2nd Spanish Fluid Mechanics Conference
Barcelona (2 - 5 July 2023)

Cavity flow induced by a flexible membrane in an oscillatory channel flow: case study of syringomyelia

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In this work, we study the flow inside a Syringomyelia, modelled by a cavity separated from a channel with an elastic membrane. The oscillatory flow of the channel is transmitted to the cavity through the deformation of the membrane. We study the structure of the time-dependent and stationary flow induced inside the cavity and the deformation of the membrane under different problem conditions.

1 Introduction

Syringomyelia is a condition in which fluid accumulates in the spinal cord in or near the central canal, forming macroscopic fluid-filled cavities called syrinxes (Elliott et al., 2013). Enlarging syrinxes often cause progressive neurological damage through a combination of direct pressure on neural tissue and ischemia. Although the exact mechanisms behind the generation of syringomyelia are still unknown, changes in hydrodynamics inside syrinxes alter the distribution of forces acting on the spinal cord and hence the stresses that can favor syrinxes enlarging.

Since the spinal cord is an elastic material, it undergoes fluid-structure interactions between the oscillatory motion of the cerebrospinal fluid and the fluid inside the developing syrinx (Linninger et al., 2016). However, the two very disparate time scales hinder the search for how these interactions can cause the development of syringomyelia in the problem: the physical processes associated with the oscillatory flow, with a period on the order of seconds, generate small changes that accumulate over thousands or millions of cycles, to cause macroscopic changes in the development of a syrinx.

In this work we study the hydrodynamics inside the syrinx and the stresses acting on its walls through a simplified model of a cavity separated from an oscillating channel flow by a flexible membrane.

2 Framework

In particular, we study the flow inside a rectangular cavity of length L, separated from a channel by an elastic membrane of thickness h_m (see Fig. 1). We consider the same height H for both the channel and the cavity. The flow inside the channel is oscillatory, with imposed Womersley velocity profiles following a sinusoidal oscillation of amplitude U and angular frequency ω at the inlet and outlet surfaces. The properties of the fluid are defined through the density ρ_f and the cinematic viscosity ν, while those of the membrane are defined by its density ρ_m, Young's modulus E, and Poisson ratio ν_m.

The problem is controlled by the following dimensionless parameters: the aspect ratio of the cavity $AR = L/H$, the Womersley number $\alpha = H\sqrt{\omega/\nu}$, the dimensionless stroke length $\varepsilon = U/\omega L$, the dimensionless membrane thickness $h^* = h_m/H$, the density ratio $\rho^* = \rho_m/\rho_f$, the Poisson ratio ν_m and the reduced velocity $U^* = U/(h_m f_n)$, where f_n is the natural frequency of the membrane.

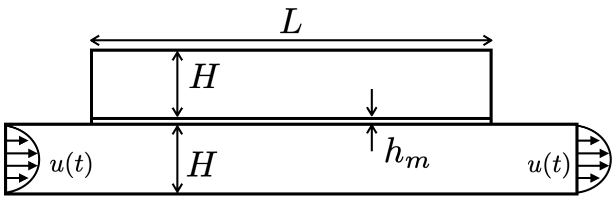

Figure 1: Scheme of the problem. Inlet and outlet velocities $u(t)$ are imposed as a Womersley profile oscillating with an angular frequency ω.

A Finite Element Method is used to solve incompressible Navier-Stokes equations for the fluid and Navier equations for the membrane. For both domains, second-order triangular elements are used in a moving mesh that adapts to the deformation of the membrane. Simulations are started from static fluid and membrane and are run until a periodic state is reached after 10-20 cycles.

3 Results

The flow inside the cavity and the deformation of the membrane have been studied for different channel flow conditions. The geometry has been fixed at $AR = 14$ and $h^* = 0.1$ and the membrane parameters at $U^* = 35$ and $\rho^* = 1$.

