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Approximating Ramping Constraints in Hydropower Scheduling

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***Abstract*—This work concerns the modeling of ramping constraints on discharge in medium-term hydropower scheduling models applied in a liberalized market context. Such models often apply a coarse time discretization to ensure reasonable computation times. Consequently, ramping constraints at a fine time-resolution are challenging to represent. To address this challenge, we derive a quadratic transition-cost term capturing the power production shifted to time periods with less favorable prices due to ramping constraints. We approximate the quadratic term by linearization so that it can be embedded in an existing hydropower scheduling tool based on stochastic linear programming. A prototype hydropower scheduling model, including the approximated transition-cost term, was tested on a realistic hydropower system in Norway. We demonstrate that the improved modeling of ramping constraints significantly impacts discharge patterns and comes at a significant, but not prohibitive, increase in computation time.**

***Index Terms*—Hydroelectric power generation, Power generation economics, Linear programming, Stochastic processes, Environmental constraints.**

I. Introduction

Environmental requirements associated with hydropower operation are changing, e.g., through proposed revisions of hydropower concessions and the implementation of EU Water Framework Directive [1]. The directive strives to ensure sustainable use of water resources, balancing the multiple uses such as hydropower, irrigation, water supply, flood control, and recreation. In this context, hydropower producers need to adjust their operational schedules so that the environmental constraints are respected. Consequently, the producers need scheduling models that represent environmental constraints in a precise and consistent manner.

This paper concerns the maximum allowed changes in water flows in a hydropower system, often referred to as maximum allowed ramping, or as ramping constraints. We focus on ramping constraints on discharge through power stations. Such constraints serve to reduce negative effects of rapid and frequent changes in the flow downstream of the hydropower outlets. This type of constraints has been studied in the existing literature [2], [3], but the approximation techniques presented here have, to the best of the authors' knowledge, not been studied before.

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We contribute to the existing literature by presenting a new technique for approximating constraints on discharge ramping within a medium-term hydropower scheduling model. The applied model is based on the stochastic dual dynamic programming (SDDP) method, and is suited for optimizing hydropower systems while treating inflow and power price as exogenous stochastic variables [4].

II. Model

A. Hydropower Scheduling Model

In this work we are concerned with medium-term hydropower scheduling in a liberalized market context. We assume that the hydropower producer is a risk-neutral price-taker trying to optimize the expected profit from sales of electricity in the day-ahead power market. For this purpose we use a prototype of the software ProdRisk [5] which is based on the SDDP algorithm [6], where stochastic market prices are treated in an outer layer based on stochastic dynamic programming (SDP). This combined SDDP/SDP model for medium-term scheduling was originally described in [7], and has later been discussed in [4], and subject to recent research, e.g., related to multiple market prices [8] and maintenance scheduling [9]. In the following we present a high-level description of the optimization problem, before emphasizing the treatment of ramping constraints. Note that several technical features in ProdRisk are omitted in the presentation for brevity. We refer to these previous works [4], [7]–[9] for a comprehensive description of the combined SDDP/SDP algorithm.

A simplified version of the optimization problem to be solved is formulated as

$$\max \mathbb{E} \left\{ \sum_{t=1}^{T} \boldsymbol{\lambda}_t^{\intercal} \mathbf{x}_t + \Phi(\mathbf{s}_T) \right\} \quad , \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, considering uncertainty in inflows and market prices, and $\Phi(\mathbf{s}_T)$ the end of horizon valuation of state variables in $\mathbf{s}_T$. For all stages t in the planning horizon $1 \cdots T$, a vector $\mathbf{x}_t$ is defined, comprising all decision variables for that stage. Associated with $\mathbf{x}_t$ there is a price vector $\boldsymbol{\lambda}_t$.

