Mathematics and Online Education

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Mathematics and Online Education

- A brief overview of distance learning

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A brief overview of distance learning

The *best* way to learn Mathematics is by teacher guidance in the time-honored judicious mixture of:

- explanation and orientation by the teacher,
- self-study of the theory,
- training by solving progressively more challenging exercises,
- resolution of problems,
- work in individual and group projects,
- evaluation(s) of progress

Preferably, these items should by developed in a **face-to-face** situation between teacher and students and of students among themselves.

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When this is not possible, **distance learning** educational situations are the alternative.

A brief overview of distance learning

Phases of distance learning:

(i) Post office phase (1728, then 19th to late 20th Century)

(ii) Radio and telephone days (from 1930s to early 21st Century)

(iii) Television era (from late 1960s to early 21st Century)

(iv) Internet epoch (from the 1990s onward)

- email
- e-learning platforms
- MOOCs
- videoconference systems
- autonomous systems (using AI)
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Historically, earlier phases used to survive in more recent ones.

Presently, only phase (iv) exists in any significant way.

Learning mathematics online

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...infusing face-to-face learning with some of the positive characteristics of contemporary Distance Learning !

Learning mathematics online

For distance learning **mathematics** the positive characteristics of contemporary Distance Learning are, among possibly others, the following:

- permanent access to discussions and explanations posted in foruns
- automatic feedback and correction of exercises
- staged progress of knowledge acquisition (e.g., through gamification)
- repeatibility of verbal explanations (videos)
- easy access to materials (software, books, papers)
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Aditionally to these, there is the time honoured characteristic of distance learning: spatial and temporal flexibility.

An example from Lisbon: *Universidade Aberta*

Universidade Aberta (UAb) is the only state owned university in Portugal exclusively dedicated to distance learning.

UAb currently has 13 mathematicians and offers programmes in Mathematics at the levels of: **life long learning** (2), **undergraduate** (2), **master** (2), **doctoral** (1), and several undergraduate and graduate service courses in UAb study programmes.

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We take opportunity of the flexibility of distance learning also to get collaboration of colleagues outside the UAb for teaching specific courses in postgraduate programmes. For instance, the PhD programmes in Applied Mathematics and Modelling and the now closed Computational Algebra have had, along its several editions, teaching collaborators from: **Canada**, **France**, **Greece**, **Spain**, **United Kingdom**, and **United States**.

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A few other occasional collaborations of colleagues from Portuguese universities and polytechnical schools also take place.

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Evaluation depend on the level of the course:

- in undergraduate programmes individual assignements along the semestre are complemented with a final examination (usually on-site; a pilot experiment is under way to make it also online only).
- in postgrad (master and doctoral) programmes there are individual and group assignments that can be complemented by a face-to-face (or zoom) final discussion.

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The following printscreens illustrate the way a course in the PhD programme is run.
Arriving at the Ordinary Differential Equations course one sees...

