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Diversified vs Specialized Private Equity Funds

A Study of Risk-Adjusted Performance using Insights from Simulations and Empirical Data

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Abstract

Private equity funds face constraints on the number and type of investments they can hold due to the time and resources required for counseling activities. This may lead to specialization in a specific industry or region and leave the funds vulnerable to significant idiosyncratic risk. This research analyzes the risk-adjusted performance of specialized and diversified private equity funds using simulations and real-life data. The simulations aimed to uncover if it was possible to detect the minimum return thresholds for specialized funds given their risk exposure. The results from the simulation model, where various types of specialized private equity funds were simulated by combining previous research with stock market data, were used to formulate a general model that estimated the minimum required IRR for private equity funds. Finally, the estimated minimum IRR was compared to the actual return of various private equity funds to analyze the return of specialized funds relative to diversified funds. The analysis revealed that the simulation model did not consistently detect the minimum return thresholds for specialized private equity funds. In addition, an analysis of historical private equity data was conducted to better understand the effect of diversification on risk and return in private equity. The analysis of historical data, using a data sample of 1,656 fully liquidated deals from Preqin, indicated that specialized funds do not consistently outperform diversified funds on a risk-adjusted basis, possibly due to diminishing returns to scale. However, it appears that the market for specialized private equity funds is relatively efficient, as investors are able to identify and invest in the most promising specialized funds, resulting in competitive returns. The prevalence of specialized private equity funds may be due to the diverse investment needs and strategies of limited partners or investors in private equity funds.

Keywords – Finance, Private Equity, Diversification, Risk-Adjusted Performance, Simulations

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1 Introduction

A distinctive feature of private equity funds is that they engage in acquisitions and provide financial and value-added services to their investments (Hellmann and Puri, 2002; Metrick and Yasuda, 2010). However, they often face constraints on the number and type of investments they can hold due to the time and resources required for these counseling activities (Gompers and Lerner, 2001; Diller and Kaserer, 2009). As a result, it may not be feasible for private equity funds to hold well-diversified portfolios, contrary to the recommendations of modern portfolio theory. This can lead to specialization in a specific industry or region, leaving the funds vulnerable to significant idiosyncratic risk not captured by a risk premium (Markowitz, 1952). This thesis seeks to uncover if specialized private equity funds provide sufficient returns given their increased risk exposure. In the first part of this thesis, a simulation model was built to determine if it was possible to detect the minimum return thresholds at which specialized and diversified private equity funds have the same risk-adjusted performance. In the second part of the thesis, the actual risk-adjusted performance of private equity funds was analyzed using historical data and compared to the estimated minimum thresholds.

The simulation part presents a new method for exploring the relationship between diversification and performance. In order to model the dynamics between diversification and returns, correlation data from comparable public companies was used to simulate various types of private equity funds. A univariate regression analysis was used on the return outputs of the simulated private equity funds to formulate a general model that aimed to estimate the minimum required return thresholds for specialized funds. However, using the estimation model on historical data showed that the estimated minimum IRRs from the model could not be used to reliably identify the relative performance of specialized funds to diversified funds on a risk-adjusted basis. This suggests that using simulation based on stock market data may not be feasible to detect the minimum return thresholds or analyze how diversification in private equity funds affects returns.

As the simulations were not feasible for identifying the minimum required returns for specialized funds, a thorough analysis of the risk-adjusted historical performance of private equity funds was also conducted. The risk-adjusted performance of specialized and diversified funds was analyzed based on a methodology proposed in Phillips (2018). The results indicated that no specialized funds consistently outperformed diversified funds across neither regions nor segments. The inconsistency in outperformance may be due to diminishing returns to scale, the idea that as a fund grows in size, the returns it generates may decrease, as argued by Berk and Green (2004). In addition, it appears that the market for specialized private equity funds is relatively efficient, as investors are able to identify and invest in the most promising specialized funds, resulting in competitive returns. This may be due to the fact that limited partners (LPs), or investors in private equity funds, often have diverse investment objectives and multi-asset portfolios where private equity constitutes only a portion. As such, it may be beneficial for LPs to diversify their portfolio by investing in a range of specialized private equity funds. Overall, the prevalence of specialized private equity funds can be attributed to the diverse investment needs and strategies of LPs.

Although conventional finance theory suggests holding diversified portfolios to reduce idiosyncratic risks, many sector-specialized private equity funds still exist today. A possible explanation for this phenomenon could be that sector-specific private equity funds achieve superior returns, despite the increased idiosyncratic risks. This research, therefore, constructed different specialized and diversified funds and simulated their performance. The aim of the simulations was to use the return outputs to formulate a general model that estimates the minimum return thresholds for specialized funds such that the risk-adjusted performance is equal between specialized and diversified funds. By comparing the estimated minimum IRR with the actual exit IRR of funds, one could determine whether or not simulations can detect outperformance by specialized funds. One of the reasons simulations are helpful is that, in practice, it can be challenging to access accurate correlation data of PE portfolio companies and thus understand how diversification affects fund performance in private equity. To model the relationship between diversification and returns in the simulations, previous research was combined with data from similar public companies as a proxy for the private equity portfolio companies.

The portfolio companies of the simulated specialized funds, which only invested in a single sector, had higher average pairwise input correlations. The simulated diversified funds invested in multiple industries and, as a result, had lower input correlations. The value of the portfolio companies within each fund was simulated using a Geometric Brownian Motion. Each portfolio company's initial and exit values were then used to calculate the overall fund performance. The returns for the specialized funds, which had a higher correlation, were larger than those of diversified funds, as expected. However, the total risk was also higher, as measured by the standard deviation of all the simulated IRRs within each specialized fund. Thus, the simulation outputs in this research aligned with the expectations that a higher correlation increased the total risk of the fund.

The relationship between correlation and returns, as discovered from the simulation data, was used to formulate a general estimation model to see if one could estimate the minimum required IRRs for specialized funds using simulations. The minimum required IRR is a measure to estimate the returns the specialized funds must achieve to compensate for the increased risks associated with less diversification. Before formulating the estimation model, the risk- and market-adjusted performance of the diversified funds had to be computed. The measure was computed using a methodology suggested by Phillips (2018), modifying the approach slightly to fit the aim of the estimation model better. Then the minimum required IRRs of the simulated specialized funds were computed by benchmarking them against the risk- and market-adjusted performance of the diversified funds. As hypothesized, the simulation outputs suggested that an increase in correlation between portfolio companies in a fund increases the minimum required IRR.

After formulating the estimation model, the real-life historical deal-level data obtained from Preqin was investigated. First, to construct specialized funds using deal-level data, the data was sorted by the portfolio company's primary industry, whether it was in the buyout or venture capital segment, and whether it had operations in the United States or Europe. Then, the various specialized funds and the diversified fund were compared using performance measures with varying risk adjustments, including a modified version of Phillips' proposed IRR, to determine which specialized fund had outperformed the diversified fund. Using the general estimation model formulated with the simulation outputs, the estimated minimum required IRR for the specialized funds was computed using the standard deviation of each fund.

The estimation model for detecting the minimum return thresholds of specialized private equity funds was revealed to be flawed. This was determined by comparing the model's estimated minimum IRRs with the actual exit IRRs of various funds in the US and EU, including buyouts and venture capital segments. The model correctly detected the outperformance of three out of four funds in the US buyouts, one out of four in the EU buyouts, and two out of three in the EU venture capital segment. The inaccuracy of the model may be due to the use of implied correlations, which possibly is a poor indicator of the actual correlation. Another explanation may be that the estimation model is based on unrealistic assumptions.

The first performance measure used to compare the performance of specialized versus diversified funds was the reported exit IRRs. Even though this research primarily focuses on risk-adjusted measures, an analysis of the exit IRR was interesting as Gompers et al. (2016) found that BO managers used IRR without risk adjustments as the primary measure to evaluate investments. The analysis of exit IRR gave mixed results as different sector funds outperformed the diversified fund in the buyout and venture capital segment, which prompted a more thorough performance analysis.

The second performance measure in the analysis was the market-adjusted IRR, which is comparable to the PME brought forward by Kaplan and Schoar (2005). Generally speaking, earlier research such as Phillips (2018) primarily used the S&P 500 to assess market risk. This is because it serves as a general alternative benchmark for many LPs' investments (Harris et al., 2015). By modifying the approach of Phillips (2018), however, this study sets itself apart by adjusting the exit IRRs with more suitable stock market indices when controlling for market returns. Similar to the exit IRRs, the market-adjusted IRR analysis also provided mixed results. Regardless, it is noteworthy that after adjusting for market returns, the same funds that demonstrated superior performance in the exit IRR analysis continued to exhibit relative outperformance compared to the diversified fund in both the US buyout and EU venture capital segments. For EU buyouts, every specialized fund underperformed relative to the diversified fund after adjusting for market risk.

There is mixed evidence on whether or not adjusting for market returns sufficiently accounts for the systematic risk (Sorensen and Jagannathan, 2015; Korteweg and Nagel, 2016). Therefore, as an additional risk adjustment, the various funds' risk- and market-adjusted IRR were computed by adjusting the market-adjusted IRRs for the underlying risks, measured by the standard deviation across all deals in the sector of the analyzed fund. The underlying risk adjustment significantly affected the results of the earlier analyses, as specialized funds that underperformed in the previous analysis now outperformed the diversified funds and vice versa. However, the analysis still finds differences within and between the segments, suggesting that investors should invest with caution in specialized funds as they do not consistently compensate for the additional risk. The results suggest that a diversified investment approach may better align GP's and LP's interests while mitigating potential agency issues. Additionally, diversified funds are expected to outperform specialized funds over the long term, as the latter may not consistently deliver sufficient risk-adjusted returns (Carhart, 1997; Sharpe, 1966).

The remainder of this research is structured as follows. First, Section 2 reviews related literature on the central topic of diversification and performance. Next, Section 3 explains the risk-adjusted performance measure proposed by Gordon Phillips in detail. Section 4 addresses the simulations and the formulation of the estimation model. Next, the historical data and the methodology are presented in Section 5, whereas the results and discussion are presented in Section 6. Following this, Section 7 draws the main conclusions and presents limitations and suggestions for further research.

2 Related Literature

Lossen (2006) estimates the effect of diversification on the performance of private equity funds over the dimensions of industry and regions. Using a multivariate regression analysis on fund-level data from USA and Europe, the result shows that the fund's rate of return increases with industry diversification. However, the results show the opposite for geographical diversification, finding no significant effects of diversifying across countries. Despite the limited sample size of 100 and a possible sample biased towards larger BO funds, Lossen finds no significant return premiums on the specialized industry or country funds compared to diversified funds. Using only US data, Ljungqvist and Richardson (2003) explore diversification by looking at 73 PE funds between 1981 and 1993 using sample data based on the records from a prominent institutional investor. With excess return as their primary performance measure, the study finds no significant relationship between the portfolio's number of represented industries, the percentage of allocated capital towards the dominant industry, and the investment's IRRs. These findings might not be representable for the industry, as the data sample might reflect a self-selection bias.

Using global data, Humphery-Jenner (2013) uses a sample of 1505 PE funds in a multivariate regression to test different hypotheses regarding the benefits and drawbacks of diversification. In relation to corporate finance literature stating that diversification could be a source of value destruction for individual public companies (Berger and Ofek, 1995; Denis et al., 2002; Eckbo and Thorburn, 2000; Aw and Chatterjee, 2004), various hypotheses are presented to explore whether PE funds are better suited for diversification. The research, which contains venture capital and buyout funds, proposes that industry or geographical diversification positively relates to higher IRRs. Furthermore, the article presents evidence that industry diversification in prior private equity funds can lead to a higher IRR for subsequent funds. This suggests that knowledge spillovers resulting from diversification can enhance the performance of private equity funds.

Focusing on the BO segment, Huss and Steger (2020) finds results that contradict Humphery-Jenner (2013) in terms of industry diversification. The research finds that funds investing in fewer industries tend to have greater IRRs. This can be supported further by previous research claiming that more specialized funds have the supporting knowledge and skills to make superior investment decisions and hence create more value (Das et al., 2004). Huss and Steger (2020) find no statistically significant relationship regarding geographical diversification, which is consistent with discoveries in Lossen (2006). In contrast to previous literature, Huss and Steger (2020) also uses a beta-adjusted PME as a dependent variable to control whether other factors, such as timing and differences in market risk exposure, may explain the finding between industry focus and performance. The findings reveal that the relationship is no longer statistically significant, indicating that the performance contribution of a more focused investment strategy may be relatively modest.

Cumming and Dai (2010) look at the performance measure of exit speed and value growth in their research on diversification within VC funds. Based on a sample of US Venture Capital investments between 1980 and June 2009, the result supports the findings from Huss and Steger (2020), suggesting that less diversification in industries, meaning a higher degree of specialization, has positive effects on the performance of venture capital funds. One key finding is that VCs frequently exhibit local bias in their investment strategies. This can be explained by the fact that having a local presence results in more efficient time management and the application of local knowledge. Which ultimately benefits the fund more than spending time and resources on diversifying across geographies and industries. Given this, we often find a limited number of investments in VCs, which often restricts diversification possibilities (Bernile et al., 2007). These findings can be further explained by Cumming and Dai (2010), Jääskeläinen et al. (2006), and Gifford (1997), which emphasizes that an increased number of portfolio companies would mean less time and focus on each company and industry.

Focusing on the UK, Cressy et al. (2014) used the HHI to measure the diversification of VC funds across industries and regions. This allowed for examining the effect of diversification on fund performance while controlling for other factors, such as the quality of portfolio companies and economic conditions. According to the research, geographical diversification has a beneficial impact on the performance of VC funds. However, industry diversification has a negative impact. The latter may suggest that different industrial sectors are exposed to their own idiosyncratic risk and characteristics, which demands more specialized strategies and management to realize the underlying values in the VC investment fully. These findings are consistent with results from Knill (2009), and Gompers et al. (2009), who using U.S. data, suggest that more specialized venture capital funds are better positioned to achieve superior performance. In light of Cressy et al. (2014) findings, the results imply that investing in various industries is more challenging for VCs than just investing in different countries.

While traditional finance literature promotes risk reduction as the primary benefit of diversification (Markowitz, 1952), Buchner et al. (2017) emphasizes its role as a strategic factor in VC funds portfolio construction. The study found that diversification positively affected VC fund performance, as diversified VC funds had higher returns and lower risk than undiversified VC funds. These results, however, are only significant for experienced VC managers, which goes well in line with the evidence from Humphery-Jenner (2013)) and Metrick and Yasuda (2010), claiming that prior diversification experience and the accumulated internal expertise makes higher future return for subsequent funds. Overall, the results of this study suggest that diversification can be an effective strategy for VC funds to improve performance and reduce risk.

The research cited provide valuable insights. However, there is still no consensus on the effect of diversification on the performance of PE funds. This paper contributes to this research gap by employing a more straightforward methodology to account for underlying risk to explore the impact of diversification across industries and countries on the IRR performance of PE funds.

3 Risk-Adjusted Performance

To investigate how the increased risk of less diversification impacts fund performance in private equity, a risk-adjusted performance metric is needed. The challenge in private equity is that both the firm value of portfolio companies and the value of the overall PE funds rarely have daily observations, which makes it difficult to measure the risk level using standard deviations. However, IRR data computed using investment and divestment dates of portfolio companies within PE funds are reported more frequently and easier to compute. By using the standard deviations of these reported IRRs, Gordon Phillips proposes a new method to risk-adjust the average and median IRRs of a private equity firm (Phillips, 2018). In his research, the measure is used to compare the risk-adjusted performance of different private equity firms. The method involves aggregating the IRR of all portfolio companies within the private equity firm and calculating the standard deviation of those IRRs to risk-adjust the firm's performance. The proposed measure is called the risk- and market-adjusted IRR and will serve as the basis for the risk-adjusted performance analysis in this research. Since this study seeks to compare different types of specialized sector funds rather than comparing various private equity firms, the authors take a slightly modified approach of Phillips' methods to fit the aim of this research better. The modified approach for the simulations is explained in Section 4, while the method used for the historical data is explained in Section 5. The remainder of this section elaborates on the methodology, as proposed by Phillips in his article, in greater detail.

