palais (1978) and Kessler (1981), whereby space becomes physically unusable. Notice that the negative externality affecting the final output is modeled in a different way from the standard environmental externality on Earth, where it is assumed that the stock of pollution affects to output (productivity) or household's utility negatively in a direct way. Here, the negative externality affects to the firms' stock of capital assets (satellites) directly.

Debris follows an accumulation process depending on how new debris is produced in each period. In modeling the debris accumulation process, we consider two main sources: launches and collisions. Different from any other source of pollution, the dynamics of orbital debris includes a self-propagating mechanism, whereby pollution generates additional pollution. That is, debris collides not only with satellites but also with other pieces of debris, producing additional debris. The law of motion of debris is given by,

$$D_{t+1} = (1-\delta_d)D_t + \gamma X_t + \omega L_t + \chi \delta_s S_t + \upsilon \theta D_t^2 \tag{8}$$

where $\gamma > 0$ is the amount of new debris generated by the destruction of a satellite, $\omega > 0$ is the amount of debris generated per launch, $0 < \chi < 1$ is the percentage of derelict satellites that remains in the orbit, and $\upsilon > 0$ is the quantity of new debris generated by self-collisions. As above, we assume that the probability of collision is proportional to the quantity of debris. It is assumed that the debris generated per launch include explosions and fragmentations produced by last-stages rockets. The parameter δ_d $(0 < \delta_d < 1)$ represents the decay rate of debris. This decay rate mainly depends on atmospheric drag and therefore, is a function of the altitude of the orbit. The higher the altitude (with respect to the Earth) of the orbit, the lower the decay rate. Additionally, non-functional satellites (end-of-life satellites) represents a type of debris if they are not removed from orbit. Usually, abandoned satellites can be removed from their orbits in two different ways, depending on their altitude, by using the last available fuel. They can be moved to a graveyard orbit at the end of their operational life to avoid collisions with operational satellites and the generation of new debris if their altitude is high. If their altitude is low, they can be sent back down to a disposal orbit where the atmospheric drag reduces their altitude until they burn up on reentry into the Earth's atmosphere, or, if they survive the burning, to the spacecraft cemetery located in the South Pacific Ocean (a mid-point between New Zealand, the Antarctica, and Chile). However, some dead satellites do not have such capacity (they have run out of gas) and remain in their initial orbit. This is represented by the term $\chi \delta_s S_t$, where the parameter χ represents the fraction of non-operational satellites that remain in the initial orbit and are not moved to a graveyard orbit (for the case of geosynchronous spacecraft) or to a disposal orbit (for low orbit satellites) at the end of their operational life.[9] Finally, collisions among debris can also considered, represented by the term $\upsilon \theta D_t^2$, where the probability of collision (θD_t) multiplies the stock of debris. Expression (8) can be extended by including additional exogenous factors producing debris, such as military tests with direct-ascending anti-satellite (ASAT) missiles.

Next, we use the model to study two scenarios that can represent two stages in the exploration and exploitation of outer space: the first stage, representing the early space race, during which little space debris was generated and the risk of collision was negligible, and the present second stage, during which the quantity of debris is significant and a positive, albeit small, risk of collision exists. In this second stage, firms that operate satellites are aware of the risk of collision and include the costs of the possible destruction of satellites in their profit maximization decisions.

2.2. Decentralized market with no debris

First, we consider the case of a decentralized market in which firms maximize profits without considering the risk of satellite destruction by debris. This scenario is intended to represent the early stages of space exploration during which the amount of debris was very small and the risk of collision was near zero. This first scenario is taken as a benchmark for a space without congestion and externalities. The only risk is a natural risk of collision with natural meteoroids not considered in the model. With no debris, we arrive at the standard results of the standard neoclassical investment model.

Each firm maximizes discounted profits (1) subject to the restriction of the satellite accumulation process (4) in which the number of

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[8] We do not consider collisions among operational satellites owned by different operators. Given the possibility of satellites with collision avoidance capabilities, debris monitoring appears crucial. Increasing Space Situational Awareness interventions would represent an important although partial solution to the destruction of working satellites by orbital debris. Although this is not a solution per se to the problem of debris, would be an important complement to other mitigation techniques.

[9] To simplify the calibration of the model, it is assumed that there are no derelict satellites left in orbit, i.e., $\chi = 0$. This assumption can be perfectly true in LEO orbit, where natural drag is high. On the other hand, derelict satellites can potentially explode creating more debris, a possibility not considered by the model. Also failures can occur preventing the execution of end-of-life disposal maneuvers. For GEO, satellites are massive and expensive and it is expected that they can be inserted in a disposal orbit at higher altitude.

satellites destroyed by collision is zero ($x_{i,t} = 0$), where $S_0 > 0$ is given. The first-order conditions for optimality are,

$$\left(\frac{1}{1+r}\right)^{t+1} A_{t+1} S_{t+1}^{\alpha-1} = \lambda_t - \lambda_{t+1}(1-\delta_s) \tag{9}$$

$$\left(\frac{1}{1+r}\right)^{t} c = \lambda_t \eta \tag{10}$$

and the transversality condition is,

$$\lim_{t\to\infty} \lambda_t \left(\frac{1}{1+r}\right)^{t} S_t = 0 \tag{11}$$

where λ_t is the shadow price for constraint (4) under the no-debris no-collision risk scenario, and is given by,

$$\lambda_t = \frac{c}{\eta}\left(\frac{1}{1+r}\right)^{t} \tag{12}$$

representing the discount value of additional future profits due to one additional launch, where the shadow marginal cost of the launch is equal to the discounted value of the average cost of one launch per new operating satellite in orbit. The shadow cost of the launch can be reduced by a decline in the unitary cost per launch or by increasing the number of satellites per launch.

The equilibrium condition for the optimal number of satellites is given by,

$$\left(\frac{1}{1+r}\right) A_{t+1} S_{t+1}^{\alpha-1} = \frac{c}{\eta}\left[1-\left(\frac{1}{1+r}\right)(1-\delta_s)\right] \tag{13}$$

where the equilibrium quantity of satellites in the non-debris scenario (denoted by the superscript nd) at any time is given by,

$$S^{nd} = \left(\frac{c(r+\delta_s)}{\eta A}\right)^{\frac{1}{\alpha-1}} \tag{14}$$

It results that $\partial S^{nd}/\partial c < 0$, $\partial S^{nd}/\partial A > 0$, and $\partial S^{nd}/\partial \eta > 0$. As the cost of launching a satellite declines, the optimal satellites in orbit increases. On the other hand, the stock of satellites depends positively on the number of satellites per launch. Notice that the ratio c/η represents the average launch cost per satellite. The cost of launching a satellite can decline because of a decline in the cost of a launch or due to an increase in the number of satellites per launch. The increase in launch systems' payload capacity and design changes that reduce the size and weight of satellites (micro-satellites) are presented by an increase in the parameter η.

Equivalently, the equilibrium number of launches, which is a proportion of the optimal stock of satellites, is given by,

$$L^{nd} = \frac{\delta_s}{\eta}\left(\frac{c(r+\delta_s)}{\eta A}\right)^{\frac{1}{\alpha-1}} \tag{15}$$

and hence the ratio S^{nd}/L^{nd} is governed by the parameters ratio η/δ_s. A reduction in launch costs will increase both the quantity of satellites and the number of launches proportionally, except if launch system technologies lead to a reduction in the number of satellites per launch or in the case of design changes to extend satellites' life-span.

2.3. Decentralized market with debris collision risk

The second stage of the human-use of outer space implies the existence of a significant quantity of debris generated from the previous stage, meaning that the risk of collision is positive. However, in this scenario we assume that firms do not take any action to mitigate the risk of collision and simply take the probability of destruction of a satellite as exogenously given. This myopic behavior of firms operating in space is justified by the nature of this market (a global commons) with no regulation and that a operator perceives its own launching behavior as a negligible factor of the risk of collision in a perfectly competitive market. In this scenario firms maximizes (1) subject to (4) and (5). The first-order conditions for optimality are,

$$\left(\frac{1}{1+r}\right)^{t+1} A_{t+1} S_{t+1}^{\alpha-1} = \lambda_t - \lambda_{t+1}(1-\delta_s) - \mu_{t+1}\theta D_{t+1} \tag{16}$$

$$\left(\frac{1}{1+r}\right)^{t} c = \lambda_t \eta \tag{17}$$

$$\lambda_t = -\mu_t \tag{18}$$

where the Lagrangian multiplier λ_t represents to the shadow marginal cost of launching a satellite, and the multiplier μ_t represent the shadow cost of a destroyed satellite (the cost of foregone space assets). The first-order condition (18) states that the shadow cost of launching a satellite is equal to the negative of the shadow cost of the loss of a satellite through a collision with debris. The equilibrium condition for the optimal number of satellites (denoted by a superscript d), at any time, is given by

$$S^{d} = \left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} \tag{19}$$

resulting in a (negative) function of the amount of debris. In this scenario as debris accumulates, a reduction in the number of satellites in orbit is observed, given the risk of collision and destruction. As long as the probability of collision is low enough, the negative impact of debris on satellite activity will also be very low. However, as the stock of debris increases, the probability of collision during the life of a satellite escalates, reducing the equilibrium quantity of satellites.[10] In computing the equilibrium, we rule out the possibility of the destruction of all satellites in the period, following Adilov et al. (2018). Therefore, we assume that $\theta D_t < 1$, in order to have a positive number of satellites in the equilibrium.