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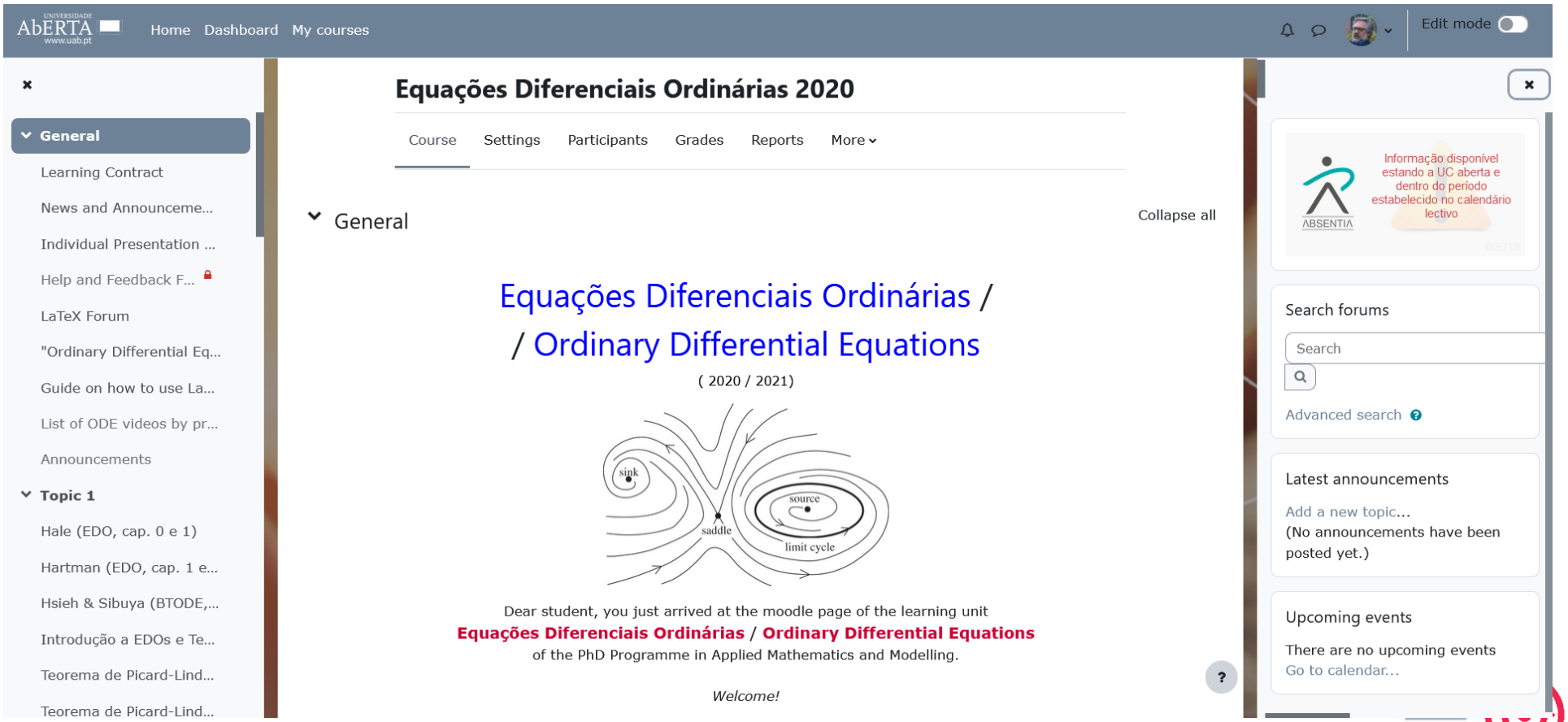