3.1 Market-Adjusted IRR

Before adjusting for the underlying risk, which in the study is defined as the standard deviations of all reported portfolio company IRRs within a private equity firm, Phillips proposes to first account for market returns in all portfolio company IRRs. To compute the market-adjusted IRR, Phillips adjusts the reported IRRs with the public stock market returns over the same period as the holding period of the portfolio company, comparable to Kaplan and Schoar (2005). The market return for individual portfolio companies in a private equity fund is then calculated as follows:

$$Market - Adjusted\, IRR_i = \; IRR_i - \; Market\; Return_{i,(t)} \tag{3.1}$$

Where i a portfolio company and (t) is the holding period of portfolio company i.

For example, suppose the reported IRR of a portfolio company A (PC A) is 30%, and the stock market return, measured with the S&P 500 during the holding period of the portfolio company, is 10%. The market-adjusted IRR of portfolio company A is then:

$$Market - Adjusted\ IRR_{PC\ A} = 30\% - 10\% = 20\%$$

The average and median market-adjusted IRRs of all portfolio companies are used to measure the overall performance of the private equity firm. Suppose that a private equity firm has three individual funds that only have one portfolio company each in their funds (PC A, PC B, and PC C). Suppose that the portfolio companies' IRR is 30%, 20%, and 10%, respectively, and that they all have the same holding period as portfolio company A such that the market return is 10%. The average market-adjusted IRR for the private equity firm is computed as follows:

$$Average\ Market{-}Adjusted\ IRR_{PE\ Firm} = \overline{(30\% - 10\%) + (20\% - 10\%) + (10\% - 10\%)} = 10\%$$

3.2 Underlying Risk

Phillips computes the standard deviation across all portfolio companies within a private equity firm to measure the underlying risk. The formula for underlying risk in a private equity firm is thereby:

$$Standard\ Deviation\ of\ IRRs = \sqrt{\frac{\sum \left(IRR_i - \overline{IRR}\right)^2}{n-1}} \tag{3.2}$$

Where i is a portfolio company, $\overline{IRR}$ is the average IRR of all portfolio companies at the private equity firm, and n is the number of portfolio companies at the private equity firm. Using the same private equity firm as in the example above, the underlying risk for all the portfolio companies is computed as the standard deviation of all three IRRs:

$$Standard\ Deviation\ of\ IRRs_{PE\ Firm} = \sqrt{\frac{(30\% - 20\%)^2 + (20\% - 20\%)^2 + (10\% - 20\%)^2}{3-1}}$$

$$Standard\ Deviation\ of\ IRRs_{PE\ Firm} = 10\%$$

3.3 Risk- and Market-Adjusted IRR

With the average market-adjusted IRR and the standard deviation of IRRs derived, the risk- and market-adjusted IRR for the private equity firm can be computed. The formula for the average risk- and market-adjusted IRR is:

$$\textit{Risk- and Market-Adjusted IRR}_{PE\ Firm} = \frac{Average\ Market - Adjusted\ IRR_{PE\ Firm}}{Standard\ Deviation\ of\ IRRs_{PE\ Firm}} \tag{3.3}$$

Consider the private equity firm from the earlier examples. Using the market-adjusted IRR and the standard deviation of IRR computed for the firm in earlier examples, the average risk- and market-adjusted IRR for the private equity firm can be calculated as follows:

$$\textit{Risk- and Market-Adjusted IRR}_{PE\ Firm} = \frac{Average\ Market - Adjusted\ IRR_{PE\ Firm}}{Standard\ Deviation\ of\ IRRs_{PE\ Firm}}$$

$$\textit{Risk- and Market-Adjusted IRR}_{PE\ Firm} = \frac{10\%}{10\%} = 1.00$$

The interpretation of the private equity firm's risk- and market-adjusted IRR is that for each unit of underlying risk (measured by the standard deviation), one gets 1.00 unit in returns. Phillips uses the risk- and market-adjusted IRR to compare the performance of different private equity firms. By comparing the risk- and market-adjusted IRR of these firms, he can determine which has performed the best on a risk-adjusted basis.

4 Simulations

4.1 Purpose and Design

This research used correlation data from public companies to simulate the performance of fictive private equity funds. The purpose of the simulations was to use the returns of the simulated funds to formulate a general model that attempts to detect the minimum return thresholds for specialized private equity funds. More specifically, the outputs from the simulations were used to formulate a model for estimating the minimum IRR, given the correlation between a specialized fund's portfolio companies. The minimum IRR is the threshold of returns that justifies the additional risk of less diversification in specialized funds. Using the estimation model formulated from the simulation outputs, the minimum IRR was computed for several diversified and specialized private equity funds using historical data. If the estimated minimum required IRRs was lower than the actual exit IRR historical data, the model suggests a relative outperformance for the specialized fund compared to the diversified. To assess whether the model actually could detect any outperformance by estimating the return thresholds, the true relative risk- and market-adjusted performance of specialized funds were computed and compared to see if they yielded the same results as the estimation model.

The simulation model was developed in Python, so testing various scenarios and assessing the results would be intuitive and straightforward. Data from the stock market and prior research were used to determine the various simulation parameter. The simulated sector funds were specialized in chosen industries based on the number of observations in the historical data sample to ensure a degree of coherence and consistency in the research. The sector funds were simulated independently for the buyout and venture capital segments, but no distinction was made based on regional differences. As a result, the estimation model constructed using the simulated outputs will not differ between regions and, therefore, will act as a general model to be used regardless of the geographical focus of the fund.

4.2 Model Description

This section presents the underlying mathematical theory and general assumptions of the simulation model.

4.2.1 GBM

A Geometric Brownian Motion refers to a continuous-time stochastic process in which the random quantity's logarithm moves in a Brownian motion with drift. Since it assumes that the constant drift is accompanied by random shocks, it is a common method of modeling the future value of a financial asset (Black and Scholes, 1973). To directly simulate the path of Geometric Brownian Motion, one can equip the following formula derived from the mathematical process presented in Appendix A1.1:

$$S_{t_{i+1}} = S_{t_i} e^{\left(r-\frac{1}{2}\sigma^2\right)(t_{i+1}-t_i)+\sigma(W(t_{i+1})-W(t_i))}$$

Where S_{t_i} is the value of of a portfolio company in time t_i, while $S_{t_{i+1}}$ is the value of a portfolio company in time t_{i+1} and represents a Geometric Brownian Motion. Furthermore,

$$\left(W\left(t_{i+1}\right) - W\left(t_i\right)\right) \sim N\left(0, t_{i+1} - t_i\right)$$

represents a Wiener Process, which thus can be rewritten as the following equation:

$$S_{t_{i+1}} = S_{t_i} e^{\left(r-\frac{1}{2}\sigma^2\right)(t_{i+1}-t_i)+\sigma\sqrt{(t_{i+1}-t_i)}Z_i} \tag{4.1}$$

The GBM formula's error term Z_i takes into account the possibility of correlation between several random variables. The drift, r, and σ are the expected return and the volatility of the investment. In order to address the correlation between investments in the same fund, Cholesky Factorization was employed in this thesis and is detailed in Appendix A1.1.

4.2.2 Standard Monte Carlo

The Monte Carlo method is a statistical technique that relies on the principles of probability theory, particularly the concept of random sampling. This principle states that a sample of data drawn from a population is typically representative of the population as a whole. In addition, the Monte Carlo method relies on several probability theorems, including the Law of Large Numbers and the Central Limit Theorem. The former states that the average of a large sample of random events will be close to the expected value, whereas the latter states that the distribution of many random variables will be approximately normal. Further mathematical details about these theorems can be found in Appendix A1.2.

Given the unobservable market values and the illiquid nature of PE as an asset class, the MC simulations provide several advantages for this research. To begin with, one can set up a risk-free environment to test out various scenarios with arbitrary alterations and modifications. This allows an in-depth understanding of the simulated situation's underlying dynamics and how correlations interfere with performance. Furthermore, simulations allow for the detailed examination and visualization of a phenomenon over a predetermined period. Combined with the ability to run the simulations multiple times and with various input variables, simulations can provide more accuracy in understanding the phenomenon and allow for a more deeper understanding of the simulated scenario.

In this research, we forecast high-dimensional time series, which involves simulating repeated measurements over time. In this context, the Monte Carlo method would be the most suitable approach as we are simulating the value of each portfolio company over an extended holding period. This was done using a Geometric Brownian Motion (See Equation 4.1). The equation estimates the value of a portfolio company in period t_{i+1}.

4.2.3 Model Assumptions

The simulation model was based on a set of assumptions regarding the static and predetermined parameters. According to Metrick and Yasuda (2010), the average lifetime for private equity funds was ten years, which was the life of all the simulated funds. The journal article also suggests that most private equity firms invest most of the committed capital in the first years of the fund's lifetime before divesting towards the end. Based on these results, the investment period was the first five years, while the divestment period was the last five years. The number of portfolio companies in every simulation was ten, with a new investment every six month. Likewise, the holding period was five years for all the portfolio companies. A constant holding period assumes that the fund exit the investments without considering the asset's value at the exit date, which may be unrealistic. The total committed capital was \$ 100 million, and for simplicity, the amount invested in each portfolio company was set to $ 10 million. Each fund was simulated 10 000 times to ensure that the effects of Law of Large numbers was applied. For a complete overview of the different parameters, see Appendix A2.

4.3 Input Parameters

4.3.1 Expected Return

Given the aim of the simulations, the expected return input was equal across all the funds in each segment (i.e., Buyout and Venture Capital). By using the same expected return input for all simulations in each segment, we can attribute any differences in performance solely to the influence of correlation rather than potential variations in expected return. According to Cambridge Associates (2021), the expected annualized return for buyout and venture capital funds are 13.64% and 14.97%, respectively. Since the simulations calculated the value of the portfolio company every month, the reported expected returns were divided by 12, yielding 1.14% for buyout funds and 1.25% for venture capital funds.

4.3.2 Volatility

The volatility in the simulation model was based on previous research from Metrick and Yasuda (2010). In the article, the authors indicated that the average volatility of a buyout investment was 60% while that of a venture capital investment was 90%. The increased volatility in venture capital was consistent with research that implies that portfolio companies in venture capital funds are more similar to small-cap companies, which often are subject to more risk (Chen et al., 2012). Again, the given percentages had to be divided by the square root of 12 to transform the volatility into monthly values. In the simulation, the volatility was 17.32% for buyout funds and 25.98% for venture capital funds.

4.3.3 Correlation

The method for calculating the correlation varied depending on whether the fund was a buyout or venture capital fund and whether the fund was diversified or specialized. The average pairwise correlation for the diversified funds was based on Metrick and Yasuda

(2010), which estimated a 20% correlation for buyout funds and 50% for venture capital funds. The average pairwise correlation between each portfolio company varied for each fund based on their respective industry specialization.

In order to compute the input correlation for the specialized buyout funds, the average correlation between public companies in the same industries had to be derived. A Bloomberg terminal was used to identify the stock market indices that accurately reflected and best represented the characteristics of the simulated funds. Each index's 30 largest constituents, measured by market capitalization, were identified and plotted in a 30x30 correlation matrix. The different indices used to compute correlations are presented in Appendix A9.1.

The average correlation in the computed correlation matrix was the average pairwise correlation used in each simulation for the specialized buyout funds. Due to limited data, the correlation for specialized venture capital funds was scaled up using the estimated buyout correlations for the same industries. This approach may not provide the most realistic scenario for the average pairwise correlation. However, it ensures that the correlation in the specialized funds in venture capital is higher than the diversified. This is important as portfolio companies operating in the same industry tend to be exposed to similar unsystematic risk, leading to higher correlations between their performance.

All the relevant input parameters discussed in this subsection are summed in the following tables:

Buyout	Symbol	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Expected Return (drift)	r	1.14%	1.14%	1.14%	1.14%	1.14%
Standard Deviation (volatility)	σ	17.32%	17.32%	17.32%	17.32%	17.32%
Average Correlation	ρ	20.00%	35.67%	36.25%	48.37%	44.50%

Venture Capital	Symbol	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Expected Return (drift)	r	1.25%	1.25%	1.25%	1.25%	1.25%
Standard Deviation (volatility)	σ	25.98%	25.98%	25.98%	25.98%	25.98%
Average Correlation	ρ	50.00%	61.18%	62.18%	82.96%	76.33%

4.4 Simulation Outputs

One entire simulation means determining the investment value from the deal date to the exit date for all ten portfolio companies in a fund using the Geometric Brownian Motion formula (See Equation 4.1). For a visual representation of single fund simulations with different correlation values, see Figure A3.1 and A3.2 in the Appendix. Ten investments were made during the existence of a fund, which means that 100 000 portfolio company values were calculated for each fund. The model assumes that each portfolio company only has two cash flows: one cash outflow at the investment date and one cash inflow at the exit date. The total cash flows during a lifetime of a fund may look similar to the table below:

Table 4.1: Fund Cash Flow Overview (In millions)

Portfolio Company	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Entry Date	01/20x1	07/20x1	01/20x2	07/20x2	01/20x3	07/20x3	01/20x4	07/20x4	01/20x5	07/20x5
Cash Outflow	-10,00	-10,00	-10,00	-10,00	-10,00	-10,00	-10,00	-10,00	-10,00	-10,00
Exit Date	01/20x6	07/20x6	01/20x7	07/20x7	01/20x8	07/20x8	01/20x9	07/20x9	01/20x10	07/20x10
Cash Inflow	4,78	1,13	16,50	23,68	15,18	4,49	4,98	19,87	11,46	26,49

The IRR of each simulated fund was then calculated using the aggregated cash flows from all portfolio companies in the fund. The IRR is a rate of return metric for evaluating investment performance and is defined as the discount rate that makes an investment's net present value equal to zero. By comparing the IRRs of the diversified fund against the IRRs of specialized funds, one can validate the assumption that higher correlation leads to increased risk and, thus, increased returns. In order to calculate the IRR, we can solve the following formula for IRR by setting the NPV equal to zero:

$$NPV = \sum_{n=0}^{N} \frac{C_n}{(1+IRR)^n} \tag{4.2}$$

Where C_n denotes the cash flow linked to period n and NPV denotes the net present value.

4.4.1 IRR

The descriptive statistics of the computed IRRs for each fund are presented in Tables A3.1 and A3.2 in the Appendix. To better compare and contrast the different funds, the IRR performance data are presented visually:

Figure 4.1: Buyout Funds: Average, Median, and Standard Deviation of IRR

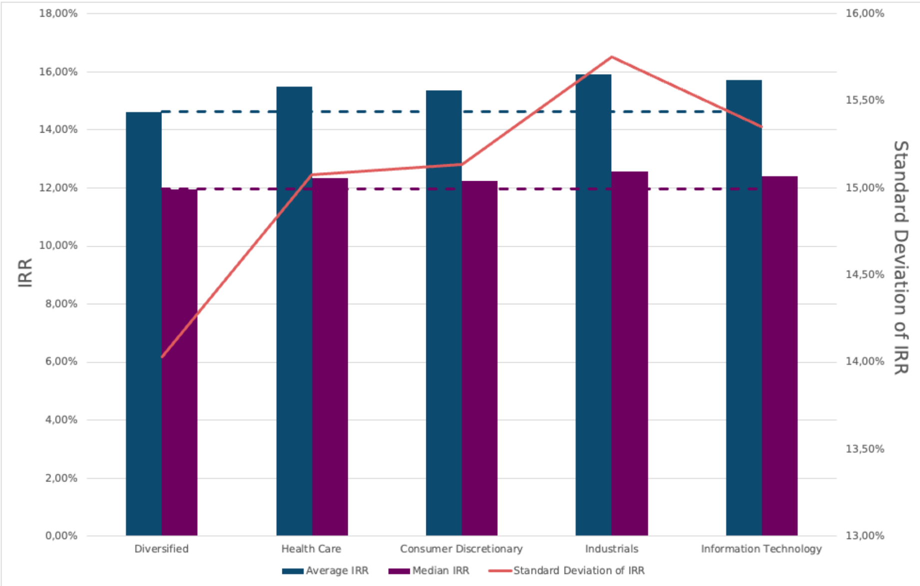
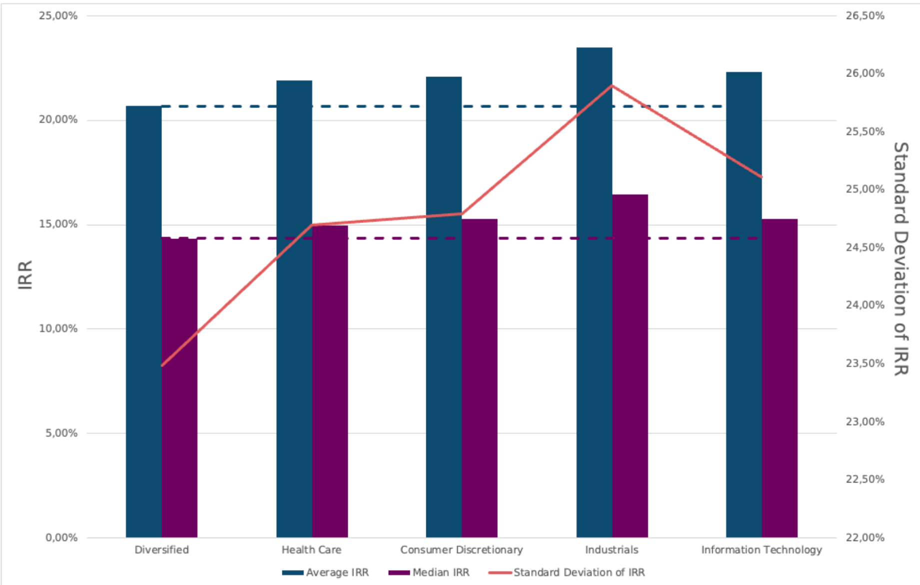


Figure 4.2: Venture Capital: Average, Median, and Standard Deviation of IRR



As expected, the visual representations of the simulated output show that all specialized funds outperform the diversified funds in both segments. This was true for both average and median, as the specialized funds are above the dotted lines. As mentioned, the only parameter that differentiates these simulations was the input correlation which indicates that, ceteris paribus, a higher average pairwise correlation between the portfolio companies yields higher average and median IRRs for the fund as a whole. This argument is further emphasized when comparing the buyout funds with their equal counterparts in the venture capital funds. All the funds in the venture capital segment have higher average and median IRRs compared to similar buyout funds, which corresponds with the higher expected return and higher correlation within the VC segment.