The equilibrium quantity of satellites destroyed by collisions can also be expressed as a function of the risk of collision as,

$$X^{d} = \theta D^d\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} \tag{20}$$

The relationship between the number of satellites destroyed by collisions and the quantity of debris can be positive or negative, depending on how debris affects the stock of satellites, given that

$$\frac{\partial X^d}{\partial D^d} = \theta\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} + \frac{c\theta^2 D^d}{(\alpha-1)\eta A}\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{2-\alpha}{\alpha-1}} \lesseqgtr 0 \tag{21}$$

whereas the first term is positive and the second is negative. The relationship between satellites destroyed and debris depends on the following condition:

$$1 \lesseqgtr \frac{c\theta D^d}{(1-\alpha)\eta A}\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{1-\alpha} \tag{22}$$

that is, for a low level of debris, the relationship between the debris and the number of satellites destroyed by collision is positive. However, once the debris reaches a threshold level, the relationship turns out to be negative, as the optimal number of operational satellites in orbit declines and not all destroyed satellites are replaced. On the other hand, the equilibrium number of launches is given by,

$$L^{d} = \left(\frac{\delta_s+\theta D^d}{\eta}\right)\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} \tag{23}$$

Taking the derivative of launches with respect to debris results,

$$\frac{\partial L^d}{\partial D^d} = \frac{\theta}{\eta}\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} + \left(\frac{(\delta_s+\theta D^d)c\theta}{(\alpha-1)\eta^2 A}\right)\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{2-\alpha}{\alpha-1}} \lesseqgtr 0 \tag{24}$$

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[10] Notice that in reality, the probability of collision does not only depend on the amount of debris but also on whether debris are tracked or not by ground surveillance systems, and on the collision avoidance capabilities of the satellite.

Table 1
Basic data of activity in outer space.

Successful launches	6200
Successful satellites launches in Earth orbit	13,100
Satellites in Earth's orbit	8410
Operating satellites	5600
Debris tracked by SSN	31,150
Incidents resulting in fragmentation	630
Debris > 10 cm	36,500
Debris between 1 cm and 10 cm	1,000,000
Debris between 1 mm and 1 cm	130,000,000

Source: ESA (10 May 2022)

Again, the first term is positive, whereas the second is negative. Operating, the sign of the relationship can be positive or negative depending on the amount of debris and on the probability of collision,

$$\theta \lesseqgtr \left(\frac{\delta_s + \theta D^d}{1-\alpha}\right)\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{1-\alpha} \tag{25}$$

As the amount of debris increases, more satellites are destroyed by collision, and initially, that leads to an increases in the number of launches necessary to replace destroyed satellites. However, as the debris continues to increase, the number of launches reduces, as the optimal amount of satellites declines, and hence fewer destroyed satellites need to be replaced. The exact form of that relationship depends on the value of the parameter θ.

Comparing the two scenarios, we find that $S^{nd} > S^d$; that is, the risk of collision reduces the number of satellites in orbit. The larger the amount of debris, the smaller the optimal quantity of satellites. However, the number of launches could be lower or higher to replace the satellites that are lost through collisions. The difference $L^d - L^{nd}$ is given by

$$L^d - L^{nd} = \left(\frac{\delta_s + \theta D^d}{\eta}\right)\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} - \frac{\delta_s}{\eta}\left(\frac{c(r+\delta_s)}{\eta A}\right)^{\frac{1}{\alpha-1}} < 0 \tag{26}$$

which is negative and implies that the number of launches decreases as the amount of debris increases. Therefore, firms must face two costs from the risk of collision: the loss of operating satellites, and the proportion of new launches required just to replace losses.

Finally, from the law of motion of debris we can obtain a simple expression for the maximum number of satellites before the Kessler syndrome occurs. The steady state quantity of debris, excluding self-propagation, as a function of the quantity of satellites, is given by:

$$D^d = \frac{\frac{\omega}{\eta}\delta_s S^d}{\delta_d - \left(\gamma\theta + \frac{\omega\theta}{\eta}\right) S^d} \tag{27}$$

For the above expression to be positive (a necessary condition for ruling out explosive trajectories in debris), the condition $\delta_d > \left(\gamma\theta + \frac{\omega\theta}{\eta}\right) S^d$ must hold. This condition can be interpreted as a condition for the Kessler syndrome, similar to the one developed by Adilov et al. (2018). Adilov et al. (2018) derived a physical and an economic Kessler conditions for the quantity of orbital debris that makes space physically unusable and economically unprofitable, respectively, and showed that "the space becomes economically unprofitable before it becomes physically unusable". Here, we present a physical Kessler condition in term of the maximum number of satellites, given by,

$$S^{Kessler} = \frac{\delta_d}{\theta(\gamma + \frac{\omega}{\eta})} \tag{28}$$

The question here is how this physical Kessler threshold compares to the optimal quantity of satellites that maximizes profits. In our model, the Kessler syndrome is only avoided if $S^d < S^{Kessler}$. Given the optimal number of satellites in the steady state, combining the economic optimality condition and the physical threshold, we find that,

$$\left(\frac{c(r+\delta_s+\theta D^d)}{\eta A}\right)^{\frac{1}{\alpha-1}} < \frac{\delta_d}{\gamma\theta + \frac{\omega\theta}{\eta}} \tag{29}$$

From that condition, we conclude that the economic condition for the quantity of satellites is below the Kessler threshold when the following condition for the amount of debris holds,[11]

$$D^d > \frac{\eta A\left(\frac{\delta_d}{\gamma\theta + \frac{\omega\theta}{\eta}}\right)^{\alpha-1} - c(r+\delta_s)}{c\theta} \tag{30}$$

3. Calibration of the model

To make the model operational to carry out simulations, we proceed first to calibrate the parameters. Two types of parameters are present: physical parameters related to space and spacecraft characteristics, and economic parameters related to the production and profit functions. Given the accelerated changes in the space industry, we calibrate the parameters of the model to the most recent data available representing the exploitation of outer space at present. Table 1 shows some key data about human activity in the Earth's orbits and the amount of debris, as estimated by the ESA (European Space Agency) in May 2022. From the beginning of space exploration, a total of around 6,200 successful launches have been realized. A launch can include more than one satellite or spacecraft. Indeed, the relationship between launches and new satellites in orbit is changing dramatically nowadays due to the use small and micro satellites, and to the higher power and payload capacity of launch systems. The number of satellites in Earth orbit is over 8410 of which 5600 are operational. The total number of pieces of debris tracked by the United States Space Surveillance Network (SSN) is 31,150. The number of registered incidents, including break-ups, explosions, collisions, or anomalous events, resulting in fragmentation is about 630. The biggest incident was the collision on February 10, 2009 of an active US communications satellite (Iridium 33), with a defunct Russian military communications satellite (Kosmos 2251). Both satellites were destroyed in the collision, producing a total of around 2,200 pieces of new tracked debris with a size of at least 5 cm (NASA, 2007). However, the most important incident was intentional (an anti-satellite military test), resulting in the destruction of the Fengyun-1C (a Chinese satellite) on January 1, 2011, by a kinetic weapon producing an estimated 3,037 pieces of new tracked debris. Most of the activity takes place at Low-Earth-Orbit (LEO, between 200 and 2000 km), and at Geostationary Orbit (GEO, at 35,786 km).