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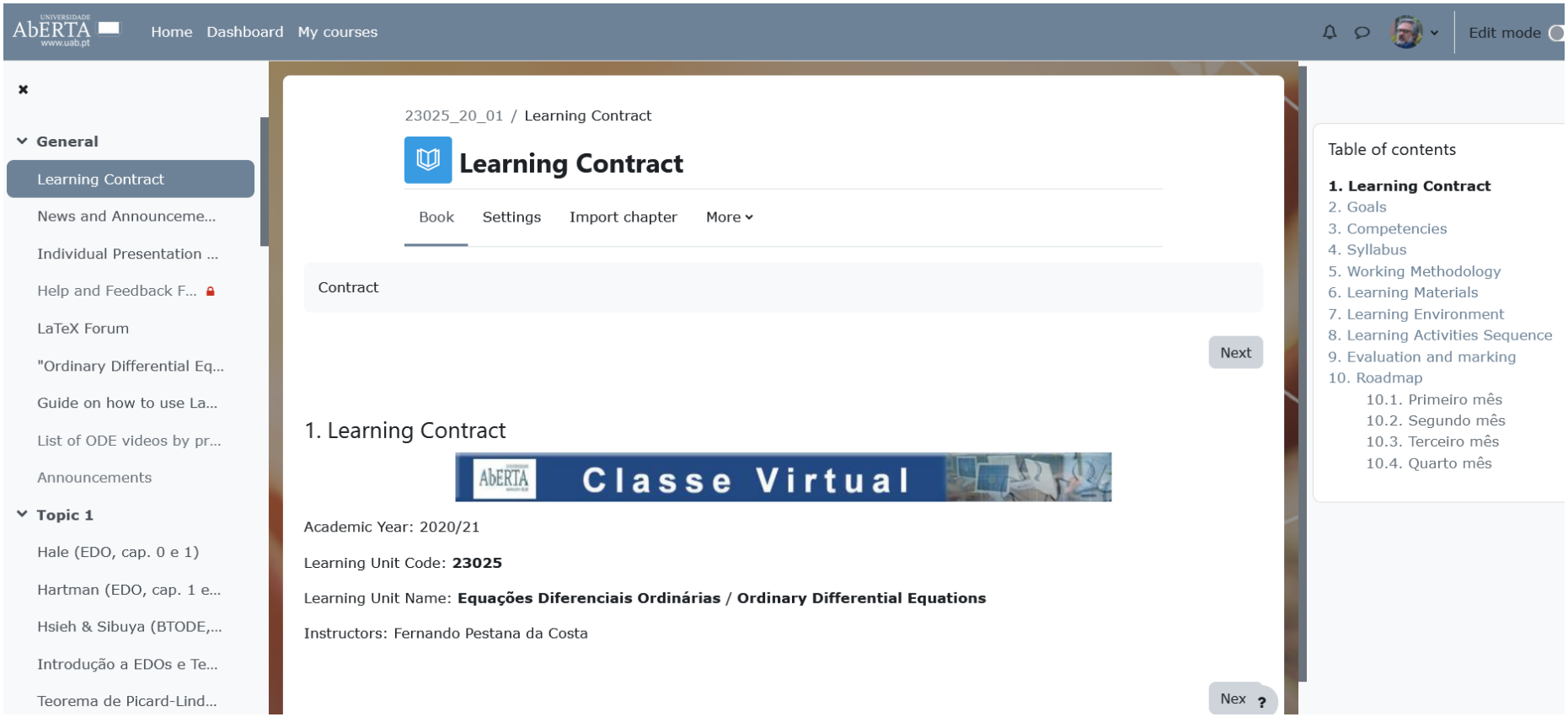

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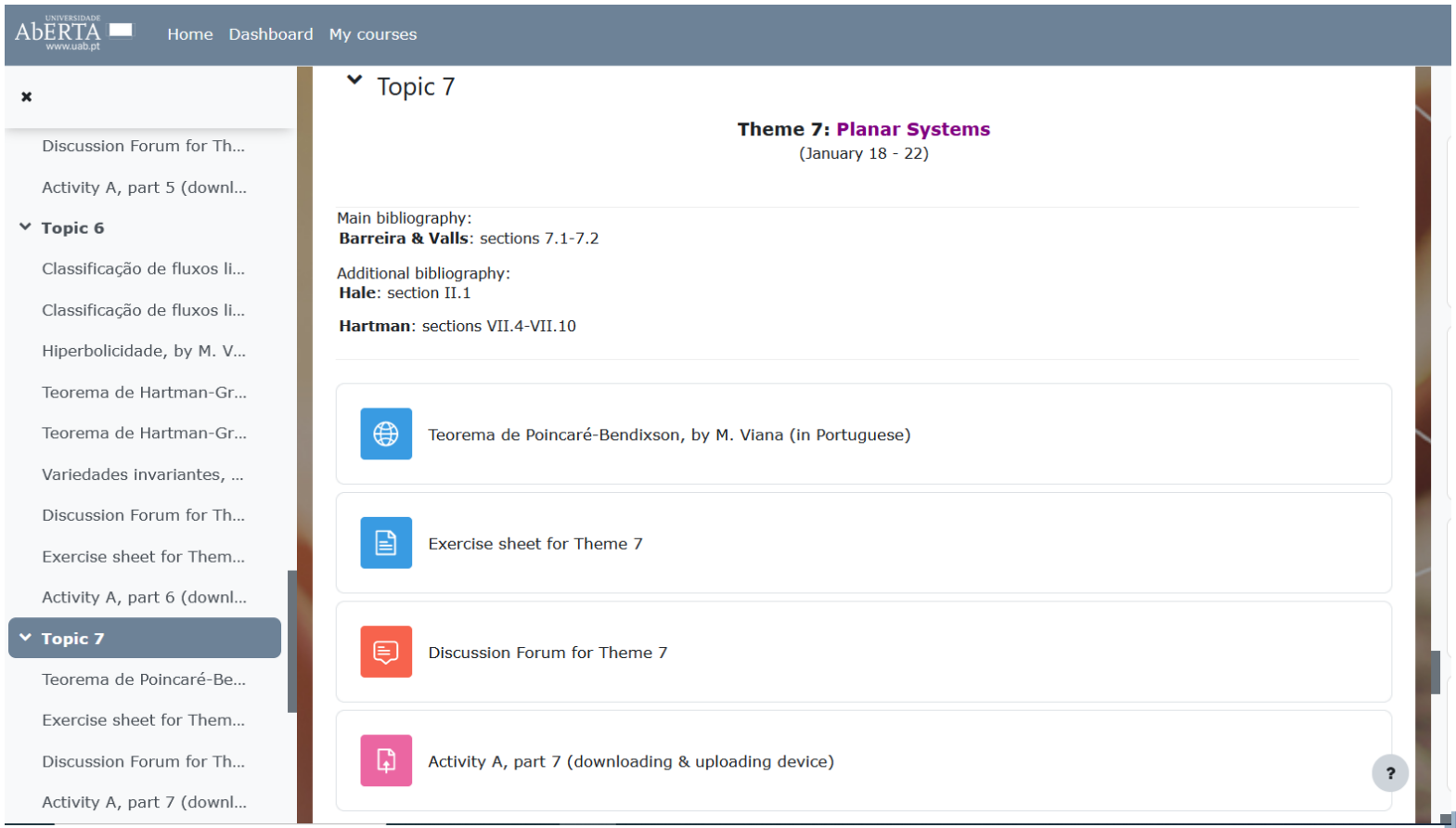

