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Fractional order PD controller design for third order plants ıncluding time delay

Zaman gecikmesi içeren üçüncü derece sistemler için kesir dereceli PD denetleyici tasarımı

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Abstract

Due to the lack of integral operator, proportional derivative controllers have difficulties in providing stability and robustness. This difficulty is especially felt in higher order systems. In this publication, analytical design method of fractional proportional derivative controllers is presented to ensure the stability of third order systems with time delay. In this method, it is aimed to achieve the frequency characteristics of a standard control system to ensure stability. It is aimed to provide the desired gain crossover frequency, phase crossover frequency and phase margin properties of the system. In this way, the stability and robustness of the system can be obtained by choosing the appropriate values. The reason for choosing a fractional order controller is that the controller parameters to provide these features can be tuned more accurately. In order for the obtained stability to be robust to unexpected external effects, it is aimed to flatten the system phase. In the literature, phase flattening is performed by setting the phase derivative to zero at a specified frequency value. This can lead to mathematical complexity. In this publication, the phase flattening process is provided graphically by correctly selecting the frequency characteristics given above. Thus, an accurate and reliable controller design method is presented, avoiding mathematical complexity. The effectiveness of the proposed method has been demonstrated on three different models selected from the literature. The positive contribution of the method to the system robustness has been proven by changing the system gain at certain rates.

Keywords: FOPD, Analytical controller design, TOPTD, Phase flattening.

Öz

İntegral operatörünün eksikliğinden dolayı, oransal türev denetleyiciler kararlılık ve dayanıklılığı sağlama konularında zorlanabilmektedir. Bu zorluk, özellikle yüksek dereceli sistemlerde kendini daha çok hissettirmektedir. Bu yayında, zaman gecikmesi içeren üçüncü derece sistemlerin kararlılığının sağlanması için kesir dereceli oransal türev denetleyicilerin analitik tasarım yöntemi sunulmuştur. Bu yöntemde kararlılığın sağlanması için standart bir kontrol sisteminin sahip olduğu frekans özelliklerine ulaşılması hedeflenmiştir. Sistemin istenen kazanç kesim frekansı, faz kesim frekansı ve faz payı özelliklerini sağlaması hedeflenmiştir. Bu şekilde uygun değerler seçilerek sistemin kararlılığı ve dayanıklılığı elde edilebilecektir. Kesir dereceli bir denetleyicinin seçilme sebebi de bu özelikleri sağlayacak denetleyici parametrelerinin daha doğru şekilde ayarlanabilmesidir. Elde edilen kararlılığın beklenmeyen dış etkilere karşı dayanıklı olması için de sistem fazının düzleştirilmesi hedeflenmiştir. Literatürde faz düzleştirme işlemi, faz türevinin belirlenen bir frekans değerinde sıfırlanması ile gerçekleştirilmektedir. Bu da matematiksel karmaşıklığa yol açabilmektedir. Bu yayında ise faz düzleştirme işlemi yukarıda verilen frekans özelliklerinin doğru şekilde seçilmesi ile grafiksel olarak sağlanmaktadır. Böylece matematiksel karmaşıklıktan kaçınılarak, doğru ve güvenilir bir denetleyici tasarım yöntemi sunulmuştur. Önerilen yöntemin etkinliği literatürden seçilmiş üç farklı model üzerinde gösterilmiştir. Yöntemin sistem dayanıklılığına pozitif katkısı ise sisteme kazancının belli oranlarda değiştirilmesi ile ispatlanmıştır.

Anahtar Kelimeler: FOPD, Analitik denetleyici tasarımı, TOPTD, Faz düzleştirme.

1 Introduction

A better understanding of fractional mathematics has led to the frequent use of this new approach in controller design methods. As known, fractional order controllers have emerged as generalized forms of classical controllers [1]-[3]. The frequently encountered Proportional-Integral-Derivative (PID) controllers have been updated as $PI^{\lambda}D^{\mu}$ (FOPID) so that the degrees of integral and derivative operators can be real numbers [4],[5]. With the lack of derivative or integral operators in these controllers, PI^{λ} (FOPI) and PD^{μ} (FOPD) controllers have formed [6],[7]. It is possible to find numerous studies related to the aforementioned controllers [8]-[10].