The standard deviation of the IRR was also higher for the sector funds than the diversified funds, visualized with solid lines. These results support the hypothesis that higher correlations between portfolio companies increase the overall fund risk, measured by IRR. In line with expectations, the specialized funds need to outperform the diversified fund by achieving IRRs above a certain threshold to compensate for the market returns and the increase in risk. These threshold were computed through the estimation model.

4.5 Estimation Model

For a sector fund, it is worth specializing as long as the returns compensate sufficiently for the increased risk. Each of the funds simulated had a different average correlation between the portfolio companies in the fund, which resulted in the varying levels of fund risk measured by the standard deviation. By using the diversified fund as a benchmark, one can compute the minimum level of returns, given the correlation between the portfolio companies of the specialized fund, that justifies the additional risk. The level of minimum returns is set such that the specialized and diversified funds deliver the same returns when accounting for market returns and underlying risk. To compute the minimum required IRR for specialized funds, the diversified fund's risk- and market-adjusted performance must be calculated first. Since the specialized funds were benchmarked against the diversified fund, the minimum required IRR of specialized funds depended on the diversified fund's risk- and market-adjusted performance. By using the minimum required returns from the simulated specialized funds, a general model for estimating the minimum return thresholds using historical data could be formulated.

Since this research aims to compare different types of specialized sector funds rather than comparing various private equity firms, the methodology to compute risk- and market-adjusted IRR for the diversified fund slightly differs from the general method detailed in Section 3. The approach used to calculate the risk- and market-adjusted IRR in the simulation part is described below.

4.5.1 Market-Adjusted IRR

To compute a market-adjusted IRR, Phillips adjusts the computed IRRs with the public stock market returns over the same period as the portfolio company's holding period. However, estimating the public stock market returns over the same period as the holding period is challenging in the simulations. Consequently, this study assumes a market return of 8.88% in all simulations, as this was the annualized average return of the S&P 500 in the last 20 years as of November 2022. The assumed market return is subtracted from the fund IRRs computed in Section 4.4.1 to compute the market-adjusted IRR of the simulated funds. The market-adjusted return for a fund in one simulation is therefore given by:

$$Market - Adjusted\ IRR_{(i,n)} = \ IRR_{(i,n)} - \mathbf{8.88\%} \tag{4.3}$$

Where i is a private equity fund in the simulations and n is the n:th simulation of that private equity fund. Since every individual fund is computed 10 000 times in the simulation, the average market-adjusted IRR of a fund is given by:

$$Average\ Market - Adjusted\ IRR_{Sim\ Fund} = \frac{\sum_{n=0}^{10\ 000} Market - Adjusted\ IRR_{(i,n)}}{10\ 000}$$

In addition to the average, the median market-adjusted IRR is also computed for the funds. The resulting market-adjusted IRR for the diversified funds is presented below:

Table 4.2: Market-Adjusted IRR for Diversified Funds

	Buyout Diversified	**Venture Capital** Diversified
Average Market-Adjusted IRR	5.74%	11.80%
Median Market-Adjusted IRR	3.07%	5.45%

4.5.2 Risk- and Market-Adjusted IRR

The computation of the underlying risk is also slightly different from what was outlined in Section 3. Phillips (2018) uses the standard deviation of all portfolio companies' IRR within a PE firm to compute the risk. However, this research utilizes the standard deviation of the 10 000 simulated IRRs of each simulated fund to compute the underlying risk. The underlying risk of a simulated fund is given by:

$$Standard\ Deviation\ of\ IRR_{Sim\ Fund} = \sqrt{\frac{\left(\sum_{n=0}^{10\ 000} IRR_{(i,n)} - \overline{IRR_i}\right)^2}{10\ 000 - 1}}$$

Where i is a private equity fund in the simulations and n is the n:th simulation of that private equity fund. The risk- and market-adjusted IRR in the simulations are then given by:

$$Risk\text{-}\ and\ Market\text{-}adjusted\ IRR_{Sim\ Fund} = \frac{Market - Adjusted\ IRR_{Sim\ Fund}}{Standard\ Deviation\ of\ IRR_{Sim\ Fund}} \tag{4.4}$$

Using the market-adjusted IRRs in Table 4.2 and the standard deviations across all simulations for a fund from Tables in Appendix A3.1 and A3.2, both the average and median risk- and market-adjusted IRRs for the diversified funds were computed below:

Table 4.3: Simulated Risk- and Market-Adjusted IRRs for Diversified Buyout and Venture Capital Fund

	Buyout Diversified	**Venture Capital** Diversified
Average Risk- and Market-Adjusted IRR	0.4092	0.5058
Median Risk- and Market-Adjusted IRR	0.2190	0.2354

With the benchmark risk-and market-adjusted IRR computed, the minimum required IRR for the specialized funds could be derived. By rearranging Equation 4.4 and substituting with the figures presented in Table 4.3, we get the following equations:

Buyout Funds

$$Minimum\ Required\ Average\ IRR_i = (0.4092 \cdot \sigma_i) + 8.88\% \tag{4.5}$$

$$Minimum\ Required\ Median\ IRR_i = (0.2190 \cdot \sigma_i) + 8.88\% \tag{4.6}$$

Venture Capital Funds

$$Minimum\ Required\ Average\ IRR_i = (0.5058 \cdot \sigma_i) + 8.88\% \tag{4.7}$$

$$Minimum\ Required\ Median\ IRR_i = (0.2354 \cdot \sigma_i) + 8.88\% \tag{4.8}$$

Where σ_i is the standard deviation of the specialized fund i. By substituting σ_i with the standard deviation of the various simulated specialized funds, the following minimum required IRRs were computed:

Table 4.4: Minimum Required IRRs for the Specialized Buyout Funds

	Health Care	Consumer Discretionary	Industrials	Information Technology
Minimum Required Average IRR	15.05%	15.07%	15.33%	15.16%
Minimum Required Median IRR	12.18%	12.19%	12.33%	12.24%

Table 4.5: Minimum Required IRRs for the Specialized Venture Capital Funds

	Health Care	Consumer Discretionary	Industrials	Information Technology
Minimum Required Average IRR	21.29%	21.34%	21.90%	21.50%
Minimum Required Median IRR	14.61%	14.64%	14.90%	14.71%

4.5.3 Formulating the Estimation Model

Given the input correlation between portfolio companies and the computed minimum required IRR for the specialized funds, a univariate linear regression was utilized to formulate a general model to estimate the minimum required IRRs for specialized funds. Such a model would be helpful to LPs as a guideline to investigate if GPs that run sector-specific funds have outperformed diversified funds, given the correlation between the portfolio companies in the sector-specific funds. For example, suppose the minimum required IRR estimated from the general model is lower than the actual IRR of the sector-specific fund. In that case, the estimated model suggests that the GP has achieved sufficient returns given the increased risk of less diversification. Conversely, the estimation model suggests the opposite if the minimum required IRR is higher than the actual IRR. The general equation for the minimum required IRR is stated below:

$$Predicted\ Minimum\ required\ IRR_i = \beta_0 + \rho_i \cdot \beta_1 \tag{4.9}$$

Where β_0 is the estimated constant, ρ_i is the average pairwise correlation between portfolio companies in specialized fund i, and β_1 is the estimated coefficient for the effect of correlation on the minimum required IRR for the specialized fund. In the regression analysis from the simulation data, the input correlation was the independent variable, and the minimum required IRR presented in Tables 4.4 and 4.5 was the dependent variable. The regression analysis yielded the following estimation model for the minimum required IRR for buyout and venture capital funds, visualized with a scatter plot and line of best fit in Appendix Figures A4.1 and A4.2:

Buyout Funds

$$Minimum\ Required\ Average\ IRR_i = (0.0236 \cdot Correlation_i) + 0.1417 \tag{4.10}$$

$$Minimum\ Required\ Median\ IRR_i = (0.0126 \cdot Correlation_i) + 0.1171 \tag{4.11}$$

Venture Capital Funds

$$Minimum\ Required\ Average\ IRR_i = (0.0319 \cdot Correlation_i) + 0.1922 \tag{4.12}$$

$$Minimum\ Required\ Median\ IRR_i = (0.0149 \cdot Correlation_i) + 0.1365 \tag{4.13}$$

Quality data on private equity funds are scarce in today's market, and data about the correlation between portfolio companies within a private equity fund is hard to come by. Since the correlation between portfolio companies was the independent variable in the *Minimum Required IRR* equations above, a model to compute the implied correlation between assets in a fund given the standard deviation of IRRs of portfolio companies in the same sector was needed. The general equation for implied correlation is stated below:

$$\widehat{\rho_i} = \beta_0 + \sigma_i \cdot \beta_1 \tag{4.14}$$

Where $\widehat{\rho_i}$ is the implied correlation of specialized fund i, β_0 is the estimated constant, σ_i is the standard deviation of IRR of the portfolio companies in specialized fund i, and β_1 is the estimated coefficient for the effect of standard deviation on implied correlation.

For the regression of the simulated data, the standard deviation of simulated IRRs from Appendix Tables A3.1 and A3.2 was the independent variable, while the input correlation was the dependent variable. Since the expected return and volatility were the same for all the simulated funds, the only parameter affecting the standard deviation of the IRR was the correlation. That yielded the following equations for implied correlation for buyout and venture capital funds, visualized with a scatter plot and line of best fit in Appendix Figures A4.3 and A4.4:

Buyout Funds

$$Implied\ Correlation_i = (16.87 \cdot Standard\ Deviation\ of\ IRR_i) - 2.1727 \tag{4.15}$$

Venture Capital Funds

$$Implied\ Correlation_i = (14.28 \cdot \ Standard\ Deviation\ of\ IRR_i) - 2.8778 \tag{4.16}$$

The equations for minimum required IRR and implied correlation were utilized when the estimation model was tested empirically on historical data later in the research.

5 Historical Data

5.1 Purpose and Design

In this part of the thesis, the performance of specialized funds is compared to that of diversified funds using historical data. Since the simulations conducted in Section 4 only provide partial insight into how diversification affects the performance of private equity funds, a thorough analysis of historical data can reveal more about the relationship between these factors in practice. The main objective is to evaluate whether specialized funds, which tend to be less diversified, have achieved sufficient returns relative to diversified funds. To this end, various IRR measures are calculated and compared between the funds. The reported exit IRR was the first measure used to compare the performance of specialized and diversified funds. Then, the reported exit IRRs were adjusted for market returns before finally, the risk- and market-adjusted IRR, based on Phillips (2018) (See Section 3), were computed to compare the performance of the specialized and diversified funds on a risk-adjusted basis.

As explained in Section 4, by comparing the model's estimated minimum IRRs with the actual historical performance, an expectation of the relative performance of specialized funds is derived. The expectations can be compared to the results from the analysis of risk- and market-adjusted IRRs to evaluate the accuracy of the estimation model. This was essential in determining whether simulations could be a valuable tool to derive the minimum return thresholds and thus detect any outperformance for private equity funds.

5.2 Data

5.2.1 Data Source

The research studies the risk-adjusted performance using data provided by Preqin. Some of Preqin's methods for gathering data are web data extraction, direct talks with fund managers, institutional investors, and industry professionals, web research from credible sources, FOIA requests, and manager-initiated data contributions (Phalippou, 2010). Most of the information comes from the FOIA request, where investors report quarterly statistics on cash invested, realizations, and net asset values. However, these FOIA requests may cause Preqin missing information on high-performing funds that choose not to accept public pension funds as investors due to their vulnerability to FOIA requests. This suggests that the information provided by Preqin may not be comprehensive, as it may not include data on certain funds that have chosen to protect their privacy in this way (Harris et al., 2014).

It is also important to point out that Preqin does not treat its information anonymously, unlike other sources such as CEPRES. This feature is critical to have in mind, as data providers might have higher incentives toward positive reporting bias. This could mean that the reported data is less reliable and representative of its intended use because it gives PE firms more incentives to manipulate the funds cash flows. Additionally, it enhances survivorship bias and selection bias by incentivizing underperforming PE firms to cease reporting (Harris et al., 2014). Nevertheless, prior research investigating the quality of the different commercial data sources finds that they tend to yield similar aggregated returns (Brown et al., 2015). Furthermore, comparing Burgiss, Preqin, and Cambridge Associates, Harris et al. (2014) shows how the data sources yield qualitatively and quantitatively similar performance results. Accordingly, the research claims that there is little support for performance selection bias and that Preqin is suitable for academic research.

Overall, Preqin is generally considered a reliable data source for private equity information, although it is essential to recognize that it may have some biases that could affect its representativeness. Despite this, Preqin is still highly regarded due to its extensive deal-level data on private equity transactions and its widespread use and credibility within the industry.

5.2.2 Descriptive Statistics

The historical analysis utilized a sample of 1,656 fully liquidated private equity deals, as identified by Preqin as "Exit IRR" deals. Exit IRR is a performance measure that accounts for fees earned by the GP and is calculated based on the sum of cash contributions, distributions, and the final exit value of the portfolio company. Using fully liquidated deals eliminates potential concerns about the reliability of the estimated net asset values (NAVs) of unrealized investments and the timing of NAV reporting.

When we examine the reported exit values, we can see that VC deals have outpaced BO deals in terms of mean and median value growth. The high standard deviations observed in the data indicate that the size of the deals varies significantly. It is important to stress that the values for both investment size and exit size are only representative for some of the samples, as it only applies to some of the deals where the GPs have reported it on their initiative. Therefore, the figures presented should be viewed as general insights rather than precise values for the two segments.

The geographical split of investments shown in Table 5.1 indicates that the sample is characterized by a large proportion (51.21%) of investments in Europe (including the UK). This somewhat contradicts the samples in earlier research, which tends to have an overweight of US observations. The remaining deals are relatively split between the US (23.97%) and the Rest of the world (24.82%). The BO Deals follow approximately similar relative distributions as in All Deals. For VC Deals, there is a more even distribution between Europe (45.10%) and the Rest of the world (40.52%). For the US, however, the deals only amount to 14.38% (22 observations) for the whole segment.

The statistics in Table 5.1 also show the different industries represented. Health Care, Consumer Discretionary, Industrials, and Information Technology are the primary industries of this research as they have the highest number of exit IRR-reported deals. The remaining exit IRR deals are aggregated into Others. Our sample shows that the Information Technology sector received the largest share of VC investments, at 34.64%. This is in conjunction with the increased focus on the IT sector in venture capital the recent years McKinsey & Company (2022). Buyout investments, excluding Others, are primarily concentrated in the industries of Consumer Discretionary (25.95%) and Industrials (16.37%). The figures also provide information about the duration of the deals.