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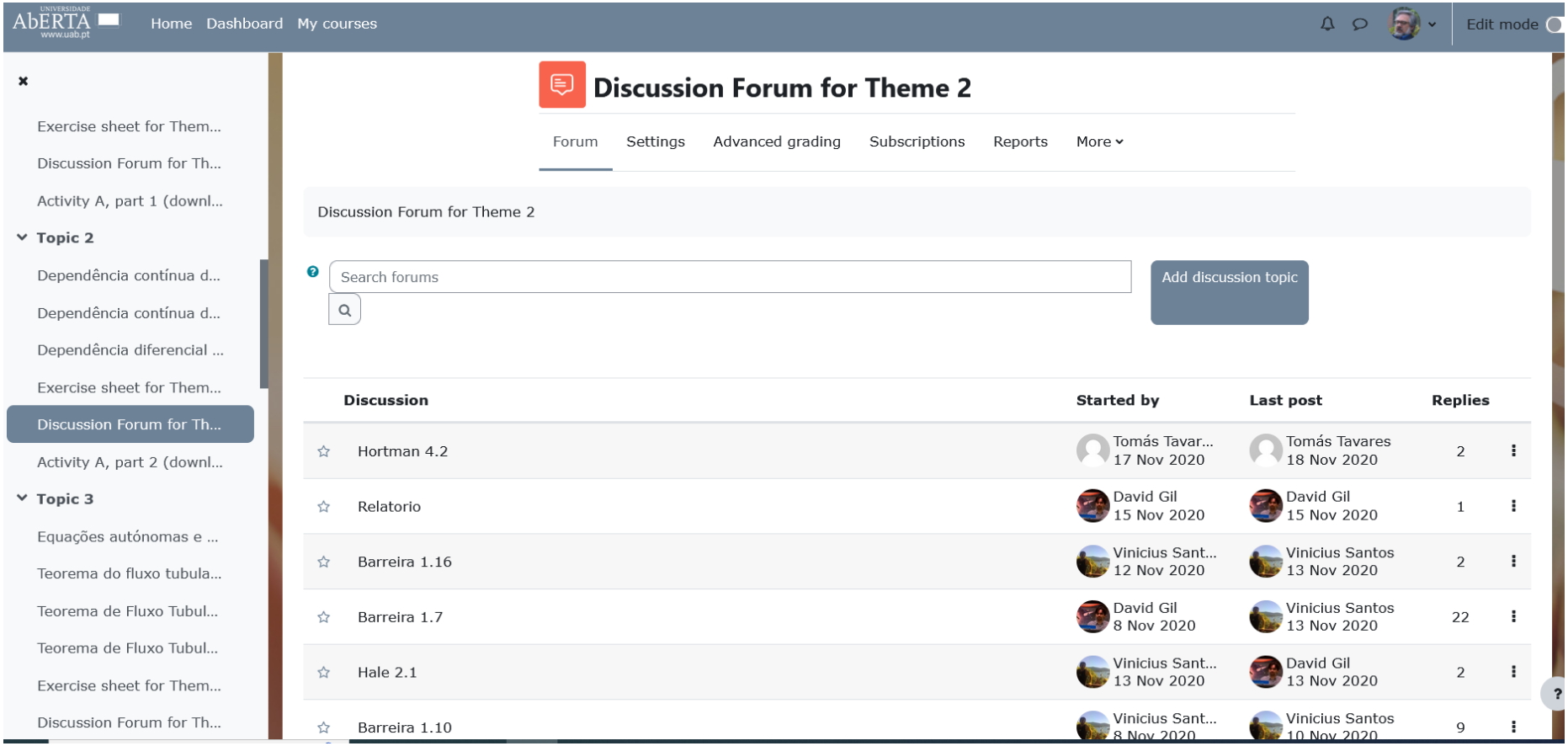

