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Scalable method for administration of resource technologies under stochastic procedures

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ABSTRACT

During the development of the S3Unica project (Smart Specialisation University Campus) and its application in the ASSET project (Advanced Systems Studies for Energy Transition), both within the European Commission, the resolution of the distributed energy generation model was proposed through the creation of an algorithm that would allow the shared market between producers and consumers. From this premise arose the need to create a replicable system to resolve this situation in the new shared generation environment, using low-cost technologies. This work develops the scalable method for resource management technologies (SMART), based on stochastic procedures, which generates microgrids with an integrated energy market. The interest of this work is based on the incorporation of real-time analysis, applying stochastic methods, and its fusion with probabilistic predictive methods that evolve and harmonise the results. The fact that the process is self-learning also enables the use of metadomotic as a tool for both comfort improvement and energy sharing. The most important results developed were the design of the internal scheme of the low-cost SMART control device together with the developments of both individual and collective resolution algorithms. By achieving the incorporation of internal and external producers in the same numerical procedure, the distributed and hybrid generation models are solved simultaneously.

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1. Introduction

Since the mathematician Benoît Mandelbrot developed the concept of fractal in 1975, there have been many theories that have tried to replicate geometries at any scale without limit of continuity. From this point of view there are many disciplines, in the field of new technologies, which assimilate this concept with scalability.

Both the S3Unica project (Smart Specialisation University Campus) and the ASSET project (Advanced Systems Studies for Energy Transition), both within the European Commission, are currently under development. The relationship between them is established in order to transfer the knowledge developed from the universities, on smart grids and resolution algorithms, to consumer cooperatives that are beginning to form all over Europe [1]. While in the past the literature has had to approach this issue from a theoretical point of view, the generalisation of self-consumption and cogeneration systems now make it possible to put into practice real models that resolve shared consumption. Today's greater capacity for collecting and interpreting of big data makes it possible to generate new algorithms and mathematical procedures, such as those presented here, to solve dynamic models between producers and consumers. Currently, energy communities are the most interested in creating internal markets to balance both the consumption and production of each of their members [2]. In them, each component could have several sources of sustainable energy production (photovoltaic, wind, geodesic, etc.) with different availability and storage capacity [3]. For this reason, a distributed energy generation model [4] is proposed and its corresponding resolution through algorithms that allow for a shared market between producers and consumers [5].

The present paper deals with the design of geometric and statistical algorithms that simulate a unit of energy consumption (housing, industry, city, etc.) and that are distributed by means of scalability to any larger set.

In this scalable method for resource management technologies, which we will call SMART, stochastic procedures are used, allowing the creation of microgrids with an integrated energy market. Furthermore, it takes as a starting point the definition of the basic unit of energy consumption, which can be composed of three elements: energy generation sources, control devices and sensors or storage systems. Through them, an algorithm is generated that chooses the most appropriate option to minimise the energy cost of the system.

Moreover, in order to be adapted to a real model, the presence of the three main actors will be necessary: energy production sources, consumers and accumulators. The most efficient real-time selection between energy supply and demand will lead to an optimal balance of the use of resources and, therefore, a firm step towards the sustainability of the populations [6].

It is very important to be able to optimise energy consumption through the use of SMART technology, as it is possible to control the energy consumed and devices can be programmed to operate at times when electricity costs less [7].

It can be seen that at present it is necessary to have a large amount of under-utilised generation in order to meet peak demand or hours of peak consumption, and to match this with one's own generation [8].

In addition, the participation of "customers" in the electricity system is very necessary, since this way it is possible to verify the real amount of load that exists in the network at a given time [9], i.e., this allows electricity systems to be more reliable and efficient, providing guarantees when offering a stable energy supply that avoids overloads in certain sectors during peak hours [10] (which would trigger the cut-off of energy supply to users). The problem is to efficiently model the real demand in a given period [11], since the load has a stochastic character, i.e. its real behaviour cannot be determined, only approximations can be made according to previously obtained data [12]. For this reason it is necessary, in a smart grid, the dynamic participation of the users [13].

In order to have smart energy management, changes are required not only in the way energy is supplied but also in how the energy market behaves [14], how dispersed it is [15] and what knowledge can be extracted from it [16]. In addition, for the use of SMART technology, it is important to maintain end-user comfort, thus requiring investment in control [17], communication [18] and decision technologies [19].

The article is structured as follows: Section 2 defines the basic concepts used and presents the interconnection diagram of the SMART scheme. Furthermore, mathematically models the previously defined concepts for both discrete and continuous time. Section 3 proposes a resolution method using differentiable curves, applied to both the individual and the collective case. Section 4 introduces the factors of predictive knowledge and passive learning, by incorporating stochastic notation into the problem. Section 5 introduces the economic factor to condition the objective function to this parameter, presenting its axiomatics together with the resolution algorithm, both in the individual and the collective case. The last section is devoted to conclusions and future work.

2. Basic concepts and modeling

The concepts that we will use throughout the article will be introduced.