The problem in (1) is a multi-stage stochastic optimization problem, in which we assume all functional relationships to be linear or linearly approximated. The combined SDDP/SDP algorithm facilitates decomposition into linear programming (LP) problems per decision stage t, according to (2).

$$Z_t = \max \sum_{k=1}^{NK} \tau_k \lambda_k p_{hk} + \alpha_t \tag{2a}$$

$$v_{hk} - v_{h,k-1} + \Gamma_w \left(q_{hk} + s_{hk} \right) = I_{hk} \qquad \forall h, k \tag{2b}$$

$$q_{hk} = \sum_{s=1}^{NS} q_{hks} \qquad \forall h, k \tag{2c}$$

$$p_{hk} = \sum_{s=1}^{NS} \eta_s q_{hks} \qquad \forall h, k \tag{2d}$$

$$\alpha_t - \sum_{h=1}^{NH} \pi_{hc} v_{hk} \leq \beta_c \qquad k = NK, \forall c \tag{2e}$$

In this work the decision stage t covers one week where inflows and market prices are perfectly known. Each decision stage t comprises a sequence of time steps $k = 1, .., NK$. A hydropower system represented by $h = 1, .., NH$ hydropower modules (combination of reservoir and plant) is optimally scheduled by maximizing the balance between here-and-now profit and future expected profit. The here-and-now profit is represented by the product of power price λ_k (in €/MWh) and the scheduled generation p_{hk} (in MW) summed over all hydropower modules and all time steps within the week, and multiplied by the duration of each time step τ_k (in hours). The future expected profit is represented by α_t (in €).

The reservoir balance in (2b) ensures that the reservoir storage volume v_{hk} (in Mm^3) is balanced with discharge q_{hk} (in m^3/s), spillage s_{hk} (in m^3/s) and inflow I_{hk} (in Mm^3). Γ_w converts from m^3/s to Mm^3. A hydropower production function is presented in (2d), converting discharge q_{hks} at segments $s = 1, .., NS$ to power (in MW). The efficiency coefficients η_s (in $MW/m^3/s$) are assumed to be decreasing with increasing s to present production as a concave function of discharge. Benders cuts in (2e), including cut coefficients π_{hc} (in €/Mm^3) and right-hand side β_c (in €), represent the expected future profit α_t. Additional constraints and variable bounds may be included depending on the type of system and study.

B. Ramping Constraints

Ramping constraints on discharge are formulated as

$$-\tau_k \Delta_R^- \leq q_k - q_{k-1} \leq \tau_k \Delta_R^+ \ , \tag{3}$$

where Δ_R^- and Δ_R^+ are the maximum allowed ramping rates for downward (-) and upward (+) ramping (in $m^3/s/h$). From (3) it is clear that the allowed band for ramping increases with increasing duration of the time steps τ_k. As an example, if Δ_R^+ = 10 $m^3/s/h$ and τ_k = 6 hours, the model allows ramping up 60 m^3/s between two consecutive time steps.

With small time steps, ramping according to Δ_R can be controlled at a desired precision level. However, there are practical and computational challenges associated with it. Since the input data series, e.g., for electricity prices, typically have hourly time resolution, there are no other reasons for adopting $\tau_k < 1$ hour. Moreover, the weekly decision problems grow with the number of time steps, leading to prohibitive computation times. Certain types of ramping constraints, on the other hand, would require time resolutions of 10-15 min for sufficient accuracy.

Next, we discuss possible measures for dealing with the above-mentioned challenges with ramping constraints. A different take on ramping is presented in [10], where the scheduling is made in continuous time. Concepts from the continuous time methodology may be worthwhile further exploration in this context, but we consider it out of scope in this work. The constraint of type (3) is formulated as an inequality and is therefore not necessarily binding. Relaxation of equation (3) provides a possibility for computational speed-up. By relaxation we refer to the process of first solving the LP problem without the constraints (they are 'relaxed'). Subsequently, from the solution of the relaxed LP problem, we identify the inequalities being violated, and add these before the LP problem is re-solved. This procedure is repeated until no more constraint violations are detected. Relaxation is already used in our scheduling model when treating cuts in the weekly decision problem. Relaxation can be combined with more or less sophisticated methods for predicting which constraints to include but was not further considered in this work.

III. Transition Cost

With discrete time, the transition from a discharge rate in one time step to the next is assumed to be instantaneous, as a step-function. In reality, this transition requires some time due to the laws of physics. Regardless of the existence of ramping constraints, we therefore argue that the time discretization contributes to overestimating the capability of adjusting generation to price. Since the price is treated exogenously, it is possible to estimate a transition cost that covers the adaption of discharge from one time step to the next. This concept is described in detail next.