In coherence with Phillips (2018), this research imposes the criterion of at least ten liquidated deals in order to calculate the standard deviation more precisely. Hence, due to limited observations, the analysis of historical data will not include US VC or Industrials for VC Europe. Table 5.2 shows the distribution of BO and VC deals in terms of investment years. The exit IRR observations for both segments are largely concentrated in the past two decades.

Table 5.1: Descriptive Statistics of Data

The descriptives for the investment data provided by Preqin are included in this table. The sample data includes liquidated PE deals spanning the years 1985 through 2021 (with some differences within the two segments). The following segment definitions are used: Buyout (BO) covers all leverage buyout and growth investments. Venture capital (VC) represents all early- and late-stage venture investment activity.

	All Deals	**BO Deals**	**VC Deals**
Number of observations			
Absolute	1 656	1 503	153
Relative	100%	90.76%	9.24%
Investment Size			
Mean	159.28	173.92	15.52
Median	33.05	35.91	5.00
Std.dev	431.01	470.99	38.32
Exit Size			
Mean	372.86	401.44	92.15
Median	123.66	132.10	40.80
Std.dev	1011.76	1097.29	171.59
Regions			
US	23.97%	24.95%	14.38%
Europe	51.21%	51.83%	45.10%
Rest of the World	24.82%	23.22%	40.52%
Industry			
Health Care	12.20%	11.64%	17.65%
Consumer Discretionary	25.00%	25.95%	15.69%
Industrials	15.46%	16.37%	6.54%
Information Technology	13.29%	11.11%	34.64%
Others	34.06%	34.93%	25.49%
Investment Duration			
Mean	4.21	4.37	2.67
Median	3.56	3.76	1.63
Standard Deviation	3.04	3.06	2.82
Sign Analysis			
# of positive IRR Deals	98.86%	98.88%	98.69%
# of negative IRR Deals	1.14%	1.12%	1.31%

Table 5.2: Sample Distribution

The distribution of BO and VC deals by investment year is shown in the table below. The table includes data for all BO and VC investments made globally (All) as well as for the corresponding sub-samples of VC and BO deals broken down by the companies geographical locations.

	Buyout Deals				**Venture Capital Deals**			
	All	US	Europe	Rest of the World	All	US	Europe	Rest of the World
1947	1	0	1	0	0	0	0	0
1985	1	0	1	0	0	0	0	0
1986	3	1	1	1	0	0	0	0
1987	0	0	0	0	0	0	0	0
1988	0	0	0	0	0	0	0	0
1989	2	0	1	1	0	0	0	0
1990	2	2	0	0	0	0	0	0
1991	2	0	2	0	0	0	0	0
1992	7	1	6	0	1	1	0	0
1993	5	1	2	2	1	1	0	0
1994	12	2	9	1	1	0	0	1
1995	14	4	8	2	2	1	1	0
1996	11	1	8	2	0	0	0	0
1997	18	3	11	4	0	0	0	0
1998	34	15	15	4	1	0	0	1
1999	40	20	19	1	3	0	1	2
2000	50	18	26	6	4	0	1	3
2001	61	21	34	6	5	0	3	2
2002	71	17	47	7	2	0	1	1
2003	89	36	39	14	7	1	3	3
2004	134	45	67	22	9	3	4	2
2005	117	32	67	18	11	0	9	2
2006	109	31	57	21	8	0	6	2
2007	85	19	46	20	11	1	4	6
2008	76	18	32	26	13	1	6	6
2009	55	7	23	25	6	1	2	3
2010	75	22	39	14	14	4	8	2
2011	62	8	30	24	10	3	1	6
2012	68	10	35	23	7	2	2	3
2013	64	5	33	26	7	1	3	3
2014	76	8	46	22	2	0	2	0
2015	46	7	30	9	4	0	3	1
2016	40	7	20	13	3	0	1	2
2017	34	7	23	4	4	0	1	3
2018	21	3	12	6	7	2	3	2
2019	15	3	8	4	5	0	3	2
2020	3	0	1	2	3	0	1	2
2021	0	0	0	0	2	0	0	2

5.3 Fund Specialization

In this thesis, deal data from different industries were compiled to construct sector-specific funds. For example, all the buyout deals from the US that were characterized as Health Care deals would make the specialized fund denoted as US BO Health Care. This allows for a comprehensive analysis of the differences between the different specialized funds. Naturally, as no deal qualifies as a "diversified" deal, the diversified funds aggregate all deal data from all industries under their respective segment (BO or VC) and regional belonging (EU or US).

Through this setup, the various performance measures explained in the next section will provide insights into diversification versus specialization through three dimensions: the type of fund, the primary region of focus, and industry specialization. Given the data limitations discussed in the previous chapter, the different fund combinations analyzed in this research are illustrated in Appendix A9.1.

5.4 Performance Analysis

The following subsection describes the underlying theory behind the performance measures used in the analysis of historical data. Note that, in the formulas below, a deal from the sample is referred to as a portfolio company.

5.4.1 Exit IRR

The descriptive statistics of the reported exit IRR for buyout and venture capital funds are computed first. The computation included the average, median, and standard deviation of the reported exit IRRs for the different funds constructed. The measures were used to get a quick overview of whether the specialized funds outperformed the diversified funds without considering the underlying risk or market return.

5.4.2 Estimation Model

The estimation model requires that the implied correlation of the specialized funds is computed. The standard deviation of the reported exit IRRs was used to compute the implied correlation by substituting them into equations 4.15 and 4.16. Then, these implied correlations were substituted in the estimation model formulated in equations 4.10, 4.11, 4.12, and 4.13 to compute the minimum required IRRs for each specialized fund. By comparing the estimated minimum required IRR with the actual exit IRRs will indicate whether or not the specialized funds outperformed or underperformed relative to diversified funds.

5.4.3 Market-Adjusted IRR

The exit IRRs was adjusted by considering the market returns as the first risk factor. As mentioned earlier, the methods proposed by Phillips were modified in the analysis of the historical data to fit the aim of this research better. The simulations (section 4) used a fixed market return of 8.88%. However, as Phillips suggests, the reported exit IRRs of portfolio companies in the historical data were adjusted with the market returns in the same period as the holding period of the portfolio company. Unlike Phillips, who uses the S&P 500 returns to market to adjust all portfolio company exit IRRs, this research uses the market returns of various stock market indices. For each of these industry-specialized funds, a stock market index that covered the same industry was used to adjust the market returns of each portfolio company's exit IRR within the fund. The stock market indices used to market-adjust the returns of the diversified and specialized funds from the historical data are listed in Appendix A9.2.

The daily value of relevant indices was obtained using a Bloomberg Terminal for the longest period possible. To adjust the reported exit IRR with the proper market return, it was essential to identify the value of the index at both the investment date and the divestment date. Then the stock market return in the relevant industry index between these dates was computed and substracted from the reported exit IRR to compute the market-adjusted IRR for the portfolio companies within the fund. The formula for the market-adjusted IRR for a single portfolio company within a fund in the historical data analysis is:

$$Market - Adjusted\ IRR_{i,c} = \ IRR_{i,c(t)} - \ Market\ Return_{i,(t)}$$

Where c denotes a individual portfolio company that operates in industry i, and t represents the holding period of the portfolio company c. $Market\ Return_{i,(t)}$ is therefore the return in the period t of a stock market index covering industry i .

The average market-adjusted IRR of the overall fund is a result of the individual portfolio company market-adjusted IRRs within the fund and is computed as follows:

$$Average\ Market - Adjusted\ IRR_f = \frac{\sum_{n=0}^{N} Market - Adjusted\ IRR_{i,c}}{N}$$

Where f makes up the specialized fund within industry i, c denotes a individual portfolio company that operates in industry i, and N is the number of portfolio companies within industry i. The median market-adjusted IRR for the funds was also computed.

5.4.4 Risk- and Market-Adjusted IRR

The underlying risk in the historical data of this research was defined as the standard deviation of all reported exit IRRs of portfolio companies within a fund. This is different from the definition used by Phillips (2018), who defined underlying risk as the standard deviation of the IRR for all portfolio companies within a private equity firm. Hence, the underlying risk of a fund in the historical data is formulated as follows:

$$Standard\ Deviation\ of\ IRR_f = \sqrt{\frac{\left(\sum_{n=0}^{N} IRR_{(i,c)} - \overline{IRR_{i,f}}\right)^2}{N-1}}$$

Where f is a specialized fund within industry i, c denotes a individual portfolio company within fund f that operates in industry i, and N is the number of portfolio companies within fund f. The risk- and market-adjusted IRR in the historical are then given by:

$$Average\ Risk\text{-}\ and\ Market\text{-}Adjusted\ IRR_f = \frac{Average\ Market - Adjusted\ IRR_f}{Standard\ Deviation\ of\ IRR_f}$$

$$Median\ Risk\text{-}\ and\ Market\text{-}Mdjusted\ IRR_f = \frac{Median\ Market - Adjusted\ IRR_f}{Standard\ Deviation\ of\ IRR_f}$$

This method enables an insightful comparison between the risk-adjusted performance of different private equity funds by considering the variation across all portfolio companies within the said fund. By comparing the risk- and market-adjusted IRRs of specialized and diversified funds, one can determine whether specialized funds, shown to have a higher risk in previous simulation outputs, have provided sufficient returns for the increased risk.

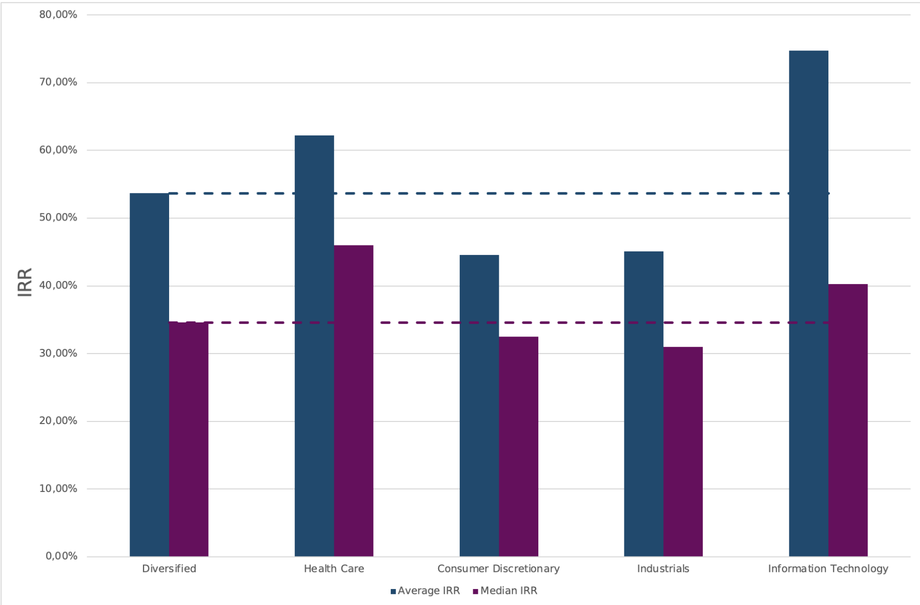
The risk- and market-adjusted IRR was also used to evaluate the performance of various funds across different regions. In Section 6.5, we present the performance of different industries through a Global fund, a US fund, and a Europe fund. However, as mentioned in Section 5.2, the lack of VC data limits this analysis to the buyout segment only.

6 Results and Discussion

6.1 Exit IRR

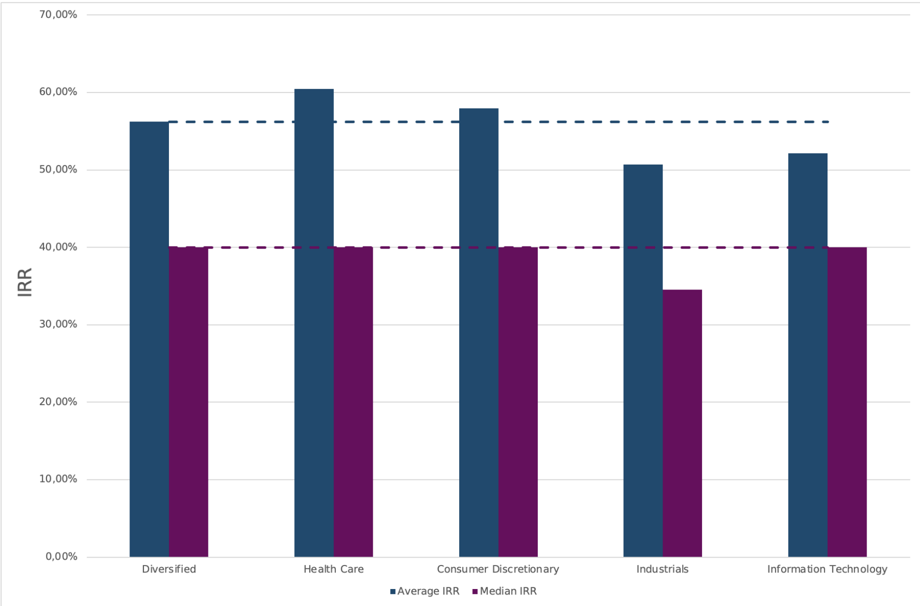
The descriptive statistics of the computed Exit IRRs are presented in Tables A5.1, A5.2, and A5.3 in the Appendix. To better compare and contrast the different funds, the IRR performance data are presented visually:

Figure 6.1: US Buyout Deals: Exit IRR



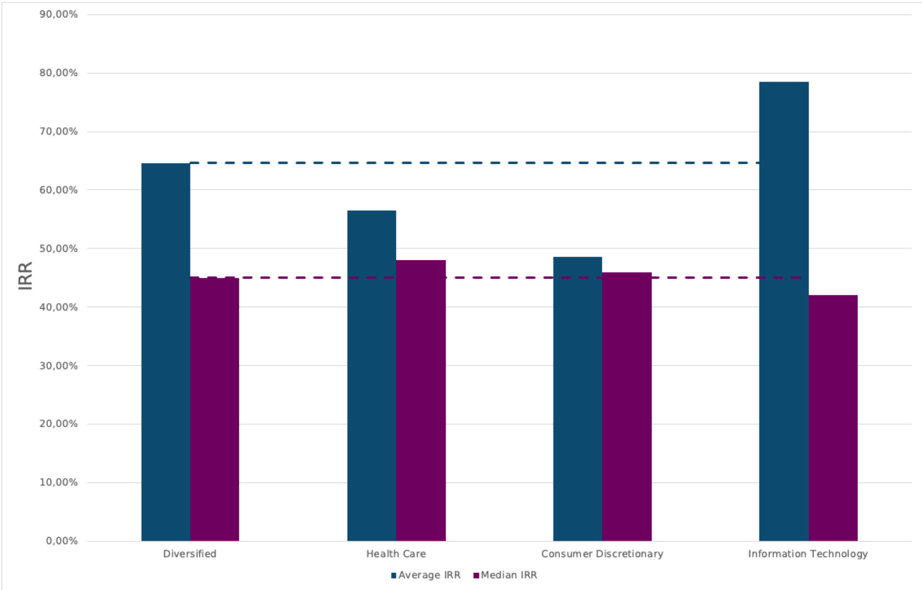
In the diagrams, the blue columns represent the average values, while the purple column represents the median values. The diagram shows that the Information Technology fund had the highest IRR among US buyout funds, with an average of 74.77%. This was followed by the Health Care fund, which had an average IRR of 62.18%. The Diversified fund had a lower average IRR of 53.67%, and Consumer Discretionary and Industrials had even lower average IRRs of 44.60% and 45.07%, respectively. By examining the median figures, we can see that the Health Care fund is the top performer, while the Information Technology fund has a significant drop in performance but still has a higher IRR than the Diversified fund. Overall, the average and median figures suggest that investing in specialized funds focused on health care and information technology provided higher returns than investing in an industry-diversified fund.

Figure 6.2: EU Buyout Deals: Exit IRR



The average IRR of the Diversified fund in EU buyouts was 56.22%. In contrast to US buyouts, the Information Technology fund was the second worst-performing industry in EU buyouts, with an average IRR of 52.16%. The worst-performing sector within EU buyout funds was Industrials, which had an average IRR of 50.72%. On the other hand, the Health Care fund performed well, being the top performer amongst the European buyout funds with an average IRR of 60.43%. Consumer Discretionary also performed better than the Diversified fund, with an average IRR of 57.93%. Looking at the average figures, the results show that specialized funds in health care and consumer discretionary sectors outperformed the diversified fund. However, when considering the median IRR, a different picture emerges. Here, the results show that all funds had an equal performance with a median IRR of 40%, except the Industrials fund which stood out with a median IRR of 34.50%. This suggests that none of the specialized funds outperformed the diversified fund, with the only underperforming fund being Industrials.