Definition 2.1. Each element (device, house, industry, etc.) that spends energy is called **consumption unit**, and is denoted by C, being the value of C the energy consumed by it. The set of consumers is denoted as:

$$C_1, C_2, \dots, C_n \tag{1}$$

Definition 2.2. Each renewable source (solar, wind, geothermal, etc.) that produces clean energy is called a **production unit**, and is denoted as P, with the value of P being the energy produced by it. In case the energy comes from power companies

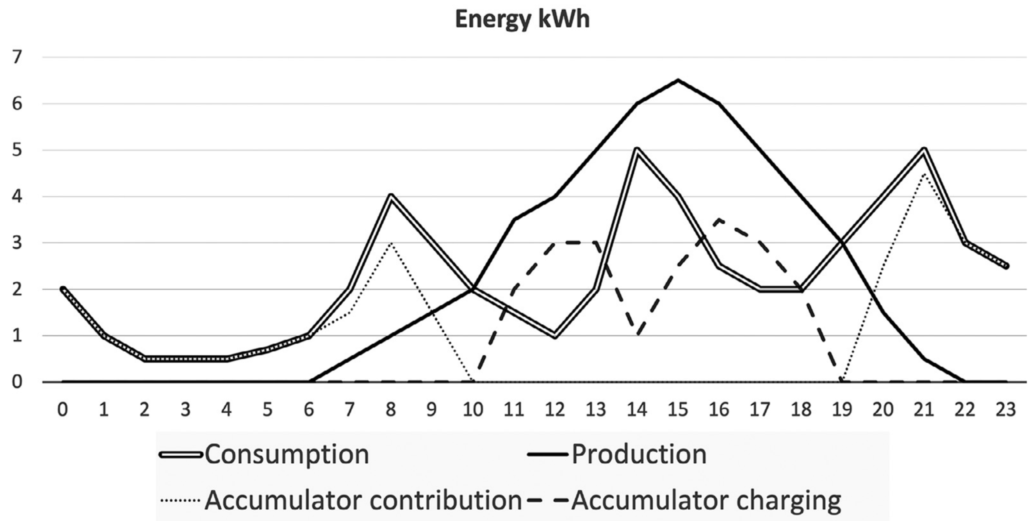

Fig. 1. Diagram of the use of energy sources throughout the hours of the day.

or other sources, and therefore it is not known whether it comes from renewable sources or not, it is denoted as O and referred as **units generated by other sources**. The sets of external and internal producers are denoted as:

$$O_1, O_2, \ldots, O_q \tag{2}$$

$$P_1, P_2, \ldots, P_m \tag{3}$$

Definition 2.3. Each element (battery, electric vehicle, hot water tank, etc.) capable of storing energy, with the value of A being the energy accumulated in it, is called an **accumulator unit**, and is denoted as A. The set of accumulators is denoted as

$$A_1, A_2, \ldots, A_l \tag{4}$$

Where accumulation has two clearly differentiated phases: when it provides energy for consumption (in Fig. 1 from 19:00 one day to 10:00 the next) and when it receives this energy and stores it (from 10:00 to 19:00).

If time is denoted by the variable t, and it is taken into account that the electricity market is conditioned by variable quotations depending on the time of day (and depicted in Fig. 1), the working sets will be denoted as:

$$\left\{ \begin{array}{c} C_1(t), C_2(t), \ldots, C_n(t) \\ O_1(t), O_2(t), \ldots, O_q(t) \\ P_1(t), P_2(t), \ldots, P_m(t) \\ A_1(t), A_2(t), \ldots, A_l(t) \end{array} \right\} \forall t \in H = \{0, 1, \ldots, 23\} \tag{5}$$

In the following diagram (Fig. 2) the priorities of production, consumption and storage can be seen, with a << + >> for the different parameters in the higher priced instances and << - >> for the remaining cheaper ones. It shows the process of external arrival of energy from other producers (external companies in hybrid generation, neighbours in distributed generation or both) and how this energy is added to the internal, produced or stored energy.

Once the external energy has been added to the internal energy produced by renewables, together with the energy provided by other users from their accumulators, the energy needs (consumption) are analysed. If this can be covered without external non-renewable sources (sufficient consumption), then the stored energy is analysed. Otherwise (insufficient consumption), external sources of any kind are used.

In the analysis of stored energy (storage), two situations are observed: if consumption does not require stored energy (sufficient), a surplus is produced and, therefore, its exit to other markets; otherwise (consumption requires stored energy, a situation that has been called insufficient) the stored energy goes on to collaborate with consumption, generating a loop back to the previous step.

In case of energy deficit, and before increasing consumption from external sources, an intelligent management (observing which loads can be disconnected) is carried out as a preventive measure to avoid unnecessary consumption.

The setting of these intervals depends on the user of the installation. Beforehand, the hourly energy price is divided into three sub-intervals of equal length between the minimum and maximum price. From this, the user can choose the most economical range << - >> as the one consisting only of the first third or decide to opt for the first two thirds. The rest will be considered as the most expensive range << + >>.