Systems expressed with time-delayed plants are widely used in the literature. First, second, or third-order time-delay plants successfully represent real processes [8],[10]-[13]. Different controller design strategies for different plants have been offered, with fractional order controllers demonstrating their benefits [14]-[17]. This paper presents an approach for designing FOPD controllers for third-order time-delay (TOPTD) models. These models have been successfully used in modeling real processes in the industry. For example, TOPTD models have been identified using a technique devised for the

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Hammerstein-Wiener process in [18]. For an electric air heater, a PI and PID controller design is used [19], the pressure of a steam boiler is controlled using a PI controller [20] and an analytical FOPI design scheme for TOPTD plants can be given [21]. PID controller design for high-order systems with time delay and parametric uncertainty using robust state feedback can be found in [22] and for a class of linear systems, an unique fractional order controller design algorithm is presented in [23].

The purpose of the controller design is to provide stability and to ensure that this stability is resistant to unexpected disturbances. A useful way to provide robustness against disturbances is to have the system phase curve flattened at a certain frequency value. This method is frequently used in the literature [24]-[26].

In this study, the phase flattening method was also used to ensure robustness, but a different path was followed for this. In the mentioned studies, when the derivative of the phase equals to zero at the target frequency value, phase flattening is accomplished [27],[28]. Thus, precautions are taken against sudden changes in the phase curve. While this method is useful, it brings computational difficulties. The approach given in this paper follows a graphical way of looking. Detailed information on the method can be found in the following sections.

The FOPD controller is chosen in this paper because of two reasons. The first one is its challenging structure coming from the deficiency of the integral operator. The second one is the number of the parameters to be tuned [12]. These parameters are k_p , k_d and μ. The parameters will be tuned to achieve the gain crossover frequency, the phase crossover frequency and the phase margin. This equalizes the number of the equations to be analytically solved [12].

The majority of comparable research in the literature aim to provide the gain crossover frequency as desired [27],[28]. In spite of this, the controller is tuned in this work to fulfill both gain and phase crossover frequencies, as well as the phase margin. This is one of the contributions that this paper brings. Also, the robustness is provided by a graphical way of looking. The method intends to tune the FOPD controller which is relatively compelling.

The remaining sections are planned in the following way. Section 2 gives the basics of the method. The FOPD controller parameters are derived in Section 3. Section 4 has illustrative instances and Section 5 includes the conclusions.

2 The Bode diagram

This section explains the design basics of the controller parameters. A general representation of a Bode diagram is given in Figure 1.

The major goal of the strategy presented in this work is to assure that the developed controller is stable and robust at the same time. Flattening the phase curve in the figure at the appropriate frequency is one way used in the literature to achieve this. The phase curve is often flattened at this frequency value by equating the derivative of the phase to zero at the gain crossover frequency, according to studies in the literature [27],[28]. While this method is useful, it makes computation quite difficult. The method proposed in this publication takes a different route. According to the method presented here, the gain crossover frequency, the phase crossover frequency, and the phase margin of the system can be fixed to the desired values. Thus, a flattening can be achieved depending on these values. For example, when the difference between the gain crossover frequency and the phase crossover frequency is increased and the phase margin is decreased, between the two crossover frequencies, the phase curve will be considerably flattened. Thus, while increasing system robustness, no computational cost will be incurred.