Figure 6.3: EU Venture Capital Deals: Exit IRR



The average IRRs of European venture capital funds suggest that, while diversification generally leads to higher returns in the EU venture capital segment, the information technology sector appears to be an exception. More specifically, the Information Technology fund had an average IRR of 78.53%, significantly outperforming the Diversified fund's average IRR of 64.60%. By contrast, Health Care and Consumer Discretionary had average IRRs of 56.49% and 48.64%, respectively. However, when considering the median exit IRRs, the results were reversed. As the Diversified fund had a median exit IRR of 45.00%, the Health Care and Consumer Discretionary funds now outperformed with median exit IRRs of 48.00% and 46.00%, respectively. Notably, the Information Technology fund underperformed relative to the Diversified Fund with a median exit IRR of 42.00%.

For buyouts and venture capital, we generally observe high values of reported exit IRRs. Even though the findings agree that private equity investments generally generate superior and high returns relative to other alternative investments, these numbers are unusually large. One reason can be that the data set obtained from Preqin may be subject to positive reporting bias (see Section 5.2.1), resulting in an overweight of high-performing deals. This claim can be further supported by Table 5.1, that shows that under 2% of the reported exit IRRs were negative, which is noteworthy. This may also be problematic when properly evaluating the performance data, as demonstrated in how the use of average and median IRRs may yield conflicting results. Due to the limited sample size and potential for a positive skew, we follow Phillips (2018), who argues that the median IRR may be a more reliable indicator of central tendency and expected returns as it is less susceptible to the influence of positive outliers. Hence, this study will mostly focus on the median IRR for the remainder of the analysis.

6.2 Estimated Minimum Required IRR

The descriptive statistics of the historical Exit IRRs and the estimated minimum IRRs from the simulations for each fund are presented in Appendix A7. To better compare and contrast the results from the estimation model, the data are presented visually below:

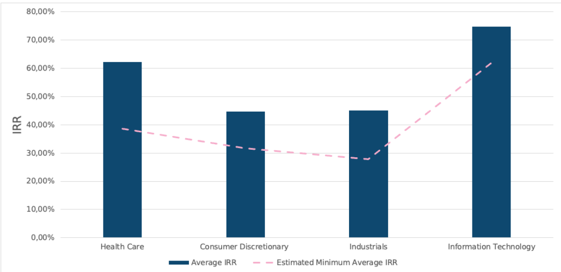


Figure 6.4: US Buyouts Estimated vs. Actual Average Exit IRR

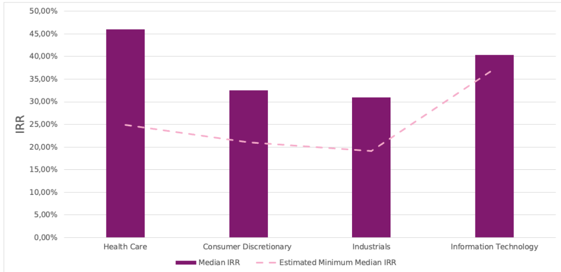


Figure 6.5: US Buyouts Estimated vs. Actual Median Exit IRR

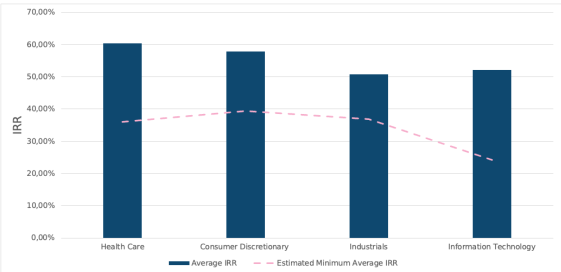


Figure 6.6: EU Buyouts Estimated vs. Actual Average Exit IRR

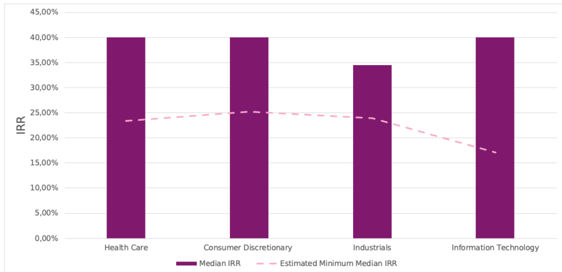


Figure 6.7: EU Buyouts Estimated vs. Actual Median Exit IRR

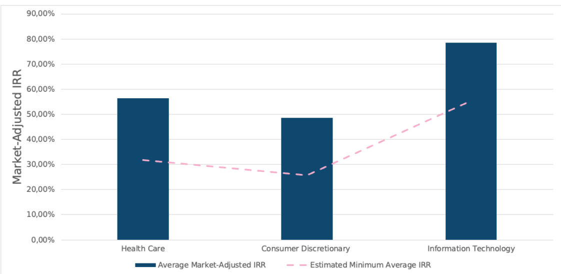


Figure 6.8: EU Venture Capital Estimated vs. Actual Average Exit IRR

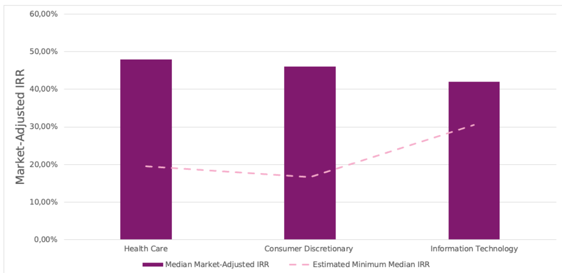


Figure 6.9: EU Venture Capital Estimated vs. Actual Median Exit IRR

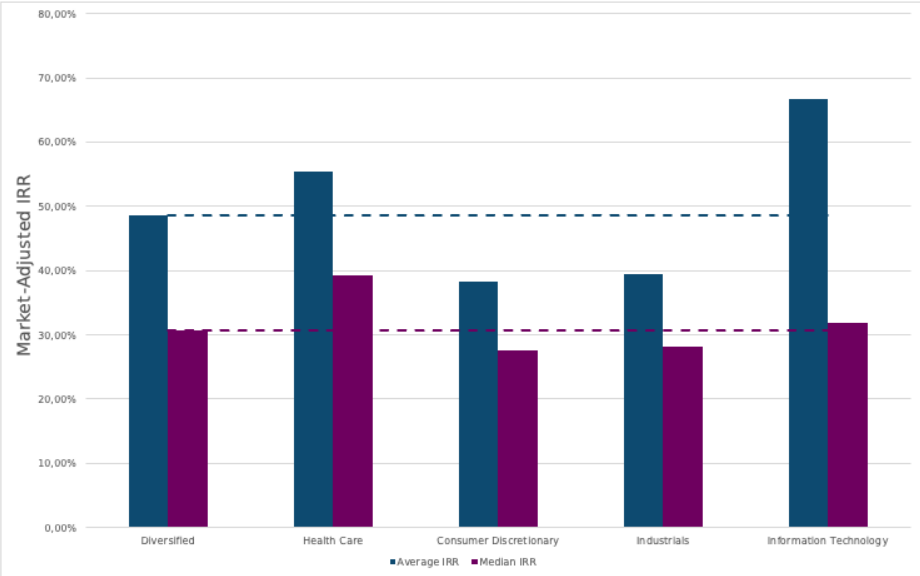
This section will compare the estimated minimum IRR to the actual exit IRR for various specialized funds to evaluate their risk-adjusted performance relative to diversified funds. As previously mentioned in Section 4, the estimation model for the minimum required IRR for specialized funds was developed based on the risk-adjusted performance of simulated funds. Therefore, the minimum IRR is what the specialized fund must achieve in IRR so that when adjusted for risk, the performance of diversified and specialized funds is at least equal. The comparison is presented through the diagrams above, with the pink lines representing the estimated minimum required IRR and the columns representing the actual historical IRRs. Suppose the column for a particular specialized fund is below the pink dotted line. In that case, the estimation model suggests the specialized fund to underperform compared to the diversified fund after considering the underlying risk of the exit IRR. Conversely, suppose the column for a specialized fund is above the dotted line. In that case, the specialized fund is estimated to outperform the diversified fund on a risk-adjusted basis.

The estimation model predicts outperformance on a risk-adjusted basis for all specialized funds from the historical data. This finding is notable because it suggests that, despite some specialized funds achieving lower exit IRRs than diversified funds in the analysis of Section 6.1, all of the specialized funds are still anticipated to outperform when evaluated using a risk-adjusted measure. To verify whether this is the case in practice, the specialized funds need to be compared with the diversified funds on a risk-adjusted basis. If the suggestions from the estimation model and the results from the risk-adjusted performance analysis coincide, one can conclude that simulations can be used to detect the minimum return thresholds for private equity funds. In the following parts of this section, the exit IRRs are therefore first adjusted for market risk before underlying risks are considered.

6.3 Market-Adjusted IRR

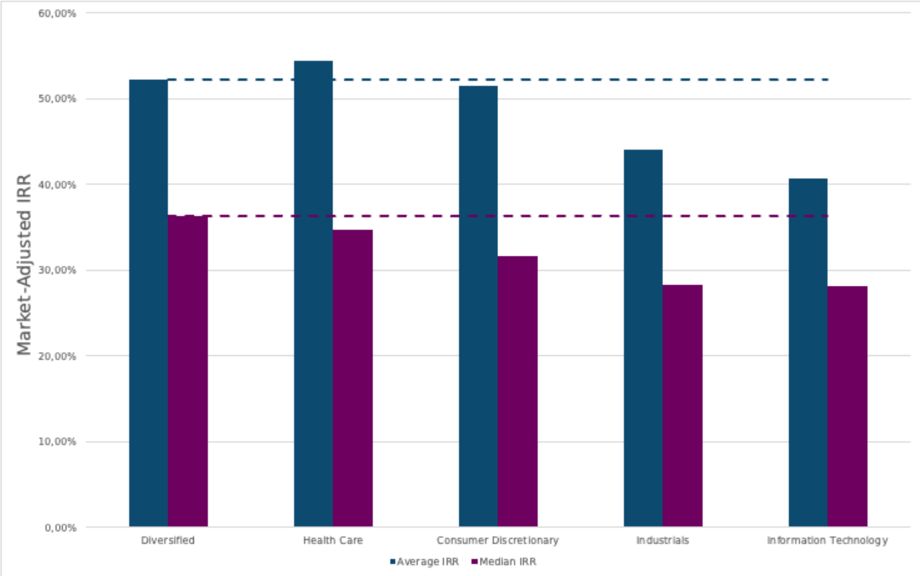
The descriptive statistics of the computed Market-Adjusted IRRs are presented in Tables A6.1, A6.2, and A6.3 in the Appendix. To better compare and contrast the different funds, the IRR performance data are presented visually:

Figure 6.10: US Buyout Deals: Market-Adjusted IRR



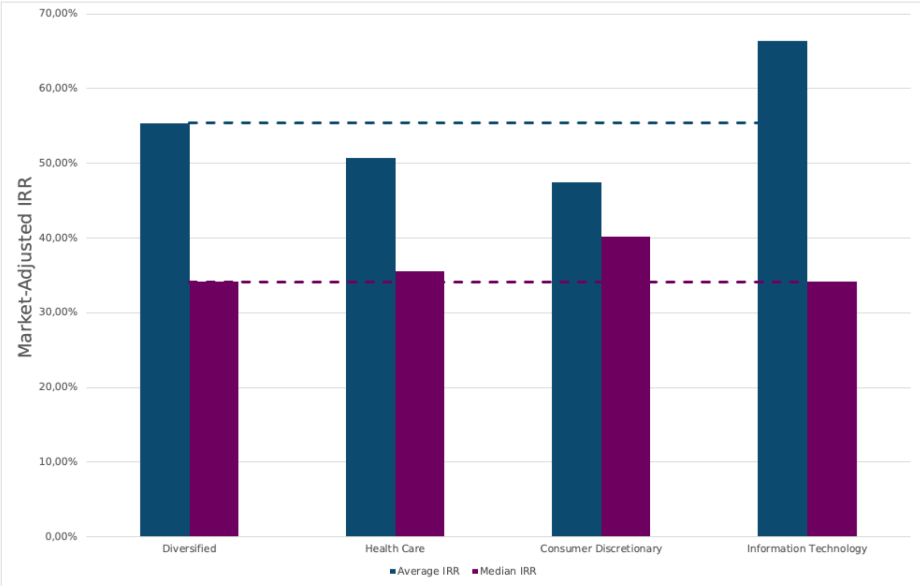
After adjusting the exit IRR for market returns for US buyout funds, the Diversified fund had a median market-adjusted IRR of 30.68%. The Health Care fund had a median IRR of 39.25% and was the top-performing fund, while the Information Technology fund was the second best-performing with a median market-adjusted IRR of 31.86%. Compared to the earlier exit IRR analysis, the health care and information technology sector still has the highest performance when adjusted for market returns, however with a lower margin relative to the diversified alternative. This suggests that some excess returns relative to the Diversified fund are compensation for the increased market risk in these sectors. The funds with the lowest median market-adjusted IRR, which still underperformed relative to the Diversified fund, were the Industrials and Consumer Discretionary funds, with 28.09% and 27.65%, respectively.

Figure 6.11: EU Buyout Deals: Market-Adjusted IRR



In the EU buyout segment, none of the specialized funds outperformed the Diversified fund after adjusting for the market returns, which is evident in the figure above. The Diversified fund had a median market-adjusted IRR of 36.30%, while the Health Care fund was the best-performing specialized fund with a median market-adjusted IRR of 34.74%. The Consumer Discretionary, Industrials, and Information Technology funds had median market-adjusted IRRs of 31.58%, 28.24%, and 28.13%, respectively. In the analysis of the median exit IRRs, Industrials was the only fund performing worse than the Diversified fund. On the other hand, all the other funds had the same returns, suggesting that some market risk was priced in the returns of specialized funds.

Figure 6.12: EU Venture Capital Deals: Market-Adjusted IRR



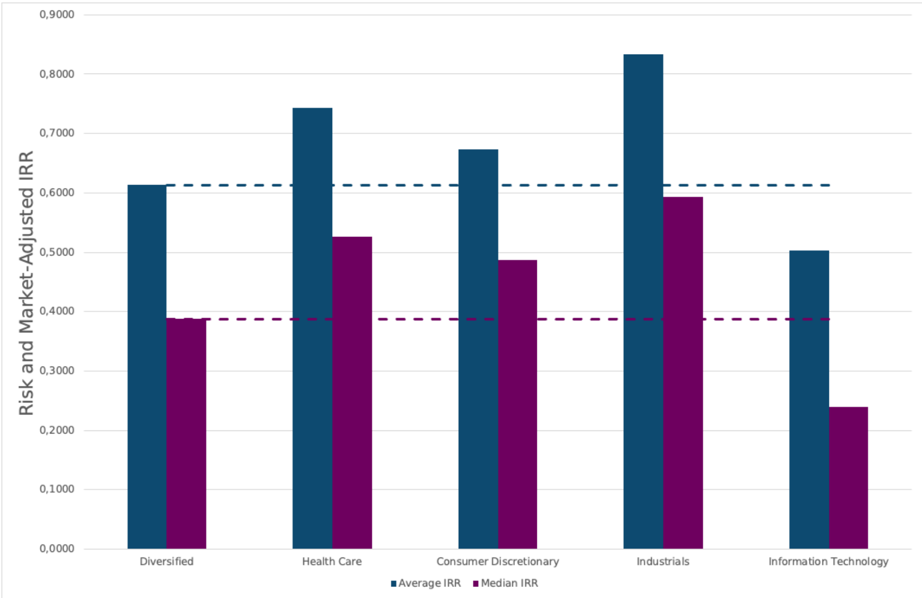
The analysis of the market-adjusted IRRs in the European venture capital market yielded divergent results compared to the analysis of exit IRR. The Diversified fund had the lowest median market-adjusted IRR at 34.12%, while the Information Technology fund, which underperformed in the earlier analysis, had a slightly higher market-adjusted IRR of 34.23%. The Health Care and Consumer Discretionary funds were the highest-performing funds, with median market-adjusted IRRs of 35.49% and 40.24%, respectively. The previous exit IRR analysis noted that the Health Care fund demonstrated the highest returns. However, when adjusted for market returns, the Consumer Discretionary fund outperformed the Health Care fund. This suggests that the reported performance of the Health Care fund may be more heavily influenced by favorable market conditions rather than the fund's active management, compared to the Consumer Discretionary fund.