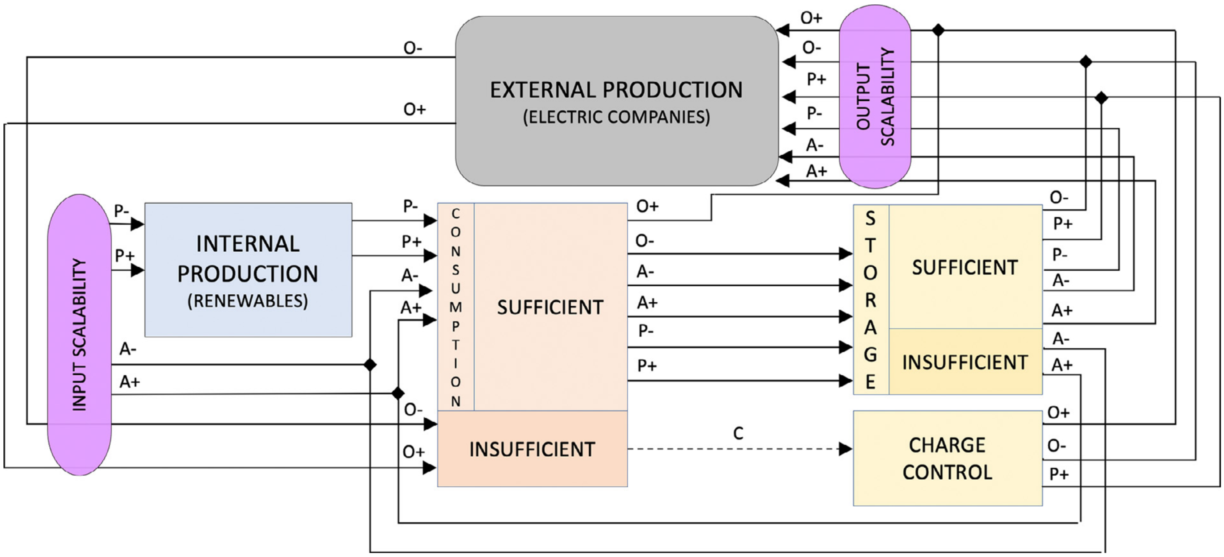

Fig. 2. SMART schedule about priorities of production, consumption and storage.

2.1. Modeling

Denoted by $\mathbf{C}(t)$, $\mathbf{O}(t)$, $\mathbf{P}(t)$ and $\mathbf{A}(t)$ for consumption, external power, renewable production and storage capacity at each point in time, it follows that:

$$\mathbf{C}(t) = \sum_{i=1}^{n} C_i(t); \quad \mathbf{O}(t) = \sum_{i=1}^{q} O_i(t); \tag{6}$$

$$\mathbf{P}(t) = \sum_{i=1}^{m} P_i(t); \quad \mathbf{A}(t) = \sum_{i=1}^{l} A_i(t) \tag{7}$$

And throughout the day the result is:

$$\mathbf{C} = \int_H C(t)\, dt; \quad \mathbf{O} = \int_H O(t)\, dt; \tag{8}$$

$$\mathbf{P} = \int_H P(t)\, dt; \quad \mathbf{A} = \int_H A(t)\, dt \tag{9}$$

The objective function that measures the most favorable situation for each study unit is one in which the production of renewables and accumulation capacity exceeds the expected consumption over time. The aim is therefore to establish:

$$\max \left\{ \int_H \Big(P(t) + A(t) - C(t) \Big)\, dt \right\} \tag{10}$$

On the estimation of the behaviour of each variable it is deduced that:

1. $C_i(t)$ will be controlled through the metadomotic [20]
2. $P_i(t)$ is based on weather forecasts
3. $A_i(t)$ is modified based on the supply prices

3. Productive curves

When dealing with interrelated functions through the variable "time", we could study their function as coordinates of a differentiable curve. For this it is assumed that:

$$\vec{\alpha}(t) = (O(t), P(t), A(t)) \tag{11}$$

The unknowns to be solved are set as $(o(t), p(t), a(t))$, indicating the proportion of energy of each type used (between 0 and 1), and are denoted as:

$$\vec{x}(t) = (o(t), p(t), a(t)) \tag{12}$$

On the other hand, since the price changes every hour, the set of values of t is discrete. Therefore the curves must be considered as "in differences" and not "differentiable".

Proposition 3.1. *The overall consumption formula is established as:*

$$C(t) = \vec{x}(t) \cdot \vec{\alpha}(t) \tag{13}$$

Proof. The dilemma arises from the fact that storage systems have a dual behaviour, as there are times when they are energy consumers and times when they supply energy. Therefore, a distinction must be made between the two situations:

1. Whenever storage is not complete, whether the price of energy is low or the production of renewables exceeds consumption, then energy can be stored. Let o_C be the proportion of other sources consumed and p_C be the proportion of renewable energy consumed. Let o_A be the proportion of other sources that is stored due to their low price and p_A be the proportion of renewable energy produced and stored. It follows that:

$$\begin{cases} o(t) = o_C(t) + o_A(t) \\ p(t) = p_C(t) + p_A(t) \end{cases} \tag{14}$$

2. If renewable production is less than consumption, then storage must provide energy.

Mathematically the two situations would look like this:

$$\begin{cases} \Big(O(t), P(t)\Big) \cdot \begin{pmatrix} o_C(t) & o_A(t) \\ p_C(t) & p_A(t) \end{pmatrix} = \Big(C(t), A(t)\Big) & \text{if } t \in O^- \vee \Big(P(t) \geq C(t)\Big) \\ \Big(O(t), P(t), A(t)\Big) \cdot \begin{pmatrix} o(t) \\ p(t) \\ a(t) \end{pmatrix} = C(t) & \text{if } P(t) < C(t) \end{cases} \tag{15}$$

The first equation calculates on the one hand the consumption due to external sources and renewable production and on the other hand the storage due to the same causes. The second calculates only the total consumption due to both external sources and renewable production and existing storage.