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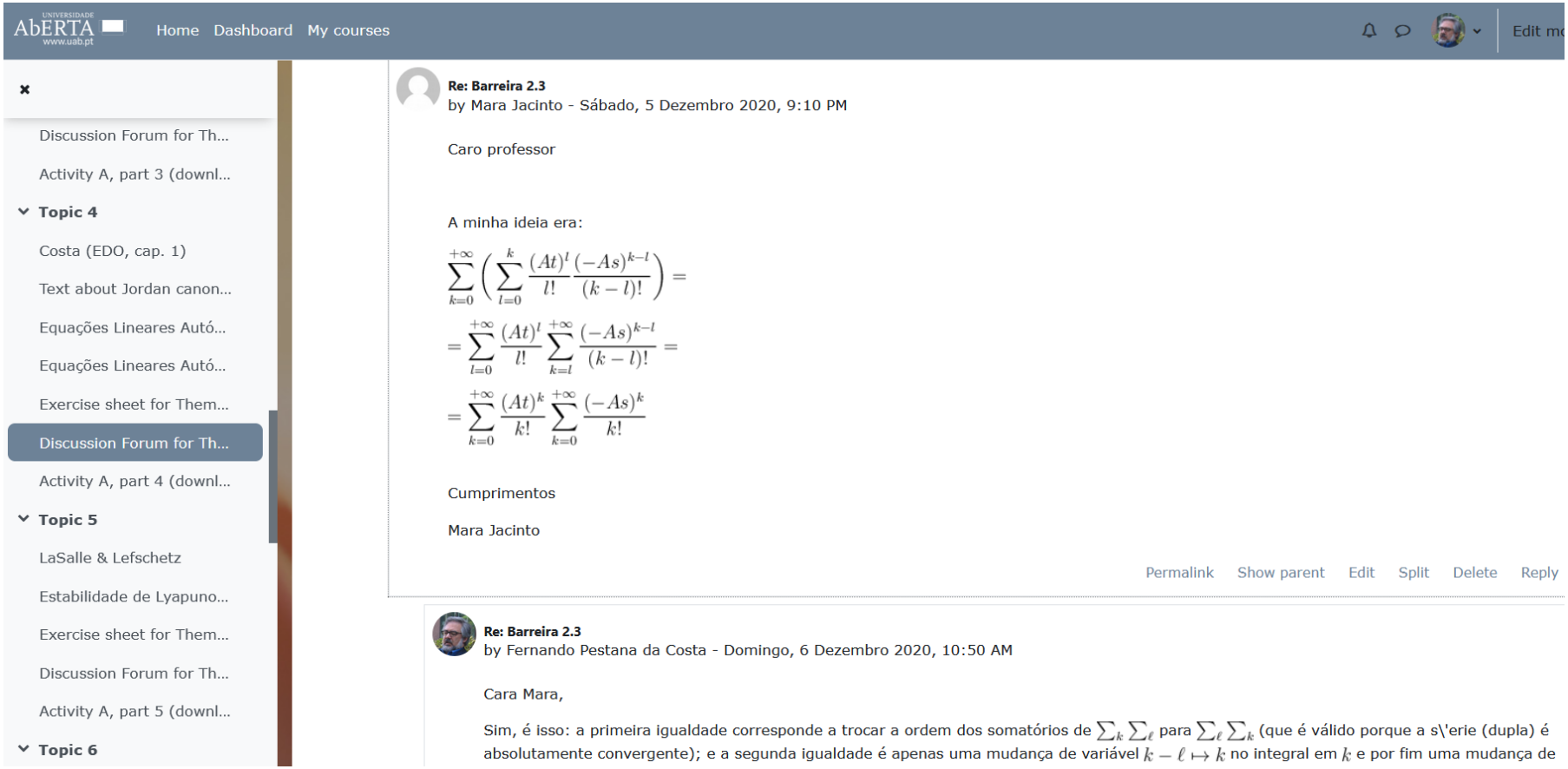