Even after adjusting the exit IRRs for the market returns, positive returns were still observed for all the funds. Theoretically, this suggests that LPs generally get enough compensation for the increased industry-specific risk of only investing in a particular sector. However, to further emphasize the discussion from Section 6.1, this could be additional evidence that there may be an issue with the representativeness of the data sample, as it is generally unlikely that all funds would consistently outperform the market with this magnitude. Nevertheless, the findings provide valuable information when comparing the different funds relative to each other.

6.4 Risk- and Market-Adjusted IRR

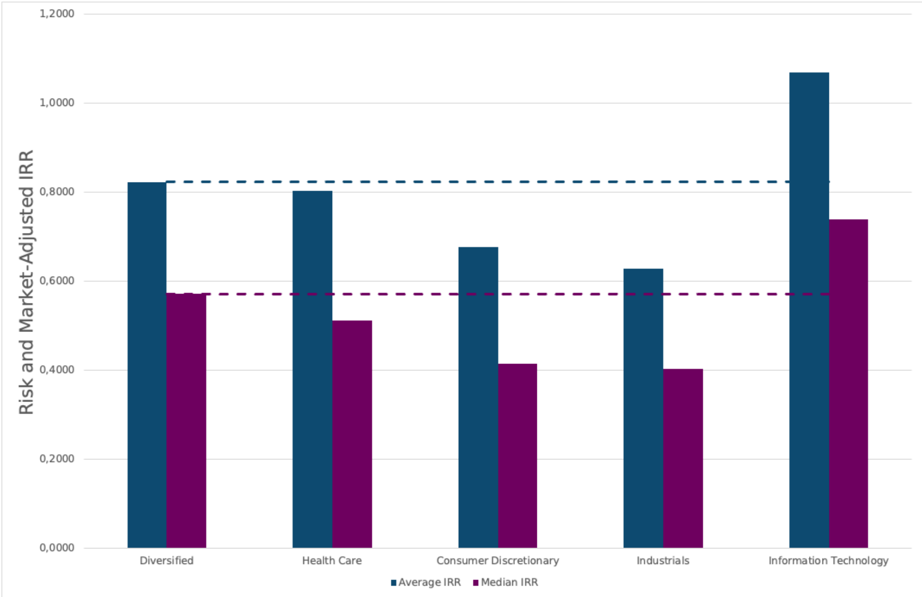
The descriptive statistics of the computed Market-Adjusted IRRs for each strategy and funds are presented in Tables A8.1, A8.2, and A8.3 in the Appendix. To better compare and contrast the different funds, the IRR performance data are presented visually:

Figure 6.13: US Buyout Deals: Risk- and Market-Adjusted IRR



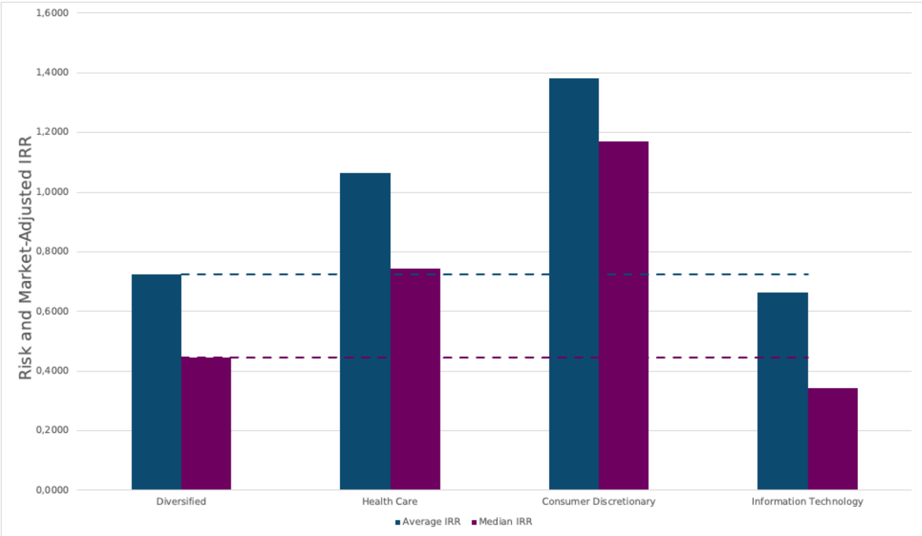
In the US buyout segment, the median risk- and market-adjusted IRR for the Diversified fund was 0.3874. Among the specialized funds, the Information Technology fund had the lowest median risk- and market-adjusted IRR at 0.2399. On the other hand, the Health Care, Consumer Discretionary, and Industrials funds all had higher median risk- and market-adjusted IRRs than the Diversified fund, with values of 0.5267, 0.4865, and 0.5932, respectively. After adjusting for underlying risk, the results suggest that all specialized industry funds, except for Information Technology, outperformed the Diversified fund on a risk- and market-adjusted basis. As previously mentioned, the estimation model predicted that every specialized fund would outperform the Diversified fund on a risk-adjusted basis. However, this prediction proved to be incorrect, as the Information Technology fund underperformed.

Figure 6.14: EU Buyout Deals: Risk- and Market-Adjusted IRR



In the EU buyout segment, the Information Technology fund was the only specialized fund that outperformed the Diversified fund. This is noteworthy, as the Information Technology fund had previously underperformed the Diversified fund when only adjusting the IRR for market risk. This implies that the Information Technology fund has a much lower underlying risk than the diversified fund, which can be confirmed by the low standard deviation in Table A5.2 in the Appendix. After adjusting for the underlying risk, the Information Technology fund had a median risk- and market-adjusted IRR of 0.7382, while the Diversified fund had a risk- and market-adjusted IRR of 0.5710. The Health Care fund performed below the diversified fund with a risk- and market-adjusted IRR of 0.5117. With a risk- and market-adjusted IRR of 0.4147 and 0.4031, the Consumer Discretionary and Industrials fund was the worst-performing when adjusted for underlying risks. The estimation model predicted that all the specialized funds would outperform relative to the diversified fund, which is wrong because every specialized fund except Information Technology underperformed.

Figure 6.15: EU Venture Capital Deals: Risk- and Market-Adjusted IRR



After adjusting for the underlying risks, the Information Technology fund was still the only underperforming specialized fund in the EU venture capital segment with a median risk- and market-adjusted IRR of 0.3416. The diversified fund had a risk- and market-adjusted of 0.4469, while the Health Care fund had 0.7437. The best-performing fund was the Consumer Discretionary fund which had a risk- and market-adjusted of 1.1706. The outperformance was not as significant in the analysis of the market-adjusted IRRs, which implies that the consumer discretionary fund had a much lower underlying risk than the diversified fund. This can be confirmed by Table A5.3 in the Appendix. The estimation model predicted that all the specialized funds would outperform relative to the diversified fund, which proved to be incorrect as Information Technology underperformed.

6.5 Regional Performance

The descriptive statistics of the Risk- and Market-Adjusted IRRs for private equity funds globally, in the US, and Europe is presented in Appendix A8.1.

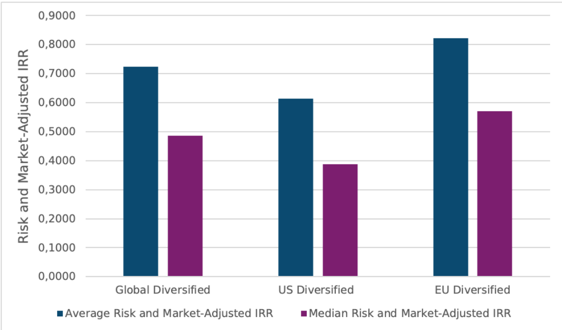


Figure 6.16: Diversified Risk-Adjusted Fund Performance

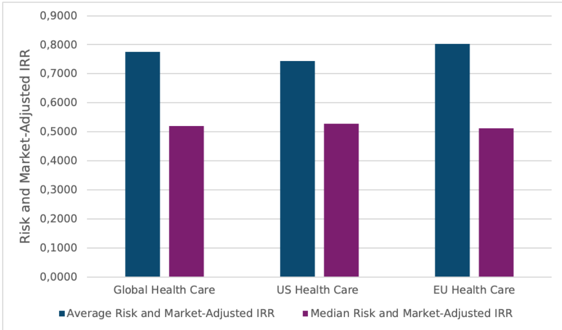


Figure 6.17: Health Care Risk-Adjusted Fund Performance

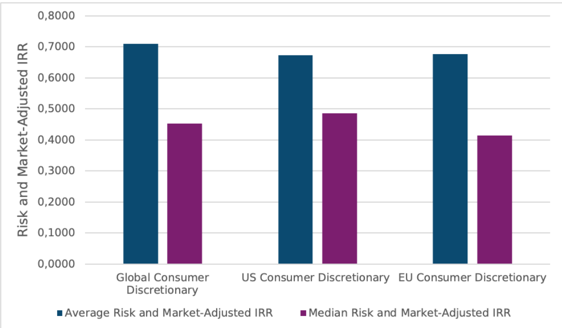


Figure 6.18: Consumer Discretionary Risk-Adjusted Fund Performance

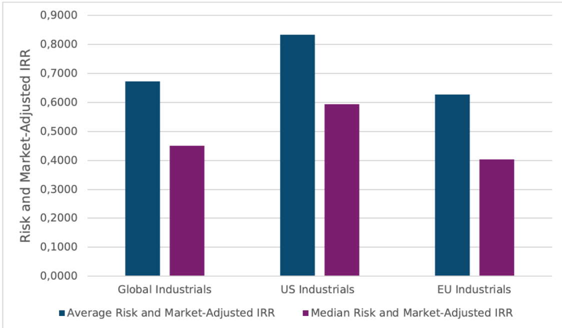


Figure 6.19: Industrials Risk-Adjusted Fund Performance

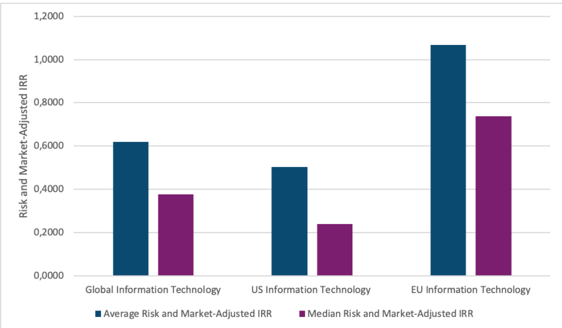


Figure 6.20: Information Technology Risk-Adjusted Fund Performance

By analyzing the median risk- and market-adjusted IRR, one can see that every global fund was outperformed by one of the other regions. The best-performing diversified fund was made of deals in Europe. This was also the case for Information Technology funds, where the Europe region significantly outperformed by a large margin. However, when looking at Industrials, the US was the highest-performing region, while Europe had the lowest median risk- and market-adjusted IRR. While the various regions reveal minimal differentiation in the Health Care sector, the US region slightly outperformed the other regions on a median risk- and market-adjusted basis. The same conclusion can be drawn upon examination of the Consumer Discretionary sector. However, it is noteworthy that the Consumer Discretionary sector was the only one in which the Global fund outperforms the US and European funds when considering average performance. These results suggest that deals in Consumer Discretionary outside the US and Europe have significantly contributed to the higher average of the global fund. However, when using median figures, the conclusion is similar to the other sectors, that the Global fund falls between the US and European funds in terms of performance.

It is crucial to note that the EU funds mainly influence the performance of the Global funds. This can be explained by more observations in this region across all sectors, as seen in Table 5.2.2. Hence, as Global reflects all observations, the remaining data from "Rest of the World" and the US are underrepresented compared to Europe. This is critical as it significantly reduces the accuracy and representativeness of the findings for geographical diversification effects.

6.6 Discussion

The present analysis suggests that the proposed estimation model for estimating the minimum return thresholds for specialized private equity funds may be flawed. This may be due to the use of implied correlations as an input variable, which assumes that the standard deviation of IRRs within a fund fully reflects the average correlation between portfolio companies within the fund. However, this assumption may not hold in practice as there may be other determinants of correlations between portfolio companies.

Additionally, the estimation model is built on outputs from simulated funds based on historical stock market data and previous research. One of the inputs in the simulation model is the expected return, which was based on reported returns in private equity indices compiled by Cambridge Associates. However, these indices may be subject to reporting biases and delays, leading to outdated and potentially inaccurate risk-return assessments.

These limitations, paired with a dataset suspected to be subject to positive reporting bias, may contribute to the model's poor performance in predicting the relative performance of specialized and diversified funds. Moreover, the average return on investment for a private equity fund was calculated using an arithmetic average of each portfolio company's returns rather than a weighted average based on the size of each portfolio company. This means that the returns from small companies with a high return on investment are given the same weight as those from larger companies with a lower return on investment. Overall, the inaccuracy of the estimation model and the possible limitations discussed above suggests that it is not possible to accurately estimate the minimum return thresholds for specialized private equity funds using the simulation model built in this thesis.

Looking at the various IRR measures, some interesting discoveries emerge. For instance, venture capital was the only segment with the same conclusions regarding which funds that out and underperformed relative to the diversified fund through all three measures. However, this was not the case for US buyouts, where funds that underperformed the diversified fund, using the median exit IRR and median market-adjusted IRR, outperformed when adjusted for underlying risk. Interestingly, this phenomenon was also present for the Information Technology fund in EU buyouts, as it only outperformed when adjusted for underlying risk. These discoveries highlight how some industries and funds may be subject to fluctuations and higher volatility in returns, which are not observable from the reported performance figures.

Our analysis of risk- and market-adjusted IRRs shows mixed results for the different segments and funds. For instance, the results of US buyouts imply that investors would be better off with a specialized investment strategy as most specialized funds outperform the diversified fund. In contrast, the results suggest that European investors are generally better off on a risk-adjusted basis holding a diversified portfolio. The venture capital findings support earlier studies claiming that VC funds with industry specialization are better positioned to achieve superior performance, as Health Care and Consumer Discretionary significantly outperform the Diversified fund. Overall, the differences within and between the segments suggest that investors should invest with caution in specialized funds as they do not consistently compensate for the additional risk.

This implication also holds in the regional analysis, where either the US or Europe sector always falls short compared to the Global fund. However, as Global never was present as the top performer, this indicates that specializing in one region may deliver higher returns per unit of risk to the LPs that have committed capital. These findings aligned with those of Lossen (2006), who discovered that a diversified fund's rate of return has no meaningful effect on performance across different regions. One possible explanation is that it is easier for private equity firms to focus on and devote time to each portfolio company when they are closer to each other, which is consistent with the findings of Cumming and Dai (2010); Jääskeläinen et al. (2006); Gifford (1997).

According to conventional finance theory, specialized private equity funds should generate higher returns to compensate for the increased risk they face compared to diversified funds. However, as the discussed results suggest, the return of specialized funds was not consistently superior to the diversified funds. One potential explanation is that the market for specialized PE funds is relatively efficient, with investors able to identify and invest in the most promising specialized funds, leading to competitive returns (Ewens et al., 2013). This would be consistent with the idea of capital market equilibrium, in which all investments are expected to generate returns that reflect their inherent risk and potential for growth. If this is the case, it may indicate that the market for specialized PE funds is functioning efficiently, with no particular investment strategy or type of fund consistently outperforming others.

Another potential explanation for seeing underperformance of specialized PE funds is the theory of diminishing returns to scale. This refers to the idea that the returns a fund generates may decrease as the fund grows in size (Berk and Green, 2004). This can occur because it becomes increasingly difficult to find high-quality investment opportunities in portfolio companies as the private equity fund grows or because the fund may have to take on less attractive investments to deploy all of its capital. Therefore, it is possible that some of the specialized funds in the analysis underperformed the diversified funds due to diminishing returns to scale as the cash inflows in the particular sector became larger (Kaplan and Schoar, 2005; Phalippou and Gottschalg, 2009). This can make it more challenging for the GPs of specialized funds to achieve the required returns for increased risk, making the specialized fund a less attractive investment for the LPs.

Another implication of diminishing return to scale is subsequent funding for new funds of the GPs. General partners often raise capital for new funds while also running current funds. According to Shobe (2016), the capital raised depends on the current funds' returns. Therefore, the GP is pressured to have a high return as early as possible, knowing that the current returns may decrease with more cash inflow. This can create incentives for the GP to invest in riskier sectors to achieve good enough returns such that LPs are willing to invest in subsequent funds Batt and Appelbaum (2021).