The standard classification of orbital debris is a function of its size and on the technical possibility of tracking it. Projections obtained using different models for debris (for example, the LEO-to-GEO Environment Debris Model, LEGEND) have estimated amounts of around 36,500 pieces of debris larger than 10 cm diameter, 1,000,000 objects between 1 cm and 10 cm, and over 130,000,000 fragments between 1 mm and 1 cm. The destruction power of debris smaller than 1 cm is estimated to be low and non-fatal in the case of a collision with a representative satellite, although they can cause serious damage in critical systems, reducing functionality and lifespan, and even disable small satellites. However, debris larger than 1 cm is potentially deadly due to the high velocity of the impact. Therefore, for the calibration of the parameters of the model we consider the estimated number

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[11] This condition is only true under the simplifying assumptions introduced in the model. Trackability of debris, differences in orbit regimes, collision avoidance capabilities, are all important components which would have a great impact on this condition.

of pieces of debris larger than 1 cm. The calibration of the physical parameters is as follows (see Table 2):

Satellite depreciation rate (δ_s). The extreme conditions in outer space and manufacturing costs determine satellites life-span in orbit. A satellite's life-span depends on the type of satellite, and on electrical, mechanical, physical and gravitational factors (Gallois, 1987). An important limitation for satellites life-span is the fuel capacity. Indeed, some derelict satellites could still be in good conditions operationally and could continue to provide services but have run out of fuel, and therefore, cannot be moved to the target orbit. The life-span varies from 6 months for CubSats (miniaturized satellites) to 15 years of GEO satellites. For LEO satellites the life-span varies from 3 to 8 years. As the model is an aggregated model for any orbit, we assume an average lifetime of 8 years, so the annual depreciation rate for satellites is fixed to 0.1733.

Debris decay rate (δ_s). In more than 60 years of space activities, about 6,200 launches have resulted in some 50,000 tracked objects in orbit, of which about 31,000 remain in space. This figures refers to objects in orbit that are regularly tracked by the US Space Surveillance Network and maintained in its catalogue, which covers objects larger than about 5–10 cm in the Low-Earth Orbit (LEO) and 30 cm to 1 m at geostationary (GEO) altitudes. Only a small fraction – about 5600 – are operational satellites. However, the total amount of debris is much larger. Estimations from the LEGEND model have reported about 1,000,000 pieces of debris larger than 1 cm. The decay rate of debris depends on several factors, including the altitude, mass, area, solar radio flux, and geomagnetic index. The most important factor is the altitude due to the atmospheric drag. The Australian Space Weather Agency (1999) estimated that the lifetime of space objects varies from 1 day at 200 km, 1 month at 300 km, 1 year at 400 km, 10 years at 500 km, 100 years at 700 km, and 1000 years at 900 km (King-Hele, 1987). On the other hand, the distribution of debris as a function of altitude is not homogeneous. The spatial density of debris shows that it is concentrated in the range 700–900 km (NASA, 2020). We use the average of this figure as a reference, and therefore, the average lifetime is estimated at around 150 years. Assuming straight-line depreciation, this results in an annual decay rate of 0.0067.[12] This is consistent with the value for the general atmospheric decay parameter of 0.0062 used by Lewis et al. (2009) in the Fast Debris Evolution (FADE) model.

Risk of collision (θ). In the history of activity in outer space, a number of collisions have been reported. Collisions can occur between pieces of debris themselves or between debris and operational satellites. A risk of collision between operating satellites also exists, but in some cases they can be avoided by maneuvering, although there is a large number of satellites with slow or not maneuver capability. Several collisions with human-made debris have been reported in recent years, but other incidents remain unknown. On February 10, 2009 an active US communications satellite (Iridium 33) collided with a non-operating Russian military communications satellite (Kosmos 2251). On January 22, 2013 a Russian small satellite (BLITS) was destroyed by a piece of debris from Fengyun-1C. On May 22, 2013, two CubeSats collided with debris (Ecuador's NEE-01 Pegaso and Argentina's CubeBug-1). A high number of probable collisions are avoided by maneuvering satellites and other spacecraft frequently. Krisko (2007) estimated an average number of catastrophic collisions (with a target and impactor larger than 10 cm) of 0.9, whereas the estimation from the DAMAGE model (Lewis et al., 2009) is 1.5, both for the period 1957–2006. As a result of these collisions, a number of pieces of debris have been generated. Farinella and Cordelli (1991) estimated a value of $\theta = 3 \times 10^{-10}$, for an estimated quantity of debris of 50,000.[13] This means a number of satellite destroyed per year of 0.2, given a probability of collision ($\theta \times 50,000$) of 1.5×10^{-5}. Kawamoto et al. (2019) estimated that the current probability of collision is much higher. The total probability of collision of objects larger than 10 cm is around 0.1 for 800–900 km orbits, 0.05 for 900–1,000 km orbits, and 0.025 for 600–700 km orbits. Following Farinella and Cordelli (1991) we take the estimation of a total probability of collision of 0.2 (i.e., one fatal collision every 5 years) as the reference, given the number of incidents observed during the last years (four collisions occurred in the period 1991 to 2009), resulting in the probability of collision is $\theta \times D = 6.6 \times 10^{-5}$. Pardini and Anselmo (2014) estimate that the risk of collision in LEO was multiplied by 4.5 between 1980 and 2010. To calibrate the risk of collision, we consider the population of pieces of debris larger than 1 cm, as they can cause deadly damage to a satellite.[14] Given a total number of potentially hazardous pieces of debris of 1,036,500, this results in a value for the risk of collision parameter of $\theta = 6.37 \times 10^{-11}$.

Number of pieces of debris per launch (ω). This is the primary source of debris generation. This parameter includes not only expended rockets and other parts discarded in the process of inserting a satellite into the target orbit but also debris generated by explosions of launch vehicles. We only consider debris larger than 1 cm. Debris smaller than 1 cm are assumed not to cause fatal damage in the case of collision. Assuming that 75% of debris is generated by launches (and the remaining 25% by other events, mainly anti-satellite military tests), this implies that launch systems are responsible for a total of 702,000 pieces of debris from 5990 launches during a period of 70 years. Dividing both figures, it results that $\omega = 117.2$. However, this figure underestimates the number of pieces of debris generated by one launch, as due to atmospheric drag, a significant number of pieces of debris already produced have decayed. Farinella and Cordelli (1991) estimated this parameter assuming an average of two unintentional explosions per year, each creating a few thousand fragments of mass greater than 1 gram, producing 70 new pieces of debris larger than 10 cm, resulting in a total number of new pieces of debris of 2059 larger than 1 cm.[15] Johnson et al. (2001) use the NASA Breakup model EVOLVE 4.0 to estimate a number of new fragments from an explosion of 238 larger than 10 cm, and 9,509 fragments larger than 1 cm. Lewis et al. (2009) estimated that the number of fragments larger than 10 cm generated by an explosion is 50, and that an average of 2.75 intact objects are added to the environment per launch. Therefore, we assume that around 150 pieces of debris larger than 1 cm generated by each launch.

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[12] As an example, the number of cataloged pieces of debris from the destruction on February 10, 2009, of Kosmos 2251, was 1347. On January 1, 2011, the cataloged pieces of debris from this satellite that remained in orbit amounted to 1273 pieces, that is, 94% of the initial debris, and this is a decay rate of around 6% in two years. Similarly, the cataloged debris from the destruction of Iridium 33 was 528 pieces, and the remaining debris in orbit was 492 pieces (a 93%) on January, 1, 2011, which represents a 7% in two years. The cataloged debris from the destruction of Fengyun-1C was estimated to be 3,037 pieces. On January 1, 2011, the cataloged debris in orbit from this satellite was calculated to be 2,932 pieces (97% of the initial quantity). Given that this satellite was destroyed in a military test on January 11, 2007, the decay rate is only 3% in four years. The difference is a consequence of the altitude of the orbit generating debris. Whereas the first incident took place at an altitude of 776 km, the second intentional incident occurred at an altitude of 860 km.

[13] Farinella and Cordelli (1991) estimate the number of collisions per unit cross section per year as the average collision velocity (10 km/s) divided by the volume of the circumterrestrial shell $(1800\ \text{km} \times 6 \times 10^{8}\ \text{km}^{2})$

[14] Krag et al. (2017) studied the loss of power of Sentinel-1 A in August 2016 resulting from the collision with a small piece of debris of around 1 cm.

[15] Only 3.54% of estimated pieces of debris are larger than 10 cm. The other 96.36% are between 1 cm and 10 cm. If an explosion produces 70 pieces of debris larger than 10 cm, the total number of pieces of debris larger than 1 cm is estimated to be 70/0.034=2059.