A. Deriving the Transition Cost

To supplement ramping constraints in limiting the ramping according to the provided requirements, we introduce the concept of *transition cost* (TC). The concept will be explained with references to Fig. 1. Consider a power station with a discharge ramping constraint, illustrated as the dotted line in Fig. 1. The figure shows two time steps with low (λ_{k-1}) and high (λ_k) prices (in €/MWh) and a corresponding discharge q. The LP problem formulated in the previous section, including ramping constraints in (3) allows discharge to instantaneously transit from a low rate in the one time step to a higher rate in the next, according to the solid-drawn line in Fig. 1.

We assume that the continuous ramping is symmetric around the shift between time steps and needs at least a minimum time to ramp of $2\Delta_t$. The change of discharge rate will therefore start Δ_t hours before time step k and will complete Δ_t hours after step k has started, as illustrated by the dotted line in Fig. 1. Due to the limited ramping, an additional

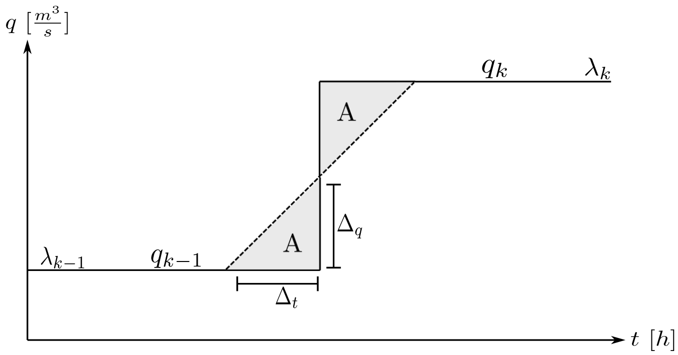

Fig. 1. Illustration of discharge ramping between two consecutive time steps.

volume A will be discharged at price λ_{k-1}, while the discharge at λ_k is reduced by a volume A. Since prices are known a priori, the economic loss (or the transition cost) of changing discharge rate between the two time steps can be estimated. We start by estimating the volume A (in m^3) in (4).

$$\Delta_t = \frac{q_k - q_{k-1}}{2\Delta_R} \tag{4a}$$

$$\Delta_q = \frac{q_k - q_{k-1}}{2} \tag{4b}$$

$$A = \frac{\Gamma_h \Delta_q \Delta_t}{2} = \frac{\Gamma_h \left(q_k - q_{k-1}\right)^2}{8\Delta_R} \tag{4c}$$

where Γ_h converts m^3/s to m^3/h.

The cost (or lost revenue) associated with the misplaced volume can be found by multiplying the volume A with the price difference and the water-to-power efficiency:

$$y_k = A\eta|\lambda_k - \lambda_{k-1}| = C\left(q_k - q_{k-1}\right)^2, \tag{5}$$

where $C = \frac{\eta|\lambda_k - \lambda_{k-1}|}{\Delta_R}$. We assume that η corresponds to the best efficiency point so that the cost (y_k) is an upper bound on the "true" cost.

Note that we do not account for the misplaced volume in (2) and (3): y_k is solely an additional cost element reflecting the lost revenue due to ramping limitations in between time steps. We add the cost elements in (5) to the objective so that (2a) is replaced with (6).

$$Z_t = \max \sum_{k=1}^{NK} \Big(\tau_k \lambda_k p_{hk} - y_k\Big) + \alpha_t \tag{6}$$

B. Approximating the Transition Cost

The LP problem formulated in Section II now becomes a quadratic optimization problem due to (5). For computational efficiency, it is crucial for the scheduling model to maintain the weekly decision problems as LP problems. Thus, we linearize the cost component, as described below and illustrated in Fig. 2.