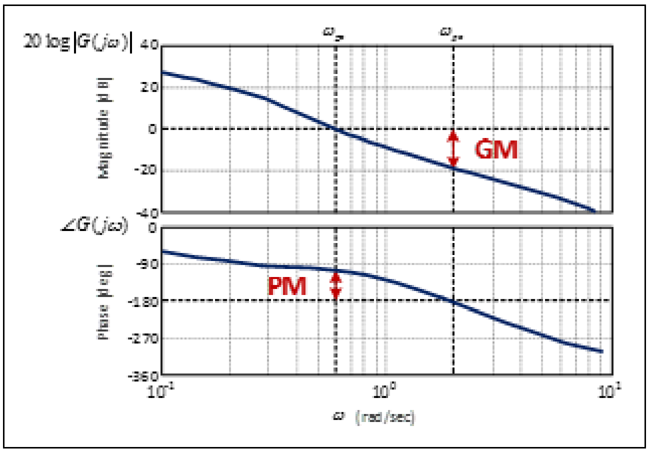

Figure 1. An example Bode diagram.

3 FOPD design for TOPTD models

The design steps of the FOPD controller for TOPTD model are given in this section. The following equations stand for the general representation of a FOPD controller and a TOPTD plant respectively.

$$C(s)=k_p+k_d s^{\mu} \tag{1}$$

$$P(s)=\frac{K}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}\mathrm{e}^{-Ls} \tag{2}$$

Thus, the system is $G(s)=C(s)P(s)$ and its frequency response is $G(j\omega)=C(j\omega)P(j\omega)$. Here, $P(j\omega)$ and $C(j\omega)$ are written as follows,

$$P(j\omega)=\left(\begin{array}{l}-\dfrac{K\left(-1+T_2T_3\omega^2+T_1\left(T_2+T_3\right)\omega^2\right)}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)\left(1+T_3^2\omega^2\right)}\\ +j\left(\dfrac{K\omega\left(-T_2-T_3+T_1\left(-1+T_2T_3\omega^2\right)\right)}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)\left(1+T_3^2\omega^2\right)}\right)\end{array}\right)\mathrm{e}^{-jL\omega} \tag{3}$$

$$C(j\omega)=k_p+k_d\omega^{\mu}\cos\left(\pi\mu/2\right)+jk_d\omega^{\mu}\sin\left(\pi\mu/2\right) \tag{4}$$

After that, both the plant and the controller's magnitude and phase may be put down as,

$$\left|P(j\omega)\right|=\sqrt{\frac{K^2}{\left(1+T_1^2\omega^2\right)\left(1+T_2^2\omega^2\right)\left(1+T_3^2\omega^2\right)}} \tag{5}$$

$$\angle P(j\omega)=-\arctan\left(\frac{\omega\left(-T_2-T_3+T_1\left(-1+T_2T_3\omega^2\right)\right)}{-1+T_2T_3\omega^2+T_1\left(T_2+T_3\right)\omega^2}\right)-L\omega \tag{6}$$

$$\left|C(j\omega)\right|=\sqrt{\left(k_p+k_d\omega^{\mu}\cos\left(\pi\mu/2\right)\right)^2+\left(k_d\omega^{\mu}\sin\left(\pi\mu/2\right)\right)^2} \tag{7}$$

$$\angle C(j\omega) = \arctan\left(\frac{k_d\omega^{\mu}\sin(\pi\mu/2)}{k_p + k_d\omega^{\mu}\cos(\pi\mu/2)}\right) \tag{8}$$

After finding the frequency responses of the plant and the controller, the controller parameters could be calculated. The controller parameters have to satisfy the following phase properties of a standard control system.

System gain and phase values at the gain crossover frequency are respectively,

$$\left|G(j\omega_{gc})\right| = 1 \tag{9}$$

$$\angle G(j\omega_{gc})\big| = PM - \pi \cdot \tag{10}$$

Similarly, system gain and phase at the phase crossover frequency are,

$$\left|G(j\omega_{pc})\right| = 10^{GM/20} \tag{11}$$

$$\angle G(j\omega_{pc})\big| = -\pi \cdot \tag{12}$$

After the brief information, there will be presented two controllers in this section.