To unify both notations, $a(t)$ can be taken to reach negative values when storage units become consumers. In such a case, if denoted by $-a(t)$, one has that:

$$\Big(O(t), P(t), A(t)\Big) \cdot \begin{pmatrix} o(t) \\ p(t) \\ -a(t) \end{pmatrix} = C(t) \tag{16}$$

being:

$$C(t) + a(t) \cdot A(t) = o(t) \cdot O(t) + p(t) \cdot P(t) =$$

$$= \Big[o_C(t) + o_A(t)\Big] \cdot O(t) + \Big[p_C(t) + p_A(t)\Big] \cdot P(t) =$$

$$= o_C(t) \cdot O(t) + p_C(t) \cdot P(t) + o_A(t) \cdot O(t) + p_A(t) \cdot P(t) =$$

$$= \Big(O(t), P(t)\Big) \cdot \begin{pmatrix} o_C(t) & o_A(t) \\ p_C(t) & p_A(t) \end{pmatrix}$$

Therefore, if it is considered that, given $\Big\{o(t), p(t), |a(t)|\Big\} \in [0, 1]$ it follows that:

$$\Big(O(t), P(t), A(t)\Big) \cdot \begin{pmatrix} o(t) \\ p(t) \\ a(t) \end{pmatrix} = C(t)$$

or, with geometric notation:
$C(t) = \vec{x}(t) \cdot \vec{\alpha}(t)$ ■ □

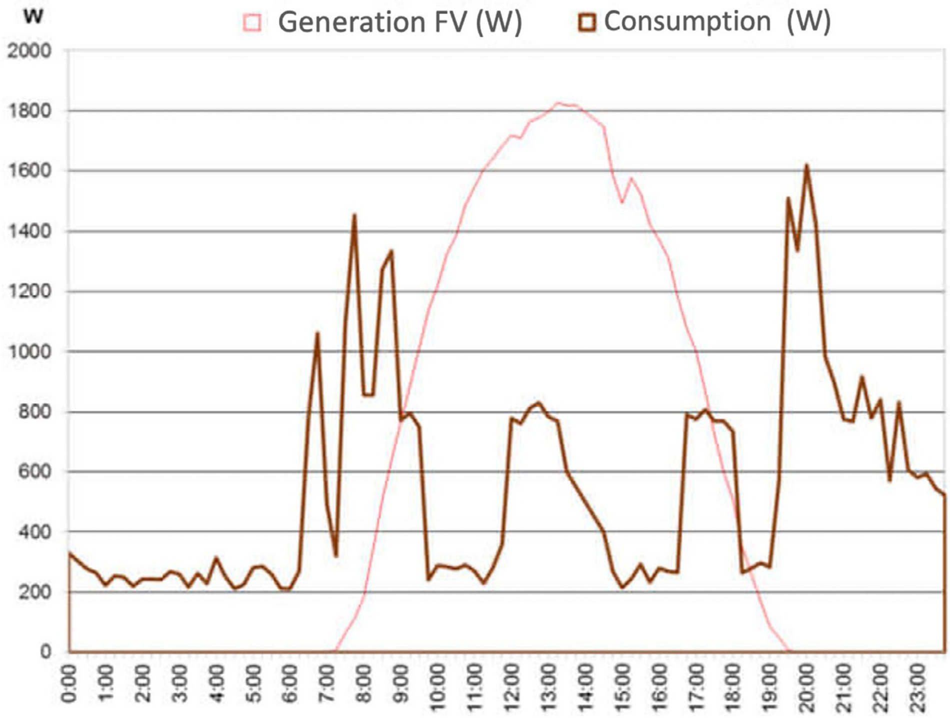

Fig. 3. Renewable production and energy consumption.