First the range of N possible Δq values are defined, as illustrated by the dots along the horizontal axis in Fig. 2. For each discrete point we compute both the exact transition cost y^*, according to (5), and the derivative $\frac{\partial y}{\partial \Delta q}|_{\Delta q^*}$. By forming

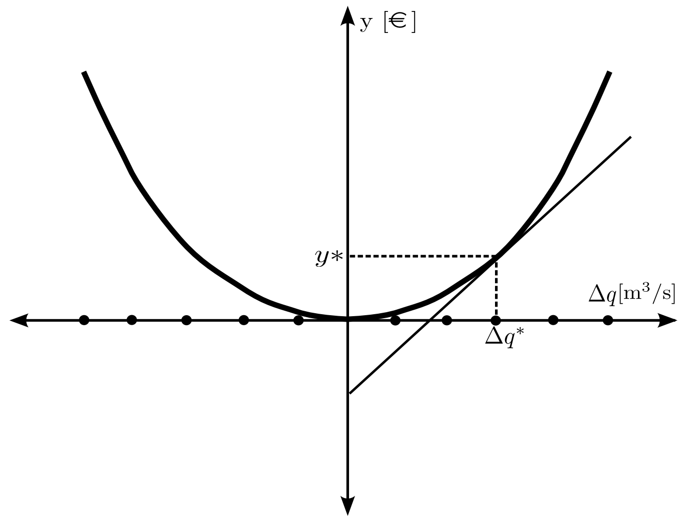

Fig. 2. Linear approximation of the transition cost.

a first-order Taylor expansion around this operating point, a linear constraint can be formulated:

$$y_k \geq y_k^* + \frac{\partial y}{\partial \Delta q}|_{\Delta q^*}\Big(\Delta q - \Delta q^*\Big). \tag{7}$$

We refer to (7) as transition cost cuts (TC-cuts) in the following. A total of N TC-cuts can be computed for each price period prior to solving the LP problems. As shown in Fig. 2, the TC cuts approximate y_k from below. Note that the bound $y_k \geq 0$ ensures a non-negative y.

C. Comments on Accuracy and Simplifications

The 'true' cost of the ramping constraint can be found by formulating an optimization model with ramping constraints and infinitesimal time discretization. Apart from the underestimation due to discretization errors related to the TC cuts, we argue that the presented approach tends to overestimate the 'true' cost for two reasons. First, we require that the transition from one discharge rate to the next is centered around the time-shift between the two time steps, as illustrated in Fig. 1. This is more restrictive than a finely discretized optimization model with ramping constraints, since the latter can decide on the ramping trajectory more freely. Second, we assume that the best efficiency point is used in (5), leading to an upper bound on the cost y_k.

The presented approach is based on the assumption that the ramping direction follows the change in power price between two consecutive time steps. For complex systems there may be situations where this assumption does no hold true, i.e., that the model finds that ramping up (resp. down) as a response to a decreasing (resp. increasing) price is favorable. Such situations where not discovered in our case study in Section IV.

IV. Case Study

A prototype of the SDDP-based ProdRisk scheduling model was prepared with the implementation of ramping constraints and TC-cuts. The model uses CPLEX as the optimization

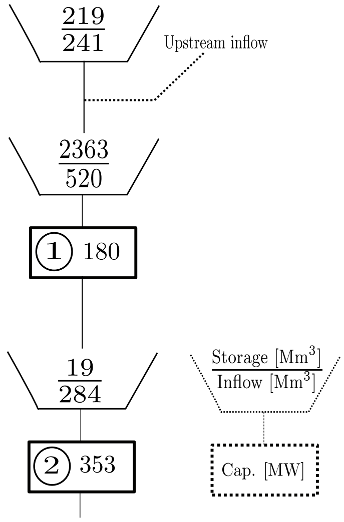

Fig. 3. Hydropower system topology and technical characteristics.

solver and was run with parallel processing. In our presentation, we will mainly focus on the numerical results and briefly report on computations times.

A. System and Case Description

The hydropower system in Røssåga is located in Northern Norway and comprises two large power stations with significant upstream reservoir capacity. An illustration of the system topology, including the major reservoirs and power stations, is provided in Fig. 3. A minimum discharge constraint of 30 m^3/s applies downstream the power stations.

We consider a ramping requirement associated with the outlet of the downstream power station labeled ② in Fig. 3. The constraint limits ramping to be at most 7.5 m^3/s per 15 min. This translates to 30 m^3/s per hour and 90 m^3/s per 3-hour block. The maximum discharge through power station 2 is 160 m^3/s.