3.1 The first controller

Gain and phase of the system may be stated using Eqs. 9 and 10 as,

$$\begin{aligned}&\left|G(j\omega_{gc})\right| = \\ &\sqrt{\left(k_p + k_d\omega_{gc}^{\mu}\cos(\pi\mu/2)\right)^2 + \left(k_d\omega_{gc}^{\mu}\sin(\pi\mu/2)\right)^2} \\ &\times\sqrt{\frac{K^2}{\left(1+T_1^2\omega_{gc}^2\right)\left(1+T_2^2\omega_{gc}^2\right)\left(1+T_3^2\omega_{gc}^2\right)}}\end{aligned} \tag{13}$$

$$\begin{aligned}&\angle G(j\omega_{gc}) = \\ &\arctan\left(\frac{k_d\omega_{gc}^{\mu}\sin(\pi\mu/2)}{k_p + k_d\omega_{gc}^{\mu}\cos(\pi\mu/2)}\right) \\ &-\arctan\left(\frac{\omega_{gc}\left(-T_2 - T_3 + T_1\left(-1+T_2T_3\omega_{gc}^2\right)\right)}{-1+T_2T_3\omega_{gc}^2 + T_1\left(T_2+T_3\right)\omega_{gc}^2}\right) - L\omega_{gc}\end{aligned} \tag{14}$$

Solving Eq. 13 and Eq. 14 together with the information $\left|G(j\omega_{gc})\right| = 1$ and $\angle G(j\omega_{gc}) = PM - \pi$, k_p and k_d parameters of the first controller can be found as given below.

$$\begin{aligned}k_p = &\pm\frac{\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}\sqrt{1+T_3^2\omega_{gc}^2}}{K\sqrt{1+\tan(\varphi_1)^2}} \\ &\mp\frac{\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}\sqrt{1+T_3^2\omega_{gc}^2}\cot(\pi\mu/2)\tan(\varphi_1)}{K\sqrt{1+\tan(\varphi_1)^2}}\end{aligned} \tag{15}$$

$$k_d = \pm\frac{\omega_{gc}^{-\mu}\sqrt{1+T_1^2\omega_{gc}^2}\sqrt{1+T_2^2\omega_{gc}^2}\sqrt{1+T_3^2\omega_{gc}^2}\csc(\pi\mu/2)\tan(\varphi_1)}{K\sqrt{1+\tan(\varphi_1)^2}} \tag{16}$$

Here, φ_1 is,

$$\varphi_1 = PM - \pi + \arctan\left(\frac{\omega_{gc}\left(-T_2 - T_3 + T_1\left(-1+T_2T_3\omega_{gc}^2\right)\right)}{-1+T_2T_3\omega_{gc}^2 + T_1\left(T_2+T_3\right)\omega_{gc}^2}\right) + L\omega_{gc} \tag{17}$$

3.2 The second controller

Again considering Eq. 11 and Eq. 12, the gain and phase of the system can be written as given in Eq. 13 and Eq. 14. Then, solving these equations together with the information $\left|G(j\omega_{pc})\right| = 10^{GM/20}$ and $\angle G(j\omega_{pc})\big| = -\pi$, k_p and k_d parameters of the second FOPD controller can be found as given below.

$$\begin{aligned}k_p = &\pm\frac{10^{GM/20}\sqrt{1+T_1^2\omega_{pc}^2}\sqrt{1+T_2^2\omega_{pc}^2}\sqrt{1+T_3^2\omega_{pc}^2}}{K\sqrt{1+\tan(\varphi_2)^2}} \\ &\mp\frac{10^{GM/20}\sqrt{1+T_1^2\omega_{pc}^2}\sqrt{1+T_2^2\omega_{pc}^2}\sqrt{1+T_3^2\omega_{pc}^2}\cot(\pi\mu/2)\tan(\varphi_2)}{K\sqrt{1+\tan(\varphi_2)^2}}\end{aligned} \tag{18}$$

$$k_d = \pm\frac{10^{GM/20}\omega_{pc}^{-\mu}\sqrt{1+T_1^2\omega_{pc}^2}\sqrt{1+T_2^2\omega_{pc}^2}\sqrt{1+T_3^2\omega_{pc}^2}\csc(\pi\mu/2)\tan(\varphi_2)}{K\sqrt{1+\tan(\varphi_2)^2}} \tag{19}$$