These findings provide several practical implications for GPs and LPs. As we find that specialized funds do not always provide outperformance, there seem to be varying degrees of benefits to be gained from investing in private equity funds that specialize in specific industries or countries. This is likely due to the fact that such specialization only sometimes translates into superior investment decision-making or the provision of greater value-adding services to portfolio companies. This suggests that the potential benefit of specialized private equity firms in dealing with the information asymmetry and agency problems prevalent in the selection and management of private companies may be limited. Hence by pursuing a diversified investment strategy, GPs can better align their interests with those of the LPs and reduce the potential for agency problems that may arise at a lower cost.

This thought can also be supported by research on equity mutual fund performance. As our diversified funds contain all deals in their respective segment, one can think of it as a passive index fund and the specialized funds as active mutual funds. One of the earliest findings in this area was Sharpe (1966), who found that most mutual funds do not outperform the market consistently. Furthermore, he found that the returns of mutual funds are primarily determined by the market's overall performance rather than the fund manager's skill, supporting the idea of the market being efficient. This idea can be further emphasized by Carhart (1997), who suggests that mutual funds that perform well in one period are not likely to perform better in the future. These findings go well in line with the discoveries in this research. Implying that in a long-term perspective, the diversified index fund is expected to perform best, as we cannot expect specialized funds to always deliver a sufficient risk-adjusted return.

Following the same reasoning, without considering management fees, the implications for LPs would be that the investor should carefully consider investment strategies and track records of the fund managers to consider how the funds fit with their overall investment goals and risk tolerance if picking a specialized strategy. However, given that specialized funds do not always provide superior returns the question of why many specialized funds exist today and why LPs should invest in specialized funds still have ambiguous explanations. The answer to this question is commonly debated in several areas of modern finance research today. Some possible reasons could be that LPs choose to invest in a specialized private equity fund to diversify their portfolio and add exposure to a specific industry or sector. In addition, private equity investment projects can provide increased access to specific investment opportunities that aligns well with the investment strategy of the LP. LPs can also engage with experienced GPs believed to generate solid returns and sufficiently satisfy the investors interests and risk preferences.

7 Conclusion

7.1 Main Conclusions

This research aimed to discover if specialized PE funds outperformed diversified PE funds on a risk-adjusted basis. In order to answer this research question, a combination of simulations and real-life data was used. The simulation model was constructed based on previous research and stock market data to explore the relationship between diversification and performance. The simulation demonstrated how a higher pairwise correlation between investments causes higher overall fund risk, supporting standard finance theory stating how LPs should demand excess returns from GPs managing specialized funds compared to diversified funds. In addition, a model was constructed from the simulations that estimated the minimum needed risk- and market-adjusted IRR for a specialized fund to have the same risk-return trade-offs as a diversified fund.

The proposed estimation model for predicting the performance of specialized private equity funds relative to diversified funds had low accuracy. When analyzing the risk- and market-adjusted IRR, it was found that the model's estimated measures did not align with the historical performance measures. While some specialized funds outperformed the diversified ones on a risk-adjusted basis in the historical data, the deviation between the estimated and historical performance measures was significant. Since the estimation model was based on simulations based on stock market correlation data, our findings conclude that it is not viable to use the correlation between similar public companies to understand the effect of diversification on the performance of private equity funds. As a result, the simulation model built in this thesis may not be an adequate tool to derive the minimum return thresholds for specialized funds.

Using a data sample of 1,656 fully liquidated deals from Preqin, an IRR performance analysis was conducted to explore the potential benefits of diversification in historical data. The different IRR measures gave different conclusions for different regions and industries, suggesting that specialized funds do not always provide sufficient returns as proposed in traditional finance theory. In conclusion, the available evidence does not allow us to make a definitive statement regarding a causal relationship between specialization and superior performance. This may indicate that the market for specialized private equity funds may be efficient, leading to competitive returns that reflect inherent risks and potential for growth. Alternatively, diminishing returns to scale and incomplete accounting for underlying risks may contribute to the underperformance of specialized PE funds compared to diversified ones. This emphasizes how LPs should always be critical when evaluating the reported performance in different industries and sectors when constructing an investment strategy. Thus, LPs need to consider the specific strategies and regional factors that may influence the performance of a sector rather than simply in a sector because it has historically outperformed.

7.2 Limitations

There are several limitations to our research. First, the simulation model was restricted regarding the number of input parameters as it assumes that the investment value follows a Geometric Brownian Motion. As a result, the model can become too simplified to extract valuable insights to implement in practice. Second, the results are based on a relatively small sample, with a varying degree of observations in the different industries in the buyout and venture capital segments. This inconsistency, and the possibility of the sample being biased, decrease the overall reliability of the conclusions drawn in this research.

7.3 Further Research

Going forward, it is expected that further data will become accessible. Given the high number of public LPs participating in private equity, more individual company-level data will become accessible over time, resulting in more representative insights regarding private equity investments. Thus, further research should take advantage of the better-quality data when estimating input parameters in their simulations and modeling the input correlations for the buyout and venture capital segment to estimate minimum required returns with higher accuracy. Finally, researchers could investigate in depth the practical implications of the results for investors and fund managers. This could include an analysis of the optimal allocation of capital across specialized and diversified funds and examination of the impact of fund selection on portfolio risk and return characteristics for the LP.

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Appendix

A1 Mathematical Theories

A1.1 Geometric Brownian Motion

The value of an investment can be modeled using the following stochastic differential equation:

$$dS_t/dt = (r + \sigma\epsilon_t)\, S_t \tag{.1}$$

A random investment in Equation .1 with a mean value of zero unrelated to prior investments is denoted by the symbol t. The investment's volatility is denoted by the σ, while the symbol r denotes the percentage drift. The stochastic differential equation shown below can be used to rewrite Equation .1:

$$dS_t = (rdt + \sigma dW_t)\, S_t \tag{.2}$$

where W_t is defined as the Wiener Process (detailed in the appendix).

To solve Equation .2, Itôs formula, which is employed to calculate the derivative of a time-dependent stochastic process, is applied. Itô's formula can be shown as $f(t,s) = ln(s)$. The appendix provides additional information regarding the formula. These are the partial derivatives of $f(t,s)$:

$$\frac{\partial f}{\partial t}(t,s) = 0, \quad \frac{\partial f}{\partial x}(t,s) = \frac{1}{s}, \quad \frac{\partial^2 f}{\partial s^2}(t,s) = -\frac{1}{s^2}$$

By utilizing the Itôs formula to $f(t,s)$, one gets the following results:

$$\begin{aligned} df(t,S_t) &= \tfrac{\partial f}{\partial t}(t,S_t)\,dt + \tfrac{\partial f}{\partial x}(t,S_t)\,dS_t + \tfrac{1}{2}\tfrac{\partial^2 f}{\partial s^2}(t,S_t)\,d\langle S\rangle_t \\ &= 0 \cdot dt + \tfrac{1}{S_t}\left(rS_t dt + \sigma S_t dW_t\right) - \tfrac{1}{2}\tfrac{1}{S_t^2}\sigma^2 S_t^2 dt \\ &= \left(r - \tfrac{1}{2}\sigma^2\right) dt + \sigma dW_t \end{aligned}$$

The solution to the SDE in Equation .2 can be obtained by rewriting the expression in its integral form and then taking the exponent:

$$S_t = S_0 e^{\left(r-\frac{1}{2}\sigma^2\right)t+\sigma W_t} \tag{.3}$$

The theories presented in this section are the basis of the Geometric Brownian Motion applied when simulating the development of investments. Most of the theories explained is sourced from Panik (2017).

A1.1.1 Cholesky Factorization

In the GBM formula, the error term considers the possible correlation between various random variables. This thesis handles the correlation between investments from the same fund using Cholesky Factorization.

A matrix A is a lower triangular if the Cholesky Factorization of $\sum$ is defined as $AA^T = \sum$. If the equation above is fulfilled, it is possible to do a Cholesky Factorization and the matrix A will be unique. For clarification, consider the following example:

$$\sum = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2 p_{1,2} \\ \sigma_1\sigma_2 p_{1,2} & \sigma_2^2 \end{pmatrix}$$

Assuming that both σ_1^1 and σ_2^1 are positive variances, a Cholesky factorization of the covariance will yield the Cholesky factor, A:

$$A = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2 p_{1,2} & \sqrt{(1-p_{1,2})}\sigma_2 \end{pmatrix}$$

A sample from a bivariate normal distribution $N(\mu, \sum)$, can be done by utilizing the Cholesky factor A defined earlier:

$$X_1 = \mu_1 + \sigma_1 Z_1$$
$$X_2 = \mu_2 + \sigma_1 p_{1,2} Z_1 + \sigma_2\sqrt{(1-p_{1,2})}Z_2$$

Where Z_1 and Z_2 are two independent standard normally distributed variables (Glasserman, 2004).

A1.1.2 Theory of Stochastic Processes

Diffusion Process A diffusion process is typically viewed as two distinct stochastic processes. The first portion is commonly referred to as the drift term, whereas the second contains the randomness. The dynamics of a diffusion process have the following form:

$$dX_t = \mu\left(t, X_t\right) dt + \sigma\left(t, X_t\right) Z_t \text{ and } X_0 = x$$

Where x is a constant, Z_t is a Gaussian disturbance term independent of events before the period t. The drift of the process is represented by μ, while the $sigma$ represents its diffusion. The drift and the diffusion of the process are deterministic. Following is a description of the Wiener Process that can be used to model Z_t.

Wiener Process The following four criteria need to hold for a stochastic process $W(t)$ to be defined as a Wiener process:

1. $W(0) = 0$ with probability 1
2. $W(t)$ has independent increments
3. For two points $0 \leq s < t$ the stochastic variable $W(t) - W(s)$ has the Gaussian distribution $N(0, t-s)$
4. W has continuous trajectories

Utilizing the Wiener Process, the diffusion process formulated above can be rewritten to:

$$dX_t = \mu\left(t, X_t\right) dt + \sigma\left(t, X_t\right) dW_t \text{ and } X_0 = x$$

(Björk, 2009)

Itôs Formula Itô processes are stochastic processes that can be expressed as the sum of a deterministic and stochastic integral. In the formula below, S_t is defined as an Itô process:

$$S_t = s + \int_0^t r(u)du + \int_0^t \sigma(u)dW_u$$

These are the initial conditions and the stochastic differential equation of S_t:

$$dS_t = r(t)dt + \sigma(t)dW(t)$$
$$S_0 = s$$

where s is a constant and r and σ are processes. The rest of this section presents some mathematical relations. The presented leading to Itôs formula is heuristic and not a formal proof. By setting $u < t$ and defining

$$\Delta t = t - u \text{ and } \Delta W_t = W_t - W_u$$

one can derive the relations defined below from $\Delta W_t \sim N(0, t-u)$.

$$\begin{aligned} E\left[\Delta W_t\right] &= 0 \\ \operatorname{Var}\left(\Delta W_t\right) &= E\left[(\Delta W_t)^2\right] = \Delta t \\ \operatorname{Var}\left((\Delta W_t)^2\right) &= 2(\Delta t)^2 \end{aligned} \tag{.4}$$

According to Itôs formula, the stochastic differential of a process $Z_t = f(t, S_t)$ is given by:

$$df\left(t, S_t\right) = \left\{\frac{\partial f}{\partial t} + r\frac{\partial f}{\partial s} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial s^2}\right\}dt + \sigma\frac{\partial f}{\partial s}dW_t$$

Utilizing the relations defined in equation .4, Itôs formula can be rewritten as:

$$f\left(t, S_t\right) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial s}dS_t + \frac{1}{2}\frac{\partial^2 f}{\partial s^2}d\langle S\rangle_t$$

where $(dt)^2 = 0, dt \cdot dW = 0$ and $(dW)^2 = dt$.

A1.2 Standard Monte Carlo

Law of Large Numbers

Assume that $x_1, x_2, ...$ is a sequence of independent identically distributed (i.i.d) random variables with a expected value of $r = E[x_i] < \infty$ and variance $\sigma^2 < \infty$, then

$$\tfrac{1}{n}\sum_{i=1}^{n} x_i \underset{n\to\infty}{\rightarrow r}$$

with a probability converging towards one. This result provides information that almost every observation of x_i, which in this research is a simulated sequence $x_1, x_2, ...$ that satisfies the following equation:

$$\frac{1}{n}\sum_{i=1}^{n} x_i \rightarrow \underset{n\to\infty}{r} = E\left[x_i\right]$$

It follows that, for large values of n, the mean provides a reliable estimate of r.

Central Limit Theorem

To evaluate how accurate the estimate produced by Standard Monte Carlo is, the central limit theorem is used. Using the same assumptions as earlier, one get the following:

$$\frac{\frac{1}{n}\sum_{i=1}^{n} x_i - r}{\sigma/\sqrt{n}} \rightarrow \underset{n\to\infty}{N}(0,1)$$

This theorem, therefore, provides an asymptotic confidence interval for r. Assuming equality for a significant number n,

$$\frac{\frac{1}{n}\sum_{i=1}^{n} x_i - r}{\sigma/\sqrt{n}} \sim N(0,1)$$

gives

$$\underbrace{\frac{1}{n}\sum_{i=1}^{n} x_i - r}_{\text{error}} \sim N\left(0, \frac{\sigma^2}{n}\right)$$

The following confidence interval can be obtained by selecting a level of confidence of $(1-\alpha)\cdot 100\%$:

$$\left[\frac{1}{n}\sum_{i=1}^{n} x_i - Z\alpha_{/2}\cdot\frac{\sigma}{\sqrt{n}}, \frac{1}{n}\sum_{i=1}^{n} x_i + Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right]$$

Where

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

is the average of the simulated portfolio company values and

$$\pm Z\alpha_{/2}\cdot\frac{\sigma}{\sqrt{n}}$$

is the degree of uncertainty, Z represents the level of confidence, and α indicates the significance level (Glasserman, 2004).

A2 Simulation Parameters

Table A2.1: Monte Carlo Simulation Parameters

The Monte Carlo simulation experiment's base case parameters are reported in this table:

Parameter	**Value**
Lifetime of funds	10 years
Investment period	5 years
Divestment period	5 years
Holding Period	5 years
Number of investments	10
Investment/Divestment months	January, July
Total committed capital	$ 100 million
Total invested in each portfolio company	$ 10 million

A3 Simulation Output

A3.1 IRR

Table A3.1: Summary of the IRR outputs for Buyout Fund Simulations

	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Input Correlation	20.00%	35.67%	36.25%	48.37%	44.50%
Min	-5.32%	-5.52%	-5.48%	-5.75%	-6.67%
25th Percentile	4.18%	4.26%	4.02%	3.93%	3.95%
75th Percentile	21.87%	23.48%	23.1%	23.96%	23.88%
Max	152.58%	135.92%	143.11%	111.36%	127.06%
Average	14.62%	15.50%	15.38%	15.93%	15.72%
Median	11.95%	12.33%	12.24%	12.58%	12.41%
Standard Deviation	14.03%	15.07%	15.14%	15.75%	15.35%

Table A3.2: Summary of the IRR outputs for Venture Capital Fund Simulations

	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Input Correlation	50.00%	61.18%	62.18%	82.96%	76.33%
Min	-6.3%	-6.91%	-6.18%	-6.71%	-6.61%
25th Percentile	3.89%	4.07%	4.5%	4.71%	4.16%
75th Percentile	30.25%	31.85%	33.32%	34.4%	32.08%
Max	259.79%	190.77%	232.77%	259.46%	190.19%
Average	20.68%	21.92%	22.09%	23.5%	22.3%
Median	14.33%	14.95%	15.27%	16.44%	15.29%
Standard Deviation	23.49%	24.7%	24.79%	25.9%	25.11%

A3.2 Simulation Path

Figure A3.1 displays one of 10 000 MC simulations for a PE fund with a low average correlation, while Figure A3.2 depicts one with a high average correlation.