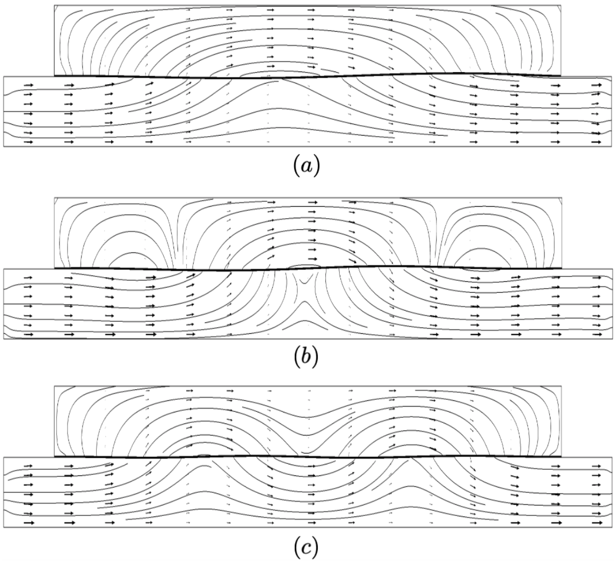

Figure 2: Flow vectors and streamlines inside the channel and the cavity at the instant of maximum deformation of the membrane for different flow conditions. (a) $\alpha = 7$, $\varepsilon = 0.028$ (b) $\alpha = 10$, $\varepsilon = 0.014$ (c) $\alpha = 14$, $\varepsilon = 0.007$. The plots are scaled for showing purposes by reducing the length by two in the horizontal axis.

When the periodic state is reached, an oscillatory flow is induced by the outter channel flow inside the cavity through the deformation of the membrane. The waves generated on the membrane move the fluid inside the cavity, coupling it with the channel flow and generating global flow patterns between the cavity and the channel (see Fig. 2). At this periodic state, the membrane and the flow in both regions oscillate at the same frequency in the range of parameters considered herein.

Depending on the channel flow conditions, different deformation modes appear at the membrane. In turn, this deformation affects the flow, generating different circulation patterns. In Fig. 2, we show the flow field and the membrane for three different sets of parameters. When the Womersley number and the stroke length are modified, the number of waves at the membrane changes and different flow cells appear.

Additionally to this main periodic flow, a secondary flow appears inside the cavity. To visualize it, we have studied the trajectories of Lagrangian particles during a large number of cycles. Along a single cycle, each particle describes a loop following the flow; however, they also experiment a net displacement $\delta << L$. The accumulation of this net displacement generates recirculation cells inside the cavity with a temporal scale several orders of magnitude bigger than the main flow oscillating period. In Fig. 3, we show the path described by Lagrangian particles inside the cavity along 250 cycles.

In addition to the numerical simulations, experiments are also performed under the same conditions. Results of the flow field established inside the cavity and the membrane deformation are also presented and compared with the numerics.

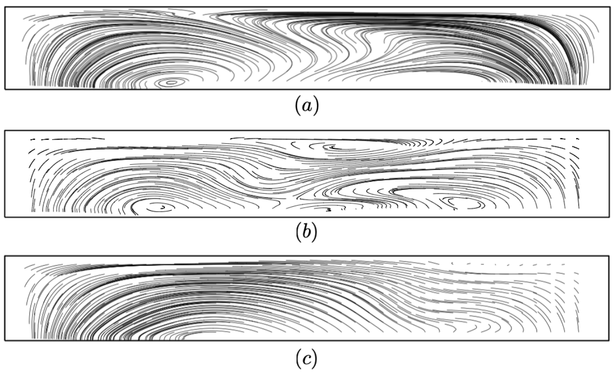

Figure 3: Structure of the secondary flow inside the cavity for the cases of Fig. 2. The lines represent the paths of the net displacement Lagrangian particles.

4 Conclusions

The flow in the channel moves the fluid inside the cavity through the deformation of the membrane. Both the main and the secondary flows inside the cavity are highly influenced by the channel flow conditions, such as the stroke length and the flow frequency. From this, we can deduce that the fluid inside the syrinx may be affected by the conditions of the cerebrospinal flow, generating flow patterns and stress conditions that can contribute to the enlarging of the cavity.

References

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Elliott NSJ, Bertram CD, Martin BA & Brodbelt AR. (2013) “Syringomyelia: A review of the biomechanics”. *J. Fluids Struct.*, **40**, 1–24.