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What can we expect for the near future?

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Presently the fashion is Artificial Inteligence (e.g. **ChatGPT** and the like.)

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Chat GPT *will* clearly be a challenge to online teaching of mathematics, with opportunities and with (many!) drawbacks, namely for the evaluation process.

However the use of ChatGPT for the production of mathematical answers by students is currently limited by its lack of accuracy in less than elementary duties.

What next?

Some recent studies have highlighted ChatGPT potentialities and problems for the teaching of mathematics. I point out to two in the reference list, namely [2] and [5].

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In [5] ("ChatGPT: A revolutionary tool for teaching and learning mathematics"), which is based on interviews, many potentialities and problems are raised, as well as hopes for future developments. I highlight just one phrase (in page 6) that shocked me as killing the *present* usefulness of ChatGPT for production of mathematical text:

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(So what's the point?!)

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> *"Contrary to many positive reports in the media about GPT-4 and ChatGPT's exam-solving abilities (a potential case of selection bias), their overall mathematical performance is well below the level of a graduate student. Hence, if your goal is to use ChatGPT to pass a graduate-level math exam, you would be better off copying from your average peer!*

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FC

factorize x^5- 4*x^4+x^2-34

‹ 2 / 2 ›

To factorize the polynomial x^5 - 4x^4 + x^2 - 34, you can start by factoring out the greatest common factor (GCF) from all terms. In this case, the GCF is 1. Then, you can look for further factorization:

x^5 - 4x^4 + x^2 - 34

= x^4(x - 4) + x^2 - 34

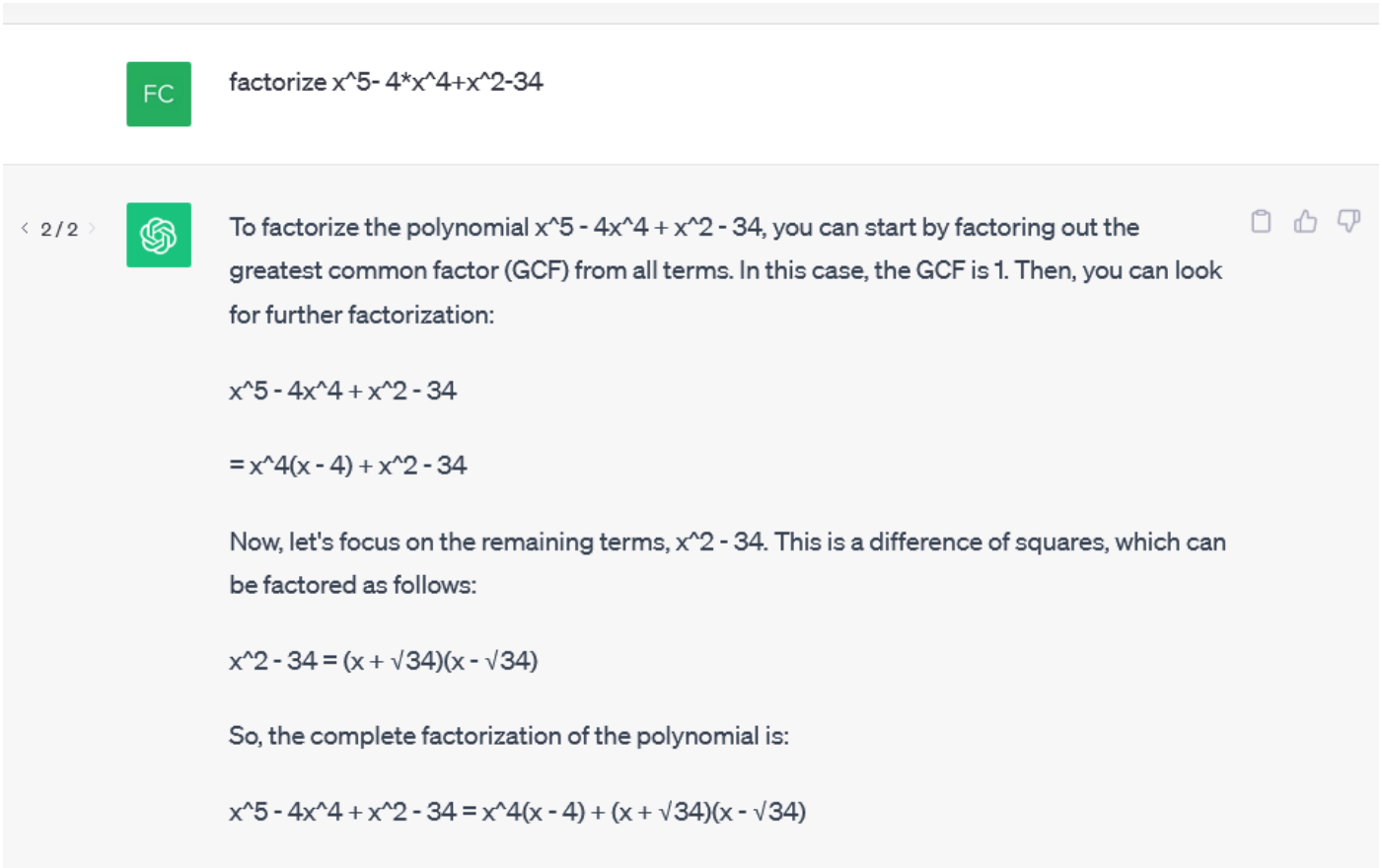
Now, let's focus on the remaining terms, x^2 - 34. This is a difference of squares, which can be factored as follows:

x^2 - 34 = (x + √34)(x - √34)

So, the complete factorization of the polynomial is:

x^5 - 4x^4 + x^2 - 34 = x^4(x - 4) + (x + √34)(x - √34)

Actually, even for elementary mathematics some answers with glaring errors can pop in:



Indeed [2] shows that many of the anwers fall behind the pass mark in most types of queries, although a notorious improvement in GPT-4, when compared with ChatGPT, is clearly noticable in the next chart.