Here, φ_2 is,

$$\varphi_2 = -\pi + \arctan\left(\frac{\omega_{pc}\left(-T_2 - T_3 + T_1\left(-1+T_2T_3\omega_{pc}^2\right)\right)}{-1+T_2T_3\omega_{pc}^2 + T_1\left(T_2+T_3\right)\omega_{pc}^2}\right) + L\omega_{pc} \tag{20}$$

In the next step, both controllers will be combined to achieve the desired properties simultaneously.

3.3 The combined controller

In this step, the common parameter μ of both controllers is calculated. Hence, two controllers could be combined into one. For this purpose, k_p parameters in Eq. 15 and Eq. 18 and k_d parameters in Eq. 16 and Eq. 19 are numerically calculated in the range of $\mu \in (0,2)$ to find the common μ satisfying both equations. This can be done by plotting equations of $10^{GM/20}$ w.r.t μ and finding the intersection point of the curves. Then, this value can be replaced in the related equations to find the combined controller.

4 Applications

This section offers examples to demonstrate the use of the suggested equations.

4.1 Example 1

The first example investigates the TOPTD plant provided from [29]. Here, parameters of the plant are given as, $K = 1s,\ T_1 = 10s,\ T_2 = 1s,\ T_3 = 0.1s,\ L = 0.1s$. It is aimed to tune the crossover frequencies as $\omega_{gc} = 0.05 rad/s$ and $\omega_{pc} = 0.80 rad/s$ and the phase margin as $30°$. In order to find the common controller to satisfy the required properties, related variables are replaced in Eq. 15 to Eq 17 and Eq. 18 to Eq. 20. Plots of $10^{GM/20}$ w.r.t μ are illustrated in Figure 2.

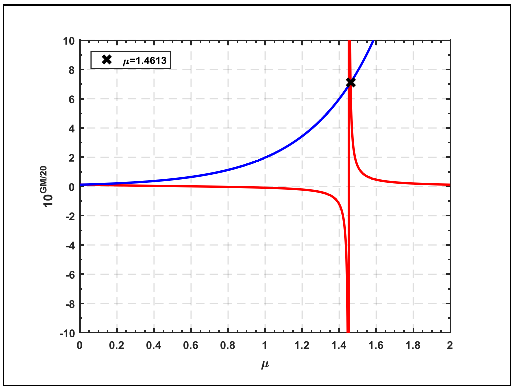

Figure 2. Plots of $10^{GM/20}$ w.r.t μ .

According to Figure 2, the common μ value for the FOPD controller is found as 1.4613. Thus, the following controller is obtained in this case.

$$C(s) = 1.418030 + 103.145s^{1.4613} \tag{21}$$

With the controller given in Eq. 21, the Bode plot of the system $G(s) = C(s)P(s)$ is obtained as given in Figure 3.

It is clear in Figure 3 that the desired phase specifications are successfully achieved with the tuned controller. Besides, the system acquired improved robustness against undesired changes in the plant gain. To prove this, step response of the system with an integrator is illustrated in Figure 4. Step responses of the system with $\pm 50\%$ iterations of the plant gain are also given in Figure 4.

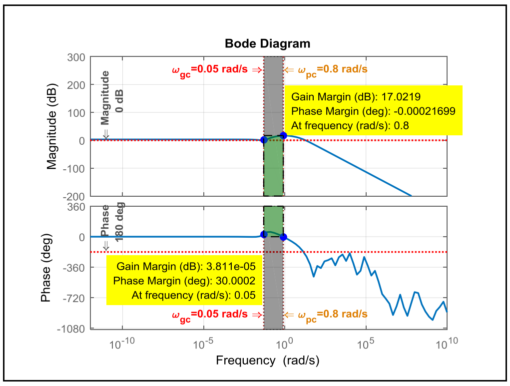

Figure 3. Bode plot with desired frequency specifications.