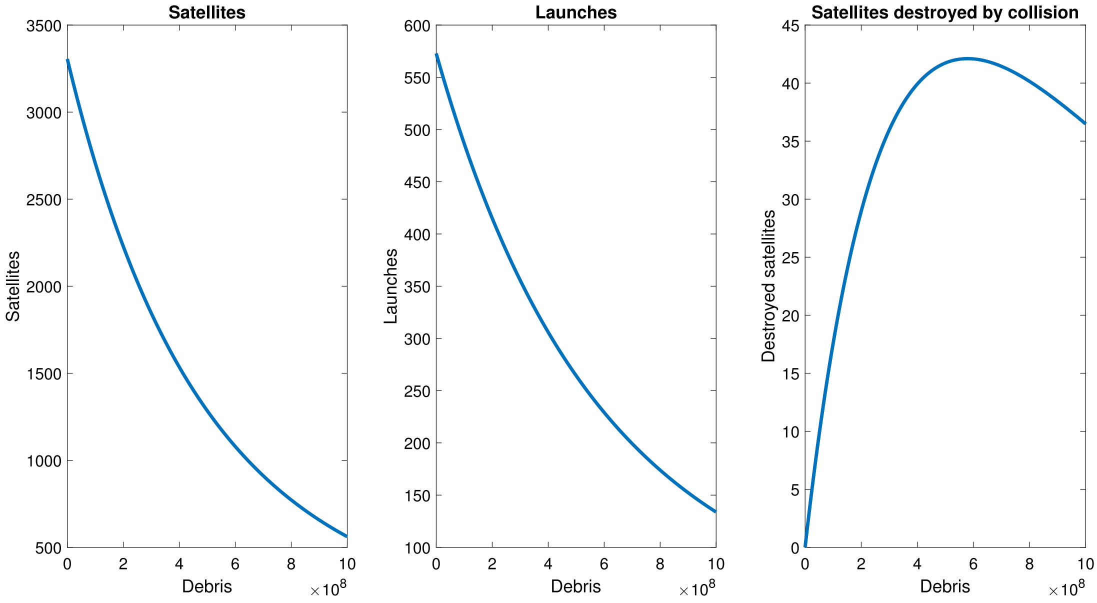

Fig. 1. Satellites, launches and satellites destroyed in steady state by collision as a function of debris.

Number of pieces of debris per collision (γ). New debris generated by collision varies greatly depending on the mass of the colliding objects. The most relevant episode was the collision on February 10, 2009, of a defunct Russian satellite (Kosmos 2251) and a US communication satellite (Iridium 33), producing an estimated total of around 2000 cataloged pieces of debris (around 3.6% of the total debris being larger than 1 cm). By contrast, the number of new pieces of debris from the collision of Sentinel-1 A is five (Joint Space Operations Center, JSpOC). Farinella and Cordelli (1991) estimated this figure from the typical mass distribution of fragments generated by hypervelocity impacts, resulting in about 10,000 fragments with a mass exceeding a few grams in the case that the largest fragment is about 10 kilograms in mass. Lewis et al. (2009) found that the number of pieces of debris per catastrophic collision is 625 (collision with debris larger than 10 cm) and 25 for a damaging collision (collision with debris between 1 and 10 cm). Given that the probability of collision is calibrated for debris larger than 1 cm, and that the estimated number of pieces of debris between 1 cm and 10 cm is 1,000,000 to an estimated number of pieces of debris larger than 10 cm of 36,500, using the amount of debris per collision estimated by Lewis et al. (2009), this implies that the number of pieces of debris larger than 1 cm resulting from a catastrophic collision is 17,748, and 710 in the case of a non-catastrophic collision. The NASA Standard Breakup model calculates the number of fragments created from the collision between two objects depending on the size and the mass, based on laboratory hypervelocity impact experiments. Johnson et al. (2001) show that results are similar from both models for a mass of the two objects of 1260 kg for a catastrophic collision. This mass is not so different from the average mass of a medium satellite (BryceTech, 2020). Assuming that the probability of collision is independent of the size of the debris, the final estimation is 1309 pieces of debris per collision. This figure is not so different from the number of pieces of debris resulting from accidental explosions estimated previously.

Derelict satellites abandoned in orbit (χ). Defunct satellites abandoned in orbit are another source of orbital debris. This occurs when satellites run out of fuel and cannot be moved to graveyard orbits. This was quite a common occurrence during the first stages of space conquest. Abandoned satellites pose considerable risk, given their mass. Indeed, one of the most harmful incident was the collision of Kosmos 2251 with Iridium 33 in February 2009. However, the number of abandoned satellites is small relative to that of other forms of debris. Additionally, new international standards for spacefaring countries and firms consider the necessity of including reserve fuel for de-orbiting maneuvers. Therefore, it is expected that the number of derelict satellites that are abandoned in orbit will tend to zero over time. Hence, we assume that this parameter is zero to simplify the simulations.

Fragments from debris collision (υ). Another source of debris generation is collision between pieces of debris. However, it is difficult to detect these collisions, except in the case of big objects such as defunct satellites or discarded rockets. Given that the number of large pieces of debris is low enough, and that in case of collision of small piece of debris additional fragments would be even smaller, we just assume that the number of fragments from debris collision is zero (no additional debris is produced) to simplify the model simulation. This source of new debris should be important in the case of the Kessler syndrome if the amount of debris reaches a threshold value with collisions in cascade.

For the economic parameters, we use standard values from the literature. The interest rate is fixed to 4% per year ($r = 0.04$), whereas the productivity parameter is normalized to one, $A = 1$. The number of satellites per launch is also normalized to one, $\eta = 1$, as we interpret the number of launches as being equivalent to the number of new satellites inserted into orbit. The technological parameter for the satellite services production function, α, is fixed to 0.85. Finally, we calibrate the launch cost internally, to match the observed values for the number of satellites using expression (17), resulting in $c = 1.2842$, for a quantity of satellites of 5600 and an amount of debris of 1,036,500 pieces.

4. Results: Quantitative simulations

Given the benchmark calibration of the model, we can obtain the implicit exact relationship between debris, launches and destroyed satellites, as given by expressions (18) and (21). Fig. 1 plots the estimated relationship between satellites, launches, and destroyed satellites for an exogenous range of debris (from zero to 10×10^8 pieces of debris larger than 1 cm). The calibrated model produces a negative relationship between debris and launches, whereas the relationship of debris with destroyed satellites is non-monotonic, positive for a low level of debris and negative for a higher level of debris. As the debris increases, the optimal number of satellites declines. Simultaneously, the number of destroyed satellites increases initially. However, destroyed satellites

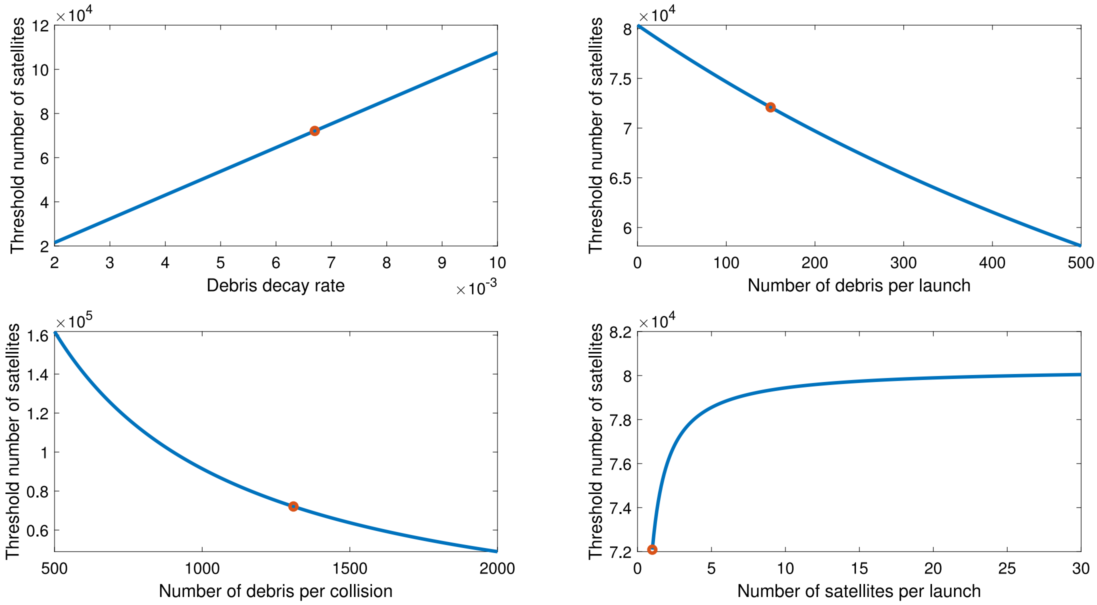

Fig. 2. Maximum number of satellites sensitivity analysis (debris decay rate, number of debris per launch, debris generated per collision and debris generated per launch). Circle represents benchmark calibration.

are not fully replaced because of the larger amount of debris, leading to a reduction in the number of launches. Second, given the lower number of launches and satellites, the number of destroyed satellites reaches a maximum and a further increase in the amount of debris, reduces the number of satellites destroyed by collisions.