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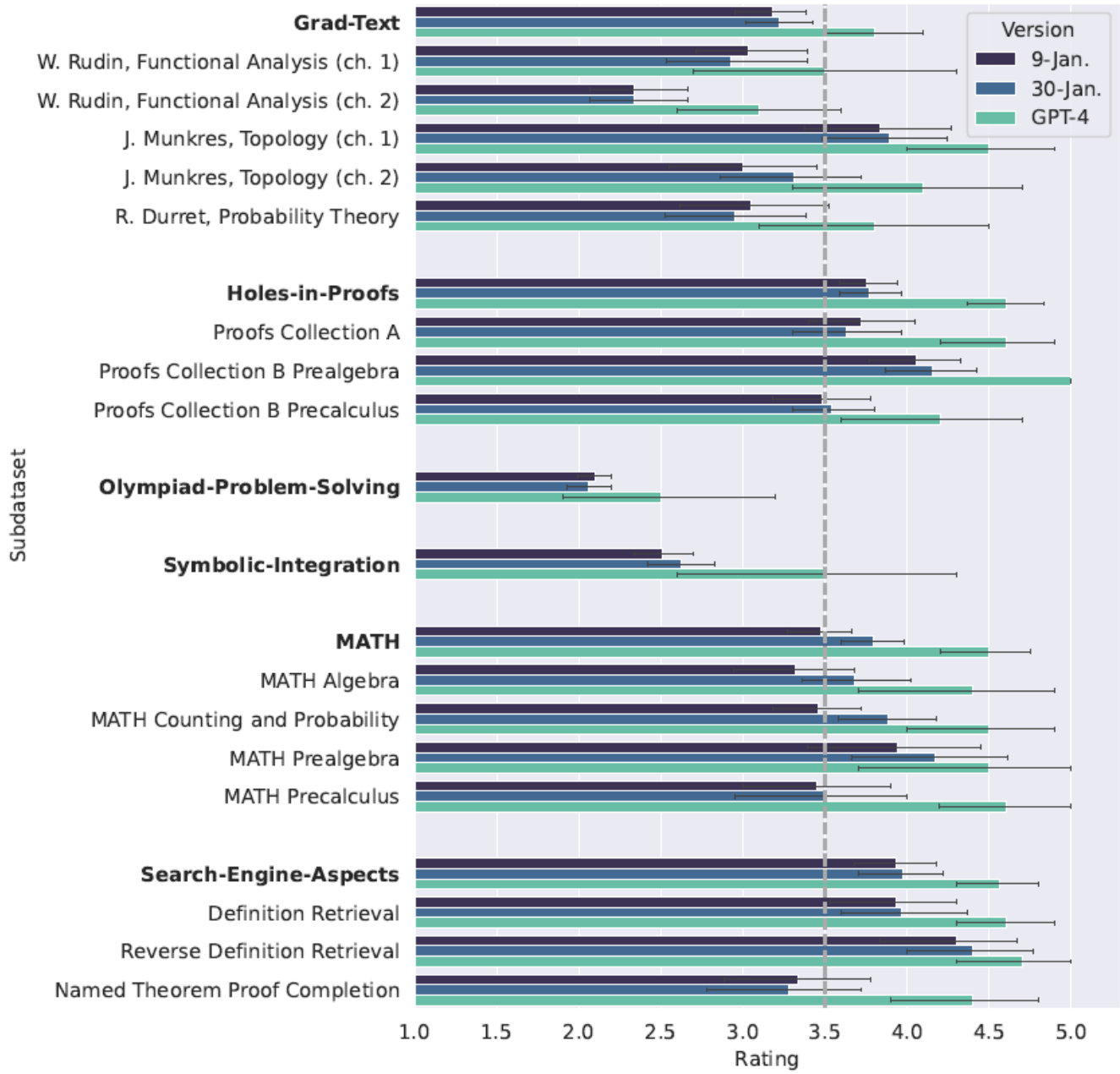
Grad-Text
W. Rudin, Functional Analysis (ch. 1)
W. Rudin, Functional Analysis (ch. 2)
J. Munkres, Topology (ch. 1)
J. Munkres, Topology (ch. 2)
R. Durret, Probability Theory
Holes-in-Proofs
Proofs Collection A
Proofs Collection B Prealgebra
Proofs Collection B Precalculus
Olympiad-Problem-Solving
Symbolic-Integration
MATH
MATH Algebra
MATH Counting and Probability
MATH Prealgebra
MATH Precalculus
Search-Engine-Aspects
Definition Retrieval
Reverse Definition Retrieval
Named Theorem Proof Completion
Subdataset
Version
9-Jan.
30-Jan.
GPT-4
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Rating

What next?

As a tentative and not too risky (actually: almost trivial) **conclusion**:

Several recent studies show that, although presently somewhat lacking, we can expect a fast improvement of Chat-X mathematical capabilities in the near future, which will likely entail important impacts in the way we teach mathematics at university, and more acutely using distance learning models.

Mathematics and Online Education

Some references:

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5. Y. Wardat et al: ChatGPT: A revolutionary tool for teaching and learning mathematics, *EURASIA J. Math. Sci. Tech. Ed.*, **19** (7), (2023) em2286

Thank you very much!

Moltes gràcies!

Muchas gracias!

Muito obrigado!