The system is optimized for a horizon of 156 weeks, using 56 3-hour time steps within the week. This is a reasonable time discretization for a medium-term scheduling, for which a reasonable balance between result quality and computation time is sought. The ramping constraint is therefore 90 m^3/s per 3-hour block. Four cases were run, as listed in Table I. The cases **NoRamp** and **Ramp** are without and with ramping constraints, respectively. The cases **Ramp-TC20** and **Ramp-TC100** are both with ramping constraints and include 20 and 100 TC-cuts, respectively.

B. Results

The converged objective function values (profit maximization) and the total computation times are shown in Table I. While the ramping constraints alone seem to reduce the objective marginally in case Ramp compared to NoRamp, the cases with TC-cuts provide significant reductions. Moreover, we find that the use of TC-cuts increases the computation time, but not dramatically, as shown in the third column in Table I.

TABLE I
OBJECTIVE AND SOLUTION TIME.

Case	Objective [M€]	Time [hr:min]
NoRamp	7221	2:14
Ramp	7220	3:11
Ramp-TC20	7169	4:14
Ramp-TC100	7159	7:34

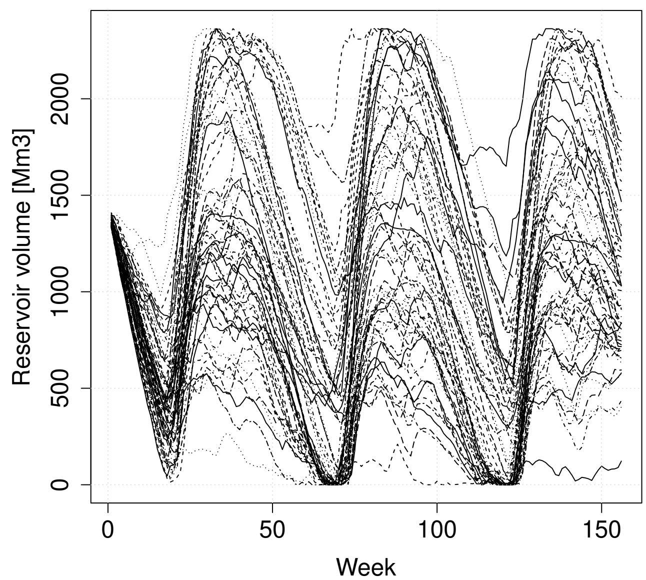

Fig. 4. Simulated reservoir trajectories for the largest reservoir.

The simulated reservoir trajectories for the largest reservoir from case Ramp are shown in Fig. 4. The trajectories obtained from the other cases differ only marginally from those in case Ramp.

We calculated time series of ramping on discharge through power station ②, and plot the duration curves for ramping obtained in cases Ramp, Ramp-TC20 and Ramp-TC100 in Fig. 5. The figure shows that the frequency of ramping (up and down) is significantly reduced when using TC-cuts. While the duration curves are similar for the two cases with TC-cuts, the Ramp-TC100 case provides a better approximation of the transition cost than Ramp-TC20, and hence the smoother curve.

Fig. 6 shows a sequence of discharge decisions for power station ② for three chosen weeks. The vertical gray, dotted lines indicate the end of each week. The use of TC-cuts in cases Ramp-TC20 and Ramp-TC100 leads to less fluctuations between the minimum discharge, best efficiency point, and maximum discharge.

V. Conclusions

A new method for approximating the cost of following ramping constraints in medium-term hydropower scheduling models was presented. Since power prices are exogenous, the

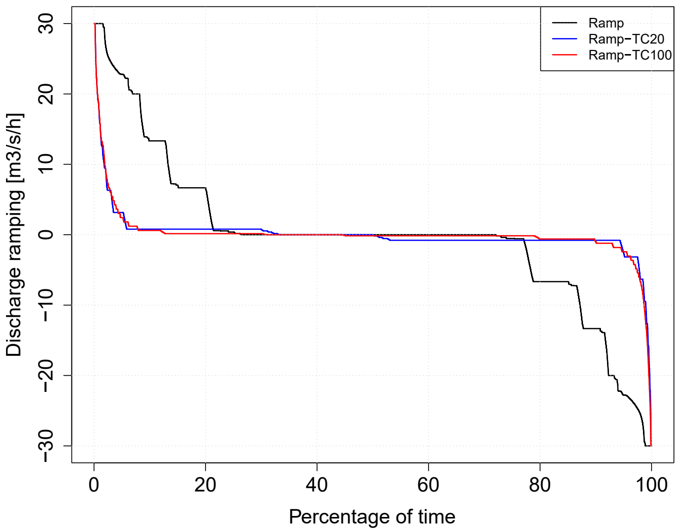

Fig. 5. Duration curves for discharge ramping.