Next, we use the calibrated model to calculate threshold values for the quantity of satellites and debris given the present human activity in outer space. First, we calculate the steady state maximum quantity of satellites in orbit before the Kessler syndrome arises, given by expression (28). This results in a maximum number of 72,090 satellites. This figure is well above the current number of operational satellites of 5600, but it is expected that the numbers will increase faster in the next years, mainly due to satellite constellations. Adilov et al. (2018) defined the physical Kessler condition in terms of the amount of debris, as the amount for which the probability of collision is one, i.e., $D^{Kessler} = 1/\theta$. Given the calibration of the parameter θ, the threshold amount of debris (larger than 1 cm) would be 1.57×10^{10} (compared with the current estimated value of 1,036,500 pieces of debris). However, the model produces an optimal quantity of satellites below one for an amount of debris of around 7.69×10^{9} (two times lower than the physical threshold), confirming Adilov et al. (2018) result indicating that the economic Kessler syndrome will occur before the physical Kessler syndrome. The steady state maximum number of satellites is a combination of five parameters: the debris decay rate, the number of pieces of debris per launch, the number of pieces of debris per collision, the number of satellites per launch, and the parameter representing the risk of collision. It is worth noting that a number of relevant factors, such as Space Situational Awareness and improving collision avoidance capabilities, are not considered in our simulations and would have a significant impact in the determination of the maximum number of satellites.

We perform a sensitivity analysis for a range of values for the first four parameters to investigate how the estimated steady state maximum number of satellites in orbit changes with respect to the benchmark calibration (represented by a circle). The main results are plotted in Fig. 2. The maximum number of satellites increases as the debris decay rate rises. For a range of the debris decay rate between 0.002 and 0.1, the range of satellites varies from 21,520 to 107,600. These figures demonstrate that the implementation of active debris removal policies could be a useful instrument to enhance the number of satellites that can be operated in the Earth's orbit. We repeat the same calculus for a range of values for the number of pieces of debris per launch from zero (representing a debris-free launch system) to 500. Although launches are the primary source of debris generation, the range for the maximum number of satellites changes slightly. For a value of 500 pieces of debris per launch, the maximum number of satellites is 58,143, whereas for a scenario with no debris per launch, the maximum number of satellites is 80,352, a figure that is only slightly above the maximum number of satellites in the baseline scenario. A larger impact on the maximum number of satellites is observed when the number of pieces of debris per collision parameter is altered. We calculate the maximum number of satellites for a range of 500 to 2000 pieces of debris per collision, resulting in a range of 48,921 to 161,820 satellites. Finally, we study the sensibility of the maximum number of satellites for a range from 1 to 30 satellites per launch. The maximum number of satellites increases up to 80,046, with little gains for further increases in the number of satellites per launch. In sum, this sensitivity analysis shows that the maximum capacity of the Earth's orbit, in aggregate, is well below 100,000 satellites for a plausible range of values of the parameters, a figure higher than the number of satellites at present, but not large enough given future projections about the number of satellites expected to be launched, especially big satellite constellations. However, given the differences among orbit regimes and satellites, this figure should be considered as a first approximation to the economic capacity of space in terms of the number of a representative satellite.

Using the calibrated model, some simulations of interest can be carried out. First, we are interesting in simulating the model for an exogenous path of launches to determine how the increasing activity by inserting more satellites into orbit affects the dynamics of debris and the risk of collision. Second, we compute the steady state quantity of satellites under four alternative scenarios. Third, we simulate the model for a decreasing exogenous path of launching costs. Finally, we study the implications of increasing the number of satellites per launch.

4.1. Exogenous path of launches

First, we simulate the model for an exogenous path of launches to describe the physical properties of the model. In this scenario, there is no economic decision about the optimal number of launches; hence, we simply look at the law of motions for the stock of satellites and debris,

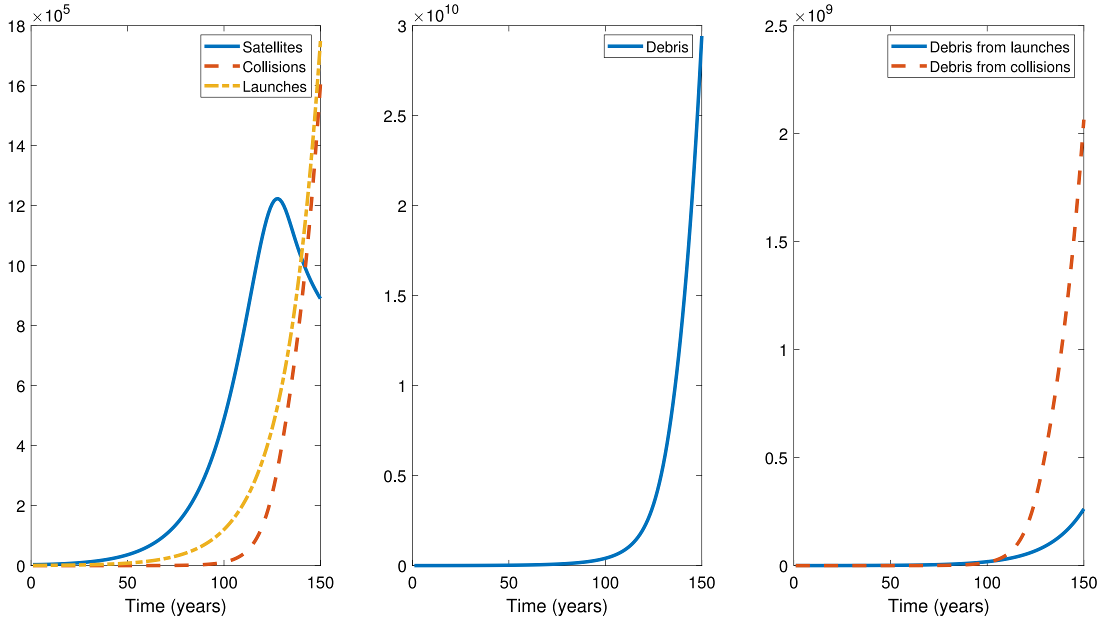

Fig. 3. Satellites, launches, satellites destroyed and debris given an exogenous launch path.

given an exogenous path of launches. The initial conditions represent space activity at present, where $S_0 = 5600$, $D_0 = 1,036,500$, $X_0 = 0.2$, $L_0 = 100$, and an average number of satellites per launch of five. It is assumed that the number of launches (or more specifically the number of launched satellites) increases at a rate of 5% per year.

Fig. 3 plots the dynamics of satellites, debris and satellite destruction depending on the exogenous path of launches. The results are similar to those obtained by Farinella and Cordelli (1991) using a two first-order differential equations model for satellites and debris. We find that the number of satellites increases initially, reaching a maximum, and then it starts to decline. This is a consequence of the increasing amount of debris, which grows exponentially, increasing the number of satellites destroyed by collision. Indeed, the number of satellites destroyed by collisions grows faster than the number of launches up to a point at which the number of satellites destroyed reaches the number of launches, where the model collapses.

The right-side plot illustrates how the different sources of new debris changes over time. As in the case of Farinella and Cordelli (1991), the main source of debris generation changes from primary debris from launches, to the secondary debris generation source from collisions. This result is obtained even if we assume no additional debris resulting from collisions between debris, consistent with the scenario advanced by Kessler and Cour-Palais (1978).

4.2. Steady state analysis

Next, we calculate steady state equilibrium values for the variables of the model for the two scenarios: no-debris, representing the initial stages of space exploration, and debris, representing the current situation. Of course, the steady state only exists by excluding a "Kessler syndrome" scenario of cascade collisions as this would imply that debris is always increasing, and therefore, a steady state only exists if $\theta D < 1$. Table 3 shows the steady state values for the key variables of the model in each scenario. Notice that in steady state, and assuming that exogenous shocks are zero (no change in the launch cost and satellite depreciation rate), the number of satellites is a constant, depending on the probability of collision. As expected, the equilibrium relationship between satellites and debris is negative, resulting in a lower quantity of satellites in the debris scenario than in the no-debris scenario. Indeed, the steady state quantity of satellites is 4.49% lower in the debris scenario than in an environment with no orbital debris. This is a considerable difference, as the risk of collision is low (the resulting probability of collision is 0.0014) for the estimated steady state value for debris of 2.25×10^7, an amount that is 21 times higher than the present estimated amount of debris. Notwithstanding the low risk of collision, the estimated number of destroyed satellites is an average of 7.97.