3.1. Individual case

The energy required for the consumption of a house must be produced through the different coordinates of $\vec{\alpha}$ of the form:

$$C(t) = o(t) \cdot O(t) + p(t) \cdot P(t) + a(t) \cdot A(t) \tag{17}$$

This expression reflects the fact that own renewable sources alone do not usually satisfy permanently all the required consumption, as can be seen in Fig. 3 (typical consumption of a domestic installation). From this it follows that:

$$C(t) = \vec{x}(t) \cdot \vec{\alpha}(t) \text{ with } t \in \{0, 1, \dots, 23\}$$

Consider next that the stored energy comes from a surplus in renewables or that external production O is in its cheapest time interval, i.e. $t \in O^-$. Then this linear combination representing such storage is expressed as:

$$A(t) = o_A(t) \cdot O(t) + p_A(t) \cdot P(t) \text{ with } o_A(t) = 0 \text{ if } t \in O^+ \tag{18}$$

Replacing:

$$C(t) = o(t) \cdot O(t) + p(t) \cdot P(t) + a(t) \cdot \Big[o_A(t) \cdot O(t) + p_A(t) \cdot P(t) \Big] =$$

$$= \Big[o(t) + a(t) \cdot o_A(t) \Big] \cdot O(t) + \Big[p(t) + a(t) \cdot p_A(t) \Big] \cdot P(t)$$

Given $\vec{x}_A(t) = (o_A(t), p_A(t), -1)$, it follows that:

$$C(t) = \Big[\vec{x}(t) + a(t) \cdot \vec{x}_A(t) \Big] \cdot \vec{\alpha}(t) \text{ with } t \in \{0, 1, \dots, 23\} \tag{19}$$

3.2. Collective case

On the other hand, when combining what happens in each house, the production curve in differences of the house i can be established as $al\vec{p}ha_i(t)$.

In this way, when sharing resources, the energy matrix $E(t) = (e_{ij}(t))$ is established, where each value $e_{ij}(t)$ indicates what type of energy is being contributed by house i to j in hour t (it can come from external electricity, produced or stored and with high or low prices for each t). In this way, the final charge for what is consumed can be coordinated. An example of matrix $E(t)$ for 3 houses would be:

$$E(t) = \begin{pmatrix} O_1(t) & 0 & 0 \\ P_1^-(t) & O_2(t) & A_3^-(t) \\ P_1^-(t) & 0 & O_3(t) \end{pmatrix} \tag{20}$$

4. Stochastic analisys and passive learning

4.1. Stochastic analisys

The metadomotic [20] makes it possible to know the uses and habits of the inhabitants with respect to energy consumption. In order to establish a similar behaviour between the days of the week, these days are ordered into three groups, differentiated on the basis of the start time of the increase in consumption: Labour (L), Weekend (W) and Random (R). Within each of these categories, three subsets are also distinguished on the basis of the different times of the year: Cold (C), Mild (M) and Hot (H). Therefore there are nine estimated average consumptions:

$$\left\{ \overline{C}_{LC}, \overline{C}_{LM}, \overline{C}_{LH}, \overline{C}_{WC}, \ldots, \overline{C}_{RH} \right\} \in \mathbb{R} \tag{21}$$

and nine different behavioural profiles or ratio functions of possible consumption:

$$\left\{ f_{LC}(t), f_{LM}(t), f_{LH}(t), f_{WC}(t), \ldots, f_{RH}(t) \right\} \in [0,1] \tag{22}$$

where $f_{XY}(t)$ is the following conditional probability:

$$f_{XY}(t) = P[C(t)|_{XY}] \text{ for each} t \in H \tag{23}$$

This provides a matrix of probable consumptions F with 24 rows (hours from 0 to 23) and 9 columns (profiles from LC to RH), where each of these columns add up to 1.

$$F = \begin{pmatrix} f_{LC}(0) & f_{LM}(0) & f_{LH}(0) & f_{WC}(0) & \cdots & f_{RH}(0) \\ f_{LC}(1) & f_{LM}(1) & f_{LH}(1) & f_{WC}(1) & \cdots & f_{RH}(1) \\ f_{LC}(2) & f_{LM}(2) & f_{LH}(2) & f_{WC}(2) & \cdots & f_{RH}(2) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{LC}(23) & f_{LM}(23) & f_{LH}(23) & f_{WC}(23) & \cdots & f_{RH}(23) \end{pmatrix} \tag{24}$$

On the other hand, there is the estimated average consumption matrix, which is square, diagonal and of order 9:

$$\overline{C} = \begin{pmatrix} \overline{C}_{LC} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \overline{C}_{LM} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \overline{C}_{LH} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \overline{C}_{RH} \end{pmatrix} \tag{25}$$

These two matrices F and $\overline{C}$ allow the facilities to be sized to cover as much time as possible. The total expected consumption over the day, using only renewable or stored energy sources, is denoted by $\overline{C}^R$, and is calculated as:

$$B = F \cdot \overline{C} \Rightarrow \overline{C}^R = \sum_{i \in H} \max_{j=1,\ldots,9} b_{ij} \quad ; \quad p(t) + a(t) = \frac{\max_{j=1,\ldots,9} b_{tj}}{\overline{C}^R} \tag{26}$$

Example 4.1. Five hours of three types of days at one time of the year are analysed:

$$F = \begin{pmatrix} 0.1 & 0 & 0 \\ 0.3 & 0.2 & 0 \\ 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.4 \\ 0 & 0.2 & 0.3 \end{pmatrix} \quad ; \quad \overline{C} = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 80 & 0 \\ 0 & 0 & 80 \end{pmatrix}$$

It is calculated $B = F \cdot \overline{C}$, $\overline{C}^R$ y $p(t) + a(t)$:

t	$F \cdot \overline{C}$			max	$p(t) + a(t)$
0	10	0	0	10	10/126
1	30	16	0	30	30/126
2	30	24	24	30	30/126
3	30	24	32	32	32/126
4	0	16	24	24	24/126
$\sum$	100	80	80	**126**	1

Therefore, a daily contribution of energy (renewable generated or stored) of $\overline{C}^R = 126$ must be ensured to cover the total consumption, ensuring in each time slot the level indicated in the column <<*max*>>.