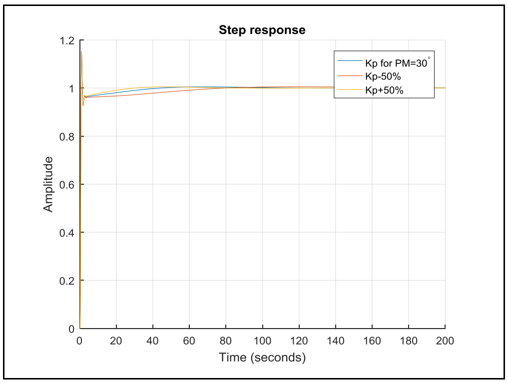

Figure 4. The original system's step response and systems with $\pm 50\%$ plant gain iterations.

As known, the proportional-derivative controller mostly yields to a steady-state error due to its lack of integral operator. To prevent this, an integrator is applied to the system to obtain its step response. Without the integrator, the system keeps its stability. However, the integrator process corrects the steady state condition of the system by approximately 40%. According to Figure 4, the tuned controller successfully prevented the system stability against gain changes up to $\pm 50\%$. This proved the effectiveness of the design scheme. We can also obtain the Bode plots and the step responses of the system for changing values of the phase margin. Figure 5 illustrates the Bode plots of the system for $PM = [10° - 90°]$. Similarly, Figure 6 shows the step responses of the systems with integrator for $PM = [10° - 90°]$.

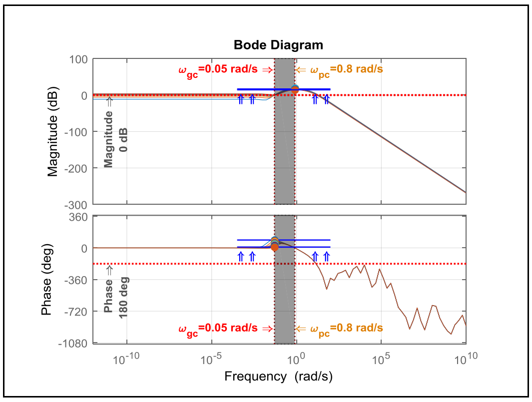

Figure 5. Bode plots of the systems for $PM = [10° - 90°]$.

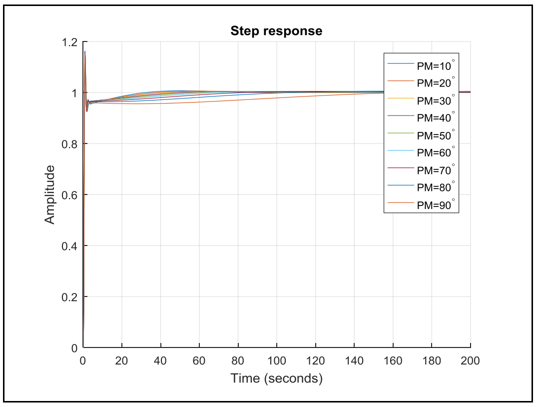

Figure 6. Step responses of the system for $PM = [10° - 90°]$.

It will be helpful to apply the method to another example.

4.2 Example 2

Let us study on the plant in the paper of Vivek et al. [30]. Parameters of the TOPTD plant are given as, $K = 0.125s,\ T_1 = 1s,\ T_2 = 1s,\ T_3 = 1s,\ L = 1s$. The crossover frequencies are aimed to be $\omega_{gc} = 0.2 rad/s$ and $\omega_{pc} = 0.8 rad/s$. The phase margin is desired to be $60°$ fort his example. The common fractional order is calculated using the intersection points of the plots of $10^{GM/20}$ w.r.t μ . These plots are given in Figure 7.

Thus the FOPD controller is found as,

$$C(s) = 1.418030 + 103.145s^{1.4613} \tag{22}$$

Figure 8 shows the system's Bode plot. The Bode's phase curve, as can be observed, satisfactorily matches the specified parameters.