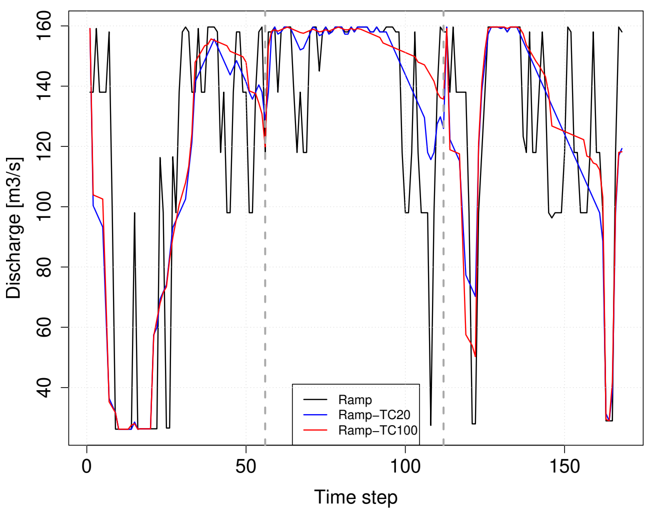

Fig. 6. Discharge for a selected sequence of three weeks.

cost can be explicitly computed and approximated by linear constraints, and this fits well within a framework based on stochastic linear programming.

A realistic case study was presented, demonstrating the impact of following ramping constraints on discharge patterns. The approximation by linear constraints contributed to significant, but not prohibitive, increases in computation times. Further work may improve the approximation procedure, e.g., by approximating the transition cost components dynamically.

REFERENCES

[1] L. E. Schäffer, A. Adeva-Bustos, T. H. Bakken, A. Helseth, and M. Korpås, "Modelling of Environmental Constraints for Hydropower Optimization Problems – a Review," in *17th International Conference on the European Energy Market*, Stockholm, Sweden, 2020.

[2] I. Guisández, "Long-term optimal hydropower reservoir operation with minimum flows and maximum ramping rates," Ph.D. dissertation, Universidad Politécnica de Madrid, 2016.

[3] I. Guisández, J. I. Pérez-Díaz, W. Nowak, and J. Haas, "Should environmental constraints be considered in linear programming based water value calculators?" *International Journal of Electrical Power & Energy Systems*, vol. 117, 2020.

[4] A. Gjelsvik, B. Mo, and A. Haugstad, *Handbook of Power Systems I*. Springer, 2010, ch. Long- and medium-term operations planning and stochastic modelling in hydro-dominated power systems based on stochastic dual dynamic programming, pp. 33–55.

[5] SINTEF. (2022) Prodrisk. [Online]. Available: https://www.sintef.no/en/software/prodrisk/

[6] M. V. F. Pereira and L. M. V. G. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical Programming*, vol. 52, pp. 359–375, 1991.

[7] A. Gjelsvik, M. M. Belsnes, and A. Haugstad, "An algorithm for stochastic medium-term hydrothermal scheduling under spot price uncertainty," in *Proc. 13th Power System Computation Conference*, Trondheim, Norway, 1999.

[8] A. Helseth, M. Fodstad, and B. Mo, "Optimal medium-term hydropower scheduling considering energy and reserve capacity markets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 3, pp. 934–942, 2016.

[9] ——, "Optimal hydropower maintenance scheduling in liberalized markets," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 6989–6998, 2018.

[10] C. Ø. Naversen, A. Helseth, B. Li, M. Parvania, H. Farahmand, and J. P. S. Catalaõ, "Hydrothermal scheduling in the continuous-time framework," *Electric Power Systems Research*, vol. 189, 2020.