Definition 4.1. The square matrices where all their values are non-negative real and both the sum of the terms by rows and by columns is 1 are called **stochastic or Markov**, being right-handed if they fulfil it only by rows and left-handed if they do it only by columns. If it is not possible to ensure that their values are non-negative, they are called quasi-stochastic.

Although it is not square, F can be considered as a **left-stochastic** matrix.

Vectors with n coordinates equal to one are considered, i.e. $\omega^n = (1, 1, \dots, 1)$. To characterise a rectangular matrix A of n rows and m columns as left stochastic it must satisfy that:

$$\omega^n \cdot A = \omega^m \tag{27}$$

and in the present case, where F is of dimension $24x9$:

$$\omega^{24} \cdot F = \omega^9 \tag{28}$$

it complies.

4.2. Passive learning

The construction of the matrices F and $\overline{C}$ requires a learning process based on previous uses and customs.

For this purpose, let us assume that the values of F and $\overline{C}$ have been constructed from a number of days of each profile, and these values are denoted as:

$$\left\{ n_{LC}, n_{LM}, n_{LH}, n_{WC}, \dots, n_{RH} \right\} \in \mathbb{N} \tag{29}$$

Then, at each end of the day, where new hourly consumption data arrives and the total consumption for that day, under any XY profile, is obtained:

$$C^*_{XY} = C^*_{XY}(0) + \dots + C^*_{XY}(23) \tag{30}$$

which will provide new knowledge about the F and $\overline{C}$ matrices, assuming f^0_{XY} and $\overline{C}^0_{XY}$ the previous values, leaving the new probabilities and consumptions as follows

$$f_{XY}(t) = \frac{f^0_{XY}(t) \cdot n_{XY} + C^*_{XY}(t)/C^*_{XY}}{n_{XY} + 1} \text{ for each } t \in H \tag{31}$$

$$\overline{C}_{XY} = \frac{\overline{C}^0_{XY} \cdot n_{XY} + C^*_{XY}}{n_{XY} + 1} \tag{32}$$

Finally: $n_{XY} = n_{XY} + 1$.

These formulas show how knowledge is acquired on the basis of the weight of past learning. Hence, it cannot be said to be a strictly stochastic process, since in such a case knowledge would only be based on the immediately previous data and not on the complete history. Therefore, there is a daily and permanent evolution of the data stored in F and $\overline{C}$.

5. Energy market

In order to achieve an energy exchange between local producers and consumers, the basis of their electricity market is established by the following axiomatic:

SMART Axiomatics

Ax.1 There is a surplus in one of the energy producers.

Ax.2 There are consumers willing to pay more for local renewable or stored energy than for external energy supplied by companies[2]

Ax.3 A renewable energy producer or energy accumulator cannot sell it externally if it is required for local consumption.[3] The first consumer is the producer or accumulator itself and, in the case of internal transfer, all producers and accumulators will contribute the energy equally.

[2] If only they were willing to pay less, companies could buy up all the surplus and corner the market.

[3] This is done to avoid a switch from self-consumer to pure producer.

5.1. Individual case

Definition 5.1. Let € be the function that gives the instantaneous price of energy and denoted by € (t). It can be assumed that the energy consumed is more expensive if it comes from external sources and zero otherwise.

The variable $\mathbf{T}(t)$ represents the balance of economic gains and losses for each value of t. This means that if it is positive, its value must be paid to others, and if it is negative, the economic compensation it represents is received. It follows that:

$$\mathbf{T}(t) = \sum_{i=1}^{n} \text{€}\ (t) C_i(t) \tag{33}$$

And throughout the day the result is:

$$\mathbf{T} = \int_H T(t)\ dt \tag{34}$$

The objective function that measures the most favorable situation for each study unit is one in which the expected consumption over time and the total cost is minimum. Therefore, it is about establishing:

$$\min \left\{ \int_H T(t)\ dt \right\} \tag{35}$$

By being able to combine different sources of internal renewable energy production (P), together with their own accumulators (A), with other external sources, the instantaneous price could be understood as:

$$\mathbf{T}(t) = \text{€}\ (t) \cdot \left[\sum_{i=1}^{n} C_i(t) - \sum_{i=1}^{m} P_i(t) - \sum_{i=1}^{l} A_i(t) \right] \tag{36}$$

5.1.1. Individual algorithm

The algorithm bases its behavior on the price of the energy consumed, in such a way that:

1. If $T(t) > 0$ then
   (a) If € (t) is high, the charges must be managed
   (b) If € (t) is low, you can act in the same way or you could store the necessary
2. If $T(t) = 0$ is only stored if € (t) is low
3. If $T(t) < 0$ then
   (a) If € (t) is low, the use of the $A_i(t)$ can be decreased, and if they were not being used, the $P_i(t)$ could also be reduced by storing or selling the excess.
   (b) If € (t) is high the surplus $P_i(t)$ is sold and, if interested, also the $A_i(t)$

5.2. Collective case

For the creation of a collective market, each of the actors participating in the market (households and electricity companies) is defined as follows:

$$\{\Omega_1, \Omega_2, \ldots, \Omega_k, \Omega_{k+1}\} \tag{37}$$

with the last element Ω_{k+1} representing the set of electricity supply companies. Let $M^P(t)$ and $M^A(t)$ matrices indicate respectively, at each position ij, how much the actor Ω_i gives up of what it produces, or of what it stores, to the actor Ω_j.