As is well known, systems can fail as a result of unanticipated load effects. A load disturbance of 10% of the plant gain is introduced to the system to demonstrate the system's resilience. Figure 9 shows the step responses of both the original and load-disturbed systems.

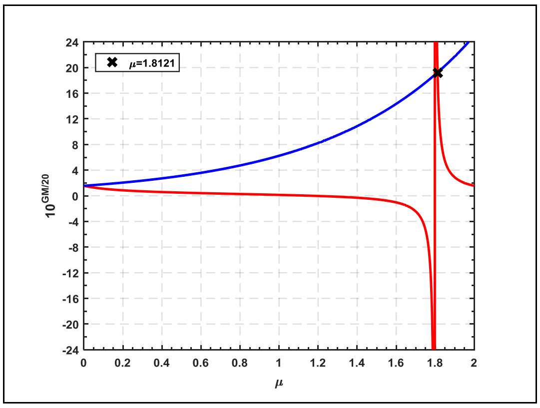

Figure 7. Plots of $10^{GM/20}$ w.r.t μ .

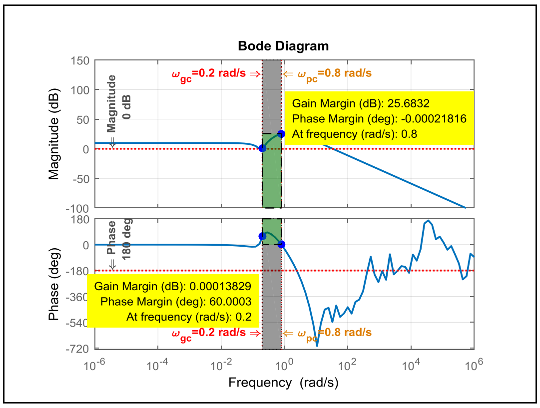

Figure 8. Bode plot with desired frequency specifications.

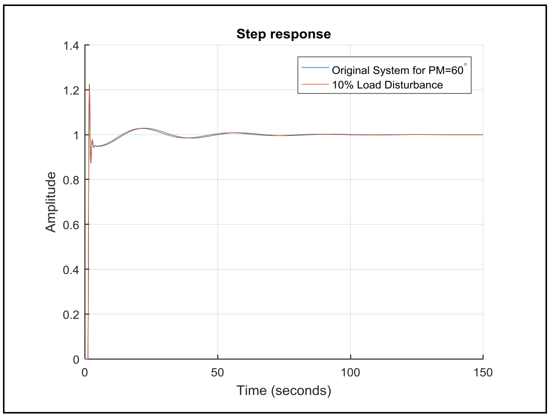

Figure 9. Step responses of the original system and the system with 10% load disturbance.

Figure 9 shows clearly that the step response almost did not show any change thus, the system gained improved robustness against unexpected disturbances. Let us consider another example.

4.3 Example 3

In the study of Sung et al., the TOPTD plant has the parameters of $K = 3.2764s$, $T_1 = 1s$, $T_2 = 1s$, $T_3 = 1s$, $L = 5s$ [31]. The FOPD controller will be tuned to achieve $\omega_{gc} = 0.12 rad/s$ and $\omega_{pc} = 0.36 rad/s$. For the PM in the interval $PM = [10° - 90°]$, Table 1 gives the FOPD controller parameters.

Table 1. FOPD controller parameters for $PM = [10° - 90°]$.

PM	k_p	k_d	μ
10°	1.187950	53.3290	1.83360
20°	1.197770	56.2820	1.83233
30°	1.171170	57.5239	1.83115
40°	1.108970	57.0173	1.82999
50°	1.013050	54.7773	1.82878
60°	0.886328	50.8721	1.82742
70°	0.732643	45.4200	1.82579
80°	0.556654	38.5862	1.82365
90°	0.363680	30.5773	1.82048

The Bode plots of the systems with the controllers in Table 1 are given in Figure 10. It can be clearly seen in Figure 10 that the gain crossover frequency is tuned to be $\omega_{gc} = 0.12 rad/s$ and the phase crossover frequency is tuned as $\omega_{pc} = 0.36 rad/s$. Similarly, the step responses of the systems with an integrator are given in Figure 11.