Table 2
Calibration of the parameters of the model.

	Parameter	Definition	Value
Physical	δ_s	Satellite depreciation rate	0.1733
	δ_d	Debris decay rate	0.0067
	θ	Risk of collision	6.37×10^{-11}
	ω	Number of pieces of debris per launch	150
	γ	Number of pieces of debris per collision	1,309
	χ	Fraction of derelict satellites	0.00
	υ	Fragments from debris collision	0.00
Economic	c	Launch cost	1.2842
	A	Productivity parameter	1.00
	η	Satellites per launch	1.00
	α	Elasticity of satellite services	0.85
	r	Interest rate	0.04

Table 3
Steady state values.

Variable	No-debris	Debris	% change
Satellites	5612	5358	−4.49
Launches	972	936	−3.70
Debris	–	2.25×10^7	–
Destroyed satellites	–	7.97	–
Satellite services	1537	1478	−3.84

The cost of the externality is calculated as forgone satellite services. Given the particular characteristic of this negative externality, debris leads to an underinvestment state, reducing the number of launches with respect to the non-debris environment. Debris increases the probability of destruction of capital assets in orbit, which reduces production and investment (new satellites launched). Compared with an environment with no debris, the social cost of debris is about 3.84% of the market, which represent a significant fraction, in spite of the low risk of collision at present. Weinzierl (2018) showed that commercial

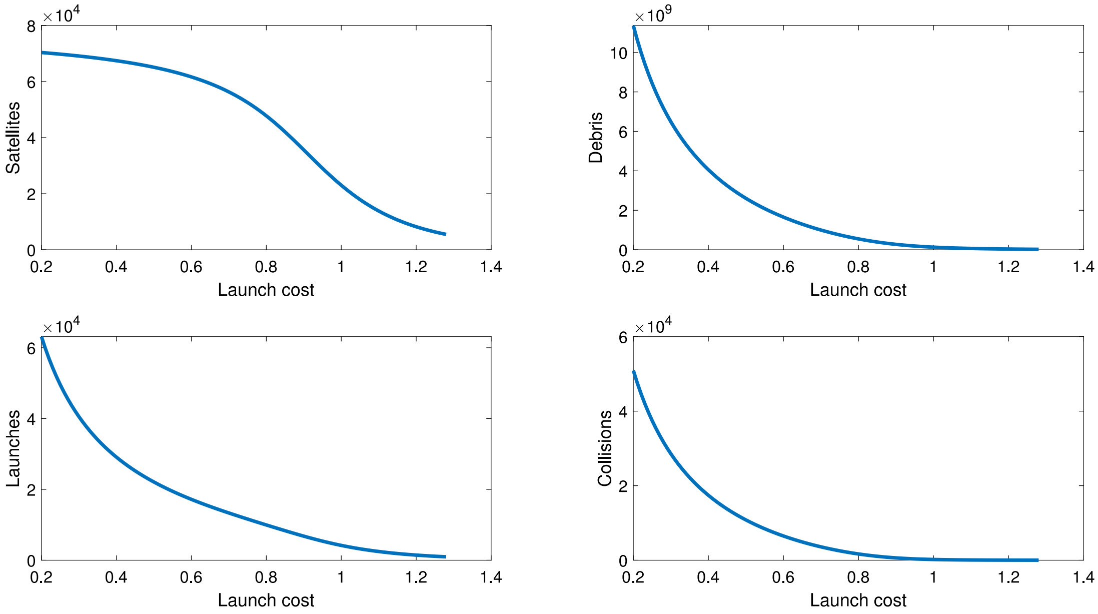

Fig. 4. Steady state values as a function of the launch cost.

activities in space generate around $300 billion in annual revenues. Thus, the social cost of debris is estimated at around $11.5 billions annually.

4.3. Launch cost reduction

An important variable driving commercial activity in outer space is the launch cost. Indeed, the enormous cost of first-generations launch systems was a formidable economic barrier to the earlier exploration of outer space, an activity limited to countries with large economic and technological resources. However, as in other industries, the costs in the space industry have been declining over time due to learning and technological progress. Nowadays, private spacefaring firms are expanding their business, introducing new technologies and new launch systems designs to reduce the costs as much as possible. Therefore, it would be of interest to study how a cost decline will expand the quantity of satellites, debris, and the probability of collision. In the benchmark calibration of the model, the launch cost parameter was calibrated internally to match the observed number of satellites in orbit of 5600, resulting in a value of 1.2842. For this sensitivity analysis, we solve the model for the optimal quantity of launches for a range of values of the launch cost from 0.2 to the calibrated value of 1.2842 (a range for a reduction in costs of approximately 80%). For that, we solve the system of equations given by (17), (18), (21) and (26) for the range of $c = [0.2 : 1.2842]$.

Fig. 4 plots the steady state values of the key variables as a function of the launch cost for the debris scenario. As expected, the lower the launch cost, the higher the number of satellites and launches. However, a higher number of launches also increases the quantity of debris, and hence, the number of satellites destroyed. We find that for a low enough launch cost, the number of launches is even higher than the stock of satellites in orbit, given the high number of satellites destroyed in collision with debris. This is because $L_c < 0$, $L_{cc} > 0$, but $S_c < 0$, $S_{cc} < 0$, as a consequence of the increase in the risk of collision as more launches occurs. This scenario will have dramatic consequences if the removal and generation of debris do not change. With a launch cost that is 80% lower than the benchmark calibration for the current situation, the number of satellites will be around 69,957, a figure close to the threshold, with a slightly lower number of satellites launched every period (54,170 per year). The reason is that the amount of debris reaches a value of 9.2×10^9, resulting in 40,860 satellites being destroyed each period. The Kessler syndrome appears (even without taking cascade collisions between pieces of debris into account) when the launch cost is zero, as all satellites are destroyed in the period and even a fraction of the newly launched satellites is destroyed. This simulation clearly illustrates that a further reduction in launch costs without a mitigation policy for orbital debris would lead to a long-run unsustainable environment in outer space.

4.4. Multiple-satellite launch systems

Finally, we investigate the implications of multiple-satellite launch systems. Satellite design and technology changes have progressively reduced the size and weight of satellites. Indeed, a new strategy pursued by spacefaring firms consists of the launching of a constellation of small satellites in a low orbit. On the other hand, launch systems are more powerful and are able to insert a heavier payload into higher orbits. These two factors, miniaturization and launchers capacity, increase the number of satellites per launch. This reduces the final cost of launching a satellite, as with the same rocket and launch costs, more than one satellite can be inserted into orbit at the same time. The number of satellites per launch is represented by the parameter η. In the benchmark calibration of the model we assumed that the number of satellites per launch was one. However, an increasing number of launches includes more than one satellite, it having become normal for a typical launch to include two or three standard satellites. The number of satellites per launch is substantially higher in the case of micro-satellites. For example, during 2020, several SpaceX Falcon 9 were launched with a payload of 60 Starlink satellites. Here, we simulate the model for a range of values of the parameter η from one to 10.

The results from this simulation exercise are plotted in Fig. 5. The optimal number of satellites increases as the number of satellites per launch increases, as this is equivalent to a decline in the per satellite launch cost (for a given value of the parameter c, representing the total launch cost). However, the relationship between the optimal number of satellites and the number of satellites per launch is a convex function, and, for a higher enough number of satellites per launch the effects of further increments in the number of satellites per launch is negligible. Increasing the number of satellites per launch increases not only the number of satellites in orbit, but also the number of launches. Indeed,