The market matrix $M(t) = M^P(t) + M^A(t)$ will represent:

1. Adding element by element of row i, the result is less than or equal to the production plus storage of each actor i and, by column j, the consumptions of each actor j.
2. Each of the elements $m_{i,k+1}$ (last column) represents the energy sold by each Ω_i to the electricity company.
3. Each of the elements $m_{k+1,j}$ (last row) represents the energy $W(t)$ purchased from the electricity company by each user Ω_j.
4. The main diagonal indicates, in each element m_{ii}, the level of self-consumption of Ω_i.

From this structure, it is necessary to establish an algorithm that allows the construction of the matrices $M^P(t)$ and $M^A(t)$, knowing the forecast vectors for each house:

1. $C(t)$: Consumption $(C_1(t), C_2(t), \ldots, C_k(t))$.
2. $P(t)$: Renewable energy production $(P_1(t), P_2(t), \ldots, P_k(t))$
3. $A(t)$: Accumulation of energy $(A_1(t), A_2(t), \ldots, A_k(t))$

The amount sold to the electricity companies being what can be considered as consumed by the electricity companies, i.e:

$$C_{k+1}(t) = \sum_{i=1}^{k} (P_i(t) + A_i(t) - C_i(t)) \tag{38}$$

if this value were positive, in which case the purchased equals zero, i.e.: $P_{k+1}(t) = 0$.

Otherwise:

$$P_{k+1}(t) = \sum_{i=1}^{k} (C_i(t) - P_i(t) - A_i(t)) \tag{39}$$

if this value were positive, in which case what is sold equals zero, i.e.: $C_{k+1}(t) = 0$.

Example 5.1. A market of three houses and one electricity company is analysed, at time t. The forecasts are assumed to be:

$$C(t) = (20, 30, 40), P(t) = (60, 20, 0) \text{ and } A(t) = (20, 20, 0)$$

The matrices could be, if no procedure is applied:

$$M^P(t) = \left(\begin{array}{ccc|c} 30 & 0 & 20 & 10 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{array}\right); \quad M^A(t) = \left(\begin{array}{ccc|c} -10 & 0 & 10 & 20 \\ 0 & 10 & 0 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$M(t) = M^P(t) + M^A(t) = \left(\begin{array}{ccc|c} 20 & 0 & 30 & 30 \\ 0 & 30 & 0 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{array}\right) \Rightarrow A(t+1) = (0, 0, 0)$$

where external sales are 40 units, purchases from companies are 10 units and the actors have consumed all the stored energy.

These 10 units requested from the electricity companies make it possible to establish the maximum recommended contract type for the customer who has requested it. Suppose that $O_3(t) = 20$ for all t.

In geometric notation it is obtained:

$$C_1(t) = 0 \cdot O_1(t) + 1/3 \cdot P_1(t) + 0 \cdot A_1(t)$$

$$C_2(t) = 0 \cdot O_2(t) + 1 \cdot P_2(t) + 1/2 \cdot A_2(t)$$

$$C_3(t) = 1/2 \cdot O_3(t) + 1/3 \cdot P_1(t) + 1/2 \cdot A_1(t)$$

5.2.1. Collective algorithm

The algorithm to be developed must satisfy the axioms stated at the beginning. One conclusion that can be drawn, in view of the Example 5.1 where these axioms are not satisfied, is that Ax.3 implies that any solution can contain either external sales or external purchases but never both at the same time.

1. Applying Ax.3 on what is produced: $m^P_{ii} = \min\{C_i(t), P_i(t)\}$.
2. Applying again Ax.3 on the accumulated: if $m^P_{ii} < C_i(t) \Rightarrow m^A_{ii} = \min\{C_i(t) - P_i(t), A_i(t)\}$.
3. By application of Ax.1 and Ax.2: all those who have surpluses, produced or stored, will cede equivalent quantities to those who need them until their needs are covered.
4. If there are still needs, they will be bought from the electricity company.
5. If all the needs have been covered locally, the surplus renewable energy can be stored (if there is capacity) or sold according to the price at the moment t.

Example 5.2. The Algorithm 5.2.1 is applied to the Example 5.1 at a cheap t instant (it is in the interest of accumulation):

$$M^P(t) = \left(\begin{array}{ccc|c} 30 & 0 & 30 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right); \quad M^A(t) = \left(\begin{array}{ccc|c} -10 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$M(t) = M^P(t) + M^A(t) = \left(\begin{array}{ccc|c} 20 & 0 & 30 & 0 \\ 0 & 30 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow A(t+1) = (30, 0, 0)$$

In this case there are no external purchases or sales

$$P_{k+1}(t) = C_{k+1}(t) = 0$$

and almost all the previously stored energy, $A(t+1) = (30, 0, 0)$, is conserved for the following instant $t+1$.