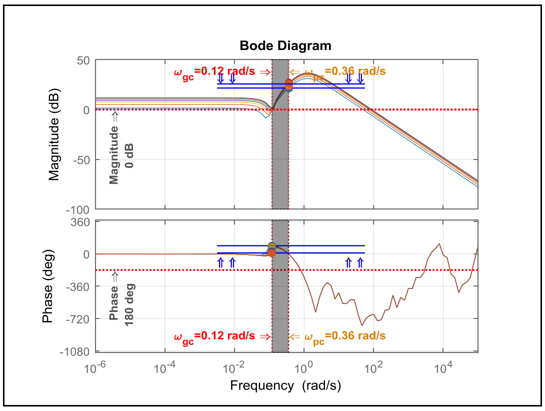

Figure 10. Bode plots of the systems in Table 1.

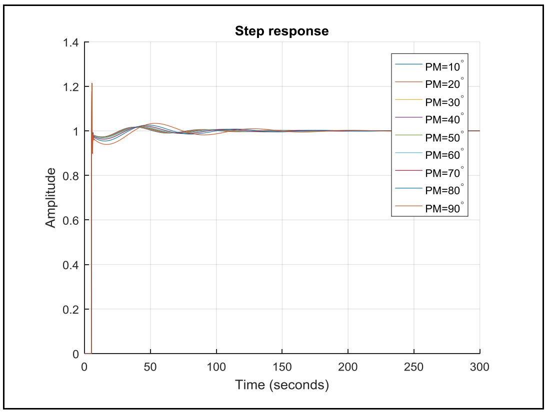

Figure 11. Step responses of the systems with the controllers in Table 1.

According to Figure 11 the system stability is assured and this shows the reliability of the proposed method.

It would be enlightening to give information about the selection of the frequency values. In the first two examples, the desired phase margins were selected intentionally. It has been observed that 30° and 60° values for the phase margin were frequently encountered in the literature. As mentioned before, setting the crossover frequencies within an increased range and decreasing the phase margin will considerably flatten the phase curve. With the values in these examples, it is aimed to show the effect of the proposed method with various phase margin values. Besides, the third example evaluates the method for $PM=[10°-90°]$. As known, every system has its own stable frequency interval. Here, the gain and phase crossover frequency values were inspired from similar studies in the literature. Also, these values were tested with the well-known Stability Boundary Locus (SBL) method. Main reason for selecting the frequency values were to prove the efficiency of the proposed method for different conditions.

5 Conclusions

A frequency domain-based method is presented to tune the FOPD controller. The tuning method relies on the properties of a standard control system. For stability issues, the gain crossover frequency, the phase crossover frequency and the phase margin could freely be tuned with this method. Tuning these specifications properly will also influence the robustness of the system in that the Bode plot's phase curve will be somewhat flattened with correct adjustment. Flattening of the phase curve is widely used in existing studies. The number of parameters to be tuned is one of the reasons why a fractional order controller was chosen. These parameters are the gain crossover frequency, the phase crossover frequency and the phase margin. As the number of these parameters is three, the fractional order proportional derivative controller has three parameters. As known, tuning the proportional derivative controller is a challenging issue because of the lack of the integral operator. The analytically tuned controller in this paper is applied on three different third-order plus time delay plants provided from the literature and the effect is proved.

6 Contribution statements by authors

Uğur DEMİROĞLU contributed to this work in the headings of concept generation and literature review. Bilal ŞENOL was involved in the design and evaluation of the outcomes. Radek MATUŠU was responsible for double-checking the spelling and substance of the article.

7 Approval by the ethics committee and a conflict of interest statement

The ethics committee does not need to approve the essay, and there are no conflicts of interest with anybody or any institution.

8 References

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