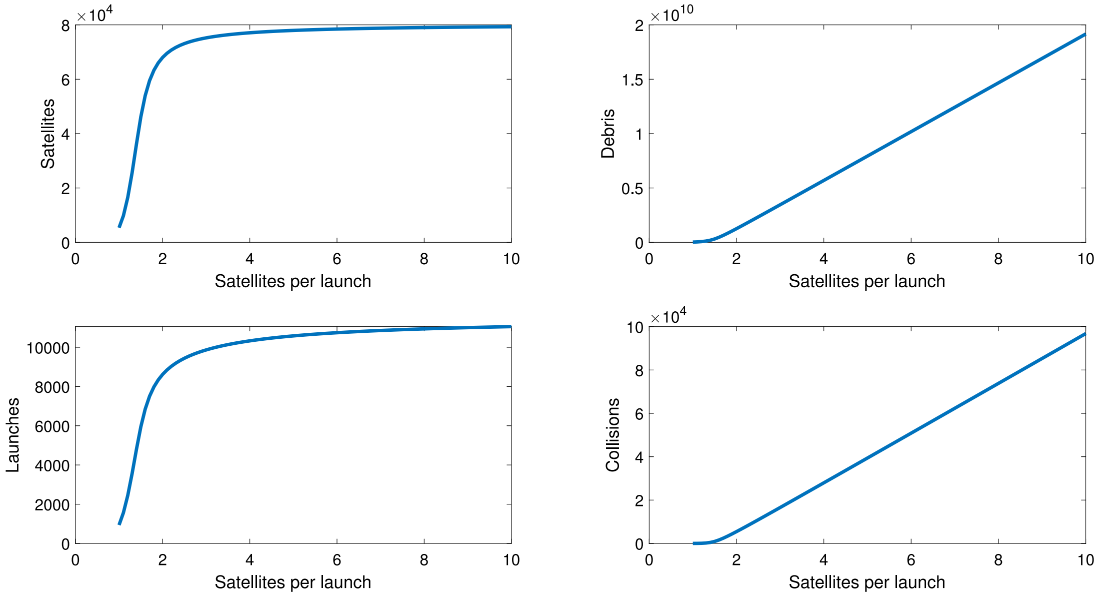

Fig. 5. Steady state as a function of the number of satellites per launch.

the number of launches shows a similar path to that of satellites. For an average of 10 satellites per launch, the steady state number of satellites is 79,314, approaching the threshold value. The number of launches is of 11,054, with a total of 96,794 satellites destroyed each period, as the amount of debris reaches a value of 1.9×10^{10}. This means that all satellites plus a fraction of the new launches are destroyed each period. The optimal number of satellites in the steady state reaches a maximum close to the threshold, even if the number of satellites per launch increases further. A similar maximum, about 10,000 launches (each with 10 satellites) is observed. The explanation is that the lower launch cost is compensated for by the higher number of destroyed satellites. Indeed, as both the number of launches and the number of satellites increases, so does the number of collisions (satellites destroyed). Finally, given the higher number of launches and the larger number of collisions, the debris increases steadily.

5. Concluding remarks

This paper presents a dynamic investment model for satellites in which outer space is a polluted environment presenting a risk of collision and destruction of satellites. The model considers an unregulated market in which orbits are a common source and human activity in outer space produces a negative externality in the form of space debris. Orbital debris is potentially hazardous and can destroy operating satellites in the case of collision. Debris is generated by launches, accidental explosions of rockets, anti-satellite military tests, collisions among debris, and collisions with satellites, among other sources. One of the particular characteristics of the negative externality of debris compared to other forms of pollution is that it affects to the stock of capital assets directly. This leads to an underinvestment situation with respect to an optimal environment with no debris.

We use the calibrated model to evaluate alternative scenarios. First, we compare the current scenario with debris with an ideal benchmark scenario with no debris. As expected, this negative externality reduces the activity in the market, decreasing the number of satellites, when the expected loss from a satellite destruction is taken into account by firms. The larger the quantity of debris, the smaller the optimal quantity of satellites. The model produces a simple expression for the maximum quantity of satellites to prevent the Kessler syndrome, depending on a combination of parameters. Given the baseline calibration of the model, the estimated aggregate threshold value for the quantity of satellites is about 72,000, a figure that is much larger than the current population of satellites but not so high as to prevent it from been reached in the near future given the expected launches of several satellite constellations containing thousands of satellites each. A sensitivity analysis is carried out, resulting in little variability of that threshold value. The consequences of launch cost declines and the increase in the number of satellites per launch are also investigated.

The results show that without debris mitigation both passive and active policies, outer space will collapse in the near future, with the destruction of a large number of satellites through collisions with debris. Even without a further reduction in launch costs, the amount of debris will increase before stabilizing, reducing the optimal number of operating satellites with respect to the present observed figure, and causing the destruction of about three satellites per year. As highlighted in the analysis done in this paper, the economic Kessler threshold for the number of satellites is lower than the physical Kessler threshold, so we would expect that space operators are interesting in the implementation of debris mitigation strategies well before the cascade effect. Further analyses in this direction are needed as debris mitigation policies, both active and passive, must be designed and implemented to expand the maximum limit for the number of satellites, reduces the number of satellites destroyed by collisions, and mitigate the production of new debris. Finally, it is worth noting that our model is an aggregate approximation of the orbital market. The results presented here are sensitive to the altitude of the satellites. The lower the altitude, the fewer the negative consequences of debris, as atmospheric drag is a drain of orbital pollution. The altitude of new satellites, especially those in large satellite constellations, will be the key to predicting the economic implications of human activity in outer space, and hence, a more disaggregated model distinguishing between LEO and GEO worth be developed.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

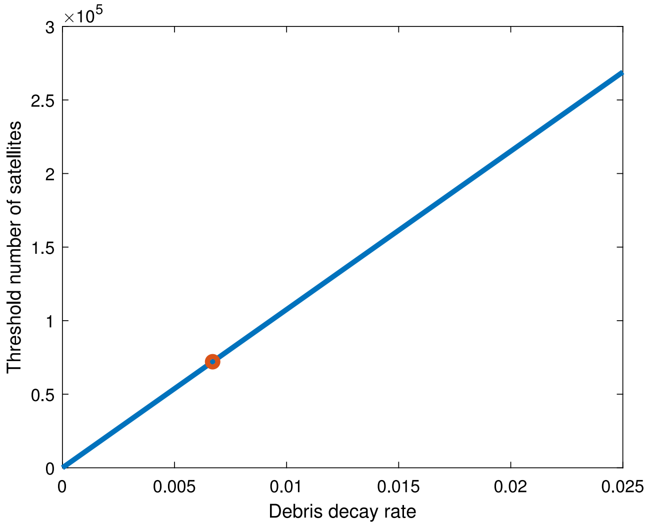

Fig. A.1. Threshold number of satellites as a function of the debris decay rate. The circle denotes the baseline value.

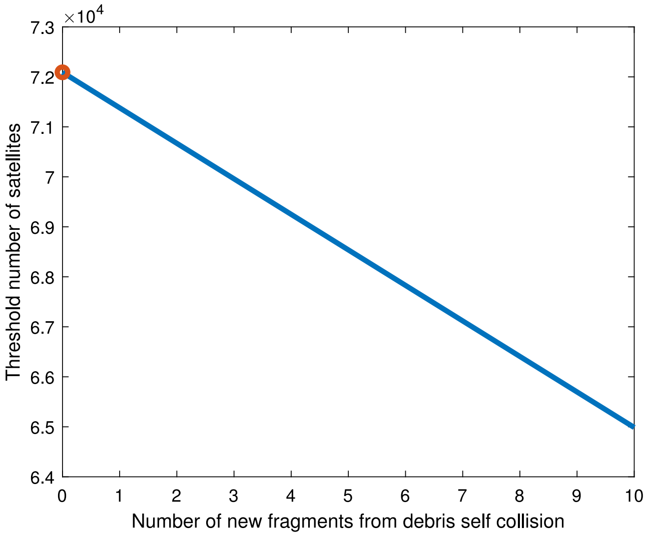

Fig. A.2. Threshold number of satellites as a function of new fragments from debris-to-debris collisions. The circle denotes the baseline value.

Appendix

This appendix extends the analysis done in the paper in two directions. First, we consider alternative values for the debris decay rate. The debris decay rate is a key parameter to calculate the threshold for the quantity of satellites to prevent the economical Kessler syndrome. The decay rate of orbital debris depends on the altitude, where atmospheric drag is high for low altitude but declines for higher altitude orbits and on other factors (mass, area, and solar radio flux). In the baseline calibration, the natural debris decay rate takes a value of 0.0067. This is an average value for all Earth's orbits, and it depends on the distribution of debris across orbits, with most debris concentrated at 700–900 km.

Fig. A.1 plots the relationship between the maximum number of satellites and the debris decay rate for a range of values between 0 and 0.025. For a debris decay rate of zero (all debris remains forever in orbit), the steady-state maximum number of satellites is zero. In this case, the amount of debris is always increasing, rendering the space unusable in the long-run. As the debris decay rate becomes positive, also the steady-state maximum number of satellites becomes positive. For a debris decay rate of 0.025, the steady-state maximum number of satellites is $S^{Kessler} = 269{,}000$). Therefore, increasing the debris decay rate allows expanding the usage of the space with more satellites. This result indicates that changes in the design of satellites and in launch systems to minimize the creation of debris together with the implementation of active debris removal policies are important instruments to prevent the Kessler syndrome while allowing to increase the number of satellites in orbit.