In geometric notation it is obtained:

$$C_1(t) = 0 \cdot O_1(t) + 1/3 \cdot P_1(t) + 0 \cdot A_1(t)$$

$$C_2(t) = 0 \cdot O_2(t) + 1 \cdot P_2(t) + 1/2 \cdot A_2(t)$$

$$C_3(t) = 0 \cdot O_3(t) + 1/2 \cdot P_1(t) + 1/2 \cdot A_2(t)$$

Example 5.3. The Algorithm 5.2.1 is applied to the Example 5.1 at an expensive time t (external sales are of interest):

$$M^P(t) = \left(\begin{array}{ccc|c} 20 & 0 & 20 & 20 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right); \quad M^A(t) = \left(\begin{array}{ccc|c} 0 & 0 & 10 & 10 \\ 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$M(t) = M^P(t) + M^A(t) = \left(\begin{array}{ccc|c} 20 & 0 & 30 & 30 \\ 0 & 30 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow A(t+1) = (0, 0, 0)$$

In this case there are 30 units of external sales, $C_{k+1}(t) = 30$, no purchases, $P_{k+1}(t) = 0$, from companies and all stored energy has been consumed, $A(t+1) = (0, 0, 0)$.

The geometric notation would be the same as in the previous example.

5.2.2. Stochastic notation

In the case where there is a producer market, i.e. where the renewable energy generated, together with storage, exceeds consumption, stochastic notation can be incorporated into the matrices.

To do so, it is sufficient to divide each column i of $M(t)$ by the value of $C_i(t)$ with $t \in \{1, 2, \dots, k, k+1\}$. In this case a left stochastic matrix appears which, being square and sum one by columns, will be denoted as $S(t)$.

In the Example 5.3 is obtained:

$$S(t) = \left(\begin{array}{ccc|c} 1 & 0 & 3/4 & 1 \\ 0 & 1 & 1/4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

5.3. Estimated consumption in real time

If the stochastic development of the probability matrix F is to be realised in real time, a global estimation procedure must be established through the control of consumption sensors. In the present case, the <<smart plugs with remote price control>> developed and patented by the Andalusian Institute of Domotic and Energy Efficiency [21] will be used as sensors for managing individual loads, forming an advanced metering infrastructure.

This requires the establishment of servers (q) for each grouping of users dependent on the same suppliers, with an index (n) for each house and (s) for the corresponding sensor within that house. Let z_t^{qns} be the state of a given qns sensor, €_t^q the price reported by the q server, Θ^q the total set of prices in the q cluster, Δ_t the set of possible states and π_t^{qns} the distribution of stationary states in Markov chains, for each time instant t. We have that the real time probability of going from state $z_t^{qns} = i$ to state $z_{t+1}^{qns} = j$ when the price of energy goes from €_t^q to €_{t+1}^q will be:

$$P\left(z_{t+1}^{qns} = i \mid z_t^{qns} = j, \text{€}_t^q, \text{€}_{t+1}^q\right) \tag{40}$$

Therefore, the probability of state changes in real time will be:

$$\overset{\pi_{t+1}^{qns}(z_{t+1}^{qns}=i)}{\sum_{j\in\Delta_t, \text{€}_t^q\in\Theta_t^q}} P\left(z_{t+1}^{qns} = i \mid z_t^{qns} = j, \text{€}_t^q, \text{€}_{t+1}^q\right) \cdot P(\text{€}_t^q) \cdot P(\text{€}_{t+1}^q) \cdot \pi_t^{qns}(z_t^{qns} = i) \tag{41}$$

If programming is required, the algorithm developed by P. Moreno and M. García [22] can be applied.

6. Conclusions and future work

The notation introduced in this paper allows the following conclusions to be drawn:

1. **Replicability**: The SMART algorithm developed for each dwelling and represented in Fig. 2 allows its direct replicability through interconnections, where the data output of one control unit serves as input to the next one.
2. **Duality**: The introduction of storage units are unified as consumption or production units depending on the time. This has been noted in parallel as positive or negative consumers, reducing the nomenclature.
3. **Differentiability**: With reference to the difference equations appearing in the production curves, an operational method is being developed for their resolution in real time.
4. **Stochastic analysis**: The resolution of efficient consumption uses algorithms based on stochastic analysis using quasi-stochastic and stochastic-left matrices. The Markov matrices are therefore established by means of consumption sensors, with different probabilities of state change based on the hourly price differences stored in the corresponding servers.

The challenges derived from this paper invite its application on a real community of producers and consumers. To this end, a research project has been initiated to apply the technologies to a pilot house and, based on the results, to extrapolate their application to a community made up of twenty houses. This project is called CONSCIOUS HOUSES and has been endowed with 5 million euros by the Andalusian Regional Government.

Data Availability

Scalable Method for Administration of Resource Technologies under stochastic procedures

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