Figure A3.1: One fund simulation with correlation = 0.05

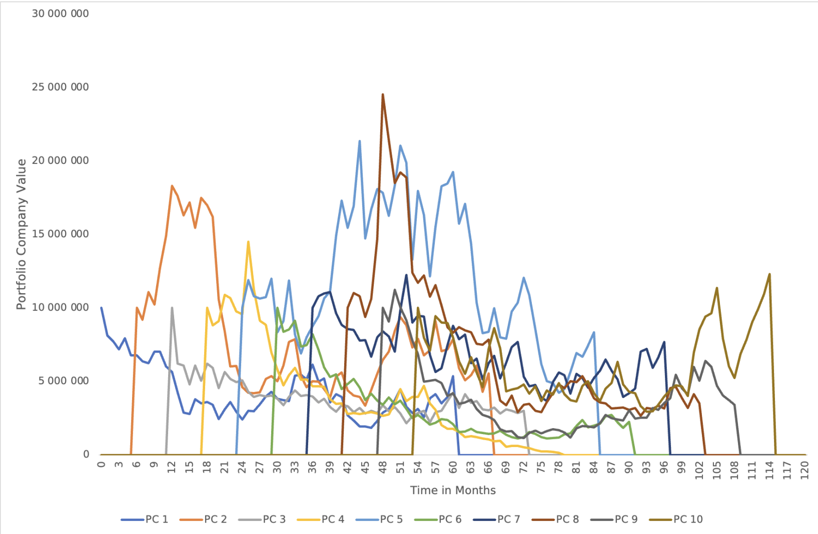
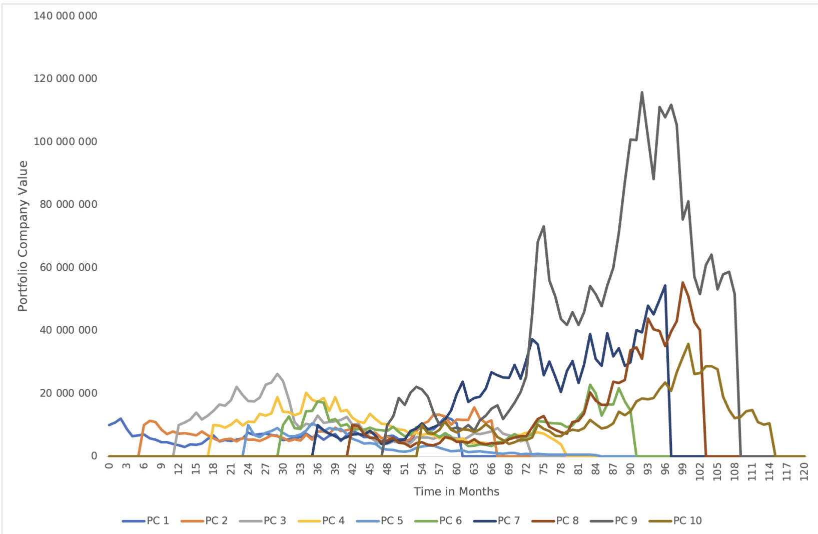


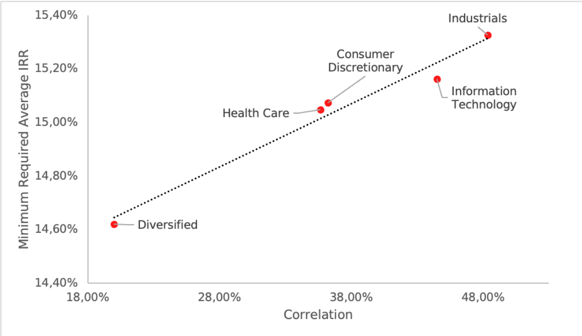
Figure A3.2: One fund simulation with correlation = 0.95



A4 Scatter Plots from Estimation Methodology

A4.1 Scatter Plot for Minimum Required IRR

Figure A4.1: Buyout Funds: Scatter Plot and Regression Line for Minimum Average and Median IRR



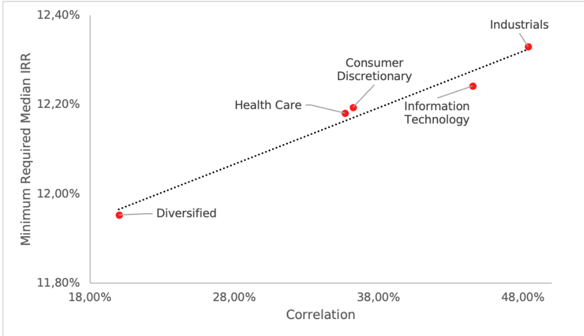
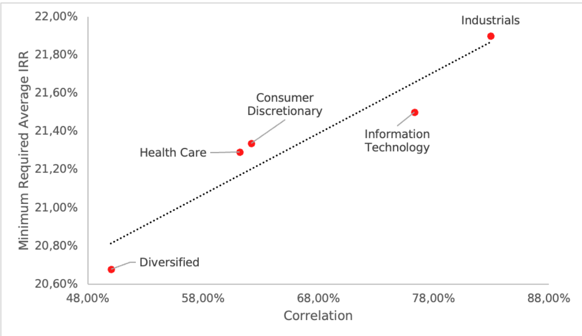
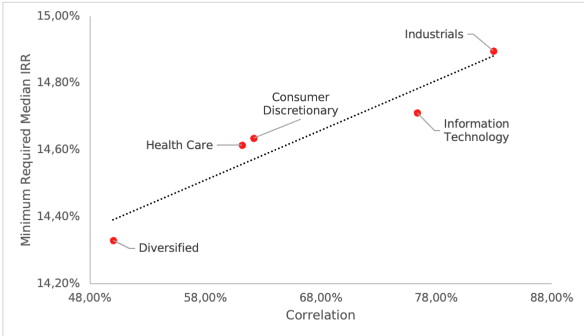


Figure A4.2: Venture Capital Funds: Scatter Plot and Regression Line for Minimum Average and Median IRR





A4.2 Scatter Plot for Implied Correlation

Figure A4.3: BO Funds: Scatter Plot for Correlation and Standard Deviation

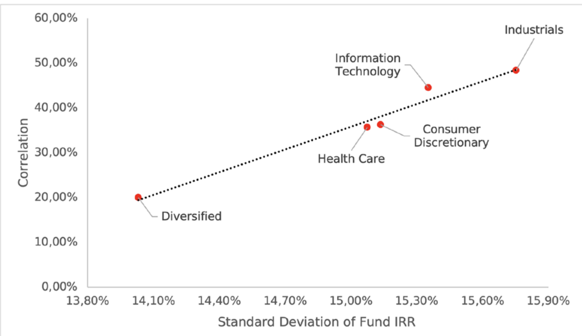
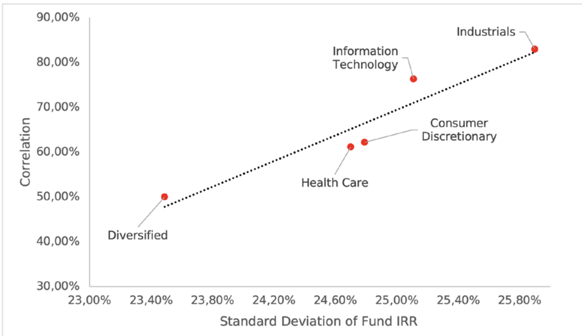


Figure A4.4: VC Funds: Scatter Plot for Correlation and Standard Deviation



A5 Exit IRR

Buyout Deal Data

Table A5.1: Descriptive Statistics of Exit IRR for US Buyout Deal Data

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	-100.00%	4.00%	-100.00%	-63.00%	4.00%
25th Percentile	23.00%	25.00%	20.00%	23.00%	25.90%
75th Percentile	60.00%	73.50%	54.00%	51.00%	76.50%
Max	824.00%	500.00%	330.00%	201.00%	824.00%
Average	53.67%	62.18%	44.60%	45.07%	74.77%
Median	34.60%	46.00%	32.50%	31.00%	40.30%
Standard Deviation	79.43%	74.61%	55.95%	48.05%	132.71%

Table A5.2: Descriptive Statistics of Exit IRR for EU Buyout Deal Data

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	-60.00%	0.31%	0.51%	-60.00%	-9.82%
25th Percentile	26.00%	30.00%	26.00%	22.30%	27.00%
75th Percentile	62.00%	75.00%	59.00%	57.75%	68.50%
Max	790.00%	400.00%	500.00%	684.00%	208.00%
Average	56.22%	60.43%	57.93%	50.72%	52.16%
Median	40.00%	40.00%	40.00%	34.50%	40.00%
Standard Deviation	65.16%	67.96%	72.89%	71.63%	37.83%

Venture Capital Deal Data

Table A5.3: Descriptive Statistics of Exit IRR for EU Venture Capital Deal Data

Venture Capital	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	0.50%	2.00%	0.71%	N/A	0.50%
25th Percentile	26.50%	34.00%	27.00%	N/A	20.00%
75th Percentile	66.00%	70.00%	58.00%	N/A	66.50%
Max	400.00%	150.00%	134.00%	N/A	400.00%
Average	64.60%	56.49%	48.64%	N/A	78.53%
Median	45.00%	48.00%	46.00%	N/A	42.00%
Standard Deviation	75.05%	39.03%	34.98%	N/A	102.10%

A6 Market-Adjusted IRR

Buyout Deal Data

Table A6.1: Descriptive Statistics of Market-Adjusted IRR for US Buyout Deal Data

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	-110.19%	-9.35%	-109.45%	-58.54%	-6.68%
25th Percentile	17.00%	20.51%	11.91%	15.98%	21.03%
75th Percentile	52.19%	67.38%	47.11%	49.19%	69.05%
Max	816.80%	493.76%	311.22%	196.01%	816.80%
Average	48.59%	55.39%	38.24%	39.45%	66.82%
Median	30.68%	39.25%	27.63%	28.09%	31.86%
Standard Deviation	79.18%	74.51%	56.80%	47.35%	132.81%

Table A6.2: Descriptive Statistics of Market-Adjusted IRR for EU Buyout Deal Data

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	-27.60%	-5.06%	-27.15%	-25.00%	-15.22%
25th Percentile	22.46%	20.15%	17.37%	16.61%	13.33%
75th Percentile	60.07%	66.39%	52.41%	53.05%	62.16%
Max	770.56%	396.88%	503.25%	677.15%	181.26%
Average	52.28%	54.50%	51.49%	44.00%	40.72%
Median	36.30%	34.74%	31.58%	28.24%	28.13%
Standard Deviation	63.57%	67.88%	76.15%	70.07%	38.10%

Venture Capital Deal Data

Table A6.3: Descriptive Statistics of Market-Adjusted IRR for EU Venture Capital Deal Data

Venture Capital	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Min	-11.33%	-11.96%	-5.8%	N/A	-15.39%
25th Percentile	12.88%	23.75%	35.85%	N/A	9.86%
75th Percentile	59.84%	60.08%	62.58%	N/A	49.05%
Max	368.68%	175.16%	132.88%	N/A	368.68%
Average	55.39%	50.76%	47.52%	N/A	66.35%
Median	34.12%	35.49%	40.24%	N/A	34.23%
Standard Deviation	76.36%	47.72%	34.37%	N/A	100.21%

A7 Estimated Minimum Required IRR

Buyout

Table A7.1: Descriptive Statistics of Estimated Minimum Required IRR for US

Buyout	Health Care	Consumer Discretionary	Industrials	Information Technology
Average Exit IRR	62.18%	44.60%	45.07%	74.77%
Median Exit IRR	46.00%	32.50%	31.00%	40.30%
Estimated Minimum Average IRR	38.70%	31.65%	27.89%	61.90%
Estimated Minimum Median IRR	24.84%	21.07%	19.06%	37.26%

Table A7.2: Descriptive Statistics of Estimated Minimum Required IRR for EU

Buyout	Health Care	Consumer Discretionary	Industrials	Information Technology
Average Exit IRR	60.43%	57.93%	50.72%	52.16%
Median Exit IRR	40.00%	40.00%	34.50%	40.00%
Estimated Minimum Average IRR	36.06%	39.35%	36.93%	24.21%
Estimated Minimum Median IRR	23.43%	25.19%	23.89%	17.09%

Venture Capital

Table A7.3: Descriptive Statistics of Estimated Minimum Required IRR for EU

Venture Capital	Health Care	Consumer Discretionary	Information Technology
Average Exit IRR	56.49%	48.64%	45.72%
Median Exit IRR	48.00%	46.00%	50.00%
Estimated Minimum Average IRR	31.73%	25.71%	55.42%
Estimated Minimum Median IRR	19.47%	16.67%	30.50%

A8 Risk and Market-Adjusted IRR

Buyout Deal Data

Table A8.1: Risk and Market-Adjusted IRRs for US Buyout Funds

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Average Risk and Market-Adjusted IRR	0.6136	0.7434	0.6734	0.8331	0.5031
Median Risk and Market-Adjusted IRR	0.3874	0.5267	0.4865	0.5932	0.2399

Table A8.2: Risk and Market-Adjusted IRRs for EU Buyout Funds

Buyout	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Average Risk and Market-Adjusted IRR	0.8224	0.8028	0.6761	0.6279	1.0685
Median Risk and Market-Adjusted IRR	0.5710	0.5117	0.4147	0.4031	0.7382

Venture Capital Deal Data

Table A8.3: Risk and Market-Adjusted IRRs for EU Venture Capital Funds

Venture Capital	Diversified	Health Care	Consumer Discretionary	Industrials	Information Technology
Average Risk and Market-Adjusted IRR	0.7254	1.0637	1.3825	N/A	0.6621
Median Risk and Market-Adjusted IRR	0.4469	0.7437	1.1706	N/A	0.3416

A8.1 Regional Performance Data

Buyout Deal Data

Table A8.4: Risk and Market-Adjusted IRRs for Diversified Funds

	Global Diversified	US Diversified	EU Diversified
Average Risk and Market-Adjusted IRR	0.7242	0.6136	0.8224
Median Risk and Market-Adjusted IRR	0.4852	0.3874	0.5710

Table A8.5: Risk and Market-Adjusted IRRs for Health Care Funds

	Global Health Care	US Health Care	EU Health Care
Average Risk and Market-Adjusted IRR	0.7744	0.7434	0.8028
Median Risk and Market-Adjusted IRR	0.5200	0.5267	0.5117

Table A8.6: Risk and Market-Adjusted IRRs for Consumer Discretionary Funds

	Global Consumer Discretionary	US Consumer Discretionary	EU Consumer Discretionary
Average Risk and Market-Adjusted IRR	0.7100	0.6734	0.6761
Median Risk and Market-Adjusted IRR	0.4529	0.4865	0.4147

Table A8.7: Risk and Market-Adjusted IRRs for Industrials Funds

	Global Industrials	US Industrials	EU Industrials
Average Risk and Market-Adjusted IRR	0.6732	0.8331	0.6279
Median Risk and Market-Adjusted IRR	0.4508	0.5932	0.4031

Table A8.8: Risk and Market-Adjusted IRRs for Information Technology Funds

	Global Information Technology	US Information Technology	EU Information Technology
Average Risk and Market-Adjusted IRR	0.6201	0.5031	1.0685
Median Risk and Market-Adjusted IRR	0.3768	0.2399	0.7382

A9 Indicies and Fund Specializations

A9.1 Simulation Indicies

Table A9.1: Indices used to compute correlations for the different buyout funds

Fund	**Index**
Diversified	S&P 500
Health Care	S&P 500 Health Care
Consumer Discretionary	S&P 500 Consumer Discretionary
Industrials	S&P 500 Industrials
Information Technology	S&P 500 Information Technology

A9.2 Historical Data Indicies

Table A9.2: Indices used to market adjust US Buyout

Fund	**Index**
Diversified	S&P 500
Health Care	S&P 500 Health Care
Consumer Discretionary	S&P 500 Consumer Discretionary
Industrials	S&P 500 Industrials
Information Technology	S&P 500 Information Technology

Table A9.3: Indices used to market adjust EU Buyout

Fund	**Index**
Diversified	MSCI Europe
Health Care	MSCI Europe Health Care
Consumer Discretionary	MSCI Europe Consumer Discretionary
Industrials	MSCI Europe Industrials
Information Technology	MSCI Europe Information Technology

Table A9.4: Indices used to market adjust EU Venture Capital

Fund	**Index**
Diversified	MSCI Europe Small Cap
Health Care	MSCI Europe Small Cap Health Care
Consumer Discretionary	MSCI Europe Small Cap Consumer Discretionary
Industrials	N/A
Information Technology	MSCI Europe Small Cap Information Technology

A9.3 Fund Specializations

Figure A9.1: Analyzed Private Equity Funds

Three different dimensions of fund categorization are examined in this thesis. The various funds are put together based on whether it is a BO or VC fund, which region they belong to, and the industry in which the fund is invested. For example, one fund analyzed would be a US Buyout fund specializing in Heath Care.