Second, we extend the law of motion of debris by including the possibility of debris-to-debris collisions as an additional source of debris creation. In this case, the debris' law of motion is given by,

$$D_{t+1} = (1-\delta_d)D_t + \gamma X_t + \omega L_t + \upsilon D_t^2$$

where $\upsilon > 0$. Given that $X_t = \theta D_t S_t$ and that $L_t = (\delta_s + \theta D_t)S_t/\eta$, the above expression can be written as,

$$D_{t+1} = (1-\delta_d)D_t + \gamma\theta D_t S_t + \frac{\omega(\delta_s + \theta D_t)}{\eta} S_t + \upsilon D_t^2$$

In steady state, the above expression we have a second-order equation for debris as a function of the quantity of satellites,

$$\delta_d D = \gamma\theta D S + \frac{\omega\delta_s}{\eta} S + \frac{\omega\theta}{\eta} D S + \upsilon D^2$$

Operating, we get,

$$D\left[\delta_d - \left(\gamma\theta + \frac{\omega\theta}{\eta}\right)S - \upsilon D\right] = \frac{\omega\delta_s}{\eta} S$$

where the term in brackets in the left side must be positive. From that condition, we obtain the following threshold value for the maximum number of satellites now as a function of the stock of debris,

$$S^{Kessler} = \frac{\delta_d - \upsilon D}{\gamma\theta + \frac{\omega\theta}{\eta}}$$

Fig. A.2 plots the maximum number of satellites for the current stock of debris ($D = 1{,}036{,}500$) as a function of the number of new fragments created by debris self collisions, υ. We use a range of values for the parameter υ from zero (baseline case) to 10. The threshold number of satellites is a negative function of the number of new fragments created by debris-to-debris collisions.

References

Adilov, N., Alexander, P.J., Cunningham, B.M., 2015. An economic analysis of earth orbit pollution. Environ. Resour. Econ. 60 (1), 81–98.

Adilov, N., Alexander, P.J., Cunningham, B.M., 2018. An economic Kessler Syndrome: A dynamic model of earth orbit debris. Econom. Lett. 166 (1), 79–82.

Adilov, N., Alexander, P.J., Cunningham, B.M., 2020. The economics of orbital debris generation, accumulation, mitigation, and remediation. J. Space Saf. Eng. 7, 447–450.

Australian Space Weather Agency, 1999. Satellite Orbital Decay Calculations. Sidney, Australia.

Béal, S., Deschamps, M., Moulin, H., 2020. Taxing congestion of the space commons. Acta Astronaut. 177, 313–319.

Bernhard, P., Deschamps, M., Zaccour, G., 2023. Large satellite constellations and space debris: Exploratory analysis of strategic management of the space commons. European J. Oper. Res. 304, 1140–1157.

BryceTech, 2020. Smallsats by the numbers 2020. In: Bryce Space and Technology.

BryceTech, 2022. Start-up space. Update on Investment in Commercial Space Ventures 2022. BryceTech, LLC.

Farinella, P., Cordelli, A., 1991. The proliferation of orbiting fragments: a simple mathematical model. Sci. Glob. Secur. 2, 365–378.

Gallois, P., 1987. Life expectancy of communication satellites. Electron. Power 33 (9), 547–550.

Grzelka, Z., Wagner, J., 2019. Managing satellite debris in low-earth orbit: Incentivizing ex ante satellite quality and ex post take-back programs. Environ. Resour. Econ. 74 (1), 319–336.

Guyot, J., Rouillon, S., 2022. Sustainable management of space activity in low Earth orbit. Manuscript.

Hardin, G., 1968. The tragedy of the commons. Science 162, 1243–1248.

Johnson, N.L., Krisko, P.H., Liou, J.C., Anz-Meador, P.D., 2001. NASA's new breakup model of EVOLVE 4.0. Adv. Space Res. 28 (9), 1377–1384.

Kawamoto, S., Nagaoka, N., Hanada, T., 2019. Impact on collision probability by post mission disposal and active debris removal. In: First International Orbital Debris Conference.

Kessler, D.J., Cour-Palais, B.G., 1978. Collision frequency of artificial satellites: The creation of a debris belt. J. Geophys. Res. 83 (6), 2637–2646.

King-Hele, D., 1987. Satellite orbits in an atmosphere. In: Theory and Applications. Glasgow, Blackie.

Klima, R., Bloembergen, D., Savani, R., Tuyis, K., Hennes, D., Izzo, D., 2016. Space debris removal: A game theoretic analysis. Games 7 (3), 20.

Krag, H., Serrano, M., Braun, V., Kuchynka, P., Catania, M., Simiski, J., Schimmerohn, M., Marc, X., Kujiper, D., Shurmer, I., O'Connell, A., Otten, M., Muñoz, I., Morales, J., Wermuth, M., McKissock, D., 2017. A 1 cm space debris impact onto the Sentinel-1A solar array. Acta Astronaut. 137, 434–443.

Krisko, P., 2007. The predicted growth of the low-Earth orbit space debris environment - An assessment of future risk for spacecraft. J. Aerosp. Eng. 221 (6), 975–985.

Letizia, F., Colombo, C., Lewis, H.G., Krag, H., 2017. Extending the ecog space debris index with fragmentation risk estimation. In: Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 2017. published by the ESA Space Debris Office.

Lewis, H.G., Swinerd, G.G., Newland, R.J., Saunders, A., 2009. The fast debris evolution model. Adv. Space Res. 44, 568–578.

Liou, J.C., Johnson, N.L., 2006. Risks in space from orbiting debris. Science 311, 340–341.

Macauley, M.K., 2015. The economics of space debris: Estimating the costs and benefits of debris mitigation. Acta Astronaut. 115, 160–164.

NASA, 2007. Detection of debris from Chinese ASAT test increases; One minor fragmentation event in second quarter of 2007. Orbital Debris Q. Rev. 11 (3), 1–2.

NASA, 2020. The tracked objects in Low Earth Orbit: 2000–2020. Orbital Debris Q. Rev. 24 (4), 11.

OECD, 2020. Space sustainability: The economics of space debris in perspective. In: OECD Science, Technology and Industry Policy Papers. p. 87.

O'Neill, G.K., 1977. The High Frontier: Human Colonies in Space. Willian Morrow and Company, New York.

Ostrom, E., 2010. Beyond markets and states, polycentric governance of complex economic systems. Am. Econ. Rev. 100 (1), 1–33.

Pardini, C., Anselmo, L., 2014. Review of past on-orbit collisions among cataloged objects and examination of the catastrophic fragmentation concept. Acta Astronaut. 100, 30–39.

Patel, P., 2019. 4 Products to manufacture in orbit. In: IEEE Spectrum 2019.

Peeters, W., 2010. From suborbital space tourism to commercial personal spaceflight. Acta Astronaut. 66, 1625–1632.

Prater, T., Werkheiser, N., Ledbetter, F., 2018. Toward a multimaterial fabrication laboratory: In-space manufacturing as an enabling technology for long-endurance human space flight. J. Br. Interplanet. Soc. 71, 27–35.

Rao, A., Burgess, M.G., Kaffine, D., 2020. Orbital-use fees could more than quadruple the value of the space industry. Proc. Natl. Acad. Sci. USA 117 (23), 12756–12762.

Ross, S.D., 2002. Near-Earth Asteroid Mining. California Institute of Technology, Unpublished manuscript.

Rouillon, S., 2020. A phsyco-economic model of low Earth orbit management. Environ. Resour. Econ. 77 (4), 695–723.

Salop, S.C., 1979. Monopolistic competition with outside goods. Bell J. Econ. 10 (1), 141–156.

Sandler, T., Shulze, W., 1981. The economics of outer space. Nat. Resour. J. 21 (2), 371–393.

Snow, M.S., 1975. Investment cost minimization for communications satellite capacity: Refinement and application of the Chenery-Manne-Srinivasan Model. Bell J. Econ. 6 (2), 621–643.

Weinzierl, M., 2018. Space, the final economic frontier. J. Econ. Perspect. 32, 173–192.

Weinzierl, M., Acocella, A., 2016. Blue origin, NASA, and new space. In: Harvard Business School Case n. 012-716. pp. 1–34.

Weinzierl, M., Acocella, A., Yamazaki, M., 2016. Astroscale, space debris, and earth's orbital commons. In: Harvard Business School Case n. 037-716. pp. 1–24.